Fully-heavy Tetraquarks in a strongly interacting medium

Jiaxing Zhao(赵佳星)

In collaboration with : Dr. Shuzhe Shi(施舒哲) and Prof. Pengfei Zhuang(庄鹏飞)

Based on: JX. Zhao, ShZh. shi, and PF. Zhuang, Phys.Rev.D 102 (2020) 11, 114001

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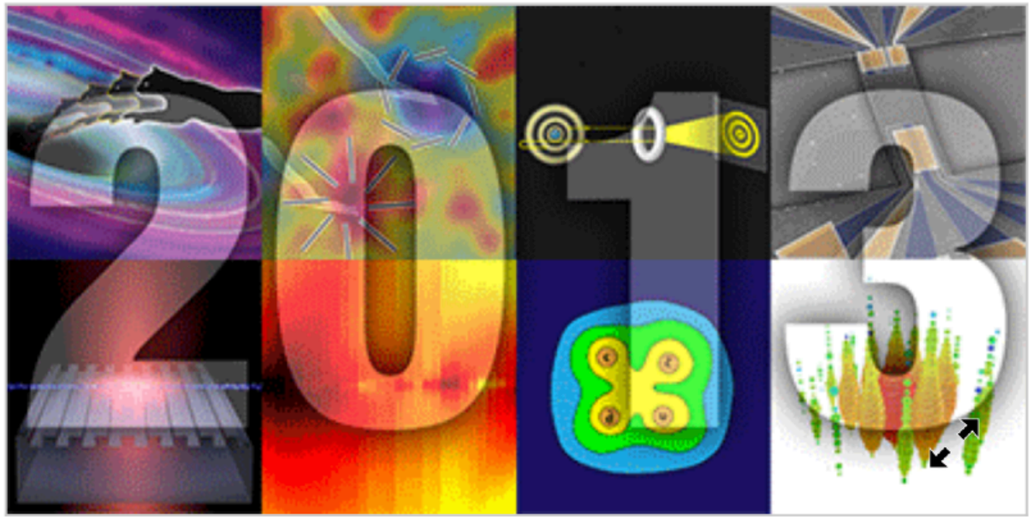
Outline

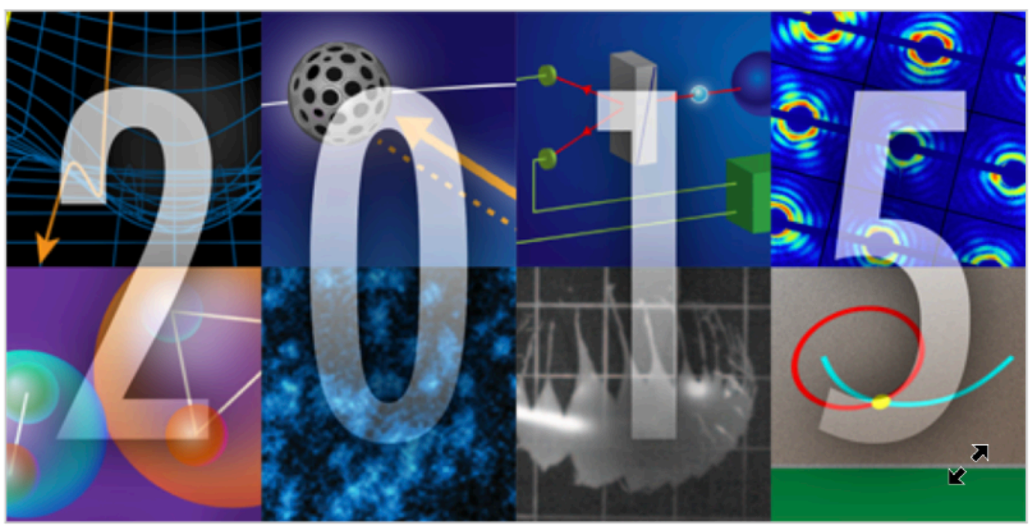
- *Brief introduction about heavy flavor exotic hadrons and Heavy ion collisions*
- *Study the static properties of fully-heavy Tetraquarks in vacuum and finite temperature.*
- *Production of fully-heavy Tetraquarks in heavy ion collisions*
- *Summary and outlook*

Heavy Flavor Exotic Hadrons

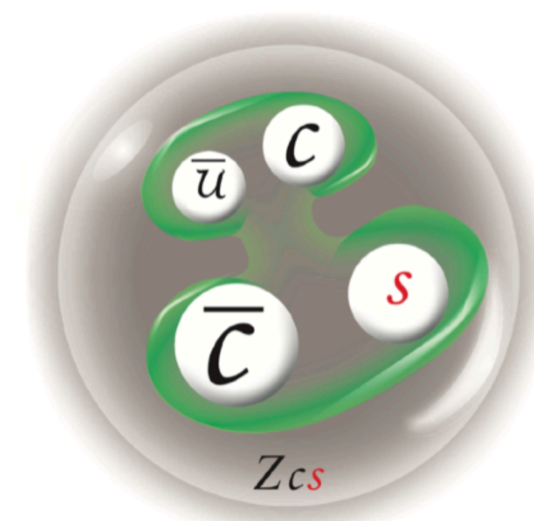


Research highlights of 2013:
Discovery of Zc(3900) at BESIII and Belle

2020: Discovery of Full-heavy Tetraquarks X(6900) at LHCb



Research highlights of 2015:
Discovery of Pentaquark at LHCb



2021: Discovery of Zcs(3985) at BESIII and Zcs(4000), Zcs(4220) at LHCb

A good platform to study nature of strong interaction.
Deepen our understanding of the complicated non-perturbative behavior of QCD in low energy regions.

Heavy Flavor Exotic Hadrons in Exp.

- *e+e- collisions at BESIII, Babar, Belle and CLEO*

  Very clean experimental environment and various production mechanisms

- *p-antiproton at Tevatron and pp collisions at LHC*

  Decay of b hadrons (B and B_s mesons as well as the Λ_b baryon)

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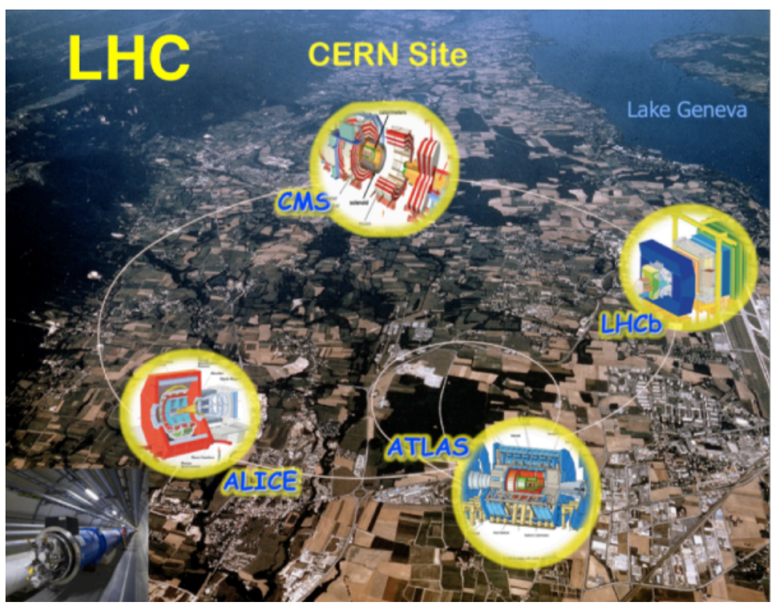
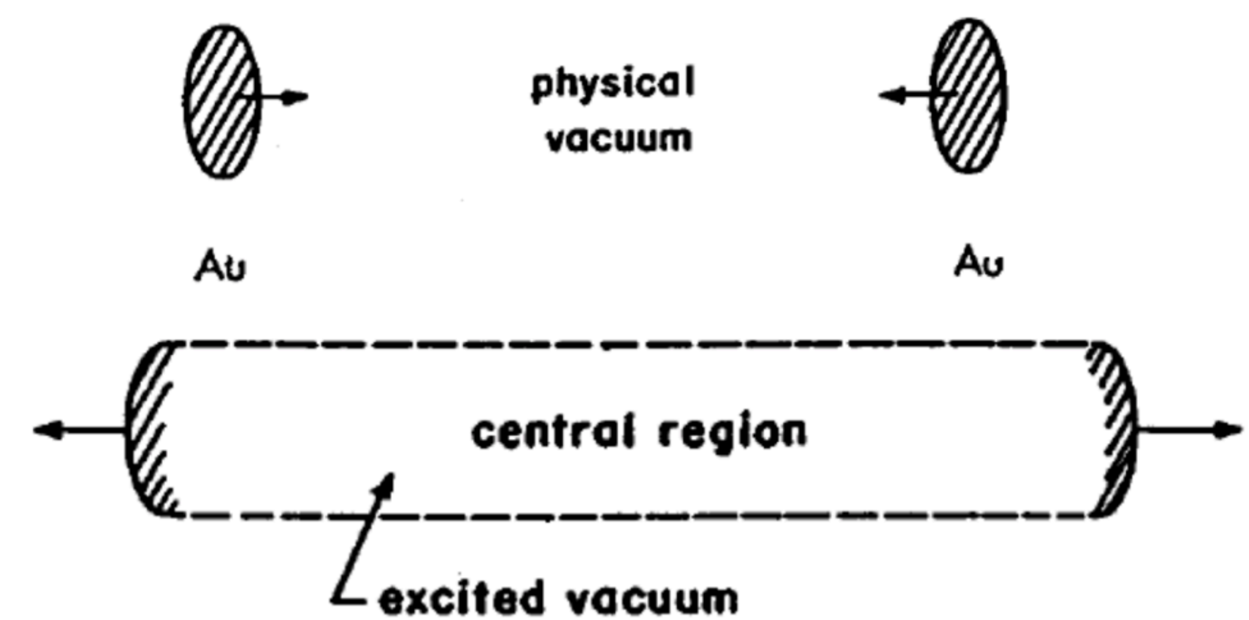
- *Relativistic Heavy-ion Collisions at RHIC and LHC*





T. D. Lee

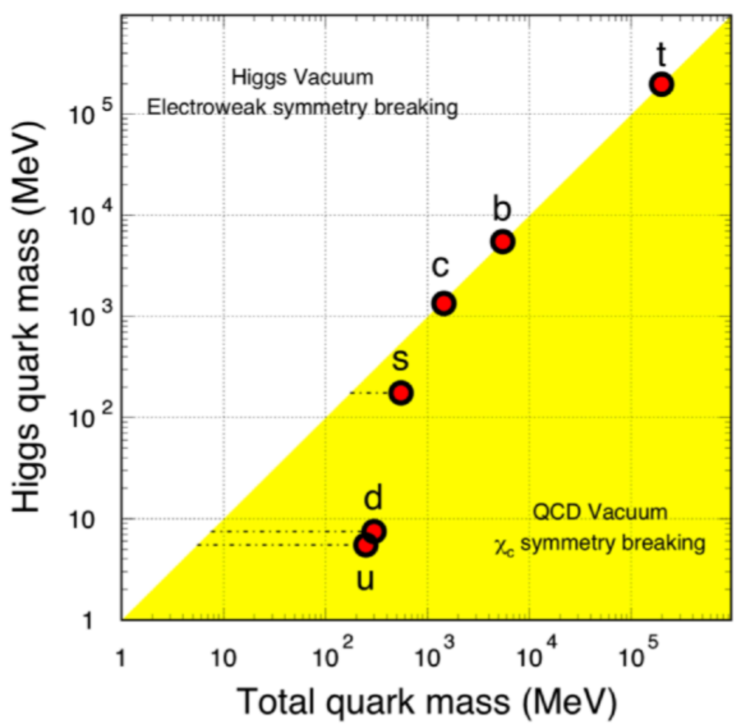
PbPb, $\sqrt{s_{NN}}$ *~ 2.76-5.02TeV*

A new state of matter: Quark-Gluon Plasma(QGP) !

QGP: color deconfinement, chiral restoration, strong coupling("prefect liquid"),…

Heavy Flavor: a sensitive probe of QGP

- *Heavy flavor can be used to probe and study the QGP !*



- *Mc, Mb >> Λ_{QCD}, produced by initial hard scattering and can be described by pQCD.*
- *Mass not change in QGP medium, number conserved. strong interaction with the hot medium.*
- *Heavy flavor hadrons production on the boundary of QCD phase transition. Clean decay mode and easy to distinguish*

***JX. Zhao K. Zhou, ShL. Chen, PF. Zhuang, Prog. Part. Nucl. Phys.* 114 (2020) 103801.**

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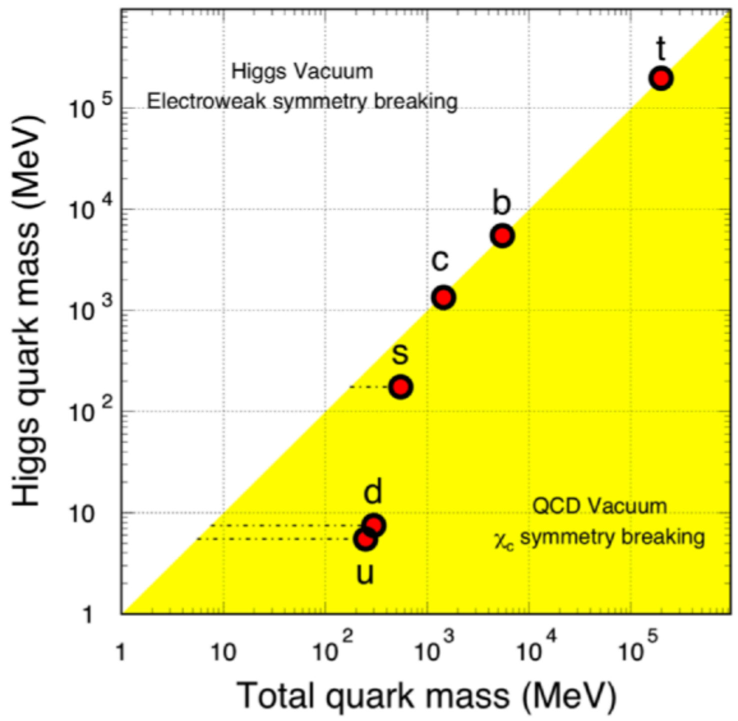

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JX. Zhao, PF. Zhuang, Few Body Syst. 58 (2017) 2, 100.
***ExHIC Collaboration, Sungtae Cho et al, Phys. Rev. Lett. 106 (2011) 212001*.**
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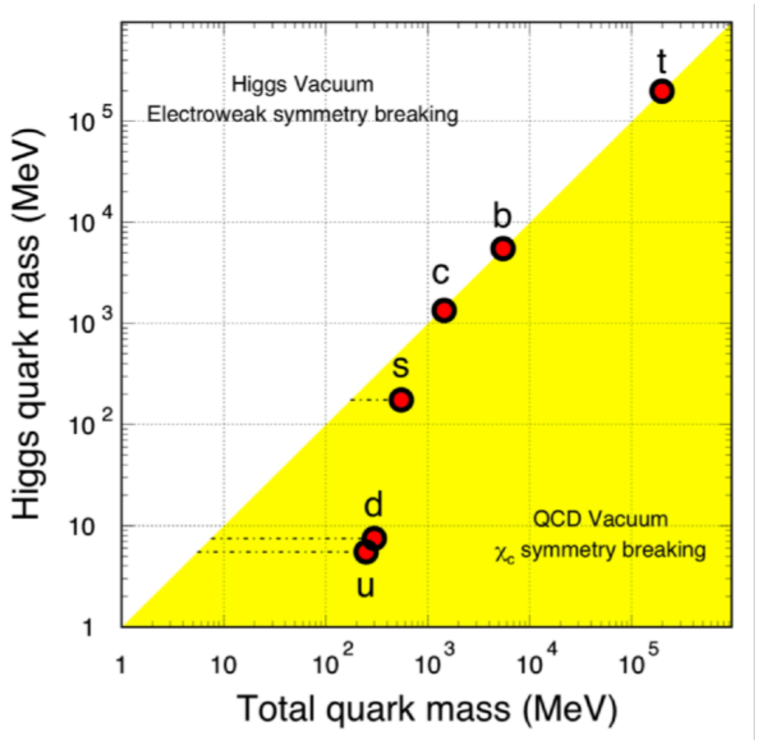

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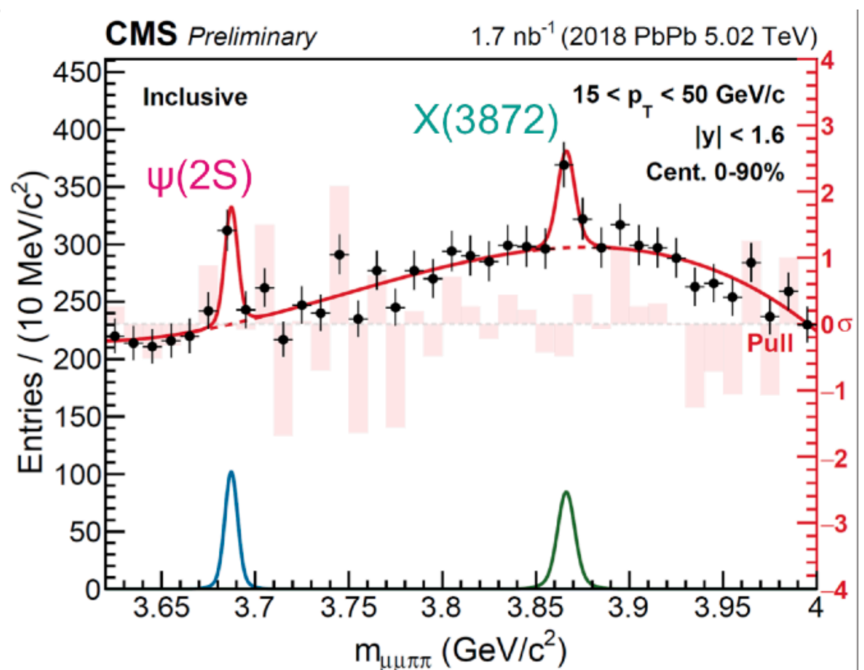

CMS Collaboration, arXiv: 2102,13048.

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*First evidence of **X(3872)** production in heavy ion collisions!*

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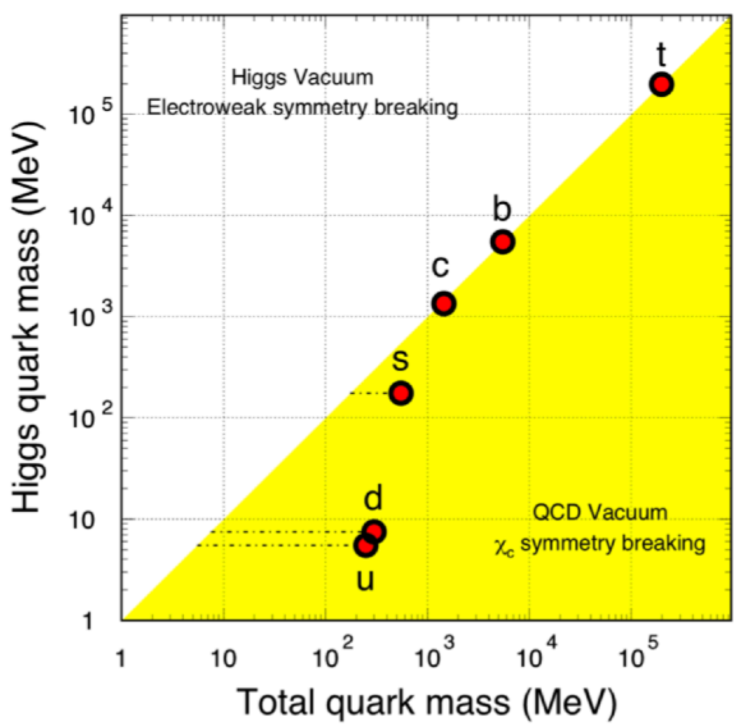

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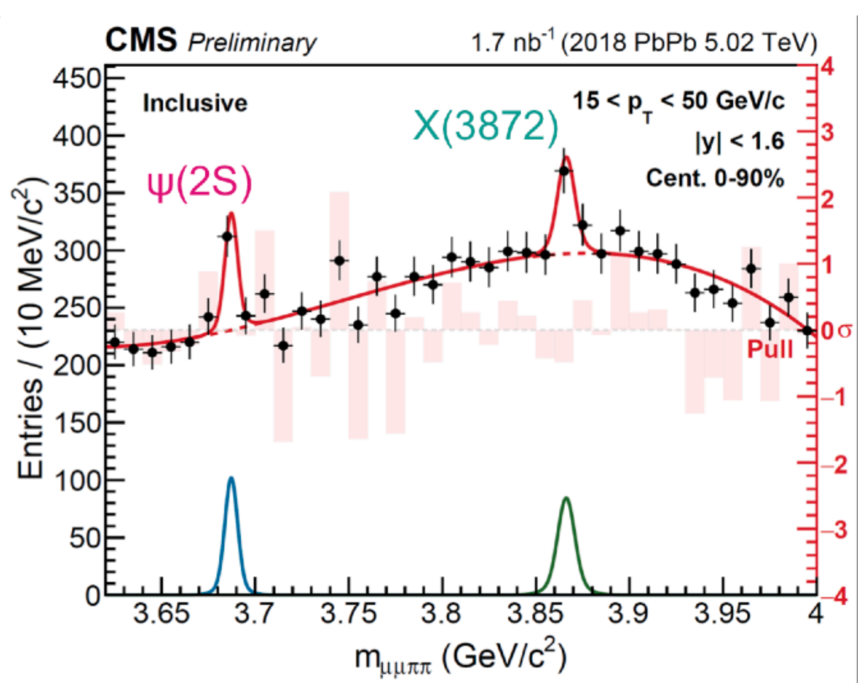

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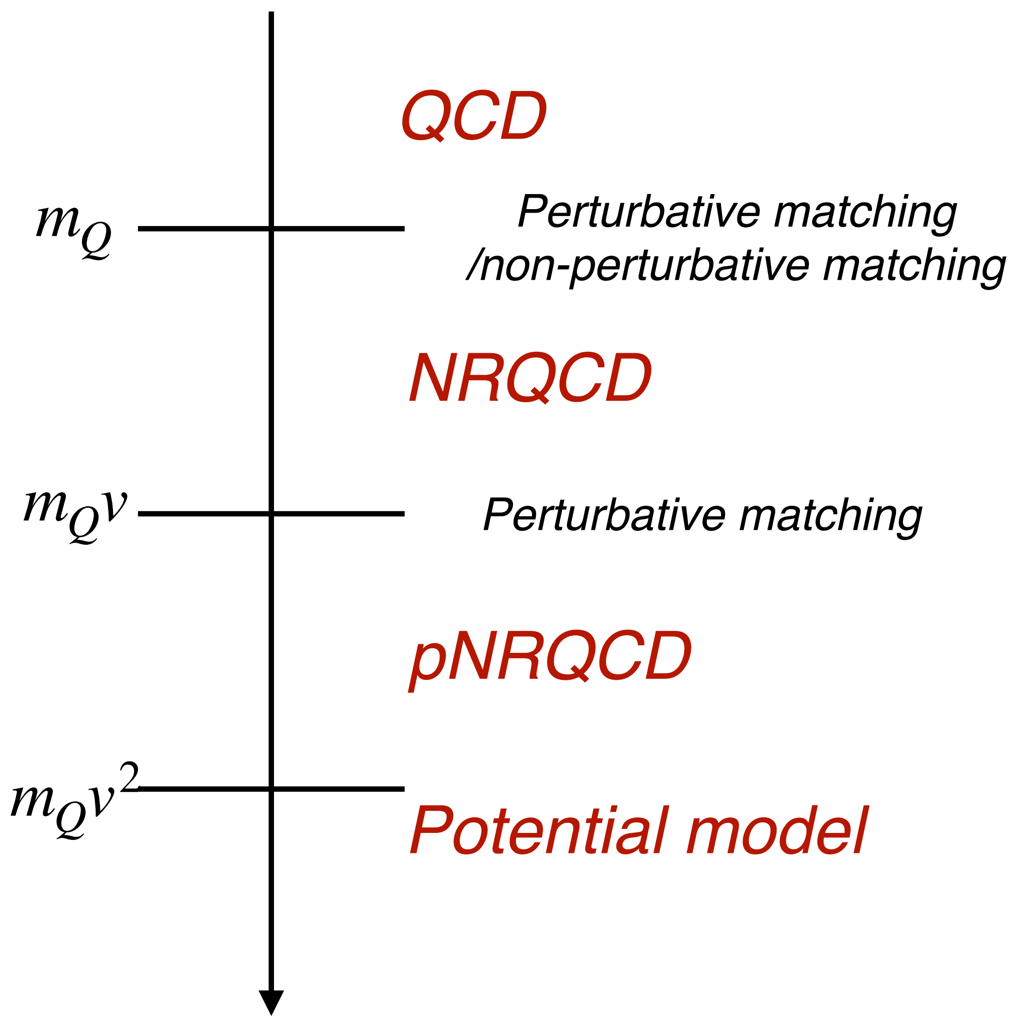
Fully-heavy Tetraquark properties and production in QGP

Heavy Flavor Effective Theory

$m_c \sim 1.5 GeV, m_b \sim 4.7 GeV$

Separation of scales:

$m_Q \gg m_Q v \gg m_Q v^2$



"Top - down"

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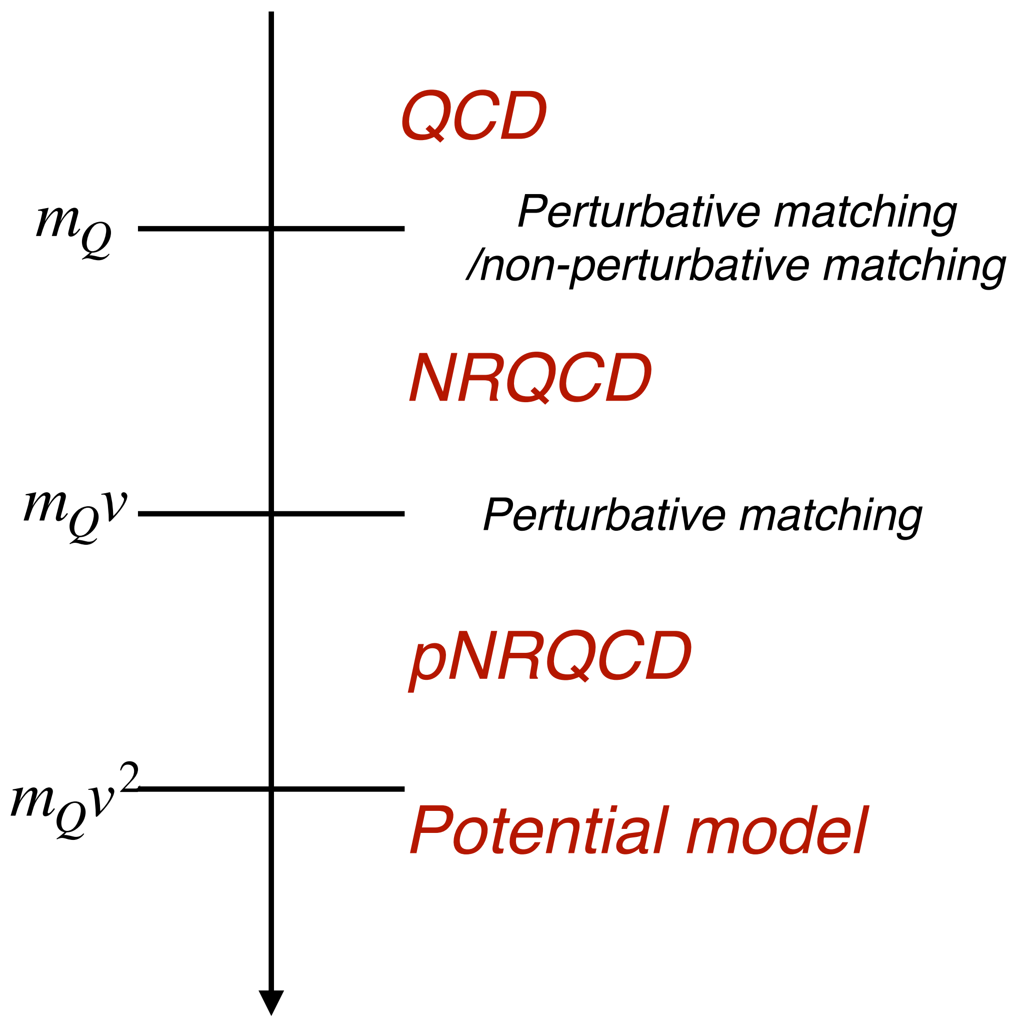


"Top - down"

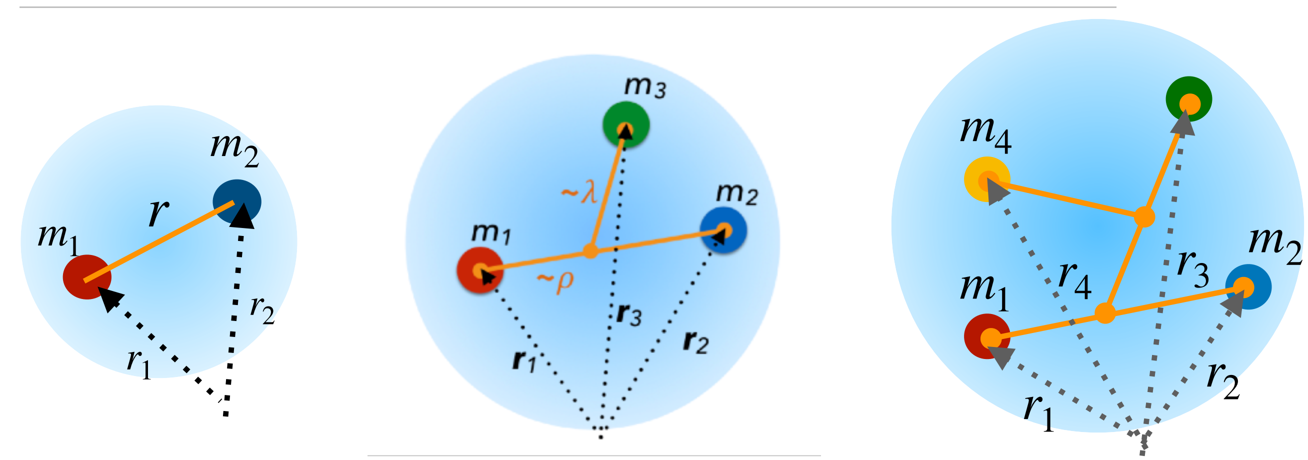
N-Body Schroedinger Equation Framework

$$\left(\sum_{i=1}^{N} \frac{\hat{p}_i^2}{2m_i} + \sum_{i<j} V_{ij} \right) \Psi(\mathbf{r}_1, \ldots, \mathbf{r}_N) = E\Psi(\mathbf{r}_1, \ldots, \mathbf{r}_N)$$

Jacobi coordinates :

$$\mathbf{R} = \frac{1}{M} \sum_{i=1}^{N} m_i \mathbf{r}_i,$$

$$\mathbf{x_j} = \sqrt{\frac{M_j m_{j+1}}{M_{j+1}\mu}} \left(\mathbf{r}_{j+1} - \frac{1}{M_j} \sum_{i=1}^{j} m_i \mathbf{r}_i \right)$$



Then, factorize the N-body motion into a center-of-mass motion and a relative motion

$$\Psi(\mathbf{r}_1, \ldots, \mathbf{r}_N) = \Theta(\mathbf{R})\Phi(\mathbf{x}_1, \ldots, \mathbf{x}_{N-1}),$$

N-Body Schroedinger Equation Framework

Further, N-1 relative coordinates can be transformed to a single hyperradial coordinate and 3N-4 hyperangular coordinates.

$$(\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_{N-1}) \rightarrow (\rho, \alpha_{N-1}, \ldots, \alpha_2, \theta_1, \phi_1, \ldots, \theta_{N-1}, \phi_{N-1})$$

$$\rho = \sqrt{\mathbf{x}_1^2 + \ldots + \mathbf{x}_{N-1}^2} \qquad \sin\alpha_i = x_i/\rho_i \qquad \hat{x}_i = (\theta_i, \phi_i)$$

N. Barnea, et al. Phys. Rev. C 61.054001(2000)

FBS Colloquium. Few-Body System 25, 199-238(1998)

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The relative motion is controlled by :

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$$\left[\frac{1}{2\mu}\left(-\frac{1}{\rho^{3N-4}}\frac{d}{d\rho}\rho^{3N-4}\frac{d}{d\rho} + \frac{\hat{K}_{N-1}^2}{\rho^2}\right) + V(\rho, \Omega)\right]\Phi(\rho, \Omega) = E_r\Phi(\rho, \Omega),$$

$$\hat{K}_{N-1}^2 = -\frac{\partial^2}{\partial\alpha_{N-1}^2} + \frac{(3N-9)-(3N-5)\cos(2\alpha_{N-1})}{\sin(2\alpha_{N-1})}\frac{\partial}{\partial\alpha_{N-1}} + \frac{1}{\cos^2\alpha_{N-1}}\hat{K}_{N-2}^2 + \frac{1}{\sin^2\alpha_{N-1}}\hat{l}_{N-1}^2,$$

$$\hat{K}_{N-1}^2\mathcal{Y}_\kappa(\Omega) = K(K+3N-5)\mathcal{Y}_\kappa(\Omega).$$ *hyper-angular momentum operator*

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$$\Phi(\rho, \Omega) = \sum_\kappa R_\kappa(\rho)\mathcal{Y}_\kappa(\Omega)$$ *hyper-spherical harmonic function expansion*

$$\left[\frac{1}{2\mu}\left(\frac{1}{\rho^{3N-4}}\frac{d}{d\rho}\rho^{3N-4}\frac{d}{d\rho} - \frac{K(K+3N-5)}{\rho^2}\right) + E_r\right]R_\kappa = \sum_{\kappa'} V_{\kappa\kappa'}R_{\kappa'}$$

Now, we apply this tool to deal with fully-heavy Tetraquark states !

Symmetric Analysis

Pauli exclusion principle requires the wave-function to be anti-symmetric when exchanging two identical fermions

$$\Psi = \psi \cdot \not{\phi}_f \cdot \chi_s \cdot \phi_c$$

Color $(3_c \otimes 3_c) \otimes (\bar{3}_c \otimes \bar{3}_c) = \bar{3}_c \otimes 3_c \oplus 6_c \otimes \bar{6}_c \oplus \bar{3}_c \otimes \bar{6}_c \oplus 6_c \otimes 3_c$

$$|\phi_1\rangle = |(QQ)_{\bar{3}_c}(\bar{Q}\bar{Q})_{3_c}\rangle, \quad |\phi_2\rangle = |(QQ)_{6_c}(\bar{Q}\bar{Q})_{\bar{6}_c}\rangle$$

Spin $2 \otimes 2 \otimes 2 \otimes 2 = 1 \otimes 1 \oplus 1 \otimes 3 \oplus 3 \otimes 1 \oplus 3 \otimes 3$

$$s = 0: \quad |\chi_1\rangle = |(QQ)_0(\bar{Q}\bar{Q})_0\rangle_0, \quad |\chi_2\rangle = |(QQ)_1(\bar{Q}\bar{Q})_1\rangle_0$$

$$s = 1: \quad |\chi_3\rangle = |(QQ)_0(\bar{Q}\bar{Q})_1\rangle_1, \quad |\chi_4\rangle = |(QQ)_1(\bar{Q}\bar{Q})_0\rangle_1, \quad |\chi_5\rangle = |(QQ)_1(\bar{Q}\bar{Q})_1\rangle_1$$

$$s = 2: \quad |\chi_6\rangle = |(QQ)_1(\bar{Q}\bar{Q})_1\rangle_2$$

So, for $cc\bar{c}\bar{c}$ *and* $bb\bar{b}\bar{b}$

$$J^{PC} = 0^{++}: \quad |\phi_1\chi_2\rangle \ \ \& \ \ |\phi_2\chi_1\rangle$$

$$J^{PC} = 1^{+-}: \quad |\phi_1\chi_5\rangle$$

$$J^{PC} = 2^{++}: \quad |\phi_1\chi_6\rangle$$

Solving the Coupled Equations

$J^{PC} = 0^{++}$:

$$\Phi(\rho,\Omega) = \sum_{\kappa} R_{\kappa}^{(1)}(\rho)\mathcal{Y}_{\kappa}(\Omega)\,|\,\phi_1\chi_2\rangle + R_{\kappa}^{(2)}(\rho)\mathcal{Y}_{\kappa}(\Omega)\,|\,\phi_2\chi_1\rangle$$

$$-\frac{1}{2\mu}\left(\frac{d^2}{d\rho^2} + \frac{8}{\rho}\frac{d}{d\rho} - \frac{K(K+7)}{\rho^2}\right)R_{\kappa}^{(1)} + \sum_{\kappa'} V_1^{\kappa\kappa'} R_{\kappa'}^{(1)} + \sum_{\kappa'} V_m^{\kappa\kappa'} R_{\kappa'}^{(2)} = E_r R_{\kappa}^{(1)}$$

$$-\frac{1}{2\mu}\left(\frac{d^2}{d\rho^2} + \frac{8}{\rho}\frac{d}{d\rho} - \frac{K(K+7)}{\rho^2}\right)R_{\kappa}^{(2)} + \sum_{\kappa'} V_2^{\kappa\kappa'} R_{\kappa'}^{(2)} + \sum_{\kappa'} V_m^{\kappa\kappa'} R_{\kappa'}^{(1)} = E_r R_{\kappa}^{(2)}$$

$J^{PC} = 1^{+-}\ ,\ 2^{++}$:

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$$V^{\kappa\kappa'} = \int V(\rho,\Omega)\mathcal{Y}_{\kappa}^{*}(\Omega)\mathcal{Y}_{\kappa'}(\Omega)d\Omega$$ *potential matrix element in angular momentum space*

- *Focus on the states with L=M=0, choose all hyperspherical harmonic functions with hyperangular quantum number K ≤ 3.*
- *Numerically solve the coupled differential equations with inverse power method*

Heavy Quark Potential

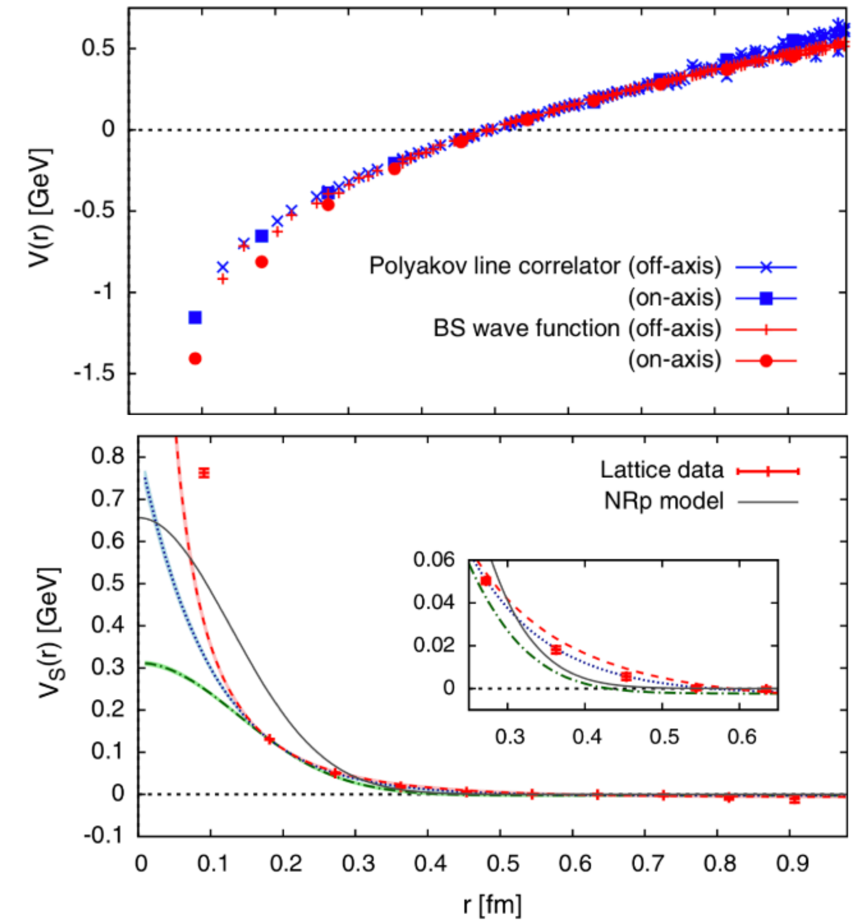
One-Gluon Exchange (OGE)

$$V_{ij}(|\mathbf{r}_{ij}|) = -\frac{1}{4}\lambda_i^a \cdot \lambda_j^a \left(V_{ij}^c(|\mathbf{r}_{ij}|) + V_{ij}^{ss}(|\mathbf{r}_{ij}|)\mathbf{s}_i \cdot \mathbf{s}_j \right)$$

$\lambda_i^a (a = 1,...,8)$ *SU(3) Gell-Mann matrices*

Cornell potential

$$V_{ij}^c(|\mathbf{r}_{ij}|) = -\frac{\alpha}{|\mathbf{r}_{ij}|} + \sigma |\mathbf{r}_{ij}|$$



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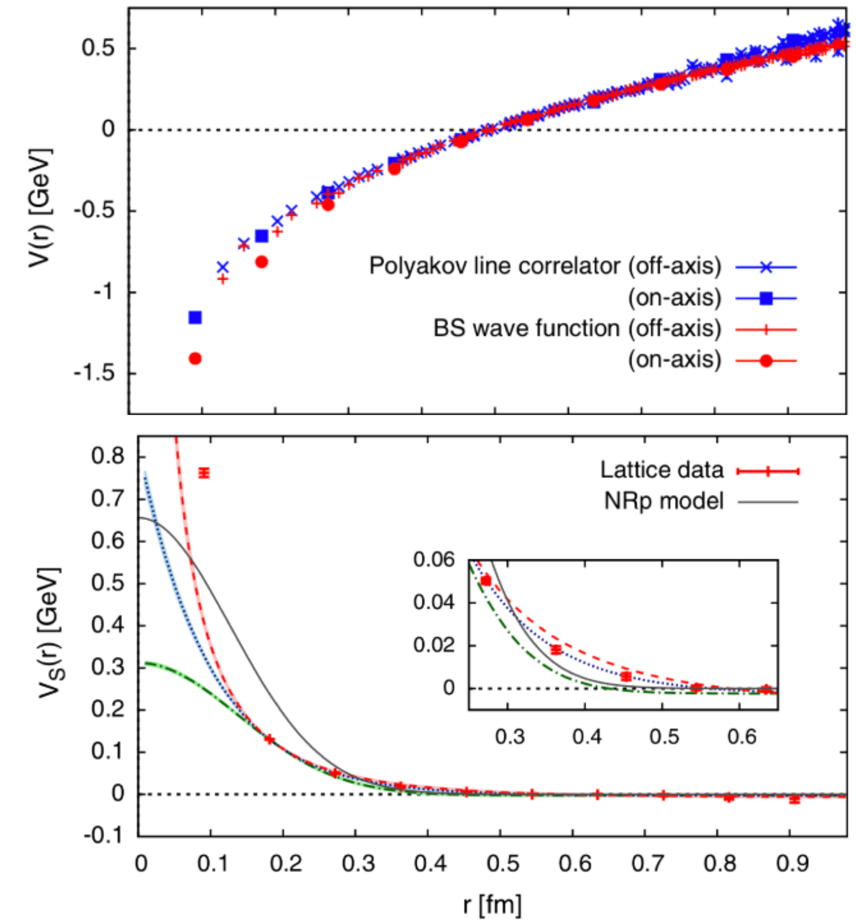

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Parameters

Fixed by quarkonium mass in vacuum.

m_b	m_c	α	σ	γ	β_b	β_c
4.7 GeV	1.29 GeV	0.308	0.15 GeV^2	1.982 GeV	0.239 GeV	1.545 GeV

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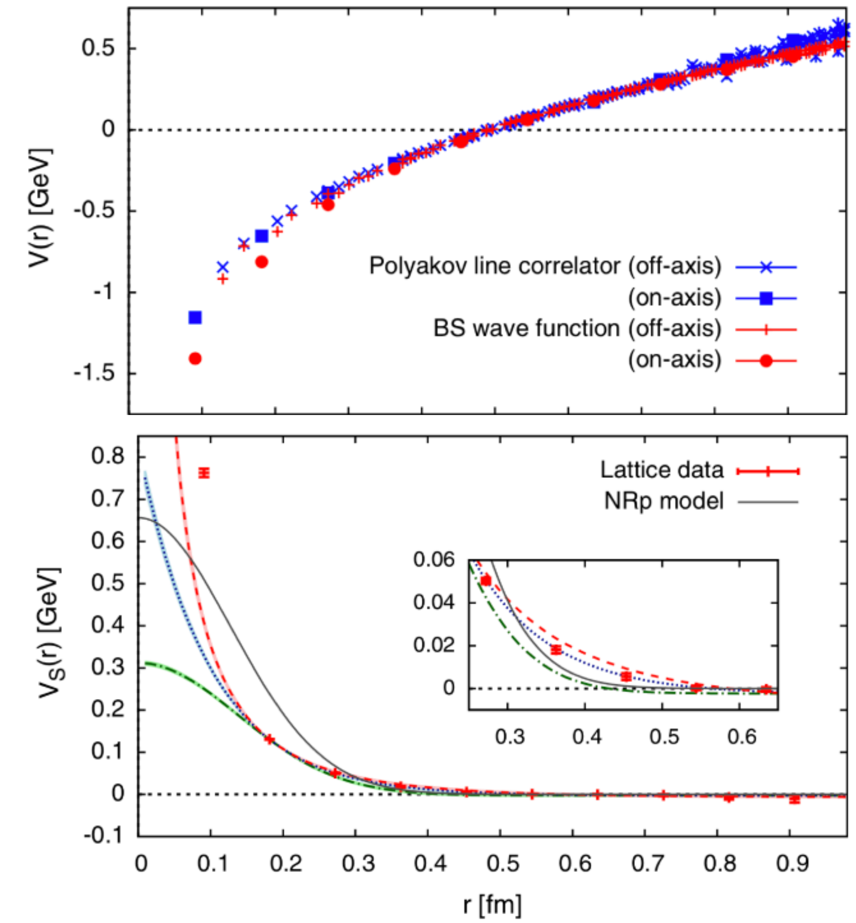

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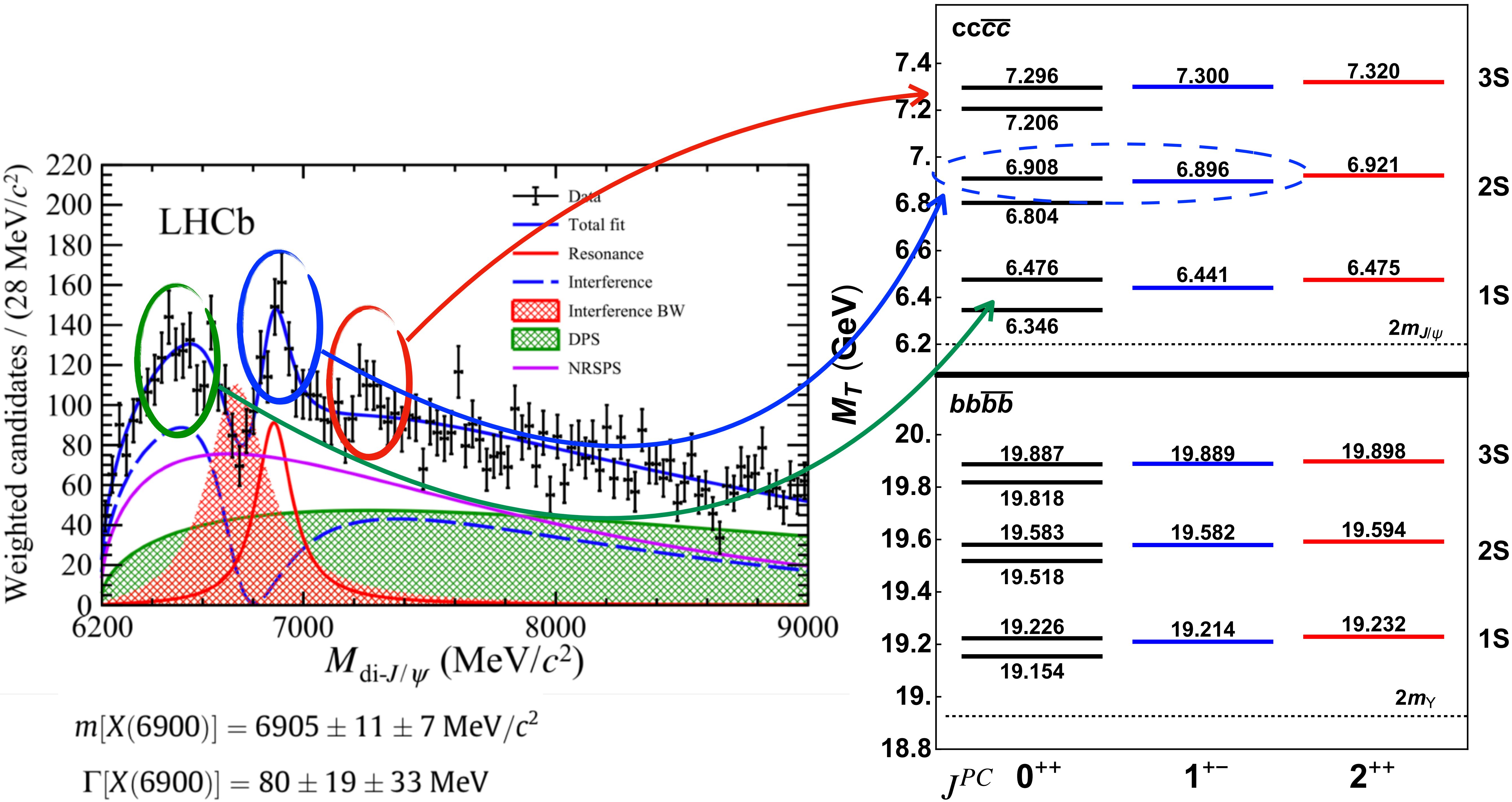
State	η_c	J/ψ	$h_c(1P)$	$\chi_c(1P)$	$\eta_c(2S)$	$\psi(2S)$	$\chi_c(2P)$
M_E (GeV)	2.981	3.097	3.525	3.556	3.639	3.696	3.927
M_T (GeV)	2.968	3.102	3.480	3.500	3.654	3.720	4.000

State	η_b	$\Upsilon(1S)$	$h_b(1P)$	$\chi_b(1P)$	$\eta_b(2S)$	$\Upsilon(2S)$	$\chi_b(2P)$
M_E (GeV)	9.398	9.460	9.898	9.912	9.999	10.023	10.269
M_T (GeV)	9.397	9.459	9.845	9.860	9.957	9.977	10.221

Agree very well with the experiment data !

Results



$$m[X(6900)] = 6905 \pm 11 \pm 7\ \mathrm{MeV}/c^2$$

$$\Gamma[X(6900)] = 80 \pm 19 \pm 33\ \mathrm{MeV}$$

Other potential states.

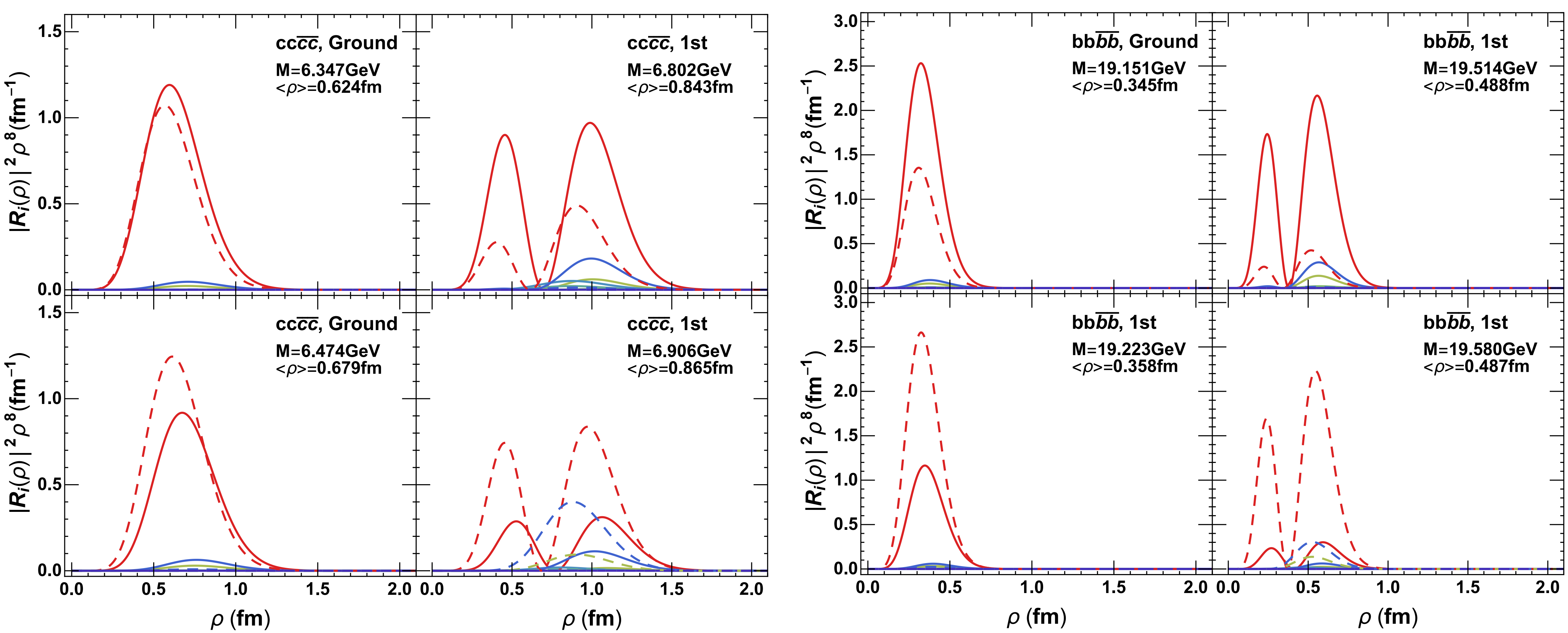
LHCb Collaboration, Science Bulletin, 2020, 65(23)1983-1993

Results

JX Zhao, ShZh. shi, and Pf. Zhuang, Phys.Rev.D 102 (2020) 11, 114001

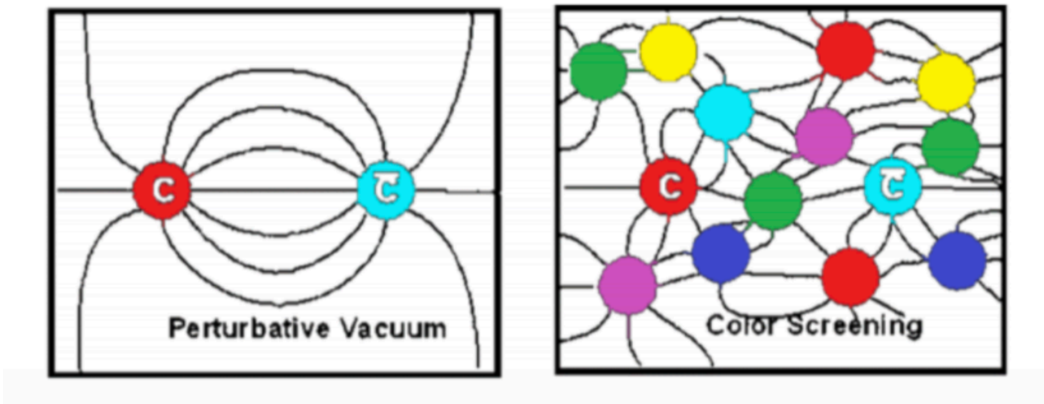
TABLE III: The calculated tetraquark mass M_T and the root-mean-squared radius $r_{\rm rms}$ for the ground and radial-excited states, $1S$, $2S$ and $3S$ of $cc\bar{c}\bar{c}$ and $bb\bar{b}\bar{b}$ with quantum numbers $J^{PC} = 0^{++}$, 1^{+-}, and 2^{++}.

	J^{PC}	0^{++}						1^{+-}			2^{++}		
	State	$1S$		$2S$		$3S$		$1S$	$2S$	$3S$	$1S$	$2S$	$3S$
$cc\bar{c}\bar{c}$	M_T(GeV)	6.346	6.476	6.804	6.908	7.206	7.296	6.441	6.896	7.300	6.475	6.921	7.320
	$r_{\rm rms}$(fm)	0.323	0.351	0.445	0.457	0.550	0.530	0.331	0.446	0.547	0.339	0.452	0.552
$bb\bar{b}\bar{b}$	M_T(GeV)	19.154	19.226	19.518	19.583	19.818	19.887	19.214	19.582	19.889	19.232	19.594	19.898
	$r_{\rm rms}$(fm)	0.180	0.186	0.259	0.259	0.328	0.325	0.181	0.257	0.324	0.183	0.259	0.326



Heavy Quark Potential at Finite-temperature

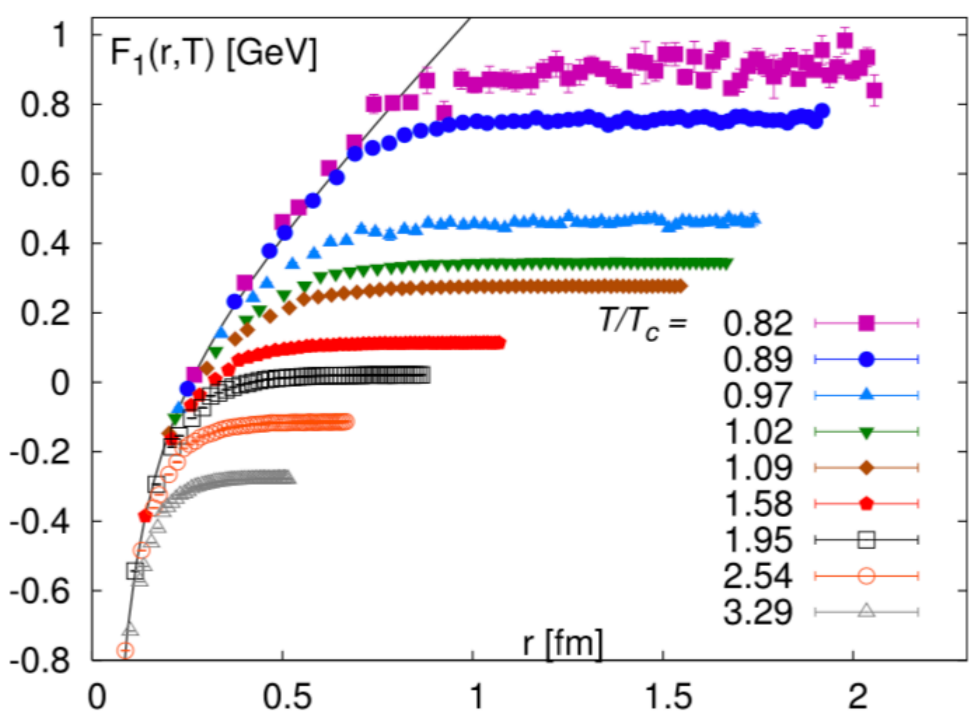
Color Screening in hot dense medium:



Hard Thermal Loop (HTL):

$$-\frac{\alpha}{r} \rightarrow -\frac{\alpha}{r}e^{-m_D r} \qquad m_D^2 = \frac{1}{3}g^2T^2\left(N_c + \frac{N_f}{2}\right)$$

Lattice QCD results:



P. Petreczky, J. Phys. G 37 (2010) 094009.

Results

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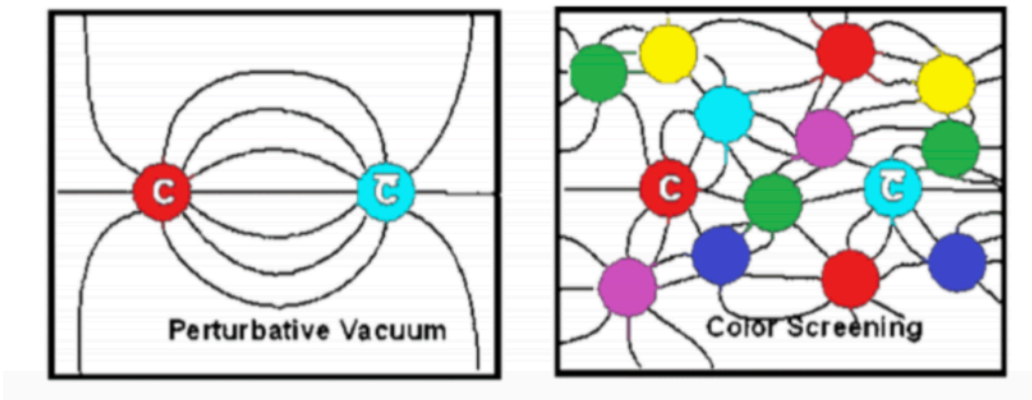

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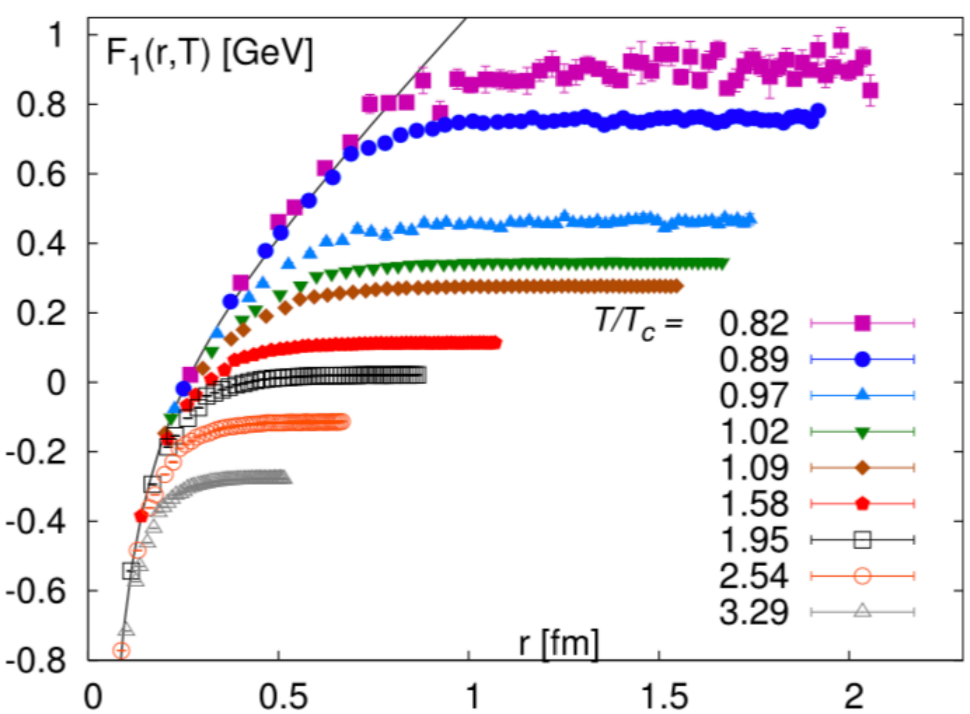

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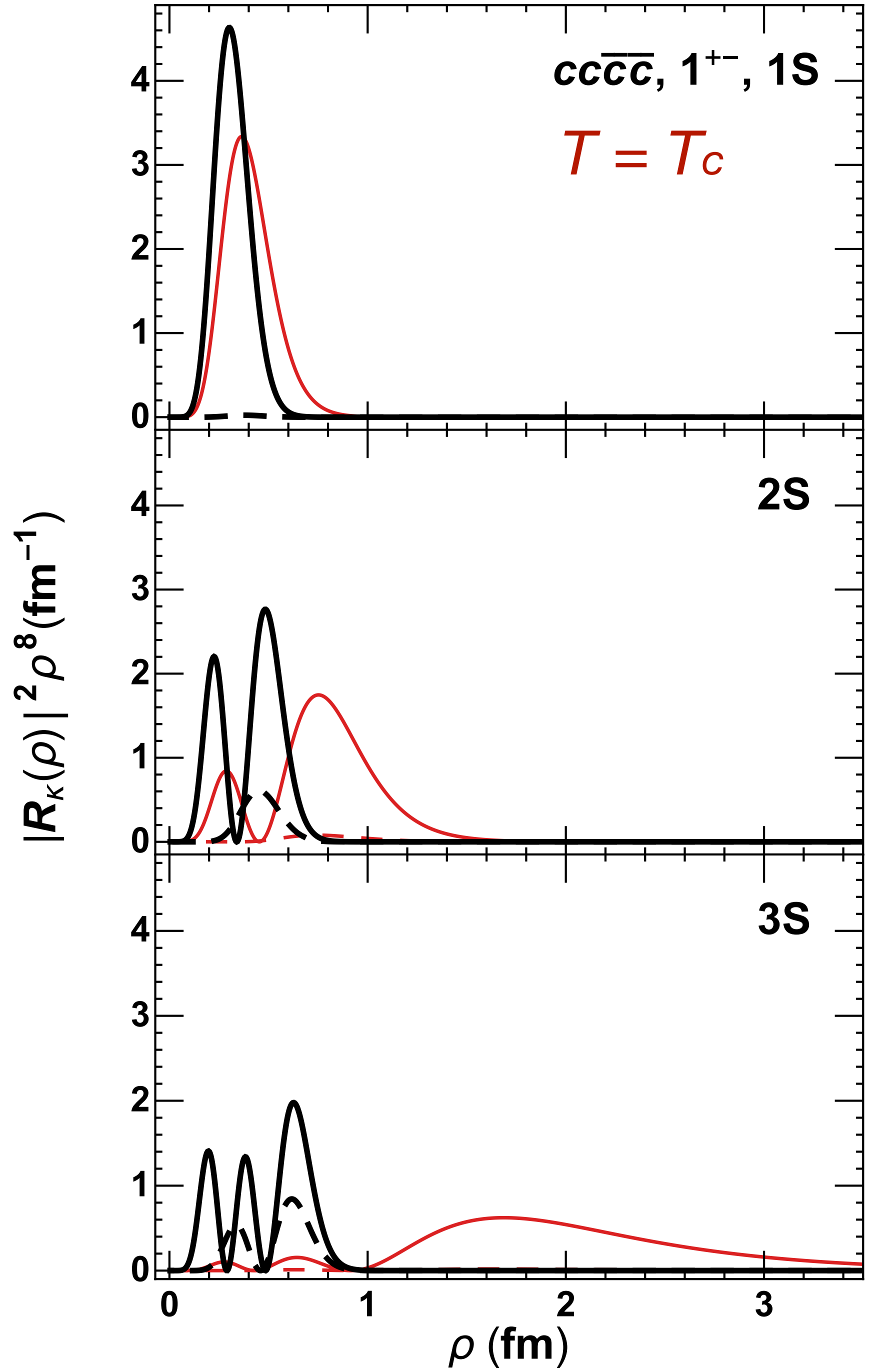
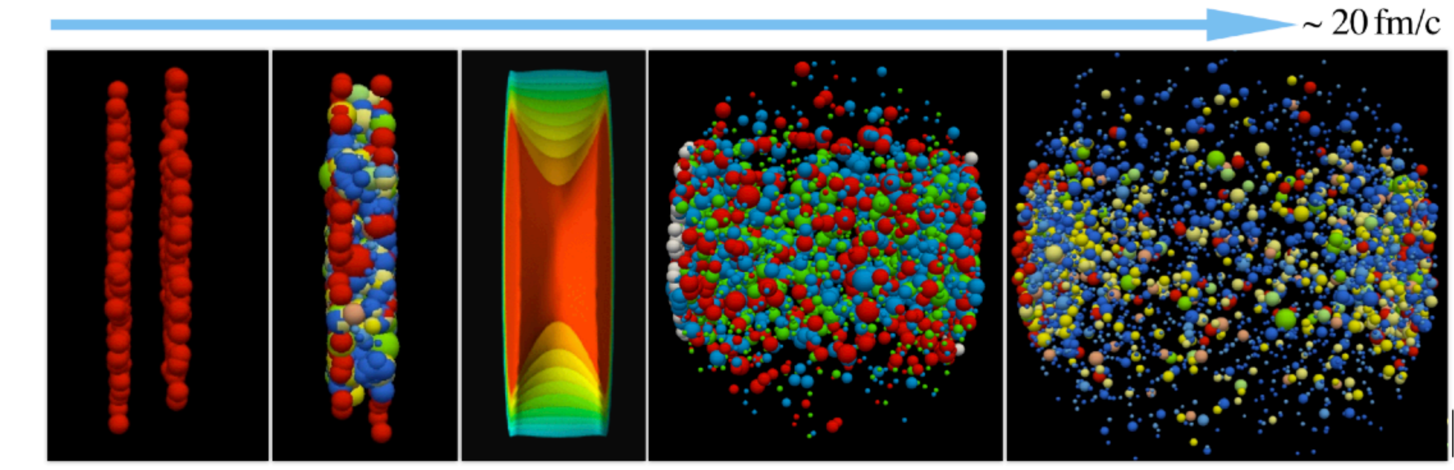
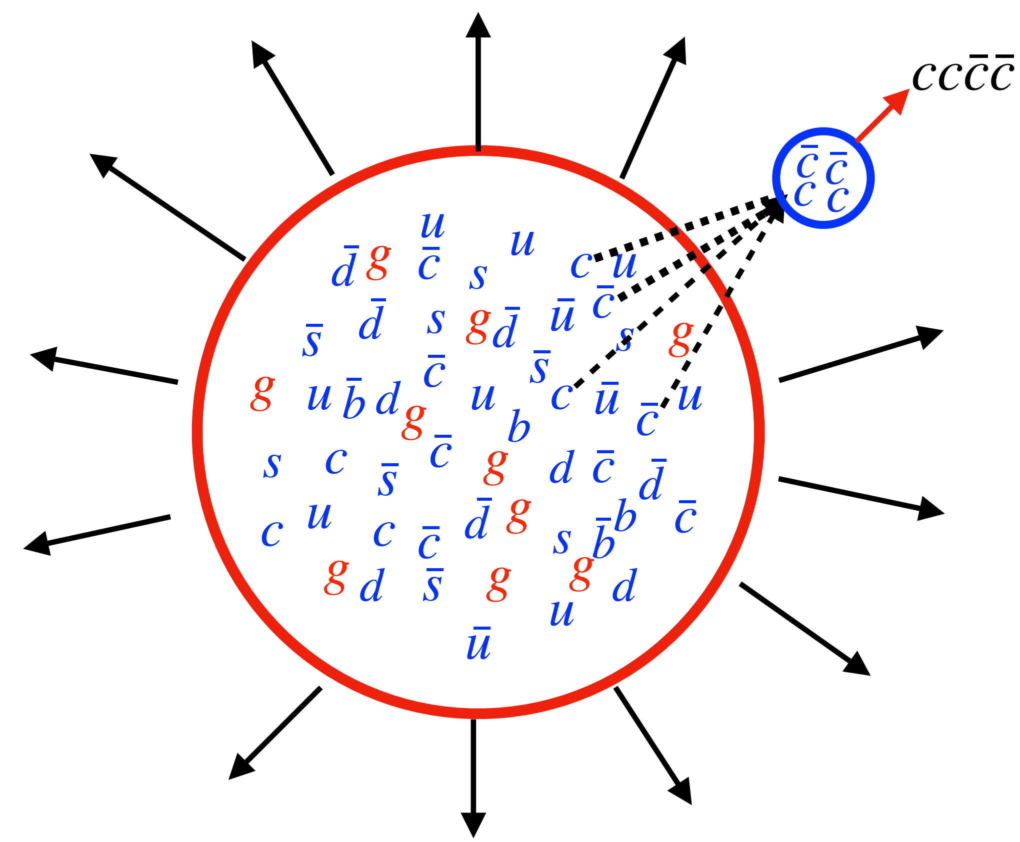
Wave function at QCD phase transition temperature Tc



	$cc\bar{c}\,\bar{c}$			$bb\bar{b}\,\bar{b}$		
	1S	2S	3S	1S	2S	3S
T_d/T_c	1.08	1.02	1.0	2.40	1.85	1.30

Dynamical production (color recombination on the phase boundary)





Heavy flavor initial production: pQCD

Evolve in the QGP medium: Langevin equation

Dynamic production: Coalescence model

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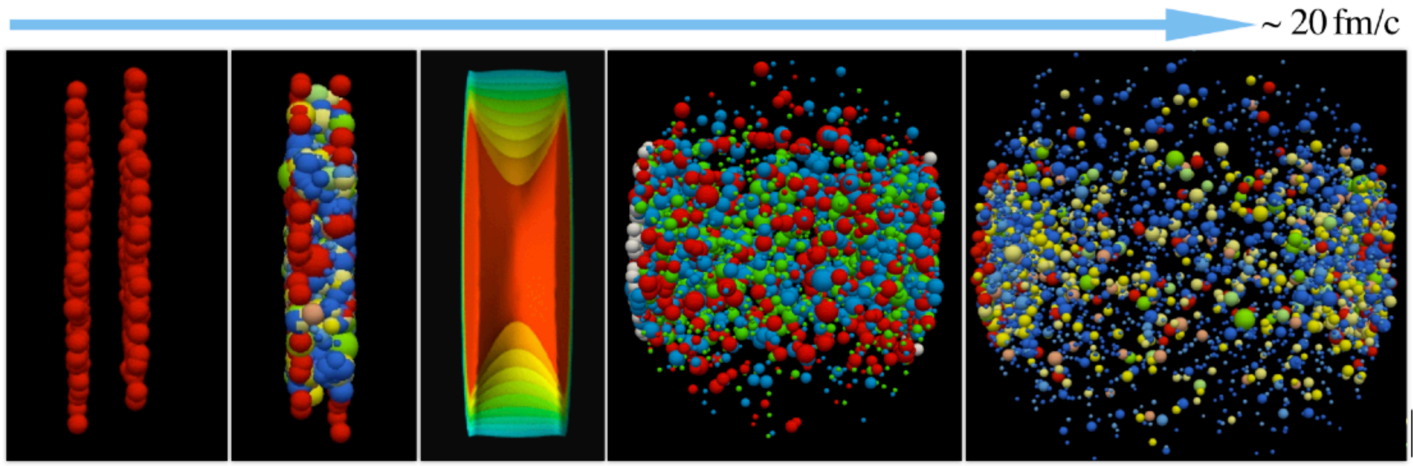
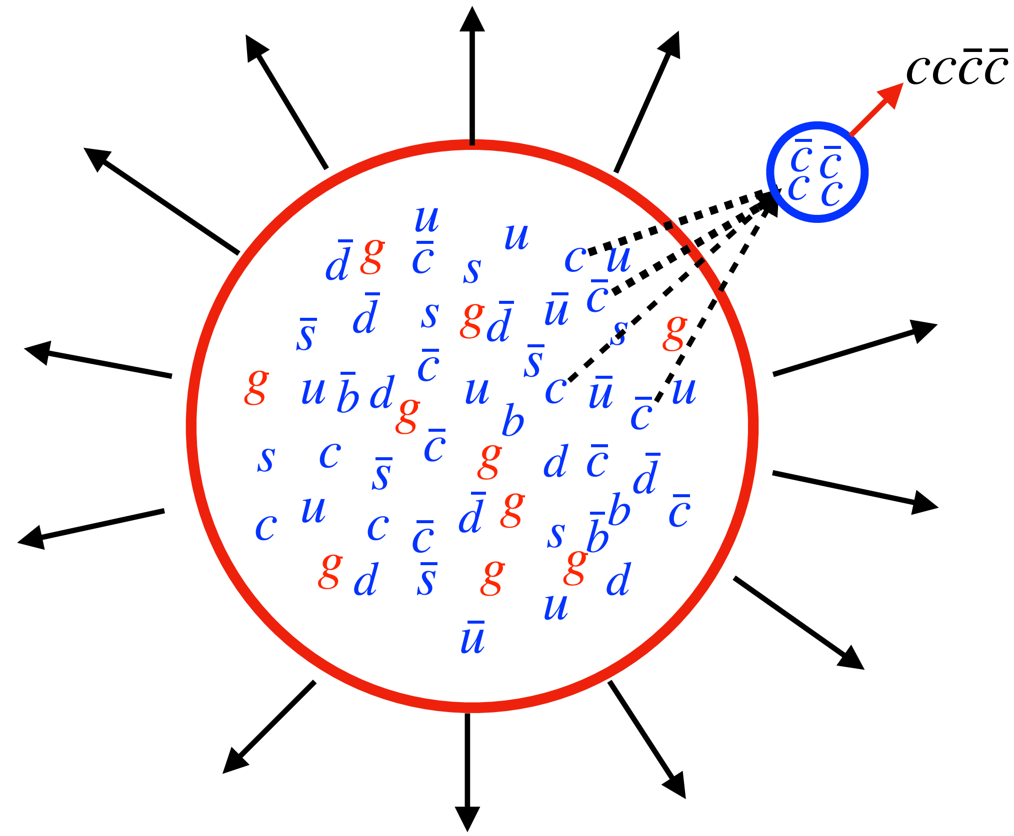




Heavy flavor initial production: pQCD

Evolve in the QGP medium: Langevin equation

Dynamic production: Coalescence model

$$\frac{dN}{d^2\mathbf{P}_T d\eta} = C\int_\Sigma \frac{P_\mu d\sigma_\mu}{(2\pi)^3}\int \frac{d^9\mathbf{x}d^9\mathbf{y}}{(2\pi)^9} F(r_1,r_2,r_3,r_4,p_1,p_2,p_3,p_4)W(\mathbf{x},\mathbf{p})$$

V. Greco, C. M. Ko and R. Rapp, Phys. Lett. B 595, 202 (2004).
D. Molnar and S. A. Voloshin, Phys. Rev. Lett. 91, 092301 (2003).
R. J. Fries, B. Muller, C. Nonaka and S. A. Bass. Phys. Rev. C68, 044902(2004).

- *The hadronization hypersurface is determined by hydrodynamics.*
- *The Wigner function can self-consistently be determined by the wavefunction.*

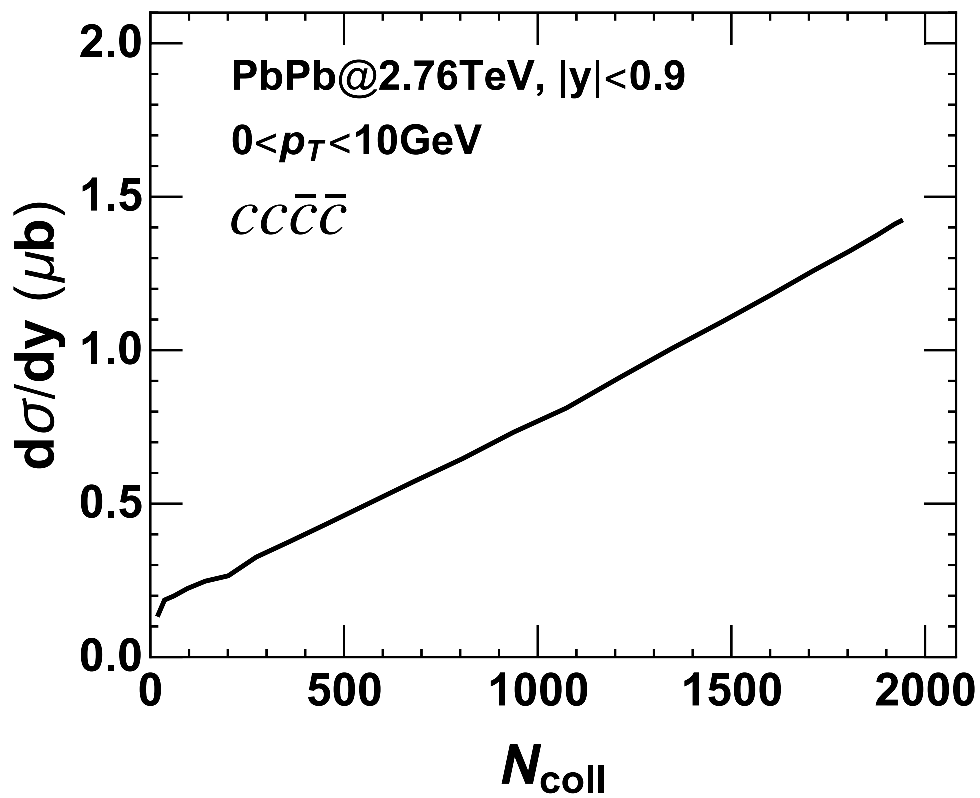
$$W(\mathbf{x},\mathbf{p},T) = \int d^9\mathbf{y}e^{-i\mathbf{p}\cdot\mathbf{y}}\Phi\left(\mathbf{x}+\frac{\mathbf{y}}{2},T\right)\Phi\left(\mathbf{x}-\frac{\mathbf{y}}{2},T\right).$$

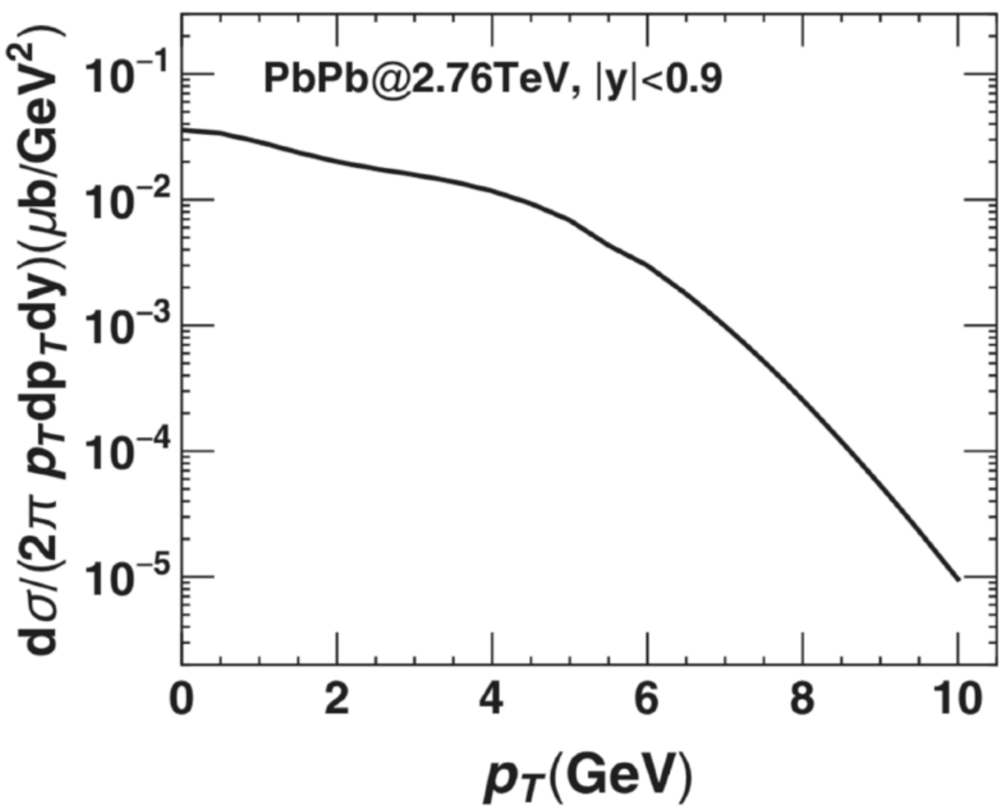
- *Heavy quark distribution functions*

$$F(r_1,r_2,r_3,r_4,p_1,p_2,p_3,p_4) = \frac{1}{4}f_Q(r_1,p_1)\,f_Q(r_2,p_2)\,f_{\bar{Q}}(r_3,p_3)\,f_{\bar{Q}}(r_4,p_4)$$

Results

Fully-heavy Tetraquark state production in heavy-ion collisions!





1 Pb+Pb collisions ~ 2000 p+p collisions.

$$\left.\frac{d\sigma}{N_{coll}dy}\right|_{AA} \approx 770pb$$ *in AA at 5.02TeV*

$$\left.\frac{d\sigma}{dy}\right|_{pp} = 78pb$$ *in pp at 7TeV*

Marek Karliner el al, Phys.Rev.D 95 (2017) 3, 034011.
Ruilin Zhu, arXiv: 2010.09082.

JX Zhao, ShZh. shi, and Pf. Zhuang, Phys.Rev.D 102 (2020) 11, 114001

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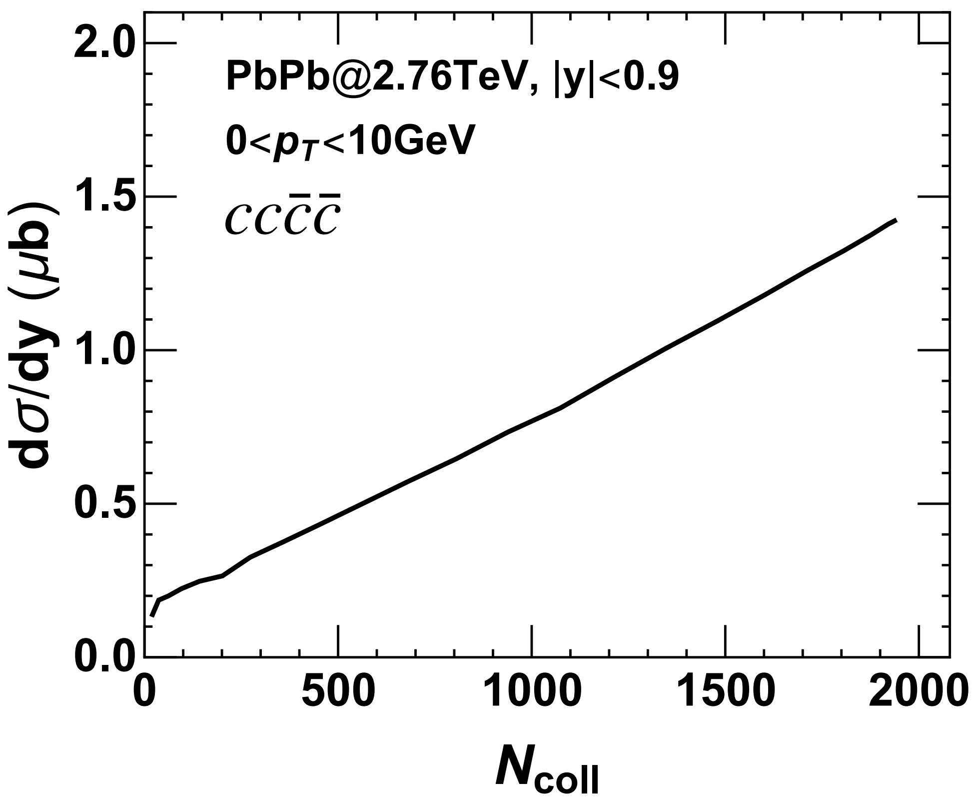

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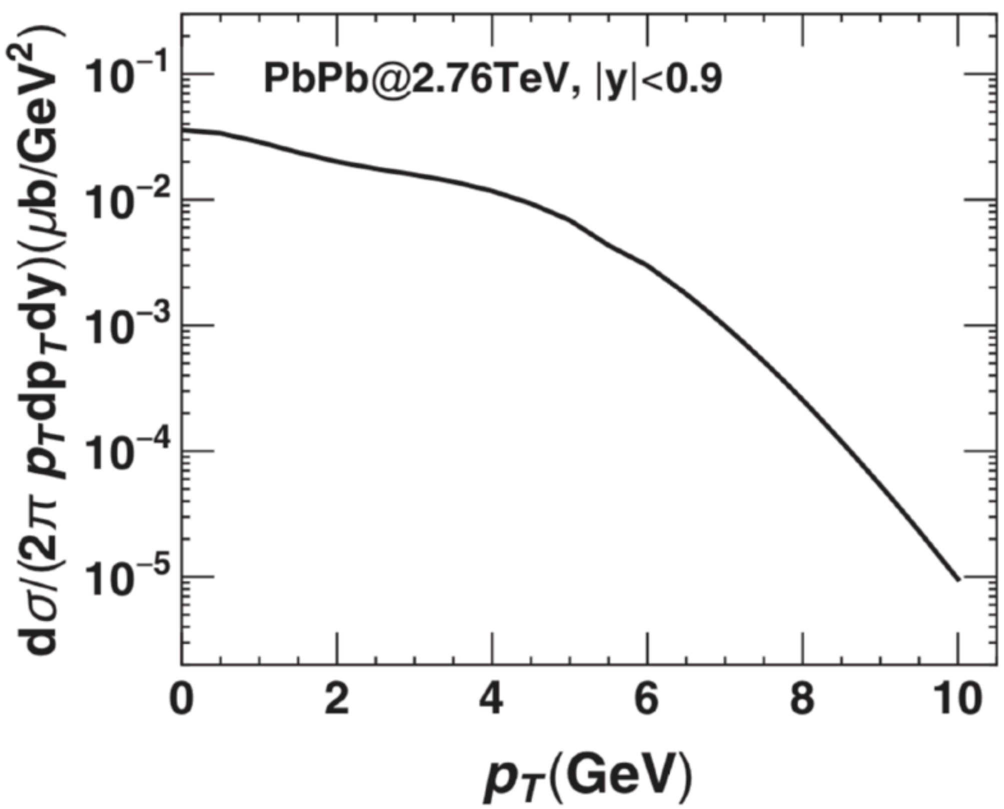

The four-lepton decay channel

$$X(cc\bar{c}\bar{c}) \rightarrow l_1^+ l_2^- l_3^+ l_4^-$$

can be well separated from the bulk back ground and makes it possible to find such exotic states in heavy-ion collision even in low pt region!

JX Zhao, ShZh. shi, and Pf. Zhuang, Phys.Rev.D 102 (2020) 11, 114001

Summary

We studied fully-heavy Tetraquark in vacuum and finite-temperature via 4-body Schroedinger equation.

1 Pb+Pb collisions ~ 2000 p+p collisions.
Many of charm, anti-charm quarks produced.
"New hadronization mechanism": color recombination.

yield significantly enhanced !

This supply a way to searching for fully-heavy Tetraquark states in the experiment.

Outlook

- *The angle excited(L=1, 2…) states of fully-heavy Tetraquark states.*
- *Searching for new observables of fully-heavy Tetraquark states in heavy-ion collisions.*

Thanks for your attention!

Backup

hyper-spherical harmonic function expansion (L = M = 0)

$$\mathcal{Y}_1 = \sqrt{\frac{105}{32}}\frac{1}{\pi^2},$$

$$\mathcal{Y}_2 = \sqrt{\frac{385}{6}}\frac{3}{16\pi^2}(3\cos(2\alpha_3) - 1),$$

$$\mathcal{Y}_3 = \sqrt{\frac{385}{2}}\frac{3}{8\pi^2}\cos(2\alpha_2)\cos^2(\alpha_3),$$

$$\mathcal{Y}_4 = -\sqrt{\frac{385}{2}}\frac{3}{4\pi^2}\cos\alpha_2 \sin\alpha_2 \cos^2\alpha_3 \times [\cos\theta_1 \cos\theta_2 + \cos(\phi_1 - \phi_2)\sin\theta_1 \sin\theta_2],$$

$$\mathcal{Y}_5 = -\sqrt{\frac{385}{2}}\frac{3}{4\pi^2}\cos\alpha_2 \cos\alpha_3 \sin\alpha_3 \times [\cos\theta_1 \cos\theta_3 + \cos(\phi_1 - \phi_3)\sin\theta_1 \sin\theta_3],$$

$$\mathcal{Y}_6 = -\sqrt{\frac{385}{2}}\frac{3}{4\pi^2}\sin\alpha_2 \cos\alpha_3 \sin\alpha_3 \times [\cos\theta_2 \cos\theta_3 + \cos(\phi_2 - \phi_3)\sin\theta_2 \sin\theta_3],$$

$$\mathcal{Y}_7 = i\sqrt{5005}\frac{3}{8\pi^2}\sin\alpha_2 \cos\alpha_2 \sin\alpha_3 \cos^2\alpha_3 \times [\cos\theta_3 \sin\theta_1 \sin\theta_2 \sin(\phi_1 - \phi_2) - \sin\theta_3 \cos\theta_2 \sin\theta_1 \sin(\phi_1 - \phi_3) + \sin\theta_3 \cos\theta_1 \sin\theta_2 \sin(\phi_2 - \phi_3)].$$

$$V_1 = \langle \phi_1\chi_2 | \sum_{i<j} V_{ij} | \phi_1\chi_2 \rangle = \frac{2}{3}(V^c_{12} + V^c_{34}) + \frac{1}{3}(V^c_{13} + V^c_{14} + V^c_{23} + V^c_{24}) + \frac{1}{6}(V^{ss}_{12} + V^{ss}_{34}) - \frac{1}{6}(V^{ss}_{13} + V^{ss}_{14} + V^{ss}_{23} + V^{ss}_{24}),$$

$$V_2 = \langle \phi_2\chi_1 | \sum_{i<j} V_{ij} | \phi_2\chi_1 \rangle = -\frac{1}{3}(V^c_{12} + V^c_{34}) + \frac{5}{6}(V^c_{13} + V^c_{14} + V^c_{23} + V^c_{24}) + \frac{1}{4}(V^{ss}_{12} + V^{ss}_{34}),$$

$$V_m = \langle \phi_1\chi_2 | \sum_{i<j} V_{ij} | \phi_2\chi_1 \rangle = \langle \phi_2\chi_1 | \sum_{i<j} V_{ij} | \phi_1\chi_2 \rangle = -\frac{\sqrt{6}}{8}(V^{ss}_{13} + V^{ss}_{14} + V^{ss}_{23} + V^{ss}_{24}),$$

$$V = \langle \phi_1\chi_5 | \sum_{i<j} V_{ij} | \phi_1\chi_5 \rangle = \frac{2}{3}(V^c_{12} + V^c_{34}) + \frac{1}{3}(V^c_{13} + V^c_{14} + V^c_{23} + V^c_{24}) + \frac{1}{6}(V^{ss}_{12} + V^{ss}_{34}) - \frac{1}{12}(V^{ss}_{13} + V^{ss}_{14} + V^{ss}_{23} + V^{ss}_{24}),$$

Many excellent review papers :

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E. Klempt, JM. Richard, Rev. Mod. Phys. 82 (2010) 1095–1153.

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A. Ali, JS. Lange, S. Stone. Prog. Part. Nucl. Phys. 97 (2017) 123–198.

FK. Guo, C. Hanhart, U. Meißner, Q. Wang, Q. Zhao. Rev. Mod. Phys. 90 (2018) 015004.

YR. Liu, HX. Chen, W. Chen, X. Liu, SL. Zhu. Prog. Part. Nucl. Phys. 107 (2019) 237–320.

N. Brambilla, S. Eidelman, C. Hanhart, A. Nefediev, CP. Shen, CE. Thomas, A. Vairo, ChZh. Yuan. Phys. Rept. 873 (2020) 1-154.

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