The Discussion of the Singularities in Triangle and Box Single Loop

Jia-Jun Wu

Collaborators: Qi Huang, Chao-Wei Shen

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中国科学院大学
University of Chinese Academy of Sciences

Content

- Background of Triangle Singularity
- What is Triangle Singularity ?
- How to search Triangle Singularity?
- How about Box Singularity
- Summary

Background of Triangle Singularity

L. D. Landau, Nucl. Phys. 13, no.1, 181-192 (1960)
S. Coleman, R.E. Norton, Nuovo Cim. **1965**, 38, 438 - 442,
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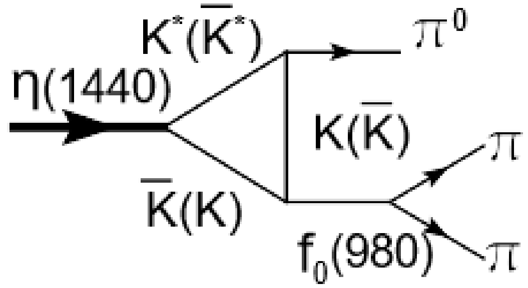

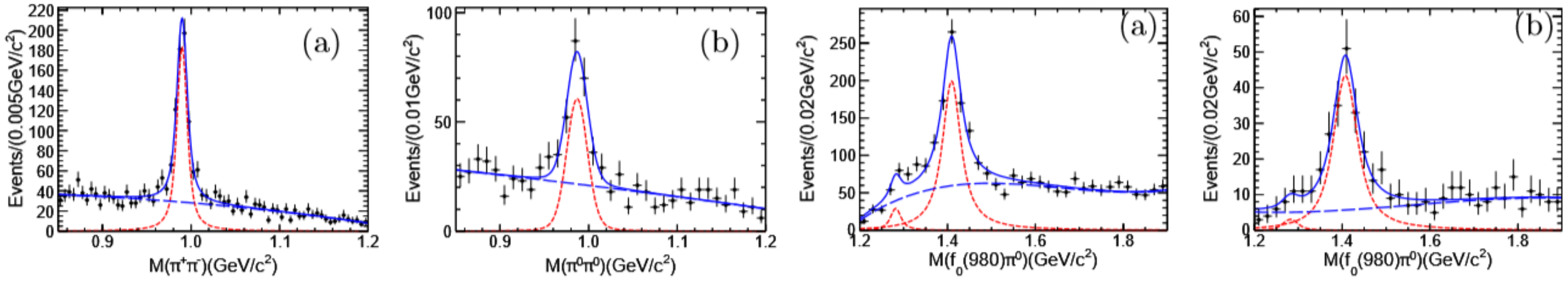

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PPNP 2020, 112, 103757

Structures	Processes	Loops	I/F	Refs.
2.1 GeV [141]	$\gamma p^+ \to N^*(2030) \to K^+\Lambda(1405)$	$K^*\Sigma\pi$	I	[142]
2.1 GeV	$\pi^- p^+ \to K^0\Lambda(1405)$, $pp \to pK^+\Lambda(1405)$	$K^*\Sigma\pi$	I	[143]
1.88 GeV	$\Lambda_c^+ \to \pi^+\pi^0\pi\Sigma$	$\bar{K}^*N\bar{K}$	I	[144, 145][a]
$N(1700)$ [10]	$N(1700) \to \pi\Delta$	$\rho N\pi$	I	[146]
$N(1875)$ [10]	$N(1875) \to \pi N(1535)$	$\Sigma^*K\Lambda$	I	[147]
$\Delta(1700)$ [148–150]	$\gamma p \to \Delta(1700) \to \pi N(1535) \to p\pi^0\eta$	$\Delta\eta p$	I	[151]
2.2 GeV [152]	$\Lambda_c^+ \to \pi^0\phi p$	$\Sigma^*K^*\Lambda$	F	[153]
1.66 GeV [154, 155]	$\Lambda_c^+ \to \pi^+K^-p$	$a_0\Lambda\eta, \Sigma^*\eta\Lambda$	F	[156]
$P_c(4450)$ [35]	$\Lambda_b^0 \to K^-J/\psi p$	$\Lambda(1890)\chi_{c1}p$	F	[157–160][b]
		$N(1900)\chi_{c1}p$	F	[159]
peaks relevant for P_c	$\Lambda_b^0 \to K^-J/\psi p$	$\bar{D}_{sJ}\Lambda_c^{(*)}\bar{D}^{(*)}$	F	[36, 158]

Structures	Processes	Loops	I/F	Refs.
$\rho(1480)$ [78, 79]	$\pi^-p \to \phi\pi^0 n$	$K^*\bar{K}K$	I	[80, 81]
$\eta(1405/1475)$ [82–86]	$\eta(1405/1475) \to \pi f_0$	$K^*\bar{K}K$	I	[87–91][a,b]
$f_1(1420)$ [92]	$f_1(1285) \to \pi a_0/\pi f_0$	$K^*\bar{K}K$	I	[89, 93–95][b]
$a_1(1420)$ [96, 97]	$a_1(1260) \to f_0\pi \to 3\pi$	$K^*\bar{K}K$	I	[97–99]
1.4 GeV [100]	$J/\psi \to \phi\pi^0\eta/\phi\pi^0\pi^0$	$K^*\bar{K}K$	I	[101][b]
1.42 GeV	$B^- \to D^{*0}\pi^- f_0(a_0), \tau \to \nu_\tau\pi^- f_0(a_0)$	$K^*\bar{K}K$	I	[102, 103]
	$D_s^+ \to \pi^+\pi^0 f_0(a_0), \bar{B}_s^0 \to J/\psi\pi^0 f_0(a_0)$	$K^*\bar{K}K$	I	[104, 105]
$f_2(1810)$ [10]	$f_2(1640) \to \pi\pi\rho$	$K^*\bar{K}^*K$	I	[106]
1.65 GeV	$\tau \to \nu_\tau\pi^- f_1(1285)$	$K^*\bar{K}^*K$	I	[107]
1515 MeV	$J/\psi \to K^+K^- f_0\,(a_0)$	$\phi\bar{K}K$	I	[108]
2.85 GeV, 3.0 GeV	$B^- \to K^-\pi^- D_{s0}^*/K^-\pi^- D_{s1}$	$K^{*0}D^{(*)0}K^+$	I	[109, 110]
5.78 GeV	$B_c^+ \to \pi^0\pi^+ B_s^0$	$\bar{K}^{*0}B^+\bar{K}$	F	[111]
[4.01, 4.02] GeV	$[\bar{D}^{*0}D^{*0}] \to \gamma X$	$D^{*0}\bar{D}^{*0}D^0$	I	[112]
4015 MeV	$e^+e^- \to \gamma X$	$D^{*0}\bar{D}^{*0}D^0$	I	[113, 114]
4015 MeV	$B \to KX\pi$, $pp/p\bar{p} \to X\pi$+anything	$D^{*0}\bar{D}^{*0}D^0$	I	[115, 116]
$\Upsilon(11020)$ [117, 118]	$e^+e^- \to Z_b\pi$	$B_1(5721)\bar{B}B^*$	I	[119, 120]
3.73 GeV	$X \to \pi^0\pi^+\pi^-$	$D^{*0}\bar{D}^0D^0$	F	[121]
[4.22, 4.24] GeV	$e^+e^- \to \gamma J/\psi\phi/\pi^0 J/\psi\eta$	$D_{s0(s1)}^*\bar{D}_s^{(*)}D_s^{(*)}$	F	[122]
[4.08, 4.09] GeV	$e^+e^- \to \pi^0 J/\psi\eta$	$D_{s0(s1)}^*\bar{D}_s^{(*)}D_s^{(*)}$	F	[122]
$Z_c(3900)$ [31, 32]	$e^+e^- \to J/\psi\pi^+\pi^-$	$D_1\bar{D}D^*$	F	[119, 123–127][c]
		$D_0^*(2400)\bar{D}^*D$	F	[128, 129]
$Z_c(4020, 4030)$ [33, 130]	$e^+e^- \to \pi^+\pi^- h_c(\psi')$	$D_{1(2)}\bar{D}^{(*)}D^{(*)}$	F	[125]
$X(4700)$ [131, 132]	$B^+ \to K^+J/\psi\phi$	$K_1(1650)\psi'\phi$	F	[133]
$Z_c(4430)$ [30, 134]	$\bar{B}^0 \to K^-\pi^+J/\psi$	$\bar{K}^{*0}\psi(4260)\pi^+$	F	[135]
$Z_c(4200)$ [136, 137]	$\bar{B}^0 \to K^-\pi^+\psi(2S)$	$\bar{K}_2^*\psi(3770)\pi^+$	F	[135]
	$\Lambda_b^0 \to p\pi^- J/\psi$	$N^*\psi(3770)\pi^-$	F	[135]
$X(4050)^\pm$ [138]	$\bar{B}^0 \to K^-\pi^+\chi_{c1}$	$\bar{K}^{*0}X\pi^+$	F	[139]
$X(4250)^\pm$ [138]	$\bar{B}^0 \to K^-\pi^+\chi_{c1}$	$\bar{K}_2^*\psi(3770)\pi^+$	F	[139]
$Z_b(10610)$ [34]	$e^+e^- \to \Upsilon(1S)\pi^+\pi^-$	$B_J^*\bar{B}^*B$	F	[128]

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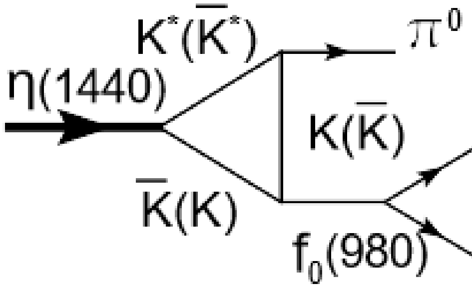
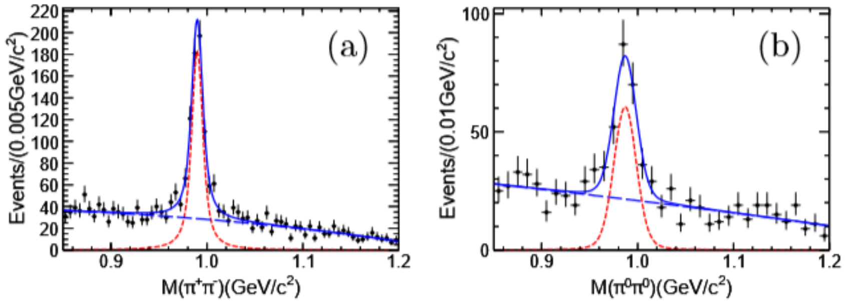
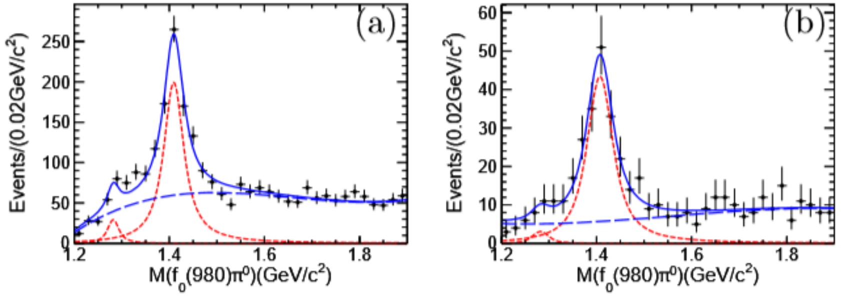
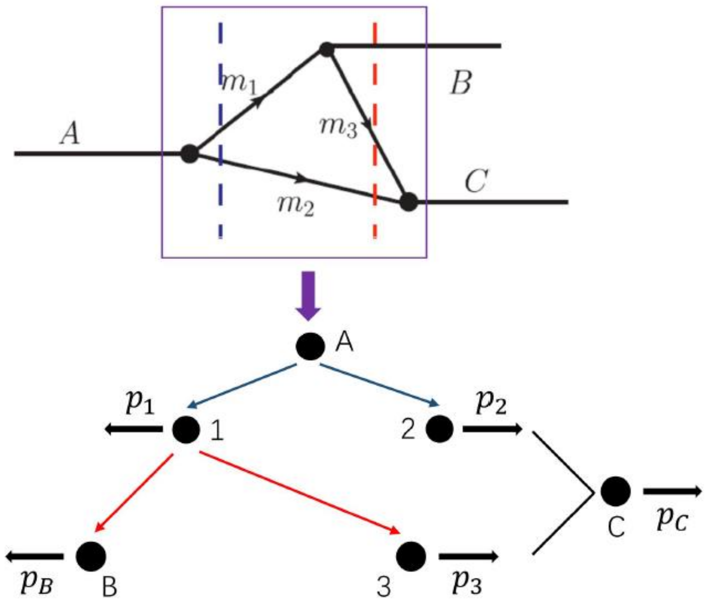






What is Triangle Singularity?



$$\int d^4q \frac{1}{(P-q)^2-m_1^2+i\epsilon}\frac{1}{q^2-m_2^2+i\epsilon}\frac{1}{(p_c-q)^2-m_3^2+i\epsilon}$$

Actually Happened Process

(1) All particles are on mass-shell;

(2) Particle 3 catch up particle 2.

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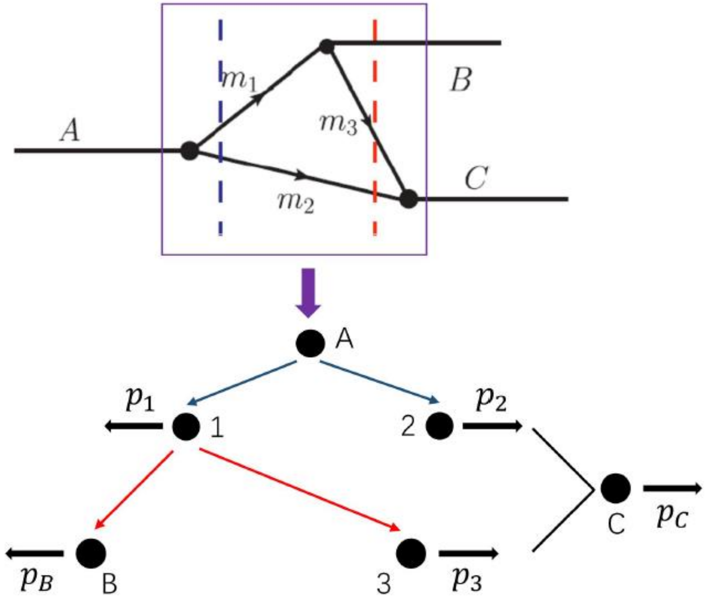

$$\int d^4q \frac{1}{(P-q)^2 - m_1^2 + i\epsilon} \frac{1}{q^2 - m_2^2 + i\epsilon} \frac{1}{(p_c - q)^2 - m_3^2 + i\epsilon}$$

$$\sim \int d^3\vec{q}\,[Res1(m_A + \omega_1(\vec{q}) - i\epsilon) + Res2(\omega_2(\vec{q}) - i\epsilon) + Res3(E_c + \omega_3(\vec{p}_c - \vec{q}) - i\epsilon)]$$

$$\sim \int d^3\vec{q}\left[\frac{1}{m_A - \omega_1(\vec{q}) - \omega_2(\vec{q}) + i\epsilon} \; \frac{1}{E_c - \omega_2(\vec{q}) - \omega_3(\vec{p}_C - \vec{q}) + i\epsilon} f(\vec{q}) + h(\vec{q})\right]$$

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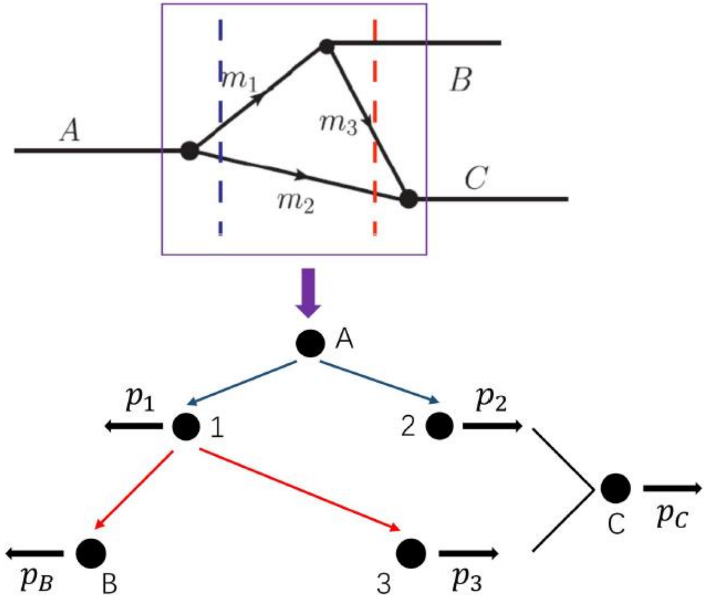
$$q_{on} = \frac{\sqrt{(m_A^2 - (m_1 + m_2)^2)(m_A^2 - (m_1 - m_2)^2)}}{2m_A}$$



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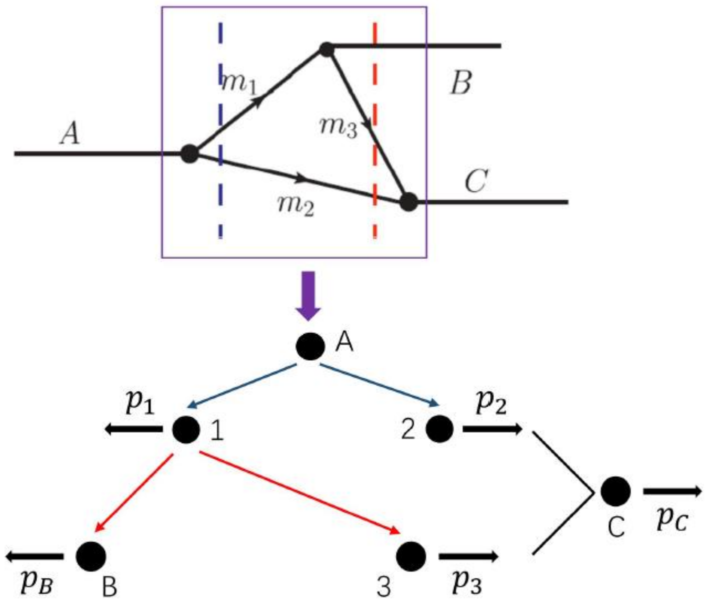

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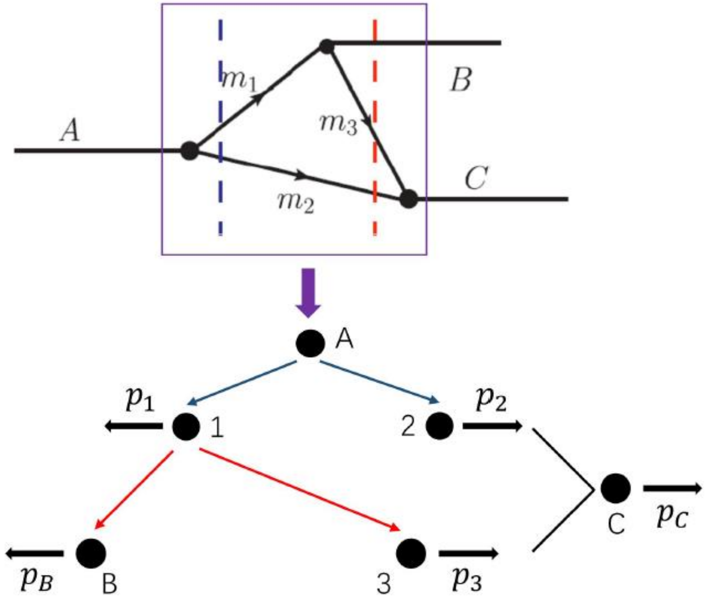

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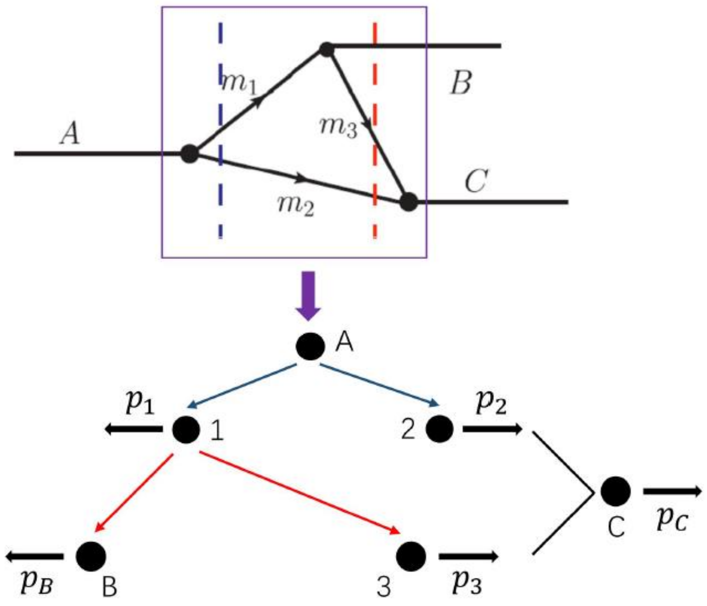

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$$\sim \int d^3\vec{q}\,[Res1(m_A+\omega_1(\vec{q})-i\epsilon)+Res2(\omega_2(\vec{q})-i\epsilon)+Res3(E_c+\omega_3(\vec{p}_c-\vec{q})-i\epsilon)]$$

$$\sim \int d^3\vec{q}\left[\frac{1}{m_A-\omega_1(\vec{q})-\omega_2(\vec{q})+i\epsilon}\frac{1}{E_c-\omega_2(\vec{q})-\omega_3(\vec{p}_C-\vec{q})+i\epsilon}f(\vec{q})+h(\vec{q})\right]$$

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$$\left(\frac{q_{on}}{\omega_2}+\frac{q_{on}-|\vec{p}_C|(-1\text{ or }1)}{\omega_3}\right)i\epsilon'+i\epsilon=0$$

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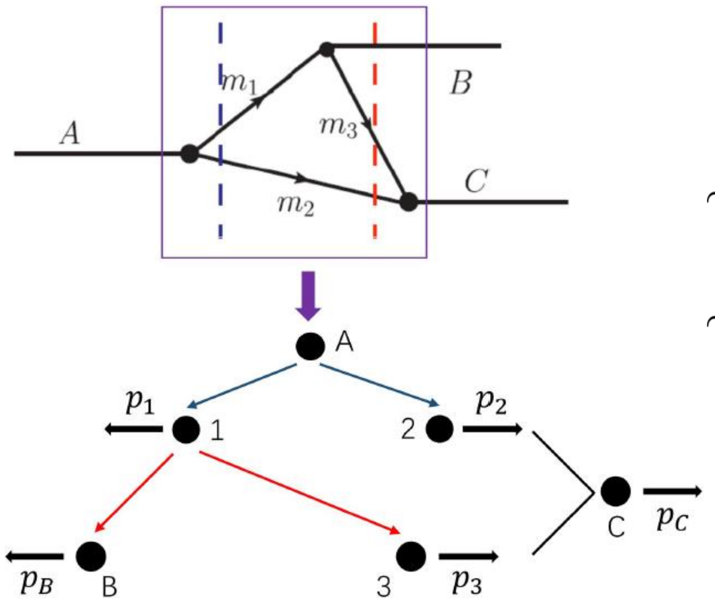

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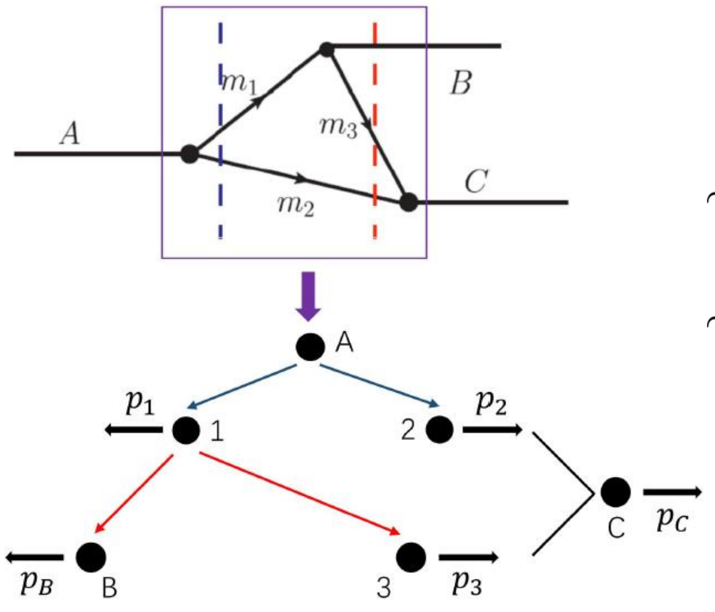

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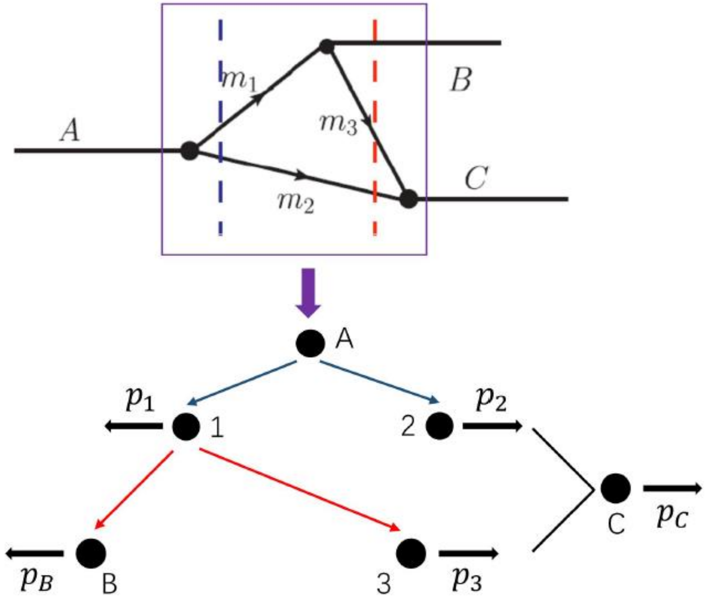
The condition equation of the six masses， but maybe two solutions.

What is Triangle Singularity?



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$$\sqrt{\left(E_C-\sqrt{q_{on}^2+m_2^2}\right)^2-m_3^2}-(|\vec{p}_C|-|q_{on}|)=0$$

The condition equation of the six masses

Why Triangle Singularity interesting ?

1. It is a pure kinematic effect
   -> Model independent
2. The effect of Loop
   -> Understand hadronic Loop contribution
3. Provide a peak structure
   -> May mixing with resonance
4. To extract the nature of hadron
   -> Study the coupling in the special point
5. ……

How to search Triangle Singularity?

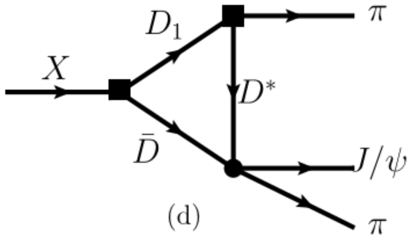
1.Threshold

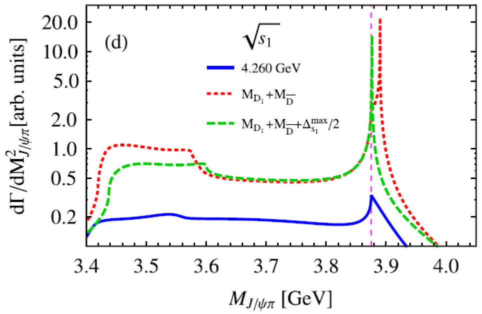
2.Width of the internal particle of the loop

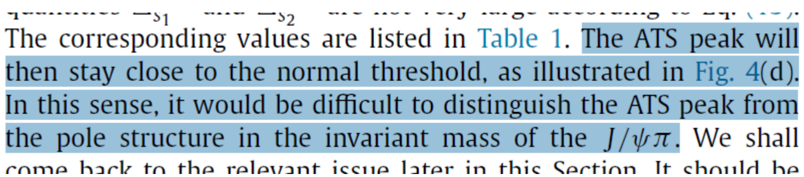
3.Unknow vertex

How to search Triangle Singularity?

1.Threshold



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X. H. Liu, M. Oka and Q. Zhao,
PLB 753, 297-302 (2016)

2.Width of the internal particle of the loop

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University of Chinese Academy of Sciences

How to search Triangle Singularity?

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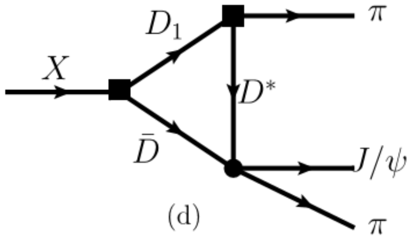

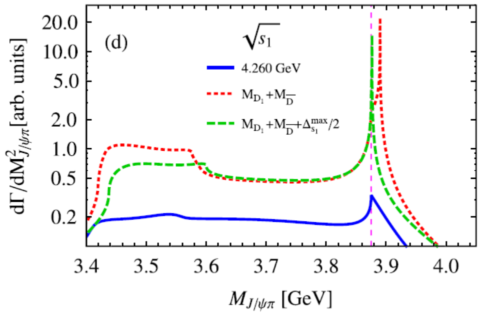

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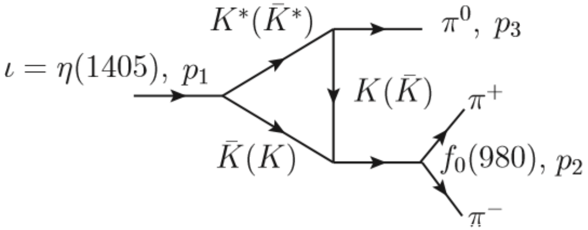

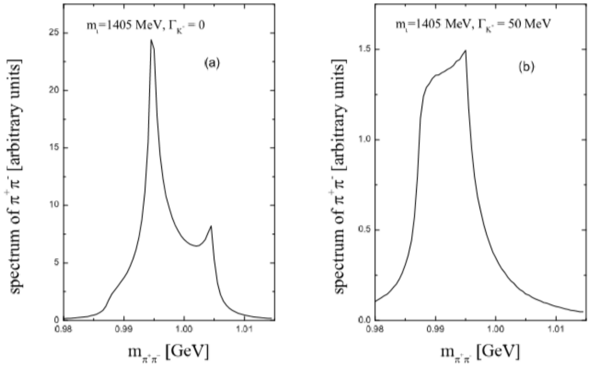

N.N. Achasov, A.A. Kozhevnikov, Sobolev IM, G.N. Shestakov *PRD* 92 (2015) 3, 036003

3.Unknow vertex

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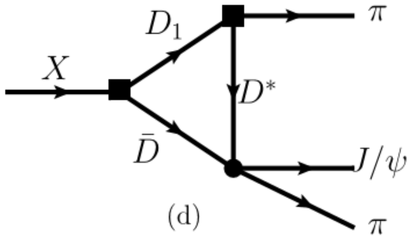

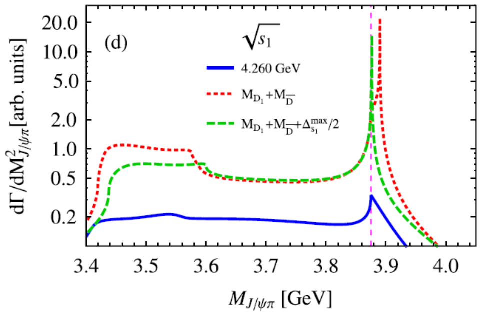

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X. H. Liu, M. Oka and Q. Zhao, PLB 753, 297-302 (2016)

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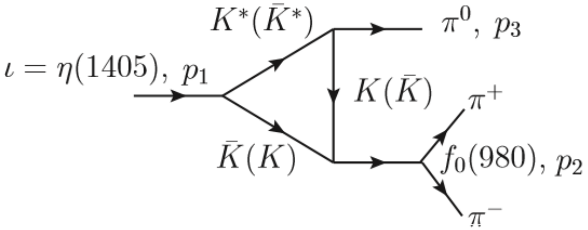
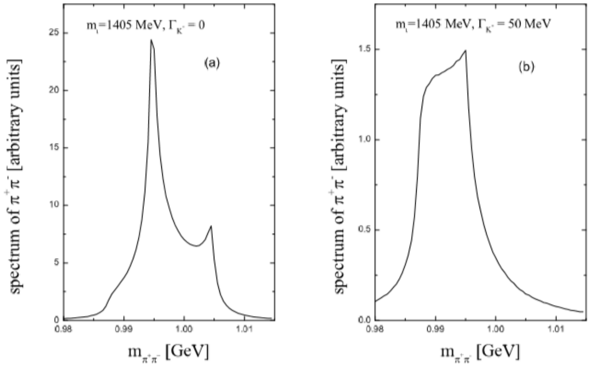
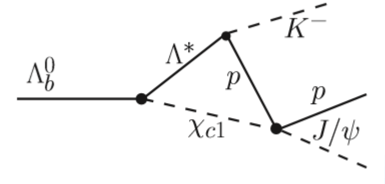


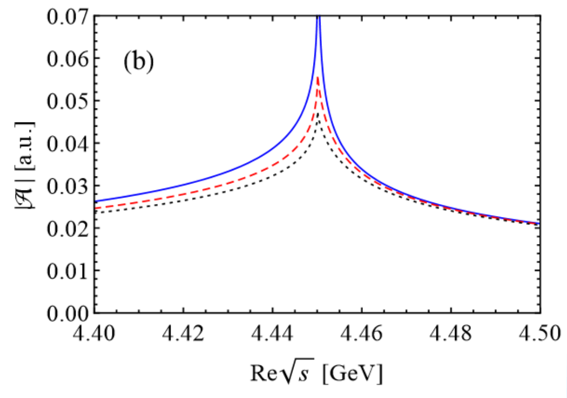


N.N. Achasov, A.A. Kozhevnikov, Sobolev IM, G.N. Shestakov *PRD* 92 (2015) 3, 036003

3.Unknow vertex

$\Lambda_b \rightarrow (c\bar{c})\Lambda^*$
$\rightarrow (c\bar{c})\Lambda\, K^-$
$\rightarrow J/\psi p K^-$





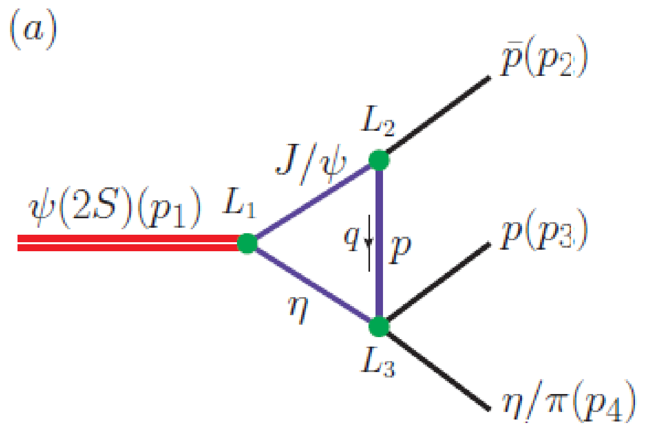
F.-K. Guo, U.-G. Meissner, W. Wang, Z. Yang, PRD92, 071502 (2015)

Our proposal

1.Threshold

2.Width of the internal particle of the loop

3.Unknow vertex



Our proposal

1.Threshold

Far away from pη threshold. Singularity point is 1.563 GeV of p η(π) invariant mass.

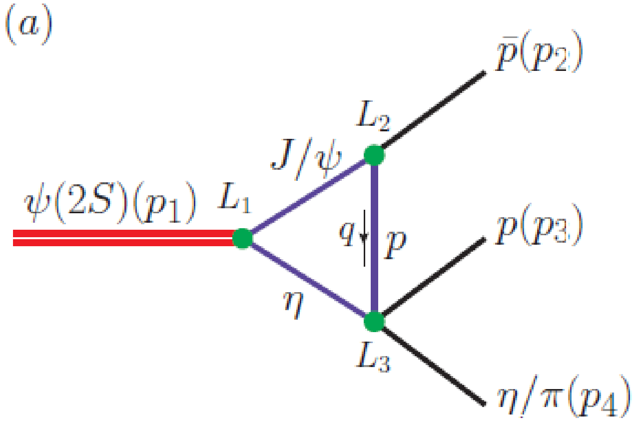
2.Width of the internal particle of the loop

All narrow internal particles, J/ψ, p, η

3.Unknow vertex

All vertices are constrained from experimental data

Our proposal



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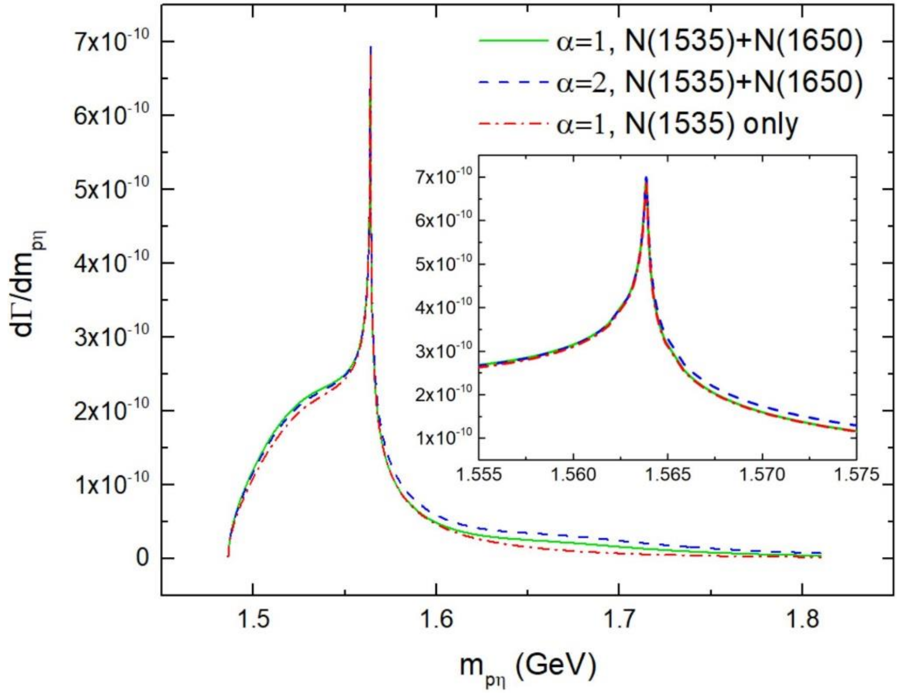

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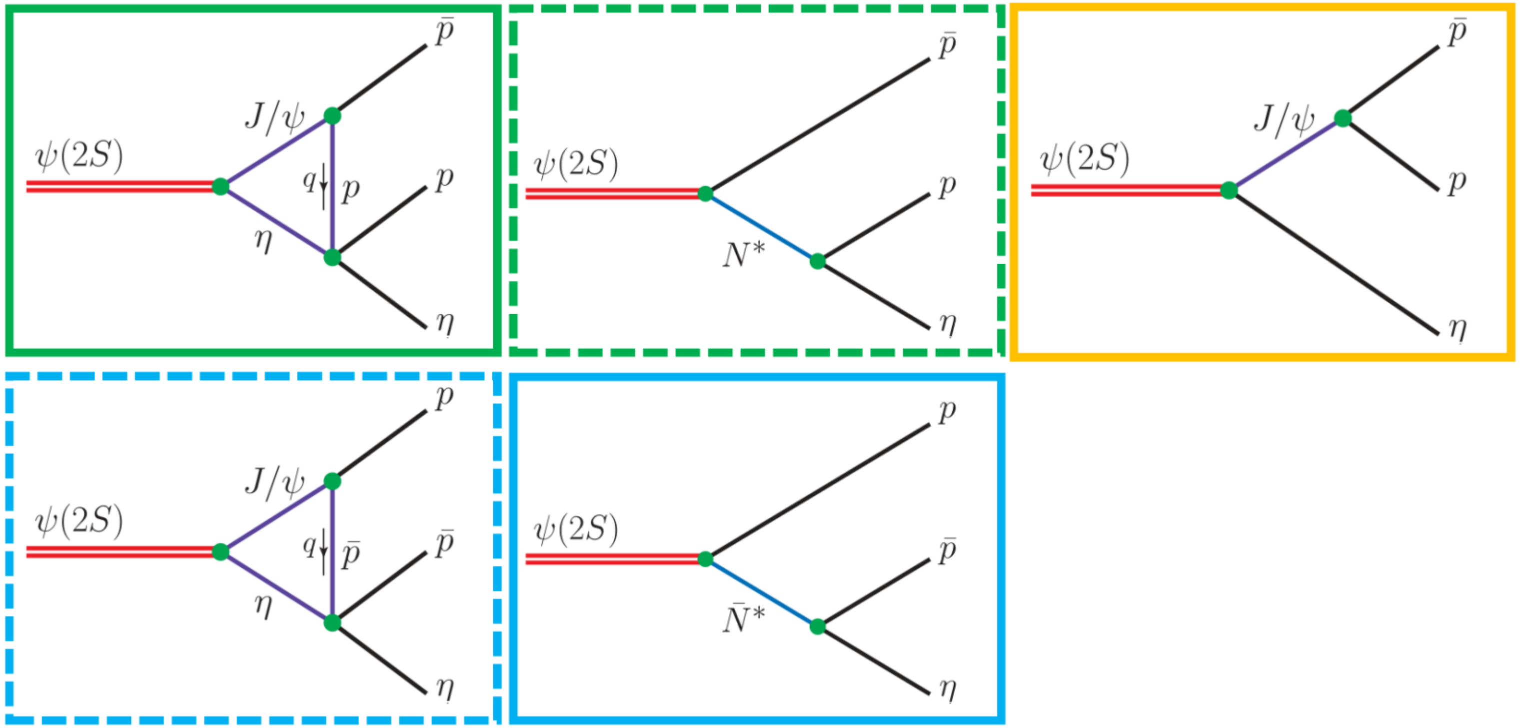
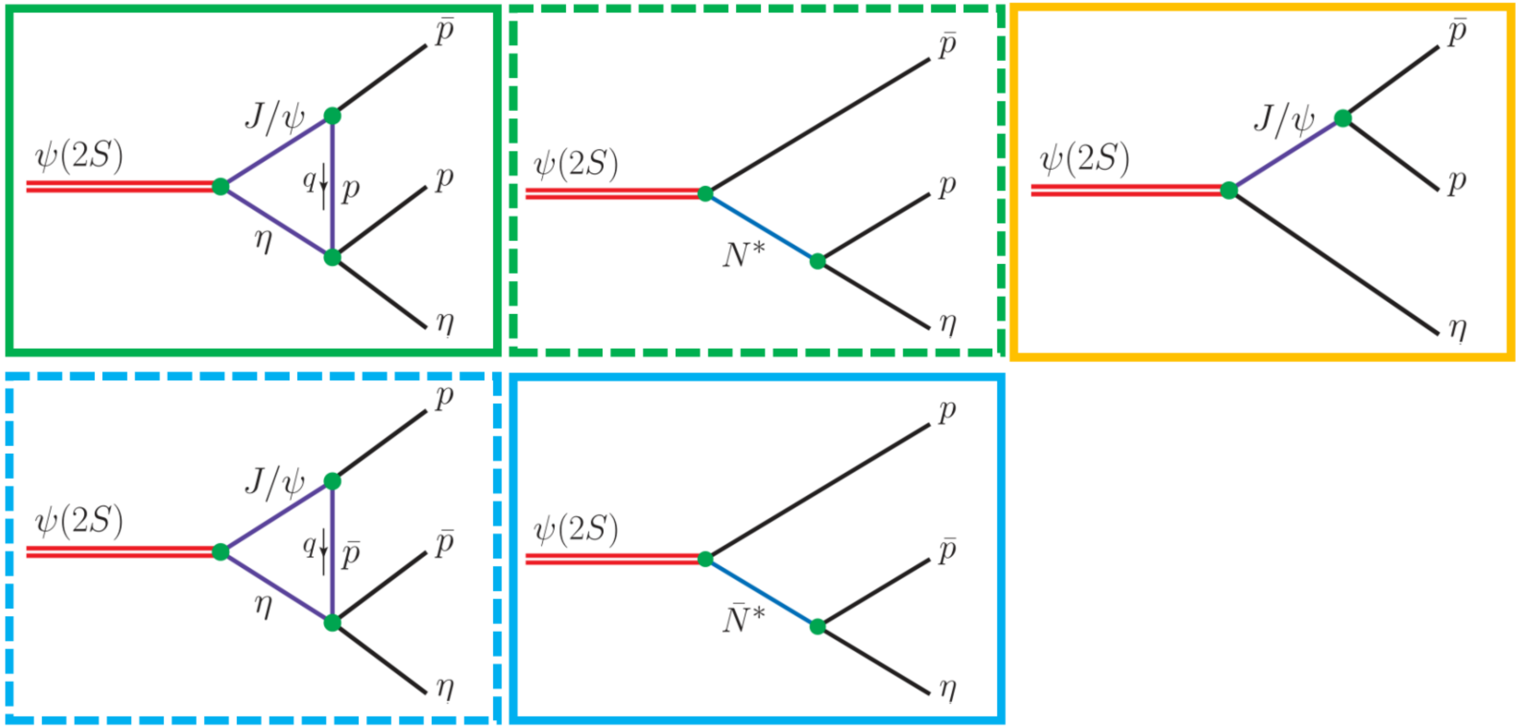
Our proposal

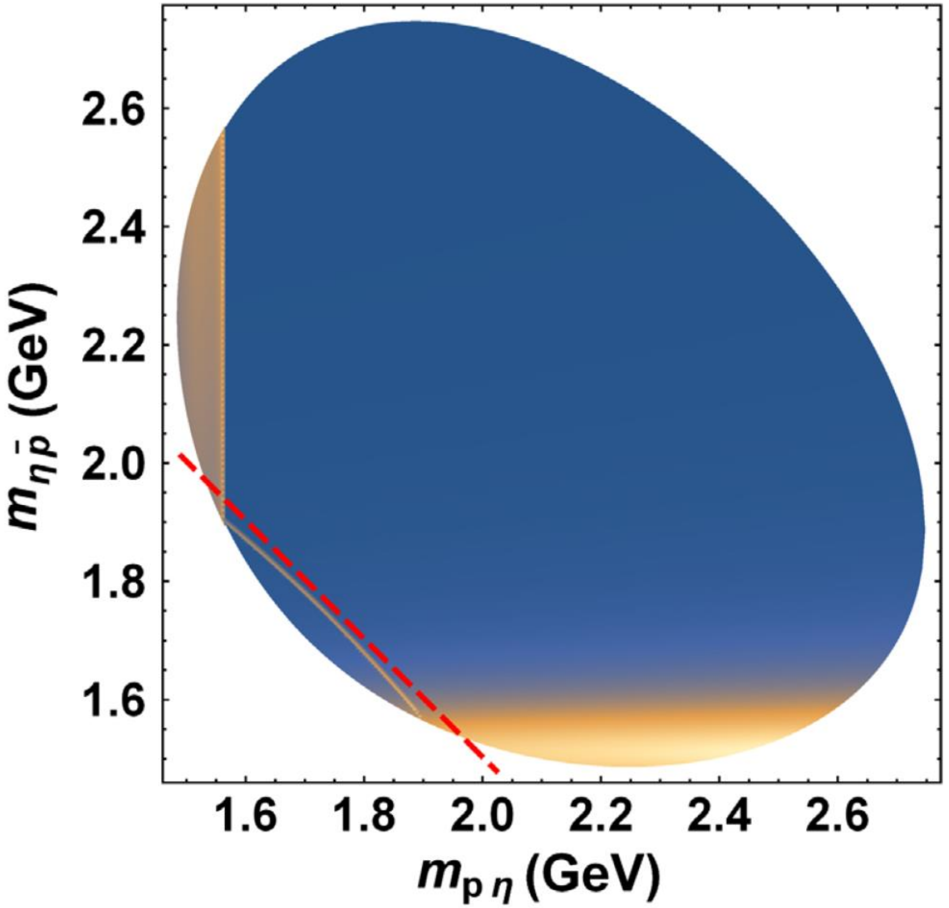
Signal + Background



Our proposal

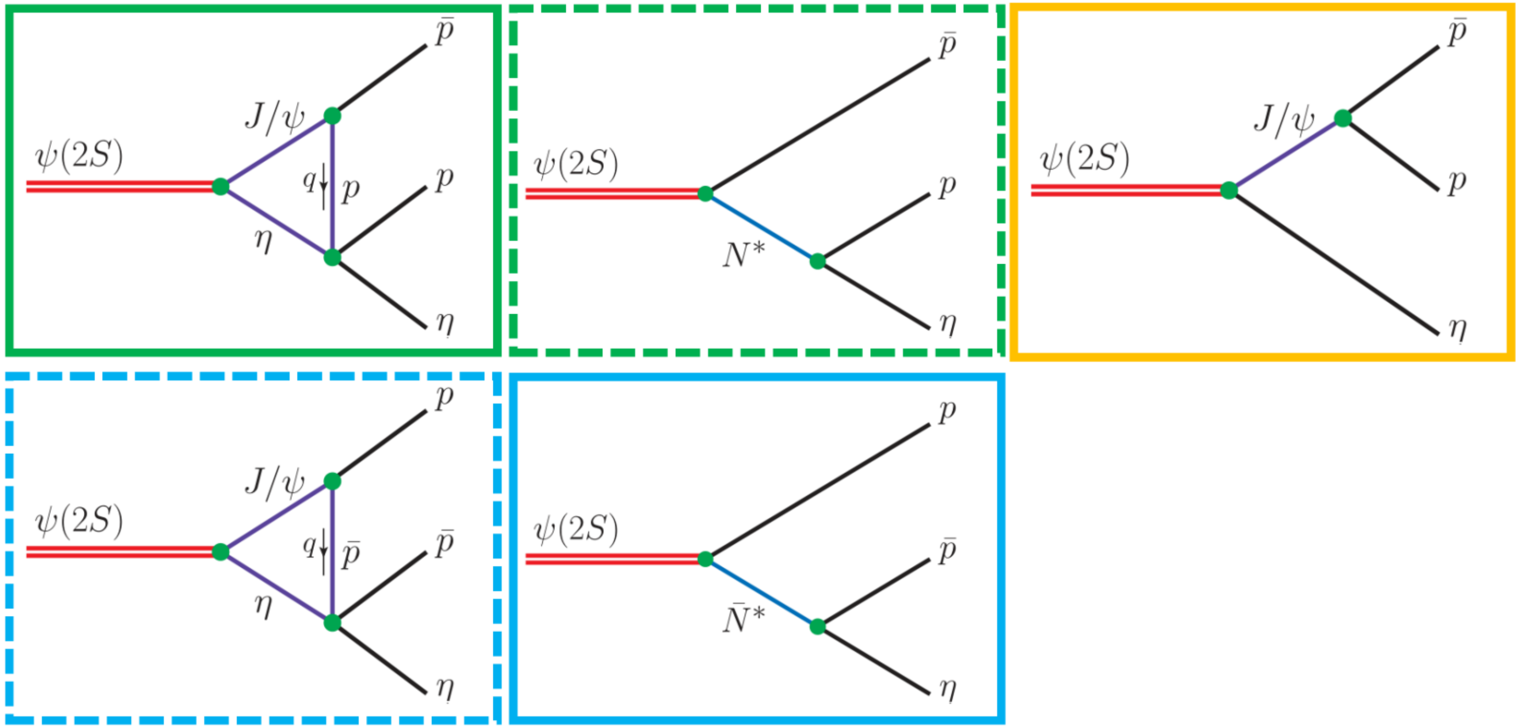
Signal + Background

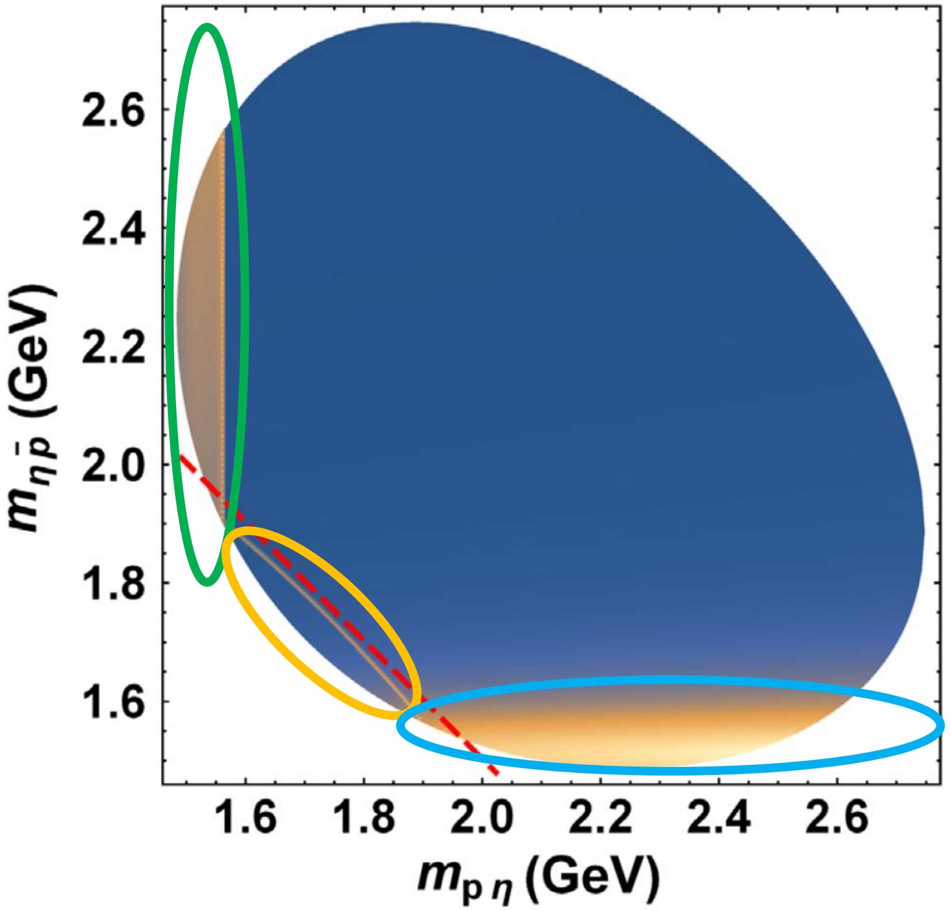




Our proposal

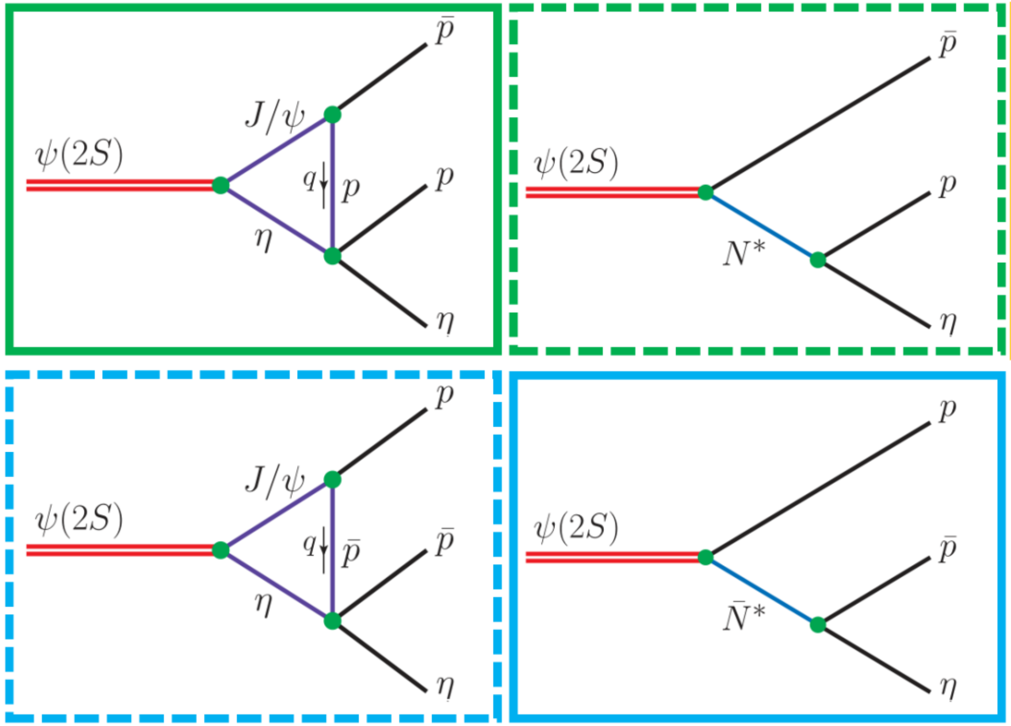
Signal + Background

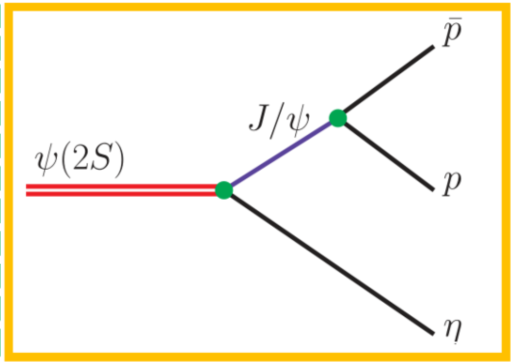




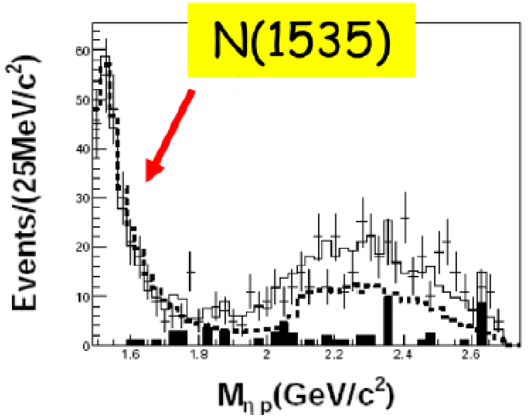
Our proposal

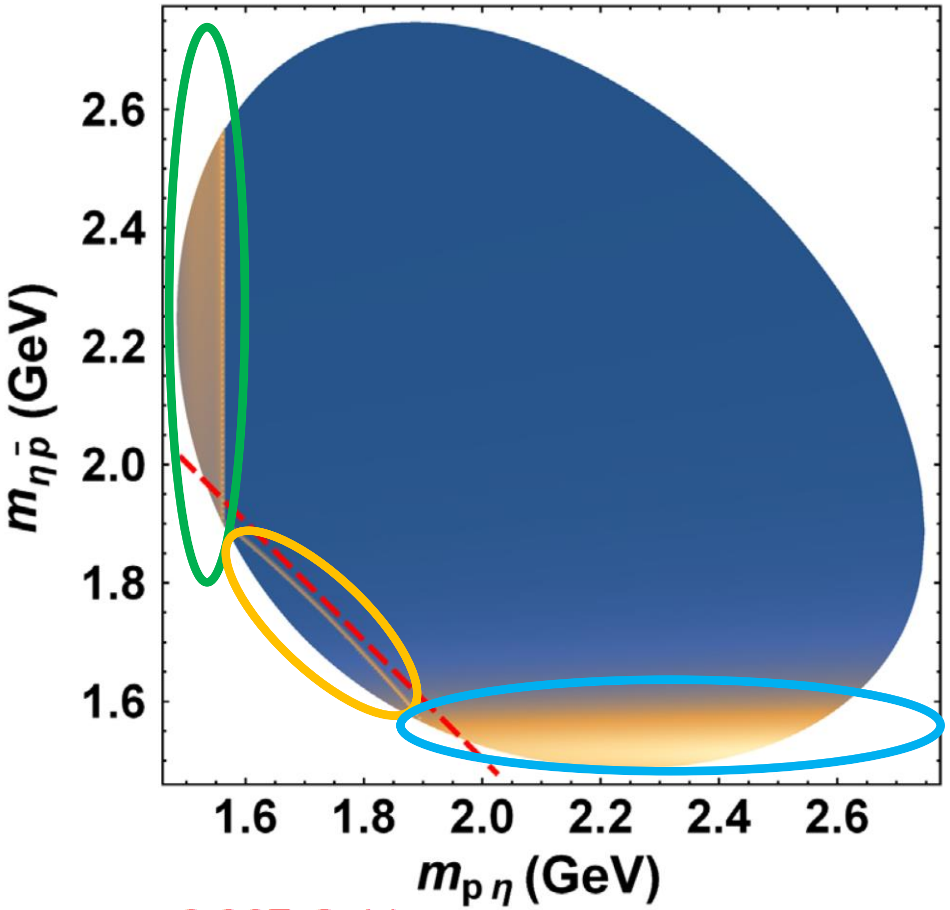
Signal + Background





Tree diagram: Dominant!
BESIII: PRD 88, 032010(2013)



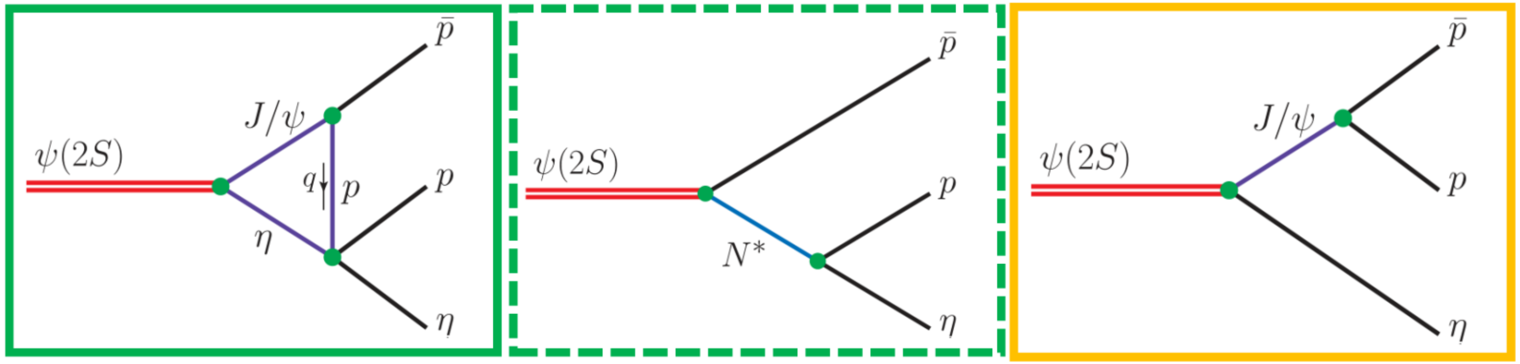


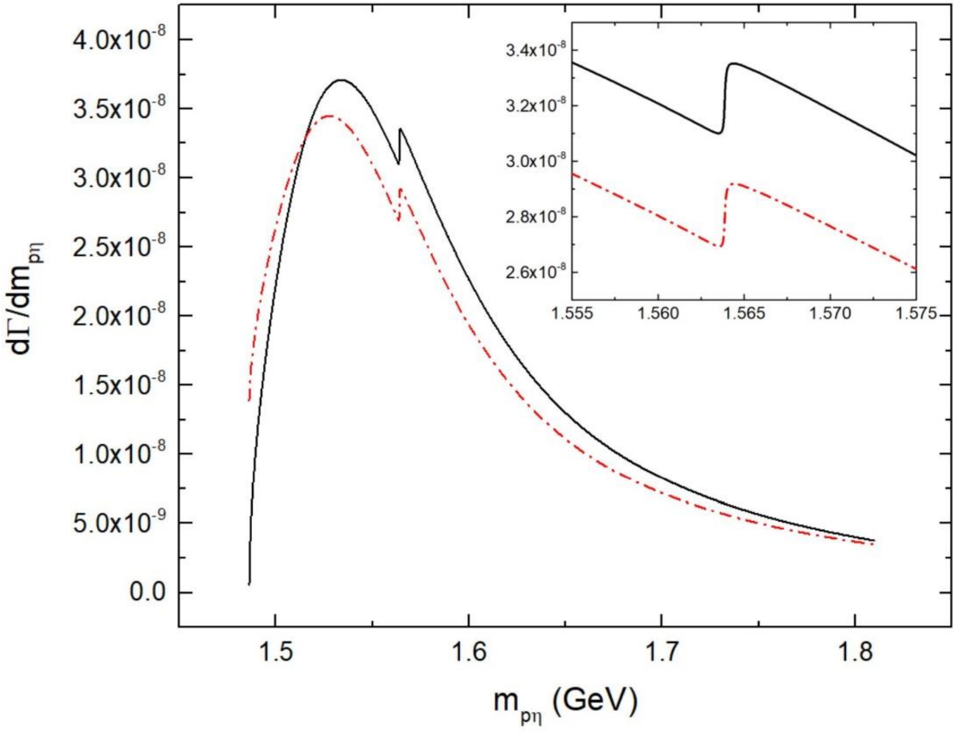
$m_{p\bar{p}} < 3.067$ GeV.

By follow BES' s cut, we can remove the contribution of J/psi-> $p\bar{p}$, at the same time we can avoid the problem from the **Schmid theorem**.

Our proposal

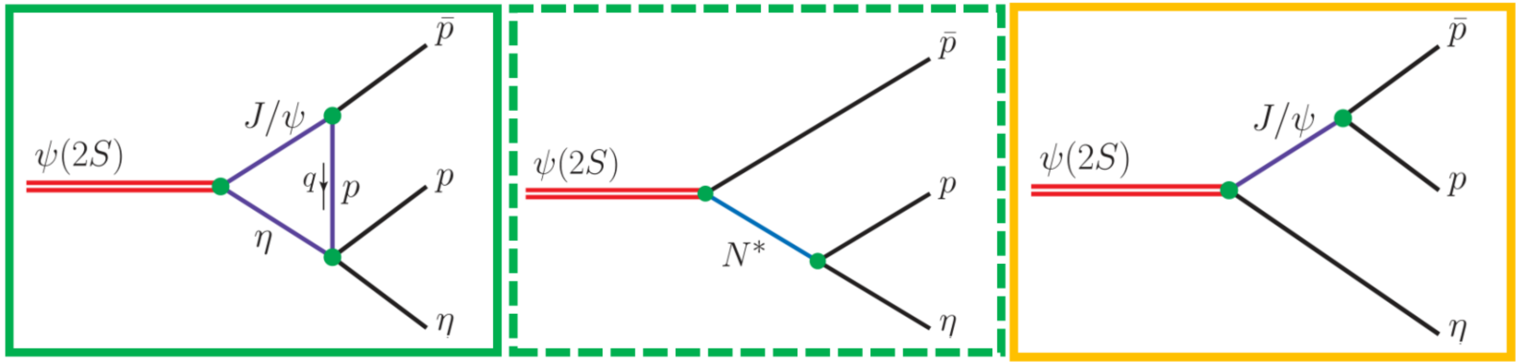
Signal + Background

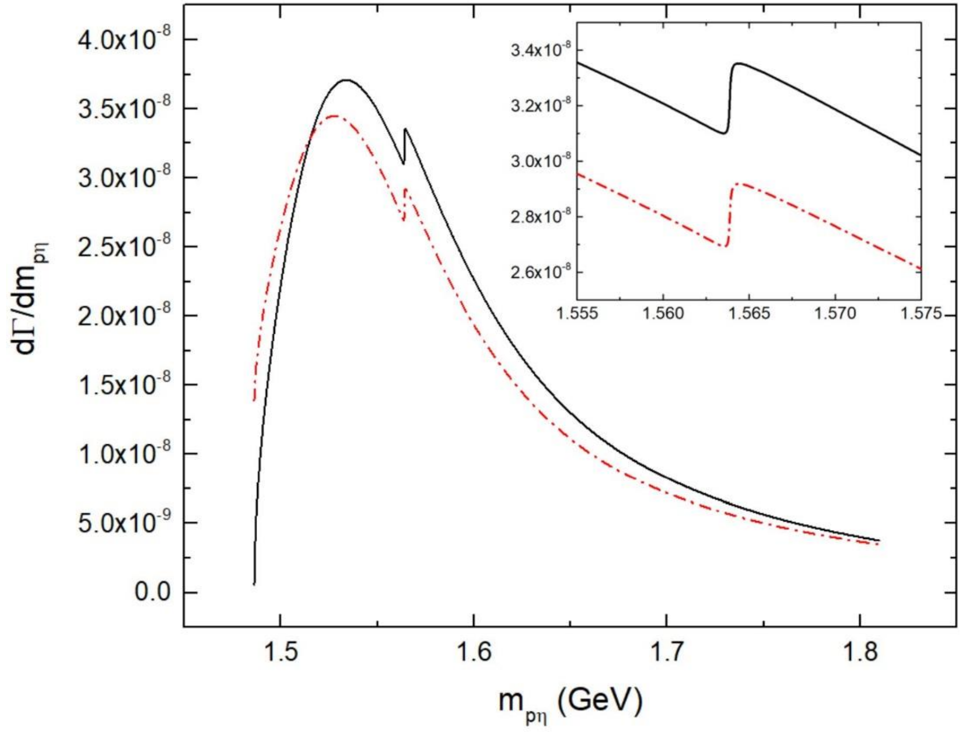




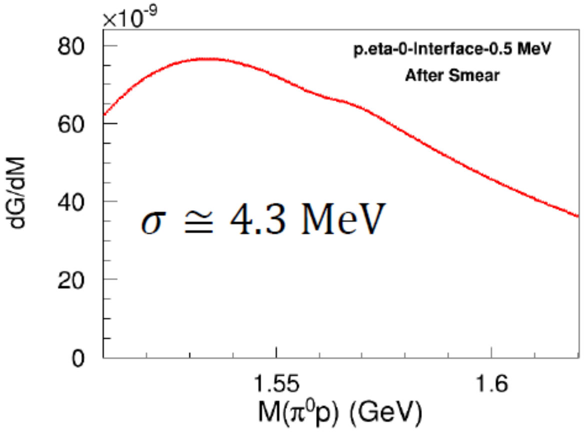
Our proposal

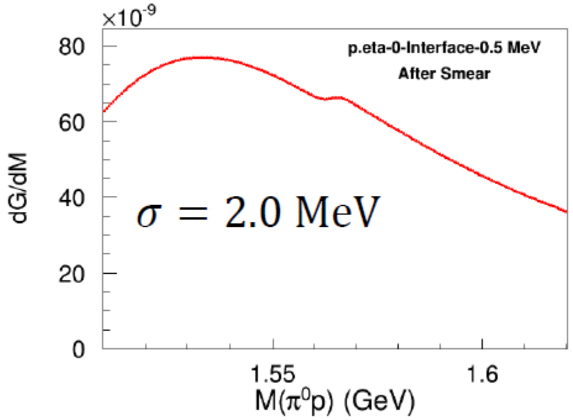
Signal + Background





Different resolution assumption

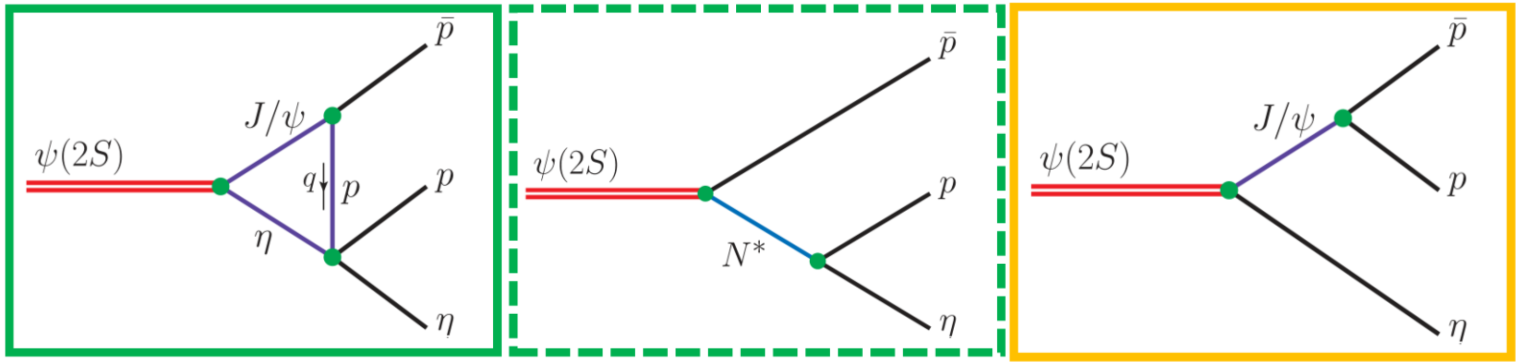




From Xiao-Rui Lyu and Zi-Yi Wang

Our proposal

Signal + Background



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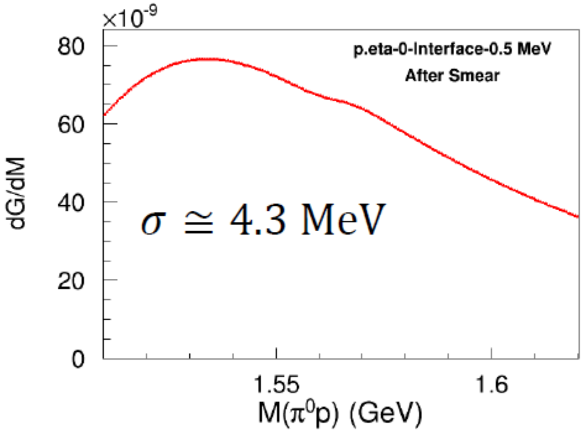
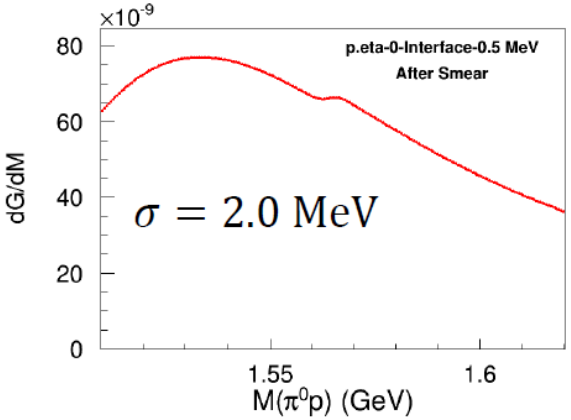
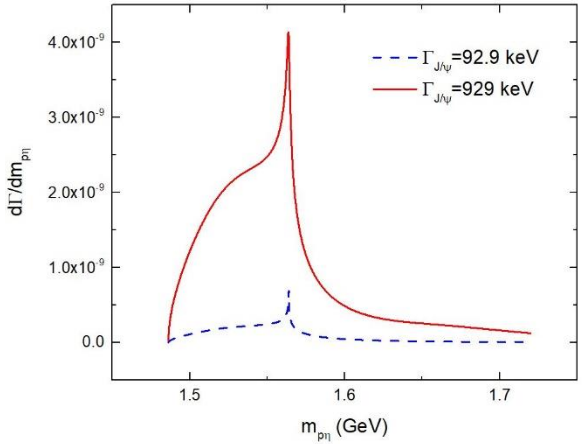
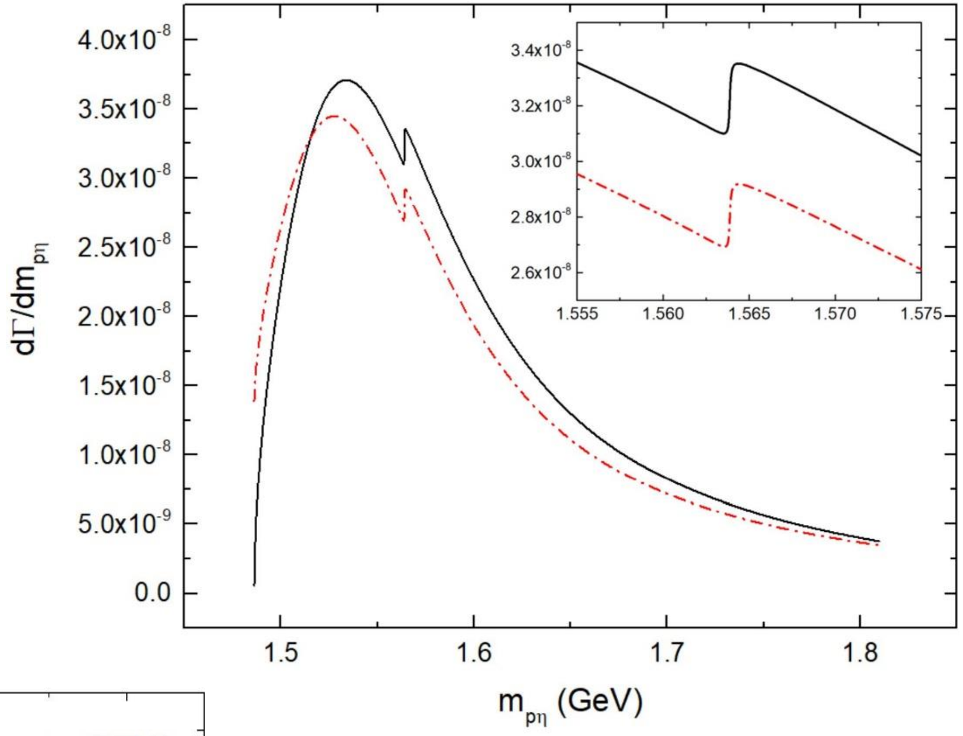








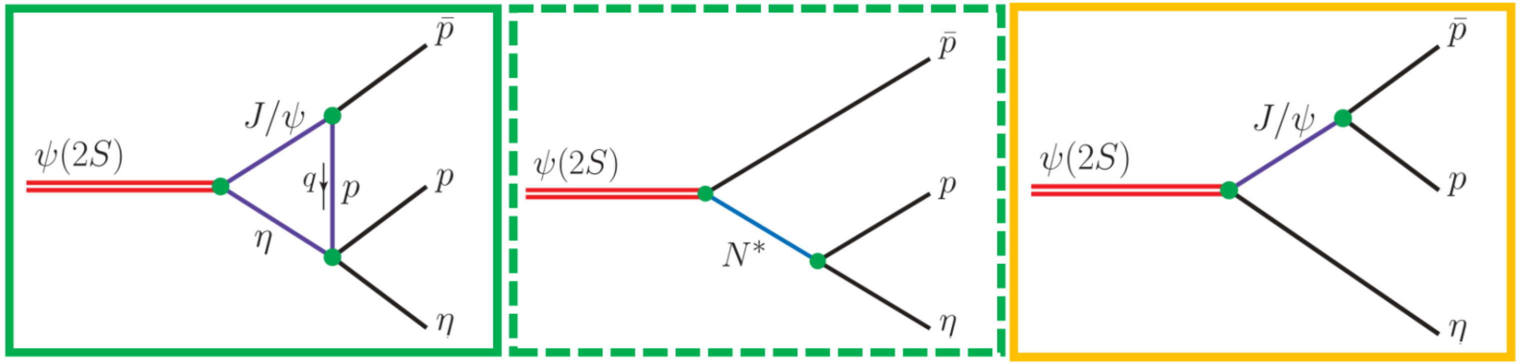
A lesson, the intermediate decay particle's width is too small !!!

1. Narrow Peak
2. Weak strength

From Xiao-Rui Lyu and Zi-Yi Wang

Our proposal

Signal + Background



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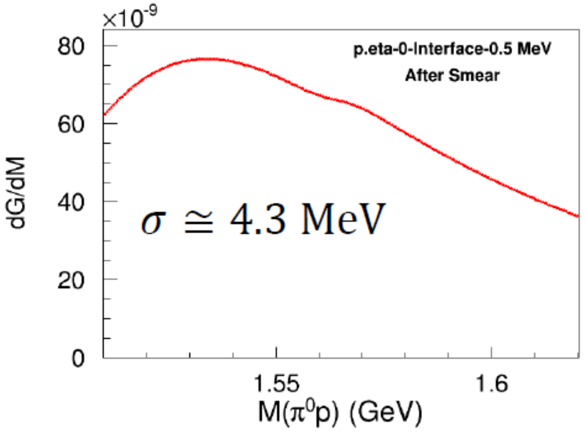
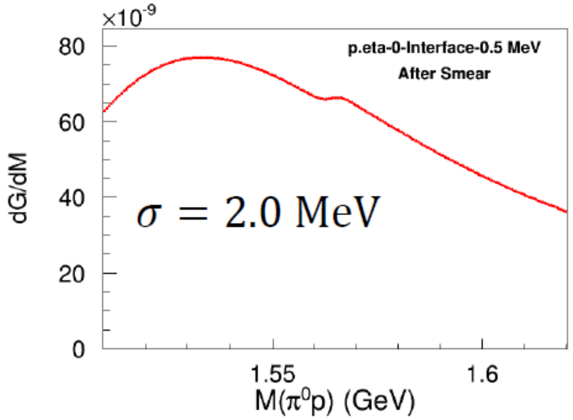
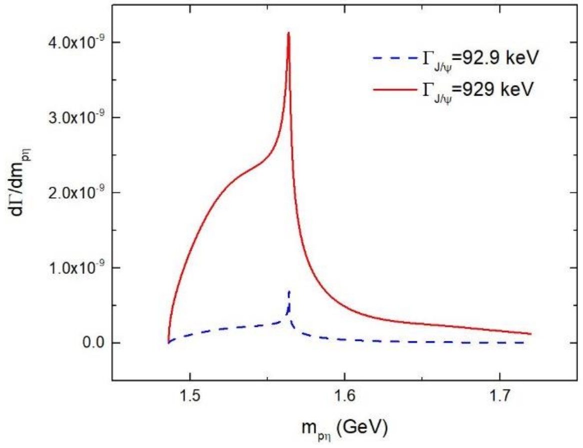
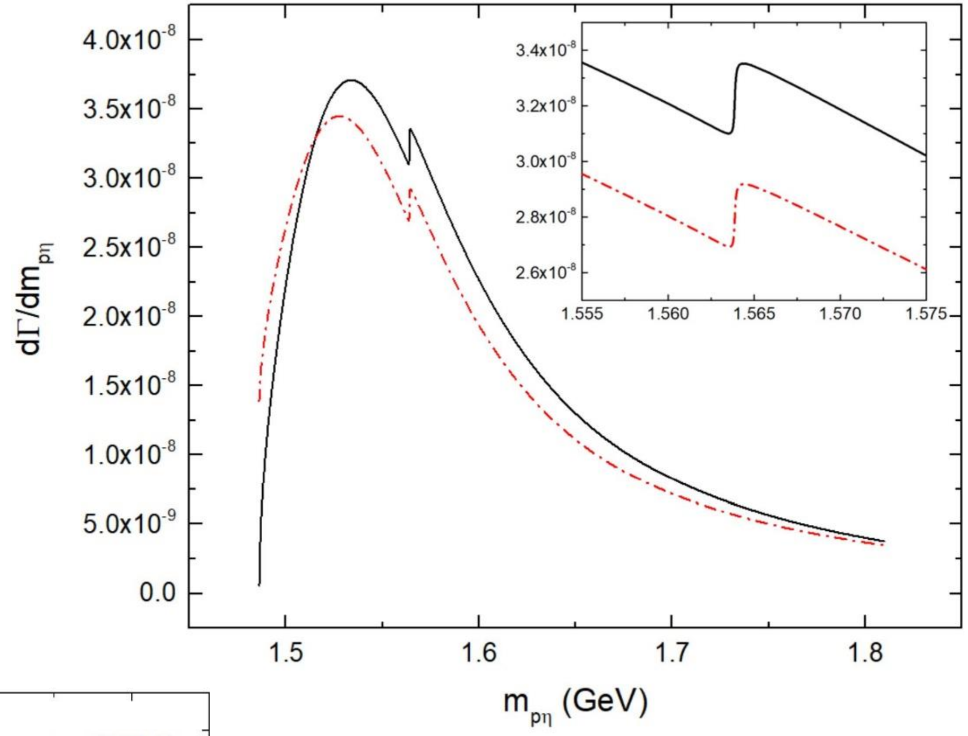








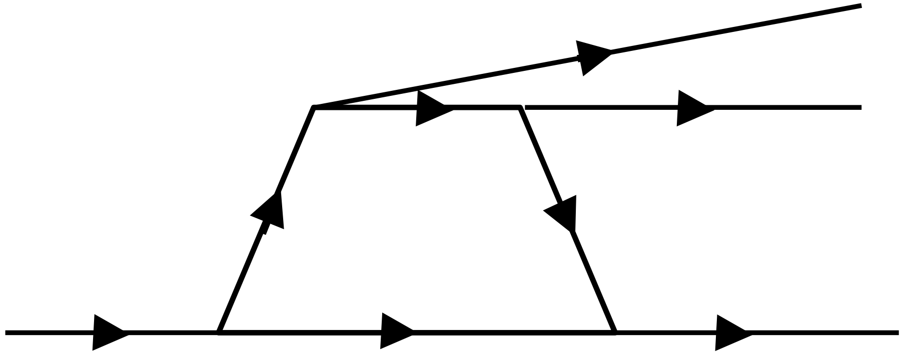
A lesson, the intermediate decay particle's width is too small !!!

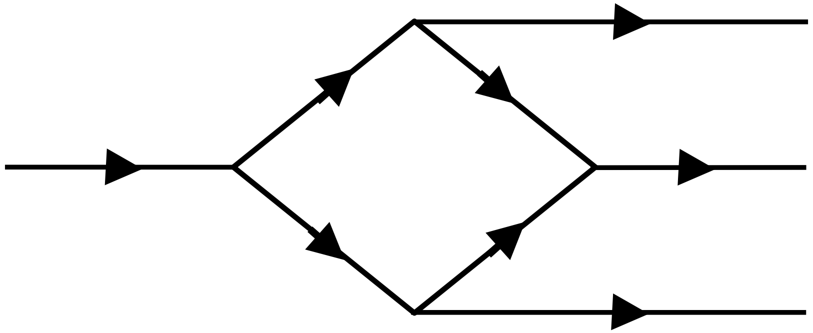
1. Narrow Peak
2. Weak strength

The better choice of width is around 2 MeV.

From Xiao-Rui Lyu and Zi-Yi Wang

How about Box Singularity





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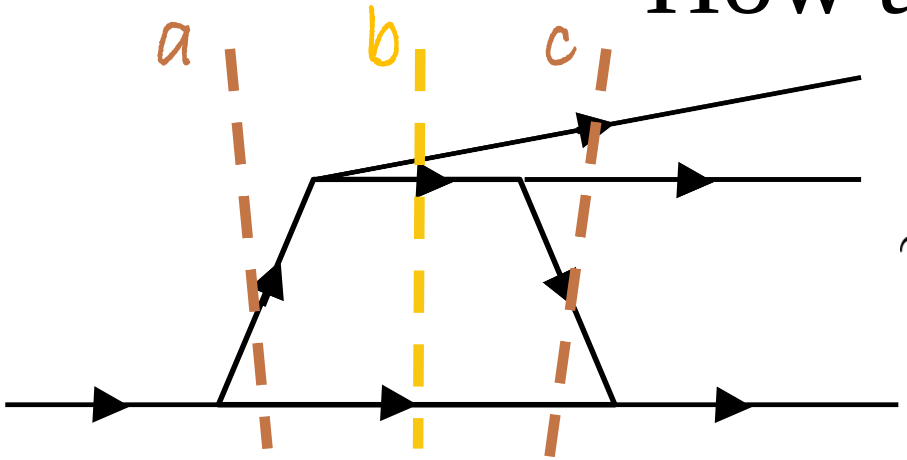
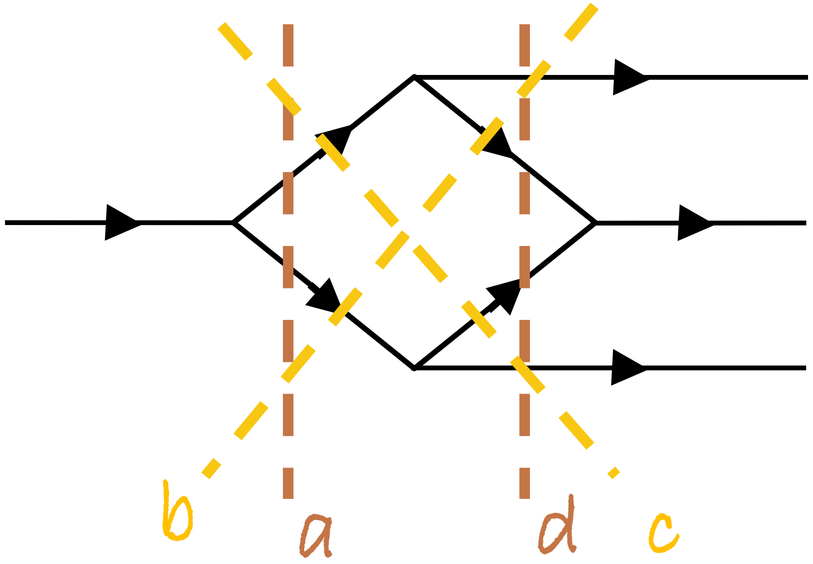


$$\sim \int d^3\vec{q}\,\frac{1}{a\,b\,c}$$

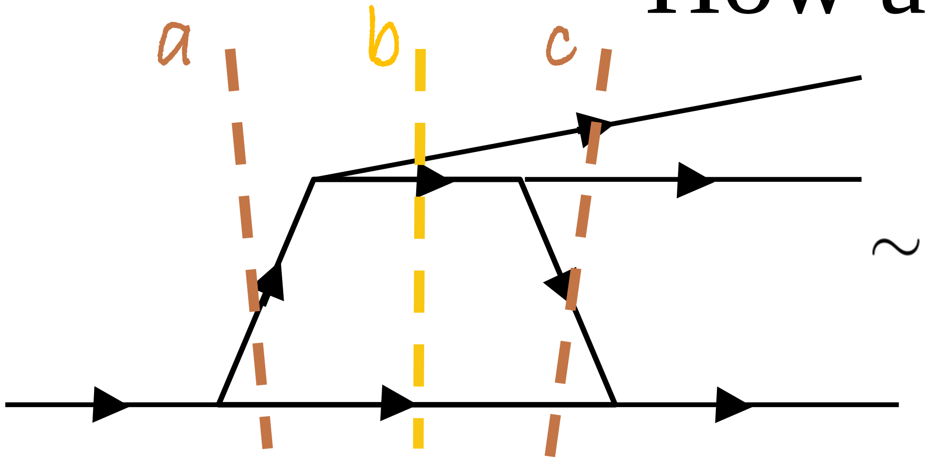


$$\sim \int d^3\vec{q}\,\frac{a+d}{a\,b\,c\,d}$$

$$a+d=b+c$$

preliminary

How about Box Singularity

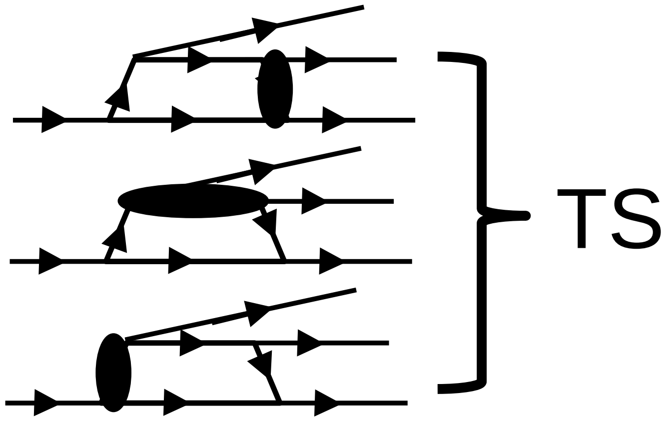


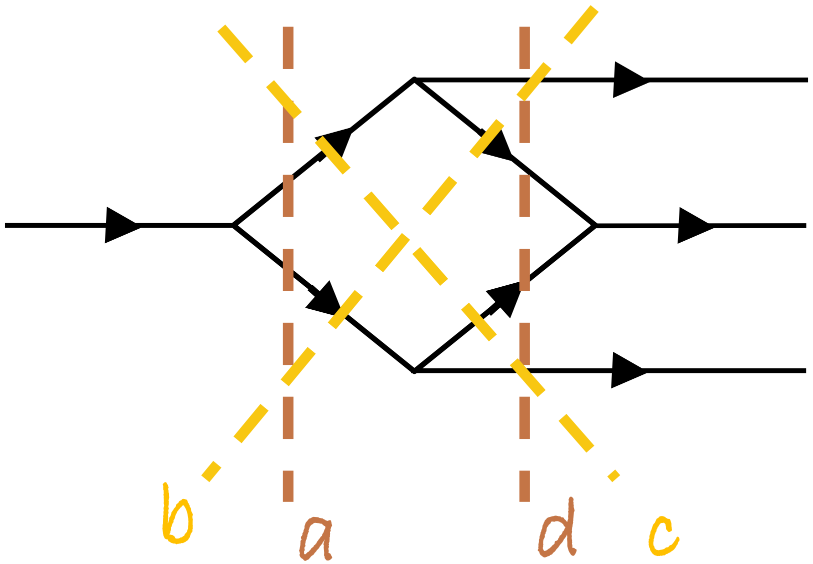
$$\sim \int d^3\vec{q}\,\frac{1}{a\,b\,c}$$

$a,\, b \to 0 \,\&\, c \nrightarrow 0$

$a,\, c \to 0 \,\&\, b \nrightarrow 0$

$b,\, c \to 0 \,\&\, a \nrightarrow 0$





$$\sim \int d^3\vec{q}\,\frac{a+d}{a\,b\,c\,d}$$

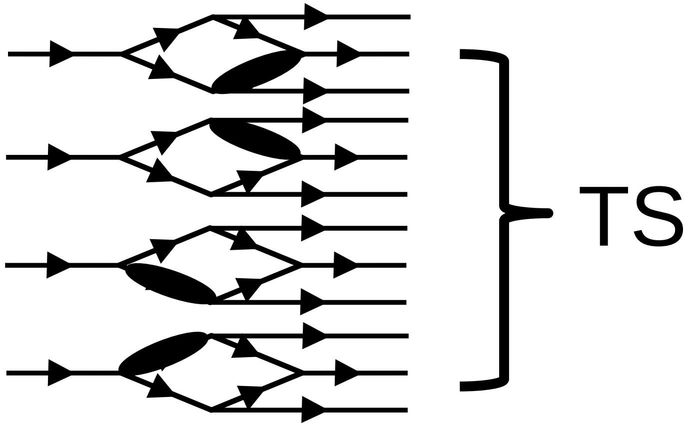
$$a + d = b + c$$

$a,\, b \to 0 \,\&\, c,\, d \nrightarrow 0$

$a,\, c \to 0 \,\&\, b,\, c \nrightarrow 0$

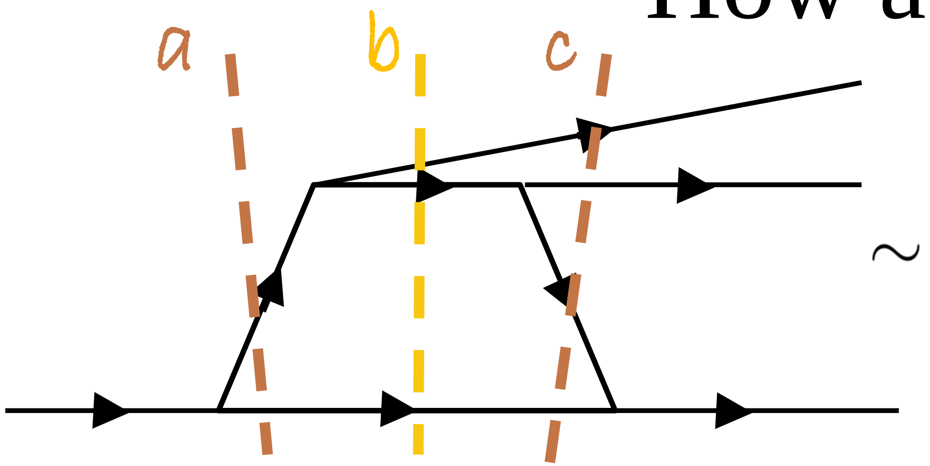
$b,\, d \to 0 \,\&\, a,\, c \nrightarrow 0$

$c,\, d \to 0 \,\&\, a,\, b \nrightarrow 0$



preliminary

How about Box Singularity



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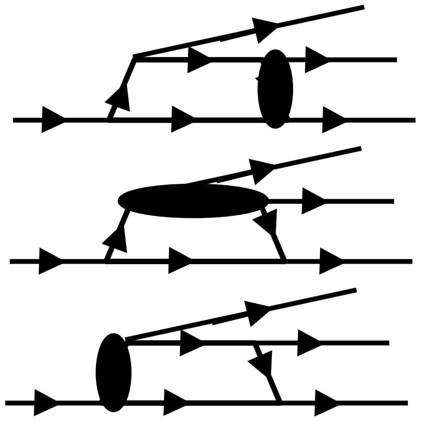
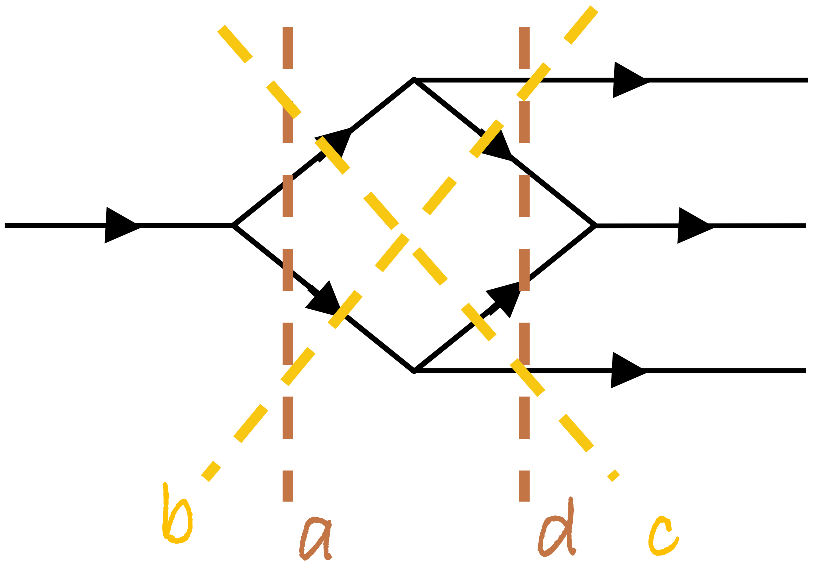


TS



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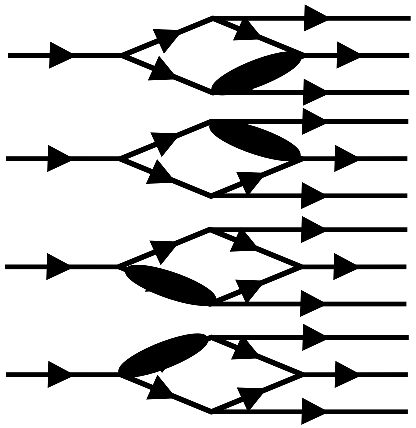
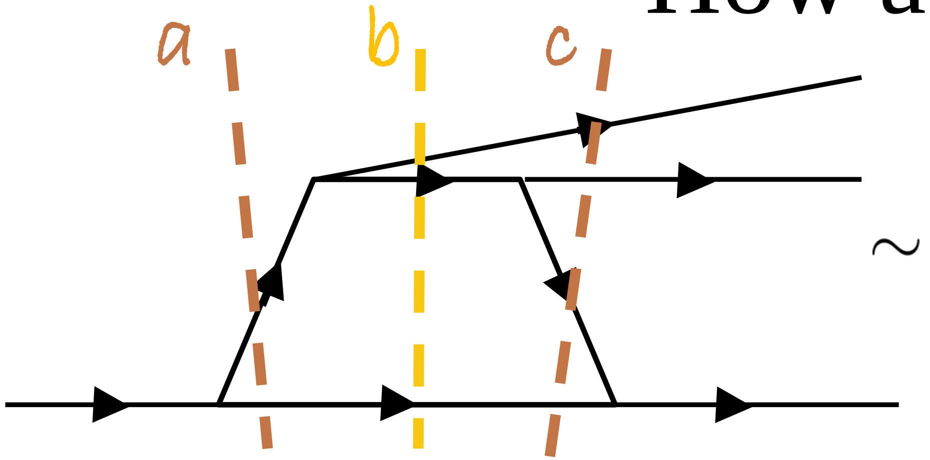


TS

Convergence!!!

preliminary

How about Box Singularity



$$\sim \int d^3\vec{q}\,\frac{1}{a\,b\,c}$$

$a,\ b,\ c \to 0???$

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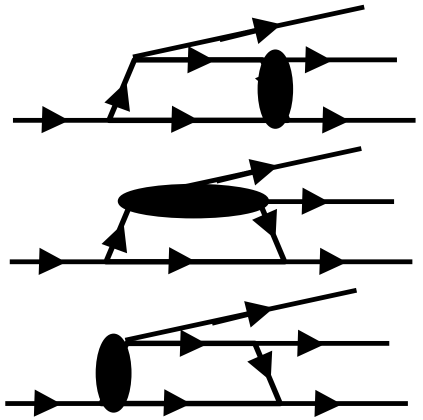

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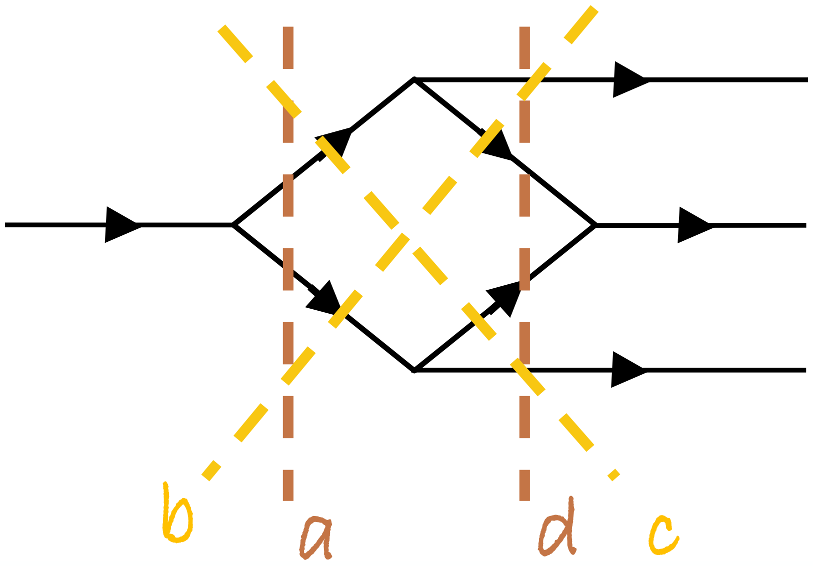

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$a,\ b \to 0\ \&\ c,\ d \not\to 0$

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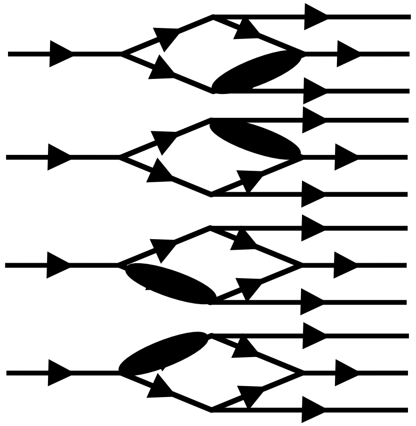

TS

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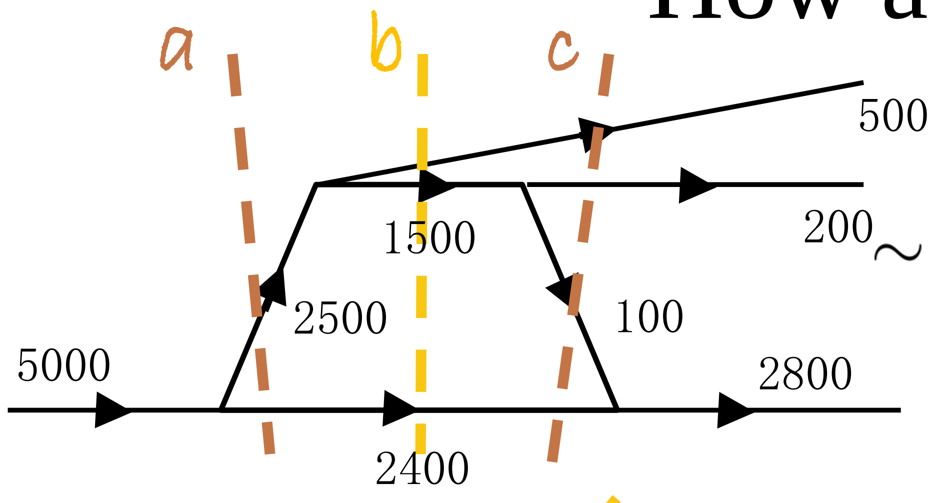
Convergence!!!

preliminary

How about Box Singularity



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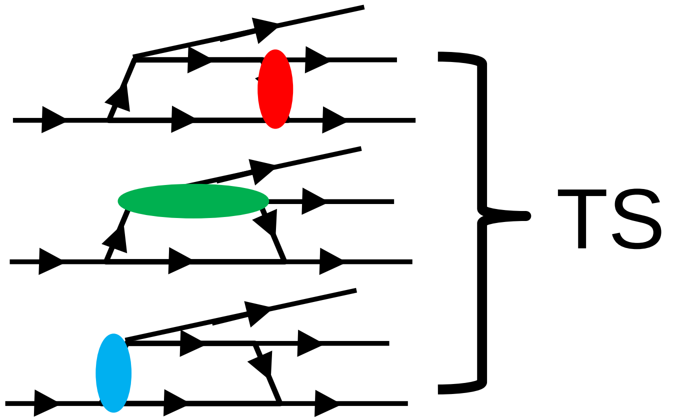
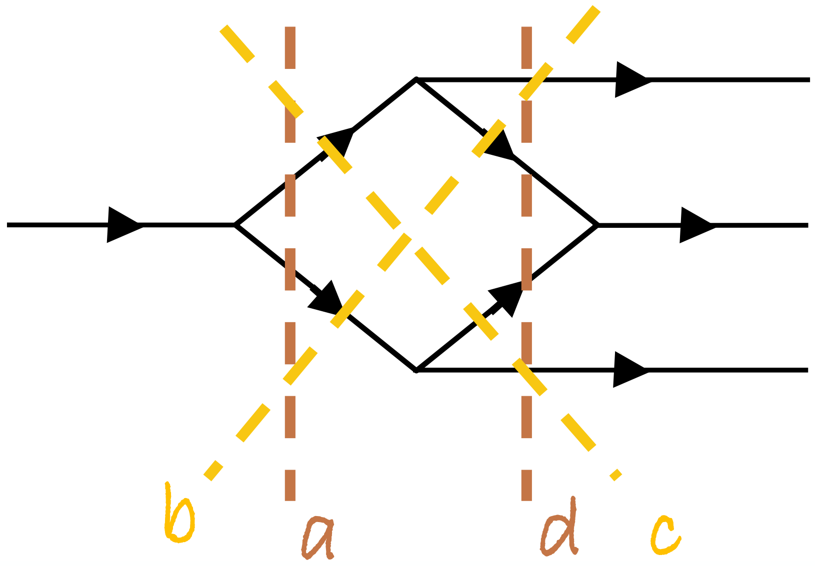
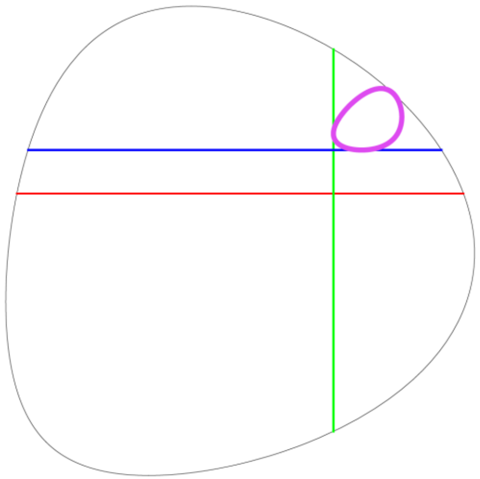
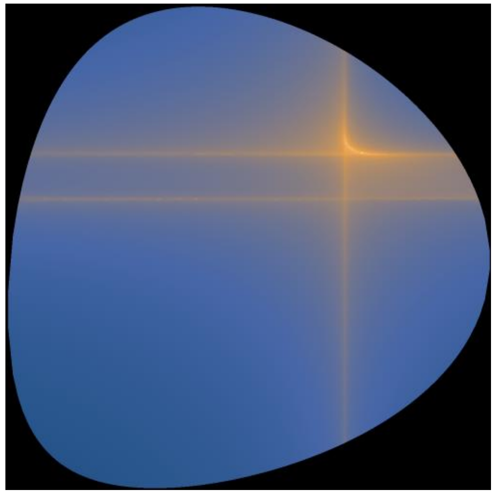


$a,\ b,\ c \to 0$ test



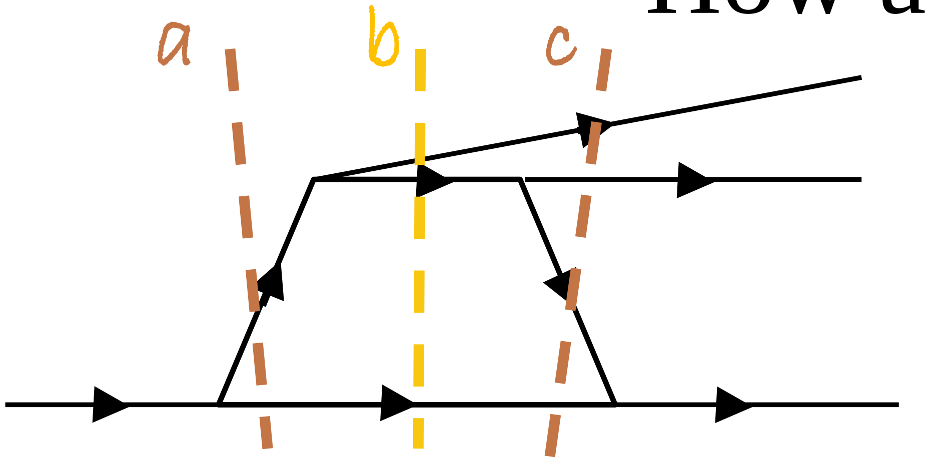


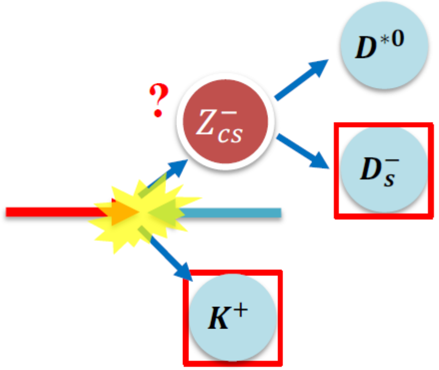


It shows a, b, c -> 0 is not enough for extracting the divergence range of box singularity !

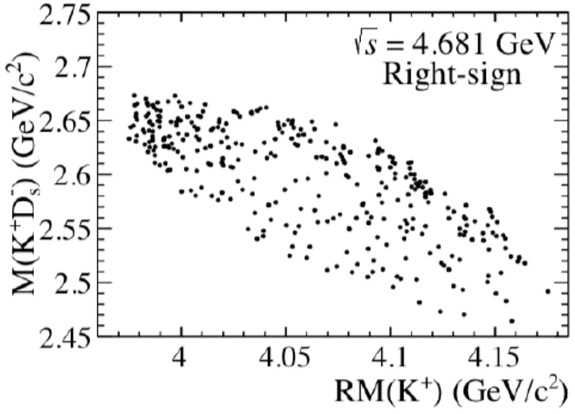
preliminary

How about Box Singularity

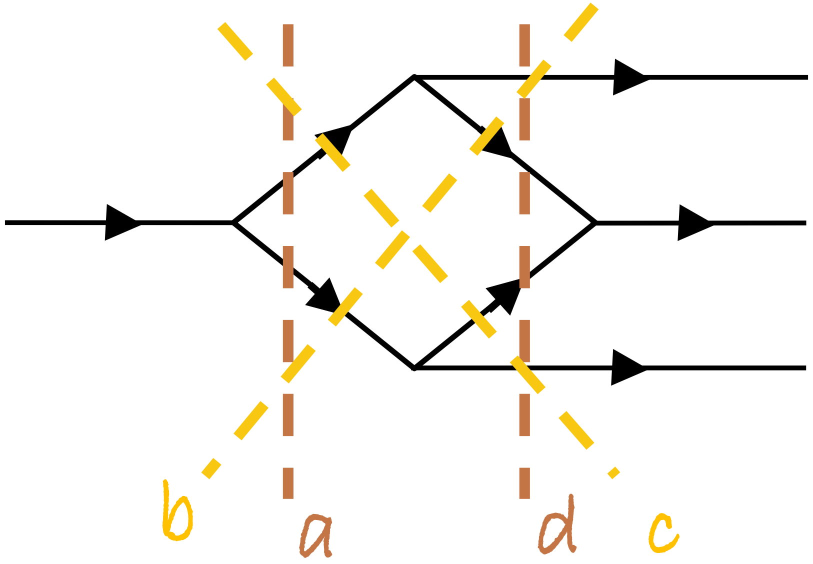


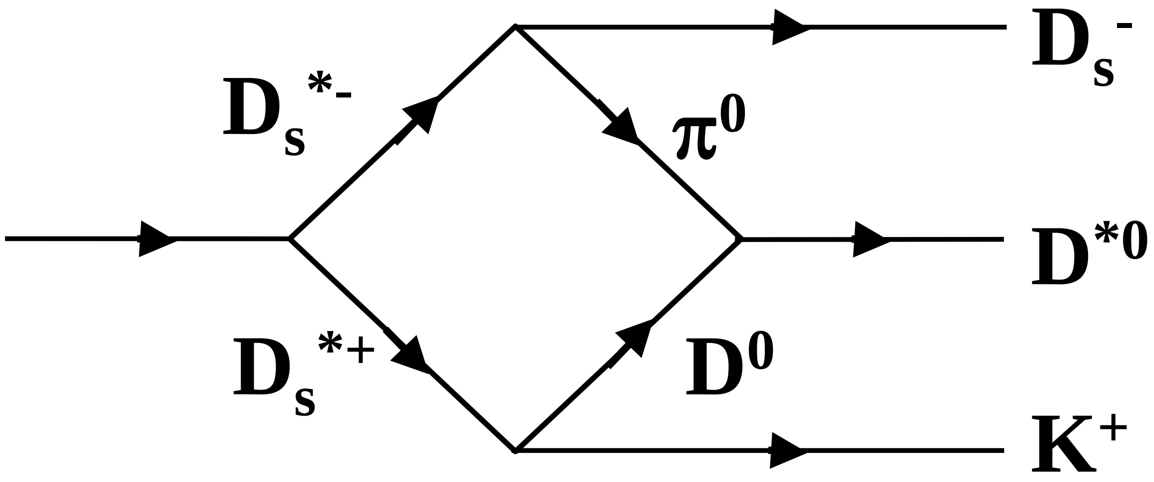


$e^+e^- \to K^+ D_s^- D^{*0}$



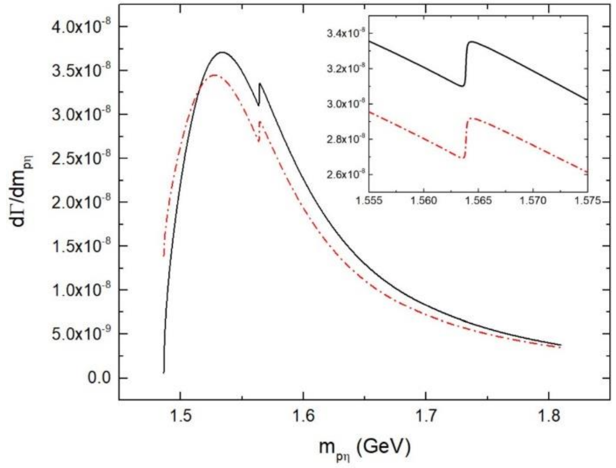
From
Peirong’ s Talk





Summary

- We discuss how to detect the Triangle Singularity, we should consider the following three point,
  1.Threshold
  2.Width of the internal particle of the loop
  3.Unknow vertex



- We propose a process $\psi(2s) \to p\,\bar{p}\,\eta$ through J/ψ, p, η loop.

- We need to find new processes to detect Triangle Singularity.

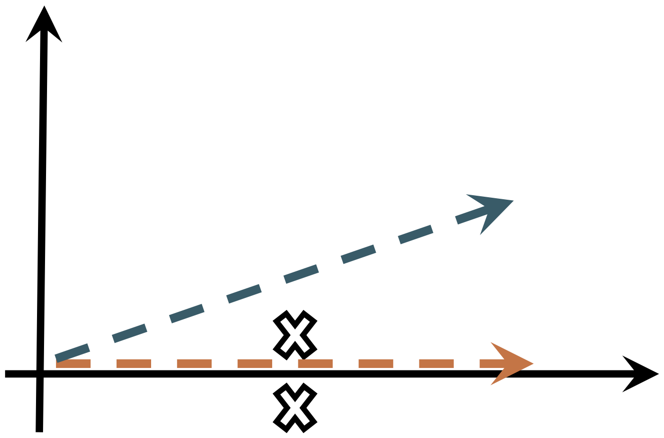
- For the box singularity, it needs more hard work ……

Backup

$$\sim \int d^3\vec{q}\left[\frac{1}{m_A-\omega_1(\vec{q})-\omega_2(\vec{q})+i\epsilon}\frac{1}{E_c-\omega_2(\vec{q})-\omega_3(\vec{p}_C-\vec{q})+i\epsilon}f(\vec{q})+h(\vec{q})\right]$$

$$q_a = q_{on} + i\epsilon$$

$$E_C - \sqrt{q^2+m_2^2} - \sqrt{\vec{p}_C^2+q^2+m_3^2-2|\vec{p}_C|q\cos\theta} + i\epsilon = 0$$



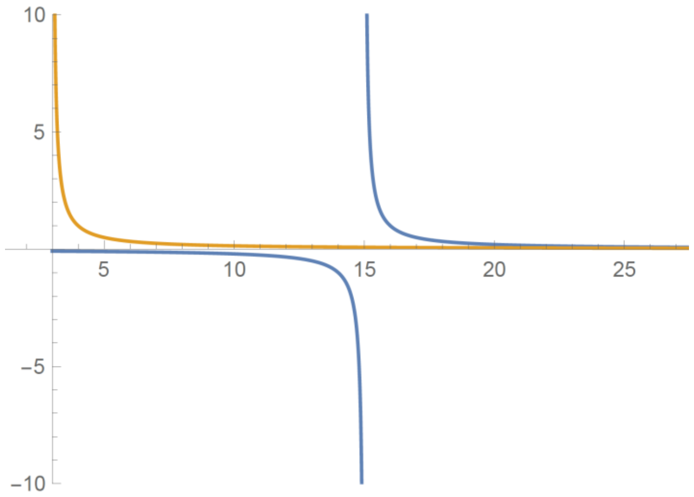
If Integral Divergence,
it should require the pole at

$q_b = q_{on} - i\epsilon'$
$\cos\theta = -1 \text{ or } 1$

1. In the complex plane, the integral routine will be fixed between two singularity.

$$\int \frac{dx}{(x+i\epsilon)^2} \to \text{Convergence} \qquad \int \frac{dx}{(x+i\epsilon)(x-i\epsilon)} \to \text{Divergence}$$

$$q_b = q_{on} - i\epsilon'$$

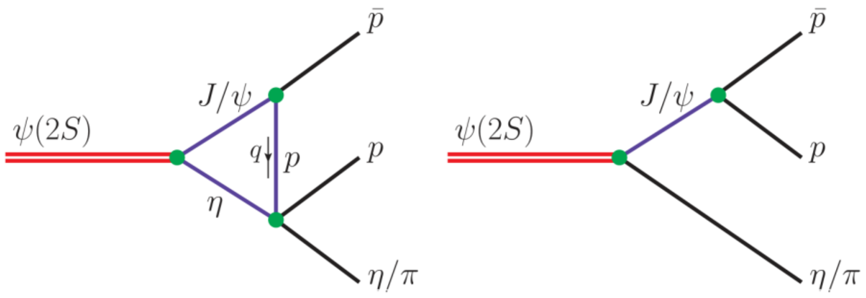


2. For $cos\theta$, it is a one order singularity, thus, it will be convergence except at the edge.

$$\cos\theta = -1 \text{ or } 1$$

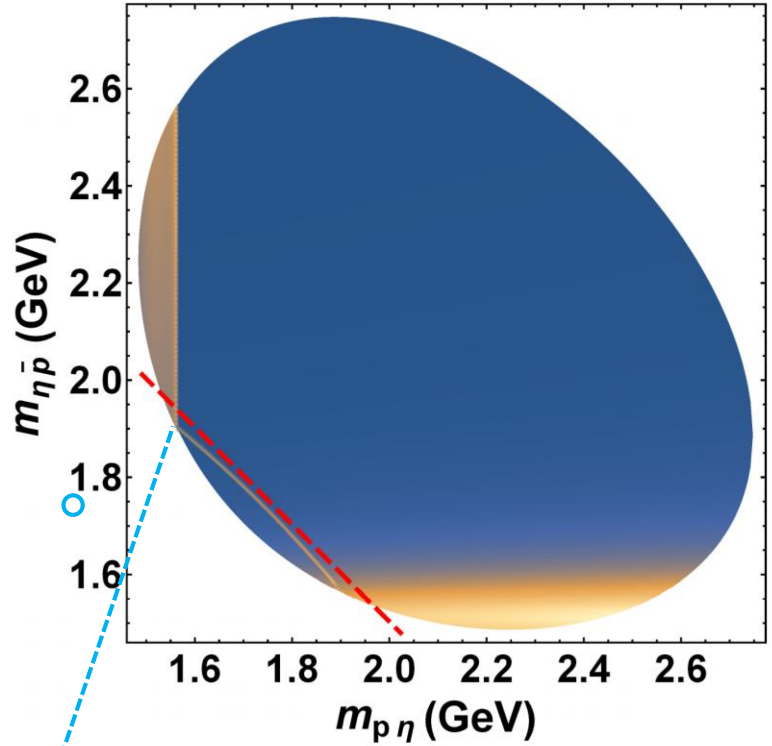
TS in $\psi(2S) \to p\bar{p}\eta$ process vs Schmid theorem

Effect of the tree diagram $\psi(2S) \to \eta(J/\psi \to p\bar{p})$



Schmid theorem:

$$\left|t^{\text{Tree}}_{J/\psi} + t^{\text{Loop}}_{\text{elastic}}\right|^2 = \left|t^{\text{Tree}}_{J/\psi} e^{i\delta}\right|^2 = \left|t^{\text{Tree}}_{J/\psi}\right|^2.$$



Schmid theorem can't be applied directly here:

1. $p\eta \to p\eta$ is not purely elastic process, $p\pi$ channel must couples with it, which is effectively included in the imaginary part of N^* propagator.

2. $\psi(2S) \to \bar{p}(N^* \to p\eta)$ will modify the amplitude to $\left|t^{\text{Tree}}_{J/\psi} + t^{\text{Loop}} + t^{\text{Tree}}_{N^*}\right|^2$, where $t^{\text{Loop}} \neq t^{\text{Loop}}_{\text{elastic}}$.

3. Contribution of $\psi(2S) \to \eta(J/\psi \to p\bar{p})$ can be removed by applying a cut $m_{p\bar{p}} < m_{J/\psi}$.

Red dashed line:

BESIII: PRD 88, 032010,

$m_{p\bar{p}} < 3.067$ GeV.

Improvement: Selection of internal particles

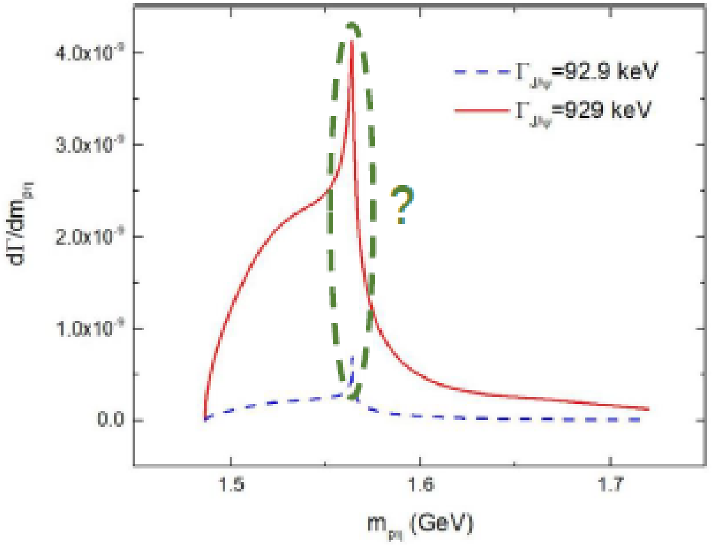
A lesson: Widths of internal particles should not be too narrow !

Or:

1. Peak of TS is too sharp to be detected by experiment. ➡ Requirement of the resolution is too high.

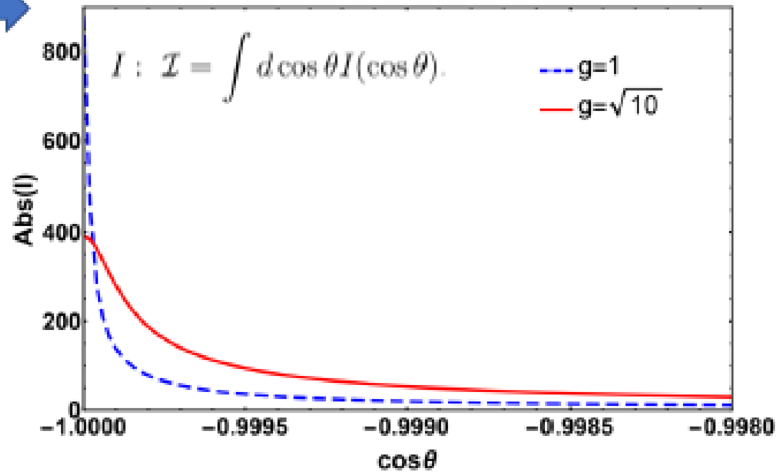
2. The intensity of the peak is weakened.

Fix $B(J/\psi \to p\bar{p})$, change width of J/ψ.



$@m_{TS}$ ➡

$$\mathcal{I} \equiv g \int \frac{d^4q}{(2\pi)^4} \frac{\mathcal{F}(p_3+p_4-q, m_\eta, \Lambda_\eta)}{(p_3+p_4-q)^2 - m_\eta^2 + i m_\eta \Gamma_\eta} \times \frac{\mathcal{F}(p_2+q, m_{J/\psi}, \Lambda_{J/\psi})}{(p_2+q)^2 - m_{J/\psi}^2 + i m_{J/\psi}\Gamma_{J/\psi}} \times \frac{\mathcal{F}(q, m_p, \Lambda_p)}{q^2 - m_p^2 + i m_p \Gamma_p},$$



Blue/Red:

$\cos\theta = -1$: $p_{J/\psi}^2 - m_{J/\psi}^2 = 0$

difference of $g/\Gamma_{J/\psi} \sim \sqrt{10}$

$\cos\theta > -1$: $p_{J/\psi}^2 - m_{J/\psi}^2 \neq 0$

difference of $g \sim 1/\sqrt{10}$

$\mathcal{I}$:

Blue: 0.360

Red: 0.514

Balance:

Strength,

Width,

Significance.

Our next step.

University of Chinese Academy of Sciences