Chiral effective Lagrangian for doubly charmed baryons and its application

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2nd Joint Theory/Experiment Workshop on Hadron and Heave Flavor Physics

Lanzhou University

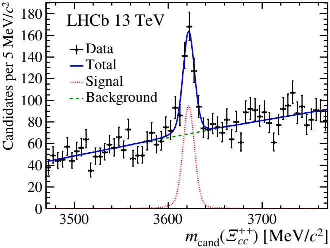
March 25-28, 2021

Based on [P.-C. Qiu and De-Liang Yao, PRD103(2021)034006]
[P.-C Qiu and De-Liang Yao, in progress]

Outline

I. Introduction

- In 2002, the SELEX Collaboration firstly reported that the Ξ_{cc}^{+} state was observed with measured mass 3519 ± 2 MeV [M. Mattson, PRL89(2002)112001].
- Other experimental groups: FOCUS [S. Ratti, Nucl.Phys.B, Proc.Suppl.115(2003)33], BABAR [B. Aubert, PRD74(2006)011103], Belle [R. Chistov, PRL97(2006)162001], LHCb [R. Aaij, JHEP12(2013)090].
- In 2017, the LHCb Collaboration announced the observation of the Ξ_{cc}^{++}, via the decay mode $\Lambda_c^{+}K^{-}\pi^{+}\pi^{+}$ [R. Aaij, PRL119(2017)112001].



- Reported value of mass(Ξ_{cc}^{++}): $3621.40 \pm 0.72 \pm 0.27 \pm 0.14$ MeV

Inspired by a theoretical work [F.S.Yu, H.Y.Jiang, R.H.Li, C.D.Lü, W.Wang, and Z.X.Zhao, CPC42(2018)]

Status of theoretical studies

- Various theoretical studies on the properties of doubly charmed (DC) baryons:
  - Heavy quark effective theory, e.g. [J. Korner, M. Kramer, and D. Pirjol, Prog.Part.Nucl.Phys.33(1994)787]
  - Quark model, e.g. [D. Ebert, R. Faustov, V. Galkin, and A. Martynenko, PRD66(2002)014008][L.Y. Xiao, K.L. Wang, Q.F. Lu, X.H. Zhong, and S.L. Zhu, PRD96(2017)094005]
  - Effective potential method, e.g. [M. Karliner and J. L. Rosner, PRD90(2014)094007]
  - Lattice QCD, e.g. [L. Liu, et al, PRD81(2010)094505] [Z. S. Brown, et al, PRD90(2014)094507]
  - Light-front approach, e.g. [W. Wang, Z.-P. Xing, and J. Xu, EPJC77(2017)800][W. Wang, F.-S. Yu, and Z.-X. Zhao, EPJC77(2017)781]
  - Chiral effective field theory
  - etc.

Theoretical studies

- Chiral effective field theory (or Chiral perturbation theory)
  - Heavy baryon approach
    - Axial charge, etc. [H.S. Li, L. Meng, Z. W. Liu, and S. L. Zhu, PLB 777(2018)] [L.Y. Xiao, K.L. Wang, Q.f. Lu, X. H. Zhong, and S. L. Zhu, PRD 96(2017)]
  - Covariant formalism with EOMS scheme
    - Masses [Z.F. Sun and M.J. Vicente Vacas, PRD93(2016)] [D. L. Yao, PRD97(2018)]
    - Electromagnetic form factors, etc. [Hiller Blin, Z. F. Sun, and Vicente Vacas, PRD98(2018)] [R.X. Shi, Y. Xiao and L.S. Geng, PRD100(2019)]
  - The spectroscopy of the DC baryons [tree level and unitarization] [Z. H. Guo, PRD96(2017)074004] [M. J. Yan, et al, PRD98 (2018)]
- Our work:
  - Complete and minimal chiral effective Lagrangian up to $O(p^4)$ [P. C. Qiu and D. L. Yao, PRD103(2021)034006]
  - Scattering lengths at one-loop level $\longrightarrow$ DC baryon spectrum [P. C. Qiu and D. L. Yao, work in progress]

II. Construction of the effective Lagrangian

Procedure

Steps for the construction of Lagrangian:

- Expansion in terms of GB masses and external momenta
- Symmetry requirements: Lorentz, P, C, h.c., Chiral symmetry
- Properties of building blocks and Clifford algebra
- Linear relations for elimination

Chiral symmetry and Degrees of freedom

- The underlying Lagrangian:

$$\mathscr{L} = \mathscr{L}^0_{\rm QCD} + \bar{q}\gamma^\mu(v_\mu + \gamma_5 a_\mu)q - \bar{q}(s - i\gamma_5 p)q$$

which is invariant under local $SU(3)_L \times SU(3)_R$ transformations.

- Meson field U:

$$U = u^2 = e^{\frac{i\Phi}{F_0}}$$

where

$$\Phi = \sum_{a=1}^{8} \Phi_a \lambda_a \equiv \begin{pmatrix} \pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}\bar{K}^0 & -\frac{2}{\sqrt{3}}\eta \end{pmatrix}$$

which transforms as under chiral transformation

$$U \to V_R U V_L^+ \qquad V_R, V_L \in SU(3)$$

- Baryon field ψ:

$$\psi = \begin{pmatrix} \Xi_{cc}^{++} \\ \Xi_{cc}^{+} \\ \Omega_{cc}^{+} \end{pmatrix}$$

It transforms as

$$\psi \to h(V_R, V_L, U)\psi$$

where the compensator $h(V_R, V_L, U)$ is defined by

$$h(V_R, V_L, U) = (\sqrt{V_R U V_L^\dagger})^\dagger V_R u$$

Building blocks

- Ingredients:

$$u_\mu = i\{u^\dagger(\partial_\mu - ir_\mu)u - u(\partial_\mu - il_\mu)u^\dagger\}$$
$$f^{\pm}_{\mu\nu} = uF^L_{\mu\nu}u^\dagger \pm u^\dagger F^R_{\mu\nu}u$$
$$\chi_\pm = u^\dagger\chi u^\dagger \pm u\chi^\dagger u$$

where

$$\chi = 2B_0(s+ip), \quad B_0 = -\langle 0|\bar{q}q|0\rangle/3F^2$$
$$F^R_{\mu\nu} = \partial_\mu r_\nu - \partial_\nu r_\mu - i[r_\nu, r_\mu], \quad r_\mu = v_\mu + a_\mu$$
$$F^L_{\mu\nu} = \partial_\mu l_\nu - \partial_\nu l_\mu - i[l_\nu, l_\mu], \quad l_\mu = v_\mu - a_\mu$$

which transforms as

$$X \to h(V_R, V_L, U)Xh^\dagger(V_R, V_L, U)$$

under chiral transformation

- Covariant derivative D^{μ}:

$$D_{\mu} = \partial_{\mu} + \Gamma_{\mu},$$
$$\Gamma_{\mu} = \frac{1}{2}\{u^{\dagger}(\partial_{\mu} - ir_{\mu})u + u(\partial_{\mu} - il_{\mu})u^{\dagger}\}$$

Any building block X:

$$[D_{\mu}, X] \to h[D_{\mu}, X]h^{\dagger}$$

The baryon field ψ:

$$D_{\mu}\psi \to hD_{\mu}\psi$$

	P	C	H.c.	p	c	h	Cdim
u_μ	$-u^\mu$	u_μ^T	u_μ	1	0	0	1
$\overrightarrow{D}_\mu$	$\overrightarrow{D}^\mu$	$\overleftarrow{D}_\mu^T$	$\overleftarrow{D}_\mu$	0	0	0	1
χ_+	χ_+	χ_+^T	χ_+	0	0	0	2
χ_-	$-\chi_-$	χ_-^T	$-\chi_-$	1	0	1	2
$f_+^{\mu\nu}$	$f_{+\mu\nu}$	$-(f_+^{\mu\nu})^T$	$f_+^{\mu\nu}$	0	1	0	2
$f_-^{\mu\nu}$	$-f_{-\mu\nu}$	$(f_-^{\mu\nu})^T$	$f_-^{\mu\nu}$	1	0	0	2
$h^{\mu\nu}$	$-h_{\mu\nu}$	$(h^{\mu\nu})^T$	$h^{\mu\nu}$	1	0	0	2

- The definition of p, c and h:

$$(\bar\psi X\psi)^P = (-1)^p(\bar\psi X\psi)$$
$$(\bar\psi X\psi)^C = (-1)^c(\bar\psi X\psi)$$
$$(\bar\psi X\psi)^\dagger = (-1)^h(\bar\psi X\psi)$$

	p	c	h	Cdim
i	0	0	1	0
1	0	0	0	0
γ_5	1	0	1	1
γ_μ	0	1	0	0
$\gamma_5\gamma_\mu$	1	0	0	0
$\sigma_{\mu\nu}$	0	1	0	0
$g_{\mu\nu}$	0	0	0	0
$\epsilon_{\mu\nu\rho\tau}$	1	0	0	0
$\overrightarrow{D}_\mu\psi$	0	1	1	0

Elimination of redundant terms

- Property of traces
- Schouten identity

$$\epsilon^{\mu\nu\lambda\tau}A^{\rho}+\epsilon^{\nu\lambda\tau\rho}A^{\mu}+\epsilon^{\lambda\tau\rho\mu}A^{\nu}+\epsilon^{\tau\rho\mu\nu}A^{\lambda}+\epsilon^{\rho\mu\nu\lambda}A^{\tau}=0$$

- Covariant derivatives and Bianchi identity

$$[D_\mu,[D_\nu,D_\lambda]]+[D_\nu,[D_\lambda,D_\mu]]+[D_\lambda,[D_\mu,D_\nu]]=0$$

- Cayley-Hamilton relation
- Equations of motion

$$(i\not{D}-M_0+\frac{g_A}{2}\not{u}\gamma_5)\psi=0,\quad \bar{\psi}(i\overleftarrow{\not{D}}+M_0-\frac{g_A}{2}\not{u}\gamma_5)=0$$

- The relativistic Lagrangian we obtained:

$$\mathscr{L}_{M\psi} = \mathscr{L}_{M\psi}^{(1)} + \mathscr{L}_{M\psi}^{(2)} + \mathscr{L}_{M\psi}^{(3)} + \mathscr{L}_{M\psi}^{(4)} + \cdots$$

where

$$\mathscr{L}_{M\psi}^{(1)} = \bar{\psi}(i\not{D} - M_0)\psi + \frac{g_A}{2}\bar{\psi}\not{u}\gamma_5\psi$$

$$\begin{aligned}\mathscr{L}_{M\psi}^{(2)} = & \; b_1\bar{\psi}\psi\langle\chi_+\rangle + b_2\bar{\psi}\widetilde{\chi}_+\psi + b_3\bar{\psi}u^2\psi + b_4\bar{\psi}\psi\langle u^2\rangle \\ & + b_5(\bar{\psi}\{u^\mu, u^\nu\}\{D_\mu, D_\nu\}\psi + H.c.) \\ & + b_6(\bar{\psi}\{D_\mu, D_\nu\}\psi\langle u^\mu u^\nu\rangle + H.c.) \\ & + i b_7\bar{\psi}[u^\mu, u^\nu]\sigma_{\mu\nu}\psi + b_8\bar{\psi}f_+^{\mu\nu}\sigma_{\mu\nu}\psi\end{aligned}$$

$$\mathscr{L}_{M\psi}^{(3)} = \sum_{i=1}^{32} c_i\bar{\psi}O_i^{(3)}\psi \qquad \mathscr{L}_{M\psi}^{(4)} = \sum_{i=1}^{218} d_i\bar{\psi}O_i^{(4)}\psi$$

- The non-relativistic Lagrangian we obtained:

$$\mathscr{L}_{M\psi} = \mathscr{L}^{(1)}_{M\psi} + \mathscr{L}^{(2)}_{M\psi} + \mathscr{L}^{(3)}_{M\psi} + \mathscr{L}^{(4)}_{M\psi} + \cdots$$

where

$$\begin{aligned}
\mathscr{L}^{(1)}_{M\psi} &= \bar{H}(iv \cdot D + g_A S \cdot u)H \\
\mathscr{L}^{(2)}_{M\psi} &= \bar{H}\{\mathcal{A}^{(2)} + \gamma_0 \mathcal{B}^{(1)\dagger}\gamma_0 \mathcal{C}^{(0)-1}\mathcal{B}^{(1)}\}H \\
\mathscr{L}^{(3)}_{M\psi} &= \bar{H}\{\mathcal{A}^{(3)} + \gamma_0 \mathcal{B}^{(1)\dagger}\gamma_0 \mathcal{C}^{(0)-1}\mathcal{B}^{(2)} + \gamma_0 \mathcal{B}^{(1)\dagger}\gamma_0 \mathcal{C}^{(1)-1}\mathcal{B}^{(1)} \\
&\quad + \gamma_0 \mathcal{B}^{(2)\dagger}\gamma_0 \mathcal{C}^{(0)-1}\mathcal{B}^{(1)}\}H \\
\mathscr{L}^{(4)}_{M\psi} &= \bar{H}\{\mathcal{A}^{(4)} + \gamma_0 \mathcal{B}^{(1)\dagger}\gamma_0 \mathcal{C}^{(0)-1}\mathcal{B}^{(3)} + \gamma_0 \mathcal{B}^{(1)\dagger}\gamma_0 \mathcal{C}^{(1)-1}\mathcal{B}^{(2)} \\
&\quad + \gamma_0 \mathcal{B}^{(1)\dagger}\gamma_0 \mathcal{C}^{(2)-1}\mathcal{B}^{(1)} + \gamma_0 \mathcal{B}^{(2)\dagger}\gamma_0 \mathcal{C}^{(0)-1}\mathcal{B}^{(2)} \\
&\quad + \gamma_0 \mathcal{B}^{(2)\dagger}\gamma_0 \mathcal{C}^{(1)-1}\mathcal{B}^{(1)} + \gamma_0 \mathcal{B}^{(3)\dagger}\gamma_0 \mathcal{C}^{(0)-1}\mathcal{B}^{(1)}\}H
\end{aligned}$$

Effective Lagrangian up to $O(p^4)$

Some explicit operators in our $O(p^3)$ and $O(p^4)$ Lagrangians

TABLE IV. Terms in the $O(q^3)$ relativistic and nonrelativistic Lagrangians.

i	$O_i^{(3)}$	$\hat{O}_i^{(3)}$
1	$\{u_\mu,\{u^\mu,u^\nu\}\}\gamma_5\gamma_\nu$	$-2\{u_\mu,\{u^\mu,S\cdot u\}\}$
2	$[u_\mu,[u^\mu,u^\nu]]\gamma_5\gamma_\nu$	$-2[u_\mu,[u^\mu,S\cdot u]]$
3	$u^\nu\gamma_5\gamma_\nu\langle u^2\rangle$	$-2S\cdot u\langle u^2\rangle$
4	$u_\mu\gamma_5\gamma_\nu\langle u^\mu u^\nu\rangle$	$-2u_\mu\langle u^\mu S\cdot u\rangle$
5	$\{u^\mu,\{u^\nu,u^\rho\}\}\gamma_5\gamma_\mu D_{\nu\rho}+\text{H.c.}$	$16m^2\{S\cdot u,(v\cdot u)^2\}\}$
6	$u^\mu\gamma_5\gamma_\mu\langle u^\nu u^\rho\rangle D_{\nu\rho}+\text{H.c.}$	$8m^2S\cdot u\langle(v\cdot u)^2\rangle$
7	$i\epsilon_{\mu\nu\rho\tau}\{[u^\mu,u^\nu],u^\rho\}\gamma^\tau$	$i\epsilon_{\mu\nu\rho\tau}\{[u^\mu,u^\nu],u^\rho\}v^\tau$
8	$i\epsilon_{\mu\nu\rho\tau}\gamma^\tau\langle[u^\mu,u^\nu]u^\rho\rangle$	$i\epsilon_{\mu\nu\rho\tau}v^\tau\langle[u^\mu,u^\nu]u^\rho\rangle$
9	$i\epsilon_{\mu\nu\lambda\tau}\{u^\mu,\{u^\nu,u^\rho\}\}\sigma^{\lambda\tau}D_\rho+\text{H.c.}$	$-4im\epsilon_{\mu\nu\lambda\tau}\{u^\mu,\{u^\nu,v\cdot u\}\}[S^\lambda,S^\tau]$
10	$i\epsilon_{\mu\nu\lambda\tau}u^\mu\sigma^{\lambda\tau}\langle u^\nu u^\rho\rangle D_\rho+\text{H.c.}$	$-4im\epsilon_{\mu\nu\lambda\tau}u^\mu\langle u^\nu v\cdot u\rangle[S^\lambda,S^\tau]$
11	$i[u_\mu,h^{\mu\nu}]\gamma_\nu$	$i[u_\mu,h^{\mu\nu}]v_\nu$
12	$i[u^\mu,h^{\nu\rho}]\gamma_\mu D_{\nu\rho}+\text{H.c.}$	$-4im^2[v\cdot u,h^{\nu\rho}]v_\nu v_\rho$
13	$i\{u^\mu,h^{\nu\rho}\}\sigma_{\mu\nu}D_\rho+\text{H.c.}$	$-4im\{u^\mu,h^{\nu\rho}\}[S_\mu,S_\nu]v_\rho$
14	$i\sigma_{\mu\nu}\langle u^\mu h^{\nu\rho}\rangle D_\rho+\text{H.c.}$	$-4im[S_\mu,S_\nu]\langle u^\mu h^{\nu\rho}\rangle v_\rho$
15	$\{u^\mu,\tilde{\chi}_+\}\gamma_5\gamma_\mu$	$-2\{S\cdot u,\tilde{\chi}_+\}$
16	$u^\mu\gamma_5\gamma_\mu\langle\chi_+\rangle$	$-2S\cdot u\langle\chi_+\rangle$
17	$\gamma_5\gamma_\mu\langle u^\mu\tilde{\chi}_+\rangle$	$-2\langle S\cdot u\tilde{\chi}_+\rangle$
18	$i\gamma_5\gamma_\mu[D^\mu,\tilde{\chi}_-]$	$-2i[S\cdot D,\tilde{\chi}_-]$
19	$i\gamma_5\gamma_\mu\langle[D^\mu,\chi_-]\rangle$	$-2i\langle[S\cdot D,\chi_-]\rangle$
20	$[\tilde{\chi}_-,u^\mu]\gamma_\mu$	$[\tilde{\chi}_-,v\cdot u]$
21	$i[u_\mu,f_+^{\mu\nu}]\gamma_5\gamma_\nu$	$-2i[u_\mu,f_+^{\mu\nu}]S_\nu$
22	$\epsilon_{\mu\nu\rho\tau}\{u^\mu,f_+^{\nu\rho}\}\gamma^\tau$	$\epsilon_{\mu\nu\rho\tau}\{u^\mu,f_+^{\nu\rho}\}v^\tau$
23	$\epsilon_{\mu\nu\rho\tau}\gamma^\tau\langle u^\mu f_+^{\nu\rho}\rangle$	$\epsilon_{\mu\nu\rho\tau}v^\tau\langle u^\mu f_+^{\nu\rho}\rangle$
24	$\epsilon_{\nu\rho\lambda\tau}[u^\mu,f_+^{\nu\rho}]\sigma^{\lambda\tau}D_\mu+\text{H.c.}$	$-4m\epsilon_{\nu\rho\lambda\tau}[v\cdot u,f_+^{\nu\rho}][S^\lambda,S^\tau]$
25	$i[D_\mu,f_+^{\mu\nu}]D_\nu+\text{H.c.}$	$2m[D_\mu,f_+^{\mu\nu}]v_\nu$
26	$i[u_\mu,f_-^{\mu\nu}]\gamma_\nu$	$i[u_\mu,f_-^{\mu\nu}]v_\nu$
27	$\epsilon_{\mu\nu\rho\tau}\{u^\mu,f_-^{\nu\rho}\}\gamma_5\gamma^\tau$	$-2\epsilon_{\mu\nu\rho\tau}\{u^\mu,f_-^{\nu\rho}\}S^\tau$
28	$\epsilon_{\mu\nu\rho\tau}\gamma_5\gamma^\tau\langle u^\mu f_-^{\nu\rho}\rangle$	$-2\epsilon_{\mu\nu\rho\tau}S^\tau\langle u^\mu f_-^{\nu\rho}\rangle$
29	$i\{u^\mu,f_-^{\nu\rho}\}\sigma_{\mu\nu}D_\rho+\text{H.c.}$	$-4im\{u^\mu,f_-^{\nu\rho}\}[S_\mu,S_\nu]v_\rho$
30	$i\sigma_{\mu\nu}\langle u^\mu f_-^{\nu\rho}\rangle D_\rho+\text{H.c.}$	$-4im[S_\mu,S_\nu]v_\rho\langle u^\mu f_-^{\nu\rho}\rangle$
31	$[D_\mu,f_-^{\mu\nu}]\gamma_5\gamma_\nu$	$-2S_\nu[D_\mu,f_-^{\mu\nu}]$
32	$[D^\lambda,f_-^{\mu\nu}]\gamma_5\gamma_\mu D_{\nu\lambda}+\text{H.c.}$	$8m^2S_\mu[v\cdot D,f_-^{\mu\nu}]v_\nu$

TABLE V. Terms in the $O(q^4)$ relativistic and nonrelativistic Lagrangians.

i	$O_i^{(4)}$	$\hat{O}_i^{(4)}$
1	$\{u^\mu,\{\{u_\mu,u^\nu\},u_\nu\}\}$	$\{u^\mu,\{\{u_\mu,u^\nu\},u_\nu\}\}$
2	$\{u^\mu,[[u_\mu,u^\nu],u_\nu]\}$	$\{u^\mu,[[u_\mu,u^\nu],u_\nu]\}$
3	$[u^\mu,\{[u_\mu,u^\nu],u_\nu\}]$	$[u^\mu,\{[u_\mu,u^\nu],u_\nu\}]$
4	$\langle\{u^\mu,\{u_\mu,u^\nu\}\}u_\nu\rangle$	$\langle\{u^\mu,\{u_\mu,u^\nu\}\}u_\nu\rangle$
5	$\langle[u^\mu,[u_\mu,u^\nu]]u_\nu\rangle$	$\langle[u^\mu,[u_\mu,u^\nu]]u_\nu\rangle$
6	$u^\mu\langle u^2u_\mu\rangle$	$u^\mu\langle u^2u_\mu\rangle$
7	$u^\mu u_\mu\langle u^2\rangle$	$u^2\langle u^2\rangle$
8	$u^\mu u^\nu\langle u_\mu u_\nu\rangle$	$u^\mu u^\nu\langle u_\mu u_\nu\rangle$
9	$i\{u^\mu,\{u_\mu,[u^\nu,u^\lambda]\}\}\sigma_{\nu\lambda}$	$2\{u^\mu,\{u_\mu,[u^\nu,u^\lambda]\}\}[S_\nu,S_\lambda]$
10	$i[u^\mu,[u_\mu,[u^\nu,u^\lambda]]]\sigma_{\nu\lambda}$	$2[u^\mu,[u_\mu,[u^\nu,u^\lambda]]][S_\nu,S_\lambda]$
11	$i[u^\nu,\{u_\mu,\{u^\mu,u^\lambda\}\}]\sigma_{\nu\lambda}$	$2[u^\nu,\{u_\mu,\{u^\mu,u^\lambda\}\}][S_\nu,S_\lambda]$
12	$i[u^\nu,[u_\mu,[u^\mu,u^\lambda]]]\sigma_{\nu\lambda}$	$2[u^\nu,[u_\mu,[u^\mu,u^\lambda]]][S_\nu,S_\lambda]$
13	$i\langle[u_\mu,u_\nu]u^2\rangle\sigma^{\mu\nu}$	$2[S^\mu,S^\nu]\langle[u_\mu,u_\nu]u^2\rangle$
14	$i\langle[u_\mu,\{u^\lambda,u_\nu\}]u_\lambda\rangle\sigma^{\mu\nu}$	$2[S^\mu,S^\nu]\langle[u_\mu,\{u^\lambda,u_\nu\}]u_\lambda\rangle$
15	$iu^\mu\langle u_\mu[u_\nu,u_\lambda]\rangle\sigma^{\nu\lambda}$	$2[S^\nu,S^\lambda]u^\mu\langle u_\mu[u_\nu,u_\lambda]\rangle$
16	$i[u^\mu,u^\nu]\langle u^2\rangle\sigma_{\mu\nu}$	$2[u^\mu,u^\nu][S_\mu,S_\nu]\langle u^2\rangle$
17	$i[u^\mu,u^\lambda]\langle u^\nu u_\lambda\rangle\sigma_{\mu\nu}$	$2[u^\mu,u^\lambda][S_\mu,S_\nu]\langle u^\nu u_\lambda\rangle$
18	$\{u^\mu,\{u_\mu,\{u^\nu,u^\lambda\}\}\}D_{\nu\lambda}+\text{H.c.}$	$-8m^2\{u^\mu,\{u_\mu,(v\cdot u)^2\}\}$
19	$[u^\mu,[u_\mu,\{u^\nu,u^\lambda\}]]D_{\nu\lambda}+\text{H.c.}$	$-8m^2[u^\mu,[u_\mu,(v\cdot u)^2]]$
20	$\{u^\nu,\{u_\mu,\{u^\mu,u^\lambda\}\}\}D_{\nu\lambda}+\text{H.c.}$	$-4m^2\{v\cdot u,\{u_\mu,\{u^\mu,v\cdot u\}\}\}$
21	$\{u^\nu,[u_\mu,[u^\mu,u^\lambda]]\}D_{\nu\lambda}+\text{H.c.}$	$-4m^2\{v\cdot u,[u_\mu,[u^\mu,v\cdot u]]\}$
22	$\langle\{u_\mu,\{u^\lambda,u_\nu\}\}u_\lambda\rangle D^{\mu\nu}+\text{H.c.}$	$-4m^2\langle\{v\cdot u,\{u^\lambda,v\cdot u\}\}u_\lambda\rangle$
23	$\langle\{u_\mu,u_\nu\}u^2\rangle D^{\mu\nu}+\text{H.c.}$	$-8m^2\langle(v\cdot u)^2u^2\rangle$
24	$u^\mu\langle u_\nu u^2\rangle D_\mu{}^\nu+\text{H.c.}$	$-4m^2v\cdot u\langle v\cdot uu^2\rangle$
25	$u^\lambda\langle\{u_\mu,u_\nu\}u_\lambda\rangle D^{\mu\nu}+\text{H.c.}$	$-8m^2u^\lambda\langle(v\cdot u)^2u_\lambda\rangle$
26	$u^2\langle u_\nu u_\lambda\rangle D^{\nu\lambda}+\text{H.c.}$	$-4m^2u^2\langle(v\cdot u)^2\rangle$
27	$\{u^\mu,u^\nu\}\langle u^2\rangle D_{\mu\nu}+\text{H.c.}$	$-8m^2(v\cdot u)^2\langle u^2\rangle$
28	$\{u^\mu,u^\nu\}\langle u_\nu u_\lambda\rangle D_\mu{}^\lambda+\text{H.c.}$	$-4m^2\{v\cdot u,u^\nu\}\langle u_\nu v\cdot u\rangle$
29	$i\{u^\lambda,\{u^\rho,[u^\mu,u^\nu]\}\}\sigma_{\mu\nu}D_{\lambda\rho}+\text{H.c.}$	$-8m^2\{v\cdot u,\{v\cdot u,[u^\mu,u^\nu]\}\}[S_\mu,S_\nu]$
30	$i[u^\lambda,[u^\rho,[u^\mu,u^\nu]]]\sigma_{\mu\nu}D_{\lambda\rho}+\text{H.c.}$	$-8m^2[v\cdot u,[v\cdot u,[u^\mu,u^\nu]]][S_\mu,S_\nu]$
31	$i[u^\mu,\{u^\lambda,\{u^\rho,u^\nu\}\}]\sigma_{\mu\nu}D_{\lambda\rho}+\text{H.c.}$	$-8m^2[u^\mu,\{v\cdot u,\{v\cdot u,u^\nu\}\}][S_\mu,S_\nu]$
32	$i[u^\mu,[u^\lambda,[u^\rho,u^\nu]]]\sigma_{\mu\nu}D_{\lambda\rho}+\text{H.c.}$	$-8m^2[u^\mu,[v\cdot u,[v\cdot u,u^\nu]]][S_\mu,S_\nu]$
33	$i\langle\{[u_\mu,u_\nu],u_\lambda\}u_\rho\rangle\sigma^{\mu\nu}D^{\lambda\rho}+\text{H.c.}$	$-8m^2\langle\{[u_\mu,u_\nu],v\cdot u\}v\cdot u\rangle[S^\mu,S^\nu]$
34	$i\langle[u_\mu,\{u_\lambda,u_\nu\}]u_\rho\rangle\sigma^{\mu\nu}D^{\lambda\rho}+\text{H.c.}$	$-8m^2\langle[u_\mu,\{v\cdot u,u_\nu\}]v\cdot u\rangle[S^\mu,S^\nu]$
35	$iu^\mu\langle[u_\nu,u_\lambda]u_\rho\rangle\sigma^{\nu\lambda}D_\mu{}^\rho+\text{H.c.}$	$-8m^2v\cdot u\langle[u_\nu,u_\lambda]v\cdot u\rangle[S^\nu,S^\lambda]$
36	$i[u^\mu,u^\nu]\langle u_\lambda u_\rho\rangle\sigma_{\mu\nu}D^{\lambda\rho}+\text{H.c.}$	$-8m^2[u^\mu,u^\nu][S_\mu,S_\nu]\langle(v\cdot u)^2\rangle$
37	$i[u^\mu,u^\nu]\langle u_\lambda u_\rho\rangle\sigma_\mu{}^\lambda D_\nu{}^\rho+\text{H.c.}$	$-8m^2[u^\mu,v\cdot u][S_\mu,S^\lambda]\langle u_\lambda v\cdot u\rangle$
38	$\{u^\mu,\{u^\nu,\{u^\lambda,u^\rho\}\}\}D$[illegible] $+\text{H.c.}$	$384m^4(v\cdot u)^4$

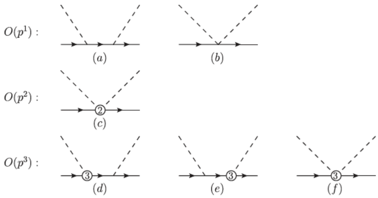
P. C. Qiu and D. L. Yao, Phys. Rev. D **103** (2021) 034006

III. One-loop analysis of the interactions between DC baryons and GBs

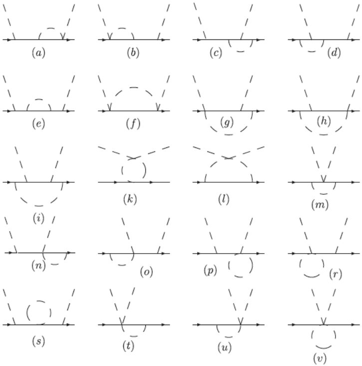
Feynman diagrams

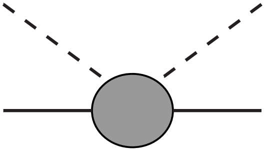
All relevent diagrams up to $O(p^3)$ for the processes of $\psi_{cc}\phi \to \psi_{cc}\phi$

- Tree diagrams:



- Loop diagrams:





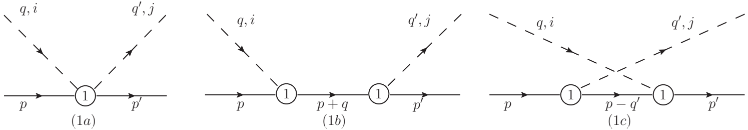
- The standard decomposition of scattering amplitude:

$$
\begin{aligned}
T =& T_{Gell-Mann} \cdot T_{Dirac} \\
=& \chi_{b'}^{\dagger} \lambda^{i} \cdots \lambda^{j} \chi_{b} \cdot \bar{u}(p')[A + \frac{1}{2}(\not{q} + \not{q}')B]u(p)
\end{aligned}
$$

where $\chi_{b'}$, χ_b are the isospinors of the baryons.

- For T_{Dirac}: obtain A, B (different graphs)
- For $T_{Gell-Mann}$: the specific forms (different graphs) and values (different processes)

- Tree diagrams at $O(p^1)$



- Explicit expressions of A, B amplitudes

	1a	1b	1c
$A^{(1)}$	0	$\frac{2g_A^2 m}{F^2}$	$\frac{2g_A^2 m}{F^2}$
$B^{(1)}$	$-\frac{2}{F^2}$	$\frac{g_A^2(3m^2+s)}{F^2(m^2-s)}$	$\frac{g_A^2(2M^2+5m^2-s-t)}{F^2(2M^2+m^2-s-t)}$

Scattering amplitudes of $O(p^1)$

The general scattering processes can be classified into 7 independent channels with definite (S, I)

- $T_{Gell-Mann}$ classified by (S, I) quantum numbers:

TABLE I. The processes are classified by strangeness (S) and isospin (I).

(S,I)	Processes	$\mathcal{F}^{(1a)}$	$\mathcal{F}^{(1b)}$	$\mathcal{F}^{(1c)}$
$(-2,\frac{1}{2})$	$\Omega_{cc}\bar{K}\to\Omega_{cc}\bar{K}$	$\frac{1}{4}$	0	$\frac{1}{2}(\Xi_{cc})$
$(0,\frac{3}{2})$	$\Xi_{cc}\pi\to\Xi_{cc}\pi$	$\frac{1}{4}$	0	$\frac{1}{2}(\Xi_{cc})$
$(-1,1)$	$\Omega_{cc}\pi\to\Omega_{cc}\pi$	0	0	0
	$\Xi_{cc}\bar{K}\to\Xi_{cc}\bar{K}$	0	0	0
	$\Omega_{cc}\pi\to\Xi_{cc}\bar{K}$	$\frac{1}{4}$	0	$\frac{1}{2}(\Xi_{cc})$
$(-1,0)$	$\Xi_{cc}\bar{K}\to\Xi_{cc}\bar{K}$	$-\frac{1}{2}$	$1(\Omega_{cc})$	0
	$\Omega_{cc}\eta\to\Omega_{cc}\eta$	0	$\frac{1}{3}(\Omega_{cc})$	$\frac{1}{3}(\Omega_{cc})$
	$\Xi_{cc}\bar{K}\to\Omega_{cc}\eta$	$\frac{\sqrt{3}}{4}$	$-\sqrt{\frac{1}{3}}(\Omega_{cc})$	$\frac{1}{2\sqrt{3}}(\Xi_{cc})$
$(1,0)$	$\Xi_{cc}K\to\Xi_{cc}K$	$-\frac{1}{4}$	0	$-\frac{1}{2}(\Omega_{cc})$
$(1,1)$	$\Xi_{cc}K\to\Xi_{cc}K$	$\frac{1}{4}$	0	$\frac{1}{2}(\Omega_{cc})$
$(0,\frac{1}{2})$	$\Xi_{cc}\pi\to\Xi_{cc}\pi$	$-\frac{1}{2}$	$\frac{3}{4}(\Xi_{cc})$	$-\frac{1}{4}(\Xi_{cc})$
	$\Xi_{cc}\eta\to\Xi_{cc}\eta$	0	$\frac{1}{12}(\Xi_{cc})$	$\frac{1}{12}(\Xi_{cc})$
	$\Omega_{cc}K\to\Omega_{cc}K$	$-\frac{1}{4}$	$\frac{1}{2}(\Xi_{cc})$	0
	$\Xi_{cc}\pi\to\Xi_{cc}\eta$	0	$\frac{1}{4}(\Xi_{cc})$	$\frac{1}{4}(\Xi_{cc})$
	$\Xi_{cc}\pi\to\Omega_{cc}K$	$-\frac{\sqrt{3}}{4\sqrt{2}}$	$\frac{\sqrt{3}}{2\sqrt{2}}(\Xi_{cc})$	0
	$\Xi_{cc}\eta\to\Omega_{cc}K$	$-\frac{\sqrt{3}}{4\sqrt{2}}$	$\frac{1}{2\sqrt{6}}(\Xi_{cc})$	$-\frac{1}{\sqrt{6}}(\Omega_{cc})$

- The $O(p^2)$ amplitudes can be derived analogously:
  - Dirac part: $A^{(2)}$ and $B^{(2)}$;
  - Gell-Mann part: Classified by (S, I) in the table below.

TABLE II. The processes are classified by strangeness (S) and isospin (I).

(S,I)	Processes	$\mathcal{F}_1^{(2)}$	$\mathcal{F}_2^{(2)}$	$\mathcal{F}_3^{(2)}$	$\mathcal{F}_4^{(2)}$	$\mathcal{F}_5^{(2)}$	$\mathcal{F}_6^{(2)}$	$\mathcal{F}_7^{(2)}$
$(-2,\frac{1}{2})$	$\Omega_{cc}\bar{K}\to\Omega_{cc}\bar{K}$	$2M_\pi^2-4M_K^2$	$-\frac{2}{3}(M_\pi^2+M_K^2)$	2	4	-8	-8	-2
$(0,\frac{3}{2})$	$\Xi_{cc}\pi\to\Xi_{cc}\pi$	$-2M_\pi^2$	$-\frac{4}{3}M_\pi^2$	2	4	-8	-8	-2
$(-1,1)$	$\Omega_{cc}\pi\to\Omega_{cc}\pi$	$-2M_\pi^2$	$\frac{2}{3}M_\pi^2$	0	4	0	-8	0
	$\Xi_{cc}\bar{K}\to\Xi_{cc}\bar{K}$	$2M_\pi^2-4M_K^2$	$\frac{2}{3}(2M_k^2-M_\pi^2)$	0	4	0	-8	0
	$\Omega_{cc}\pi\to\Xi_{cc}\bar{K}$	0	$-M_K^2-M_\pi^2$	2	0	-8	0	-2
$(-1,0)$	$\Xi_{cc}\bar{K}\to\Xi_{cc}\bar{K}$	$2M_\pi^2-4M_K^2$	$-\frac{2}{3}(4M_k^2+M_\pi^2)$	4	4	-16	-8	4
	$\Omega_{cc}\eta\to\Omega_{cc}\eta$	$\frac{2}{3}(M_\pi^2-4M_K^2)$	$\frac{22}{9}M_\pi^2-\frac{40}{9}M_K^2$	$\frac{8}{3}$	4	$-\frac{32}{3}$	-8	0
	$\Xi_{cc}\bar{K}\to\Omega_{cc}\eta$	0	$\sqrt{\frac{1}{3}}(5M_K^2-3M_\pi^2)$	$-\frac{2}{\sqrt{3}}$	0	$\frac{8}{\sqrt{3}}$	0	$-2\sqrt{3}$
$(1,0)$	$\Xi_{cc}K\to\Xi_{cc}K$	$-2M_\pi^2$	$\frac{2}{3}M_\pi^2+2M_k^2$	-2	4	8	-8	2
$(1,1)$	$\Xi_{cc}K\to\Xi_{cc}K$	$-2M_\pi^2$	$\frac{2}{3}M_\pi^2-2M_k^2$	2	4	-8	-8	-2
$(0,\frac{1}{2})$	$\Xi_{cc}\pi\to\Xi_{cc}\pi$	$-2M_\pi^2$	$-\frac{4}{3}M_\pi^2$	2	4	-8	-8	4
	$\Xi_{cc}\eta\to\Xi_{cc}\eta$	$\frac{2}{3}(M_\pi^2-4M_K^2)$	$\frac{8}{9}(M_K^2-M_\pi^2)$	$\frac{2}{3}$	4	$-\frac{8}{3}$	-8	0
	$\Omega_{cc}K\to\Omega_{cc}K$	$-2M_\pi^2$	$\frac{2}{3}M_\pi^2-2M_K^2$	2	4	-8	-8	2
	$\Xi_{cc}\pi\to\Xi_{cc}\eta$	0	$-2M_\pi^2$	2	0	-8	0	0
	$\Xi_{cc}\pi\to\Omega_{cc}K$	0	$-\sqrt{\frac{3}{2}}(M_K^2+M_\pi^2)$	$\sqrt{6}$	0	$-4\sqrt{6}$	0	$\sqrt{6}$
	$\Xi_{cc}\eta\to\Omega_{cc}K$	0	$\sqrt{\frac{1}{6}}(5M_K^2-3M_\pi^2)$	$-\sqrt{\frac{2}{3}}$	0	$4\sqrt{\frac{2}{3}}$	0	$\sqrt{6}$

Scattering amplitudes of $O(p^3)$

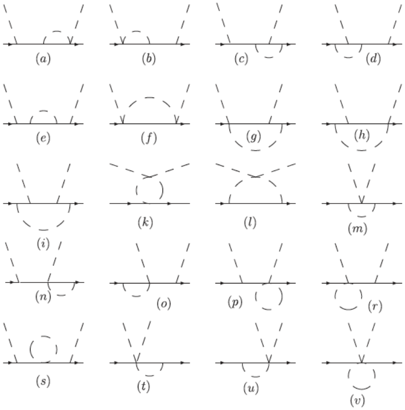
- The calculation of $O(p^3)$ tree amplitudes is finished:
  - Dirac part: $A^{(3)}$ and $B^{(3)}$;
  - Gell-Mann part: Classified by (S, I) in the table below.

TABLE III. The processes are classified by strangeness (S) and isospin (I).

(S,I)	Processes	$\mathcal{F}_1^{(3)}$	$\mathcal{F}_2^{(3)}$	$\mathcal{F}_3^{(3)}$	$\mathcal{F}_4^{(3)}$	$\mathcal{F}_5^{(3)}$	$\mathcal{F}_6^{(3)}$
$(-2,\frac{1}{2})$	$\Omega_{cc}\bar{K}\to\Omega_{cc}\bar{K}$	-4	8	-2	-2	$-4M_K^2$	$-4M_K^2$
$(0,\frac{3}{2})$	$\Xi_{cc}\pi\to\Xi_{cc}\pi$	-4	8	-2	-2	$-4M_\pi^2$	$-4M_\pi^2$
$(-1,1)$	$\Omega_{cc}\pi\to\Omega_{cc}\pi$	0	0	0	-2	0	0
	$\Xi_{cc}\bar{K}\to\Xi_{cc}\bar{K}$	0	0	0	-2	0	0
	$\Omega_{cc}\pi\to\Xi_{cc}\bar{K}$	-4	8	-2	0	$-4M_\pi^2$	$-4M_K^2$
$(-1,0)$	$\Xi_{cc}\bar{K}\to\Xi_{cc}\bar{K}$	8	-16	-4	-2	$8M_K^2$	$8M_K^2$
	$\Omega_{cc}\eta\to\Omega_{cc}\eta$	0	0	$-\frac{8}{3}$	-2	0	0
	$\Xi_{cc}\bar{K}\to\Omega_{cc}\eta$	$-4\sqrt{3}$	$8\sqrt{3}$	$\frac{2}{\sqrt{3}}$	0	$-4\sqrt{3}M_K^2$	$\frac{4}{\sqrt{3}}(M_\pi^2-4M_K^2)$
$(1,0)$	$\Xi_{cc}K\to\Xi_{cc}K$	4	-8	2	-2	$4M_K^2$	$4M_K^2$
$(1,1)$	$\Xi_{cc}K\to\Xi_{cc}K$	-4	8	-2	-2	$-4M_K^2$	$-4M_K^2$
$(0,\frac{1}{2})$	$\Xi_{cc}\pi\to\Xi_{cc}\pi$	8	-16	-2	-2	$8M_\pi^2$	$8M_\pi^2$
	$\Xi_{cc}\eta\to\Xi_{cc}\eta$	0	0	$-\frac{2}{3}$	-2	0	0
	$\Omega_{cc}K\to\Omega_{cc}K$	4	-8	-2	-2	$4M_K^2$	$4M_K^2$
	$\Xi_{cc}\pi\to\Xi_{cc}\eta$	0	0	-2	0	0	0
	$\Xi_{cc}\pi\to\Omega_{cc}K$	$2\sqrt{6}$	$-4\sqrt{6}$	$-\sqrt{6}$	0	$2\sqrt{6}M_\pi^2$	$2\sqrt{6}M_K^2$
	$\Xi_{cc}\eta\to\Omega_{cc}K$	$2\sqrt{6}$	$-4\sqrt{6}$	$\sqrt{\frac{2}{3}}$	0	$\frac{2\sqrt{2}}{\sqrt{3}}(4M_K^2-M_\pi^2)$	$2\sqrt{6}M_K^2$

Loop diagrams in process

The calculation of loop diagrams of $O(p^3)$ is ongoing.



- UV divergences:
  - dimensional regularization
  - $\overline{\mathrm{MS}} - 1$ subtraction
- PCB terms:
  - Due to internal baryon propagators appearing in the loop (the baryon mass is non-vanished in the chiral limit)
  - We prefer the EOMS scheme to remedy this issue

IV. Summary and Outlook

Summary and Outlook

- We have constructed chiral effective Lagrangians up to $O(p^4)$ in the relativistic and non-relativistic formalism
- Calculate scattering amplitudes up to $O(p^3)$ order (tree diagrams and loop diagrams)
- Utilize EOMS scheme to solve power counting breaking problem
- Obtain scattering lengths with the accuracy of one-loop order
- Compare our ChPT results with lattice data (QMHPC cluster @ HNU) to determine the unknown LECs
- Study the spectrum of the doubly charmed baryons

Thank you very much for your patience!