Hunting for fully-heavy tetraquark states

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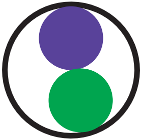
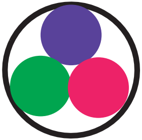
Outline

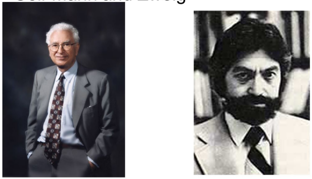
1. Background of the exotic hadron states
2. Moment sum rules for $cc\bar{c}\bar{c}$ and $bb\bar{b}\bar{b}$ tetraquarks
3. Decay properties of the $cc\bar{c}\bar{c}$ and $bb\bar{b}\bar{b}$ tetraquarks
4. Moment sum rules for $bc\bar{b}\bar{c}$ and $cc\bar{b}\bar{b}$ tetraquarks
5. Summary

Quark model


meson($q\bar{q}$)


baryon(qqq)

Gell-Mann and Zweig



- **Quark model** is established to classify hadrons: mesons ($q\bar{q}$) and baryons (qqq).
- Hadrons with exotic quantum numbers are exotic hadron states.
- **QCD** allows for hadrons outside the naive quark model. Hadron structures are more complicated in **QCD**: $\mathrm{N}_{\mathrm{quarks}} \neq 2, 3$.
- $SU(3)_c$ **gauge symmetry**: $(N_q - N_{\bar{q}})$ is divisible by 3, plus any number N_g of valence gluons can form a color singlet.

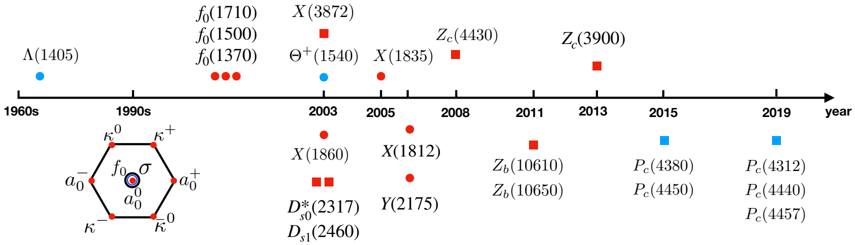
Searching for exotica

Light hadron sector:

- **Dibaryon** candidates: Deuteron, H states, $d^*(2380)$.
- **Hybrid** candidates: $\pi_1(1400)$, $\pi_1(1600)$ and $\pi_1(2015)$ (dispute).
- **Glueball** candidates: $a_0(980)$ and $f_0(980)$, a C-odd gluonic compound.
- **Tetraquark** candidates: light scalar mesons.
- **Pentaquark** candidate: $\Theta^+(1540)$ ($S = 1$, long story of appeared and disappeared)

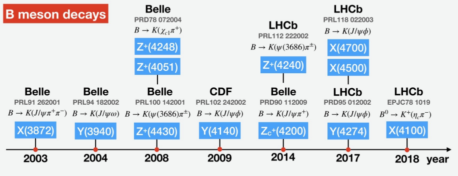
Heavy hadron sector: **breakthough in multiquarks!**

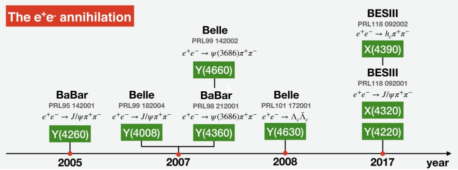
- $P_c(4380)$, $P_c(4312)$, $P_c(4440)$, $P_c(4457)$, $\mathbf{P_{cs}(4459)}$: hidden-charm pentaquark states.
- Plenty of **XYZ states**: candidates of molecules, tetraquarks, hybrids...

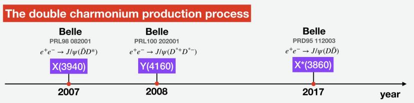


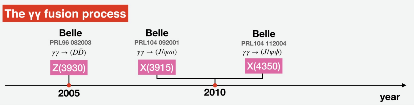
Overview of XYZ States

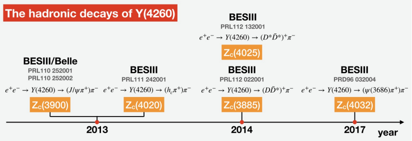
Experiments: Belle, BaBar, BESIII, CDF, CLEO, D0, LHCb...











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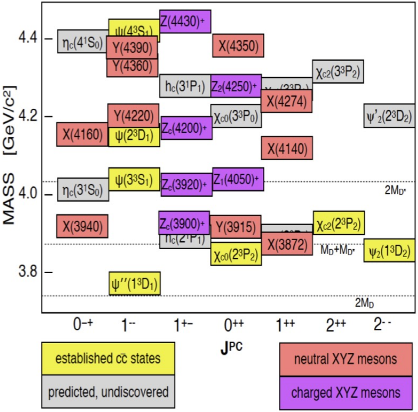
Overview of XYZ States

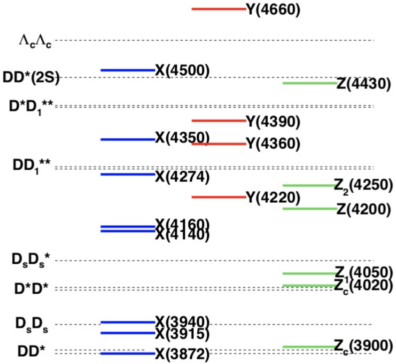


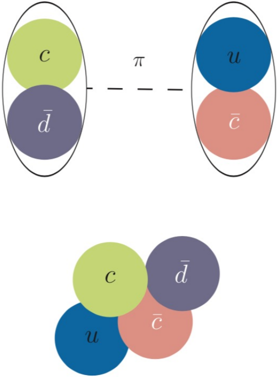
arXiv:1812.10947

- Many charmonium-like states were discovered above the open-charm thresholds.
- Their masses and decay modes are different from the pure $c\bar{c}$ charmonium states.
- Some charged Z_c states were observed, which are evidences for four-quark states $(c\bar{c}u\bar{d})$.
- They are good candidates for exotic hadron states: **molecule, tetraquark, hybrid …**

Theoretical Models

- Lots of “near-threshold” exotic resonances!
- Inside structure: tetraquark, molecule





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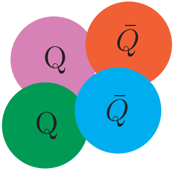
Fully-heavy tetraquarks: $QQ\bar{Q}\bar{Q}$

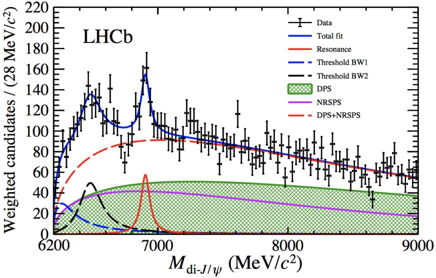
$QQ\bar{Q}\bar{Q}$ Tetraquarks:

- They are far away from the mass range of the $q\bar{q}$ and $Qq\bar{Q}\bar{q}$ hadrons, can be clearly distinguished experimentally from the spectroscopy.
- The light mesons $(\pi, \rho, \omega, \sigma ...)$ can not be exchanged between two charmonia/bottomonia.
- The binding force comes from the short-range gluon exchange.
- A molecule configuration is not favored and thus the $QQ\bar{Q}\bar{Q}$ is a good candidate for compact tetraquark.



$X(6900)$: resonance structure in J/ψ-pair mass spectrum

LHCb observed several structures in the J/ψ-pair mass spectrum **(Sci. Bull., 2020, 2020, 65)**:



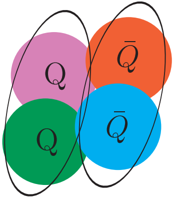
- The mass and width of $X(6900)$ are:(1) $M = 6905 \pm 11 \pm 7$ MeV, $\Gamma = 80 \pm 19 \pm 33$ MeV based on no-interference fit; (2) $M = 6886 \pm 11 \pm 11$ MeV, $\Gamma = 168 \pm 33 \pm 69$ MeV based on the simple model with interference.
- A broad structure next to threshold ranging from 6.2 to 6.8 GeV;
- Hint for another structure around 7.2 GeV with low significance.

Tetraquark Sum Rules

- Study two-point correlation function of current $J(x)$ with the same quantum numbers with hadron state:

$$\Pi(q^2) = i\int d^4x e^{iq\cdot x}\langle\Omega|T[J(x)J^\dagger(0)]|\Omega\rangle$$

- Classify states $|X\rangle$ by coupling to current $\langle\Omega|J(x)|X\rangle \neq 0$
- Currents are probes of spectrum and might not overlap with state



Pauli principle operate for diquark structure:

Interpolating currents with $J^{PC} = 0^{-+}$ and 0^{--}:

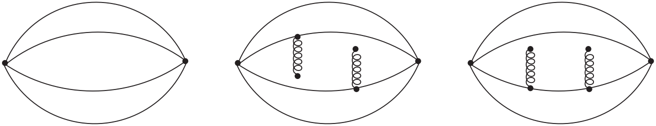
$$J_1^\pm = Q_a^T C Q_b \bar{Q}_a \gamma_5 C \bar{Q}_b^T \pm Q_a^T C\gamma_5 Q_b \bar{Q}_a C \bar{Q}_b^T\,,$$
$$J_2^+ = Q_a^T C\sigma_{\mu\nu} Q_b \bar{Q}_a \sigma^{\mu\nu}\gamma_5 C \bar{Q}_b^T\,,$$

- Hadron level: described by the dispersion relation

$$\begin{aligned}\Pi(q^2) &= \frac{(q^2)^N}{\pi}\int\frac{\mathrm{Im}\Pi(s)}{s^N(s-q^2-i\epsilon)}ds+\sum_{n=0}^{N-1}b_n(q^2)^n,\\ \rho(s) &= \frac{1}{\pi}\mathrm{Im}\Pi(s)=\sum_n\delta(s-m_n^2)\langle 0|J|n\rangle\langle n|J^\dagger|0\rangle\\ &\qquad = f_X^2\delta(s-m_X^2)+\mathrm{continuum},\end{aligned}$$

- Quark-gluon level: evaluated via operator product expansion(OPE)

$$\Pi(s)=\Pi^{pert}(s)+\Pi^{\langle GG\rangle}(s)+...,$$



- Define moments in Euclidean region $Q^2 = -q^2 > 0$:

$$M_n(Q_0^2) = \frac{1}{n!}\left(-\frac{d}{dQ^2}\right)^n \Pi(Q^2)|_{Q^2=Q_0^2}$$
$$= \int_{m_H^2}^{\infty} \frac{\rho(s)}{(s+Q_0^2)^{n+1}} ds = \frac{f_X^2}{(m_X^2+Q_0^2)^{n+1}}\left[1+\delta_n(Q_0^2)\right],$$

where $\delta_n(Q_0^2)$ contains the higher states and continuum.

- Ratio of the moments

$$r(n, Q_0^2) \equiv \frac{M_n(Q_0^2)}{M_{n+1}(Q_0^2)} = \left(m_X^2 + Q_0^2\right)\frac{1+\delta_n(Q_0^2)}{1+\delta_{n+1}(Q_0^2)}.$$

- Predict hadron mass

$$m_X = \sqrt{r(n, Q_0^2) - Q_0^2}$$

for sufficiently large n when $\delta_n(Q_0^2) \cong \delta_{n+1}(Q_0^2)$ for convergence.

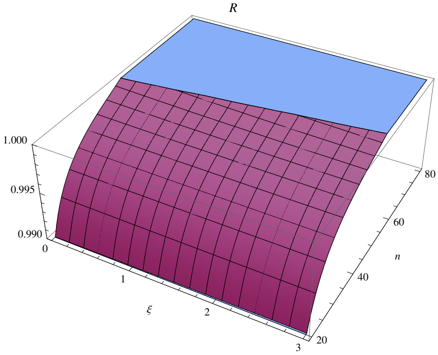
Limitations for $(n,\, \xi)$ parameter space:

$$\xi = Q_0^2/16m_c^2, \text{ for } cc\bar{c}\bar{c} \text{ system;}$$
$$\xi = Q_0^2/m_b^2, \text{ for } bb\bar{b}\bar{b} \text{ system.}$$

- Small ξ: higher dimensional condensates give large contributions to $M_n(Q_0^2)$, leading to bad OPE convergence.
- Large ξ: slower convergence of $\delta_n(Q_0^2)$. This can be compensated by taking higher derivative n for the lowest lying resonance to dominate.
- Large n: moving further away from the asymptotically free region. The OPE convergence would also become bad.
- Requiring $\Pi^{\langle GG\rangle}(s) \leq \Pi^{pert}(s)$ to obtain an upper limit n_{max}, which will increase with respect to ξ.
- Good $(n,\, \xi)$ region: the lowest lying resonance dominates the moments while the OPE series has good convergence.

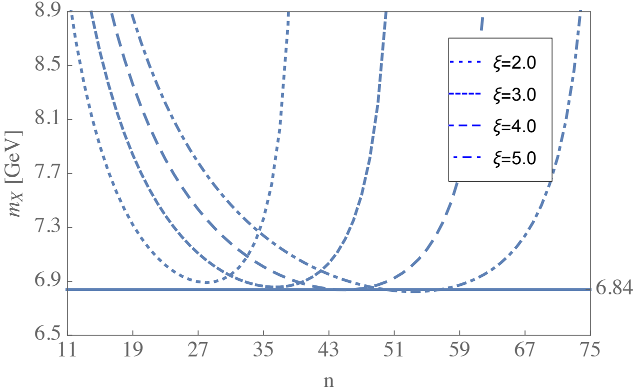
Hölder's inequality:

$$R = \frac{M_n(Q_0^2)^2}{M_r(Q_0^2)M_{2n-r}(Q_0^2)} \leq 1 \,,$$



The boundary gives $(n, \xi) = (48, 0.2), (49, 0.4), (49, 0.6), (50, 0.8)$.

Mass for $cc\bar{c}\bar{c}$ tetraquark with $J^{PC}=0^{-+}$: mass curves have plateaus at $(n,\xi)=(28,2),(36,3),(45,4),(53,5)$



$$m_{T_{cc\bar{c}\bar{c}}}=(6.84\pm0.18)\,\mathrm{GeV}.$$

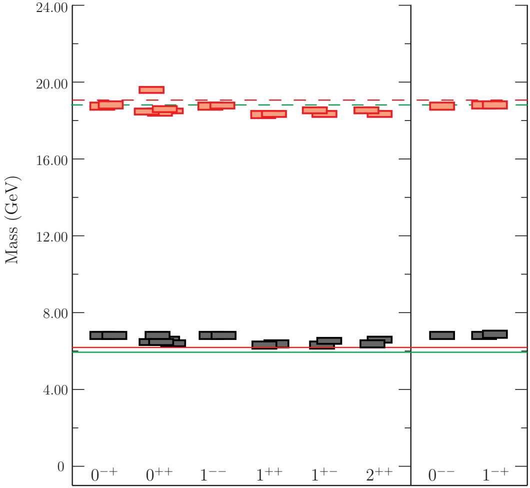
Good predictions for LHCb's observations:

PLB773(2017), 247-251

J^{PC}	Currents	m_{X_c}(GeV)	m_{X_b}(GeV)
0^{++}	J_1	6.44 ± 0.15	18.45 ± 0.15
	J_2	6.59 ± 0.17	18.59 ± 0.17
	J_3	6.47 ± 0.16	18.49 ± 0.16
	J_4	6.46 ± 0.16	18.46 ± 0.14
	J_5	6.82 ± 0.18	19.64 ± 0.14
1^{++}	$J_{1\mu}^+$	6.40 ± 0.19	18.33 ± 0.17
	$J_{2\mu}^+$	6.34 ± 0.19	18.32 ± 0.18
1^{+-}	$J_{1\mu}^-$	6.37 ± 0.18	18.32 ± 0.17
	$J_{2\mu}^+$	6.51 ± 0.15	18.54 ± 0.15
2^{++}	$J_{1\mu\nu}$	6.51 ± 0.15	18.53 ± 0.15
	$J_{2\mu\nu}$	6.37 ± 0.19	18.32 ± 0.17
0^{-+}	J_1^+	6.84 ± 0.18	18.77 ± 0.18
	J_2^+	6.85 ± 0.18	18.79 ± 0.18
0^{--}	J_1^-	6.84 ± 0.18	18.77 ± 0.18
1^{-+}	$J_{1\mu}^+$	6.84 ± 0.18	18.80 ± 0.18
	$J_{2\mu}^+$	6.88 ± 0.18	18.83 ± 0.18
1^{--}	$J_{1\mu}^-$	6.84 ± 0.18	18.77 ± 0.18
	$J_{2\mu}^-$	6.83 ± 0.18	18.77 ± 0.16

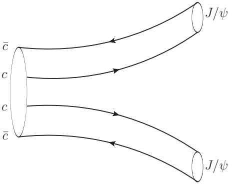
Our previous calculations in 2017 are consistent very good with the LHCb's observation:

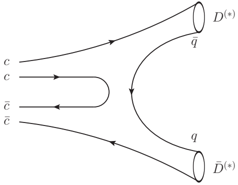
- **The masses of $cc\bar{c}\bar{c}$ tetraquarks with $J^{PC} = 0^{++}$, 2^{++} are agree with the broad structure around 6.2-6.8 GeV;**
- **The masses of $cc\bar{c}\bar{c}$ tetraquarks with $J^{PC} = 0^{-+}$, 1^{-+} are consistent with the mass of X(6900).**

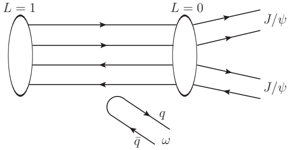

24.00
20.00
16.00
12.00
8.00
4.00
0
Mass (GeV)
0^{-+}
0^{++}
1^{--}
1^{++}
1^{+-}
2^{++}
0^{--}
1^{-+}

Decay behavior: $cc\bar{c}\bar{c}$ tetraquarks

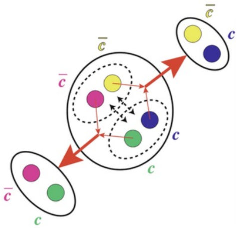
- $cc\bar{c}\bar{c} \to (ccq) + (\bar{c}\bar{c}\bar{q})$: kinematically forbidden.
- $cc\bar{c}\bar{c} \to (cqq) + (\bar{c}\bar{q}\bar{q})$: suppressed by two light quark pair creation.
- $cc\bar{c}\bar{c} \to (c\bar{c}) + (c\bar{c})$: charm quark pair rearrangement or annihilation (suppressed). Phase space is small.
- $cc\bar{c}\bar{c} \to (q\bar{c}) + (c\bar{q})$: possible via a heavy quark pair annihilation and a light quark pair creation, with large phase space.
- $cc\bar{c}\bar{c}(L=1) \to cc\bar{c}\bar{c}(L=0) + (q\bar{q})_{I=0}$: OZI forbidden.







Strong decays into di-charmonia

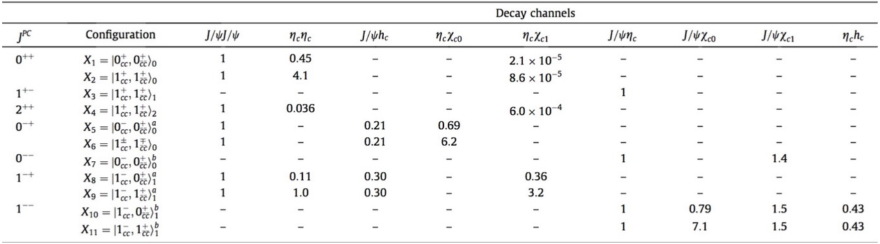


PLB773(2017), 247-251;
Sci.Bull.65,2020, 1994-2000

J^{PC}	S-wave	P-wave
0^{++}	$\eta_c(1S)\eta_c(1S)$, $J/\psi J/\psi$	$\eta_c(1S)\chi_{c1}(1P)$, $J/\psi h_c(1P)$
0^{-+}	$\eta_c(1S)\chi_{c0}(1P)$, $J/\psi h_c(1P)$	$J/\psi J/\psi$
0^{--}	$J/\psi \chi_{c1}(1P)$	$J/\psi \eta_c(1S)$
1^{++}	–	$J/\psi h_c(1P)$, $\eta_c(1S)\chi_{c1}(1P)$, $\eta_c(1S)\chi_{c0}(1P)$
1^{+-}	$J/\psi \eta_c(1S)$	$J/\psi \chi_{c0}(1P)$, $J/\psi \chi_{c1}(1P)$, $\eta_c(1S)h_c(1P)$
1^{-+}	$J/\psi h_c(1P)$, $\eta_c(1S)\chi_{c1}(1P)$	$J/\psi J/\psi$
1^{--}	$J/\psi \chi_{c0}(1P)$, $J/\psi \chi_{c1}(1P)$, $\eta_c(1S)h_c(1P)$	$J/\psi \eta_c(1S)$

Strong decays into di-charmonia

We calculate their relative branching ratios through the Fierz rearrangement:



		Decay channels								
J^{PC}	Configuration	$J/\psi J/\psi$	$\eta_c\eta_c$	$J/\psi h_c$	$\eta_c\chi_{c0}$	$\eta_c\chi_{c1}$	$J/\psi\eta_c$	$J/\psi\chi_{c0}$	$J/\psi\chi_{c1}$	$\eta_c h_c$
0^{++}	$X_1 = \lvert 0^+_{cc}, 0^+_{\bar{c}\bar{c}}\rangle_0$	1	0.45	–	–	2.1×10^{-5}	–	–	–	–
	$X_2 = \lvert 1^+_{cc}, 1^+_{\bar{c}\bar{c}}\rangle_0$	1	4.1	–	–	8.6×10^{-5}	–	–	–	–
1^{+-}	$X_3 = \lvert 1^+_{cc}, 1^+_{\bar{c}\bar{c}}\rangle_1$	–	–	–	–	–	1	–	–	–
2^{++}	$X_4 = \lvert 1^+_{cc}, 1^+_{\bar{c}\bar{c}}\rangle_2$	1	0.036	–	–	6.0×10^{-4}	–	–	–	–
0^{-+}	$X_5 = \lvert 0^-_{cc}, 0^+_{\bar{c}\bar{c}}\rangle_0^a$	1	–	0.21	0.69	–	–	–	–	–
	$X_6 = \lvert 1^\pm_{cc}, 1^\mp_{\bar{c}\bar{c}}\rangle_0$	1	–	0.21	6.2	–	–	–	–	–
0^{--}	$X_7 = \lvert 0^-_{cc}, 0^+_{\bar{c}\bar{c}}\rangle_0^b$	–	–	–	–	–	1	–	1.4	–
1^{-+}	$X_8 = \lvert 1^-_{cc}, 0^+_{\bar{c}\bar{c}}\rangle_1^a$	1	0.11	0.30	–	0.36	–	–	–	–
	$X_9 = \lvert 1^-_{cc}, 1^+_{\bar{c}\bar{c}}\rangle_1^a$	1	1.0	0.30	–	3.2	–	–	–	–
1^{--}	$X_{10} = \lvert 1^-_{cc}, 0^+_{\bar{c}\bar{c}}\rangle_1^b$	–	–	–	–	–	1	0.79	1.5	0.43
	$X_{11} = \lvert 1^-_{cc}, 1^+_{\bar{c}\bar{c}}\rangle_1^b$	–	–	–	–	–	1	7.1	1.5	0.43

- We suggest **the broad structure around 6.2-6.8 GeV to be a S-wave $cc\bar{c}\bar{c}$ tetraquark with $J^{PC} = 0^{++}$ or 2^{++}**, while **the narrow structure around 6.9 Gev to be a P-wave $cc\bar{c}\bar{c}$ tetraquark with $J^{PC} = 0^{-+}$ or 1^{-+}**.
- We propose to confirm them in the **di-η_c, $J/\psi h_c$, $\eta_c\chi_{c0}$, $\eta_c\chi_{c1}$** channels. These channels are helpful to determine their quantum numbers.

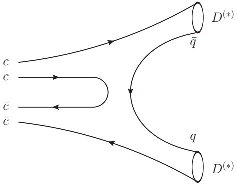
$bb\bar{b}\bar{b}$ tetraquarks

The $bb\bar{b}\bar{b}$ tetraquarks lie below two bottomonium thresholds:

- $bb\bar{b}\bar{b} \to (b\bar{b}) + (b\bar{b})$: kinematically forbidden.
- $bb\bar{b}\bar{b} \to (bbq) + (\bar{b}\bar{b}\bar{q})$: kinematically forbidden.
- $bb\bar{b}\bar{b} \to (q\bar{b}) + (b\bar{q})$: possible via a heavy quark pair annihilation and a light quark pair creation.
- $bb\bar{b}\bar{b} \to (b\bar{b}) + \gamma$: electromagnetic decay via $b\gamma_\mu\bar{b} \to \gamma$ photon production process.
- These $bb\bar{b}\bar{b}$ states are expected to be very narrow. They are good candidates for compact tetraquarks, if they do exist.

Doubly hidden-flavor $bc\bar{b}\bar{c}$ and doubly open-flavor $cc\bar{b}\bar{b}/bb\bar{c}\bar{c}$ tetraquark systems

- Pauli exclusion principle does not operate for $bc\bar{b}\bar{c}$ systems.
- Bound $bc\bar{b}\bar{c}$ tetraquarks may be more favorable than $cc\bar{c}\bar{c}$ and $bb\bar{b}\bar{b}$ systems, based on the CMI model and string-inspired Chromoelectric model.
- All $cc\bar{c}\bar{c}$, $bb\bar{b}\bar{b}$ and $bc\bar{b}\bar{c}$ tetraquarks can strongly decay by annihilating at least a pair of heavy quarks with the same flavor.
- The $cc\bar{b}\bar{b}$ tetraquarks have no annihilating decay channel! They are expected to be stable and narrow if they lie below the $2B_c$ threshold.

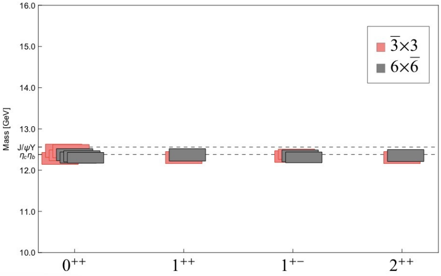


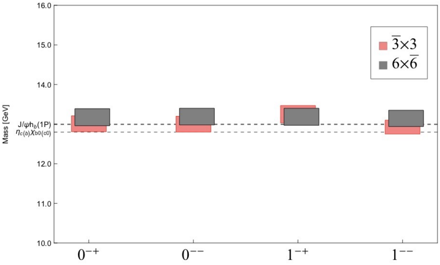
Mass spectra for the $bc\bar{b}\bar{c}$ tetraquarks

arXiv:2102.10605

J^{PC}	*Current*	Mass[GeV]	*Current*	Mass[GeV]
	$[\bar{\mathbf{3}} \otimes \mathbf{3}]_c$		$[\mathbf{6} \otimes \bar{\mathbf{6}}]_c$	
0^{++}	J_1	$12.28^{+0.15}_{-0.14}$	j_1	$12.37^{+0.15}_{-0.14}$
	J_2	$12.46^{+0.17}_{-0.15}$	j_2	$12.29^{+0.15}_{-0.12}$
	$J_{5\mu\nu}(S)$	$12.35^{+0.14}_{-0.12}$	$j_{5\mu\nu}(S)$	$12.32^{+0.15}_{-0.12}$
	$J_{5\mu\nu}(T)$	$12.45^{+0.17}_{-0.15}$	$j_{5\mu\nu}(T)$	$12.29^{+0.14}_{-0.12}$
0^{-+}	$J_{3\mu}$	$12.99^{+0.22}_{-0.18}$	$j_{3\mu}$	$13.16^{+0.23}_{-0.20}$
0^{--}	$J_{4\mu}$	$12.98^{+0.22}_{-0.18}$	$j_{4\mu}$	$13.17^{+0.23}_{-0.19}$
1^{++}	$J_{3\mu}$	$12.30^{+0.15}_{-0.14}$	$j_{3\mu}$	$12.36^{+0.16}_{-0.14}$
1^{+-}	$J_{4\mu}$	$12.32^{+0.15}_{-0.13}$	$j_{4\mu}$	$12.34^{+0.15}_{-0.14}$
	$J_{6\mu\nu}$	$12.38^{+0.13}_{-0.12}$	$j_{6\mu\nu}$	$12.30^{+0.14}_{-0.12}$
1^{-+}	$J_{5\mu\nu}$	$13.23^{+0.24}_{-0.20}$	$j_{5\mu\nu}$	$13.17^{+0.23}_{-0.20}$
1^{--}	$J_{6\mu\nu}$	$12.91^{+0.19}_{-0.16}$	$j_{6\mu\nu}$	$13.13^{+0.22}_{-0.19}$
2^{++}	$J_{5\mu\nu}$	$12.30^{+0.15}_{-0.14}$	$j_{5\mu\nu}$	$12.35^{+0.15}_{-0.14}$

Mass spectra for the $bc\bar{b}\bar{c}$ tetraquarks

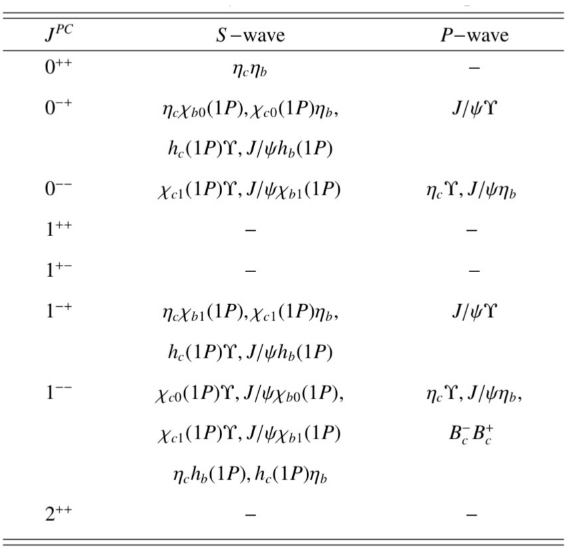




- Two thresholds for $bc\bar{b}\bar{c}$ tetraquarks: $c\bar{c}+b\bar{b}$ and $b\bar{c}+c\bar{b}$.
- The S-wave positive parity $bc\bar{b}\bar{c}$ tetraquarks lie below the two-meson thresholds, they can decay into $b\bar{b}/c\bar{c}$ + light meson final states.
- The P-wave negative parity $bc\bar{b}\bar{c}$ tetraquarks can decay into the $c\bar{c}+b\bar{b}$ and $b\bar{c}+c\bar{b}$ final states.

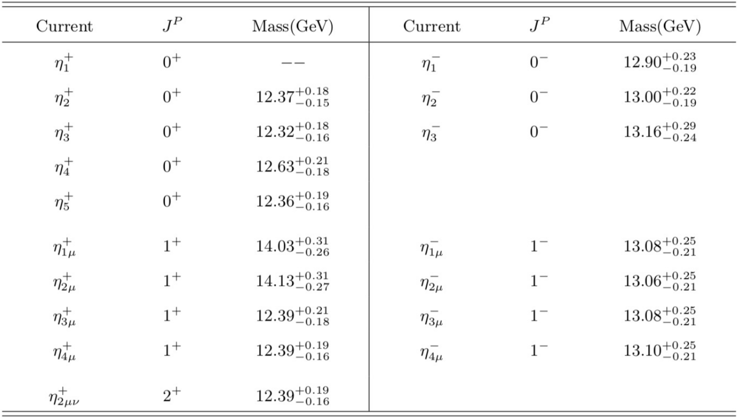
Decay properties of the $bc\bar{b}\bar{c}$ tetraquarks

arXiv:2102.10605



J^{PC}	$S-$wave	$P-$wave
0^{++}	$\eta_c\eta_b$	–
0^{-+}	$\eta_c\chi_{b0}(1P), \chi_{c0}(1P)\eta_b,$ $h_c(1P)\Upsilon, J/\psi h_b(1P)$	$J/\psi\Upsilon$
0^{--}	$\chi_{c1}(1P)\Upsilon, J/\psi\chi_{b1}(1P)$	$\eta_c\Upsilon, J/\psi\eta_b$
1^{++}	–	–
1^{+-}	–	–
1^{-+}	$\eta_c\chi_{b1}(1P), \chi_{c1}(1P)\eta_b,$ $h_c(1P)\Upsilon, J/\psi h_b(1P)$	$J/\psi\Upsilon$
1^{--}	$\chi_{c0}(1P)\Upsilon, J/\psi\chi_{b0}(1P),$ $\chi_{c1}(1P)\Upsilon, J/\psi\chi_{b1}(1P)$ $\eta_c h_b(1P), h_c(1P)\eta_b$	$\eta_c\Upsilon, J/\psi\eta_b,$ $B_c^- B_c^+$
2^{++}	–	–

Mass spectra for the $cc\bar{b}\bar{b}$ tetraquarks



Current	J^P	Mass(GeV)	Current	J^P	Mass(GeV)
η_1^+	0^+	$--$	η_1^-	0^-	$12.90^{+0.23}_{-0.19}$
η_2^+	0^+	$12.37^{+0.18}_{-0.15}$	η_2^-	0^-	$13.00^{+0.22}_{-0.19}$
η_3^+	0^+	$12.32^{+0.18}_{-0.16}$	η_3^-	0^-	$13.16^{+0.29}_{-0.24}$
η_4^+	0^+	$12.63^{+0.21}_{-0.18}$			
η_5^+	0^+	$12.36^{+0.19}_{-0.16}$			
$\eta_{1\mu}^+$	1^+	$14.03^{+0.31}_{-0.26}$	$\eta_{1\mu}^-$	1^-	$13.08^{+0.25}_{-0.21}$
$\eta_{2\mu}^+$	1^+	$14.13^{+0.31}_{-0.27}$	$\eta_{2\mu}^-$	1^-	$13.06^{+0.25}_{-0.21}$
$\eta_{3\mu}^+$	1^+	$12.39^{+0.21}_{-0.18}$	$\eta_{3\mu}^-$	1^-	$13.08^{+0.25}_{-0.21}$
$\eta_{4\mu}^+$	1^+	$12.39^{+0.19}_{-0.16}$	$\eta_{4\mu}^-$	1^-	$13.10^{+0.25}_{-0.21}$
$\eta_{2\mu\nu}^+$	2^+	$12.39^{+0.19}_{-0.16}$			

- One thresholds for $cc\bar{b}\bar{b}$ tetraquarks: $B_c^{(*)}B_c^{(*)}$.
- Except for two 1^+ states, the S-wave positive parity $cc\bar{b}\bar{b}$ tetraquarks lie below the $B_c^{(*)}B_c^{(*)}$ threshold, they are stable against the strong interaction.
- Negative parity $cc\bar{b}\bar{b}$ tetraquarks $0^-/1^- \to B_cB_c^*, B_c^*B_c^*$ in P-wave.

Summary

- We have calculated the mass spectra for the $cc\bar{c}\bar{c}$, $bb\bar{b}\bar{b}$, $bc\bar{b}\bar{c}$ and $cc\bar{b}\bar{b}$ tetraquark states, and have studied their decay properties.
- Our results suggest that **the broad structure around 6.2–6.8 GeV can be interpreted as an S-wave $cc\bar{c}\bar{c}$ tetraquark state with $J^{PC}=0^{++}$ or 2^{++}**, while **the narrow structure $X(6900)$ to be a P-wave one with $J^{PC}=0^{-+}$ or 1^{-+}**.
- We propose to confirm them in the di-η_c, $J/\psi h_c$, $\eta_c\chi_{c0}$ and $\eta_c\chi_{c1}$ channels. These channels are helpful to determine their quantum numbers.
- The **S-wave positive parity $bc\bar{b}\bar{c}$ tetraquarks lie below the two-meson thresholds**, they can decay into $b\bar{b}/c\bar{c}$ + light meson final states.
- The **S-wave positive parity $cc\bar{b}\bar{b}$ tetraquarks lie below the $B_c^{(*)}B_c^{(*)}$ threshold**, they are expected to be very stable and narrow.

Thank you for your attention!