强子结构与动力学方程

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Bound state and quantum field theory

$$\Theta_0 = \beta(\alpha)\frac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu}\,.$$

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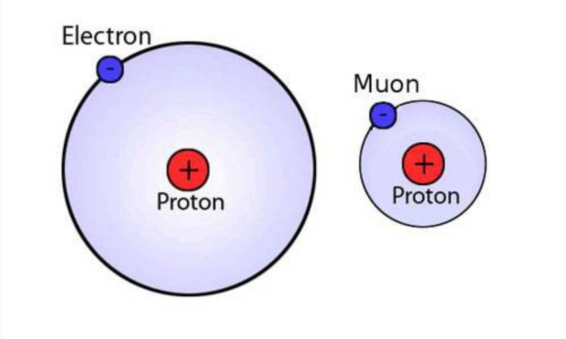
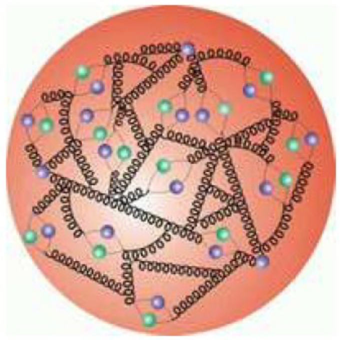


QED

Trace anomaly

- All renormalisable four-dimensional theories possess a trace anomaly;
- The size of the trace anomaly in QED must be great deal smaller than that in QCD.



QCD

Field theory Successful:

- Nonrelativistic quantum mechanics to handle bound state;
- Perturbation theory to handle relativistic effects

Field theory not Successful yet:

- Growth of the running coupling constant in the infrared region;
- Confinement;
- Dynamical Chiral Symmetry Breaking;
- Possible nontrivial vacuum structure in hadron

Continuum Schwinger function Method

Dyson, F. J. (1949), “The S Matrix In Quantum Electrodynamics,” Phys. Rev. 75, 1736.
Schwinger, J. S. (1951), “On The Green’s Functions Of Quantized Fields: 1 and 2,” Proc. Nat. Acad. Sci. 37 (1951) 452; ibid 455.

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Dyson-Schwinger Equations

Bethe-Salpeter Equations(Nambu)

Faddeev Equation

Ward-Takahashi identity

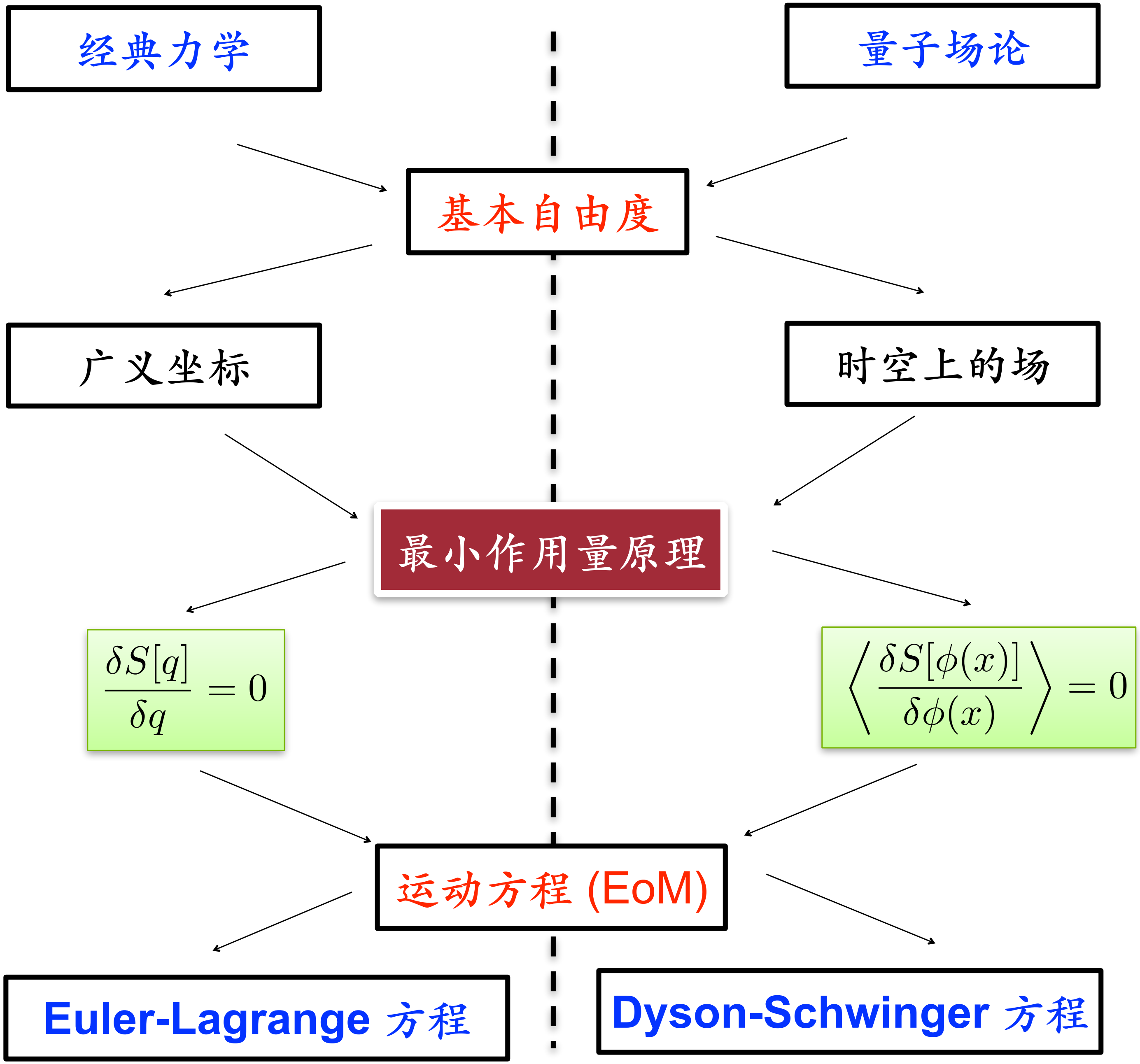
Scattering Problem

……

- They provide a systematic, symmetry-preserving approach to solving the bound-state problem in QCD;
- Predictions from CSM analyses are practically identical to those obtained via the lattice-regularized theory.

DSEs group

- Cui, *et al*, EPJA 57 (2021) 5, EPJC80 (2020) 1064
- Ding, *et al*, CPC44 (2020) 031002, PRD101(2020)054014
- Binosi, *et al*, PLB790(2019)257
- Chen, *et al*, PRD98(2018) 091505
- Gao, *et al*, PRD96 (2017) 034024
- Chang, *et al*, PLB737(2014), PRL110(2013)132001,PRL111(2013)1418002



From Sixue Qin

DSEs

$$F[G^n] = F[G^n, G^{n+1}]$$

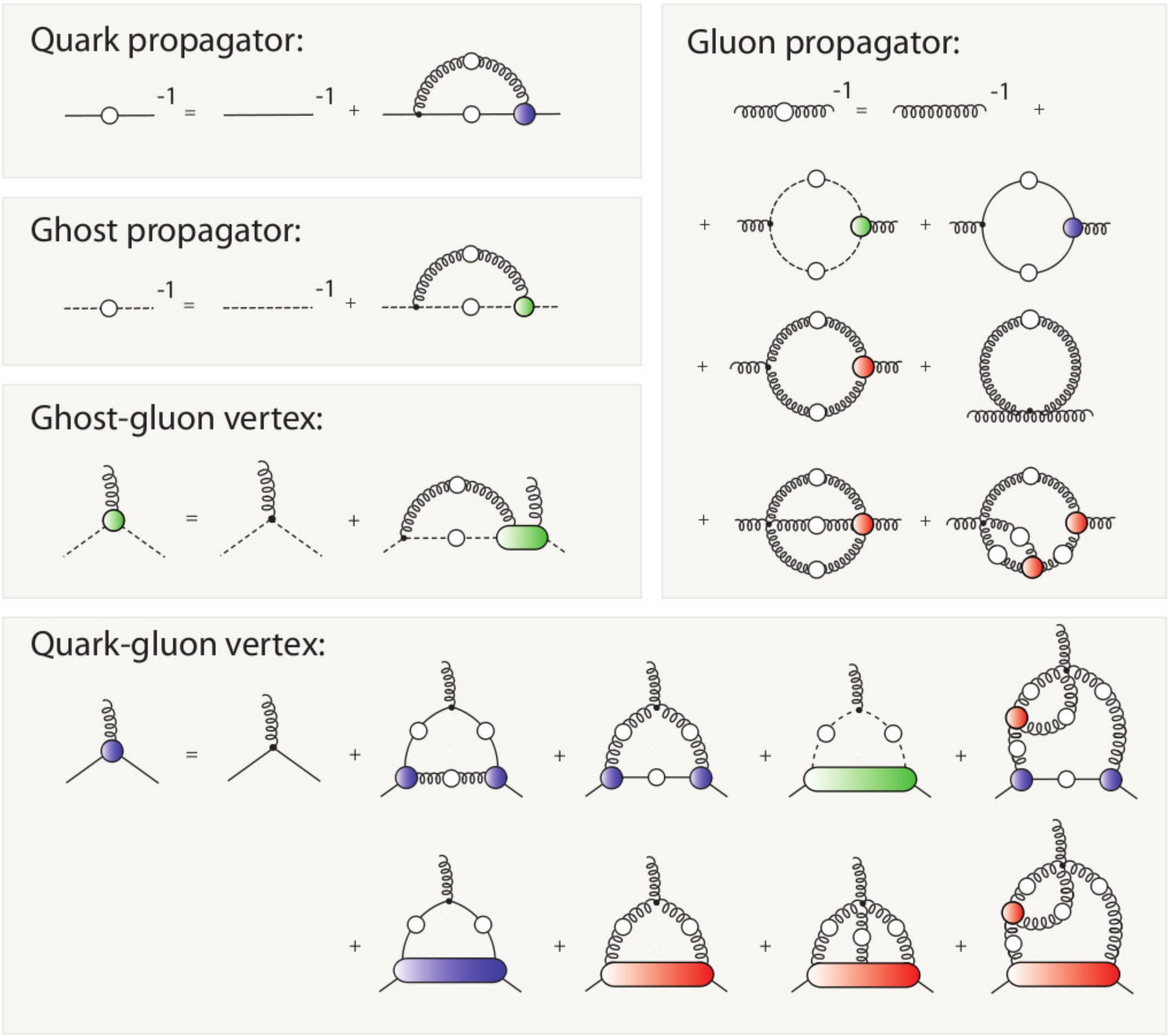
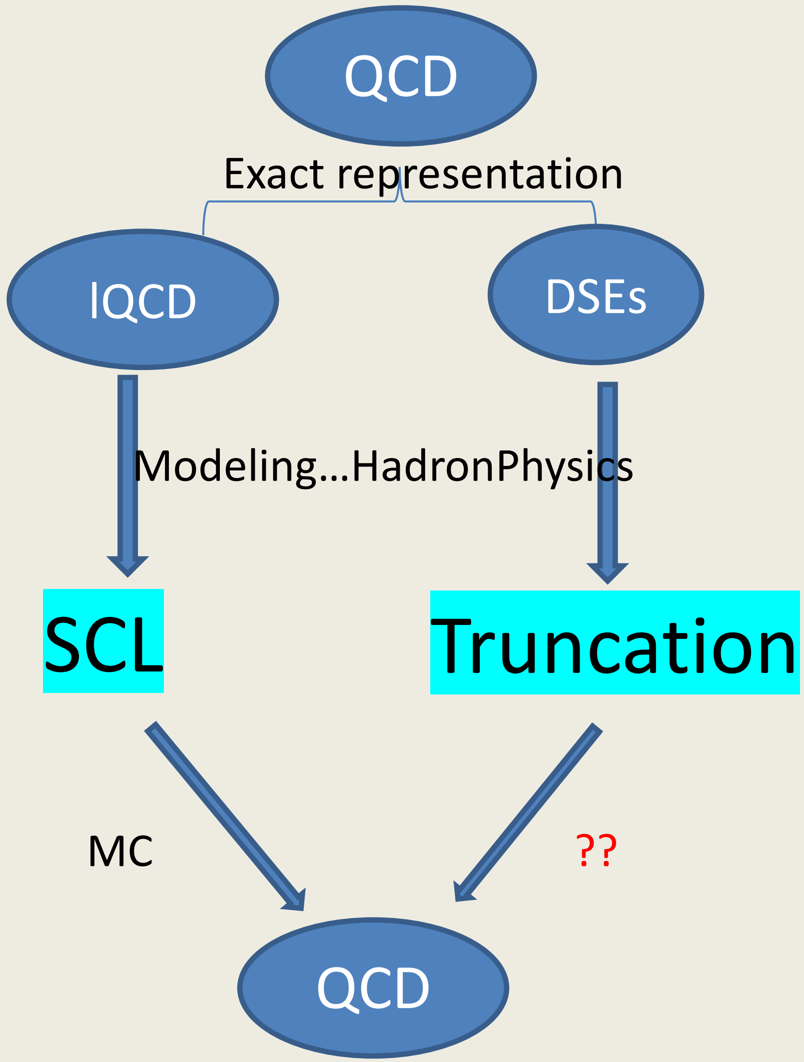


Image courtesy of Gernot Eichmann

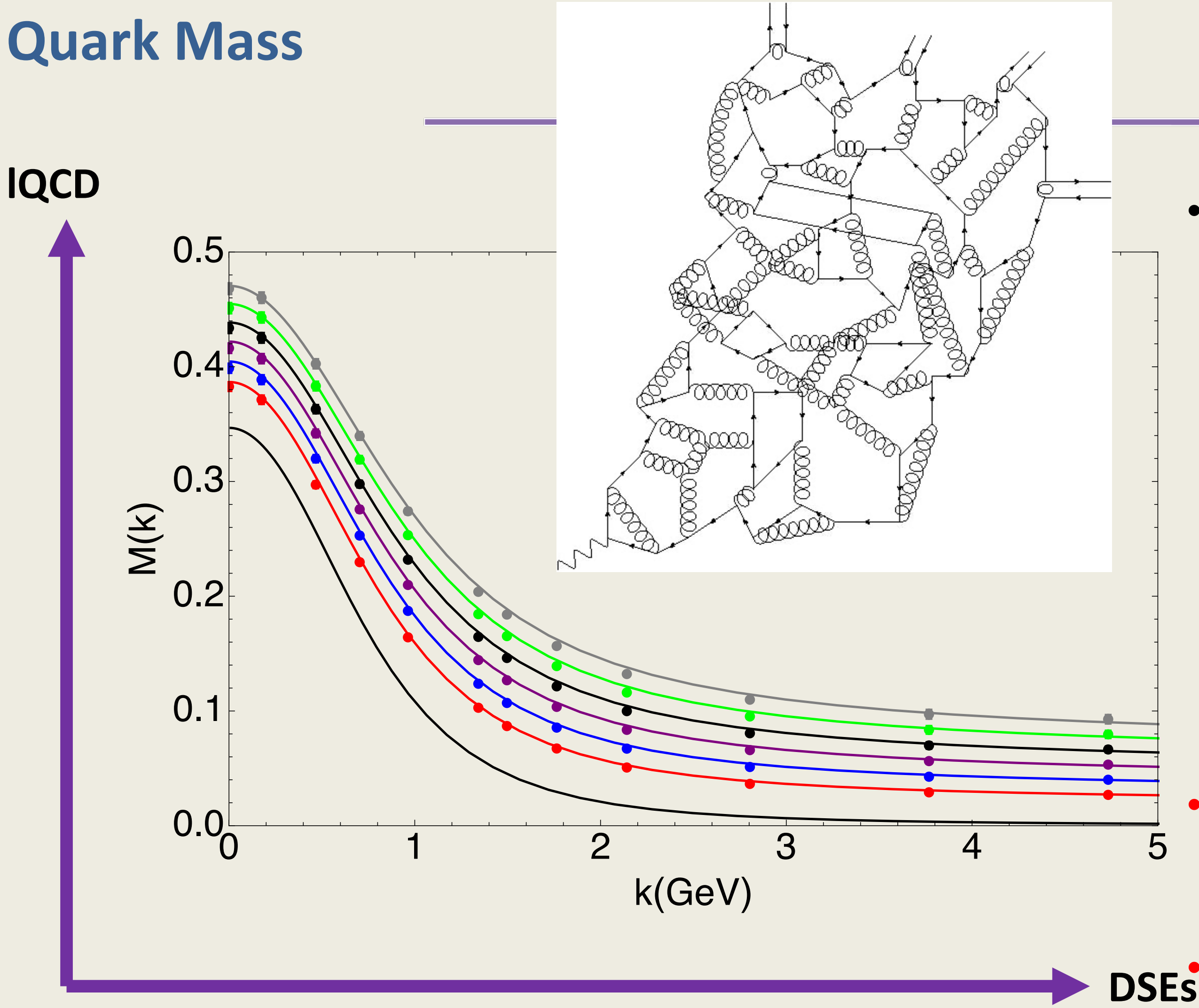
Dyson-Schwinger Equation scope

Study bound state problem within an continuum field theory



Quark Mass

Quark Mass



- Quarks progressivley become more sorphisticated as experience grew with formulating and solving the quark gap equation and as computational methods and power improved for lattcie-regularised QCD.
- Not Proper Mass/Rest Mass/Newtonian mass
- DCSB representation
- Roughly M_0...Constituent quarks(Model)

Measure Quark Mass

Maris, Roberts and Tandy, **Phys. Lett. B420**(1998) 267-273

- Pion's Bethe-Salpeter amplitude

  Solution of the Bethe-Salpeter equation

$$\Gamma_{\pi^j}(k;P) = \tau^{\pi^j}\gamma_5\Big[iE_\pi(k;P)+\gamma\cdot P F_\pi(k;P) + \gamma\cdot k\, k\cdot P\, G_\pi(k;P)+\sigma_{\mu\nu}\,k_\mu P_\nu\, H_\pi(k;P)\Big]$$

- Dressed-quark propagator $S(p)=\dfrac{1}{i\gamma\cdot p\,A(p^2)+B(p^2)}$

- Axial-vector Ward-Takahashi identity entails(chiral limit)

$$f_\pi E(k;P|P^2=0)=B(k^2)+(k\cdot P)^2\frac{d^2B(k^2)}{d^2k^2}+\dots$$

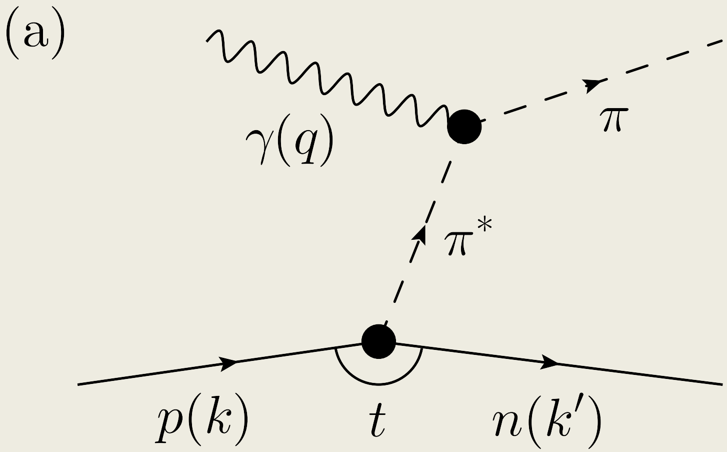
PHYSICAL REVIEW D VOLUME 43, NUMBER 5 1 MARCH 1991

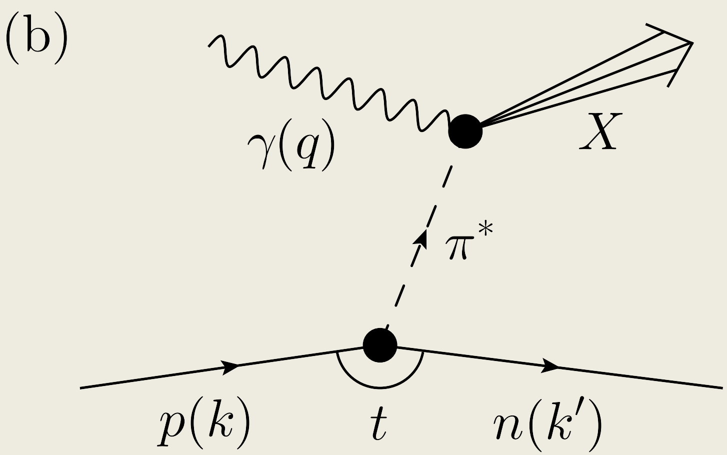
Calculation of chiral-symmetry breaking and pion properties as a Goldstone boson

Yuan-ben Dai, Chao-shang Huang, and Dong-sheng Liu
Institute of Theoretical Physics, Academia Sinica, P. O. Box 2735, Beijing, China

(Received 19 June 1990; revised manuscript received 5 November 1990)

Measure Quark Mass





Sullivan processes, in which a nucleon's pion cloud is used to provide access to the pion's

(a) elastic form factor and
(b) parton distribution functions.

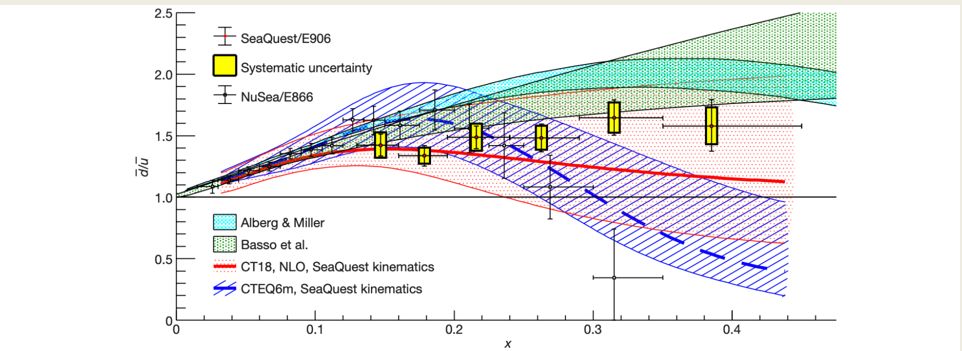


Fig. 2 | Ratios $\bar{d}(x)/\bar{u}(x)$. Ratios $\bar{d}(x)/\bar{u}(x)$ in the proton (red filled circles) with their statistical (vertical bars) and systematic (yellow boxes) uncertainties extracted from the present data based on NLO calculations of the Drell–Yan cross-sections. Also shown are the results obtained by the NuSea experiment (open black squares) with statistical and systematic uncertainties added in quadrature[4]. The cyan band shows the predictions of the meson–baryon model of Alberg & Miller[25] and the green band shows the predictions of the statistical parton distributions of Basso et al.[21]. The red solid (blue dashed) curves show the ratios $\bar{d}(x)/\bar{u}(x)$ calculated with CT18[29] (CTEQ6[35]) parton distributions at the scales of the SeaQuest results. The horizontal bars on the data points indicate the width of the bins.

Article

The asymmetry of antimatter in the proton

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Check for updates

J. Dove[1], B. Kerns[1], R. E. McClellan[1,18], S. Miyasaka[2], D. H. Morton[3], K. Nagai[2,4], S. Prasad[1], F. Sanftl[2], M. B. C. Scott[3], A. S. Tadepalli[5,18], C. A. Aidala[3,6], J. Arrington[7,19], C. Ayuso[3,20], C. L. Barker[8], C. N. Brown[9], W. C. Chang[4], A. Chen[1,3,4], D. C. Christian[10], B. P. Dannowitz[1], M. Daugherity[8], M. Diefenthaler[1,18], L. El Fassi[5,11], D. F. Geesaman[7,21], R. Gilman[5], Y. Goto[12], L. Guo[6,22], R. Guo[13], T. J. Hague[8], R. J. Holt[7,23], D. Isenhower[8], E. R. Kinney[14], N. Kitts[8], A. Klein[6], D. W. Kleinjan[6], Y. Kudo[15], C. Leung[1], P.-J. Lin[14], K. Liu[6], M. X. Liu[6], W. Lorenzon[3], N. C. R. Makins[1], M. Mesquita de Medeiros[7], P. L. McGaughey[6], Y. Miyachi[15], I. Mooney[3,24], K. Nakahara[16,25], K. Nakano[2,12], S. Nara[15], J.-C. Peng[1], A. J. Puckett[6,26], B. J. Ramson[3,27], P. E. Reimer[7]✉, J. G. Rubin[3,7], S. Sawada[17], T. Sawada[3,28], T.-A. Shibata[2,29], D. Su[4], M. Teo[1,30], B. G. Tice[7], R. S. Towell[8], S. Uemura[6,31], S. Watson[8], S. G. Wang[4,13,32], A. B. Wickes[6], J. Wu[10], Z. Xi[8] & Z. Ye[7]

Imaging Hadron Structure?

Nankai University

Imaging Hadron Structure?

u $\bar{d}$

$$u(x) = 6\,x\,(1-x)$$
$$\bar{u}(x) = 0$$
$$\bar{d}(x) = 6\,x\,(1-x)$$
$$d(x) = 0$$

$$g(x) = 0$$

→

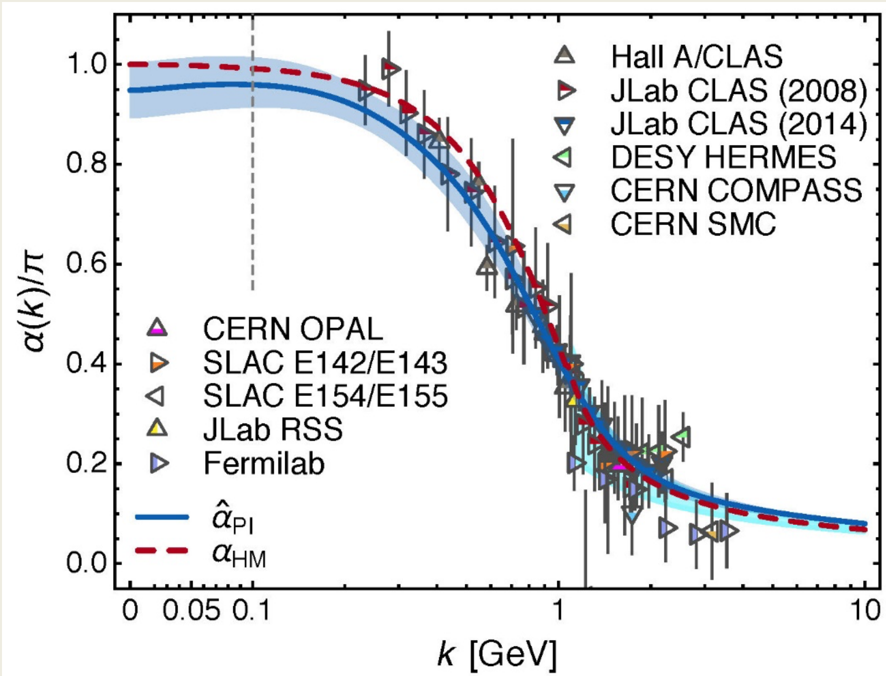
$$u_v(x) = u(x) - \bar{u}(x) = 6\,x\,(1-x)$$

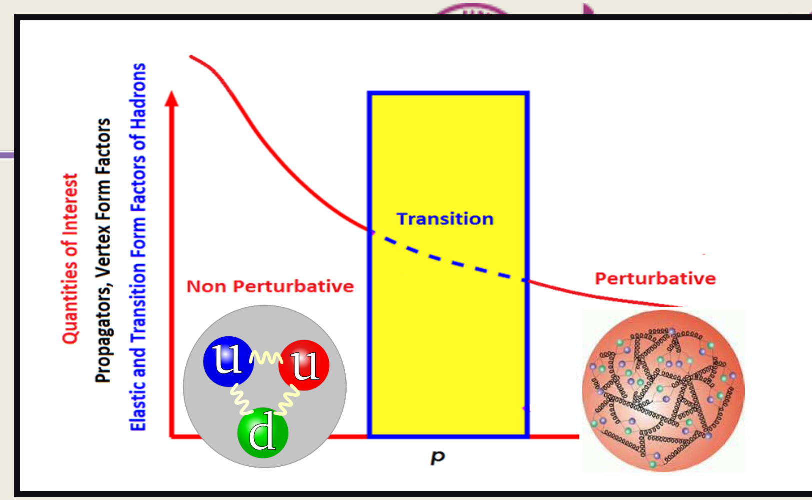
$$Singlet(x) = 2 * 6\,x\,(1-x)$$

$$g(x) = 0$$

No “glue” and No “sea”

Hadronic Scale



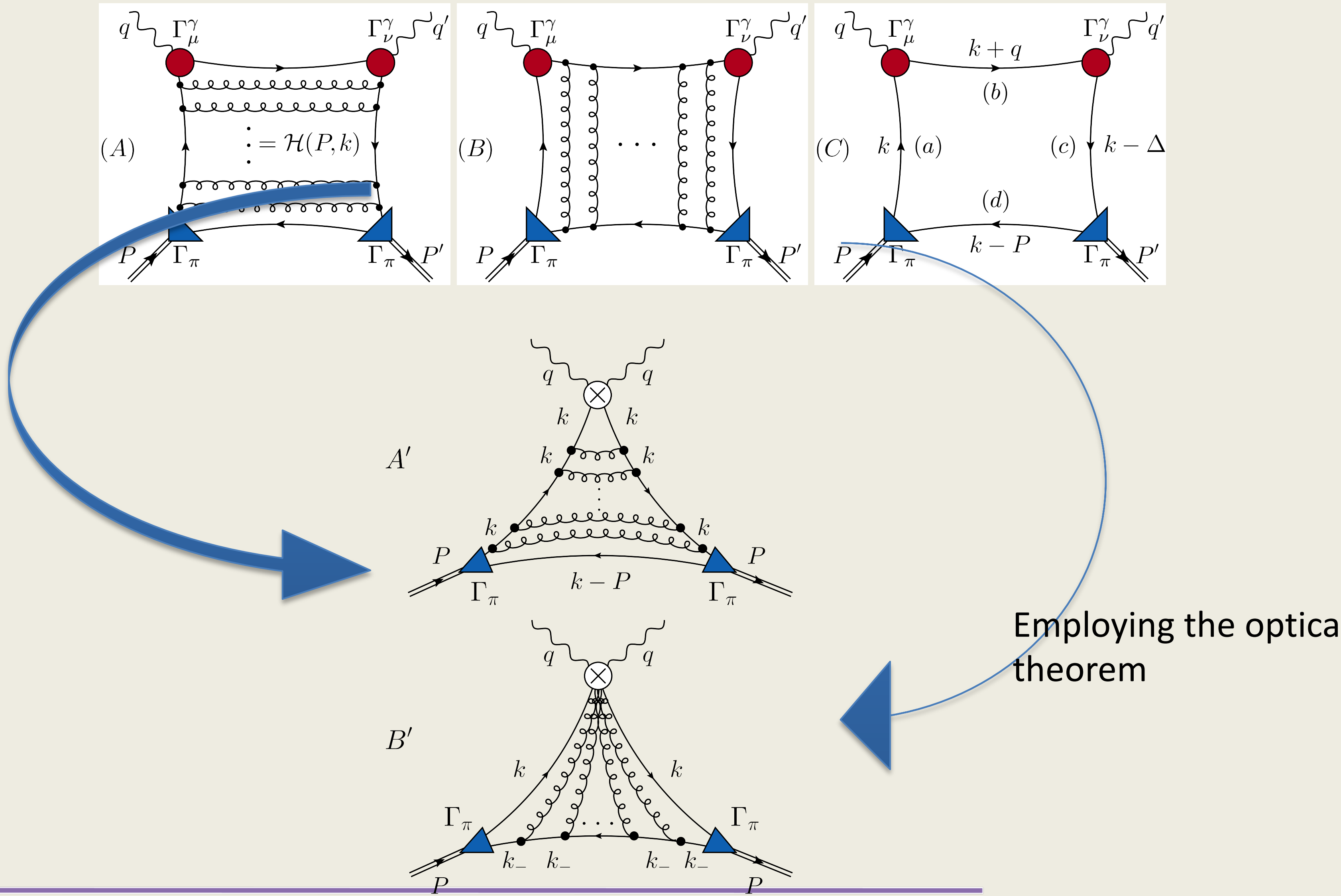


$$\alpha_{\rm PI}(k^2) = \frac{\pi \gamma_m}{\ln[(m_\alpha^2 + k^2)/\Lambda_{\rm QCD}^2]},$$

$$m_\alpha = 0.30\,{\rm GeV} \gtrsim \Lambda_{\rm QCD}.$$

- Modeling interaction and truncation approximation
- Renormalize our DSEs at the hadronic scale $\zeta = m_\alpha$
- Pure valences

Pion Compton Scattering(RL symmetry)



$$q^\pi(x;\zeta_H) = q^\pi(1-x;\zeta_H);$$

Interaction and spectrum

Interaction and spectrum

Nankai University

Abstract A symmetry-preserving treatment of a vector×vector contact interaction is used to compute spectra of ground-state $J^P = 0^\pm, 1^\pm$ $(f\bar{g})$ mesons, their partner diquark correlations, and $J^P = 1/2^\pm, 3/2^\pm$ (fgh) baryons, where $f, g, h \in \{u, d, s, c, b\}$. Results for the leptonic decay constants of all mesons are also obtained, including scalar and pseudovector states involving heavy quarks. The spectrum of baryons produced by this chiefly algebraic approach reproduces the 64 masses known empirically or computed using lattice-regularised quantum chromodynamics with an accuracy of 1.4(1.2)%. It also has the richness of states typical of constituent-quark models and predicts many baryon states that have not yet been observed. The study indicates that dynamical, nonpointlike diquark correlations play an important role in all baryons; and, typically, the lightest allowed diquark is the most important component of a baryon's Faddeev amplitude.

$$\mathcal{G} = \frac{4\pi\alpha_{IR}}{m_G^2}$$

arXiv: 2102.12568

Masses of positive- and negative-parity hadron ground-states, including htose with heavy quarks

Pei-Lin Yin, Zhu-Fang Cui, C. D. Roberts, Jorge Segovia

Interaction and spectrum

$$\mathcal{G} = \frac{8\pi^2}{\omega^4} D e^{-s/\omega^2} + \frac{8\pi^2 \gamma_m \mathcal{F}(s)}{\ln[\tau + (1 + s/\Lambda_{QCD}^2)^2]}$$

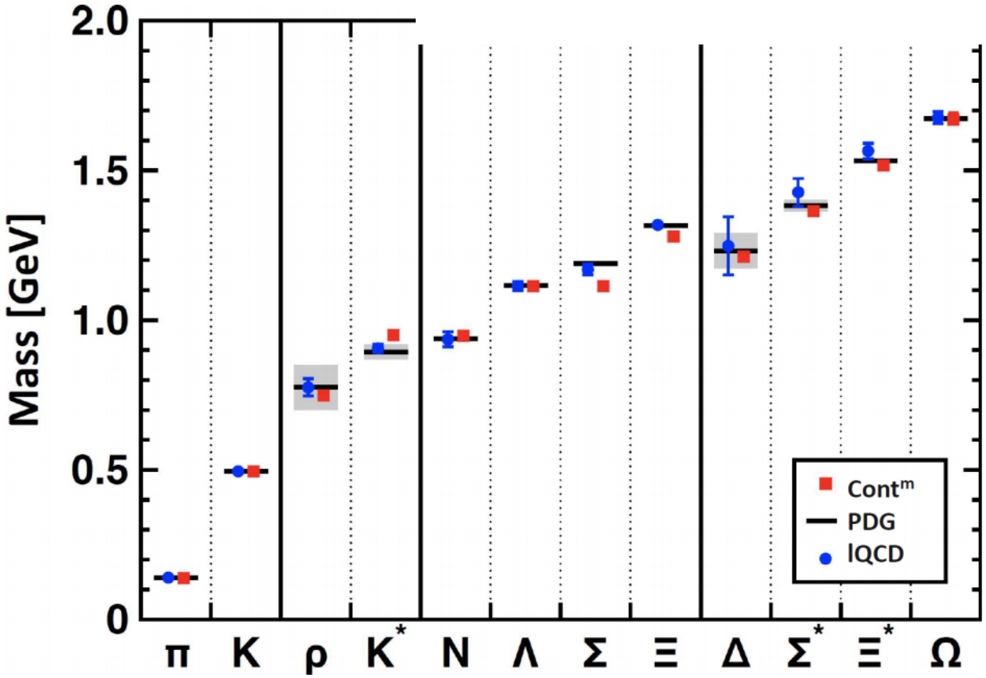


FIG. 4. Masses of pseudoscalar and vector mesons, and ground-state positive-parity octet and decuplet baryons calculated using continuum (Cont$^{\rm m}$ – squares, red) [31] and lattice [79] methods in QCD compared with experiment [34] (PDG: black bars, with decay-widths of unstable states shaded grey).

arXiv: 2008.07629

Impressions of the Continuum Bound State Problem in QCD

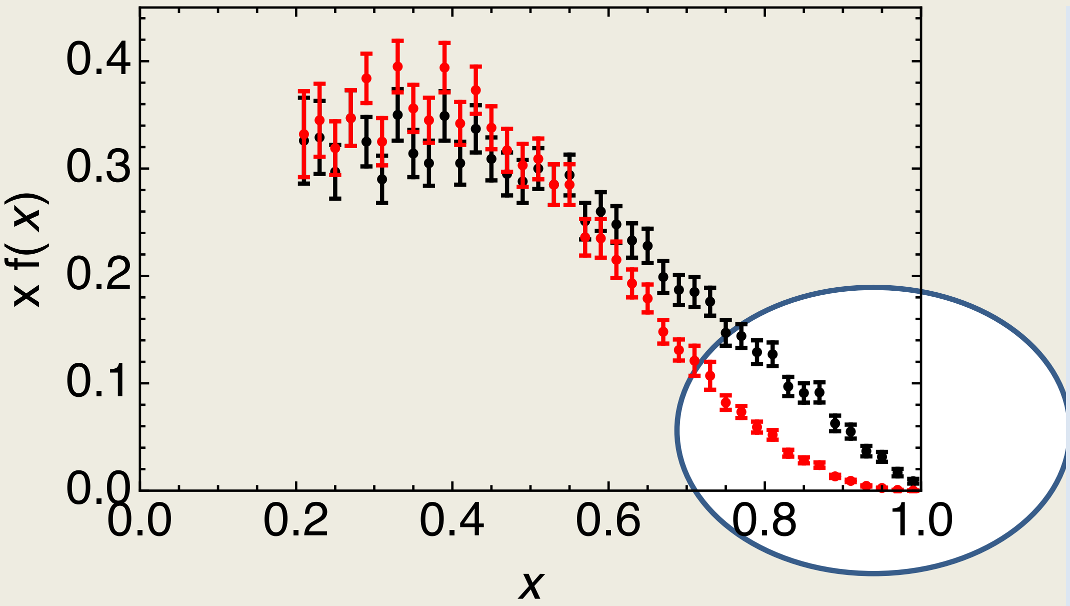
Si-xue Qin, C. D. Roberts

Hadron masses are global, volume-integrated properties, insensitive to the detail behavior of quark mass

However, this feature becomes vital for dynamical, structural properties: elastic form factor and parton distribution amplitude and functions.

Interaction and structure

Interaction←→large x



- 1989…Conway et al. Phys. Rev.D 39 (1989) 92
  Leading-order analysis of Drell-Yan data
- 2010...Aicher et al. Phys. Rev. Lett.105 (2010) 252003
  Consistent next-to-leading order anaylsis

Model and Model

- Nambu – Jona-Lasinio model, translationally invariant regularisaion
  $q^{\pi}(x) \sim (1-x)^{0}$,
  which becomes "*1*" after evolving from a low resolution scale
- NJL models with a hard cutoff & also some duality arguments:
  $q^{\pi}(x) \sim (1-x)^{1}$
- Relativistic constituent quark models:
  $q^{\pi}(x) \sim (1-x)^{0...2}$
  depending on the form of model wave function
- Instanton-based models
  $q^{\pi}(x) \sim (1-x)^{1...2}$

Interaction(example)

$$\mathcal{G} = \frac{4\pi\alpha_{IR}}{m_G^2}$$

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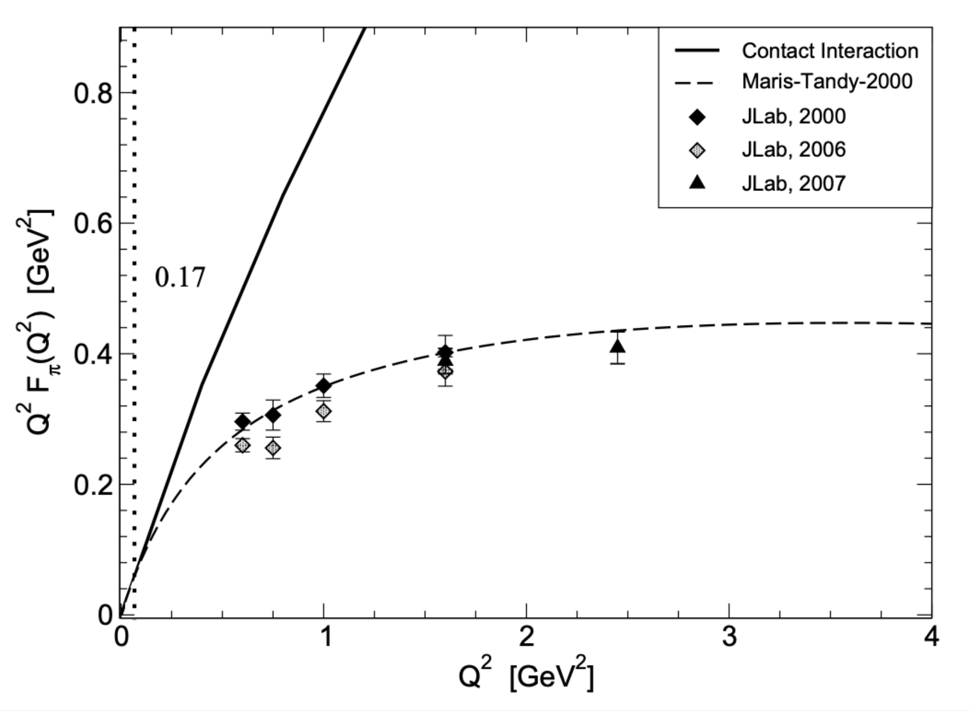
Quark Mass = constant
Pion Amplitude = constant

PDA(x)=1
PDF(x)=1



Structure of pion

Elastic form factor of pion

Interaction(example)

$$\mathcal{G} = \frac{1}{(k^2)^\tau}$$

$$PDF(x) \sim (1-x)^{2\tau-2}$$

Structure of pion

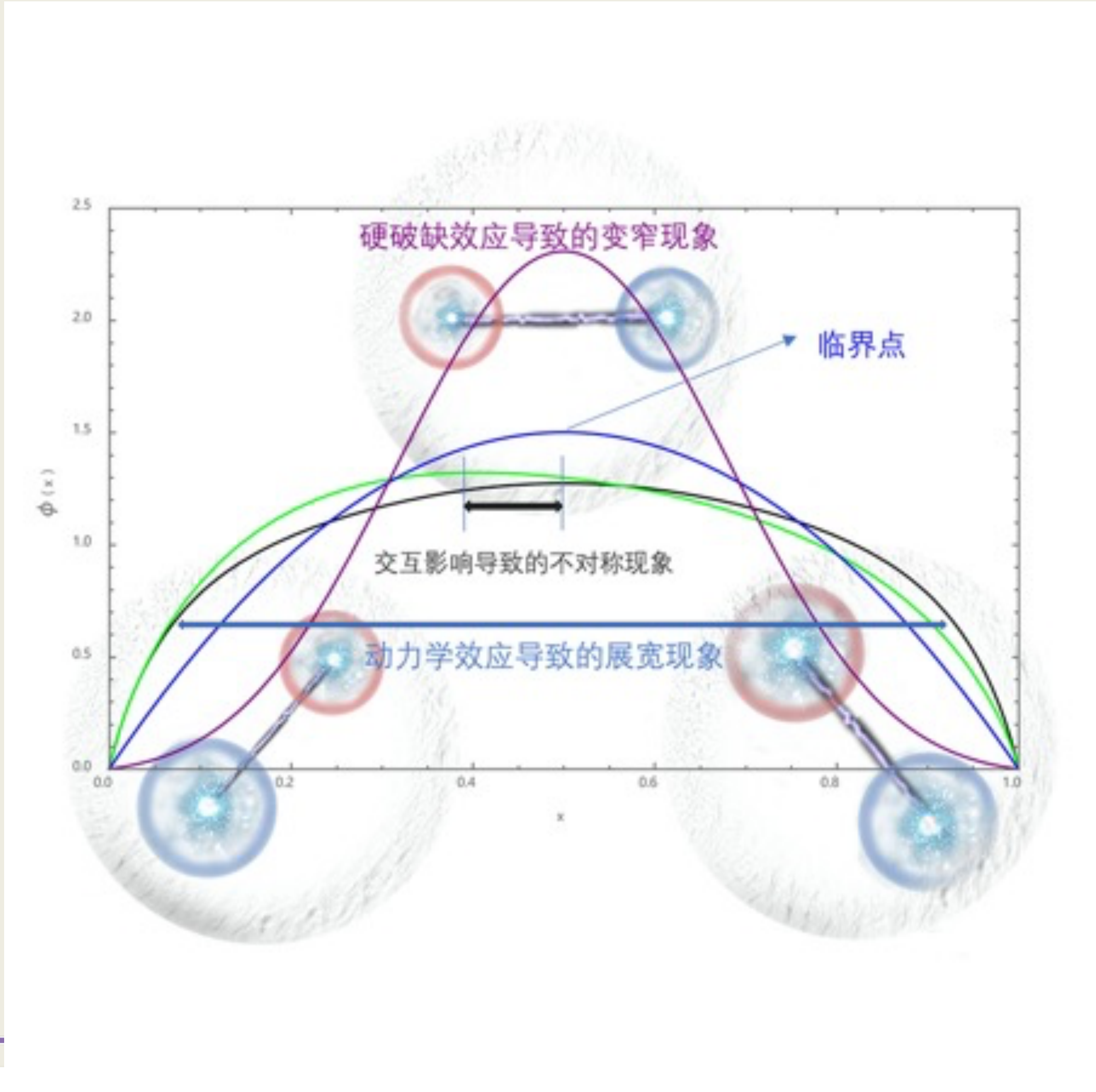
$$F(Q^2) \sim \frac{1}{(Q^2)^\tau}$$

Elastic form factor of pion

$$\mathcal{G} \sim \frac{1}{k^2 \ln(k^2)}$$

QCD one-loop interaction

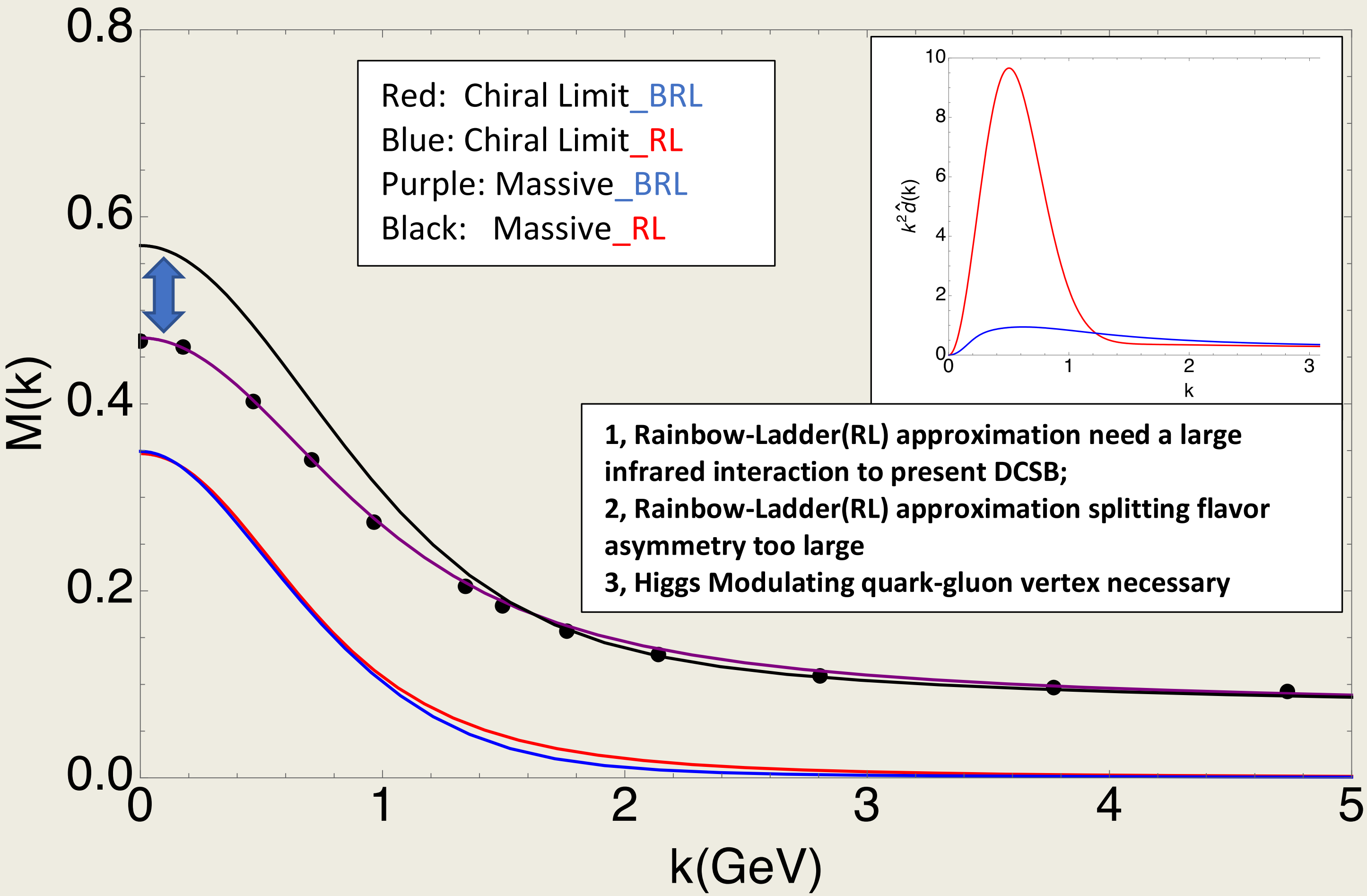
Higgs modulation of emergent mass as revealed in kaon and pion distributions



Pion and Kaon
distribution amplitudes
electromagentic form factors
structure functions

On the same footing

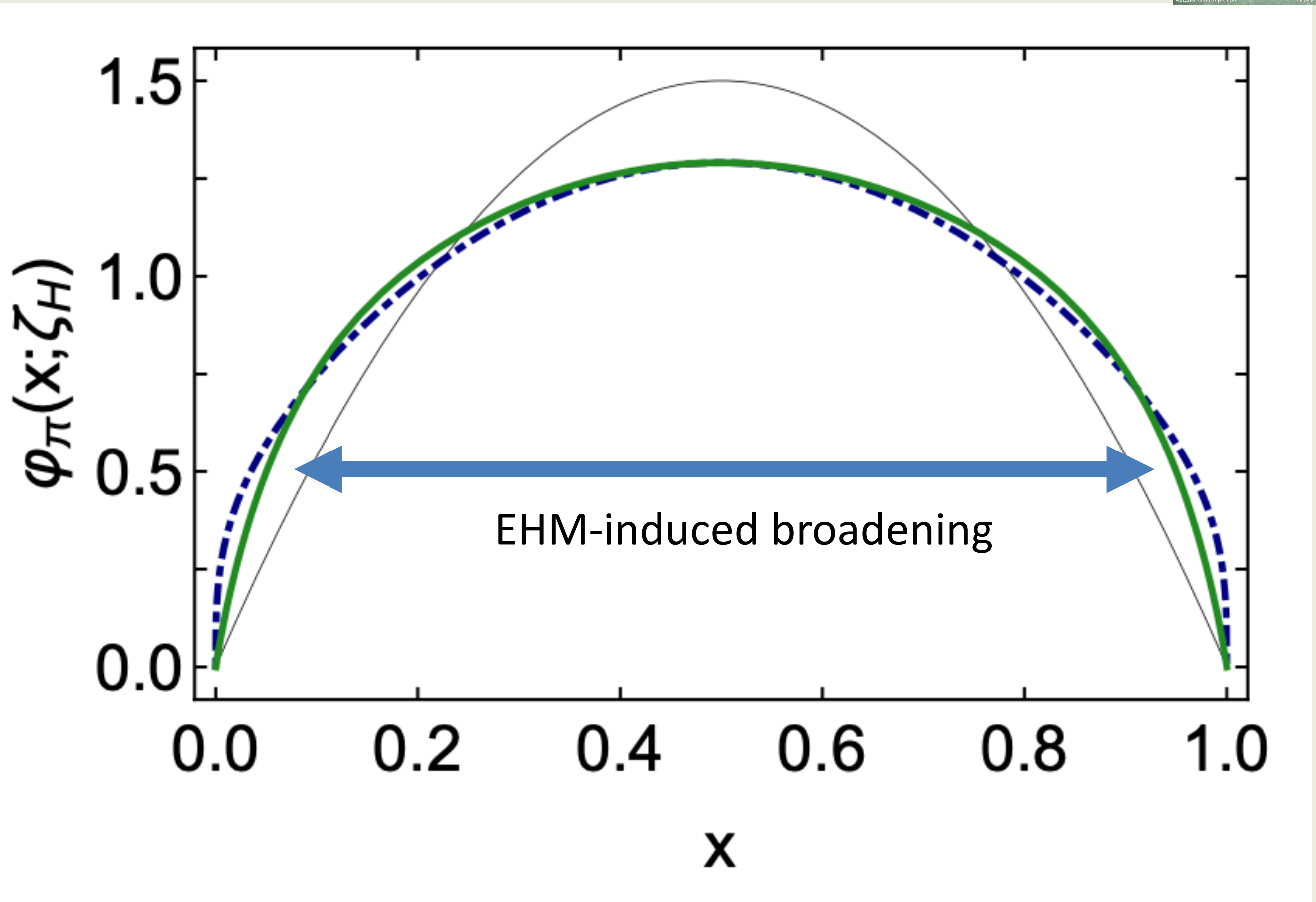
Higgs modulation of emergent mass



1, Rainbow-Ladder(RL) approximation need a large infrared interaction to present DCSB;
2, Rainbow-Ladder(RL) approximation splitting flavor asymmetry too large
3, Higgs Modulating quark-gluon vertex necessary

BRL (Put physics in the right place!!)

Higgs Boson



- Asymptotic profile $6x(1-x)$
- following these 40 years of effort, continuum phenomenology and theory agree that the pion's DA at hadronic scale is a BROAD, CONCAVE function, possessing greater support in the neighbourhood of its endpoints.
- Endpoint behavior lesson

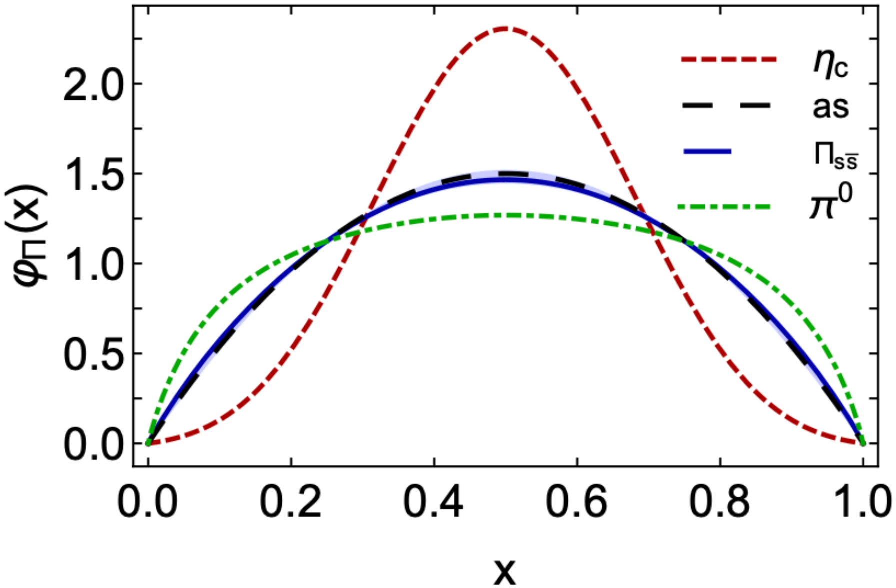
Y2013->Y2020

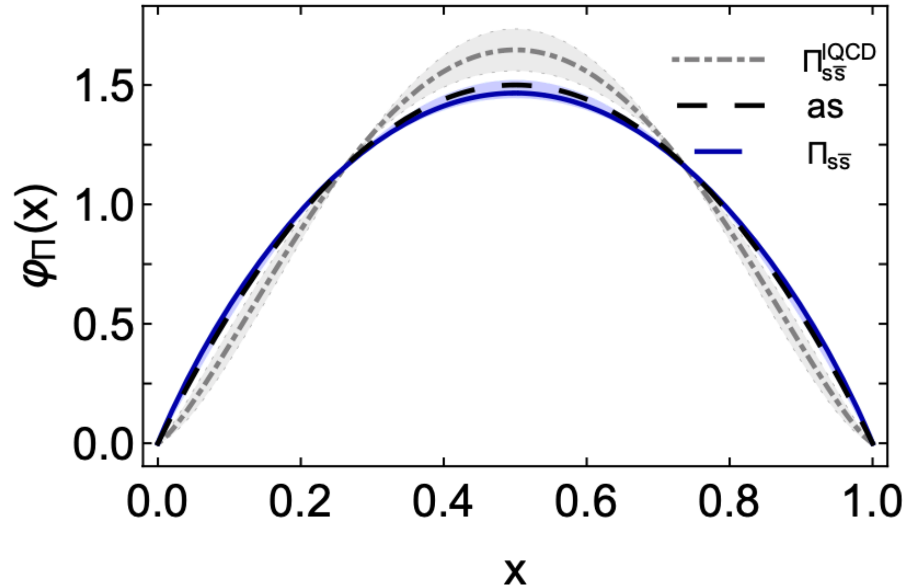
$$\varphi_\pi^{\alpha_\pi}(x;\zeta_H) = 1.81[x(1-x)]^{\alpha_\pi}\left[[1+a_2^\pi C_2^{(\alpha_\pi+1/2)}(1-2x)\right]$$

$$\varphi_\pi(x;\zeta_H) = 18.2\,x(1-x)\left[1-2.33\sqrt{x(1-x)}+1.79x(1-x)\right]$$

NG's DA (presence of Higgs)

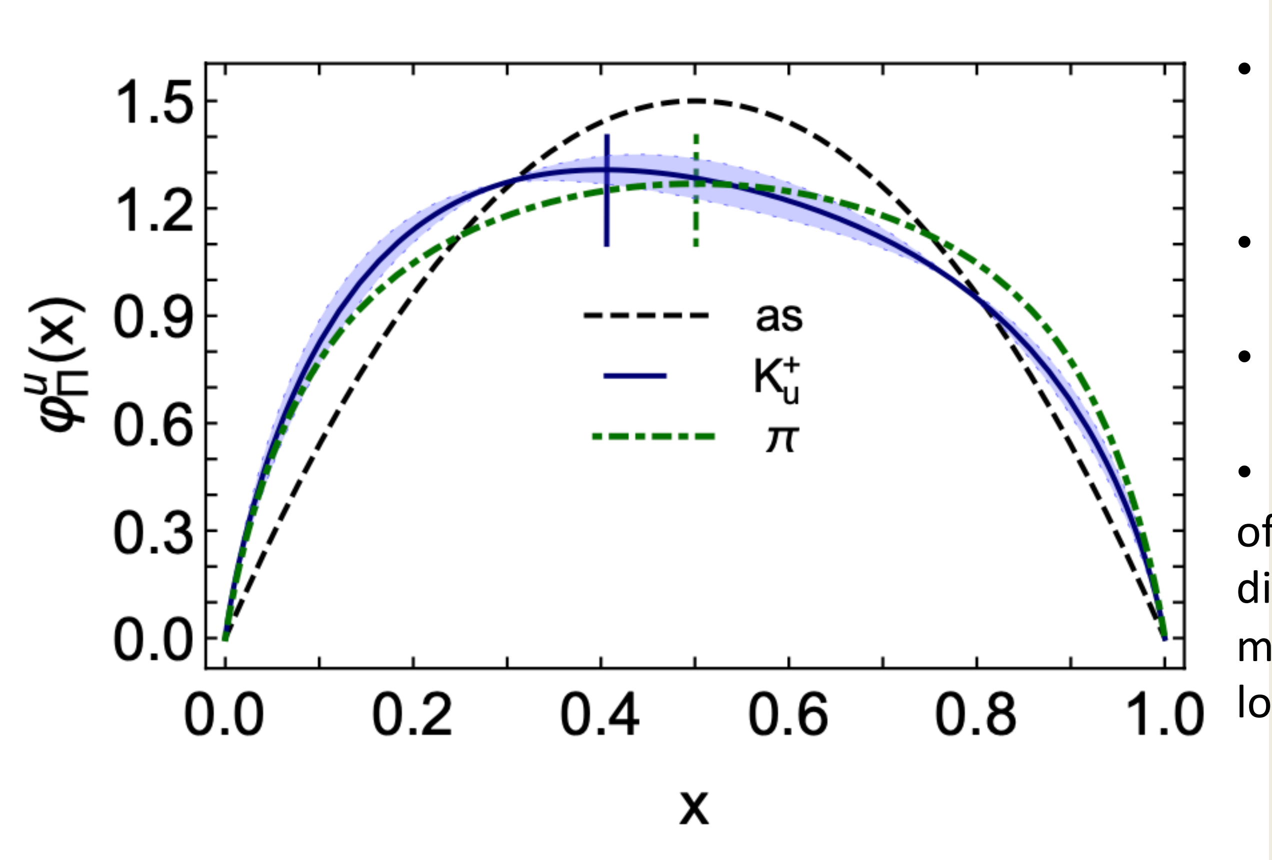
Higgs Boson ✓





- Question: when does the Higgs mechanism begin to influence mass generation (pion…eta_c)
- A critical current quark mass lies in the neighbourhood of the s-quark
- lQCD calculation(R. Zhang, *et al*, PRD102(2020)094519) and continuum analyses in QCD agree upon the existence of such critical current mass(ps boundsate mass 0.69GeV both)
- For a DA very similar to Asymptotic one, EHM and Higgs-boson couplings are playing a roughly equal role in forming the wave function

$K^+ \quad (u\bar{s})$



- Flavor asymmetry

  $f_K/f_\pi \approx 1.2 \approx M_s(0)/M_{\overline{ud}}(0).$

- Peak shifted to x=0.4

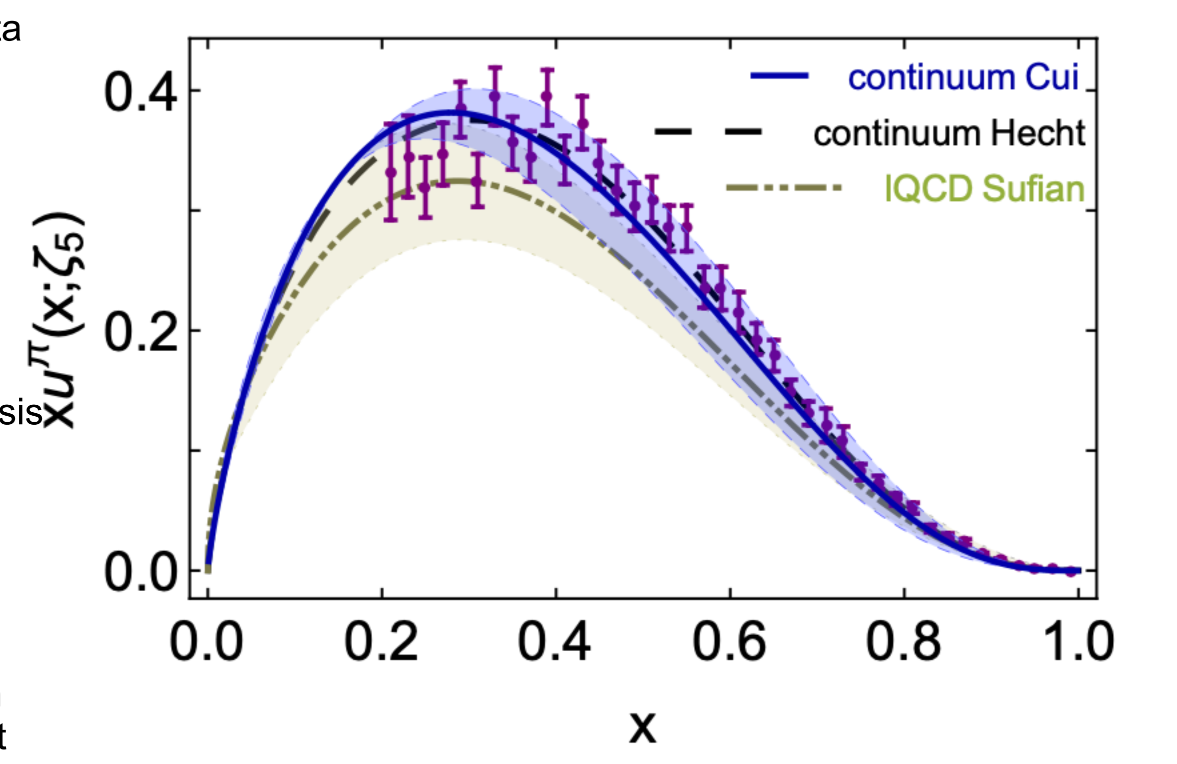
  20% to the left

- Higgs-boson modulation of EHM
- With increasing current mass of the heavier quark the distortion of this DA becomes more pronounced and its peak location moves toward x=0.

$$\langle x\rangle_\pi^{\zeta_H} \approx 0.5,\ \langle x\rangle_{K^+}^{\zeta_H} \approx 0.48,\ \langle x\rangle_D^{\zeta_H} \approx 0.32,\ \langle x\rangle_{\bar{B}}^{\zeta_H} \approx 0.19.$$

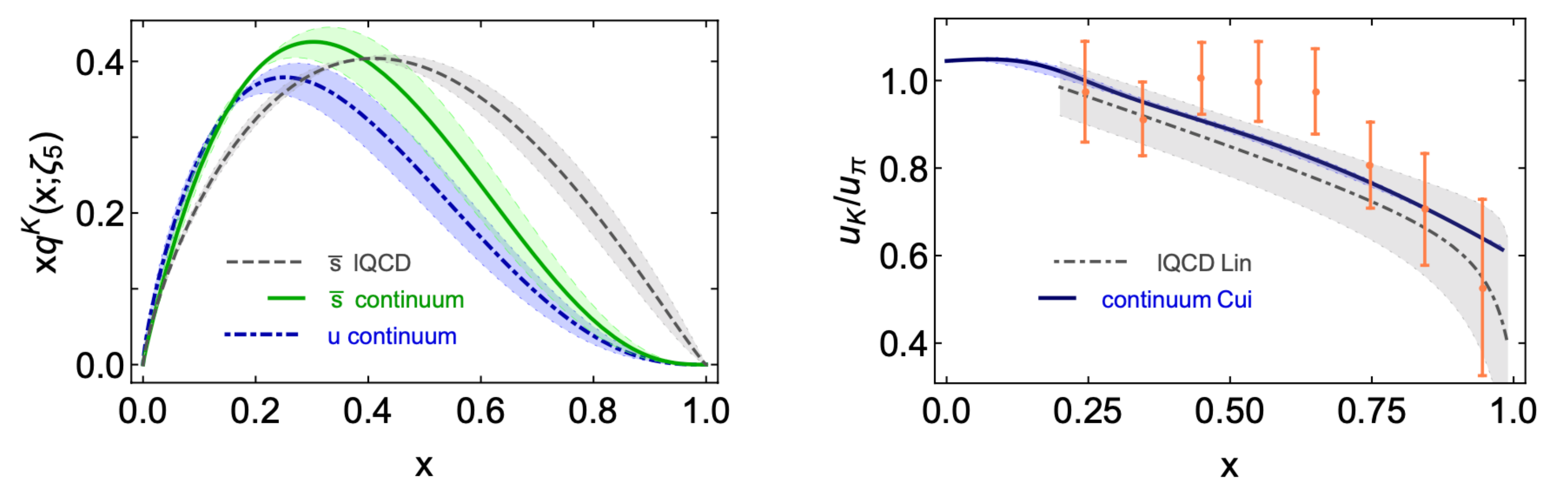
Pion's DF

- 1989…Conway et al. Phys. Rev.D 39 (1989) 92
  Leading-order analysis of Drell-Yan data
- 2000…Hecht et al. Phys. Rev.C 63 (2001)025213
  QCD connected model calculation
- 2010...Aicher et al. Phys. Rev. Lett.105 (2010) 252003
  Consistent next-to-leading order anaylsis
- 2019/04...Ding, et al.
  Continuum QCD prediction
- 2019/01...Sufian, et al.
  1st exploratory lattice-QCD calculation
  Using lattice-calculated matrix element obtained through spatially separated current-current correlations in coordinate space



Update analyses: Chang, *et al, CPC44(2020)114105*

Kaon's valence DF



	$q(\zeta_5)$	$\langle x q^K \rangle$	$\langle x^2 q^K \rangle$	$\langle x^3 q^K \rangle$
continuum [96]	u	0.19(2)	0.067(09)	0.030(5)
	$\bar{s}$	0.23(2)	0.085(11)	0.040(7)
lattice [322]	u	0.19(1)	0.080(07)	0.042(6)
	$\bar{s}$	0.27(1)	0.123(07)	0.070(6)

- lQCD is significantly harder than the continuum result
- *lattice DF behaves* $(1-x)^{1.13(16)}$

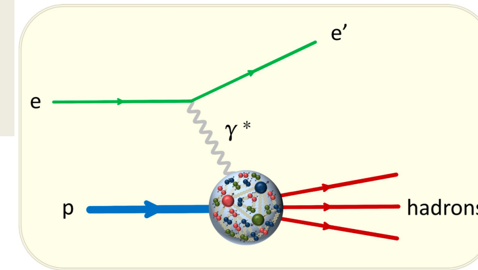
Higgs-modulation of EHM $f_K/f_\pi \approx 1.2 \approx M_s(0)/M_{\overline{ud}}(0).$

DSE: $\frac{\langle x\bar{s}\rangle^K}{\langle x\bar{u}\rangle^K} = 1.18(1)$ vs. lQCD: $\frac{\langle x\bar{s}\rangle^K}{\langle x\bar{u}\rangle^K} = 1.38(7)$

It may reasonably to anticipated that future refinements of lQCD setups, algorithms and analyses will move lattice and continuum DFs closer together

lQCD: H-W Lin, etal, arXiv:2003.14128

Thanks for your attention



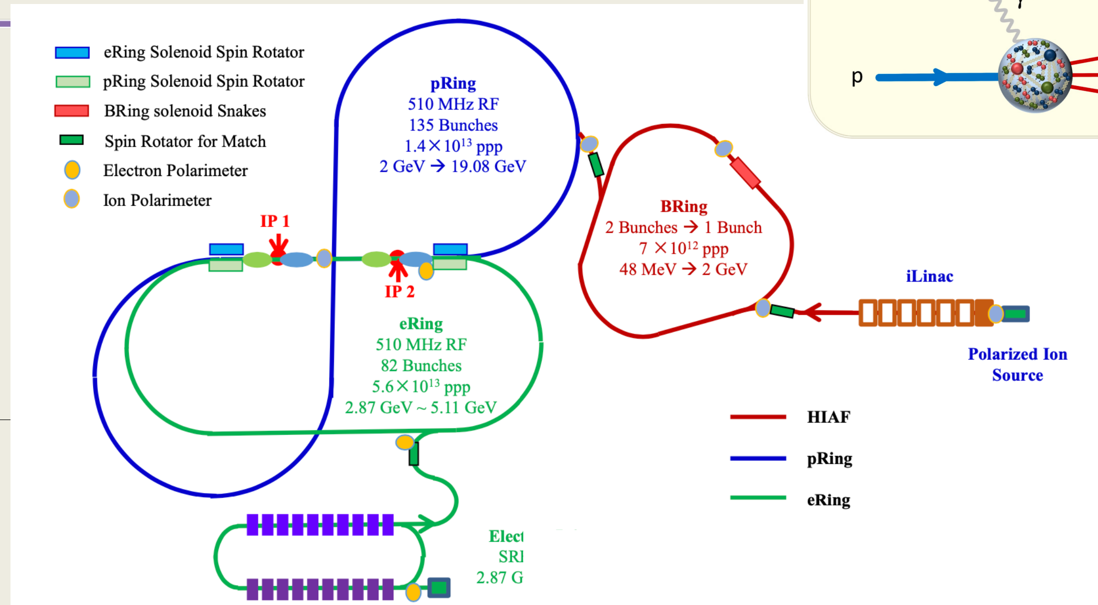
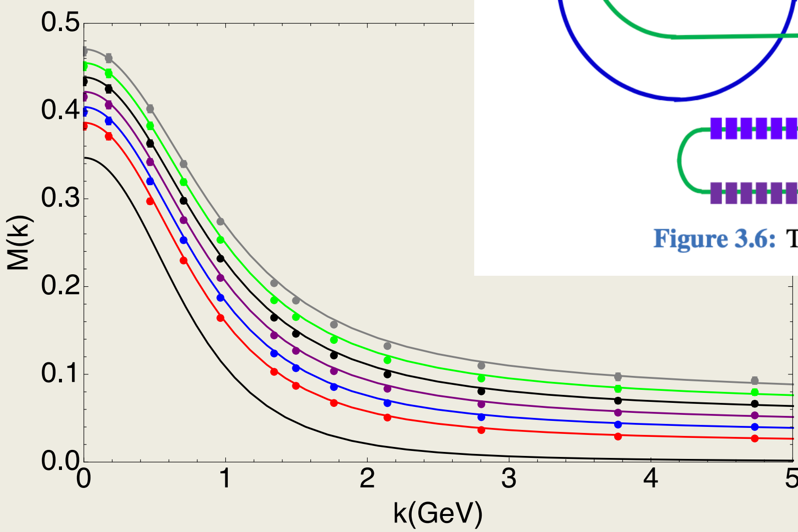
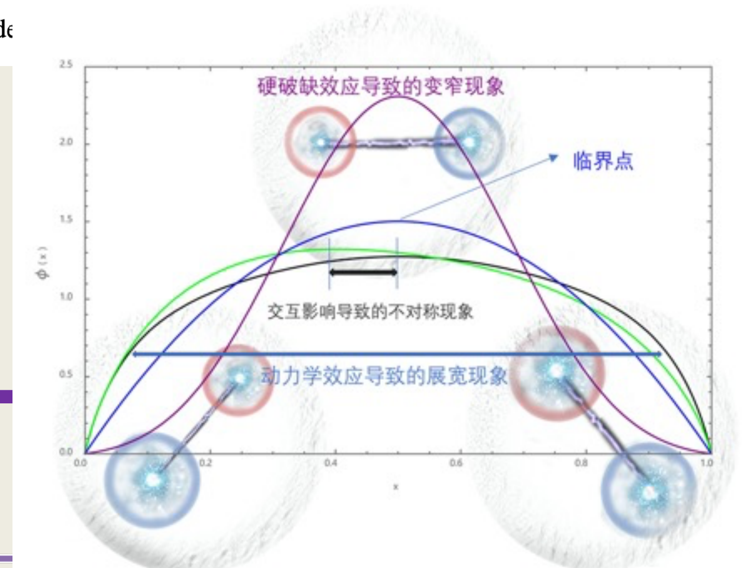


Figure 3.6: The polarization de

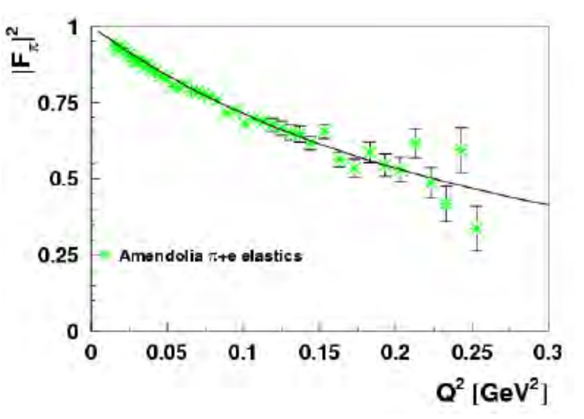
lQCD





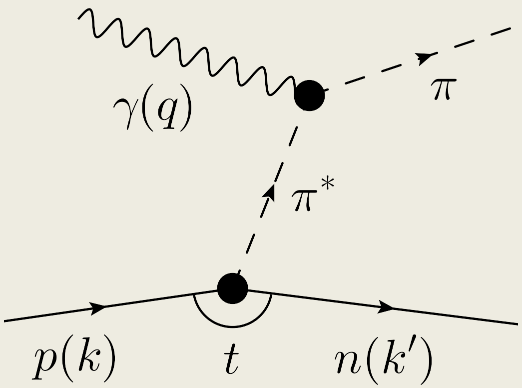
Measurement of the π^+ Form Factor

- At low Q^2, $F\pi$ can be measured directly via high energy elastic π^+ scattering from the atomic electrons
    - CERN SPS used 300 GeV pions to measure form factor up to Q^2=0.25GeV^2
      (Amedolia et al, NPB277, 168 (1986))
    - These data used to constrain the pion charge radius: $r\pi$=0.657±0.012 fm



- At larger Q^2, $F\pi$ must be measured indirectly using the "pion cloud" of the proton in exclusive pion electroproduction: p(e, e' π^+)n
    - at small –t, the pion pole process dominates the longitudinal cross section, σ_L
      (L. Favart, et al, Eur. Phys. J. A 52 (2016) 158)
    - In the Born term model, $F\pi$ appears as

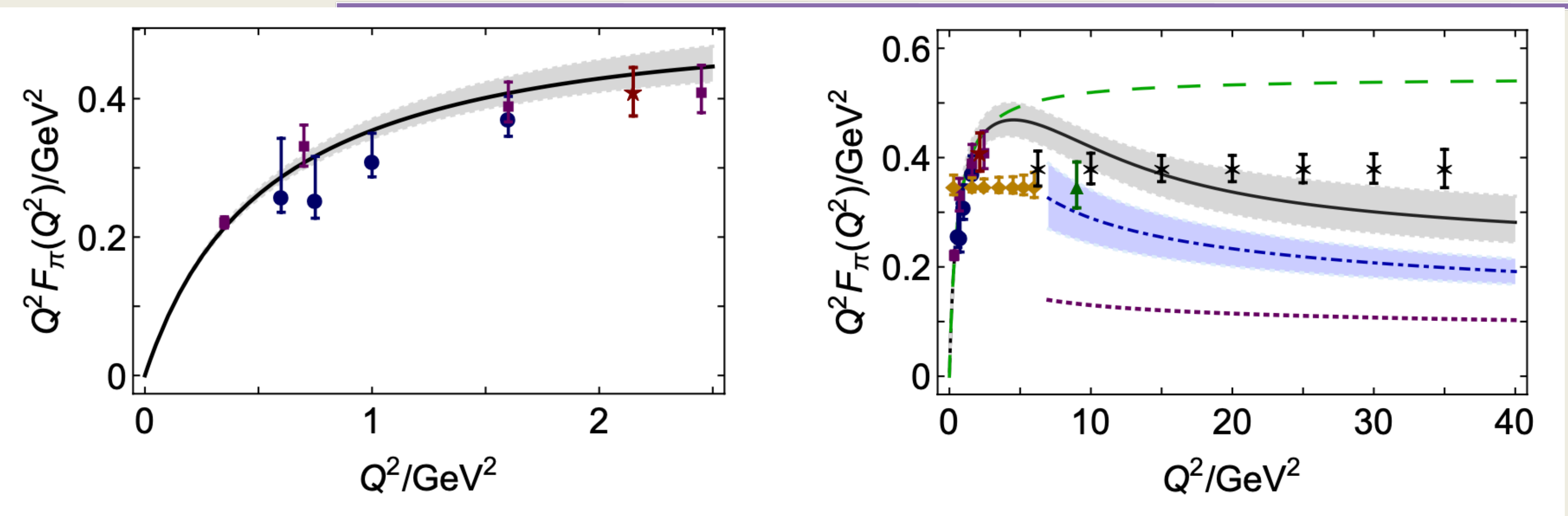
$$\frac{d\sigma_L}{dt} \propto \frac{-t}{(t-m_\pi^2)} g_{\pi NN}^2(t) Q^2 F_\pi^2(Q^2,t)$$



Sullivan process, in which a nucleon's pion cloud is used to provide access to the pion's elastic form factor

Experimental studies over the last decade have given confidence in the electroproduction method yielding the physical pion form factor----Tanja Horn

Elastic Electromagnetic Form Factors of Pion(EHM)



Jlab pion data: Black line is DSE parameter-free prediction, χ^2**/datum=1.0**.

Scaling and scaling violations: a, DSEs tracks a monopole form factor until $Q^2 \sim 6GeV^2$

b, Thereafter, scaling violation

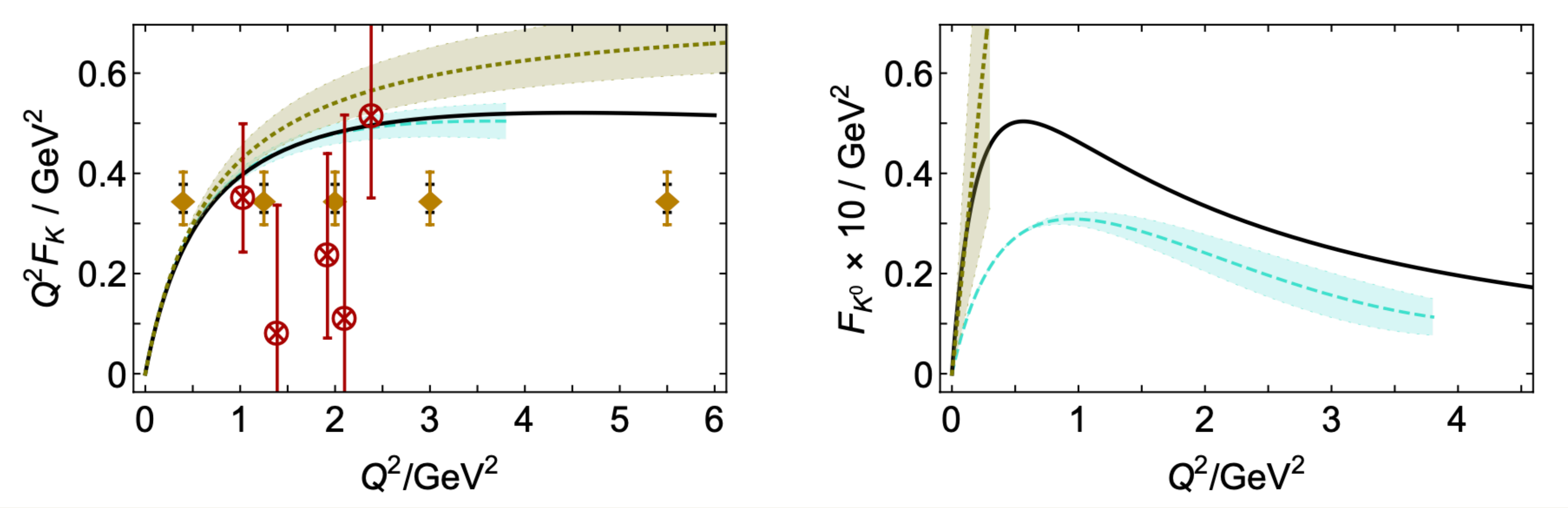
c, JLab12 at $Q^2 \sim 9GeV^2$, sufficient to validate this prediction

(measurement will be the first too have uncovered QCD scaling violations in a hard exclusive process)

pQCD and Large Q^2

Elastic Electromagnetic Form Factors of Kaon

南開大學 Nankai University



- Solid line: DSEs prediction
- Dashed turquoise curve within like coloured bands-lQCD result(PoS Lattice2018,298(2018))
- Dotted olive curve within band-monople based on kaon charge radius

✓ Continuum and lattice results for charged Kaon form factor are almost identical on Q^2<$4GeV^2$

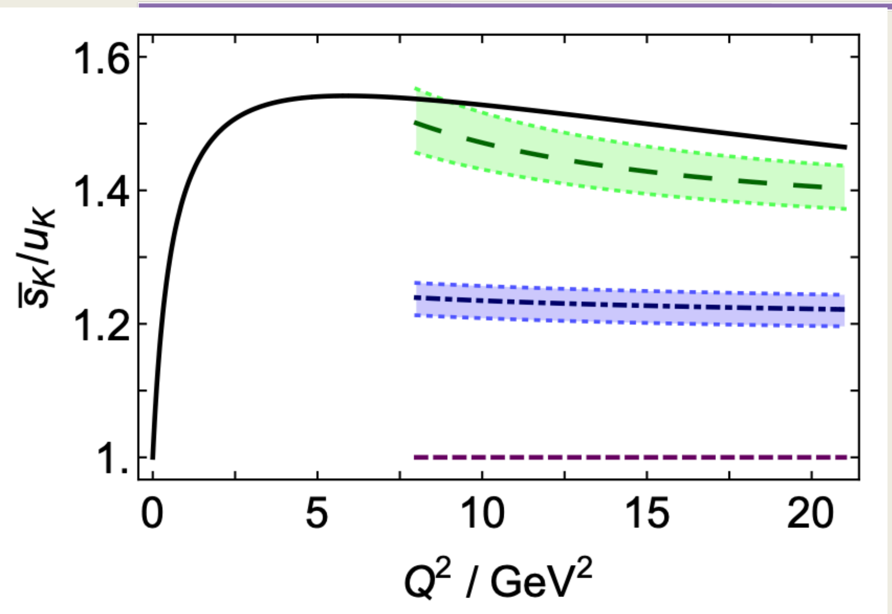
✓ Neutral kaon has a nonzero charge form factor

DSEs: $r^2_{K^0} = -(0.21fm)^2$; experiment:$r^2_{K^0} = -(0.24 \pm 0.08fm)^2$;lattice:$r^2_{K^0} = -(0.16fm)^2$

For neutral kaon the momentum dependence is similar and lQCD result is a roughly uniform two-thirds of the size of the continuum prediction

Elastic Electromagnetic Form Factors of Kaon

Nankai University



Falvor separation

Factored out the electric charges

- The ratio is unity at Q^2=0, owing to current conservation
- pQCD predicts Unity on $\Lambda_{QCD}^2/Q^2 \sim 0$
- Between these limits, a peak value of roughly 1.5 at $Q^2 \sim 6GeV^2 (\frac{f_k^2}{f_\pi^2} \approx 1.4)$, Typical for Higgs-boson modulation of EHM.
- The derivation from unity must remain significant on a very large part of the domain.