Study on the compact tetraquarks in an extended relativized quark model

Qi-Fang Lü

Hunan Normal University

Collaborators：Dian-Yong Chen, Yu-Bing Dong

Lanzhou

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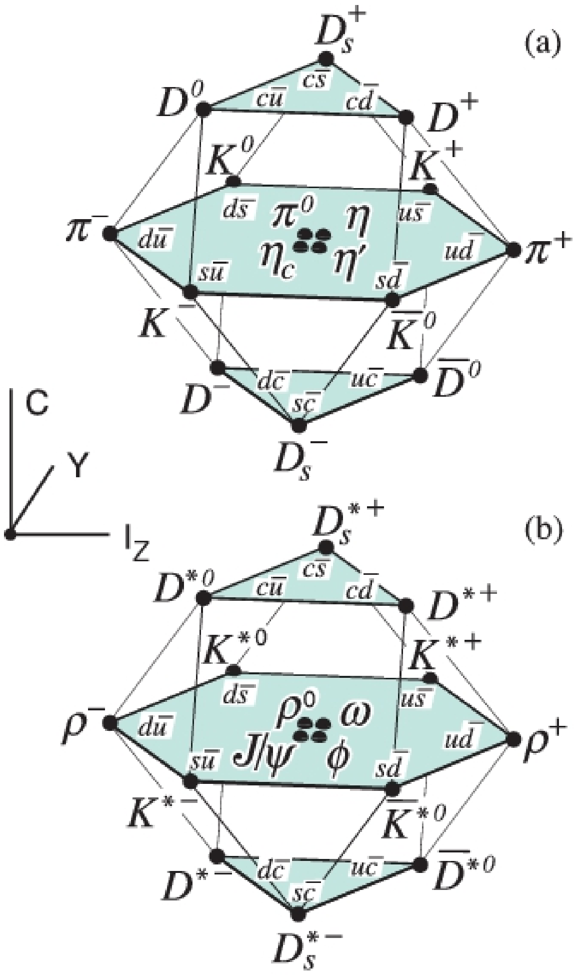
Outline

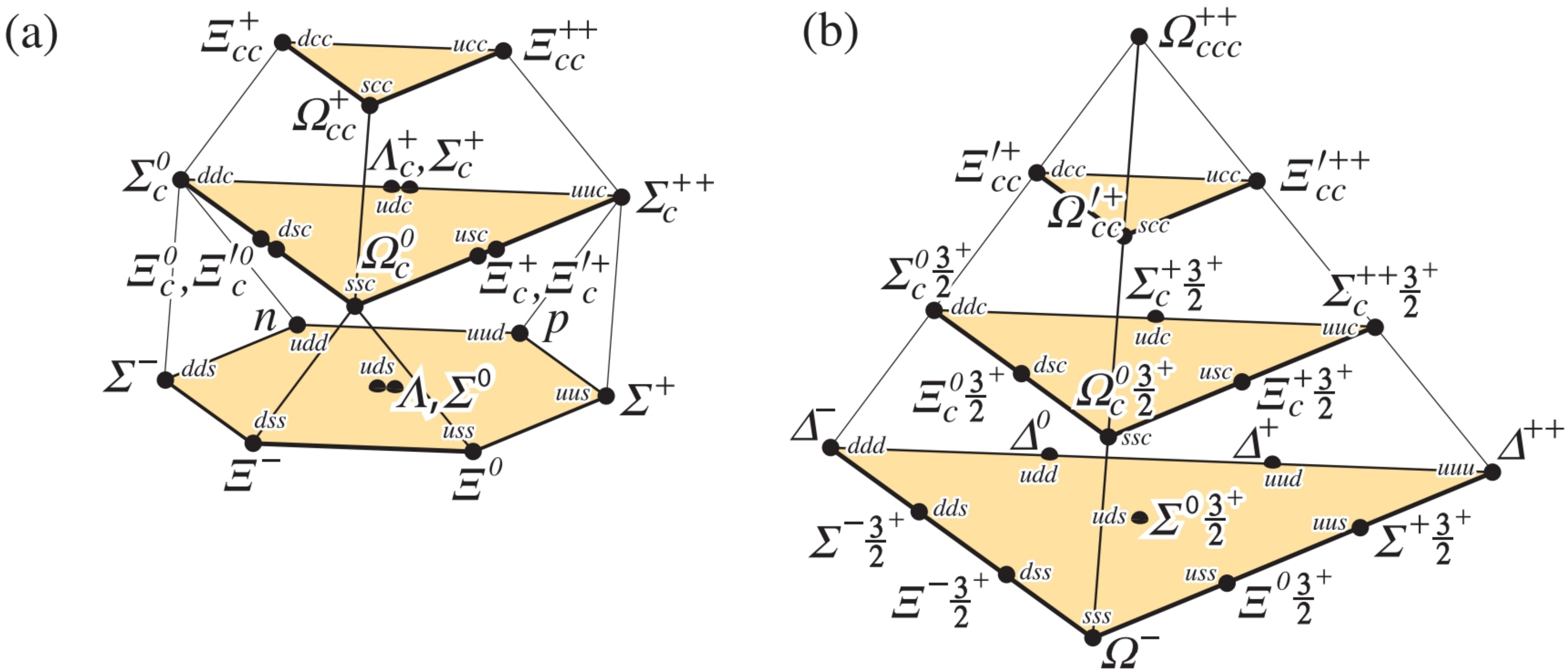
- Compact tetraquarks
- Extended relativized quark model
- Results and discussions
- Summary

Conventional mesons



Pseudoscalar and vector mesons

Conventional baryons

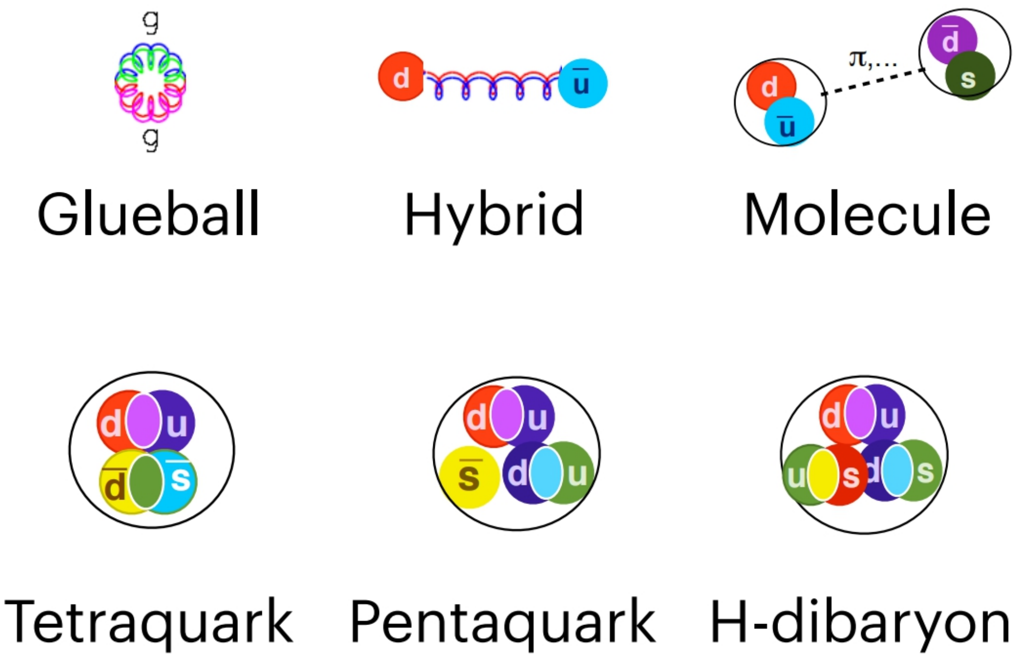


Conventional baryons

P. A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020)

Exotica

In addition to the conventional states, QCD also permits the existence of other types of hadrons, known as exotic states.



Tetraquark candidates observed recently：X(6900), X(2900), Z_{cs}(3985), Z_{cs}(4000), Z_{cs}(4220), X(4685), X(4630)

How to calculate the spectrum

- **Simple quark model**: only the color-magnetic interactions
- **Diquark models**: pointlike or not? diquark masses? input or solved? the type of interactions between diquarks?
- **Four-body calculations**:relativistic or nonrelativistic? Cornell potential or Goldstone boson exchanges?
- **Numerical methods**: simple Gaussian, multiple Gaussian, Other complete bases?

Here, we adopt the Gaussian expansion method to solve the four-body relativized Hamiltonian.

Extended relativized quark model

Extend the two-body Godfrey-Isgur model to four-body tetraquarks

$$H = H_0 + \sum_{i<j} V_{ij}^{\mathrm{oge}} + \sum_{i<j} V_{ij}^{\mathrm{conf}}, \qquad H_0 = \sum_{i=1}^{4} (p_i^2 + m_i^2)^{1/2}.$$

$$V_{ij}^{\mathrm{oge}} = \beta_{ij}^{1/2} \tilde{G}(r_{ij}) \beta_{ij}^{1/2} + \delta_{ij}^{1/2+\epsilon_c} \frac{2\boldsymbol{S_i} \cdot \boldsymbol{S_j}}{3m_i m_j} \nabla^2 \tilde{G}(r_{ij}) \delta_{ij}^{1/2+\epsilon_c}$$

with

$$\beta_{ij} = 1 + \frac{p_{ij}^2}{(p_{ij}^2 + m_i^2)^{1/2} (p_{ij}^2 + m_j^2)^{1/2}},$$

and

$$\delta_{ij} = \frac{m_i m_j}{(p_{ij}^2 + m_i^2)^{1/2} (p_{ij}^2 + m_j^2)^{1/2}}.$$

Extended relativized quark model

$$\tilde{G}(r_{ij}) = \boldsymbol{F}_i \cdot \boldsymbol{F}_j \sum_{k=1}^{3} \frac{\alpha_k}{r_{ij}} \mathrm{erf}(\tau_{kij} r_{ij}),$$

$$V_{ij}^{\mathrm{conf}} = -\frac{3}{4} \boldsymbol{F}_i \cdot \boldsymbol{F}_j \left\{ br \left[\frac{e^{-\sigma_{ij}^2 r^2}}{\sqrt{\pi} \sigma_{ij} r} + \left(1 + \frac{1}{2\sigma_{ij}^2 r^2}\right) \mathrm{erf}(\sigma_{ij} r) \right] + c \right\}.$$

S. Godfrey and N. Isgur, Phys. Rev. D 32, 189 (1985)

Qi-Fang Lü, Dian-Yong Chen, Yu-Bing Dong, Phys. Rev. D 102, 034012 (2020)

Qi-Fang Lü, Dian-Yong Chen, Yu-Bing Dong, Eur. Phys. J. C 80, 871 (2020)

Qi-Fang Lü, Dian-Yong Chen, Yu-Bing Dong, Phys. Rev. D 102, 074021 (2020)

Advantages of this model

- Describe the tetraquaks and conventional mesons in a uniform frame; self-consistency
- Unified description of different flavor sectors: heavy-heavy, heavy-light, and light-light inteactions
- Include more relativistic effects
- Four-body calculations; without diquark or other approximation

Wave functions for the compact tetraquarks

Classification according to the number of heavy quarks

$$QQ\overline{Q}\overline{Q} \quad QQ\overline{Q}\overline{q} \quad QQ\overline{q}\overline{q} \quad Qq\overline{Q}\overline{q} \quad Qq\overline{q}\overline{q} \quad qq\overline{q}\overline{q}$$

The wave functions of Color-Flavor-Spin part can be obtained by the Pauli exclusion principle.

An example: constructing the Color-Flavor-Spin wave functions for the $Q_1Q_2'\overline{q}_3\overline{q}_4'$ systems.

$Q_1Q'_2\overline{q}_3\overline{q}'_4$

Color： antisymmetric $|\bar{3}3\rangle$, symmetric $|6\bar{6}\rangle$.

$$|\bar{3}3\rangle = |(Q_1Q'_2)^{\bar{3}}(\bar{q}_3\bar{q}'_4)^3\rangle,$$

$$|6\bar{6}\rangle = |(Q_1Q'_2)^6(\bar{q}_3\bar{q}'_4)^{\bar{6}}\rangle,$$

Flavor： cc, bb, and ss are symmetric, cb, nn, and ns can be symmetric or antisymmetric.

Spatial： S-wave, always symmetric.

$Q_1 Q_2' \overline{q}_3 \overline{q}_4'$

Spin： two fermions, spin 1 is symmetric, spin 0 is antisymmetric

$$\chi_0^{00} = |(Q_1 Q_2')_0 (\bar{q}_3 \bar{q}_4')_0\rangle_0,$$

$$\chi_0^{11} = |(Q_1 Q_2')_1 (\bar{q}_3 \bar{q}_4')_1\rangle_0,$$

$$\chi_1^{01} = |(Q_1 Q_2')_0 (\bar{q}_3 \bar{q}_4')_1\rangle_1,$$

$$\chi_1^{10} = |(Q_1 Q_2')_1 (\bar{q}_3 \bar{q}_4')_0\rangle_1,$$

$$\chi_1^{11} = |(Q_1 Q_2')_1 (\bar{q}_3 \bar{q}_4')_1\rangle_1,$$

$$\chi_2^{11} = |(Q_1 Q_2')_1 (\bar{q}_3 \bar{q}_4')_1\rangle_2,$$

$Q_1Q'_2\bar{q}_3\bar{q}'_4$

System	IJ^P	Configuration		
$\{cc\}[\bar{u}\,\bar{d}]$	01^+	$\vert\{cc\}^{\bar{3}}_1[\bar{u}\,\bar{d}]^3_0\rangle_1$	$\vert\{cc\}^6_0[\bar{u}\,\bar{d}]^{\bar{6}}_1\rangle_1$	$\cdots$
$\{cc\}\{\bar{u}\,\bar{d}\}$	10^+	$\vert\{cc\}^{\bar{3}}_1\{\bar{u}\,\bar{d}\}^3_1\rangle_0$	$\vert\{cc\}^6_0\{\bar{u}\,\bar{d}\}^{\bar{6}}_0\rangle_0$	$\cdots$
	11^+	$\vert\{cc\}^{\bar{3}}_1\{\bar{u}\,\bar{d}\}^3_1\rangle_1$	$\cdots$	$\cdots$
	12^+	$\vert\{cc\}^{\bar{3}}_1\{\bar{u}\,\bar{d}\}^3_1\rangle_2$	$\cdots$	$\cdots$
$\{bb\}[\bar{u}\,\bar{d}]$	01^+	$\vert\{bb\}^{\bar{3}}_1[\bar{u}\,\bar{d}]^3_0\rangle_1$	$\vert\{bb\}^6_0[\bar{u}\,\bar{d}]^{\bar{6}}_1\rangle_1$	$\cdots$
$\{bb\}\{\bar{u}\,\bar{d}\}$	10^+	$\vert\{bb\}^{\bar{3}}_1\{\bar{u}\,\bar{d}\}^3_1\rangle_0$	$\vert\{bb\}^6_0\{\bar{u}\,\bar{d}\}^{\bar{6}}_0\rangle_0$	$\cdots$
	11^+	$\vert\{bb\}^{\bar{3}}_1\{\bar{u}\,\bar{d}\}^3_1\rangle_1$	$\cdots$	$\cdots$
	12^+	$\vert\{bb\}^{\bar{3}}_1\{\bar{u}\,\bar{d}\}^3_1\rangle_2$	$\cdots$	$\cdots$
$(cb)[\bar{u}\,\bar{d}]$	00^+	$\vert(cb)^{\bar{3}}_0[\bar{u}\,\bar{d}]^3_0\rangle_0$	$\vert(cb)^6_1[\bar{u}\,\bar{d}]^{\bar{6}}_1\rangle_0$	$\cdots$
	01^+	$\vert(cb)^{\bar{3}}_1[\bar{u}\,\bar{d}]^3_0\rangle_1$	$\vert(cb)^6_0[\bar{u}\,\bar{d}]^{\bar{6}}_1\rangle_1$	$\vert(cb)^6_1[\bar{u}\,\bar{d}]^{\bar{6}}_1\rangle_1$
	02^+	$\vert(cb)^6_1[\bar{u}\,\bar{d}]^{\bar{6}}_1\rangle_2$	$\cdots$	$\cdots$
$(cb)\{\bar{u}\,\bar{d}\}$	10^+	$\vert(cb)^{\bar{3}}_1\{\bar{u}\,\bar{d}\}^3_1\rangle_0$	$\vert(cb)^6_0\{\bar{u}\,\bar{d}\}^{\bar{6}}_0\rangle_0$	$\cdots$
	11^+	$\vert(cb)^{\bar{3}}_0\{\bar{u}\,\bar{d}\}^3_1\rangle_1$	$\vert(cb)^{\bar{3}}_1\{\bar{u}\,\bar{d}\}^3_1\rangle_1$	$\vert(cb)^6_1\{\bar{u}\,\bar{d}\}^{\bar{6}}_0\rangle_1$
	12^+	$\vert(cb)^{\bar{3}}_1\{\bar{u}\,\bar{d}\}^3_1\rangle_2$	$\cdots$	$\cdots$
$\{cc\}[\bar{u}\,\bar{s}]$	$\frac{1}{2}1^+$	$\vert\{cc\}^{\bar{3}}_1[\bar{u}\,\bar{s}]^3_0\rangle_1$	$\vert\{cc\}^6_0[\bar{u}\,\bar{s}]^{\bar{6}}_1\rangle_1$	$\cdots$
$\{cc\}\{\bar{u}\,\bar{s}\}$	$\frac{1}{2}0^+$	$\vert\{cc\}^{\bar{3}}_1\{\bar{u}\,\bar{s}\}^3_1\rangle_0$	$\vert\{cc\}^6_0\{\bar{u}\,\bar{s}\}^{\bar{6}}_0\rangle_0$	$\cdots$
	$\frac{1}{2}1^+$	$\vert\{cc\}^{\bar{3}}_1\{\bar{u}\,\bar{s}\}^3_1\rangle_1$	$\cdots$	$\cdots$
	$\frac{1}{2}2^+$	$\vert\{cc\}^{\bar{3}}_1\{\bar{u}\,\bar{s}\}^3_1\rangle_2$	$\cdots$	$\cdots$

$Q_1Q'_2\overline{q}_3\overline{q}'_4$

$\{bb\}[\bar{u}\,\bar{s}]$	$\frac{1}{2}1^+$	$\lvert\{bb\}^{\bar{3}}_1[\bar{u}\,\bar{s}]^3_0\rangle_1$	$\lvert\{bb\}^6_0[\bar{u}\,\bar{s}]^{\bar{6}}_1\rangle_1$	$\cdots$
$\{bb\}\{\bar{u}\,\bar{s}\}$	$\frac{1}{2}0^+$	$\lvert\{bb\}^{\bar{3}}_1\{\bar{u}\,\bar{s}\}^3_1\rangle_0$	$\lvert\{bb\}^6_0\{\bar{u}\,\bar{s}\}^{\bar{6}}_0\rangle_0$	$\cdots$
	$\frac{1}{2}1^+$	$\lvert\{bb\}^{\bar{3}}_1\{\bar{u}\,\bar{s}\}^3_1\rangle_1$	$\cdots$	$\cdots$
	$\frac{1}{2}2^+$	$\lvert\{bb\}^{\bar{3}}_1\{\bar{u}\,\bar{s}\}^3_1\rangle_2$	$\cdots$	$\cdots$
$(cb)[\bar{u}\,\bar{s}]$	$\frac{1}{2}0^+$	$\lvert(cb)^{\bar{3}}_0[\bar{u}\,\bar{s}]^3_0\rangle_0$	$\lvert(cb)^6_1[\bar{u}\,\bar{s}]^{\bar{6}}_1\rangle_0$	$\cdots$
	$\frac{1}{2}1^+$	$\lvert(cb)^{\bar{3}}_1[\bar{u}\,\bar{s}]^3_0\rangle_1$	$\lvert(cb)^6_0[\bar{u}\,\bar{s}]^{\bar{6}}_1\rangle_1$	$\lvert(cb)^6_1[\bar{u}\,\bar{s}]^{\bar{6}}_1\rangle_1$
	$\frac{1}{2}2^+$	$\lvert(cb)^6_1[\bar{u}\,\bar{s}]^{\bar{6}}_1\rangle_2$	$\cdots$	$\cdots$
$(cb)\{\bar{u}\,\bar{s}\}$	$\frac{1}{2}0^+$	$\lvert(cb)^{\bar{3}}_1\{\bar{u}\,\bar{s}\}^3_1\rangle_0$	$\lvert(cb)^6_0\{\bar{u}\,\bar{s}\}^{\bar{6}}_0\rangle_0$	$\cdots$
	$\frac{1}{2}1^+$	$\lvert(cb)^{\bar{3}}_0\{\bar{u}\,\bar{s}\}^3_1\rangle_1$	$\lvert(cb)^{\bar{3}}_1\{\bar{u}\,\bar{s}\}^3_1\rangle_1$	$\lvert(cb)^6_1\{\bar{u}\,\bar{s}\}^{\bar{6}}_0\rangle_1$
	$\frac{1}{2}2^+$	$\lvert(cb)^{\bar{3}}_1\{\bar{u}\,\bar{s}\}^3_1\rangle_2$	$\cdots$	$\cdots$
$\{cc\}\{\bar{s}\,\bar{s}\}$	00^+	$\lvert\{cc\}^{\bar{3}}_1\{\bar{s}\,\bar{s}\}^3_1\rangle_0$	$\lvert\{cc\}^6_0\{\bar{s}\,\bar{s}\}^{\bar{6}}_0\rangle_0$	$\cdots$
	01^+	$\lvert\{cc\}^{\bar{3}}_1\{\bar{s}\,\bar{s}\}^3_1\rangle_1$	$\cdots$	$\cdots$
	02^+	$\lvert\{cc\}^{\bar{3}}_1\{\bar{s}\,\bar{s}\}^3_1\rangle_2$	$\cdots$	$\cdots$
$\{bb\}\{\bar{s}\,\bar{s}\}$	00^+	$\lvert\{bb\}^{\bar{3}}_1\{\bar{s}\,\bar{s}\}^3_1\rangle_0$	$\lvert\{bb\}^6_0\{\bar{s}\,\bar{s}\}^{\bar{6}}_0\rangle_0$	$\cdots$
	01^+	$\lvert\{bb\}^{\bar{3}}_1\{\bar{s}\,\bar{s}\}^3_1\rangle_1$	$\cdots$	$\cdots$
	02^+	$\lvert\{bb\}^{\bar{3}}_1\{\bar{s}\,\bar{s}\}^3_1\rangle_2$	$\cdots$	$\cdots$
$(cb)\{\bar{s}\,\bar{s}\}$	00^+	$\lvert(cb)^{\bar{3}}_1\{\bar{s}\,\bar{s}\}^3_1\rangle_0$	$\lvert(cb)^6_0\{\bar{s}\,\bar{s}\}^{\bar{6}}_0\rangle_0$	$\cdots$
	01^+	$\lvert(cb)^{\bar{3}}_0\{\bar{s}\,\bar{s}\}^3_1\rangle_1$	$\lvert(cb)^{\bar{3}}_1\{\bar{s}\,\bar{s}\}^3_1\rangle_1$	$\lvert(cb)^6_1\{\bar{s}\,\bar{s}\}^{\bar{6}}_0\rangle_1$
	02^+	$\lvert(cb)^{\bar{3}}_1\{\bar{s}\,\bar{s}\}^3_1\rangle_2$	$\cdots$	$\cdots$

Matrix elements of color and spin

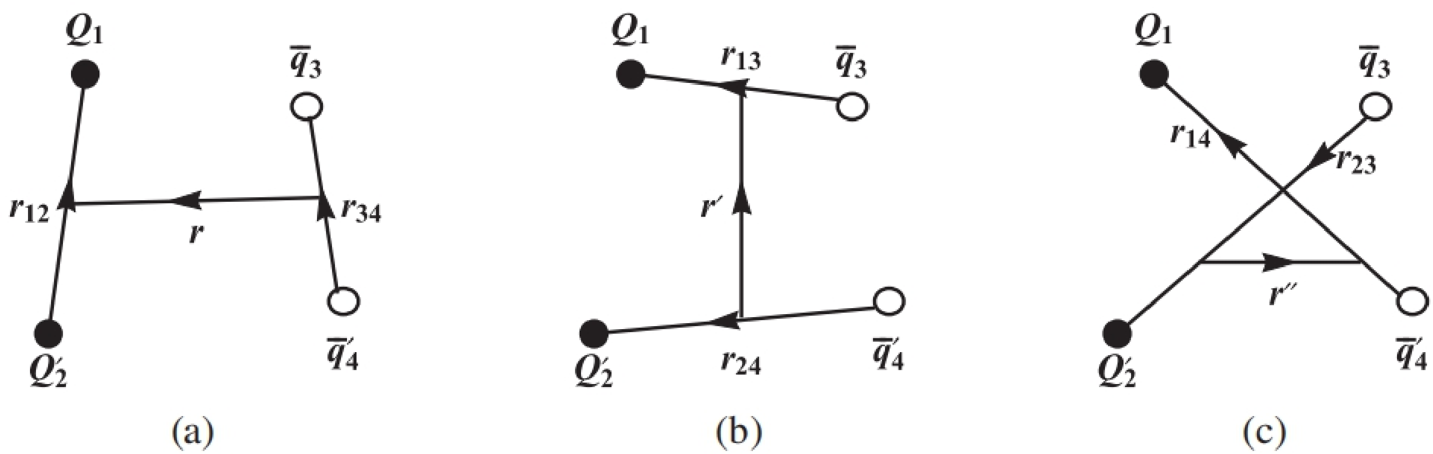
TABLE II. Color matrix elements.

$\langle \hat{O} \rangle$	$\langle F_1 \cdot F_2 \rangle$	$\langle F_3 \cdot F_4 \rangle$	$\langle F_1 \cdot F_3 \rangle$	$\langle F_2 \cdot F_4 \rangle$	$\langle F_1 \cdot F_4 \rangle$	$\langle F_2 \cdot F_3 \rangle$
$\langle \bar{3}3 \vert \hat{O} \vert \bar{3}3 \rangle$	$-2/3$	$-2/3$	$-1/3$	$-1/3$	$-1/3$	$-1/3$
$\langle 6\bar{6} \vert \hat{O} \vert 6\bar{6} \rangle$	$1/3$	$1/3$	$-5/6$	$-5/6$	$-5/6$	$-5/6$
$\langle \bar{3}3 \vert \hat{O} \vert 6\bar{6} \rangle$	0	0	$-1/\sqrt{2}$	$-1/\sqrt{2}$	$1/\sqrt{2}$	$1/\sqrt{2}$

TABLE III. Spin matrix elements.

$\langle \hat{O} \rangle$	$\langle S_1 \cdot S_2 \rangle$	$\langle S_3 \cdot S_4 \rangle$	$\langle S_1 \cdot S_3 \rangle$	$\langle S_2 \cdot S_4 \rangle$	$\langle S_1 \cdot S_4 \rangle$	$\langle S_2 \cdot S_3 \rangle$
$\langle \chi_0^{00} \vert \hat{O} \vert \chi_0^{00} \rangle$	$-3/4$	$-3/4$	0	0	0	0
$\langle \chi_0^{11} \vert \hat{O} \vert \chi_0^{11} \rangle$	$1/4$	$1/4$	$-1/2$	$-1/2$	$-1/2$	$-1/2$
$\langle \chi_0^{00} \vert \hat{O} \vert \chi_0^{11} \rangle$	0	0	$-\sqrt{3}/4$	$-\sqrt{3}/4$	$\sqrt{3}/4$	$\sqrt{3}/4$
$\langle \chi_1^{01} \vert \hat{O} \vert \chi_1^{01} \rangle$	$-3/4$	$1/4$	0	0	0	0
$\langle \chi_1^{10} \vert \hat{O} \vert \chi_1^{10} \rangle$	$1/4$	$-3/4$	0	0	0	0
$\langle \chi_1^{11} \vert \hat{O} \vert \chi_1^{11} \rangle$	$1/4$	$1/4$	$-1/4$	$-1/4$	$-1/4$	$-1/4$
$\langle \chi_1^{01} \vert \hat{O} \vert \chi_1^{10} \rangle$	0	0	$1/4$	$1/4$	$-1/4$	$-1/4$
$\langle \chi_1^{01} \vert \hat{O} \vert \chi_1^{11} \rangle$	0	0	$-\sqrt{2}/4$	$\sqrt{2}/4$	$-\sqrt{2}/4$	$\sqrt{2}/4$
$\langle \chi_1^{10} \vert \hat{O} \vert \chi_1^{11} \rangle$	0	0	$\sqrt{2}/4$	$-\sqrt{2}/4$	$-\sqrt{2}/4$	$\sqrt{2}/4$
$\langle \chi_2^{11} \vert \hat{O} \vert \chi_2^{11} \rangle$	$1/4$	$1/4$	$1/4$	$1/4$	$1/4$	$1/4$

Spatial part



The Jacobi coordinates：

$$\boldsymbol{r}_{12} = \boldsymbol{r}_1 - \boldsymbol{r}_2,$$

$$\boldsymbol{r}_{34} = \boldsymbol{r}_3 - \boldsymbol{r}_4,$$

$$\boldsymbol{r} = \frac{m_1\boldsymbol{r}_1 + m_2\boldsymbol{r}_2}{m_1 + m_2} - \frac{m_3\boldsymbol{r}_3 + m_4\boldsymbol{r}_4}{m_3 + m_4},$$

$$\boldsymbol{R} = \frac{m_1\boldsymbol{r}_1 + m_2\boldsymbol{r}_2 + m_3\boldsymbol{r}_3 + m_4\boldsymbol{r}_4}{m_1 + m_2 + m_3 + m_4}.$$

Spatial part

$$\Psi(\boldsymbol{r}_{12},\boldsymbol{r}_{34},\boldsymbol{r}) = \sum_{n_Q,n_q,n} C_{n_Q n_q n}\psi_{n_Q}(\boldsymbol{r}_{12})\psi_{n_q}(\boldsymbol{r}_{34})\psi_n(\boldsymbol{r}),$$

$$\psi_n(\boldsymbol{r}) = \frac{2^{7/4}\nu_n^{3/4}}{\pi^{1/4}} e^{-\nu_n r^2} Y_{00}(\hat{\boldsymbol{r}}) = \left(\frac{2\nu_n}{\pi}\right)^{3/4} e^{-\nu_n r^2},$$

$$\nu_n = \frac{1}{r_1^2 a^{2(n-1)}}, \quad (n = 1 - N_{\max}).$$

$$\phi_n(\boldsymbol{p}) = \frac{2^{1/4}}{\pi^{1/4}\nu_n^{3/4}} e^{-p^2/(4\nu_n)} Y_{00}(\hat{\boldsymbol{p}}) = \left(\frac{1}{2\pi\nu_n}\right)^{3/4} e^{-p^2/(4\nu_n)}.$$

$$\langle\alpha|\beta_{ij}^{1/2}\tilde{G}(r_{ij})\beta_{ij}^{1/2}|\beta\rangle = \sum_{\gamma,\delta,\rho,\lambda}\langle\alpha|\beta_{ij}^{1/2}|\gamma\rangle(N^{-1})_{\gamma\delta}\langle\delta|\tilde{G}(r_{ij})|\rho\rangle \times (N^{-1})_{\rho\lambda}\langle\lambda|\beta_{ij}^{1/2}|\beta\rangle.$$

The generalized eigenvalue problem

Homogeneous equation set

$$\sum_{j=1}^{N_{\max}^3}(H_{ij}-EN_{ij})C_j=0, \quad (i=1-N_{\max}^3).$$

We first solve this equation to get the masses of pure configurations, and then calculate the off-diagonal effects between different configurations. The final mass spectra can be obtained by diagonalizing the mass matrix of these configurations.

$QQ\overline{qq}$ system for test

- The conclusions are roughly consistent in the literature, test our model
- Wide attentions for the doubly heavy systems
- Heavy-light systems, relativistic effects
- Stable tetraquarks, below the threshold

$QQ\overline{qq}$

Previous works roughly agree with the following statements:

- The lowest $bb\bar{q}\bar{q}$ state is stable, below the open bottom strong decay threshold
- The lowest $cc\bar{q}\bar{q}$ state is above the open charm threshold

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$QQ\overline{qq}$

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Numerical stability

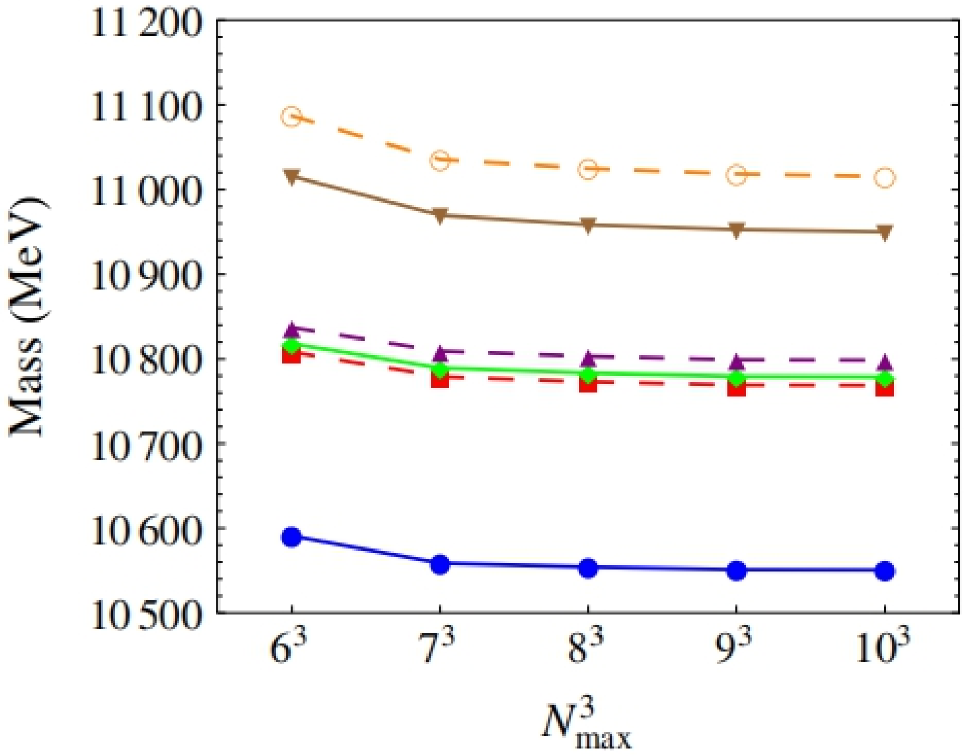
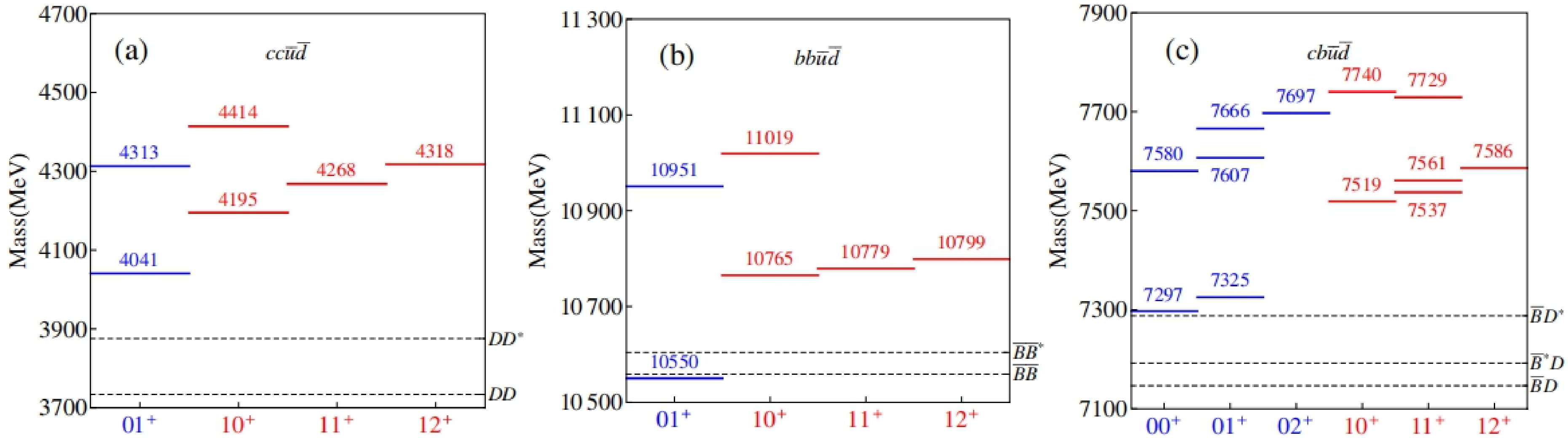
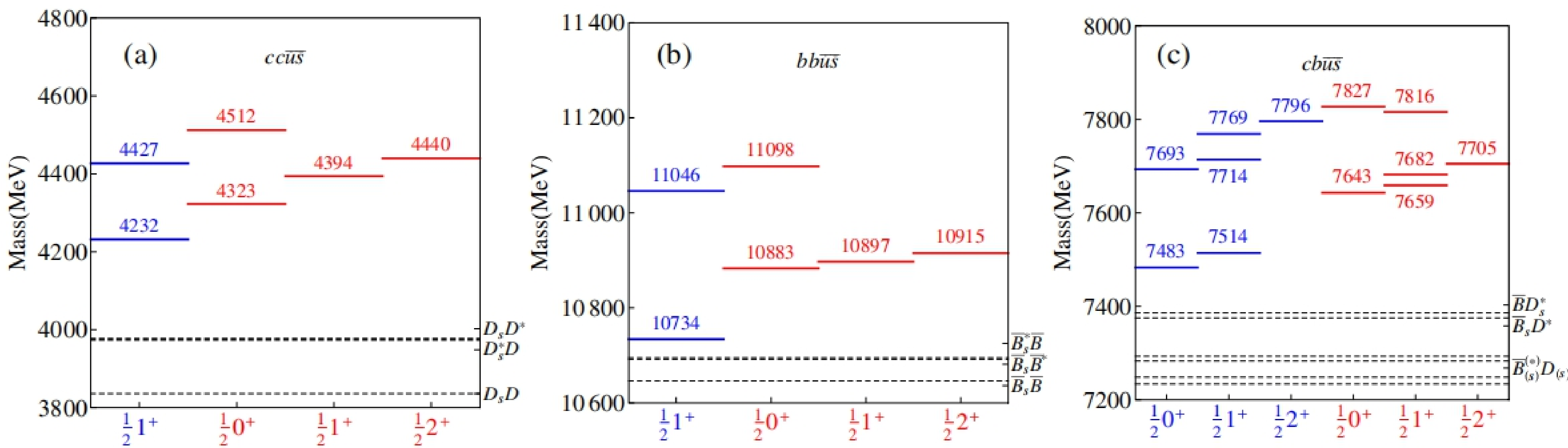


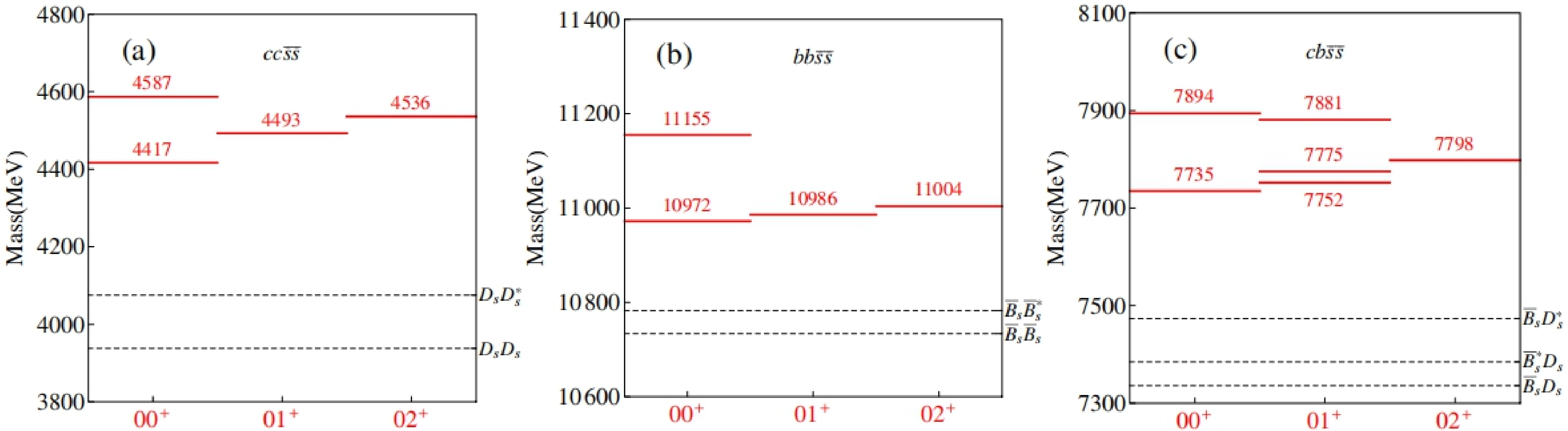
FIG. 2. Numerical stabilities for six pure configurations of $bb\bar{u}\bar{d}$ system. The blue points, red squares, green diamonds, purple triangles, brown inverted triangles, and orange circles stand for the $|\{bb\}_1^{\bar{3}}[\bar{u}\,\bar{d}]_0^3\rangle_1$, $|\{bb\}_1^{\bar{3}}\{\bar{u}\,\bar{d}\}_1^3\rangle_0$, $|\{bb\}_1^{\bar{3}}\{\bar{u}\,\bar{d}\}_1^3\rangle_1$, $|\{bb\}_1^{\bar{3}}\{\bar{u}\,\bar{d}\}_1^3\rangle_2$, $|\{bb\}_0^6[\bar{u}\,\bar{d}]_1^{\bar{6}}\rangle_1$, and $|\{bb\}_0^6\{\bar{u}\,\bar{d}\}_0^{\bar{6}}\rangle_0$ configurations.

Our results


(a) $cc\bar{u}\bar{d}$
Mass(MeV)
4700 4500 4300 4100 3900 3700
4313 4041 4414 4195 4268 4318
DD^*
DD
01+ 10+ 11+ 12+
(b) $bb\bar{u}\bar{d}$
Mass(MeV)
11 300 11 100 10 900 10 700 10 500
10951 10550 11019 10765 10779 10799
$\bar{B}\bar{B}^*$
$\bar{B}\bar{B}$
01+ 10+ 11+ 12+
(c) $cb\bar{u}\bar{d}$
Mass(MeV)
7900 7700 7500 7300 7100
7580 7297 7666 7607 7325 7697 7740 7519 7729 7561 7537 7586
$\bar{B}D^*$
$\bar{B}^*D$
$\bar{B}D$
00+ 01+ 02+ 10+ 11+ 12+


(a) $cc\bar{u}\bar{s}$
Mass(MeV)
4800 4600 4400 4200 4000 3800
4427 4232 4512 4323 4394 4440
D_sD^*
D_s^*D
D_sD
$\frac{1}{2}1^+$ $\frac{1}{2}0^+$ $\frac{1}{2}1^+$ $\frac{1}{2}2^+$
(b) $bb\bar{u}\bar{s}$
Mass(MeV)
11 400 11 200 11 000 10 800 10 600
11046 10734 11098 10883 10897 10915
$\bar{B}_s^*\bar{B}$
$\bar{B}_s\bar{B}^*$
$\bar{B}_s\bar{B}$
$\frac{1}{2}1^+$ $\frac{1}{2}0^+$ $\frac{1}{2}1^+$ $\frac{1}{2}2^+$
(c) $cb\bar{u}\bar{s}$
Mass(MeV)
8000 7800 7600 7400 7200
7693 7483 7769 7714 7514 7796 7827 7643 7816 7682 7659 7705
$\bar{B}D_s^*$
$\bar{B}_sD^*$
$\bar{B}_{(s)}^{(*)}D_{(s)}$
$\frac{1}{2}0^+$ $\frac{1}{2}1^+$ $\frac{1}{2}2^+$ $\frac{1}{2}0^+$ $\frac{1}{2}1^+$ $\frac{1}{2}2^+$

Our results



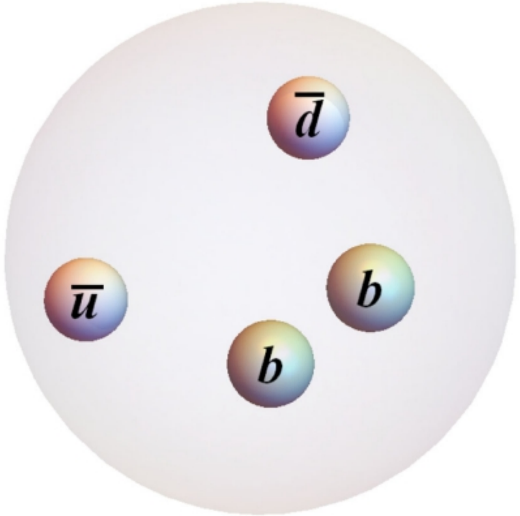


FIG. 4. The stable $IJ^P = 01^+$ $bb\bar{u}\bar{d}$ state.

Mass ratios

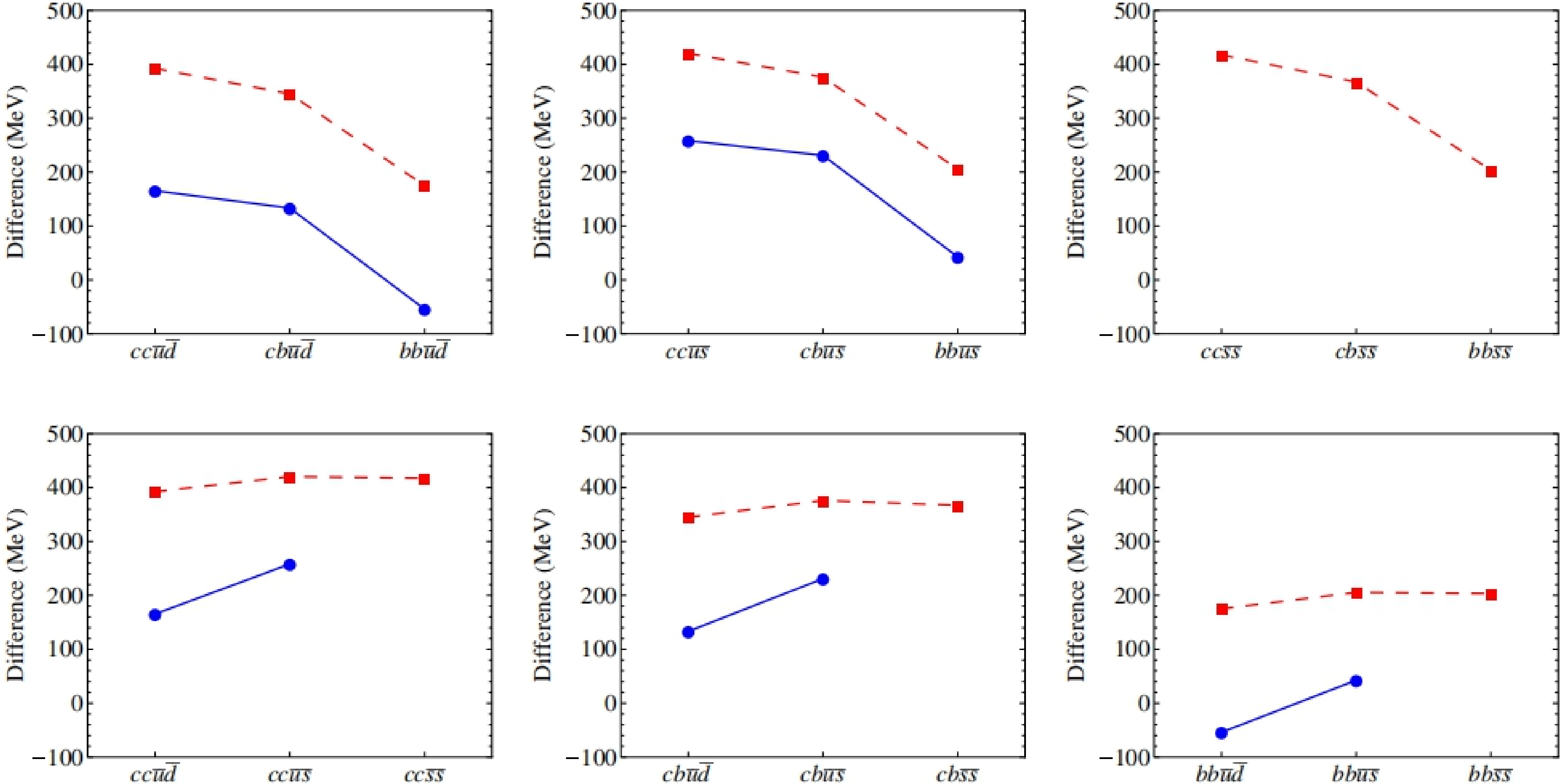


FIG. 7. Mass differences between lower $J^P = 1^+$ tetraquarks and thresholds versus the different systems. The blue points stand for the tetraquarks including antisymmetric light subsystems, and the red squares correspond to the ones with symmetric light subsystems.

With the largest mass ratio of heavy and light subsystems, m_Q/m_q, we can obtain a bound tetraquark

$QQ\bar{Q}\bar{Q}$

Are the ground states above or below the thresholds?

Two standpoints in the literature:

- Include the Cornell potential, above the thresholds, even much higher
- Without the Cornell potential, below the thresholds

The kinetic and potential terms are crucial for the exact spectrum

Before the observation of X(6900)

Predicted masses for $bb\bar{b}\bar{b}$ are below the thresholds

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Before the observation of X(6900)

Predicted masses for $bb\bar{b}\bar{b}$ are above the thresholds

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R.J. Lloyd, J.P. Vary, Phys. Rev. D 70, 014009 (2004)

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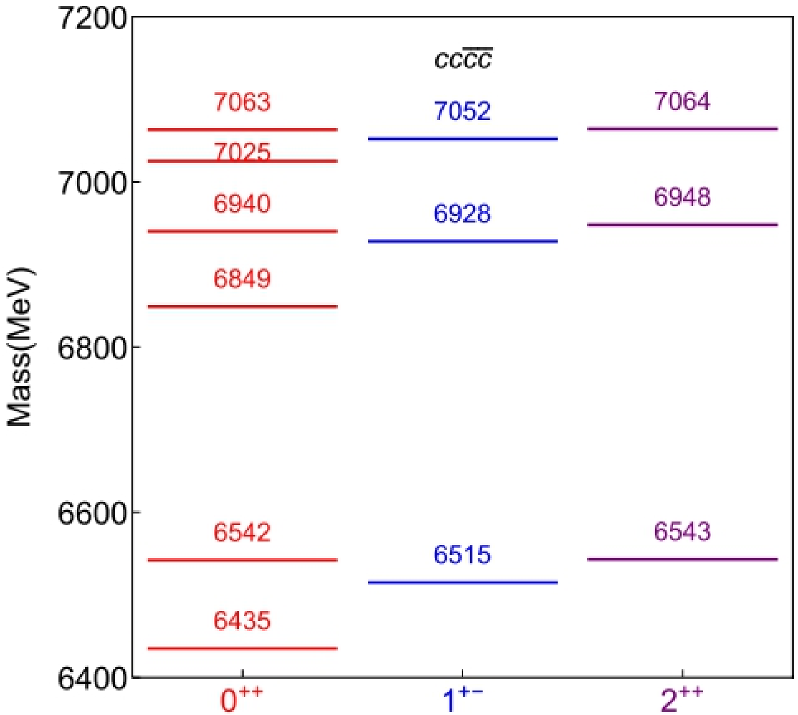
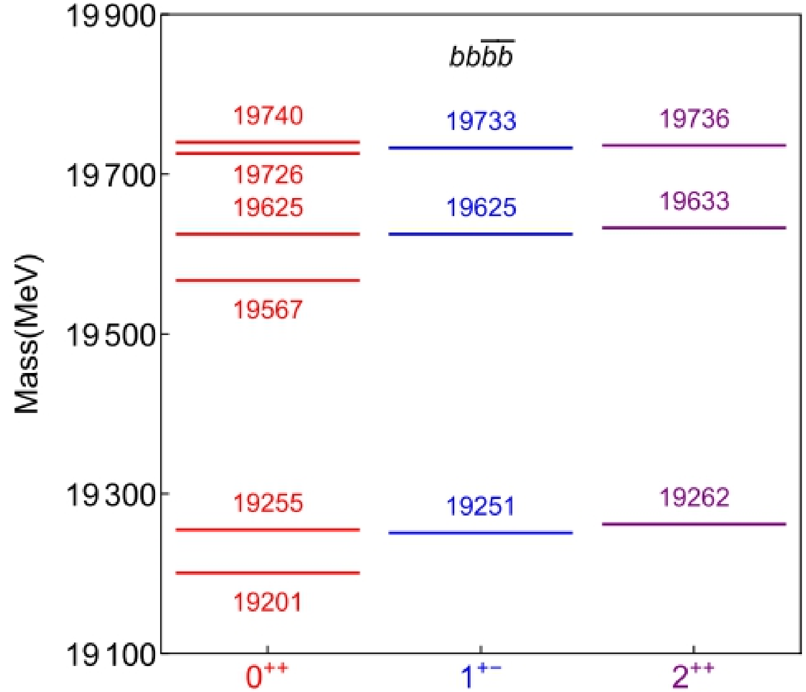
No signial for $bb\bar{b}\bar{b}$ in experiments until now.

Our results





X(6900): first radial excitations

The broad structures in 6.4~6.5GeV: ground states

Our results

$$R = \frac{\Gamma[cc\bar{c}\bar{c} \to J/\psi J/\psi]}{\Gamma[cc\bar{c}\bar{c} \to \psi(2S) J/\psi]} \quad R_{4\mu} = \frac{\Gamma[cc\bar{c}\bar{c} \to J/\psi J/\psi \to \mu^+\mu^-\mu^+\mu^-]}{\Gamma[cc\bar{c}\bar{c} \to \psi(2S) J/\psi \to \mu^+\mu^-\mu^+\mu^-]}$$

$$R[cc\bar{c}\bar{c}(6849)] = 0.113, \quad R_{4\mu}[cc\bar{c}\bar{c}(6849)] = 0.843,$$

$$R[cc\bar{c}\bar{c}(6940)] = 0.122, \quad R_{4\mu}[cc\bar{c}\bar{c}(6940)] = 0.910,$$

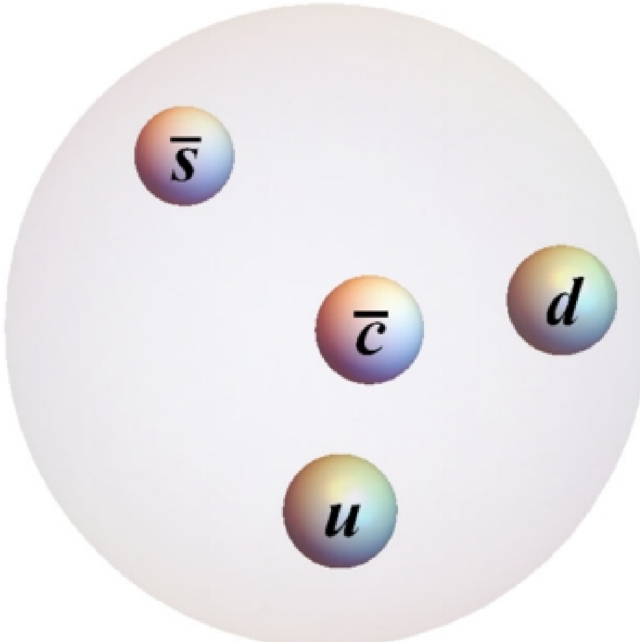
$$R[cc\bar{c}\bar{c}(6948)] = 0.075. \quad R_{4\mu}[cc\bar{c}\bar{c}(6948)] = 0.559.$$

As a radial excitaion, X(6900) can be searched in J/ψψ(2S) final states

Plenty of references for the X(6900) can be found in

https://inspirehep.net/literature?sort=mostrecent&size=25&page=1&q=refersto%3Arecid%3A1804391

$qq\bar{q}\bar{Q}$



Disfavor the X(2900) as compact tetraquark

Maybe molecules or kinematic effects

Summary

- We adopt the extended relativized quark to investigate the singly, doubly, and fully heavy tetraquarks.
- In these states, only the $IJ^P = 01^+ \, bb\bar{u}\bar{d}$ lie below the threshold.
- The X(6900): the radial excitation of fully charm tetraquarks
- The X(2900): disfavor the tetraquark interpretation
- Future works: XYZ, pentaquarks, decays ...

My homepage https://inspirehep.net/authors/1383269

Thanks for your attentions！