Production of Fully-Heavy Tetraquarks Using NRQCD Factorization

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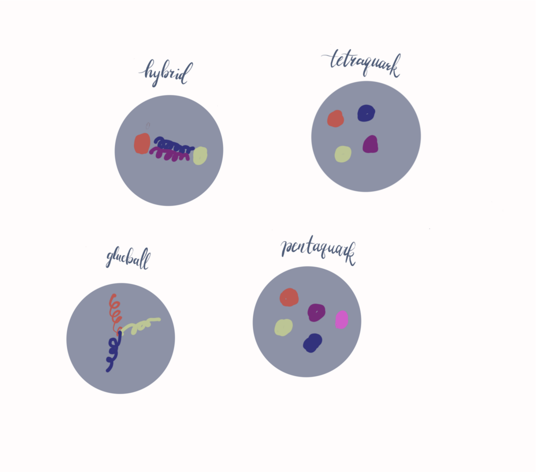
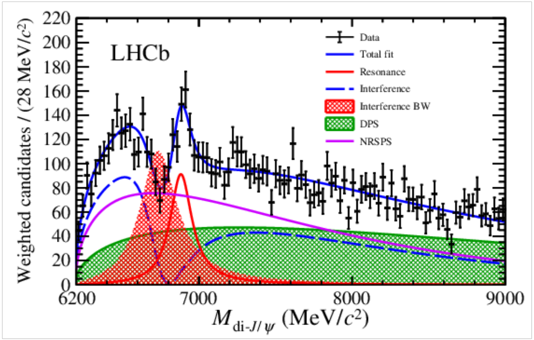
Motivation

Exotic Hadrons



Discovery of $X(6900)$



Invariant mass spectrum of J/ψ-pair candidates $(\mathrm{LHCb},\ 2020)$

- First fullly-charm tetraquark candidate
- Strong decay to two J/ψ, $C = +$

Fully-heavy Tetraquark

- Theoretical investigations on the fully heavy tetraquarks date back to late 1970s (Iwasaki, 1976; Chao, 1981).
- Phenomenological studies of spectra and decay properties: Badalian *et al.*, 1987; *et al.*, 2006; Wang, 2017,2020; W. Chen *et al.*, 2017,2018; Wu *et al.*, 2018; Liu *et al.*, 2019; Wang, Di, 2019; H.-X. Chen *et al.*, 2020; Jin *et al.*, 2020; Guo, Oller, 2020.... **See W.Chen, Huang, Z.-H. Guo, J. Wu, Liang**
- Search for the fully-bottom tetraquark on Lattice NRQCD: found no indication of any states below $2\eta_b$ threshold in the $0^{++}, 1^{+-}$ and 2^{++} channels (Hughes *et al.*, 2018).

Production Mechanism

- Duality relations: Berezhnoy *et al.*, 2011, 2012; Kaliner *et al.*, 2017
- Color evaporation model: Carvalho *et al.*, 2016; Maciuła *et al.*, 2020

Production Mechanism

- **Duality relations:** Berezhnoy *et al.*, 2011, 2012; Kaliner *et al.*, 2017
- **Color evaporation model:** Carvalho *et al.*, 2016; Maciuła *et al.*, 2020
- **NRQCD-inspired:** Y.-Q. Ma, Zhang, 2020; Feng et al., 2020; R.-L. Zhu, 2020

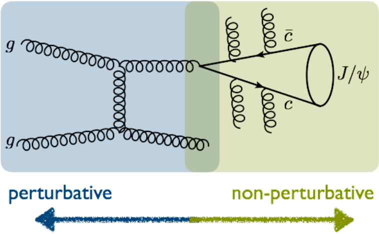
NRQCD Factorization

QCD Factorization Theorem

The inclusive production of high-$P_\perp$ hadrons in the high-energy hadron collision experiments is dominated by the fragmentation mechanism (Collins *et al.*, 1989).

$$\mathrm{d}\sigma\left[A+B\to H\left(P_\perp\right)+X\right]$$
$$=\sum_i \mathrm{d}\hat{\sigma}\left[A+B\to i\left(\frac{P_\perp}{z}\right)+X\right]\otimes \boldsymbol{D_{i\to H}}\left(z,\mu\right)+\mathcal{O}\left(\frac{1}{P_\perp^2}\right)$$

$\hat{\sigma}$: partonic cross section, $D_{i\to H}\left(z,\mu\right)$: fragmentation function



PARTICLEBITES, 2016

Fragmentation Function

Gauge-invariant operator definition for the fragmentation function (Collins, Soper, 1982)

$$D_{g\to H}(z,\mu_\Lambda) = \frac{-g_{\mu\nu} z^{d-3}}{2\pi k^+ (N_c^2-1)(d-2)} \int_{-\infty}^{+\infty} dx^- e^{-ik^+ x^-} \sum_X \left\langle 0 \middle| G_c^{+\mu}(0) \mathcal{E}^\dagger (0,0,\mathbf{0}_\perp)_{cb} \middle| H(P)+X \right\rangle \left\langle H(P)+X \middle| \mathcal{E}(0,x^-,\mathbf{0}_\perp)_{ba} G_a^{+\nu}(0,x^-,\mathbf{0}_\perp) \middle| 0 \right\rangle$$

G: field-strength tensor of gluons,
k: momentum of G,
$\mathcal{E}$: gauge link,
$d := 4-2\epsilon$: spacetime dimension,
$z := P^+/k^+$

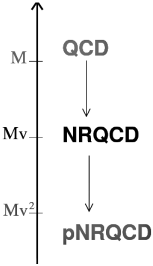
DGLAP Equation

Fragmentation function follows Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution equation

$$\mu\frac{\partial}{\partial\mu}D_{g\to T_{4c}}(z,\mu)=\frac{\alpha_s(\mu)}{\pi}\sum_{j\in\{g,c\}}\int_z^1\frac{\mathrm{d}y}{y}P_{g\leftarrow j}\left(\frac{z}{y},\mu\right)D_{g\to T_{4c}}(y,\mu)$$

$P_{g\leftarrow j}$ is the splitting kernel.

NRQCD Factorization (Bodwin, Braaten, Lepage, 1995)

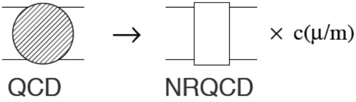


Vairo, Hadron 2011

- Quarkonium energy scale (Braaten, 1997)

	$c\bar{c}$	$b\bar{b}$	$t\bar{t}$
M	1.5 GeV	4.7 GeV	180 GeV
Mv	0.9 GeV	1.5 GeV	16 GeV
Mv^2	0.5 GeV	0.5 GeV	1.5 GeV

- Integrate out the heavy($\sim M$) degrees of freedom



Qiu, 2011

NRQCD Factorization

- For the fragmentation function $D_{g\to T_{4c}}$

$$D_{g\to T_{4c}}(z,\mu_\Lambda)=\frac{d_{3\times3}\left[g\to cc\bar{c}\bar{c}^{(J)}\right]}{m^9}\left|\left\langle T_{4c}^{(J)}\left|\mathcal{O}^{(J)}_{\bar{\mathbf{3}}\otimes\mathbf{3}}\right|0\right\rangle\right|^2+\frac{d_{6\times6}\left[g\to cc\bar{c}\bar{c}^{(J)}\right]}{m^9}\left|\left\langle T_{4c}^{(J)}\left|\mathcal{O}^{(J)}_{\mathbf{6}\otimes\bar{\mathbf{6}}}\right|0\right\rangle\right|^2+\frac{d_{3\times6}\left[g\to cc\bar{c}\bar{c}^{(J)}\right]}{m^9}2\mathrm{Re}\left[\left\langle T_{4c}^{(J)}\left|\mathcal{O}^{(J)}_{\bar{\mathbf{3}}\otimes\mathbf{3}}\right|0\right\rangle\left\langle 0\left|\mathcal{O}^{(J)\dagger}_{\mathbf{6}\otimes\bar{\mathbf{6}}}\right|T_{4c}^{(J)}\right\rangle\right]$$

- For the exclusive production

$$\sigma(e^+e^-\to T_{4c}^J+\gamma)=\frac{F^{[J]}_{3,3}}{m_c^8}(2M_{T_{4c}})\left|\left\langle T_{4c}^{(J)}\left|\mathcal{O}^{(J)}_{\bar{\mathbf{3}}\otimes\mathbf{3}}\right|0\right\rangle\right|^2+\frac{F^{[J]}_{6,6}}{m_c^8}(2M_{T_{4c}})\left|\left\langle T_{4c}^{(J)}\left|\mathcal{O}^{(J)}_{\mathbf{6}\otimes\bar{\mathbf{6}}}\right|0\right\rangle\right|^2+\frac{F^{[J]}_{3,6}}{m_c^8}(2M_{T_{4c}})2\mathrm{Re}\left[\left\langle T_{4c}^{(J)}\left|\mathcal{O}^{(J)}_{\bar{\mathbf{3}}\otimes\mathbf{3}}\right|0\right\rangle\left\langle 0\left|\mathcal{O}^{(J)\dagger}_{\mathbf{6}\otimes\bar{\mathbf{6}}}\right|T_{4c}^{(J)}\right\rangle\right]$$

NRQCD Operators

We construct the NRQCD local operators for the S-wave tetraquark with $J^{PC} = 0^{++}, 1^{+-}, 2^{++}$

$$\mathcal{O}^{(0)}_{\bar{\mathbf{3}}\otimes\mathbf{3}} = -\frac{1}{\sqrt{3}}[\psi_a^\dagger \sigma^i (i\sigma^2)\psi_b^*][\chi_c^T (i\sigma^2)\sigma^i \chi_d]\ \mathcal{C}^{ab;cd}_{\bar{\mathbf{3}}\otimes\mathbf{3}},$$

$$\mathcal{O}^{(0)}_{\mathbf{6}\otimes\bar{\mathbf{6}}} = \frac{1}{\sqrt{6}}[\psi_a^\dagger (i\sigma^2)\psi_b^*][\chi_c^T (i\sigma^2)\chi_d]\ \mathcal{C}^{ab;cd}_{\mathbf{6}\otimes\bar{\mathbf{6}}},$$

$$\mathcal{O}^{(1)i}_{\bar{\mathbf{3}}\otimes\mathbf{3}} = \frac{\mathrm{i}}{\sqrt{2}}\epsilon^{ijk}\left(\psi_a^\dagger \sigma^j \mathrm{i}\sigma^2 \psi_b^*\right)\left(\chi_c^T \mathrm{i}\sigma^2 \sigma^k \chi_d\right)\mathcal{C}^{ab;cd}_{\mathbf{3}\otimes\bar{\mathbf{3}}}$$

$$\mathcal{O}^{(2)kl}_{\bar{\mathbf{3}}\otimes\mathbf{3}} = [\psi_a^\dagger \sigma^m (i\sigma^2)\psi_b^*][\chi_c^T (i\sigma^2)\sigma^n \chi_d]\ \Gamma^{kl;mn}\ \mathcal{C}^{ab;cd}_{\bar{\mathbf{3}}\otimes\mathbf{3}}$$

$$\mathcal{C}^{ab;cd}_{\bar{\mathbf{3}}\otimes\mathbf{3}} := \frac{1}{2\sqrt{3}}(\delta^{ac}\delta^{bd} - \delta^{ad}\delta^{bc}), \quad \mathcal{C}^{ab;cd}_{\mathbf{6}\otimes\bar{\mathbf{6}}} := \frac{1}{2\sqrt{6}}(\delta^{ac}\delta^{bd} + \delta^{ad}\delta^{bc})$$

$$\Gamma^{kl;mn} := \frac{1}{2}(\delta^{km}\delta^{ln} + \delta^{kn}\delta^{lm} - \frac{2}{3}\delta^{kl}\delta^{mn})$$

NRQCD Operators

- The operators manifest the correct C-parity under the charge conjugation transformations

$$\psi \to \mathrm{i}\left(\chi^\dagger \sigma^2\right)^t, \quad \chi \to -\mathrm{i}\left(\psi^\dagger \sigma^2\right)^t$$

- We use the basis in which the quark and anti-quark pairs in the color-triplet and color-sexet, respectively. The operators can also be constructed from quark-antiquark pairs in the color-singlet and color-octet.
- These NRQCD operators in can also be inferred by performing the Foldy-Wouthuysen-Tani transformation from the QCD interpolating currents(H.-X. Chen *et al.*,2020).

Perturbative Matching

We use the perturbative matching procedure to determine the short-distance coefficients(SDCs).

- Replace the physical tetraquark state T_{4c}^J with a free 4-quark state
- Calculate both sides of factorization formula in perturbative QCD and perturbative NRQCD
- Solving the factorization formula to determine the SDCs.

The SDCs are insensitive to the long-distance physics.

Four-Quark States

For convenience, we use the eigenstates of the angular momentum, manifesting the same quantum numbers as the physical tetraquark states.

$$\left|\mathcal{T}^{J,m_j}_{\bar{\mathbf{3}}\otimes\mathbf{3}}(Q)\right\rangle = \frac{1}{2}\sum_{s_*,\lambda_*}\left\langle\frac{1}{2}\lambda_1\frac{1}{2}\lambda_2\middle|1s_1\right\rangle\left\langle\frac{1}{2}\lambda_3\frac{1}{2}\lambda_4\middle|1s_2\right\rangle\langle 1s_1 1s_2|Jm_j\rangle$$

$$\mathcal{C}^{ab;cd}_{\bar{\mathbf{3}}\otimes\mathbf{3}}\left|c_a^{\lambda_1}(q_1)c_b^{\lambda_2}(P-q_1)\bar{c}_c^{\lambda_3}(q_2)\bar{c}_d^{\lambda_4}(Q-P-q_2)\right\rangle$$

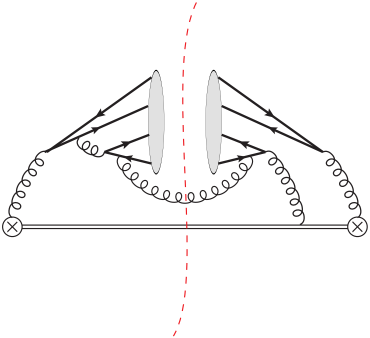
$$\left|\mathcal{T}^{0,0}_{\mathbf{6}\otimes\bar{\mathbf{6}}}(Q)\right\rangle = \frac{1}{2}\sum_{\lambda_*}\left\langle\frac{1}{2}\lambda_1\frac{1}{2}\lambda_2\middle|00\right\rangle\left\langle\frac{1}{2}\lambda_3\frac{1}{2}\lambda_4\middle|00\right\rangle$$

$$\mathcal{C}^{ab;cd}_{\mathbf{6}\otimes\bar{\mathbf{6}}}\left|c_a^{\lambda_1}(q_1)c_b^{\lambda_2}(P-q_1)\bar{c}_c^{\lambda_3}(q_2)\bar{c}_d^{\lambda_4}(Q-P-q_2)\right\rangle,$$

$pp \to T_{4c} + X$ via Fragmentation

Feynman Diagrams

- We employ the self-written program `HepLib`, which employ `Qgraf` and `GiNaC` internally to generate the Feynman diagrams (Feng *et al.*, 2021).
- There are about 100 diagrams for the amplitude.



Perturbative QCD Calculation

To project the $QQ\bar{Q}\bar{Q}$ into correct spin/color quantum number of tetraquark, we use the following projector

$$\bar{u}_i^a \bar{u}_j^b v_k^c v_l^d \to (C\Pi_\mu)^{ij} (\Pi_\nu C)^{lk} \mathcal{C}^{abcd}_{\bar{\mathbf{3}}\otimes\mathbf{3}} J^{\mu\nu}_{0,1,2}$$
$$\bar{u}_i^a \bar{u}_j^b v_k^c v_l^d \to (C\Pi_0)^{ij} (\Pi_0 C)^{lk} \mathcal{C}^{abcd}_{\mathbf{6}\otimes\bar{\mathbf{6}}}$$

C is the charge conjugate operator, $\Pi_\mu(\Pi_0)$ is the spin-triplet(singlet) projector of quarks (Petrelli *et al.*, 1997), $J^{\mu\nu}_{0,1,2}$ are the spin projectors of quark and anti-quark pairs (Braaten, Lee, 2003).

$$J_0^{\mu\nu} = \frac{1}{\sqrt{3}} \eta^{\mu\nu}(P)$$
$$J_1^{\mu\nu}(\epsilon) = -\frac{i}{\sqrt{2P^2}} \varepsilon^{\mu\nu\rho\sigma} \epsilon_\rho P_\sigma$$
$$J_2^{\mu\nu}(\epsilon) = \epsilon_{\rho\sigma} \left\{ \frac{1}{2} \left[\eta^{\mu\rho}(P)\eta^{\nu\sigma}(P) + \eta^{\mu\sigma}(P)\eta^{\nu\rho}(P) \right] - \frac{1}{3} \eta^{\mu\nu}(P)\eta^{\rho\sigma}(P) \right\}$$

SDCs

$$d_{3\times3}(g\to 0^{++}) = \frac{\pi^2\alpha_s^4}{497664z(2-z)^2(3-z)}\Big[186624 - 430272z + 511072z^2 - 425814z^3 + 217337z^4 - 61915z^5 + 7466z^6 + 42(1-z)(2-z)(3-z)(-144+634z - 385z^2 + 70z^3)\log(1-z) + 36(2-z)(3-z)\left(144 - 634z + 749z^2 - 364z^3 + 74z^4\right)\log\left(1-\frac{z}{2}\right) + 12(2-z)(3-z)\left(72 - 362z + 361z^2 - 136z^3 + 23z^4\right)\times\log\left(1-\frac{z}{3}\right)\Big].$$

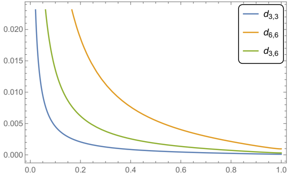
$$d_{6\times6}(g\to 0^{++}) = \frac{\pi^2\alpha_s^4}{55296z(2-z)^2(3-z)}\Big[186624 - 430272z + 617824z^2 - 634902z^3 + 374489z^4 - 115387z^5 + 14378z^6 - 6(1-z)(2-z)(3-z)(-144-2166z + 1015z^2 + 70z^3)\log(1-z) - 156(2-z)(3-z)\left(144 - 1242z + 1693z^2 - 876z^3 + 170z^4\right)\log\left(1-\frac{z}{2}\right) + 300(2-z)(3-z)\left(72 - 714z + 953z^2 - 472z^3 + 87z^4\right)\times\log\left(1-\frac{z}{3}\right)\Big].$$

$$d_{3\times6}(g\to 0^{++}) = -\frac{\pi^2\alpha_s^4}{165888z(2-z)^2(3-z)}\Big[186624 - 430272z + 490720z^2 - 394422z^3 + 199529z^4 - 57547z^5 + 7082z^6 + 6(1-z)(2-z)(3-z)(-432+3302z - 1855z^2 + 210z^3)\log(1-z) - 12(2-z)(3-z)\left(720 - 2258z + 2329z^2 - 1052z^3 + 226z^4\right)\log\left(1-\frac{z}{2}\right) + 12(2-z)(3-z)\left(936 - 4882z + 4989z^2 - 1936z^3 + 331z^4\right)\times\log\left(1-\frac{z}{3}\right)\Big].$$

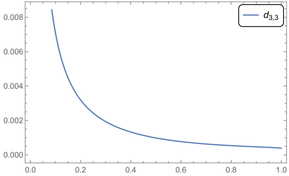
SDCs

$$
\begin{aligned}
d_{3\times3}\left(g \to 2^{++}\right) =& \frac{\pi^2\alpha_s^4}{622080z^2(2-z)^2(3-z)}\Big[2\left(46656-490536z+1162552z^2\right.\\
&\left.-1156308z^3+595421z^4-170578z^5+21212z^6\right)z\\
&+3(1-z)(2-z)(3-z)(-20304-31788z)(1296+1044z\\
&+73036z^2-36574z^3+7975z^4)\log(1-z)\Big]+33(2-z)(3-z)(1296+25)\\
&-9224z^2+9598z^3-3943z^4+725z^5)\log\left(1-\frac{z}{3}\right)\Big],\\
d_{6\times6}\left(g \to 2^{++}\right) =& d_{3\times6}\left(g \to 2^{++}\right) = 0.
\end{aligned}
$$

- There is NO IR divergence.



SDCs for T_{4c}^{0++}



SDC for T_{4c}^{2++}

Long-Distance Matrix Elements(LDMEs)

- The NRQCD LDMEs should be calculated in lattice QCD in principle since they are non-perturbative.
- We use the diquark model to calculate the LDMEs, resulting in the product of wave functions at the origin.
- The fock component of $\mathbf{6}\otimes\bar{\mathbf{6}}$ is neglected for simplicity while there are some results in literature(Lü, *et al.*, 2020; Zhao, *et al.*, 2020). **See Q.-F. Lü, Zhao**

$$\left|\left\langle T_{4c}^{0}\left|\mathcal{O}_{\bar{\mathbf{3}}\otimes\mathbf{3}}^{(0)}\right|0\right\rangle\right|^2=\frac{1}{4\pi^3}\left|R_{\mathcal{D}}(0)\right|^4\left|R_T(0)\right|^2,$$

$$\sum_{m_j}\left|\left\langle T_{4c}^{2,m_j}\left|\mathcal{O}_{\bar{\mathbf{3}}\otimes\mathbf{3}}^{(2)kl}\right|0\right\rangle\right|^2=\frac{5}{4\pi^3}\left|R_{\mathcal{D}}(0)\right|^4\left|R_T(0)\right|^2.$$

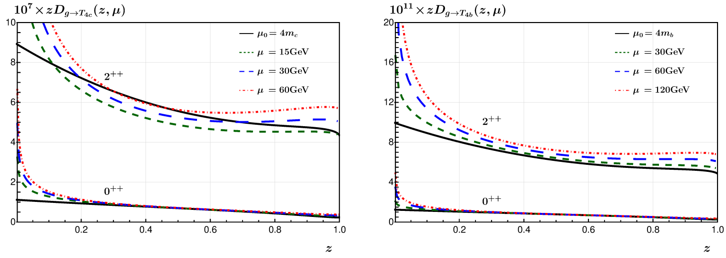
- The phenomenological results we use $(\mathrm{GeV}^{3/2})$(Kiselev *et al.*, 2002; Debastiani, Navarra, 2019):

	$R_{\mathcal{D}}(0)$	$R_{T^0}(0)$	$R_{T^2}(0)$
T_{4c}	0.523	2.902	2.583
T_{4b}(Coulomb)	0.703	5.57909	5.57909

Evolution of Fragmentation Function

Since the process is gluon dominance, the leading order splitting kernels read (n_f: number of active light quark flavors):

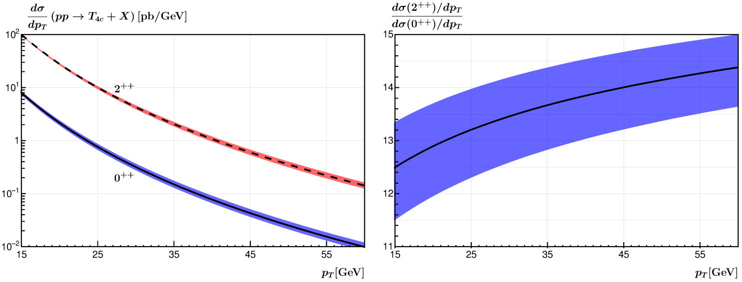
$$P_{g\leftarrow g}(z) = 6\left[\frac{(1-z)}{z} + \frac{z}{(1-z)_+} + z(1-z) + \left(\frac{11}{12} - \frac{n_\mathrm{f}}{18}\right)\delta(1-z)\right]$$



Evolution of $g \to T_{4c/4b}$ fragmentation functions.

Phenomenology at LHC

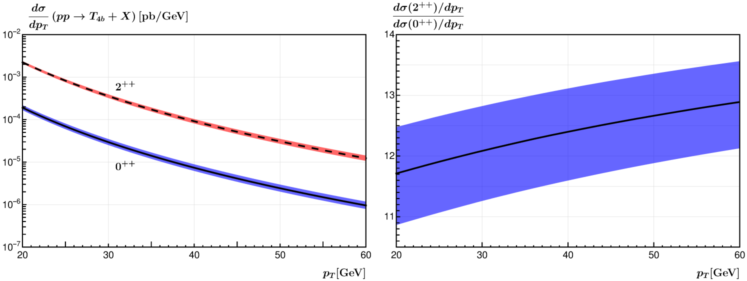
- Parameters: $\sqrt{s} = 13\,\mathrm{TeV}$; `CTEQ14` PDF sets; factorization scale $\mu \in [p_T/2, 2p_T]$
  $m_c = 1.5\,\mathrm{GeV}$



	0^{++}		2^{++}	
p_T/GeV	σ/pb	N_{events}	σ/pb	N_{events}
$[15, 60]$	33^{+4}_{-4}	$9.9^{+1.2}_{-1.2} \times 10^7$	424^{+13}_{-21}	$1.27^{+0.04}_{-0.06} \times 10^9$

Phenomenology at LHC

- Parameters: $\sqrt{s} = 13\,\mathrm{TeV}$; $\texttt{CTEQ14}$ PDF sets; factorization scale $\mu \in [p_T/2, 2p_T]$ $m_b = 4.8\,\mathrm{GeV}$



	0^{++}		2^{++}	
p_T/GeV	$\sigma/10^{-3}\mathrm{pb}$	$N_{\mathrm{events}}/10^3$	$\sigma/10^{-2}\mathrm{pb}$	$N_{\mathrm{events}}/10^4$
$[20, 60]$	$1.04^{+0.17}_{-0.15}$	$3.12^{+0.51}_{-0.45}$	$1.24^{+0.11}_{-0.11}$	$3.72^{+0.33}_{-0.33}$

$$e^+e^- \to T_{4c} + \gamma$$

Factorization Formula

The differential cross section can be expressed in terms of the differential decay rate of a virtual photon.

$$\frac{\mathrm{d}\sigma\left[e^+e^- \to \gamma(\lambda_1) + T_{4c}^J(\lambda_2)\right]}{\mathrm{d}\cos\theta} = \frac{4\pi\alpha}{s^{3/2}} \sum_{S_z=\pm 1} \frac{\mathrm{d}\Gamma\left[\gamma^*(S_z) \to \gamma(\lambda_1) + T_{4c}^J(\lambda_2)\right]}{\mathrm{d}\cos\theta}$$

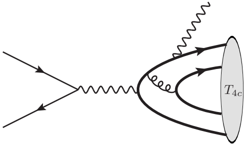
$$= \frac{4\pi\alpha}{s^{3/2}} \sum_{S_z=\pm 1} \frac{|\mathbf{p}_f|}{16\pi s} \frac{3}{4\pi} \left|\mathcal{M}^J_{\lambda_1,\lambda_2}\right|^2 \left|d^1_{S_z,\lambda}(\theta)\right|^2$$

The NRQCD factorization holds true at the helicity amplitude level.

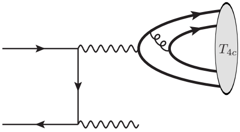
$$\mathcal{M}^J_{\lambda_1,\lambda_2} = \frac{\mathcal{A}^{3[J]}_{\lambda_1,\lambda_2}}{m_c^4} \sqrt{2M_{T_{4c}}} \langle T^J_{4c} | \mathcal{O}^{(J)}_{\bar{\mathbf{3}}\otimes\mathbf{3}} | 0 \rangle + \frac{\mathcal{A}^{6[J]}_{\lambda_1,\lambda_2}}{m_c^4} \sqrt{2M_{T_{4c}}} \langle T^J_{4c} | \mathcal{O}^{(J)}_{\mathbf{6}\otimes\bar{\mathbf{6}}} | 0 \rangle$$

Feynman Diagrams

- There are roughly 40 s-channel diagrams in total.
- Due to C-parity conservation, the t-channel process in $b)$ does not contribute.



$a)$



$b)$

SDCs

$$\mathcal{A}_{1,0}^{3[0]} = \mathcal{A}_{-1,0}^{3[0]} = -\frac{16\pi^{5/2}\alpha\alpha_s\left(10-17r+9r^2\right)}{27\sqrt{3}(3-r)(2-r)},$$

$$\mathcal{A}_{1,0}^{6[0]} = \mathcal{A}_{-1,0}^{6[0]} = -\frac{16\pi^{5/2}\alpha\alpha_s\left(10-9r+r^2\right)}{9\sqrt{3}(3-r)(2-r)},$$

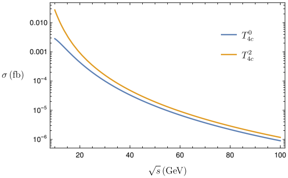
$$\mathcal{A}_{1,0}^{3[2]} = \mathcal{A}_{-1,0}^{3[2]} = \frac{128\pi^{5/2}\alpha\alpha_s}{27\sqrt{6}(3-r)},$$

$$\mathcal{A}_{1,1}^{3[2]} = \mathcal{A}_{-1,-1}^{3[2]} = \frac{512\pi^{5/2}\alpha\alpha_s}{27\sqrt{2}(3-r)}\left(\frac{m_c}{s^{1/2}}\right),$$

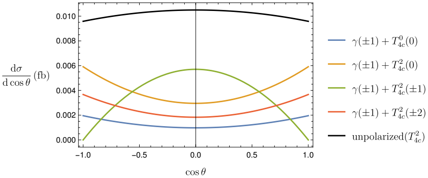
$$\mathcal{A}_{1,2}^{3[2]} = \mathcal{A}_{-1,-2}^{3[2]} = \frac{2048\pi^{5/2}\alpha\alpha_s}{27(3-r)}\left(\frac{m_c}{s^{1/2}}\right)^2.$$

At large $\sqrt{s}$ limit, the polarized cross section scales as $\sigma \propto s^{-2-|\lambda|}$, which is compatible with the helicity selection rule.

Phenomenology



Total cross sections



Angular distributions at $\sqrt{s} = 10.58$ GeV

We neglect the $\mathbf{6} \otimes \bar{\mathbf{6}}$ component and adopt the diquark model as before. The total cross sections for these exclusive processes decline quite fast with increasing $\sqrt{s}$, and at the B factory energy $\sqrt{s} = 10.58$ GeV

$$\sigma\left[e^+e^- \to T^0_{4c} + \gamma\right] \approx 0.0026 \text{ fb},$$
$$\sigma\left[e^+e^- \to T^2_{4c} + \gamma\right] \approx 0.020 \text{ fb}.$$

There would be $130\ T^0_{4c} + \gamma$ events and $1020\ T^2_{4c} + \gamma$ events at the `Belle2` experiment since the designed luminosity is about $50\,\text{ab}^{-1}$

Summary

- We propose a model-independent approach to study the production of fully heavy tetraquark, based on NRQCD factorization.
- The production rates of T_{4c} appears to be significant on the LHC due to the huge luminosity.
- The production rates at $\sqrt{s} = 10.58$ GeV are too small to be observed at `Belle 2` experiment.
- Model-independent estimates on the NRQCD matrix elements are required to make more reliable phenomenological predictions.

Thanks!