

# $\Xi_c \rightarrow \Xi$ Form Factors and $\Xi_c \rightarrow \Xi l^+ \nu_l$ Decay Rates From Lattice QCD

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W.Wang and Y.B.Yang.

March 27, 2021

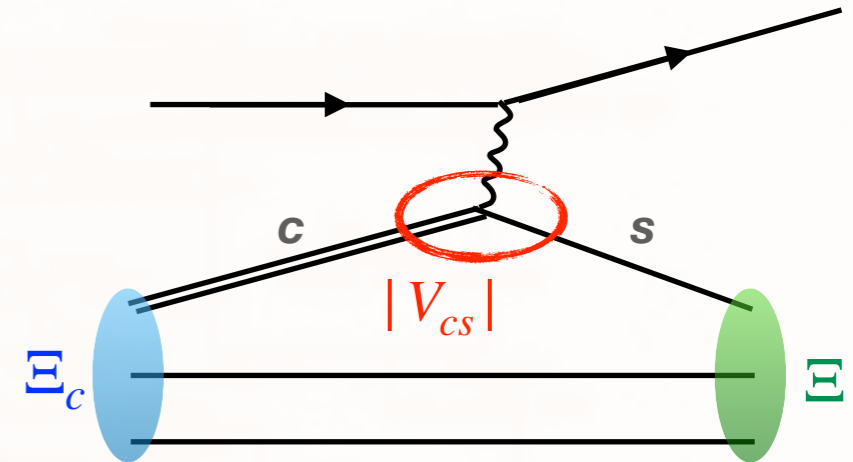
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# OUTLINE

- **Motivation**
- **Semi-leptonic  $\Xi_c$  decays and helicity-based form factors**
- **Lattice calculations of  $\Xi_c \rightarrow \Xi$  form factors**
- **Differential decay width and branching fractions from lattice results**
- **Determination of  $|V_{cs}|$**
- **Summary**

# Motivation

- Determine **CKM matrix elements**  $|V_{cs}|$ ;
- Provide a **stringent test and first-principle verification** for theoretical models;



- Important for the experimental researches of heavy baryons:

- Studies of doubly-charmed baryon  $\Xi_{cc}^{++}$  decay

R. Aaij et al. [LHCb], PRL121, 162002 (2018)

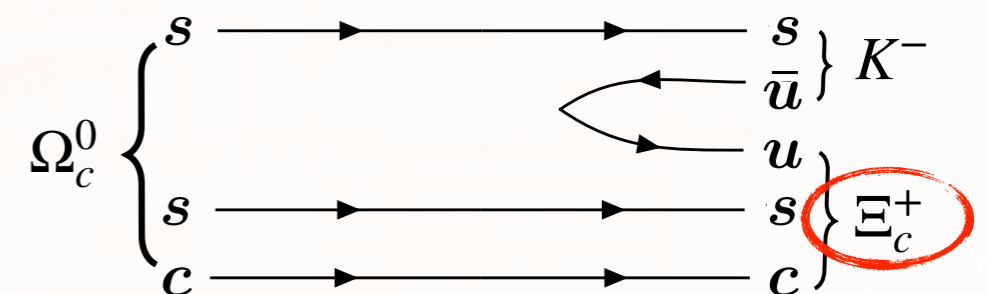
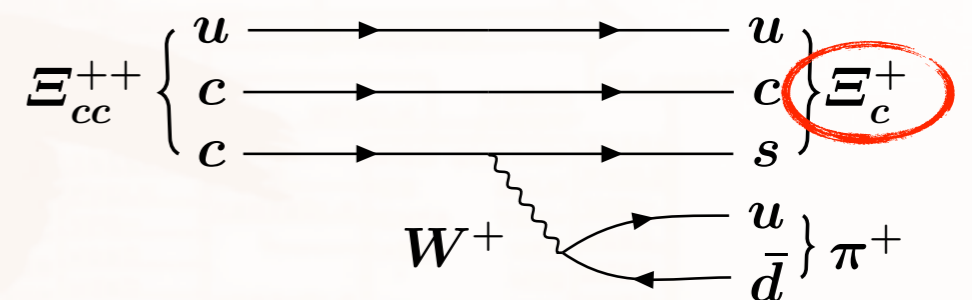
- Precision measurement of the lifetime of  $\Xi_b^0$

R. Aaij et al. [LHCb], PRL113, 032001 (2014)

- Discovery of new exotic hadron candidates  $\Omega_c$

R. Aaij et al. [LHCb], PRL118, 182001 (2017)

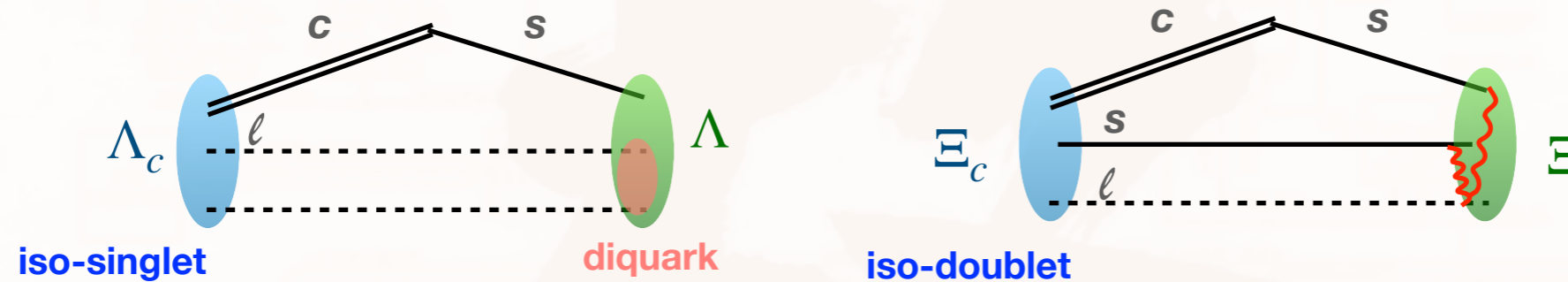
- .....



# Motivation

- $\Xi_c$  contains more **versatile decay modes**

- $\Xi_c \rightarrow \Xi$  contain different QCD dynamics with  $\Lambda_c \rightarrow \Lambda$ ;



- **A different pattern** between inclusive and exclusive decays of  $\Lambda_c$  and  $D$ :

$$\mathcal{B}(\Lambda_c^+ \rightarrow X e^+ \nu_e) = (3.95 \pm 0.34 \pm 0.09) \%$$

$$\mathcal{B}(\Lambda_c^+ \rightarrow \Lambda e^+ \nu_e) = (3.63 \pm 0.38 \pm 0.20) \%$$

**~ 1**

$$\mathcal{B}(D^0 \rightarrow X e^+ \nu_e) = (6.49 \pm 0.11) \%$$

$$\mathcal{B}(D^0 \rightarrow K^- e^+ \nu_e) = (3.542 \pm 0.035) \%$$

**~ 2**

**M. Ablikim et al. [BESIII], PRL121, 251801 (2018)**

# Motivation

## ✓ Experimental

**Belle**  $\mathcal{B}(\Xi_c^0 \rightarrow \Xi^- e^+ \nu_e) = (1.72 \pm 0.10 \pm 0.12 \pm 0.50) \%$

Y. B. Li et al. [Belle],  
arXiv:2103.06496 [hep-ex].

$\mathcal{B}(\Xi_c^0 \rightarrow \Xi^- \mu^+ \nu_\mu) = (1.71 \pm 0.17 \pm 0.13 \pm 0.50) \%$

**ALICE**  $\mathcal{B}(\Xi_c^0 \rightarrow \Xi^- e^+ \nu_e) = (2.43 \pm 0.25 \pm 0.35 \pm 0.72) \%$

J. Zhu on behalf of the ALICE  
collaboration, PoS ICHEP2020  
(2021) 524.

## ✓ Theoretical

**QCD SR**  $\mathcal{B}(\Xi_c^0 \rightarrow \Xi^- e^+ \nu_e) = (3.4 \pm 1.7) \%$

Z. X. Zhao, arXiv:2103.09436 [hep-ph].

**LF QM**  $\mathcal{B}(\Xi_c^0 \rightarrow \Xi^- e^+ \nu_e) = (3.49 \pm 0.95) \%$

C. Q. Geng et al, arXiv:2012.04147 [hep-ph].

**LCSR**  $\mathcal{B}(\Xi_c^0 \rightarrow \Xi^- e^+ \nu_e) = (2.4_{-1.0}^{+0.9}) \%$

Y. L. Liu et al, J. Phys. G 37, 115010 (2010).

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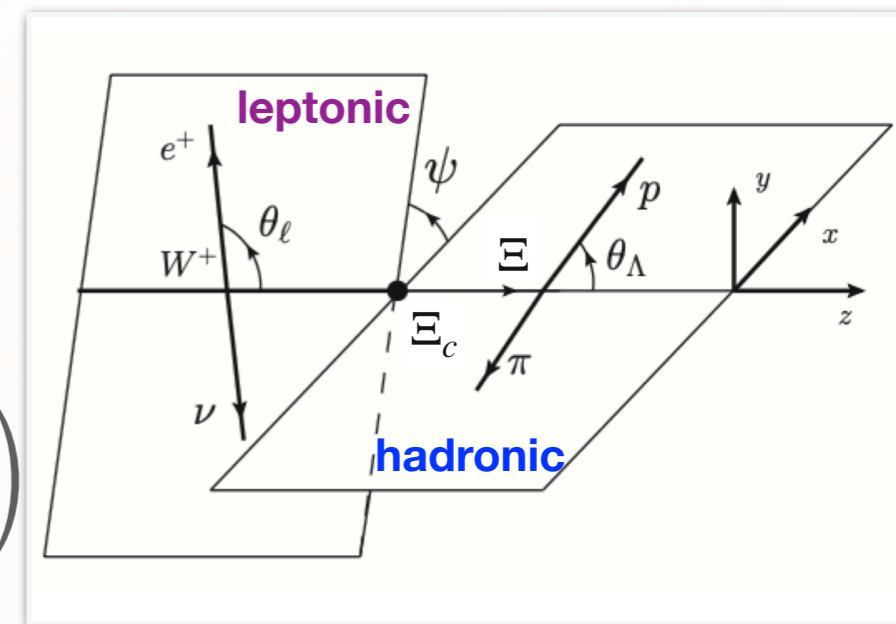
**Lattice** ?

# Semileptonic $\Xi_c$ decays:

- Helicity amplitude

$$\begin{aligned}
 \mathcal{M}(\Xi_c \rightarrow \Xi l^+ \nu_l) &= \frac{G_F}{\sqrt{2}} V_{cs} \langle \Xi | \bar{s} \gamma^\mu (1 - \gamma_5) c | \Xi_c \rangle \langle \bar{l} \nu_l | \bar{\nu} \gamma^\nu (1 - \gamma_5) l | 0 \rangle g_{\mu\nu} \\
 &= \frac{G_F}{\sqrt{2}} V_{cs} \left( H^\mu \epsilon_\mu^*(t) \times L^\nu \epsilon_\nu(t) - \sum_\lambda H^\mu \epsilon_\mu^*(\lambda) \times L^\nu \epsilon_\nu(\lambda) \right)
 \end{aligned}$$

Hadronic part      Leptonic part

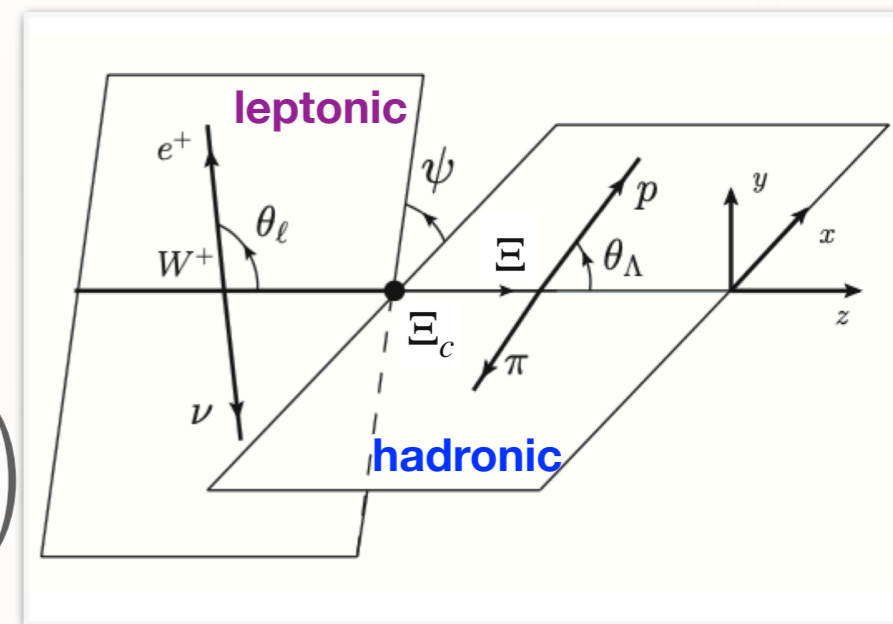


# Semileptonic $\Xi_c$ decays:

- **Helicity amplitude**

$$\begin{aligned} \mathcal{M}(\Xi_c \rightarrow \Xi l^+ \nu_l) &= \frac{G_F}{\sqrt{2}} V_{cs} \langle \Xi | \bar{s} \gamma^\mu (1 - \gamma_5) c | \Xi_c \rangle \langle \bar{l} \nu_l | \bar{\nu} \gamma^\nu (1 - \gamma_5) l | 0 \rangle g_{\mu\nu} \\ &= \frac{G_F}{\sqrt{2}} V_{cs} \left( H^\mu \epsilon_\mu^*(t) \times L^\nu \epsilon_\nu(t) - \sum_\lambda H^\mu \epsilon_\mu^*(\lambda) \times L^\nu \epsilon_\nu(\lambda) \right) \end{aligned}$$

Hadronic part      Leptonic part



- **Hadronic matrix element: Helicity-base form factor parametrization**

$$\begin{aligned} \langle \Xi(P_\Xi) | V^\mu | \Xi_c(P_{\Xi_c}) \rangle &= \bar{u}(P_\Xi, S_2) \left[ \left( m_{\Xi_c} - m_\Xi \right) \frac{q^\mu}{q^2} f_0(q^2) + \frac{m_{\Xi_c} + m_\Xi}{s_+} \left( P_{\Xi_c}^\mu + P_\Xi^\mu - \frac{q^\mu}{q^2} (m_{\Xi_c}^2 - m_\Xi^2) \right) f_+(q^2) \right. \\ &\quad \left. + \left( \gamma^\mu - \frac{2m_\Xi}{s_+} P_{\Xi_c}^\mu - \frac{2m_{\Xi_c}}{s_+} P_\Xi^\mu \right) f_\perp(q^2) \right] u(P_{\Xi_c}), \end{aligned}$$

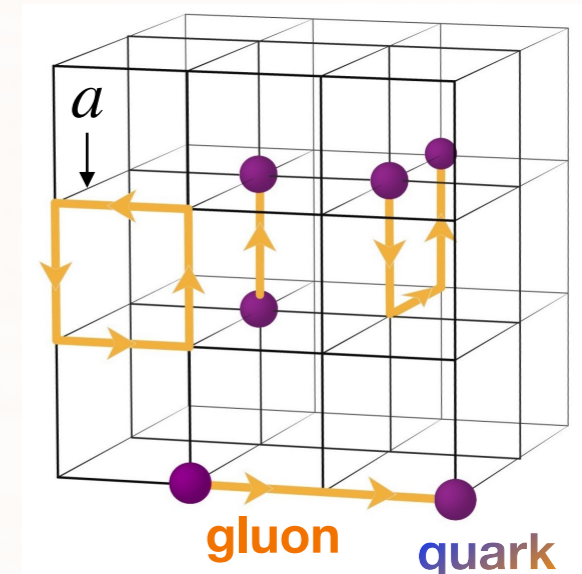
**vector**

$$\begin{aligned} \langle \Xi(P_\Xi) | A^\mu | \Xi_c(P_{\Xi_c}) \rangle &= -\bar{u}(P_\Xi, S_2) \gamma^5 \left[ \left( m_{\Xi_c} + m_\Xi \right) \frac{q^\mu}{q^2} g_0(q^2) + \frac{m_{\Xi_c} - m_\Xi}{s_-} \left( P_{\Xi_c}^\mu + P_\Xi^\mu - \frac{q^\mu}{q^2} (m_{\Xi_c}^2 - m_\Xi^2) \right) g_+(q^2) \right. \\ &\quad \left. + \left( \gamma^\mu + \frac{2m_\Xi}{s_-} P_{\Xi_c}^\mu - \frac{2m_{\Xi_c}}{s_-} P_\Xi^\mu \right) g_\perp(q^2) \right] u(P_{\Xi_c}) \end{aligned}$$

**axial-vector**

# Helicity based form factors

- **Helicity based form factors:**  $f_{\perp}, f_0, f_+, g_{\perp}, g_0, g_+$ 
  - **Non-perturbative**
  - QCD sum rules, light-cone sum rules, light-front quark model, ...  
=> **model dependent**
- **Lattice QCD: a non-perturbative theory from the first principle of QCD**
  - **Discretization, and wick rotation to Euclidean spacetime;**
  - **Quark fields on lattice sites, gauge links connecting the sites;**
  - **Pion mass to benchmark the light quark mass;**
  - **Able to calculate the non-perturbative observables:**



Credit: M. Savage@NNPSS2015

$$\langle O \rangle = \frac{1}{Z} \int D\psi D\bar{\psi} DA e^{-S_E} O$$



# Lattice setup

- This work is based on **2+1 flavor** ensembles generated with tree level tadpole improved clover fermion action and tadpole improved Symanzik gauge action;

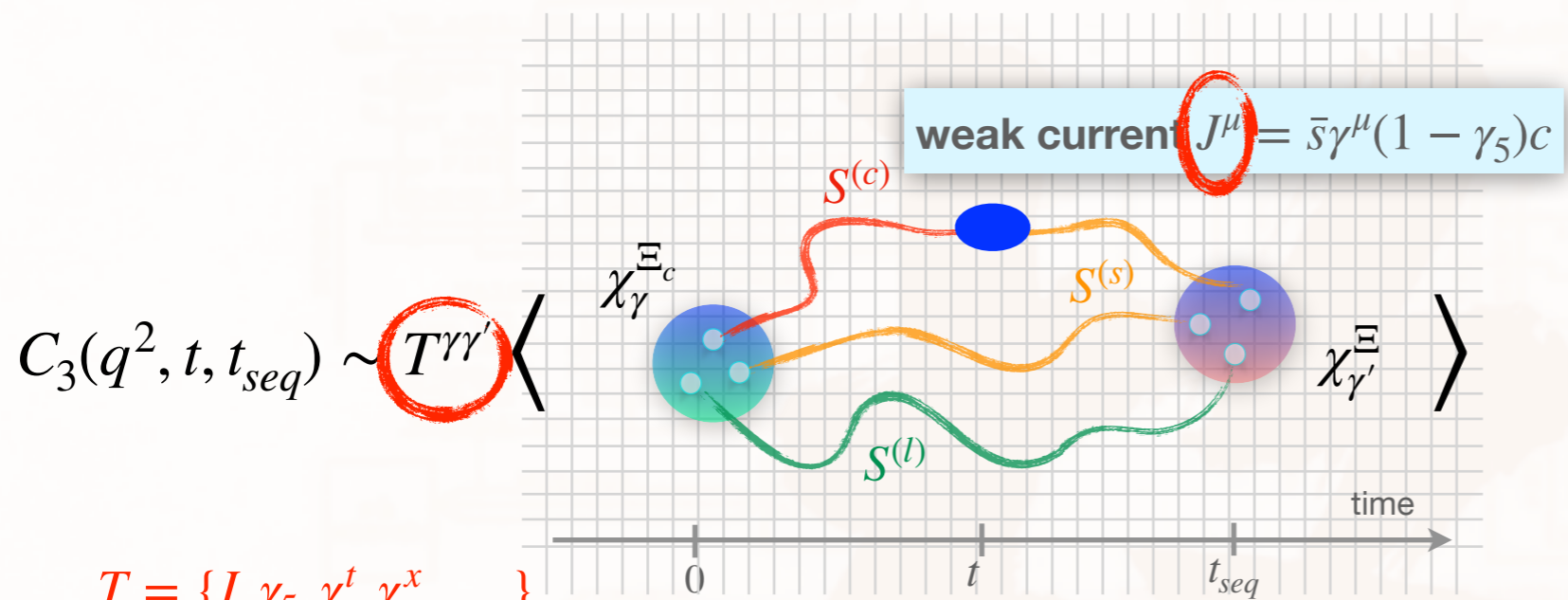
- **Basic informations of two ensembles used in this calculation;**

	$\beta = \frac{10}{g^2}$	$L^3 \times T$	a	$c_{\text{sw}}$	$\kappa_l$	$m_\pi$	$\kappa_s$	$m_{\eta_s}$
s108	6.20	$24^3 \times 72$	0.108	1.161	-0.2770	290	0.1330	640
s080	6.41	$32^3 \times 96$	0.080	1.141	-0.2295	300	0.1318	650

- Determining the **charm quark mass** by requiring the  $J/\psi$  mass to its physical value  $m_{J/\psi} = 3.96900(6)$  GeV within 0.3% accuracy:  $m_c^{s108} a = 0.485$ ,  
 $m_c^{s080} a = 0.235$ .

# Extract bare $\Xi_c \rightarrow \Xi$ form factors

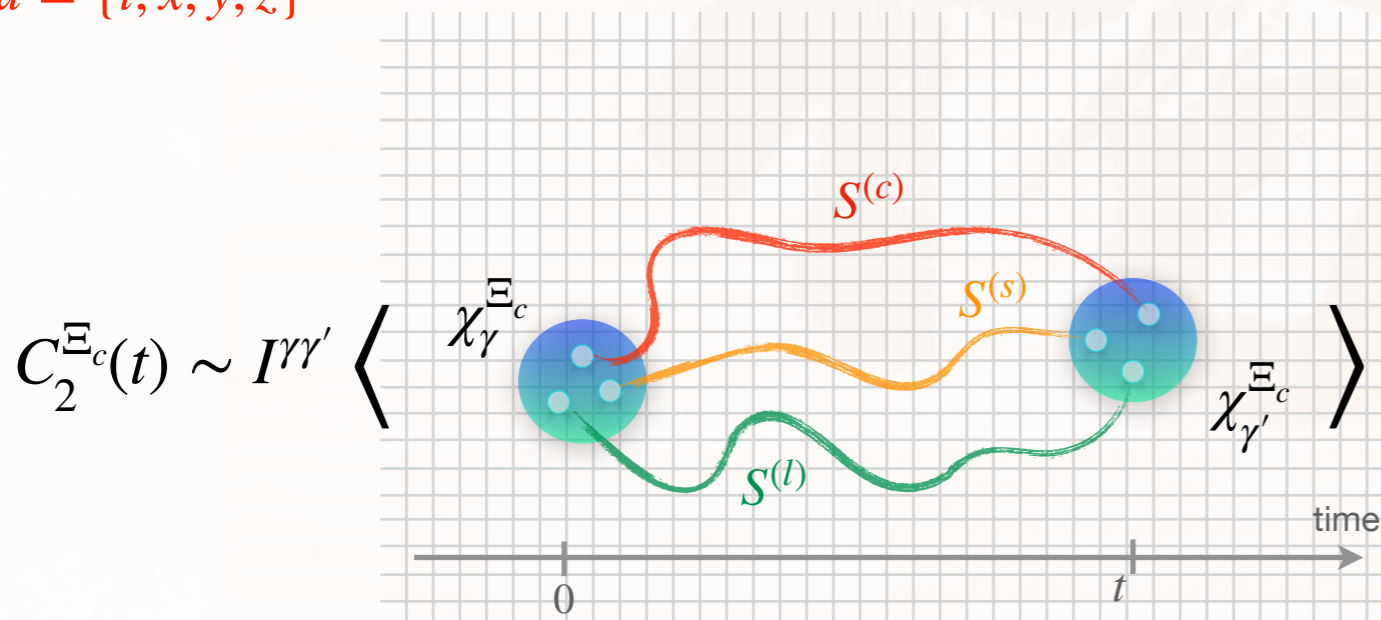
- Correlation functions on lattice



$T = \{I, \gamma_5, \gamma^t, \gamma^x, \dots\}$   
 $\mu = \{t, x, y, z\}$

- Ratios for different projections and currents:

$$R_{V/A}(T, \mu) = \sqrt{\frac{C_3^{V/A}(q^2, t, t_{seq}) C_3^{V/A}(q^2, t_{seq} - t, t_{seq})}{C_2^{B_1}(t_{seq}) C_2^{B_2}(t_{seq})}}$$



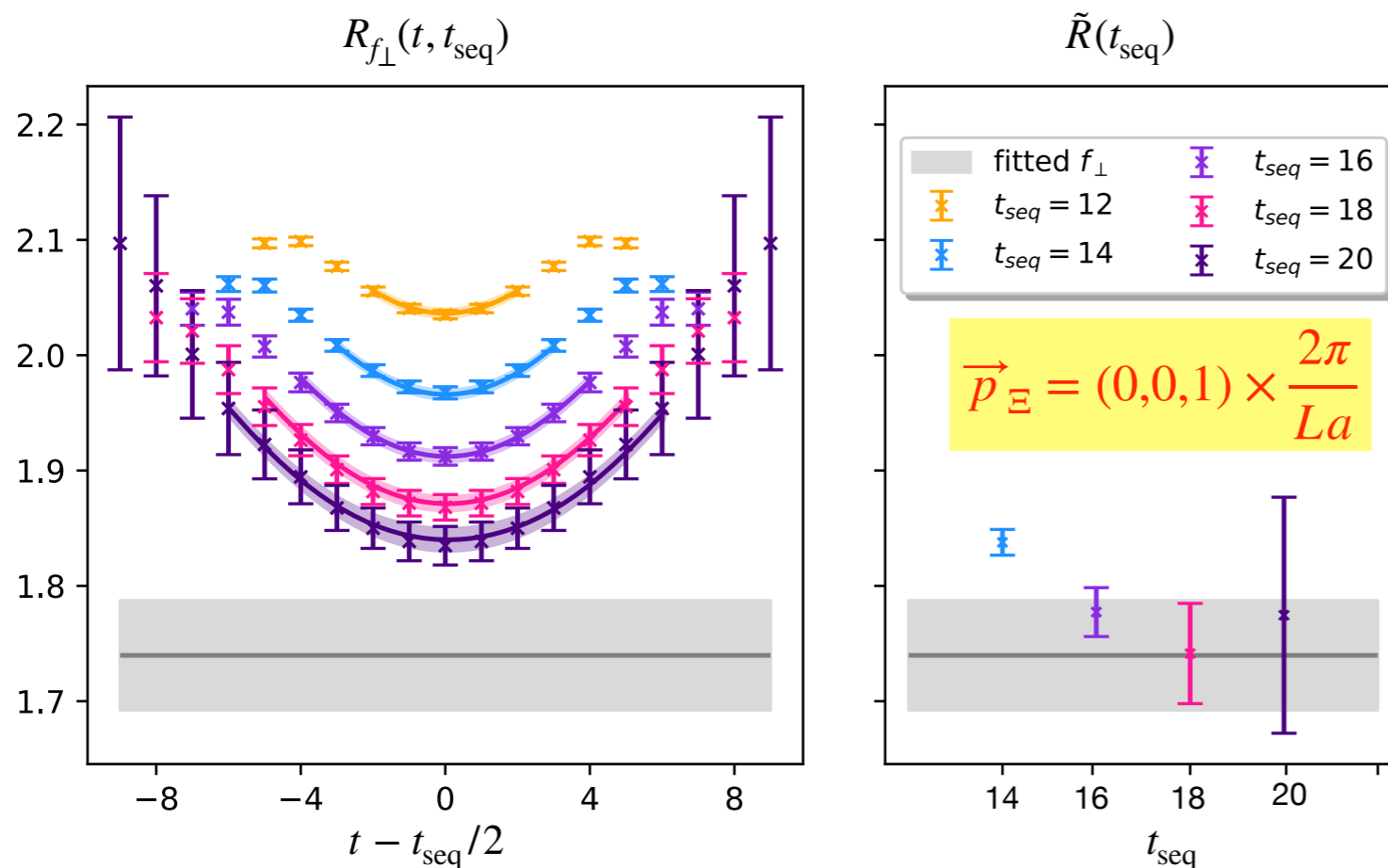
- Ratios for the six form factors can be constructed by different combinations of  $R_{V/A}(T, \mu)$ .

# Extract bare $\Xi_c \rightarrow \Xi$ form factors

- Ratios for the six form factors can be constructed by different combinations of  $R_{V/A}(T, \mu)$ :

$$R_{f_\perp} \equiv \frac{R_V(\gamma_5 \gamma^x, \gamma^y)}{4m_{\Xi_c} N_z \hat{p}} = f_\perp \left( \frac{\left( 1 + c_1 e^{-\Delta E_1 t} + c_2 e^{-\Delta E_2 (t_{\text{seq}} - t)} \right) \left( 1 + c_1 e^{-\Delta E_1 (t_{\text{seq}} - t)} + c_2 e^{-\Delta E_2 t} \right)}{(1 + d_1 e^{-\Delta E_1 t_{\text{seq}}}) (1 + d_2 e^{-\Delta E_2 t_{\text{seq}}})} \right)^{1/2}$$

contributions from excited states



- The differential summed ratio for validity check:

$$\tilde{R}(t_{\text{seq}}) \equiv \frac{SR(t_{\text{seq}}) - SR(t_{\text{seq}} - \Delta t)}{\Delta t}$$

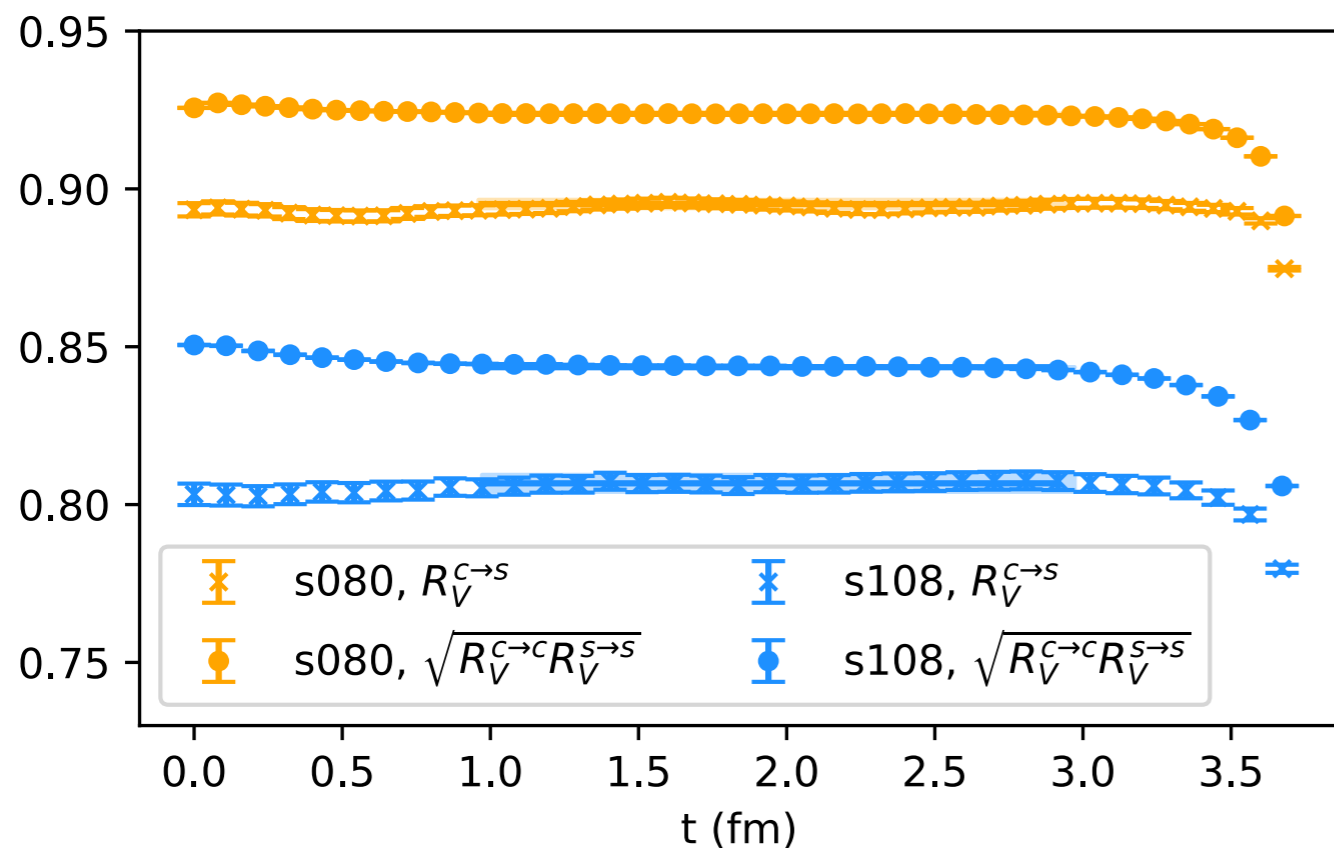
$$SR(t_{\text{seq}}) \equiv \sum_{t_c < t < t_{\text{seq}} - t_c} R_F(t, t_{\text{seq}})$$

# Renormalization factors for $c \rightarrow s$ current

- Use the ratio of **conserve current** and vector current to determine the renormalization factor  $Z_V$  for vector current:

$$R_V^{q_1 \rightarrow q_2}(t) = \frac{\langle M_1(T/2) \sum_{\vec{x}} V_{\text{cons.}}^{q_1 \rightarrow q_2}(\vec{x}, t) M_2(0) \rangle}{\langle M_1(T/2) \sum_{\vec{x}} V^{q_1 \rightarrow q_2}(\vec{x}, t) M_2(0) \rangle} = Z_V^{q_1 \rightarrow q_2} + \mathcal{O}(e^{-T/4\Delta E})$$

$$\bar{q}_1 V_{\mu}^{\text{cons.}} q_2 = \bar{q}_1(x) \frac{1 - \gamma_{\mu}}{2} U_{\hat{\mu}}(x) q_2(x + \hat{\mu}) - \bar{q}_1(x + \hat{\mu}) \frac{1 + \gamma_{\mu}}{2} U_{\hat{\mu}}^{\dagger}(x) q_2(x)$$



S. Hashimoto et al., PRD61, 014502 (1999);  
A. X. El-Khadra et al., PRD64, 014502 (2001)

- A “mostly nonperturbative” method:

$$Z_V^{c \rightarrow s} = \sqrt{Z_V^{c \rightarrow c} Z_V^{s \rightarrow s}}$$

- Differences arise from **discretization effects**, and will contribute to the **systematic errors (~3% for FFs)**.

# Renormalization factors for $c \rightarrow s$ current

- Use the off-shell quark matrix elements:

$$\frac{Z_q}{Z_A} \text{Tr}[\langle q | A^\mu | q \rangle \gamma_\mu \gamma_5] = \frac{Z_q}{Z_V} \text{Tr}[\langle q | V^\mu | q \rangle \gamma_\mu]$$

- And the “most nonperturbative” method:

RI/MOM scheme

$$Z_A^{c \rightarrow s} \equiv Z_V^{c \rightarrow s} \sqrt{\frac{\text{Tr}[\langle c | V^\mu | c \rangle \gamma_\mu] \text{Tr}[\langle s | V^\mu | s \rangle \gamma_\mu]}{\text{Tr}[\langle c | A^\mu | c \rangle \gamma_\mu \gamma_5] \text{Tr}[\langle s | A^\mu | s \rangle \gamma_\mu \gamma_5]}}$$

*G. Martinelli et al.,  
Nucl. Phys. B445, 81 (1995).*

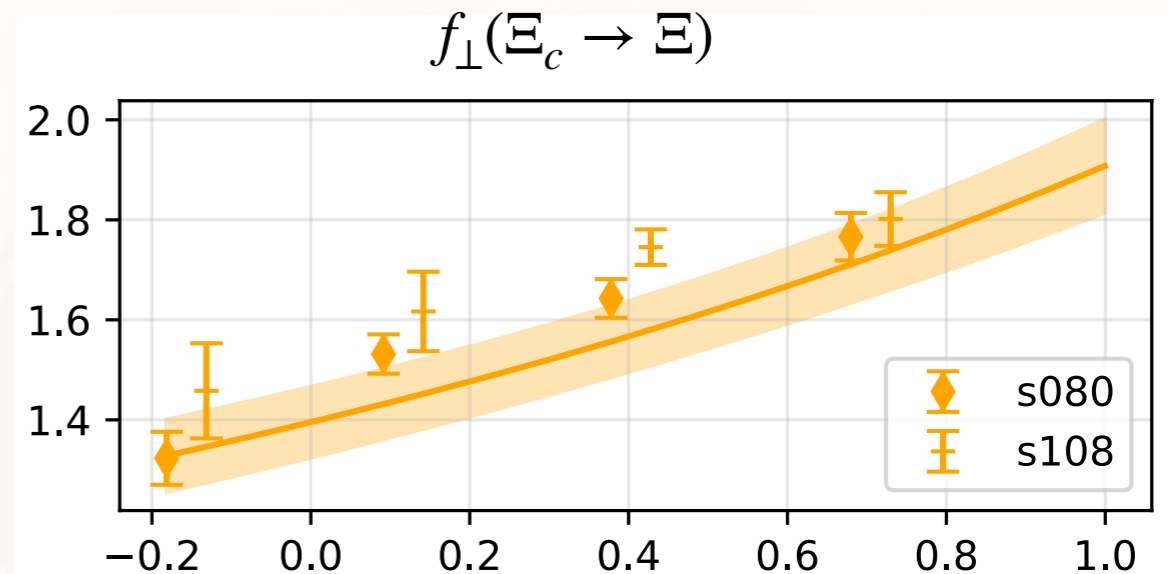
- With  $a^2 p^2$  extrapolation using three values of  $p^2$  in the range of  $a^2 p^2 = (4, 6, 8)$ , we obtained  $Z_A/Z_V = 1.010231(69)$  and  $1.020296(68)$  on s108 and s080, respectively.

# $q^2$ distribution for $\Xi_c \rightarrow \Xi$ form factors

- Extrapolate to the **continuum limit** (shaded regions);
- $z$ -expansion parametrization of form factors to obtain the  $q^2$ -distribution:

$$f(q^2) = \frac{1}{1 - q^2 / (m_{\text{pole}}^f)^2} \sum_{n=0}^{n_{\text{max}}} \left( c_n^f + d_n^f a^2 \right) \left[ z(q^2) \right]^n$$

- Use  $D_s$  meson pole mass for  $m_{\text{pole}}^{f_{\perp}}$ , ...
- Consider the **discretization effects** by estimating the  $d_n^f$  terms.



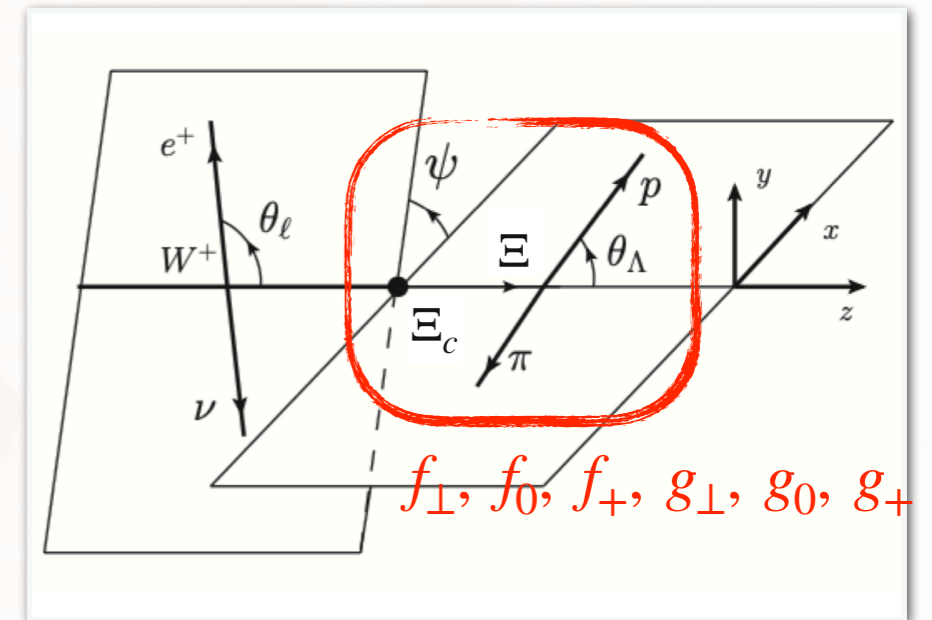
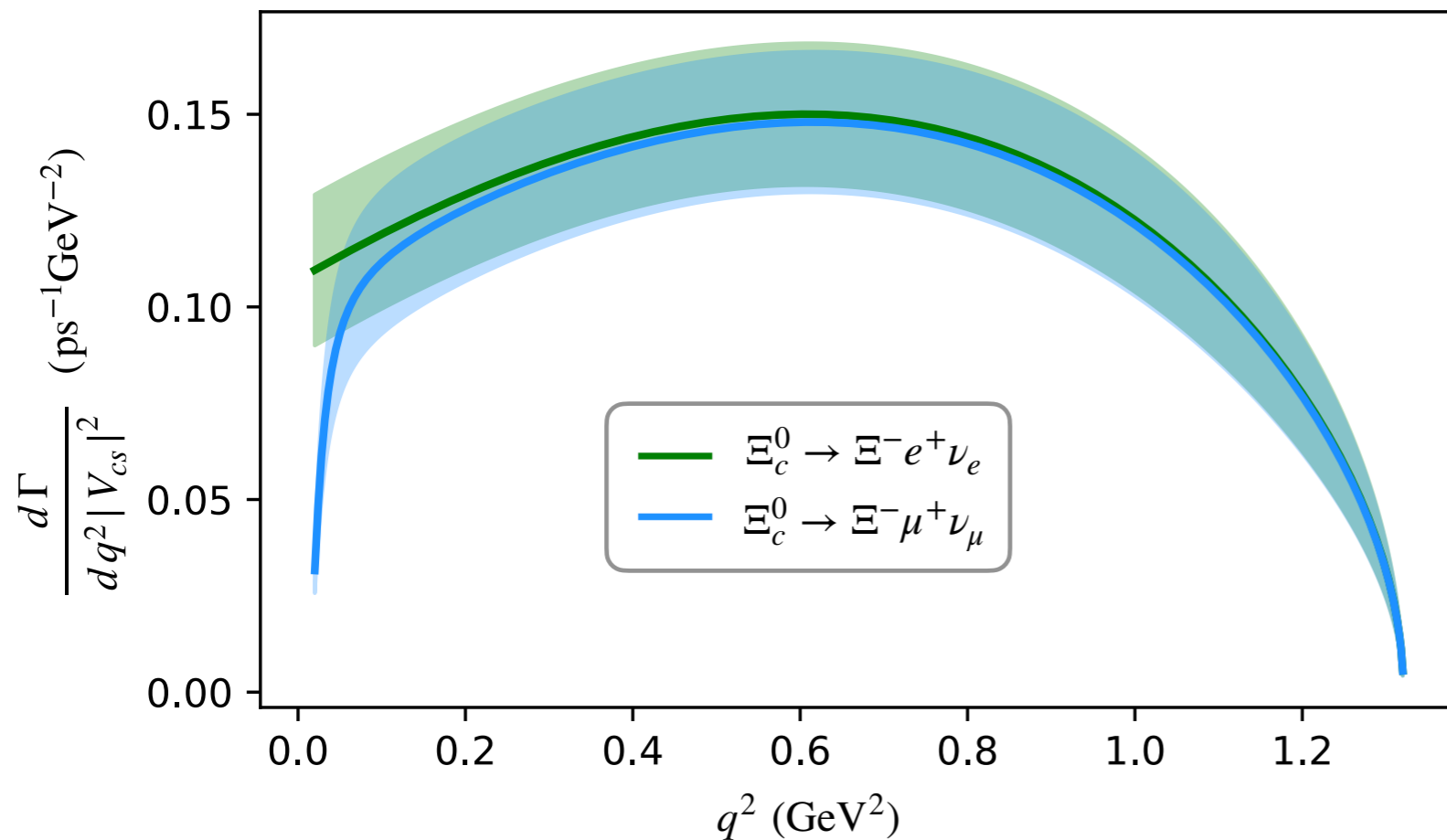
Fit results for the  $z$ -expansion parameters

	$c_0$	$c_1$	$c_2$
$f_{\perp}$	$1.51 \pm 0.09$	$-1.88 \pm 1.21$	$1.71 \pm 0.49$
$f_0$	$0.64 \pm 0.09$	$-1.83 \pm 1.22$	$0.56 \pm 0.51$
$f_+$	$0.77 \pm 0.07$	$-4.09 \pm 1.18$	$0.35 \pm 0.49$
$g_{\perp}$	$0.56 \pm 0.07$	$-0.35 \pm 1.26$	$0.15 \pm 0.29$
$g_0$	$0.63 \pm 0.07$	$-1.37 \pm 1.36$	$0.15 \pm 0.29$
$g_+$	$0.56 \pm 0.08$	$0.00 \pm 1.38$	$0.14 \pm 0.29$

# Differential decay widths

- The differential decay widths of  $\Xi_c^0 \rightarrow \Xi^- l^+ \nu_l$ :

$$\frac{d\Gamma}{dq^2 |V_{cs}|^2} \quad (\text{ps}^{-1} \text{GeV}^{-2})$$



- Results for  $\Xi_c^+ \rightarrow \Xi^0 l^+ \nu_l$  are similar.

# Branching fractions and differential decay widths

## - Predicted decay branching fractions:

$$\mathcal{B}(\Xi_c^0 \rightarrow \Xi^- e^+ \nu_e) = 2.38(0.30)(0.32)(0.07) \%$$

$$\mathcal{B}(\Xi_c^0 \rightarrow \Xi^- \mu^+ \nu_\mu) = 2.29(0.29)(0.30)(0.06) \%$$

$$\mathcal{B}(\Xi_c^+ \rightarrow \Xi^0 e^+ \nu_e) = 7.18(0.90)(0.96)(0.20) \%$$

$$\mathcal{B}(\Xi_c^+ \rightarrow \Xi^0 \mu^+ \nu_\mu) = 6.91(0.87)(0.91)(0.19) \%$$

- Statistical errors
- Systematic errors from continuum extrapolation
- Systematic errors from renormalization

## - Compare with PDG, experiment and theory:

$$(2.38 \pm 0.44) \%$$

<b>PDG</b>	$\mathcal{B}(\Xi_c^0 \rightarrow \Xi^- e^+ \nu_e) = (1.8 \pm 1.2) \%$	✓
<b>Belle</b>	$\mathcal{B}(\Xi_c^0 \rightarrow \Xi^- e^+ \nu_e) = (1.72 \pm 0.10 \pm 0.12 \pm 0.50) \%$	✓
<b>ALICE</b>	$\mathcal{B}(\Xi_c^0 \rightarrow \Xi^- e^+ \nu_e) = (2.43 \pm 0.25 \pm 0.35 \pm 0.72) \%$	✓
<b>QCD SR</b>	$\mathcal{B}(\Xi_c^0 \rightarrow \Xi^- e^+ \nu_e) = (3.4 \pm 1.7) \%$	✓
<b>LF QM</b>	$\mathcal{B}(\Xi_c^0 \rightarrow \Xi^- e^+ \nu_e) = (3.49 \pm 0.95) \%$	✓
<b>LCSR</b>	$\mathcal{B}(\Xi_c^0 \rightarrow \Xi^- e^+ \nu_e) = (2.4^{+0.9}_{-1.0}) \%$	✓

Our results fit the experimental measurements and theoretical predictions well (within 1- $\sigma$ ) !



# Determination of $|V_{cs}|$

## - From Belle measurements:

Y. B. Li et al. [Belle], arXiv:2103.06496 [hep-ex].

$$\mathcal{B}(\Xi_c^0 \rightarrow \Xi^- e^+ \nu_e) = (1.72 \pm 0.10 \pm 0.12 \pm 0.50) \%$$

$$\mathcal{B}(\Xi_c^0 \rightarrow \Xi^- \mu^+ \nu_\mu) = (1.71 \pm 0.17 \pm 0.13 \pm 0.50) \%$$

From the uncertainty of  $\Xi_c^0 \rightarrow \Xi^- \pi^+$

$$|V_{cs}| = 0.834 \pm (0.051)_{\text{stat.}} \pm (0.56)_{\text{syst.}} \pm (0.127)_{\text{exp.}}$$

Theo. error ~ 8.9%

Exp. error ~ 15.2%

## - From ALICE measurements:

J. Zhu on behalf of the ALICE collaboration, PoS ICHEP2020 (2021) 524.

$$\mathcal{B}(\Xi_c^0 \rightarrow \Xi^- e^+ \nu_e) = (2.43 \pm 0.25 \pm 0.35 \pm 0.72) \%$$

$$|V_{cs}| = 0.983 \pm (0.060)_{\text{stat.}} \pm (0.065)_{\text{syst.}} \pm (0.167)_{\text{exp.}}$$

Exp. error ~ 17.0%

### Theoretical uncertainties:

- total ~ 8.9%
- statistical ~ 6.1%
- systematic from extrapolation ~ 6.5%
- systematic from renormalization ~ 1.5%

### Experimental uncertainties:

- Belle ~ 15.2%
- ALICE ~ 17.0%

## - Compare with PDG result:

$$|V_{cs}| = 0.97320 \pm 0.00011$$

# SUMMARY

- The **first lattice QCD calculation** of  $\Xi_c \rightarrow \Xi$  form factors: predicted the **differential decay widths, branching fractions**, and extracted the **CKM matrix element  $|V_{cs}|$** ;
- The lattice calculation was done on two lattice spacings and **extrapolated to the continuum**, and finite renormalization was done by conserved vector current;
- A more precise experimental measurement will greatly improve the precision in  $|V_{cs}|$  and can be achieved in future especially by Belle-II, LHCb and other experiments.

*Thank you for your attentions!*