有效场论与强子相互作用

刘占伟　兰州大学物理科学与技术学院　2016 年至今

第二届强子与重味物理理论与实验联合研讨会，兰州 2021

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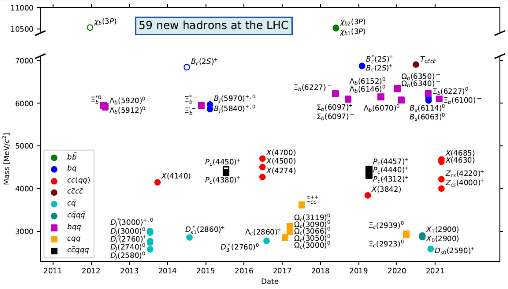
1. 手征有效场论与重味强子相互作用及可能分子态
2. 哈密顿有效场论与核子激发态

Hadron interaction is important for

hadron structure and spectrum

- molecular/multiquark states
- threshold effect
- kinetic effect
- ...



“anomaly” in the eletroweak process

- $\Delta I = 1/2$ rule. $K \to \pi\pi$: $A_0/A_2 \sim 22$
- $e^+e^- \to$ hadron pairs

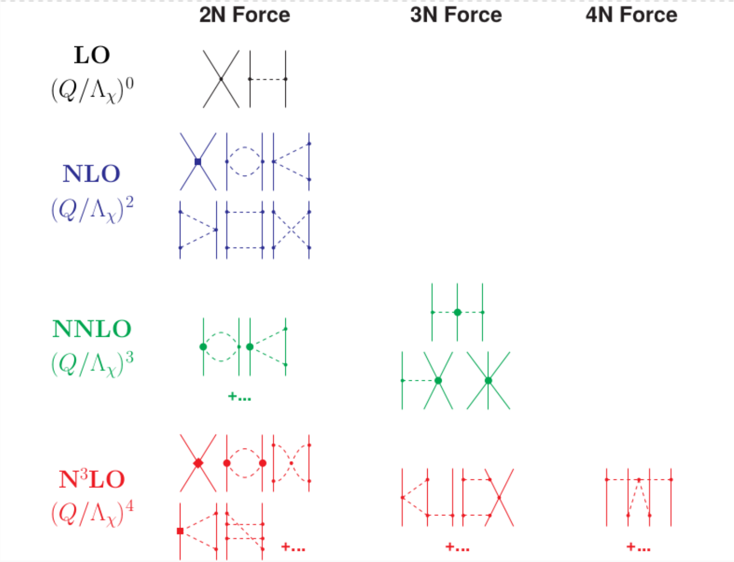
手征有效场论与重味强子相互作用及可能分子态

Study of Interactions within chiral perturbation theory (ChPT

- ChPT with respect on symmetries of QCD
- Power counting
    - NOT in power series: $\alpha_s, \alpha_s^2, \alpha_s^3, ...$
    - expanded with small momentum
    - systematically study, order by order, error controlled
    - check of standard model
- Natual extension

  2-body force, 3-body force,...
- Wide applications

Nucleon-nucleon interaction



With Heavy Meson EFT, we study the systems made up of

- DD
- D^*D
- D^*D^*

Similar for $B^{(*)}B^{(*)}$ and corresponding anti-meson pair system.

Lagrangians

- Leading order vertice

  contact terms: $D^{(*)}D^{(*)}D^{(*)}D^{(*)}$ vertice

  $D^{(*)}D^{(*)}\pi$, $D^{(*)}D^{(*)}\pi\pi$ vertice

- Next-to-leading order vertice

  they absorb divergences, provide finite higher-order corrections

$$\begin{aligned}
\mathcal{L}_{4H}^{(0)} &= D_a \operatorname{Tr}[H\gamma_\mu \bar{H}]\operatorname{Tr}[H\gamma^\mu \bar{H}] + D_b \operatorname{Tr}[H\gamma_\mu\gamma_5\bar{H}]\operatorname{Tr}[H\gamma^\mu\gamma_5\bar{H}] \\
&\quad + E_a \operatorname{Tr}[H\gamma_\mu\lambda^a\bar{H}]\operatorname{Tr}[H\gamma^\mu\lambda_a\bar{H}] + E_b \operatorname{Tr}[H\gamma_\mu\gamma_5\lambda^a\bar{H}]\operatorname{Tr}[H\gamma^\mu\gamma_5\lambda_a\bar{H}], \\
\mathcal{L}_{H\phi}^{(1)} &= -\langle (iv\cdot\partial H)\bar{H}\rangle - \langle Hv\cdot\Gamma\bar{H}\rangle + g\langle H\not{u}\gamma_5\bar{H}\rangle - \frac{1}{8}\Delta\langle H\sigma^{\mu\nu}\bar{H}\sigma_{\mu\nu}\rangle,
\end{aligned}$$

$$\begin{aligned}
\mathcal{L}_{4H}^{(2)} &= D_a^h \operatorname{Tr}[H\gamma_\mu\bar{H}]\operatorname{Tr}[H\gamma^\mu\bar{H}]\operatorname{Tr}(\chi_+) + \ldots \\
&\quad + D_a^d \operatorname{Tr}[H\gamma_\mu\tilde{\chi}_+\bar{H}]\operatorname{Tr}[H\gamma^\mu\bar{H}] + \ldots \\
&\quad + D_1^q \operatorname{Tr}[(D^\mu H)\gamma_\mu\gamma_5(D^\nu\bar{H})]\operatorname{Tr}[H\gamma_\nu\gamma_5\bar{H}] + \ldots
\end{aligned}$$

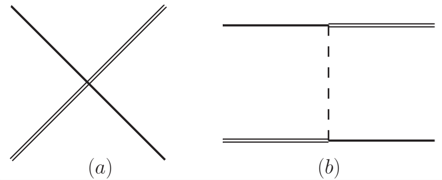
Diagrams

- Leading order

  contact, one-pion exchange

- Next-to-leading order

  two-pion exchange, renormalization to $\mathrm{D}^{(*)}\mathrm{D}^{(*)}\pi$ coupling, loop corrections to contact term, tree diagrams with NL vertice



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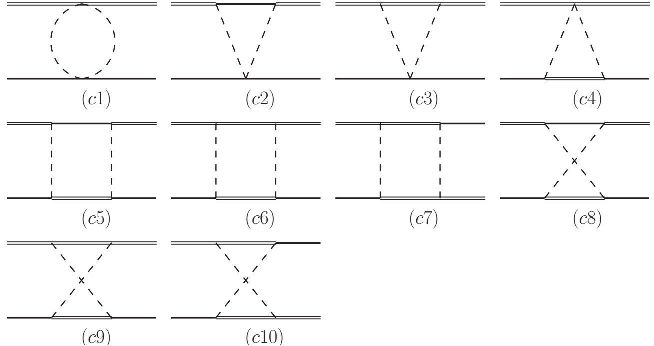

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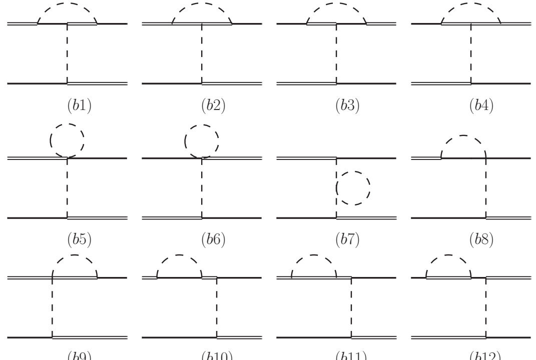

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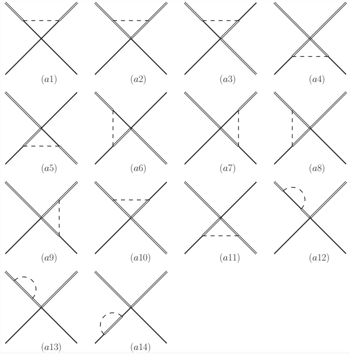
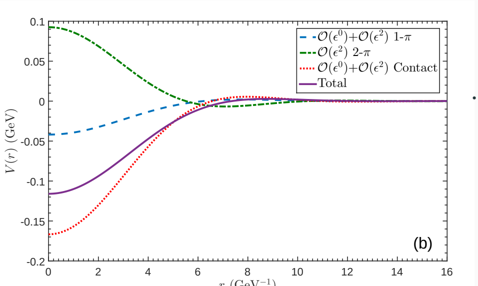


Search for new states

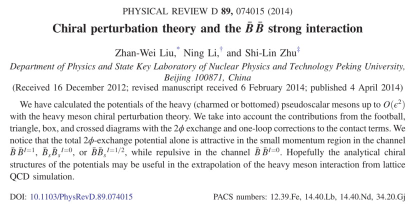
- Potentials→ partial waves, dynamical equation (momentum space)
  → T matrices → poles
- Potentials→ Fourier transform, dynamical equation (coordinate space)
  → eigenvalues of bound states for different partial waves

Example:

potentials for $\bar{B}^*\bar{B}^*$ with $I(J^P) = 0(1^+)$



- binding energies:
  $\Delta E_{\bar{B}\bar{B}^*} \simeq -12.6^{+9.2}_{-12.9}$ MeV,
  $\Delta E_{\bar{B}^*\bar{B}^*} \simeq -23.8^{+16.3}_{-21.5}$ MeV
- strong decays are forbidden because of phase space
  they can be searched in $\bar{B}\bar{B}\gamma$ or $\bar{B}\bar{B}\gamma\gamma$



PHYSICAL REVIEW D **89,** 074015 (2014)

Chiral perturbation theory and the $\bar{B}\bar{B}$ strong interaction

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Department of Physics and State Key Laboratory of Nuclear Physics and Technology Peking University,
Beijing 100871, China

(Received 16 December 2012; revised manuscript received 6 February 2014; published 4 April 2014)

We have calculated the potentials of the heavy (charmed or bottomed) pseudoscalar mesons up to $O(\epsilon^2)$ with the heavy meson chiral perturbation theory. We take into account the contributions from the football, triangle, box, and crossed diagrams with the 2ϕ exchange and one-loop corrections to the contact terms. We notice that the total 2ϕ-exchange potential alone is attractive in the small momentum region in the channel $\bar{B}\bar{B}^{I=1}$, $\bar{B}_s\bar{B}_s{}^{I=0}$, or $\bar{B}\bar{B}_s{}^{I=1/2}$, while repulsive in the channel $\bar{B}\bar{B}^{I=0}$. Hopefully the analytical chiral structures of the potentials may be useful in the extrapolation of the heavy meson interaction from lattice QCD simulation.

DOI: 10.1103/PhysRevD.89.074015 PACS numbers: 12.39.Fe, 14.40.Lb, 14.40.Nd, 34.20.Gj

PHYSICAL REVIEW D **99,** 014027 (2019)

DD^* potentials in chiral effective field theory and possible molecular states

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[1]*School of Physical Science and Technology, Lanzhou University, Lanzhou 730000, China*
[2]*Research Center for Hadron and CSR Physics, Lanzhou University & Institute of Modern Physics of CAS, Lanzhou 730000, China*
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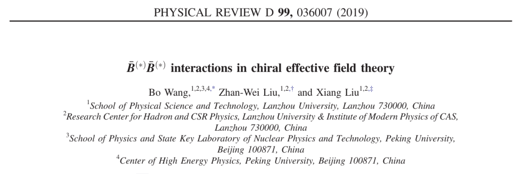

PHYSICAL REVIEW D **99,** 036007 (2019)

$\bar{B}^{(*)}\bar{B}^{(*)}$ interactions in chiral effective field theory

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[4]*Center of High Energy Physics, Peking University, Beijing 100871, China*

Our recent studies of hadron spectrum

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   arXiv 2103.03127 (2021).

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3. Exploring the exotic pentaquark states

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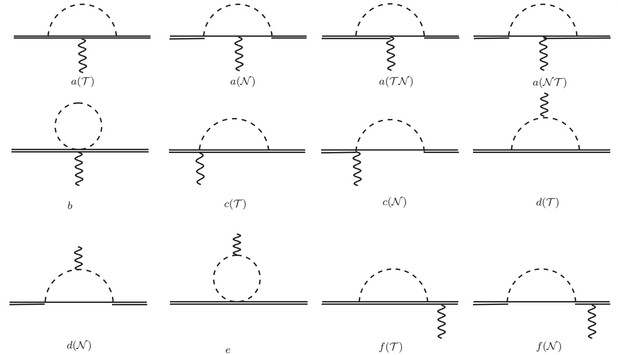
Electromagnetic properties



we have studied electromagnetic properties within ChPT

- Decuplet to octet baryon electromagnetic transitions

  EPJC 79, 66 (2019); PRD 95, 076001 (2017).
- Magnetic moments of the heavy baryons

  PLB 777, 169 (2018); PRD 98, 094013 (2018);

  PRD 98, 054026 (2018); PRD96, 076011, (2017)

哈密顿有效场论与核子激发态

Lattice QCD

- LQCD starts from the first principle of QCD
- model independent, reliable
- LQCD gives hadron spectra and quark distribution functions
  at finite volumes, large quark masses, discrete spaces
- not directly related to physical observables

Connection between Scattering Data and Lattice QCD Data

Lattice QCD

- large pion mass: extrapolation
- finite volume
- discrete space

Lattice QCD Data → Physical Data

- Lüscher Formalisms and extensions:
  - Model independent; efficient in single-channel problems
    - Spectrum → Phaseshifts; $m_{K_L} - m_{K_S}$ etc.
- Effective Field Theory (EFT), Models, etc
  - with low-energy constants fitted by Lattice QCD data

Physical Data → Lattice QCD Data

- EFT: discretization, analytic extension, Lagrangian modification
- various discretization: eg. discretize the momentum in the loop

Scattering Data and Lattice QCD data are two important sources for studying resonances.

We should try to analyse them both at the same time.

$N^*(1535)$ **with** πN **Scattering**

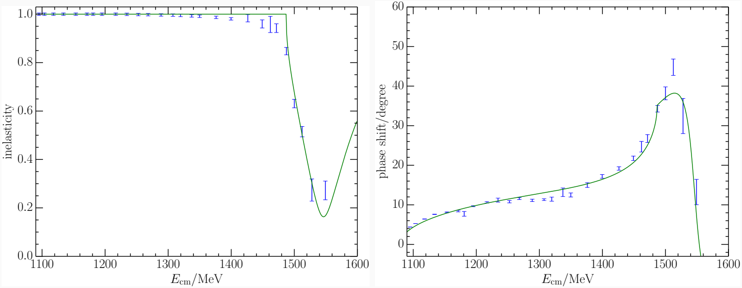
$N^*(1535)$ is the lowest resonance with $I(J^P) = \frac{1}{2}(\frac{1}{2}^-)$.

- One needs to consider the interactions

  among the bare baryon N^*_0, πN channel, and ηN channel.
- Phase shifts and inelasticities

  obtained by solving 3-dimensional reduction equation with the

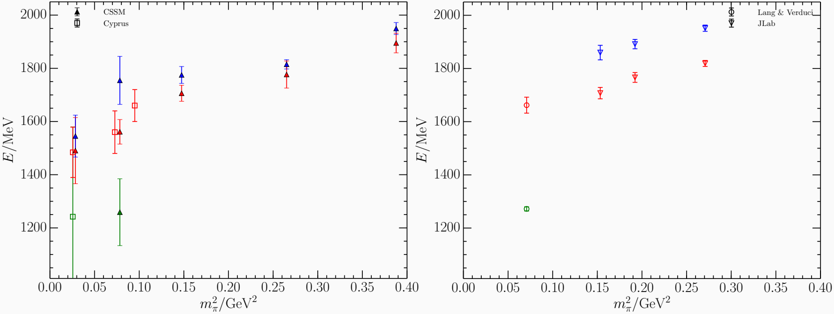
  interactions.



πN Scattering with $I(J^P) = \frac{1}{2}(\frac{1}{2}^-)$.

Spectra at Finite Volumes

3 sets of lattice data at different pion masses and finite volumes



L ≈ 3 fm

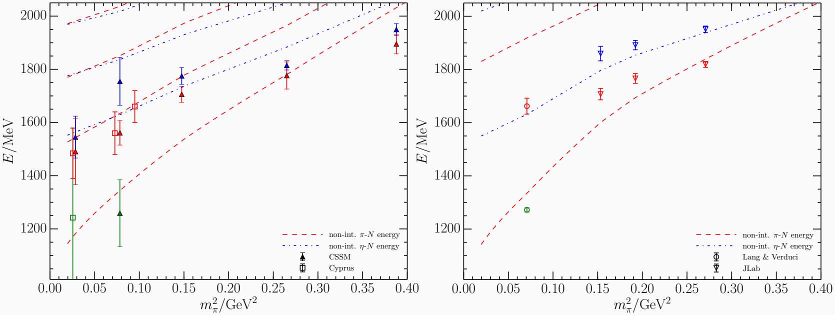
L ≈ 2 fm

Spectra with $\mathrm{I}(\mathrm{J}^\mathrm{P}) = \frac{1}{2}(\frac{1}{2}^-)$ at finite volumes

Spectra at Finite Volumes

3 sets of lattice QCD data at different pion masses and finite volumes

Non-interacting energies of the two-particle channels



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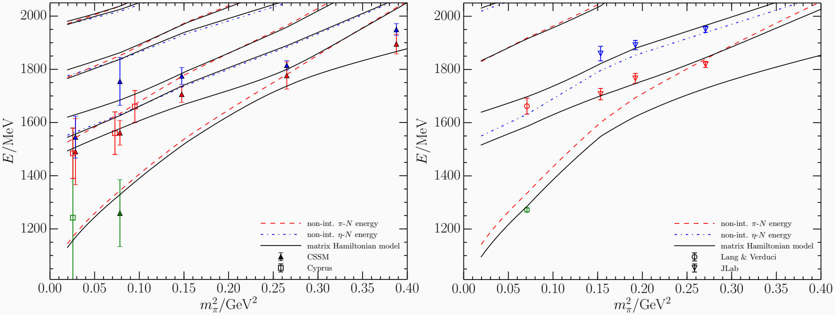
Eigenenergies of Hamiltonian effective field theory



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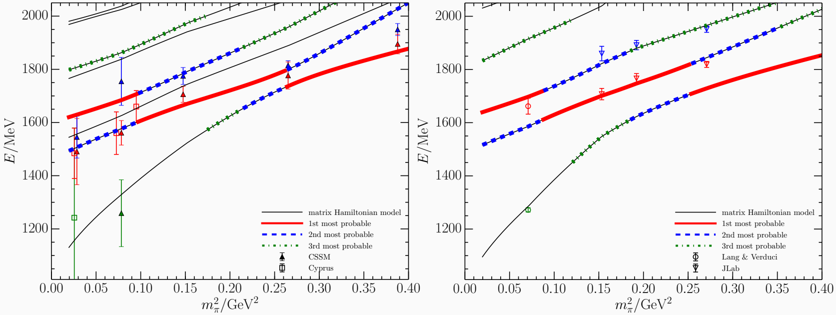
Eigenenergies of Hamiltonian effective field theory

Coloured lines indicating most probable states observed in LQCD

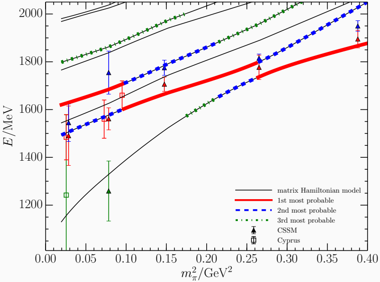


L ≈ 3 fm

L ≈ 2 fm

Spectra with $\mathrm{I(J^P)} = \frac{1}{2}(\frac{1}{2}^-)$ at finite volumes

Components of Eigenstates with L ≈ 3 fm

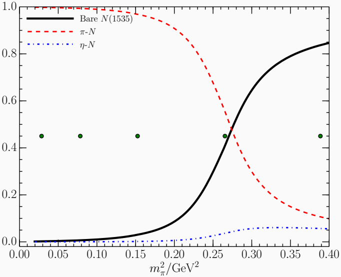


Spectra with $I(J^P) = \frac{1}{2}(\frac{1}{2}^-)$ and L ≈ 3 fm

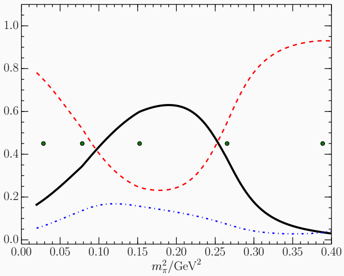
- The 1st eigenstate at light quark masses is mainly πN scattering states.
- The most probable state at physical quark mass is the 4th eigenstate.

It contains about 60% bare N*(1535), 20% πN and 20% ηN.

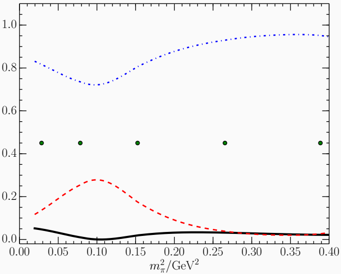
Components of Eigenstates with L ≈ 3 fm

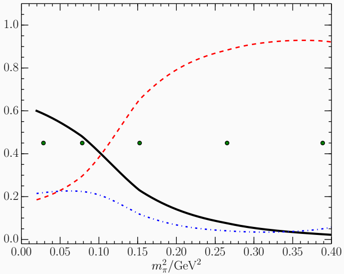


1st eigenstate



2nd eigenstate





Our recent work about Hamiltonian Effective Field Theory

- $N^*(1535)$ with Hamiltonian effective field theory

  Liu,Kamleh,Leinweber,Stokes,Thomas,Wu, PRL116, 082004 (2016)

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- $N^*(1440)$ with Hamiltonian effective field theory

  Liu,Kamleh,Leinweber,Stokes,Thomas,Wu, PRD95, 034034 (2017);

  Wu,Leinweber,**Liu**,Thomas, PRD97, 094509 (2018).

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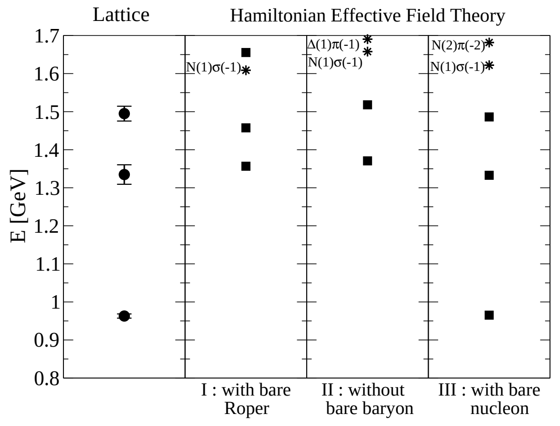
- $\Lambda(1405)$ with Hamiltonian effective field theory

  Liu,Hall,Leinweber,Thomas,Wu, PRD95, 014506 (2017).

Our results are verified

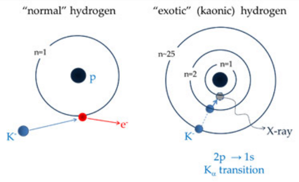


interpolating operators: N(0), N(0)σ(0), N(p)π(−p), Δ(p)π(−p). from Lang, Leskovec, Padmanath, Prelovsek, PRD95 (2017) no.1, 014510.

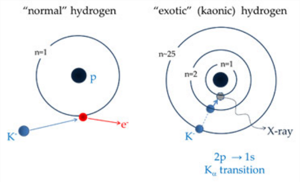
No these two higher states with N^{-P}(0)π(0)... from CMMS.

Kaonic Hydrogen

Kaonic Hydrogen



Kaonic Hydrogen



- energy shift and width of 1s level were measured at SIDDHARTA-2

$$\begin{aligned} \epsilon_{1s}^{p} &= 283 \pm 36(\mathrm{stat}) \pm 6(\mathrm{sys})\ \mathrm{eV}, \\ \Gamma_{1s}^{p} &= 541 \pm 89(\mathrm{stat}) \pm 22(\mathrm{sys})\ \mathrm{eV}, \end{aligned}$$

- they are related to the scattering length of K^-p

$$\epsilon_{1s}^{p} - \frac{i}{2}\Gamma_{1s}^{p} = \frac{-2\alpha_e^3\,\mu_{K^-p}^2\,a_{K^-p}}{1 + 2\alpha_e\,\mu_{K^-p}\,(\ln\alpha_e - 1)\,a_{K^-p}},$$

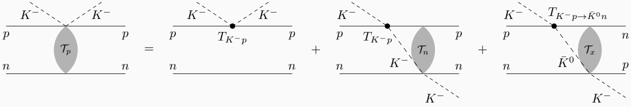
- HEFT provides

$$\epsilon_{1s}^{p} = 307\ \mathrm{eV}, \qquad \Gamma_{1s}^{p} = 533\ \mathrm{eV},$$

where $\bar{K}N$ interactions are not fine tuned.

Kaonic Deuteron without Recoil Effect

K̄NN scattering amplitude can be solved by the Faddeev equation



,...

With the static approximation,

$$a_{K^-d} = \frac{m_d}{m_K + m_d} \int d^3\vec{r}\, |\psi_d(\vec{r})|^2 \; \hat{A}_{K^-d}(r)\,,$$

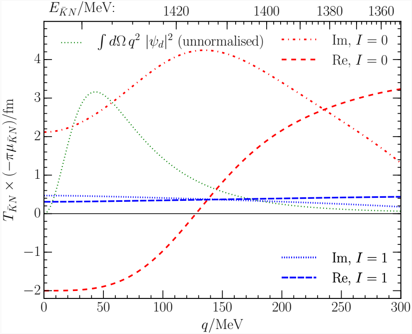
where

$$\hat{A}_{K^-d}(r) = \frac{\tilde{a}_{K^-p} + \tilde{a}_{K^-n} + (2\tilde{a}_{K^-p}\tilde{a}_{K^-n} - b_x^2)/r - 2b_x^2\tilde{a}_{K^-n}/r^2}{1 - \tilde{a}_{K^-p}\tilde{a}_{K^-n}/r^2 + b_x^2\tilde{a}_{K^-n}/r^3}\,.$$

Our results without recoil effect are similar to others

$$\epsilon_{1S}^{d}|_{\mathrm{StaticApprox}} = 855\,\mathrm{eV}, \quad \Gamma_{1S}^{d}|_{\mathrm{StaticApprox}} = 1127\,\mathrm{eV}\,.$$

Recoil Effect



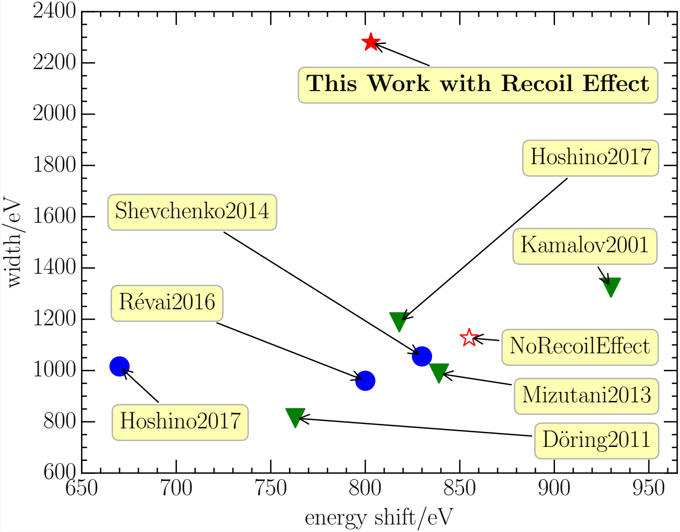
- The recoil effect is mainly from the single scattering process

$$\langle\, T^{d}_{\bar{K}N}\, \rangle \equiv \int d^3\vec{q}\, |\psi_d(\vec{q})|^2\ T_{\bar{K}N}(\vec{q}).$$

- If no Λ(1405) exists,

  this kind of recoil effect can be totally neglected.

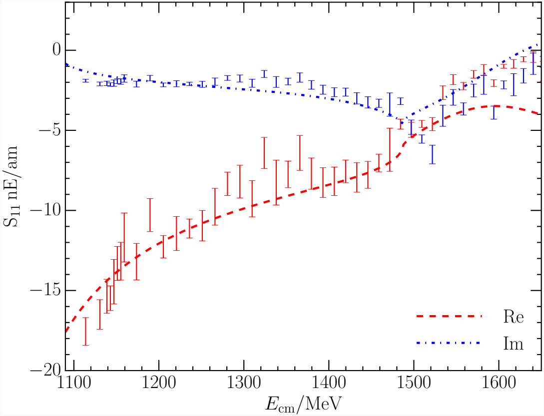
Comparison



Z.-W. Liu*, Wu,Leinweber,Thomas, Phys.Lett.B808,135652 (2020)

On progress: N(1535) **in** $\gamma + \mathrm{N} \to \pi + \mathrm{N}$

to understand the structure of N(1535) and the interactions of πN/ηN at low energies and near the resonance



preliminary result from on going work by Dan Guo and Zhan-Wei Liu.

Summary

- With Chiral effective field theory, we have studied
  - interacting potentials between two hadrons
  - binding energies of possible molecular states
  - electromagnetic properties of hadrons

- With Hamiltonian effective field theory, we have studied
  - nucleon resonances N(1535), N(1440), Λ(1405) and related interactions
  - recoil effect for the life of kaonic deuteron