The explanation of some exotic states in the $\bar{c}\bar{s}cs$ tetraquark system

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Outline

- Motivations
- the quark delocalization color screening model(QDCSM)
- The mass spectrum and Resonance states
- Summary

Motivations

➢ Experimental results

- In 2020: (LHCb Collaboration)

$$B^+ \to D^+ D^- K^+$$

A narrow resonance just below the $D_s\bar{D}_s$threshold- the $\chi_{c0}(3930)$ discovered in the $\bar{D}D$ channel.

Phys. Rev. Lett. 125, 242001(2020)

- In 2021: (LHCb Collaboration)

$$B^+ \to J/\psi\phi K^+$$

Two resonance X(4685) $J^P = 1^+$
X(4630) $J^P = 1^-$

arXiv: 2103.01803v1(2021).

2009 CDF Collaboration $B^+ \to J/\psi\phi K^+$ X(4140)

LHCb(2012), CMS(2014), D0(2014), BABAR(2015)

2010 Belle Collaboration $\gamma\gamma \to J/\Psi\phi$ X(4350) $J^{PC} = 0^{++}\ or\ 2^{++}$

2017 CDF Collaboration $B^+ \to J/\psi\phi K^+$ X(4274)

2016 LHCb Collaboration $B^+ \to J/\psi\phi K^+$ X(4140) X(4274) $J^{PC} = 1^{++}$

X(4500) X(4700) $J^{PC} = 0^{++}$

Phys. Rev. Lett. 102, 242002 (2009).
Phys. Rev. D. 85, 091103(2012).
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➢ Theoretical studies

1) Molecular interpetations

X. Liu and S.-L. Zhu, Phys. Rev. D 80, 017502 (2009); 85,019902(E) (2012).
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R. M. Albuquerque, M. E. Bracco, and M. Nielsen, Phys.Lett. B 678, 186 (2009).
G.-J. Ding, Eur. Phys. J. C 64, 297 (2009).
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- X(4140) can only be a $J^{PC} = 0^{++}$ or 2^{++} $D_s^{*+} D_s^{*-}$ molecule.

2) Compact tetraquark interpetations

Qi-Fang L and Yu-Bing Dong, Phys. Rev. D 94, 074007(2016).
J. Wu, Y. R. Liu, K. Chen, X. Liu, and S. L. Zhu, Phys.Rev. D 94, 094031 (2016).
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F. Stancu, J. Phys. G 37, 075017 (2010).
H. X. Chen, E. L. Cui, W. Chen, X. Liu, and S. L. Zhu,Eur. Phys. J. C 77, 160 (2017).
C. R. Deng, J. L. Ping, H. X. Huang, and F. Wang, Phys.Rev. D 98, 014026 (2018).
Y. F. Yang and J. L. Ping, Phys. Rev. D 99, 094032 (2018).
Z. G. Wang, Eur. Phys. J. C 77, 78 (2017).
Z. G. Wang, Eur. Phys. J. C 76, 657 (2016).

3) Conventional charmonium interpetations

P. G. Ortega, J. Segovia, D. R. Entem, and F. Fernndez, Phys. Rev. D 94, 114018 (2016).

- X(4274),X(4500),X(4700) were all defined as convention charmonium.

➢ Our aim

to see whether some exotic resonances exist in the $\bar{c}\bar{s}cs$ tetraquark system ?

The quark delocalization color screening model(QDCSM)

- General form of the Hamiltionian

$$H=\sum_{i=1}^{4}\left(m_i+\frac{\boldsymbol{p}_i^2}{2m_i}\right)-T_{CM}+\sum_{j>i=1}^{4}V(r_{ij}),$$

$$V(r_{ij})=V_{CON}(r_{ij})+V_{OGE}(r_{ij})+V_{\chi}(r_{ij})$$

$$V_{OGE}(r_{ij})=\frac{1}{4}\alpha_s^{ij}\boldsymbol{\lambda}_i^c\cdot\boldsymbol{\lambda}_j^c\left[\frac{1}{r_{ij}}-\frac{\pi}{2}\delta(\boldsymbol{r}_{ij})\left(\frac{1}{m_i^2}+\frac{1}{m_j^2}+\frac{4\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j}{3m_im_j}\right)\right],$$

$$V_{CON}(r_{ij})=-a_c\boldsymbol{\lambda}_i^c\cdot\boldsymbol{\lambda}_j^c[f(r_{ij})+V_{0_{ij}}],$$

$$f(r_{ij})=\begin{cases} r_{ij}^2 \\ \frac{1-e^{-\mu_{ij}r_{ij}^2}}{\mu_{ij}} \end{cases}$$

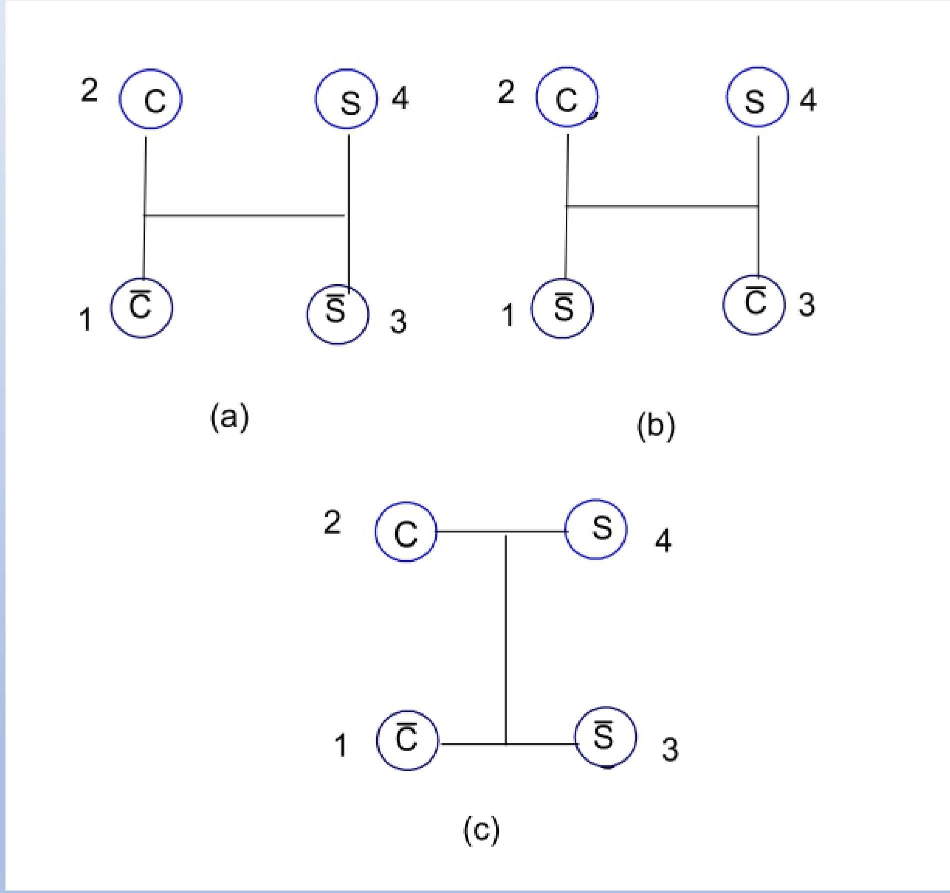
$$V_{\chi}(r_{ij})=v_{ij}^{\eta}\left[\left(\lambda_i^8\cdot\lambda_j^8\right)\cos\theta_P-(\lambda_i^0\cdot\lambda_j^0)\sin\theta_P\right]$$

$$v_{ij}^{\eta}=\frac{g_{ch}^2}{4\pi}\frac{m_\chi^2}{12m_im_j}\frac{\Lambda_\chi^2}{\Lambda_\chi^2-m_\chi^2}m_\chi\left\{(\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j)\left[Y(m_\chi r_{ij})-\frac{\Lambda_\chi^3}{m_\chi^3}Y(\Lambda_\chi r_{ij})\right]\right\}$$

The mass spectrum and Resonance states

➢ The dynamic calculation

arXiv:2103.12425



- Two configurations:

  The meson-meson structures shown in Fig(a) and Fig(b)

  The diquark-antidiquark structure shown in Fig(c)

The wave function

The multiquark system wave function is an internal product of the color, spin, flavor, and orbit terms.

a) The spin wave function.

$$
\begin{aligned}
S_0^1 &= \chi_{00}\chi_{00} \\
S_0^2 &= \sqrt{\frac{1}{3}}(\chi_{11}\chi_{1-1} - \chi_{10}\chi_{10} + \chi_{1-1}\chi_{11}) \\
S_1^3 &= \chi_{00}\chi_{11} \\
S_1^4 &= \chi_{11}\chi_{00} \\
S_1^5 &= \sqrt{\frac{1}{2}}(\chi_{11}\chi_{10} - \chi_{10}\chi_{11}) \\
S_2^6 &= \chi_{11}\chi_{11}
\end{aligned}
$$

b) The flavor wave function

$$
\begin{aligned}
F_0^1 &= c\bar{c}s\bar{s} \\
F_0^2 &= c\bar{s}s\bar{c} \\
F_0^3 &= cs\bar{c}\bar{s}
\end{aligned}
$$

c) The color wave function

The meson-meson structures:

$$[111]\times[111]=[222]\quad (1\times\bar{1})$$

$$C^1_{[111]} = \sqrt{\frac{1}{3}}(r\bar{r}+g\bar{g}+b\bar{b})$$

$$\chi^c_1 = C^1_{[111]}C^1_{[111]}$$

$$[21]\times[21]=[222]\quad (8\times\bar{8})$$

$$\begin{aligned}
C^2_{[21]} &= r\bar{b}, C^3_{[21]} = -r\bar{g} \\
C^4_{[21]} &= g\bar{b}, C^5_{[21]} = -b\bar{g} \\
C^6_{[21]} &= g\bar{r}, C^7_{[21]} = b\bar{r} \\
C^8_{[21]} &= \sqrt{\frac{1}{2}}(r\bar{r}-g\bar{g}) \\
C^9_{[21]} &= \sqrt{\frac{1}{6}}(-r\bar{r}-g\bar{g}+2b\bar{b})
\end{aligned}$$

$$\begin{aligned}
\chi^c_2 =& \sqrt{\frac{1}{8}}(C^2_{[21]}C^7_{[21]} - C^4_{[21]}C^5_{[21]} - C^3_{[21]}C^6_{[21]} \\
&+ C^8_{[21]}C^8_{[21]} - C^6_{[21]}C^3_{[21]} + C^9_{[21]}C^9_{[21]} \\
&- C^5_{[21]}C^4_{[21]} + C^7_{[21]}C^2_{[21]})
\end{aligned}$$

The diquark-antidiquark structure.

$$
\begin{aligned}
C^1_{[2]} &= rr, C^2_{[2]} = \sqrt{\frac{1}{2}}(rg+gr) \\
C^3_{[2]} &= gg, C^4_{[2]} = \sqrt{\frac{1}{2}}(rb+br) \\
C^5_{[2]} &= \sqrt{\frac{1}{2}}(gb+bg), C^6_{[2]} = bb \\
C^7_{[11]} &= \sqrt{\frac{1}{2}}(rg-gr), C^8_{[11]} = \sqrt{\frac{1}{2}}(rb-br) \\
C^9_{[11]} &= \sqrt{\frac{1}{2}}(gb-bg)
\end{aligned}
$$

$$
\begin{aligned}
C^1_{[22]} &= \bar{r}\bar{r}, C^2_{[22]} = -\sqrt{\frac{1}{2}}(\bar{r}\bar{g}+\bar{g}\bar{r}) \\
C^3_{[22]} &= \bar{g}\bar{g}, C^4_{[22]} = \sqrt{\frac{1}{2}}(\bar{r}\bar{b}+\bar{b}\bar{r}) \\
C^5_{[22]} &= -\sqrt{\frac{1}{2}}(\bar{g}\bar{b}+\bar{b}\bar{g}), C^6_{[22]} = \bar{b}\bar{b} \\
C^7_{[211]} &= \sqrt{\frac{1}{2}}(\bar{r}\bar{g}-\bar{g}\bar{r}), C^8_{[211]} = -\sqrt{\frac{1}{2}}(\bar{r}\bar{b}-\bar{b}\bar{r}) \\
C^9_{[211]} &= \sqrt{\frac{1}{2}}(\bar{g}\bar{b}-\bar{b}\bar{g})
\end{aligned}
$$

$[2]\times[22]=[222]$ $(3\times\bar{3})$

$$
\chi^c_3 = \sqrt{\frac{1}{6}}(C^1_{[2]}C^1_{[22]} - C^2_{[2]}C^{[2]}_{[22]} + C^3_{[2]}C^3_{[22]} + C^4_{[2]}C^4_{[22]} - C^5_{[2]}C^5_{[22]} + C^6_2 C^6_{22})
$$

$[11]\times[211]=[222]$ $(6\times\bar{6})$

$$
\chi^c_4 = \sqrt{\frac{1}{3}}(C^7_{[11]}C^7_{[211]} - C^8_{[11]}C^8_{[211]} + C^9_{[11]}C^9_{[211]})
$$

d) The orbital wave function

The total orbital wave functions can be constructed by coupling the orbital wave function of two internal cluster and the relative motion wave function between two clusters.

$$\psi^L = \psi_1(R_1)\psi_2(R_2)\chi_L(R)$$

$$\chi_L(R) = \sqrt{\frac{1}{4\pi}(\frac{3}{2\pi b^2})}\sum_{i=1}^{n} C_i \times \int exp[-\frac{3}{4b^2}(R-s_i)^2] Y_{LM}(\hat{s}_i) d\hat{s}_i$$

TABLE III. All possible channels for all quantum numbers

$IJ^P=00^+$			$IJ^P=01^+$			$IJ^P=02^+$		
index	$F_I^i;S_s^j;\chi_k^c$ [i;j;k]	channels	index	$F_I^i;S_s^j;\chi_k^c$ [i;j;k]	channels	index	$F_I^i;S_s^j;\chi_k^c$ [i;j;k]	channels
1	[1,1,1]	$\eta_c\eta_s$	1	[1,3,1]	$\eta_c\phi$	1	[1,6,1]	$J/\psi\phi$
2	[2,1,1]	$D_s\bar{D}_s$	2	[2,3,1]	$D_sD_s^*$	2	[2,6,1]	$D_s^*D_s^*$
3	[1,2,1]	$J/\psi\phi$	3	[1,4,1]	$J/\psi\eta_s$	3	[3,6,3]	$(cs)(\bar{c}\bar{s})$
4	[2,2,1]	$D_s^*\bar{D}_s^*$	4	[2,4,1]	$D_s^*D_s$	4	[3,6,4]	$(cs)(\bar{c}\bar{s})$
5	[3,1,3]	$(cs)(\bar{c}\bar{s})$	5	[1,5,1]	$J/\psi\phi$			
6	[3,1,4]	$(cs)(\bar{c}\bar{s})$	6	[2,5,1]	$D_s^*D_s^*$			
7	[3,2,3]	$(cs)(\bar{c}\bar{s})$	7	[3,3,3]	$(cs)(\bar{c}\bar{s})$			
8	[3,2,4]	$(cs)(\bar{c}\bar{s})$	8	[3,3,4]	$(cs)(\bar{c}\bar{s})$			
			9	[3,4,3]	$(cs)(\bar{c}\bar{s})$			
			10	[3,4,4]	$(cs)(\bar{c}\bar{s})$			
			11	[3,5,3]	$(cs)(\bar{c}\bar{s})$			
			12	[3,5,4]	$(cs)(\bar{c}\bar{s})$			

The bound state calculation

TABLE IV. The lowest-lying eigenenergies of $c\bar{c}s\bar{s}$ tetraquarks with $IJ^P = 00^+$ in the QDCSM.

Index	Channel	Threshold	E_{sc}	E_{cc}	E_{mix}
1	$\eta_{s\bar{s}}\eta_{c\bar{c}}$	3942	3944	3938	3930
2	$D_s\bar{D}_s$	3936	3938		
3	$J/\psi\phi$	4117	4119		
4	$D_s^*\bar{D}_s^*$	4224	4226		
5	$(cs)(\bar{c}\bar{s})$		4324	4219	
6	$(cs)(\bar{c}\bar{s})$		4442		
7	$(cs)(\bar{c}\bar{s})$		4405		
8	$(cs)(\bar{c}\bar{s})$		4305		

$\chi_{c0}(3930)$

- No bound state is found for the meson-meson structure or diquark-antiquark structure.
- A bound state with mass of 3930 MeV is obtained by coupling all channels of two structures.

To explore the structure of this bound state.

TABLE V: the percentages of each channel.

Index	Channel	
1	$\eta_{s\bar{s}}\eta_{c\bar{c}}$	12%
2	$D_s\bar{D}_s$	85%
3	$J/\psi\phi$	0.07%
4	$D_s^*\bar{D}_s^*$	0.26%
5	$(cs)(\bar{c}\bar{s})$	0.48%
6	$(cs)(\bar{c}\bar{s})$	0.11%
7	$(cs)(\bar{c}\bar{s})$	1.1%
8	$(cs)(\bar{c}\bar{s})$	0.12%

- **The largest contribution to forming this bound state comes from the $D_s\bar{D}_s$ channel.**
- **This bound state tends to be a molecular state.**
- **This bound state can be explained the $\chi_{c0}(3930)$ as a molecular state $D_s\bar{D}_s$**

- **Lattice QCD calculation . arXiv:2011.02542(2020)**
- **Bethe-Salpeter equation. Progr. Phys 41, 65-93 (2021).**
- **HQSS. arXiv: 2012.09813 (2021).**

TABLE V: The lowest-lying eigenenergies of $c\bar{c}s\bar{s}$ tetraquarks with $IJ^P = 01^+$ in the QDCSM.

Index	Channel	Threshold	E_{sc}	E_{cc}	E_{mix}
1	$\eta_{c\bar{c}}\phi$	4004	4006	4006	4006
2	$D_s\bar{D}_s^*$	4080	4082		
3	$J/\psi\eta_{s\bar{s}}$	4055	4057		
4	$D_s^*\bar{D}_s$	4080	4082		
5	$J/\psi\phi$	4117	4119		
6	$D_s^*\bar{D}_s^*$	4224	4226		
7	$(cs)(\bar{c}\bar{s})$		4375	4327	
8	$(cs)(\bar{c}\bar{s})$		4419		
9	$(cs)(\bar{c}\bar{s})$		4375		
10	$(cs)(\bar{c}\bar{s})$		4419		
11	$(cs)(\bar{c}\bar{s})$		4413		
12	$(cs)(\bar{c}\bar{s})$		4352		

TABLE VI: The lowest-lying eigenenergies of $c\bar{c}s\bar{s}$ tetraquarks with $IJ^P = 02^+$ in the QDCSM.

Index	Channel	Threshold	E_{sc}	E_{cc}	E_{mix}
1	$J/\psi\phi$	4117	4122	4121	4119
2	$D_s^*\bar{D}_s^*$	4224	4229		
3	$(cs)(\bar{c}\bar{s})$		4429	4420	
4	$(cs)(\bar{c}\bar{s})$		4437		

- **No bound state exists in the meson-meson structure or diquark-antidiquark structure.**

- **There is no bound state when considering all channel coupling.**

➢ Resonance states

- A real scaling method

Phys. 75, 2465 (1981)

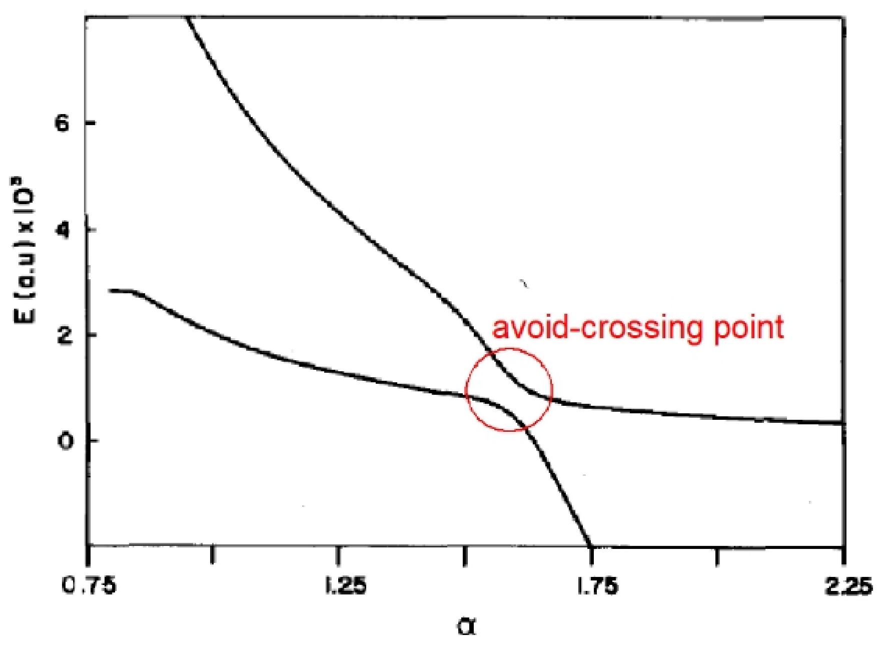
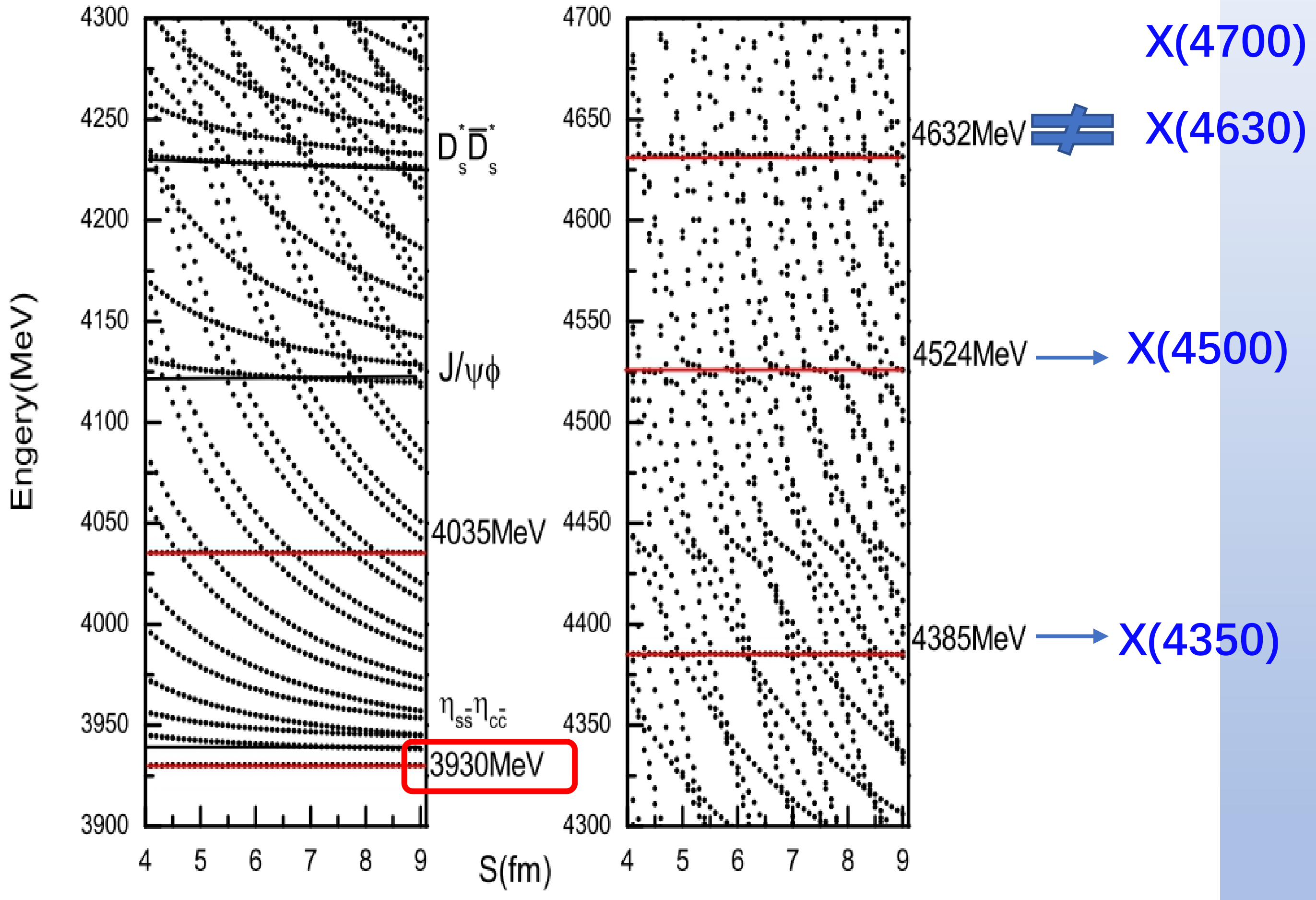


FIG. 2. Stabilization graph for the resonance.

- With the increase of the distance between two clusters , the energy per lane will tend to the threshold value.
- The resonance state will not be affected by the boundary at a large distance and should be stay stable.
- The resonance will show as an avoid-crossing structure like the fig.



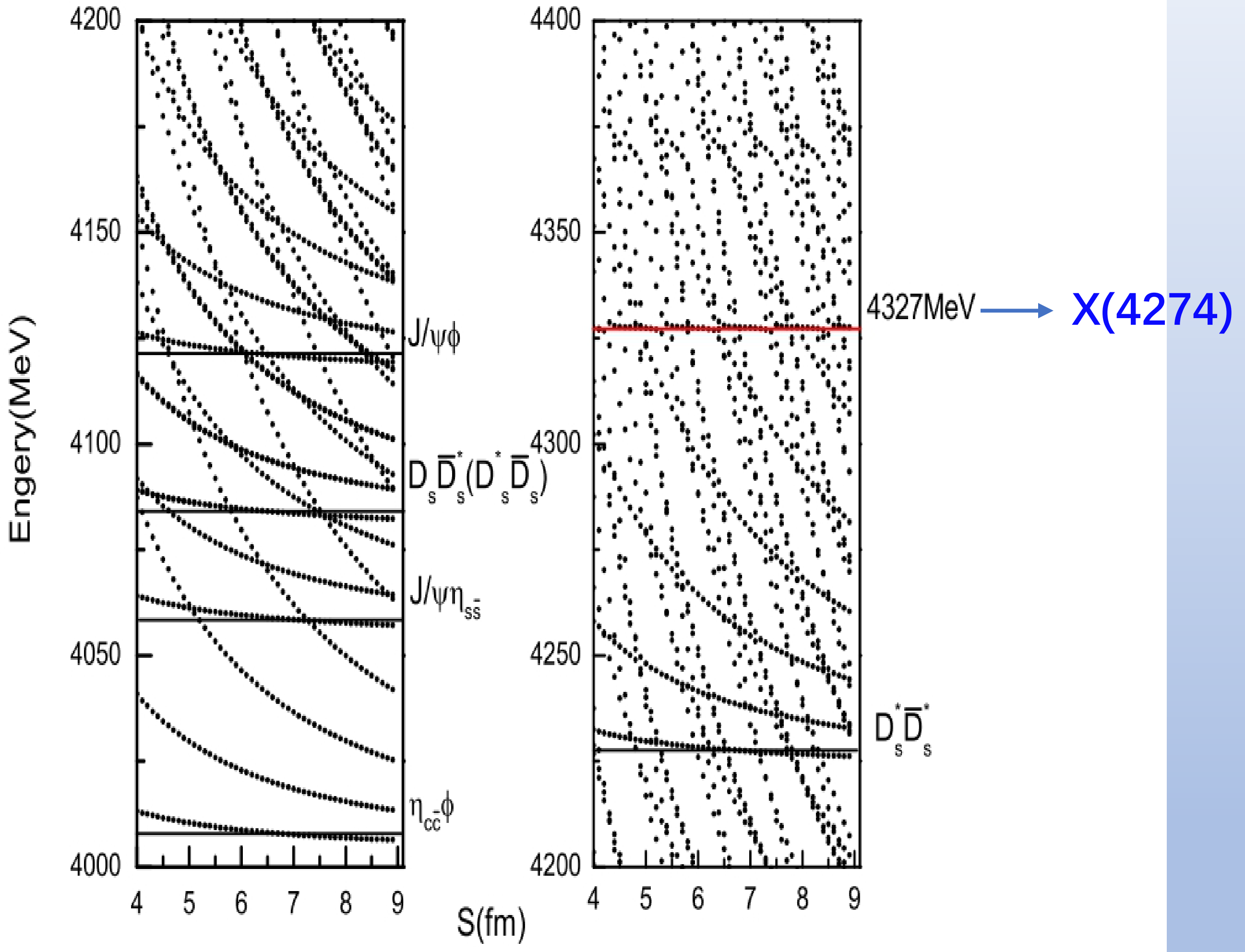
The stabilization plots of the energies of the $c\bar{c}s\bar{s}$ with $IJ^P = 00^+$ in QDCSM.

- The bound state located at the energy of 3930 MeV
- The four resonance states with the energy around 4035 MeV, 4385MeV, 4524MeV, 4632MeV.
- X(4380) → X(4350) : a compact tetraquark .

  Belle Collaboration Phys. Rev. Lett. 104, 112004 (2010).

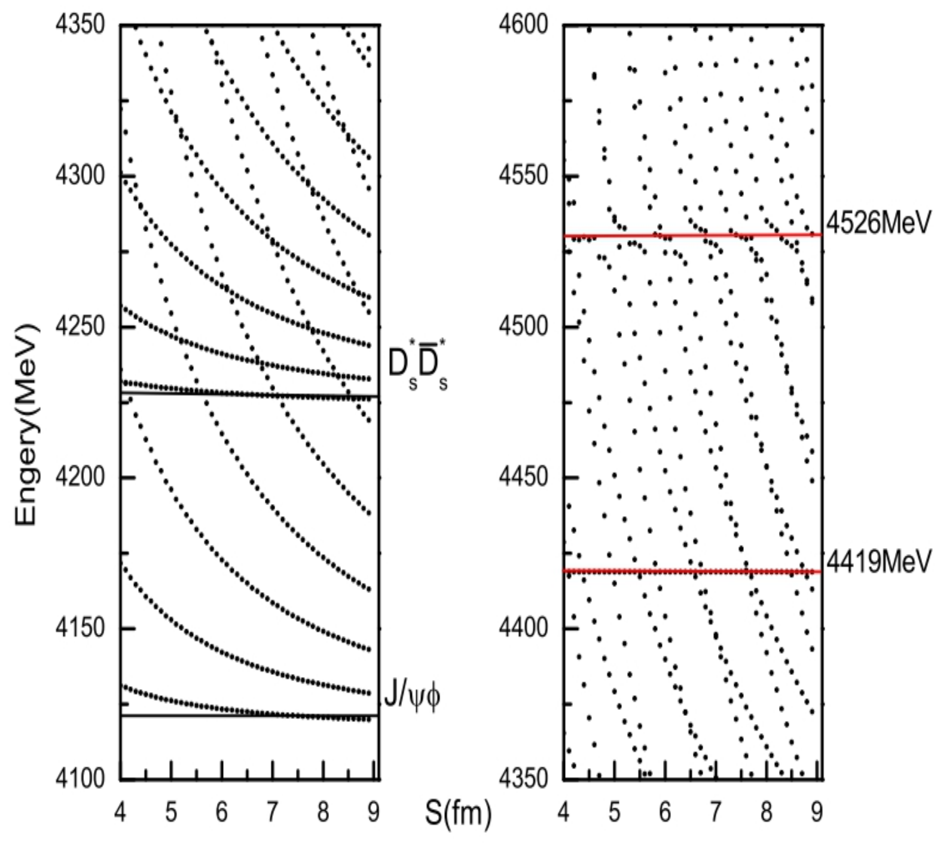
- X(4524) → X(4500): a compact tetraquark.

  LHCb Collaboration Phys. Rev. D 95, 012002 (2017).



The stabilization plots of the energies of the $c\bar{c}s\bar{s}$ with $IJ^P = 01^+$ in QDCSM.

- A resonant state is obtained at the energy around 4327 MeV.
- The resonant state around 4327 MeV is explained as the X(4274).
- This result similar to Phys. Rev. D 83, 034010 (2011).



The stabilization plots of the energies of the $c\bar{c}s\bar{s}$ with $IJ^P = 02^+$ in QDCSM.

- The two new resonance states : X(4526)

  X(4419)

Summary

- A bound state is acquired
  a bound molecular $D_s\bar{D}_s\ (IJ^P = 00^+)$ $\chi_{c0}(3930)$

- Several resonant states are obtained .
  $IJ^P = 00^+$ 4035MeV
  4385MeV → X(4350)
  4524MeV → X(4500)
  4632MeV → X(4700)
  $IJ^P = 01^+$ 4327MeV → X(4274)
  the compact tetraquark state
  $IJ^P = 02^+$ 4419MeV, 4526MeV

Thanks for your attention!