On the nature of $X(6900)$ and other structures in the LHCb di-J/ψ spectrum

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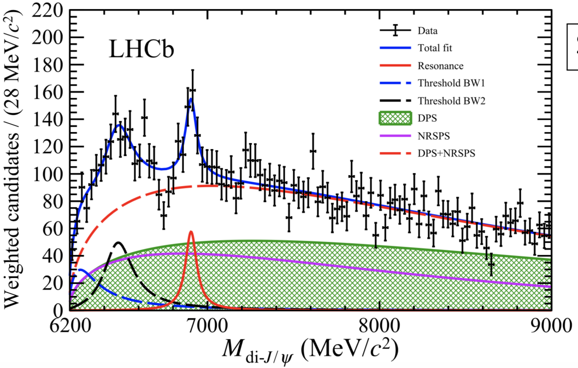
Based on [Z.-R. Liang, De-Liang Yao, work in preparation]

Outline

- Introduction
  - Background
  - Status of theoretical studies
- Theoretical Framework
  - Effective Lagrangian
  - Helicity amplitudes and partial-wave projection
  - Production amplitude and effects of final state interaction
- Numerical Results
  - Description of the LHCb data with model with three-coupled channels
  - Analyses with four-coupled channels and hints of four possible states
  - Further consideration: a combined fit
- Summary and outlook

Background



Sci. Bull. 65 (2020), 1983-1993

- LHCb reported a narrow structure around 6.9 GeV in the di-J/ψ invariant mass spectrum: X(6900).
- A possible board structure at range $[6.2\ \mathrm{GeV}, 6.8\ \mathrm{GeV}]$.
- A hint of another structure around $7.2\ \mathrm{GeV}$.

Observation of structure in the J/ψ -pair mass spectrum #2

LHCb Collaboration • Roel Aaij (NIKHEF, Amsterdam) et al. (Jun 30, 2020)

Published in: *Sci.Bull.* 65 (2020) 23, 1983-1993 • e-Print: 2006.16957 [hep-ex]

pdf links DOI cite

Status of theoretical studies

Many works have been accumulated:

- Quark model:
  - R. N. Faustov, V. O. Galkin and E. M. Savchenko, arXiv:2103.01763.
  - X. Jin, X. Liu, Y. Xue, H. Huang and J. Ping, arXiv:2011.12230.
  - Q. F. Lü, D. Y. Chen and Y. B. Dong, Eur. Phys. J. C **80** (2020) no.9, 871.
- QCD sum rules:
  - B. C. Yang, L. Tang and C. F. Qiao, arXiv:2012.04463.
  - B. D. Wan and C. F. Qiao, arXiv:2012.00454.
  - Z. G. Wang, Chin. Phys. C **44** (2020) no.11, 113106.
- NRQCD factorization:
  - F. Feng, Y. Huang, Y. Jia, W. L. Sang and J. Y. Zhang, arXiv:2011.03039.
  - Y. Q. Ma and H. F. Zhang, arXiv:2009.08376.
  - F. Feng, Y. Huang, Y. Jia, W. L. Sang, X. Xiong and J. Y. Zhang, arXiv:2009.08450.

Status of theoretical studies

- Phenomenological model:
    - Z. Zhao, K. Xu, A. Kaewsnod, X. Liu, A. Limphirat and Y. Yan, arXiv:2012.15554.
    - C. Gong, M. C. Du, B. Zhou, Q. Zhao and X. H. Zhong, arXiv:2011.11374.
    - Q. F. Cao, H. Chen, H. R. Qi and H. Q. Zheng, arXiv:2011.04347.
    - X. K. Dong, V. Baru, F. K. Guo, C. Hanhart and A. Nefediev, arXiv:2009.07795.
    - Z. H. Guo and J. A. Oller, Phys. Rev. D **103** (2021), 034024.
- ...
- **Our work** (Effective Lagrangian & Unitarization)
  Reveal possible states and explore their J^{PC} quantum numbers
    - derive potentials from effective Lagrangians
    - take coupled-channel effects into account
    - employ helicity amplitude formalism and perform partial-wave analysis

Effective Lagrangian

$$
\begin{aligned}
\mathcal{L}_{\text{eff.}} &= h_1(J/\psi\cdot J/\psi)^2 \\
&+ h_2(J/\psi\cdot J/\psi)(J/\psi\cdot\psi(2S)) \\
&+ h_3(J/\psi\cdot J/\psi)(J/\psi\cdot\psi(3770)) \\
&+ 2h_4(J/\psi\cdot\psi(2S))^2 \\
&+ 2h_4'(J/\psi\cdot J/\psi)(\psi(2S)\cdot\psi(2S)) \\
&+ h_5(J/\psi\cdot\psi(2S))(J/\psi\cdot\psi(3770)) \\
&+ h_5'(J/\psi\cdot J/\psi)(\psi(2S)\cdot\psi(3770)) \\
&+ h_6(J/\psi\cdot\psi(3770))^2 \\
&+ h_6'(J/\psi\cdot J/\psi)(\psi(3770)\cdot\psi(3770)) \\
&+ h_7(J/\psi\cdot\psi(2S))(\psi(2S)\cdot\psi(2S)) \\
&+ h_8(J/\psi\cdot\psi(3770))(\psi(2S)\cdot\psi(2S)) \\
&+ h_8'(J/\psi\cdot\psi(2S))(\psi(3770)\cdot\psi(2S)) \\
&+ 4h_9(\psi(2S)\cdot\psi(2S))^2
\end{aligned}
$$

- Four channels: $\{J/\psi J/\psi,\ J/\psi\psi(2S),\ J/\psi\psi(3770),\ \psi(2S)\psi(2S)\}$
- The Lagrangian satisfies the basic symmetries, such as Lorentz invariance, P and C parity symmetries and so on.
- The unknown couplings $h_{1,2,..9}$ and $h'_{4,5,6,8}$ need to be determined by experimental data.

Potentials for the scattering processes

- The generic form of the potentials for $V_1(p_1,\epsilon_1) + V_2(p_2,\epsilon_2) \to V_3(p_3,\epsilon_3) + V_4(p_4,\epsilon_4)$

$$V_{ij} = \mathcal{C}_1 \epsilon_1 \cdot \epsilon_2 \epsilon_3^\dagger \cdot \epsilon_4^\dagger + \mathcal{C}_2 \epsilon_1 \cdot \epsilon_3^\dagger \epsilon_2 \cdot \epsilon_4^\dagger + \mathcal{C}_3 \epsilon_1 \cdot \epsilon_4^\dagger \epsilon_2 \cdot \epsilon_3^\dagger$$

- Coefficients for different processes

V_{ij}	Channels	$\mathcal{C}_1$	$\mathcal{C}_2$	$\mathcal{C}_3$
11	$J/\psi J/\psi \to J/\psi J/\psi$	$8h_1$	$8h_1$	$8h_1$
12	$J/\psi J/\psi \to \psi(2S) J/\psi$	$2h_2$	$2h_2$	$2h_2$
13	$J/\psi J/\psi \to \psi(3770) J/\psi$	$2h_3$	$2h_3$	$2h_3$
14	$J/\psi J/\psi \to \psi(2S)\psi(2S)$	$4h_4'$	$2h_4$	$2h_4$
22	$\psi(2S) J/\psi \to \psi(2S) J/\psi$	$2h_4$	$4h_4'$	$2h_4$
23	$\psi(2S) J/\psi \to \psi(3770) J/\psi$	h_5	$2h_5'$	h_5
24	$\psi(2S) J/\psi \to \psi(2S)\psi(2S)$	$2h_7$	$2h_7$	$2h_7$
33	$\psi(3770) J/\psi \to \psi(3770) J/\psi$	$2h_6$	$4h_6'$	$2h_6$
34	$\psi(3770) J/\psi \to \psi(2S)\psi(2S)$	$2h_8$	h_8'	h_8'
44	$\psi(2S)\psi(2S) \to \psi(2S)\psi(2S)$	$8h_9$	$8h_9$	$8h_9$

Helicity amplitude

- Helicity amplitudes (81 in total), $\lambda_i = \pm 1, 0$

$$V_{\lambda_1\lambda_2\lambda_3\lambda_4} = \epsilon_3^{\rho\dagger}(p_3, \lambda_3)\epsilon_4^{\sigma\dagger}(p_4, \lambda_4) V_{\mu\nu\rho\sigma}\epsilon_1^{\mu}(p_1, \lambda_1)\epsilon_2^{\nu}(p_2, \lambda_2)$$

$$V_{\mu\nu\rho\sigma} = \mathcal{C}_1 g_{\mu\nu} g_{\rho\sigma} + \mathcal{C}_2 g_{\mu\rho} g_{\nu\sigma} + \mathcal{C}_3 g_{\mu\sigma} g_{\nu\rho}$$

According to P and C parity symmetries ($81 \to 25$)
e.g.

$$V_{++++} = V_{----}$$
$$V_{+++0} = -V_{---0} = V_{-0--} = -V_{+0++}$$

- Explicit expressions for helicity amplitudes
e.g.

$$V_{++++} = V_{----} = \mathcal{C}_1 + \frac{1}{4}(\mathcal{C}_3(-1+z_s)^2 + \mathcal{C}_2(1+z_s)^2)$$

with $z_s = \cos\theta$, θ being scattering angle .

Partial-Wave Projection

- Partial wave amplitudes

$$V^J_{\lambda_1\lambda_2\lambda_3\lambda_4}(s) = \frac{1}{2}\int_{-1}^{+1} dz_s\, V_{\lambda_1\lambda_2\lambda_3\lambda_4}(s, t(s, z_s)) d^J_{\lambda,\lambda'}(z_s)$$

  where $\lambda = \lambda_1 - \lambda_2$, $\lambda' = \lambda_3 - \lambda_4$.
  Relations due to P, C symmetries and properties of $d^J_{\lambda\lambda'}$ functions:
  e.g.
  - $V^J_{++++} = V^J_{----}$
  - $V^J_{+++0} = V^J_{---0} = V^J_{-0--} = V^J_{+0++}$
- Helicity basis → JLS basis (definite J^{PC} quantum number)

$$\mathcal{V}^J = \sum_{\substack{\lambda_1\lambda_2\\ \lambda_3\lambda_4}} U^J_{\lambda_3\lambda_4}\mathcal{V}^J_{\lambda_1\lambda_2\lambda_3\lambda_4}[U^J_{\lambda_1\lambda_2}]^\dagger$$

  - The transformation matrix

$$U^J_{\lambda_1\lambda_2} = \frac{1}{\sqrt{2S+1}}\langle S_1\lambda_1 S_2 - \lambda_2|S\lambda\rangle \quad => \boxed{\text{C-G coefficient}}$$

S-Wave Amplitude

S-wave: $L=0$, $J=L+S=S \Rightarrow$ For identical particles: $L+S = \mathrm{even}$

- $J^{PC} = 0^{++}$

$$\mathcal{V}(0^{++}) = \frac{2}{3}V^{J=0}_{++++} + \frac{2}{3}V^{J=0}_{++--} + \frac{1}{3}V^{J=0}_{0000} - \frac{4}{3}V^{J=0}_{++00}$$

- $J^{PC} = 2^{++}$

$$\begin{aligned}\mathcal{V}(2^{++}) &= \frac{2}{15}V^{J=2}_{0000} + \frac{4}{15}V^{J=2}_{00++} + \frac{4\sqrt{6}}{15}V^{J=2}_{00+-} + \frac{2\sqrt{6}}{15}(V^{J=2}_{+-++} + V^{J=2}_{-+++}) \\ &+ \frac{4\sqrt{3}}{15}(V^{J=2}_{00+0} + V^{J=2}_{000+}) + \frac{2\sqrt{3}}{15}(V^{J=2}_{+0++} + V^{J=2}_{0+++} + V^{J=2}_{0-++} + V^{J=2}_{-0++}) \\ &+ \frac{1}{5}(V^{J=2}_{+0+0} + V^{J=2}_{-0+0} + V^{J=2}_{0+0+} + V^{J=2}_{0-0+}) \\ &+ \frac{2\sqrt{2}}{5}(V^{J=2}_{+-+0} + V^{J=2}_{0++-} + V^{J=2}_{-++0} + V^{J=2}_{0-+-}) \\ &+ \frac{2}{5}(V^{J=2}_{0++0} + V^{J=2}_{0-+0} + V^{J=2}_{+-+-} + V^{J=2}_{-++-}) + \frac{1}{15}(V^{J=2}_{++++} + V^{J=2}_{--++})\end{aligned}$$

Unitarization

- Requirement of unitarity
  $\longrightarrow$ Bethe-Salpeter equation method is employed to restore unitarity.
- The unitarized amplitude under on-shell approximation

$$\mathcal{T}_J(s) = \mathcal{V}^J(s) \cdot \left[1 - \mathcal{G}(s) \cdot \mathcal{V}^J(s)\right]^{-1}$$

Graphic representation

- For coupled-channel:

$$\mathcal{V}^J(s) = \begin{pmatrix} V^J_{11}(s) & V^J_{12}(s) & V^J_{13}(s) & V^J_{14}(s) \\ V^J_{12}(s) & V^J_{22}(s) & V^J_{23}(s) & V^J_{24}(s) \\ V^J_{13}(s) & V^J_{23}(s) & V^J_{33}(s) & V^J_{34}(s) \\ V^J_{14}(s) & V^J_{24}(s) & V^J_{34}(s) & V^J_{44}(s) \end{pmatrix}$$

- two-point functions

$$\mathcal{G}(s) = \begin{pmatrix} g_1(s) & & & \\ & g_2(s) & & \\ & & g_3(s) & \\ & & & g_4(s) \end{pmatrix}$$

Unitarization

- The explicit form of two-point loop function

$$
\begin{aligned}
g_i(s) =& \frac{1}{16\pi^2}\Bigg\{ a(\mu) + \ln\frac{M_{V_1}^2}{\mu^2} + \frac{s - M_{V_1}^2 + M_{V_2}^2}{2s}\ln\frac{M_{V_2}^2}{M_{V_1}^2} \\
&+ \frac{\sigma(s)}{2s}\Big[\ln(s - M_{V_2}^2 + M_{V_1}^2 + \sigma(s)) - \ln(-s + M_{V_2}^2 - M_{V_1}^2 + \sigma(s)) \\
&+ \ln(s + M_{V_2}^2 - M_{V_1}^2 + \sigma(s)) - \ln(-s - M_{V_2}^2 + M_{V_1}^2 + \sigma(s))\Big]\Bigg\}
\end{aligned}
$$

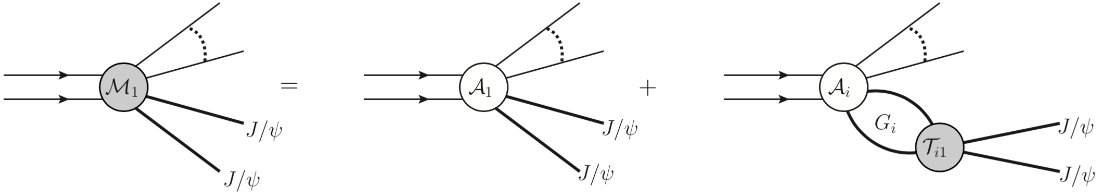
where

- $\sigma(s) = \{[s - (M_{V_2} + M_{V_1})^2][s - (M_{V_2} - M_{V_1})^2]\}^{1/2}$.
- The subtraction scale $a(\mu) = -3.0$ at renormalization scale $\mu = 1\ \mathrm{GeV}$.
- Definition of Riemann Sheet

$$
g^{\mathrm{II}}(s) = g^{\mathrm{I}}(s) + 2i\rho(s)
$$

Production Amplitudes

- Production of di-J/ψ via pp collision



- The production amplitude

$$\mathcal{M}_1(s) = \mathcal{A}_1 + \sum \mathcal{A}_i \mathcal{G}_i(s)\mathcal{T}_{i1}(s) = \mathcal{A}_1\left[1 + \sum r_i \mathcal{G}_i(s)\mathcal{T}_{i1}(s)\right]$$

  - $\gamma_i = \mathcal{A}_i/\mathcal{A}_1$
  - $\mathcal{A}_1$ represent direct production of J/ψ pairs.

The invariant mass formula

- The invariant mass

$$\frac{\mathrm{d}\mathcal{N}}{\mathrm{d}\sqrt{s}} \propto \rho(s)|\mathcal{M}_1(s)|^2 = \rho(s)|\mathcal{A}_1(s)|^2\left|\gamma + \sum_{i=1}^{3}\mathcal{G}_i(s)\mathcal{T}_{i1}(s)\right|^2$$

  with γ coherent background.
- Phase factor

$$\rho(s) = \frac{p_1(s)}{8\pi\sqrt{s}} = \frac{\lambda^{1/2}(s, m_{J/\psi}^2, m_{J/\psi}^2)}{16\pi s}$$

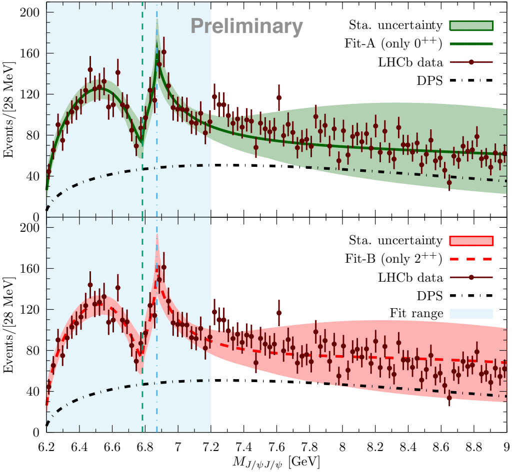
- The direct production amplitude is parameterized as

$$|\mathcal{A}_1(s)|^2 = \alpha^2 e^{-2\beta s}$$

  - Normalization factor α
  - The slope parameter β is fixed ($\beta = 0.0123$)
    [X. K. Dong, V. Baru, F. K. Guo, C. Hanhart and A. Nefediev, arXiv:2009.07795.]

Fit with three coupled-channels

- Description of LHCb data:



- consider three coupled-channels: $J/\psi J/\psi$, $J/\psi\psi(2S)$, $J/\psi\psi(3770)$;
- perform two different kinds of fits 0^{++} and 2^{++} within range [6.2 GeV, 7.2 GeV] ;
- Both fits well describe the di-J/ψ spectrum within 1-σ uncertainty.

Fit with three coupled-channels

- Pole positions and residues:
  - 0^{++}

RS	Position $\sqrt{s_{\text{pole}}}$ [MeV]	$\lvert\text{Residue}\rvert^{1/2}$ [GeV] $J/\psi J/\psi$	$J/\psi\psi(2S)$	$J/\psi\psi(3770)$
I	$6172.4^{+21.3}_{-30.4}$	$17.2^{+4.5}_{-11.7}$	$22.4^{+4.1}_{-14.0}$	$5.0^{+4.2}_{-4.3}$
II	$6191.2^{+2.5}_{-3.5}$	$4.3^{+5.5}_{-1.0}$	$5.6^{+12.4}_{-1.3}$	$1.2^{+3.9}_{-0.9}$
II	$6959.1^{+37.1}_{-58.7} - i161.2^{+125.9}_{-93.3}$	$27.8^{+7.1}_{-7.7}$	$37.2^{+4.0}_{-8.2}$	$20.3^{+16.1}_{-5.9}$

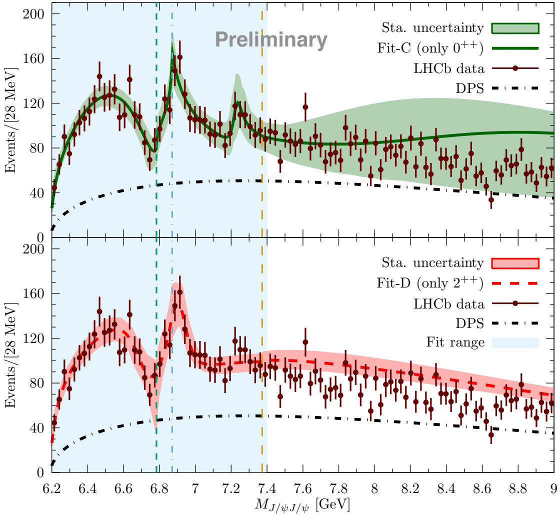
  - 2^{++}

RS	Position $\sqrt{s_{\text{pole}}}$ [MeV]	$\lvert\text{Residues}\rvert^{1/2}$ [GeV] $J/\psi J/\psi$	$J/\psi\psi(2S)$	$J/\psi\psi(3770)$
I	$6170.1^{+19.5}_{-32.1}$	$17.5^{+4.4}_{-6.0}$	$22.6^{+2.8}_{-5.9}$	$5.3^{+3.4}_{-2.9}$
II	$6190.9^{+2.4}_{-3.8}$	$4.4^{+1.0}_{-1.1}$	$5.7^{+0.8}_{-1.0}$	$1.3^{+0.9}_{-0.7}$
II	$6994.1^{+53.8}_{-92.7} - i156.9^{+150.6}_{-127.8}$	$28.4^{+8.7}_{-8.2}$	$36.9^{+4.6}_{-8.7}$	$19.7^{+13.8}_{-7.6}$

  - A near-threshold bound state is found in these two cases, referred as to $X(6200)$.
  - A peak located in 6.9 GeV appears due to the $J/\psi\psi(3770)$ threshold effects.

Fit with four coupled-channels

- Description of LHCb data:



- consider four coupled-channels $J/\psi J/\psi$, $J/\psi\psi(2S)$, $J/\psi\psi(3770)$, $\psi(2S)\psi(2S)$.
- perform two different kinds of fits 0^{++} and 2^{++} and extend the energy range to [6.2 GeV,7.4 GeV] ;
- Fit-C (0^{++}) and Fit-D (2^{++}) behave differently. A peak around 7.2 GeV is observed in 0^{++} case.

Fits with four coupled-channels

- Pole positions and residues:

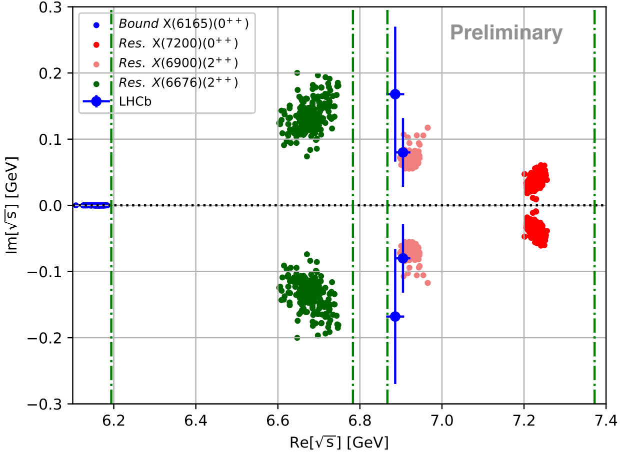
RS	Position $\sqrt{s_{\rm pole}}$ [MeV]	$\lvert\text{Residue}\rvert^{1/2}$ [GeV] $J/\psi J/\psi$	$J/\psi\psi(2S)$	$J/\psi\psi(3770)$	$\psi(2S)\psi(2S)$
I (0^{++})	$6165.6^{+18.8}_{-57.7}$	$19.8^{+5.5}_{-5.0}$	$15.8^{+6.1}_{-3.5}$	$0.7^{+0.5}_{-0.7}$	$1.9^{+1.7}_{-1.9}$
IV (0^{++})	$7225.4^{+30.9}_{-23.7} - i30.4^{+30.4}_{-21.4}$	$4.4^{+2.3}_{-2.5}$	$5.7^{+1.4}_{-2.0}$	$0.9^{+0.6}_{-0.5}$	$\boxed{37.8^{+2.1}_{-2.7}}$
II (2^{++})	$6676.7^{+70.8}_{-73.4} - i136.9^{+63.3}_{-63.0}$	$15.0^{+2.4}_{-2.9}$	$24.4^{+2.6}_{-11.6}$	$13.5^{+18.8}_{-12.0}$	$37.2^{+5.2}_{-5.5}$
IV (2^{++})	$6920.7^{+44.7}_{-22.9} - i66.1^{+51.2}_{-10.7}$	$6.2^{+5.3}_{-1.7}$	$9.6^{+3.2}_{-6.4}$	$5.2^{+4.1}_{-4.5}$	$\boxed{50.1^{+3.8}_{-1.4}}$

- Fit-C:
  (1) A resonance X(7200) ;
  (2) A bound state $X(6200)$;
  (3) Their J^{PC} numbers are 0^{++}.

- Fit-D:
  (1) A narrow resonance X(6900);
  (2) A broad resonance X(6670);
  (3) Their J^{PC} numbers are 2^{++}.

Fits with four coupled-channels

- The locations of poles:



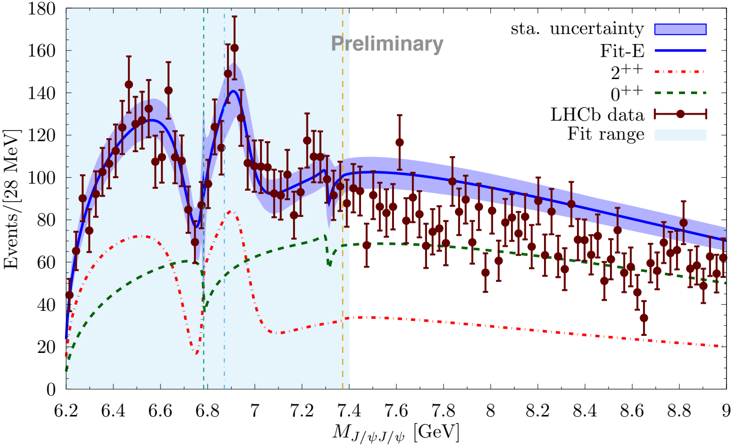
- Green → the 2^{++} X(6670) resonance
- Pink → the 2^{++} X(6900) resonance
- Red → the 0^{++} X(7200) resonance
- Blue → the 0^{++} X(6200) bound state

LHCb results (the blue error bar):

- $m = 6905 \pm 11 \pm 7$ MeV
  $\Gamma = 80 \pm 19 \pm 33$ MeV
- $m = 6886 \pm 11 \pm 11$ MeV
  $\Gamma = 168 \pm 33 \pm 69$ MeV

Combined fit

- Description of LHCb data:



- three coupled-channels $J/\psi J/\psi$, $J/\psi \psi(2S)$, $\psi(2S)\psi(2S)$
- To judge which partial-wave is dominant $\rightarrow$ both are sizeable

Combined fit

- Pole positions and residue

RS (J^{PC})	Position $\sqrt{s_{\text{pole}}}$ [MeV]	$\lvert\text{Residue}\rvert^{1/2}$ [GeV] $J/\psi J/\psi$	$J/\psi\psi(2S)$	$\psi(2S)\psi(2S)$
I (0^{++})	$6059.6^{+98.0}_{-21.0}$	$29.5^{+2.0}_{-11.2}$	$16.2^{+10.0}_{-2.6}$	$4.5^{+1.4}_{-0.4}$
II (2^{++})	$6759.2^{+84.8}_{-63.5} - i120.1^{+91.8}_{-54.6}$	$17.0^{+6.5}_{-5.8}$	$28.7^{+9.9}_{-7.0}$	$42.9^{+21.4}_{-16.6}$
III (2^{++})	$6950.0^{+32.6}_{-22.1} - i88.7^{+15.8}_{-10.8}$	$9.0^{+1.3}_{-1.6}$	$9.0^{+1.3}_{-2.2}$	$50.7^{+0.9}_{-1.5}$

  - The $X(6200)$, $X(6670)$ and $X(6900)$ still exist.
  - Their pole locations are shifted, due to the absence of the $J/\psi\psi(3770)$ channel.
  - The $X(7200)$ disappears, however, there still exist an enhancement around 7.2 GeV.
    $\rightarrow$ The $J/\psi\psi(3770)$ channel is important for the existence of the $X(7200)$ state.

Summary and Outlook

- Exploring all possible states in the range $[6.2\ \mathrm{GeV}, 7.4\ \mathrm{GeV}]$ by means of partial-wave analysis.
- Four states are found
  - X(6200): a bound state with $J^{PC} = 0^{++}$, $\sqrt{s_{\mathrm{pole}}} = 6165.6^{+18.8}_{-57.7}\ \mathrm{MeV}$
  - X(6670): a broad resonant state with $J^{PC} = 2^{++}$, $\sqrt{s_{\mathrm{pole}}} = 6676.7^{+70.8}_{-73.4} - i136.9^{+63.3}_{-63.0}\ \mathrm{MeV}$
  - X(6900): a narrow resonant state with $J^{PC} = 2^{++}$, $\sqrt{s_{\mathrm{pole}}} = 6920.7^{+44.7}_{-22.9} - i66.1^{+51.2}_{-10.7}\ \mathrm{MeV}$
  - X(7200): a narrow resonant state with $J^{PC} = 0^{++}$, $\sqrt{s_{\mathrm{pole}}} = 7225.4^{+30.9}_{-23.7} - i30.4^{+30.4}_{-21.4}\ \mathrm{MeV}$

  - The above results needs to be confirmed by more precise experimental data;
  - Study the structures of these states in future.

Thanks for your attention!