Wilson is Not Anomalous: On Gauge Anomalies In SMEFT

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HELMHOLTZ RESEARCH FOR GRAND CHALLENGES

The Standard Model EFT (SMEFT)

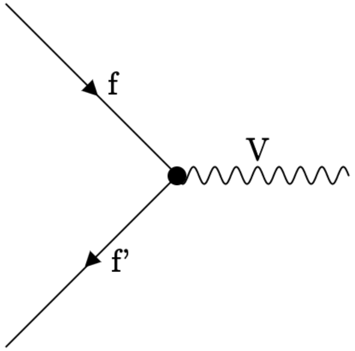
- **Field content and gauge symmetries of the SM and linearly realized EW sym.**

$$\mathcal{L}_{\mathrm{SMEFT}} = \mathcal{L}_{\mathrm{SM}} + \sum_{d>4}^{\infty} \sum_{i} \frac{c_i^{(n)}}{\Lambda^{d-4}} \mathcal{O}_i^{(n)}$$

- **We work at dimension 6 in the Warsaw basis (1008.4884)**

$$\mathcal{O}_{\phi q}^{(3)} = \left(H^\dagger i \overleftrightarrow{D}_\mu^a H\right) (\bar{q_L} \gamma^\mu \sigma^a q_L) \quad \mathcal{O}_{\phi q}^{(1)} = \left(H^\dagger i \overleftrightarrow{D}_\mu H\right) (\bar{q_L} \gamma^\mu q_L)$$

$$\mathcal{O}_{\phi u} = \left(H^\dagger i \overleftrightarrow{D}_\mu H\right) (\bar{u_R} \gamma^\mu u_R) \quad \mathcal{O}_{\phi l}^{(3)} = \left(H^\dagger i \overleftrightarrow{D}_\mu^a H\right) (\bar{l_L} \gamma^\mu \sigma^a l_L)$$



Why anomalies?

- Classical symmetries might not be quantum symmetries

Anomalous symmetry

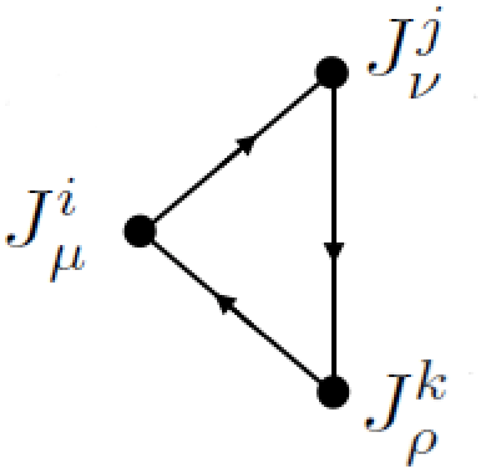
Gauge ← → **Global**

Gauge
↓
Inconsistent QFT[1]
↓
The anomaly must be cancelled

Global
↓
The anomaly doesn't bother us and does not renormalize!
↓
It gives phenomenologically important information

- How to compute an anomaly

$$\partial^\mu \left\langle 0 \middle| J^i_\mu(x)\, J^j_\nu(y)\, J^k_\rho(z) \middle| 0 \right\rangle \propto D^{ijk} \equiv \sum_{LH\psi} \frac{1}{2} \mathrm{Tr}\Big(\underbrace{T^i\{T^j,T^k\}}_{\text{Generators of: } G_i,\, G_j,\, G_k}\Big) - \sum_{RH\psi} (\text{same})$$



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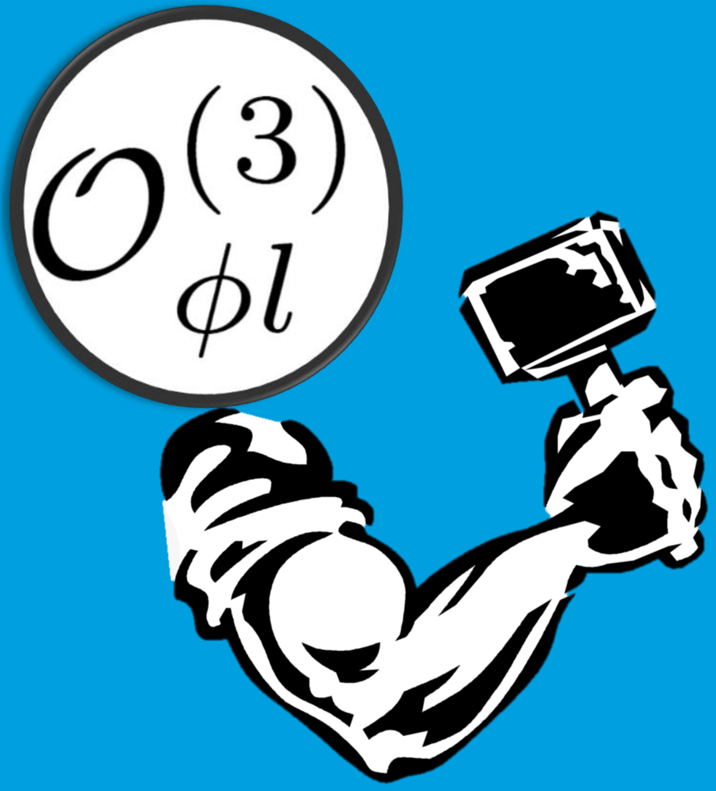
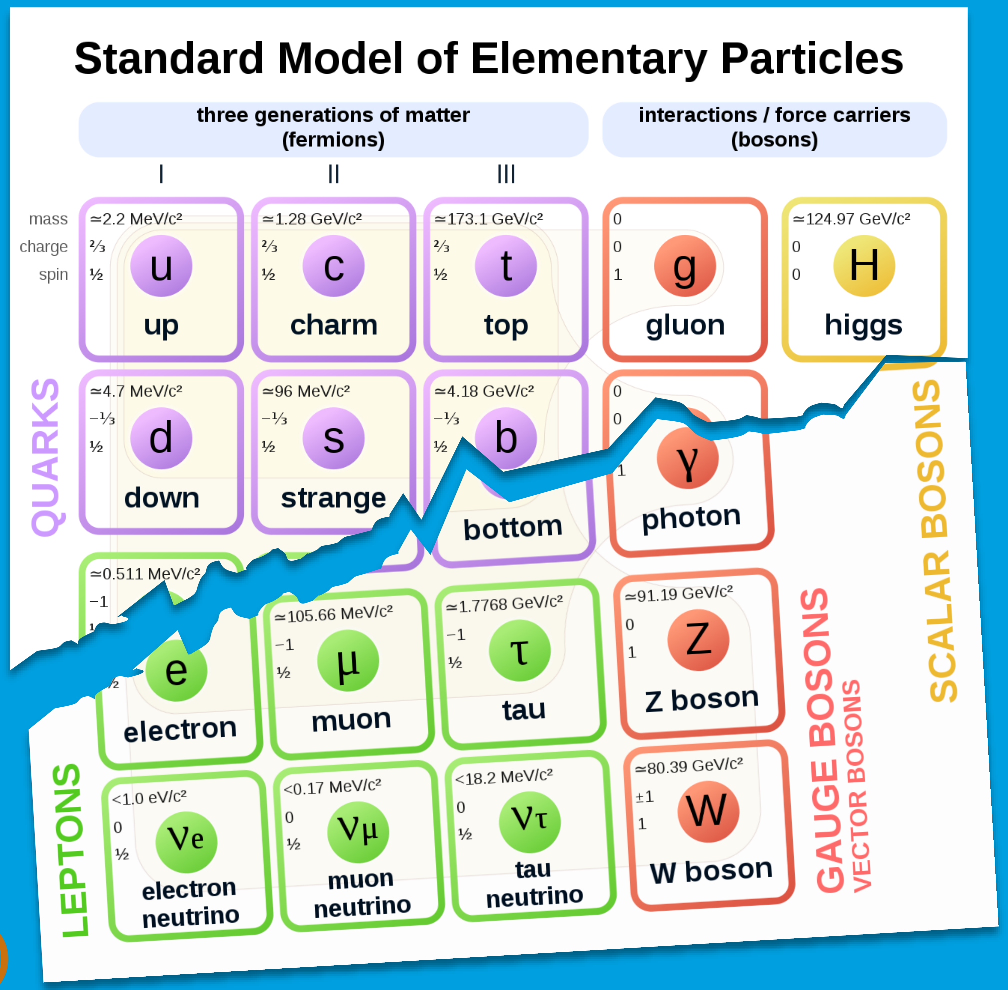
[1] See J. Preskill, *Annals Phys.* 210 (1991) 323-379

From higher-dim. operators to anomalies

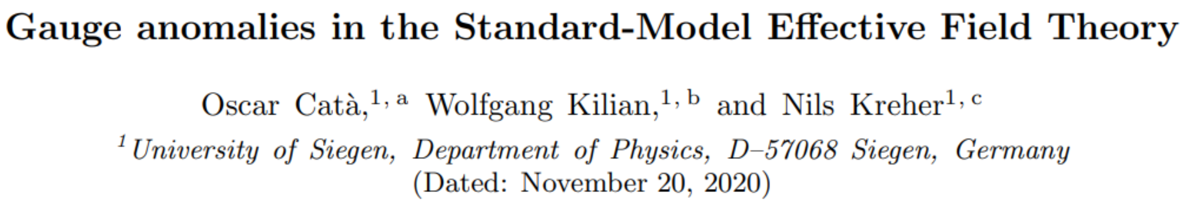




(Dramatization)

Doubts on the horizon

- **SMEFT was understood to be gauge anomaly free at any order…**



Gauge anomalies in the Standard-Model Effective Field Theory

Oscar Catà,[1, a] Wolfgang Kilian,[1, b] and Nils Kreher[1, c]

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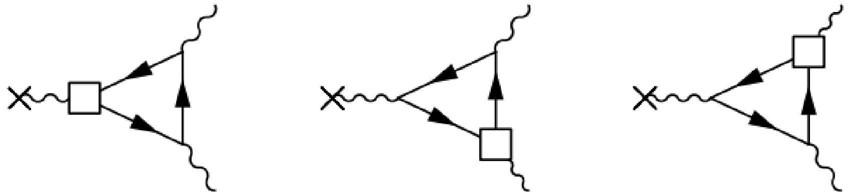
(Dated: November 20, 2020)

(arXiv 2011.09976)

- **Do chiral dim-6 operators generate gauge anomalies?**



$$c^{(3)}_{\varphi f} \equiv c^{(3)}_{\varphi_q} = c^{(3)}_{\varphi l}\,,$$

$$c^{(1)}_{\varphi f} \equiv \frac{1}{y_q} c^{(1)}_{\varphi q} = \frac{1}{y_u} c_{\varphi u} = \frac{1}{y_d} c_{\varphi d} = \frac{1}{y_l} c^{(1)}_{\varphi l} = \frac{1}{y_e} c_{\varphi e}$$

??

There are simple counterexamples

Add to the SM a singlet Weyl fermion (type 1 see-saw)

A toy model: currents

$k{=}1{,}2$

$$\mathcal{L} = -\frac{1}{4g_A^2}F^2_{A,\mu\nu} - \frac{1}{4g_B^2}F^2_{B,\mu\nu} + |D\varphi|^2 - V(|\varphi|) + i\overline{\psi}_k \not{D}\psi_k$$

$$+ i\frac{c_{L,k}}{\Lambda^2}\left(\varphi^\dagger \overleftrightarrow{D}_\mu \varphi\right)\overline{\psi}_{k,L}\gamma^\mu\psi_{k,L} + i\frac{c_{R,k}}{\Lambda^2}\left(\varphi^\dagger \overleftrightarrow{D}_\mu \varphi\right)\overline{\psi}_{k,R}\gamma^\mu\psi_{k,R}$$

$$\Big\downarrow \qquad\qquad\qquad \Big\downarrow$$

$$\sim \mathcal{O}^{(1)}_{\phi q} = \left(H^\dagger i\overleftrightarrow{D}_\mu H\right)(\bar{q_L}\gamma^\mu q_L) \qquad \sim \mathcal{O}_{\phi u} = \left(H^\dagger i\overleftrightarrow{D}_\mu H\right)(\bar{u_R}\gamma^\mu u_R)$$

$$\varphi \to e^{iq^B_\varphi \epsilon_B}\varphi\,,\ \psi_k \to e^{iq^B_k\gamma_5\epsilon_B}\psi_k \quad D_\mu\psi_k = (\partial_\mu + iq^A_k A_\mu + iq^B_k\gamma_5 B_\mu)\psi_k,\ D_\mu\varphi = (\partial_\mu + iq^B_\varphi B_\mu)\varphi$$

- **Anomaly cancellation conditions from dim.-4 operators:**

$$U(1)^2_A \times U(1)_B:\ \left(q^A_k\right)^2 q^B_k = 0\ , \qquad U(1)^3_B:\ \left(q^B_k\right)^3 = 0$$

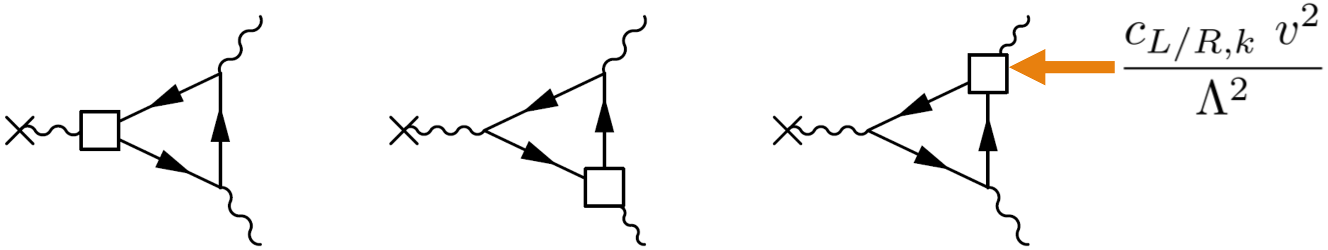
A toy model: more currents

- **Fermion currents modified by dim.-6 operators:**

$$\mathcal{L} \supset -\left(q_k^B + q_\varphi^B \frac{2c_{L,k}|\varphi|^2}{\Lambda^2}\right)\overline{\psi}_{k,L} \not{B} \psi_{k,L} - \left(-q_k^B + q_\varphi^B \frac{2c_{R,k}|\varphi|^2}{\Lambda^2}\right)\overline{\psi}_{k,R} \not{B} \psi_{k,R}$$

- **Following Catà et al, in the broken phase, we would compute:**

$$\varphi = \frac{v+h}{\sqrt{2}} e^{i\frac{\theta}{v}}$$



Which means asking for the conservation of:

$$\tilde{J}^B_\mu = 2q^B_\varphi\left(\frac{c_{L,k}}{\Lambda^2}|\varphi|^2\overline{\psi}_{k,L}\gamma_\mu\psi_{k,L} + \frac{c_{R,k}}{\Lambda^2}|\varphi|^2\overline{\psi}_{k,R}\gamma_\mu\psi_{k,R}\right) + q_k^B\overline{\psi}_k\gamma_\mu\gamma_5\psi_k$$

A toy model: even more currents

$$\tilde{J}^B_\mu = 2q^B_\varphi\left(\frac{c_{L,k}}{\Lambda^2}|\varphi|^2\overline{\psi}_{k,L}\gamma_\mu\psi_{k,L} + \frac{c_{R,k}}{\Lambda^2}|\varphi|^2\overline{\psi}_{k,R}\gamma_\mu\psi_{k,R}\right) + q^B_k\overline{\psi}_k\gamma_\mu\gamma_5\psi_k$$

- **Leading to:**

$$U(1)^2_A \times \tilde{J}^B_\mu :\ \left(q^A_k\right)^2 q^B_\varphi\left(c_{L,k} - c_{R,k}\right) = 0\ ,$$
$$U(1)_A \times U(1)_B \times \tilde{J}^B_\mu :\ q^A_k q^B_k q^B_\varphi\left(c_{L,k} + c_{R,k}\right) = 0\ ,$$
$$U(1)^2_B \tilde{J}^B_\mu :\ \left(q^B_k\right)^2 q^B_\varphi\left(c_{L,k} - c_{R,k}\right) = 0\ .$$

$$2c^{(1)}_{\varphi q} - c_{\varphi u} - c_{\varphi d} = 0$$
$$6y_\varphi c^{(1)}_{\varphi q} + 12y_q c^{(3)}_{\varphi q} + 2y_\varphi c^{(1)}_{\varphi l} + 4y_l c^{(3)}_{\varphi l} = 0$$
$$\mathcal{A} \equiv 6y_q^2 c^{(1)}_{\varphi q} + 2y_l^2 c^{(1)}_{\varphi l} - 3y_u^2 c_{\varphi u} - 3y_d^2 c_{\varphi d} - y_e^2 c_{\varphi e} = 0$$

(arXiv 2011.09976)

But $\tilde{J}^B_\mu$ is not conserved even at classical level!

- **The classically conserved current is:**

$$J^B_\mu = \underbrace{i\, q^B_\varphi\, \varphi^\dagger \overleftrightarrow{\partial}_\mu \varphi}_{\text{Forgotten piece.}} + \tilde{J}^B_\mu$$

Forgotten diagrams! Which ones?

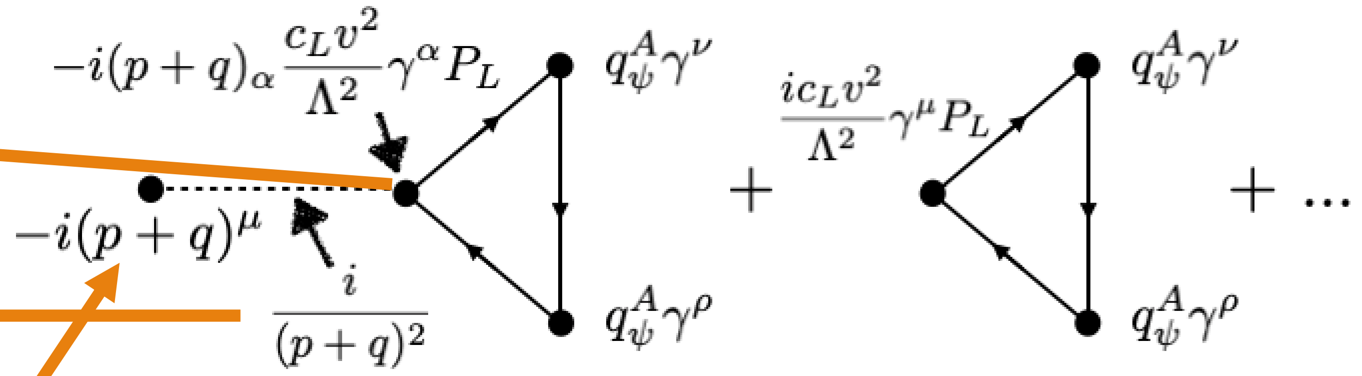
A toy model: diagrams with GBs

- **Let's compute:** $\partial^\mu \langle 0 | J^B_\mu(x) J^A_\nu(y) J^A_\rho(z) | 0 \rangle$

Broken phase: There are diagrams with Goldstone bosons (GBs)!

$-\frac{v c_{L,k}}{\Lambda^2} \partial_\mu \theta \overline{\psi}_{k,L} \gamma^\mu \psi_{k,L}$

GB propagator



$$J^B_\mu = -i q^B_\varphi \left(\varphi^\dagger \overleftrightarrow{\partial}_\mu \varphi + 2i \frac{c_{R,k}}{\Lambda^2} |\varphi|^2 \overline{\psi}_{k,R} \gamma_\mu \psi_{k,R} + 2i \frac{c_{L,k}}{\Lambda^2} |\varphi|^2 \overline{\psi}_{k,L} \gamma_\mu \psi_{k,L} \right) + q^B_k \overline{\psi}_k \gamma_\mu \gamma_5 \psi_k$$

The diagrams with GBs cancel the ones with dim-6 vertices

(After contracting with the momentum from the divergence)

No constraints on the WCs from triangle diagrams

A toy model: bosonic EFT

- Add Yukawa terms to the toy model: $\delta\mathcal{L} = -y_k \varphi \overline{\psi}_{k,L}\psi_{k,R} + h.c.$
- In the broken phase, the fermions can be integrated out to get a bosonic EFT
- In the bosonic EFT, anomalies are carried by the Wess-Zumino terms.

Will the dim.-6 operators contribute to the WC of the WZ terms?

- And the Wess-Zumino terms in the EFT are:

$$\mathcal{L}_{EFT} \supset -\underbrace{\frac{\left(q_k^A\right)^2}{16\pi^2}\frac{\theta}{v}F_A\tilde{F}_A - \frac{0}{16\pi^2}\frac{\theta}{v}F_A\tilde{F}_B - \frac{\left(q_k^B\right)^2}{24\pi^2}\frac{\theta}{v}F_B\tilde{F}_B}_{\text{Axionic terms}} - \underbrace{\frac{\left(q_k^A\right)^2 q_k^B}{6\pi^2}A_\mu B_\nu \tilde{F}_A^{\mu\nu} - \frac{0}{8\pi^2}A_\mu B_\nu \tilde{F}_B^{\mu\nu}}_{\text{Generalized Chern-Simons terms}}$$

No constraints on the WCs from triangle diagrams

SMEFT: extending the EFT technique

- **Anomalies are independent of fermion masses.**
- **Integrate out all fermions and the Higgs, keep only gauge bosons.**
- **With only dim.-4 operators:**

$$
\begin{aligned}
\mathcal{L}_{\rm EFT} \supset & -\frac{1}{16\pi^2}\frac{\pi^3}{v} B\tilde{B}\left(3\left[y_u^2 + y_Q y_u - y_d^2 - y_Q y_d\right] + y_\nu^2 + y_L y_\nu - y_e^2 - y_e y_L\right) \\
& - \frac{1}{16\pi^2}\frac{\pi^3}{v} B\tilde{W}^3 \left(\frac{3(y_d + 4y_Q + y_u)}{2} + \frac{y_e + 4y_L + y_\nu}{2}\right) \\
& - \frac{1}{8\pi^2} B_\mu W^3_\nu \tilde{B}^{\mu\nu} \frac{(y_\nu - y_e)(y_e + y_L + y_\nu) + 3(y_u - y_d)(y_d + y_Q + y_u)}{2} \\
& - \frac{1}{8\pi^2} B_\mu W^3_\nu \tilde{W}^{3,\mu\nu} \frac{3(y_u + y_d) + y_e + y_\nu}{4} \,,
\end{aligned}
$$

$$
\delta\mathcal{L}_{\rm EFT} = -\frac{\epsilon_Y}{16\pi^2}\left[\left(6y_Q^3 + 2y_L^3 - 3y_u^3 - 3y_d^3 - y_e^3 - y_\nu^3\right) B\tilde{B} + \frac{3y_Q + y_L}{2} W^3\tilde{W}^3\right] - \frac{\epsilon_3}{16\pi^2}\left(3y_Q + y_L\right) B\tilde{W}^3
$$

Usual gauge anomaly cancellation conditions on the hypercharges

SMEFT: extending the EFT technique

And the dim.-6 operators contribution?

Dim.-6 vertex

$$\mathcal{L}_{EFT} \supset -\frac{\mathcal{C}_{F_0 W^3}}{16\pi^2}\frac{\pi^3}{v}F_0^{\mu\nu}\tilde{W}^3_{\mu\nu} - \frac{\mathcal{C}_{F_0 B}}{16\pi^2}\frac{\pi^3}{v}F_0^{\mu\nu}\tilde{B}^3_{\mu\nu} - \frac{E_{A_0 BB}}{8\pi^2\Lambda^2}A_{0,\mu}B_\nu\tilde{B}^{\mu\nu} - \frac{E_{A_0 W^3 B}}{8\pi^2\Lambda^2}A_{0,\mu}W^3_\nu\tilde{B}^{\mu\nu}$$

$$-\frac{E_{A_0 B W^3}}{8\pi^2\Lambda^2}A_{0,\mu}B_\nu\tilde{W}^{3,\,\mu\nu} - \frac{E_{B W^3 F_0}}{8\pi^2\Lambda^2}B_\mu W^3_\nu\tilde{F}_0^{\mu\nu} - \frac{E_{A_0 W^3 W^3}}{8\pi^2\Lambda^2}A_{0,\mu}W^3_\nu\tilde{W}^{3,\,\mu\nu}$$

$$A_{0,\mu} \equiv v^2\left(y_\varphi B_\mu - \frac{W^3_\mu}{2}\right) - v\partial_\mu\pi^3 \qquad F_{0,\mu\nu} = v^2\left(y_\varphi B_{\mu\nu} - \frac{W^3_{\mu\nu}}{2}\right)$$

Compute all the terms, rearrange, integrate by parts and…
Ok, let's go slowly

SMEFT: extending the EFT technique

Gather all the terms proportional to $\pi^3 B\widetilde{B}$

$$\mathcal{L}_{EFT} \supset -\frac{\mathcal{C}_{F_0 B}}{16\pi^2}\frac{\pi^3}{v} F_{0,\mu\nu}\tilde{B}^{\mu\nu} - \frac{\mathcal{C}_{F_0 W}}{16\pi^2}\frac{\pi^3}{v}F_{0,\mu\nu}\tilde{W^3}^{\mu\nu} - \frac{E_{A_0BB}}{8\pi^2\Lambda^2}A_{0,\mu}B_\nu\tilde{B}^{\mu\nu} - \frac{E_{BW^3F_0}}{8\pi^2\Lambda^2}B_\mu W^3_\nu \tilde{F_0}^{\mu\nu} - \frac{E_{A_0BW^3}}{8\pi^2\Lambda^2}A_{0,\mu}B_\nu\tilde{W^3}^{\mu\nu}$$

$$\mathcal{L}_{EFT} \supset -v\, y_\varphi \frac{\mathcal{C}_{F_0 B}}{16\pi^2}\pi^3 B_{\mu\nu}\tilde{B}^{\mu\nu} + v\frac{E_{A_0BB}}{8\pi^2\Lambda^2}\partial_\mu\left(\pi^3\right)B_\nu\tilde{B}^{\mu\nu}$$

Integrate by parts

$$\mathcal{L}_{EFT} \supset -v\, y_\varphi \frac{\mathcal{C}_{F_0 B}}{16\pi^2}\pi^3 B_{\mu\nu}\tilde{B}^{\mu\nu} - v\frac{E_{A_0BB}}{16\pi^2\Lambda^2}\pi^3 B_{\mu\nu}\tilde{B}^{\mu\nu}$$

$$y_\varphi \mathcal{C}_{F_0 B} + \frac{E_{A_0BB}}{\Lambda^2}$$????

SMEFT: extending the EFT technique

Is that a zero?

$$y_\varphi \mathcal{C}_{F_0 B} + \frac{E_{A_0 BB}}{\Lambda^2}$$

$$\begin{aligned}
\mathcal{C}_{F_0 B} = -\frac{1}{3\Lambda^2}\Big[3\boxed{\Big(c_{\varphi d}\left(y_q + 2y_d\right)} - c_{\varphi u}\left(y_q + 2y_u\right) + c^{(1)}_{\varphi q}\left(y_d - y_u\right) + c^{(3)}_{\varphi q}\left(y_d + 4y_q + y_u\right)\Big) \\
+ c_{\varphi e}\left(y_l + 2y_e\right) + c^{(1)}_{\varphi l}\left(y_e - y_\nu\right) + c^{(3)}_{\varphi l}\left(y_e + 4y_l + y_\nu\right)\Big]
\end{aligned}$$

$$\begin{aligned}
E_{A_0 BB} = \boxed{c_{\varphi d}\left(y_q - y_d\right)\left(y_q + 2y_d\right)} + c_{\varphi u}\left(y_q - y_u\right)\left(y_q + 2y_u\right) - c^{(1)}_{\varphi q}\left(y_d^2 + y_d y_q - 4y_q^2 + y_q y_u + y_u^2\right) \\
- c^{(3)}_{\varphi q}\left(y_d - y_u\right)\left(y_d + y_q + y_u\right) + \frac{1}{3}c_{\varphi e}\left(y_l - y_e\right)\left(y_l + 2y_e\right) \\
- \frac{1}{3}c^{(1)}_{\varphi l}\left(y_e^2 + y_e y_l - y_l^2 + y_l y_\nu + y_\nu^2\right) - \frac{1}{3}c^{(3)}_{\varphi l}\left(y_e - y_\nu\right)\left(y_e + y_l + y_\nu\right)
\end{aligned}$$

$$y_q - y_d = y_\varphi$$

See also F. Feruglio, arXiv: 2012.13989, JHEP 03 (2021) 128

SMEFT: extending the EFT technique

Is that a zero?

$$y_\varphi \mathcal{C}_{F_0 B} + \frac{E_{A_0 BB}}{\Lambda^2} = 0$$

$$\begin{aligned}\mathcal{C}_{F_0 B} = -\frac{1}{3\Lambda^2}\Big[& 3\left(\boxed{c_{\varphi d}\left(y_q+2y_d\right)} - c_{\varphi u}\left(y_q+2y_u\right) + c^{(1)}_{\varphi q}\left(y_d-y_u\right) + c^{(3)}_{\varphi q}\left(y_d+4y_q+y_u\right)\right) \\ & + c_{\varphi e}\left(y_l+2y_e\right) + c^{(1)}_{\varphi l}\left(y_e-y_\nu\right) + c^{(3)}_{\varphi l}\left(y_e+4y_l+y_\nu\right)\Big]\end{aligned}$$

$$\begin{aligned}E_{A_0 BB} = & \boxed{c_{\varphi d}\left(y_q-y_d\right)\left(y_q+2y_d\right)} + c_{\varphi u}\left(y_q-y_u\right)\left(y_q+2y_u\right) - c^{(1)}_{\varphi q}\left(y_d^2+y_d y_q-4y_q^2+y_q y_u+y_u^2\right) \\ & - c^{(3)}_{\varphi q}\left(y_d-y_u\right)\left(y_d+y_q+y_u\right) + \frac{1}{3}c_{\varphi e}\left(y_l-y_e\right)\left(y_l+2y_e\right) \\ & - \frac{1}{3}c^{(1)}_{\varphi l}\left(y_e^2+y_e y_l-y_l^2+y_l y_\nu+y_\nu^2\right) - \frac{1}{3}c^{(3)}_{\varphi l}\left(y_e-y_\nu\right)\left(y_e+y_l+y_\nu\right)\end{aligned}$$

$$y_q - y_d = y_\varphi$$

See also F. Feruglio, arXiv: 2012.13989, JHEP 03 (2021) 128

SMEFT: extending the EFT technique

Is that a zero?

$$y_\varphi \mathcal{C}_{F_0 B} + \frac{E_{A_0 BB}}{\Lambda^2} = 0$$

$$\mathcal{C}_{F_0 B} = -\frac{1}{3\Lambda^2}\Big[3\boxed{\big(c_{\varphi d}\left(y_q + 2y_d\right)} - c_{\varphi u}\left(y_q + 2y_u\right) + c^{(1)}_{\varphi q}\left(y_d - y_u\right) + c^{(3)}_{\varphi q}\left(y_d + 4y_q + y_u\right)\big)$$
$$+ c_{\varphi e}\left(y_l + 2y_e\right) + c^{(1)}_{\varphi l}\left(y_e - y_\nu\right) + c^{(3)}_{\varphi l}\left(y_e + 4y_l + y_\nu\right)\Big]$$

$$E_{A_0 BB} = \boxed{c_{\varphi d}\left(y_q - y_d\right)\left(y_q + 2y_d\right)} + c_{\varphi u}\left(y_q - y_u\right)\left(y_q + 2y_u\right) - c^{(1)}_{\varphi q}\left(y_d^2 + y_d y_q - 4y_q^2 + y_q y_u + y_u^2\right)$$
$$- c^{(3)}_{\varphi q}\left(y_d - y_u\right)\left(y_d + y_q + y_u\right) + \frac{1}{3} c_{\varphi e}\left(y_l - y_e\right)\left(y_l + 2y_e\right)$$

The same cancellation happens with all the other terms.

All the dim-6 contributions cancel out!

No constraints on the WCs from triangles.

See also F. Feruglio, arXiv: 2012.13989, JHEP 03 (2021) 128

Conclusions

- Dim-6 chiral operators in triangles do not generate gauge anomalies.
- SMEFT at dim-6 is free of gauge anomalies coming from triangles.
- The same conclusion was obtained via different techniques by F. Feruglio in 2012.13989 (JHEP 03 (2021) 128).
- Our technique is easy to extend to higher-dimensional operators that modify the gauge couplings in a similar way.
- We use similar techniques to analyse relations between WCs and anomalies in axion (ALP) EFTs and connections to chiral extensions of the SM (see arXiv:2011.10025).

Thank you for your attention

And keep using SMEFT!

Contact

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Appendix A:
Counterexample to Catà et al results

Simple counterexample

Add to the SM a singlet Weyl fermion (type 1 see-saw).

$$\mathcal{L}^{\text{Int}}_{BSM} = -(\lambda_N)_i \bar{N} \tilde{\varphi}^\dagger \ell_{L,i}$$

Matching onto SMEFT:

$$\frac{1}{\Lambda^2}\left(c^{(1)}_{\varphi\ell}\right)_{ij} = \frac{(\lambda_N)^*_i (\lambda_N)_j}{4M_N^2}, \qquad \frac{1}{\Lambda^2}\left(c^{(1)}_{\varphi q}\right)_{ij} = 0$$

$$\frac{1}{\Lambda^2}\left(c^{(3)}_{\varphi\ell}\right)_{ij} = -\frac{(\lambda_N)^*_i (\lambda_N)_j}{4M_N^2} \qquad \frac{1}{\Lambda^2}\left(c^{(3)}_{\varphi q}\right)_{ij} = 0$$

$$c^{(3)}_{\varphi q} \neq c^{(3)}_{\varphi l} \qquad c^{(1)}_{\varphi q} \neq \frac{y_q}{y_l} c^{(1)}_{\varphi l}$$ **!!**

Appendix B:
All WZ terms for the neutral sector of SMEFT

$$\mathcal{L}_{\text{EFT}} \supset -\frac{1}{16\pi^2}\mathcal{C}_{F_0 B}\frac{\pi^3}{v}F_0\tilde{B} - \frac{1}{16\pi^2}\mathcal{C}_{F_0 W^3}\frac{\pi^3}{v}F_0\tilde{W}^3$$

$$\mathcal{L}_{\text{EFT}} \supset -\frac{E_{A_0 BB}}{8\pi^2\Lambda^2}A_{0,\mu}B_\nu\tilde{B}^{\mu\nu} - \frac{E_{A_0 W^3 B}}{8\pi^2\Lambda^2}A_{0,\mu}W^3_\nu\tilde{B}^{\mu\nu} - \frac{E_{A_0 BW^3}}{8\pi^2\Lambda^2}A_{0,\mu}B_\nu\tilde{W}^{3,\mu\nu}$$
$$- \frac{E_{BW^3F_0}}{8\pi^2\Lambda^2}B_\mu W^3_\nu\tilde{F}^{\mu\nu}_0 - \frac{E_{A_0 W^3 W^3}}{8\pi^2\Lambda^2}A_{0,\mu}W^3_\nu\tilde{W}^{3,\mu\nu}$$

$$\mathcal{C}_{F_0 B} = -\frac{1}{3\Lambda^2}\Big[3\left(c^{(1)}_{\varphi d}(2y_d + y_Q) - c^{(1)}_{\varphi u}(y_Q + 2y_u) + c^{(1)}_{\varphi Q}(y_d - y_u) + c^{(3)}_{\varphi Q}(y_d + 4y_Q + y_u)\right)$$
$$+ c^{(1)}_{\varphi e}(2y_e + y_L) + c^{(1)}_{\varphi L}(y_e - y_\nu) + c^{(3)}_{\varphi L}(y_e + 4y_L + y_\nu)\Big]$$

$$\mathcal{C}_{F_0 W^3} = \frac{1}{6\Lambda^2}\Big[3c^{(1)}_{\varphi d} + 3c^{(1)}_{\varphi u} + 12c^{(1)}_{\varphi Q} + c^{(1)}_{\varphi e} + 4c^{(1)}_{\varphi L}\Big]$$

$$\begin{aligned}
E_{A_0BB} =& c^{(1)}_{\varphi u}(y_Q - y_u)(y_Q + 2y_u) + c^{(1)}_{\varphi d}(y_Q - y_d)(y_Q + 2y_d) - c^{(1)}_{\varphi Q}\left(y_d^2 + y_d y_Q - 4y_Q^2 + y_Q y_u + y_u^2\right) \\
& - c^{(3)}_{\varphi Q}(y_d - y_u)(y_d + y_Q + y_u) + \frac{1}{3}c^{(1)}_{\varphi e}(y_L - y_e)(y_L + 2y_e) \\
& - \frac{1}{3}c^{(1)}_{\varphi L}\left(y_e^2 + y_e y_L - 4y_L^2 + y_L y_\nu + y_\nu^2\right) - \frac{1}{3}c^{(3)}_{\varphi L}(y_e - y_\nu)(y_e + y_L + y_\nu) \\
E_{A_0W^3B} =& \frac{1}{2}c^{(1)}_{\varphi u}(y_Q - y_u) + \frac{1}{2}c^{(1)}_{\varphi d}(y_d - y_Q) + c^{(1)}_{\varphi Q}(y_d - y_u) + c^{(3)}_{\varphi Q}(y_d - 2y_Q + y_u) \\
& + \frac{1}{6}c^{(1)}_{\varphi e}(y_e - y_L) + \frac{1}{3}c^{(1)}_{\varphi L}(y_e - y_\nu) + \frac{1}{3}c^{(3)}_{\varphi L}(y_e - 2y_L + y_\nu) \\
E_{A_0BW^3} =& \frac{1}{2}c^{(1)}_{\varphi u}(y_Q + 2y_u) - \frac{1}{2}c^{(1)}_{\varphi d}(y_Q + 2y_d) - \frac{1}{2}c^{(1)}_{\varphi Q}(y_d - y_u) - \frac{1}{2}c^{(3)}_{\varphi Q}(y_d + 4y_Q + y_u) \\
& - \frac{1}{6}c^{(1)}_{\varphi e}(y_L + 2y_e) - \frac{1}{6}c^{(1)}_{\varphi L}(y_e - y_\nu) - \frac{1}{6}c^{(3)}_{\varphi L}(y_e + 4y_L + y_\nu) \\
E_{BW^3F_0} =& \frac{3}{2}c^{(1)}_{\varphi u}y_u - \frac{3}{2}c^{(1)}_{\varphi d}y_d - \frac{3}{2}c^{(1)}_{\varphi Q}(y_d - y_u) - \frac{3}{2}c^{(3)}_{\varphi Q}(y_d + y_u) \\
& - \frac{1}{2}c^{(1)}_{\varphi e}y_e - \frac{1}{2}c^{(1)}_{\varphi L}(y_e - y_\nu) - \frac{1}{2}c^{(3)}_{\varphi L}(y_e + y_\nu) \\
E_{A_0W^3W^3} =& \frac{1}{12}\left(3c^{(1)}_{\varphi u} + 3c^{(1)}_{\varphi d} + 12c^{(1)}_{\varphi Q} + c^{(1)}_{\varphi e} + 4c^{(1)}_{\varphi L}\right)
\end{aligned}$$