Institute of Theoretical Physics
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Landscape of Effective Field Theories

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ITP-CAS group:

- Hao-Lin Li, Zhe Ren, Jing Shu, Ming-Lei Xiao, **JHY**, Yu-Hui Zheng, 2005.00008
- Hao-Lin Li, Zhe Ren, Ming-Lei Xiao, **JHY**, Yu-Hui Zheng, 2007.07899
- Hao-Lin Li, Zhe Ren, Ming-Lei Xiao, **JHY**, Yu-Hui Zheng, 2012.09188
- Hao-Lin Li, Jing Shu, Ming-Lei Xiao, **JHY**, 2012.11615

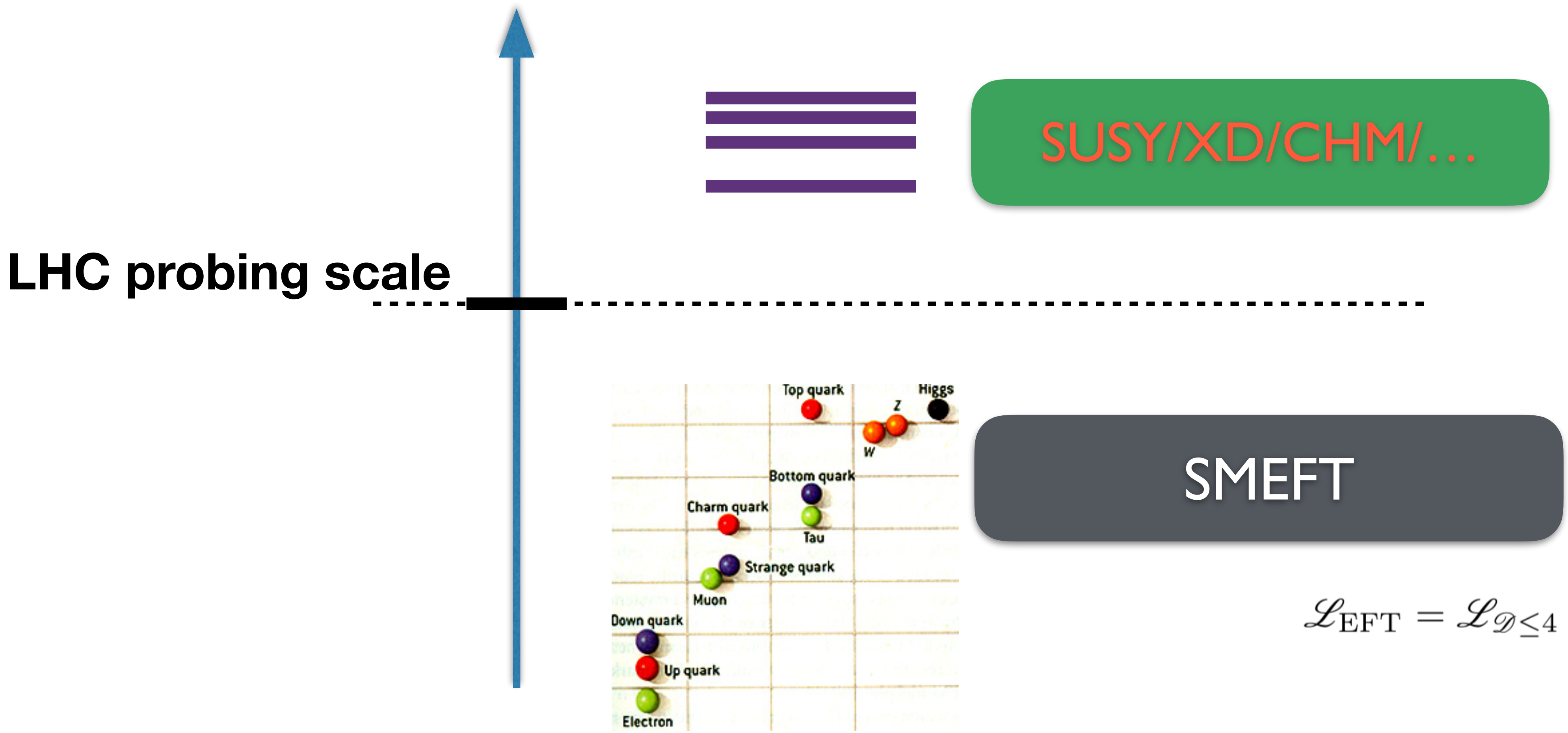
Higgs and Effective Field Theory - HEFT 2021

April 15, 2021 @ USTC

Outline

- Introduction
- Y-basis: operator as spinor tensor
- P-basis: operator with repeated field
- J-basis: UV origin of operator
- Summary and outlook

New Physics w/o New Particle

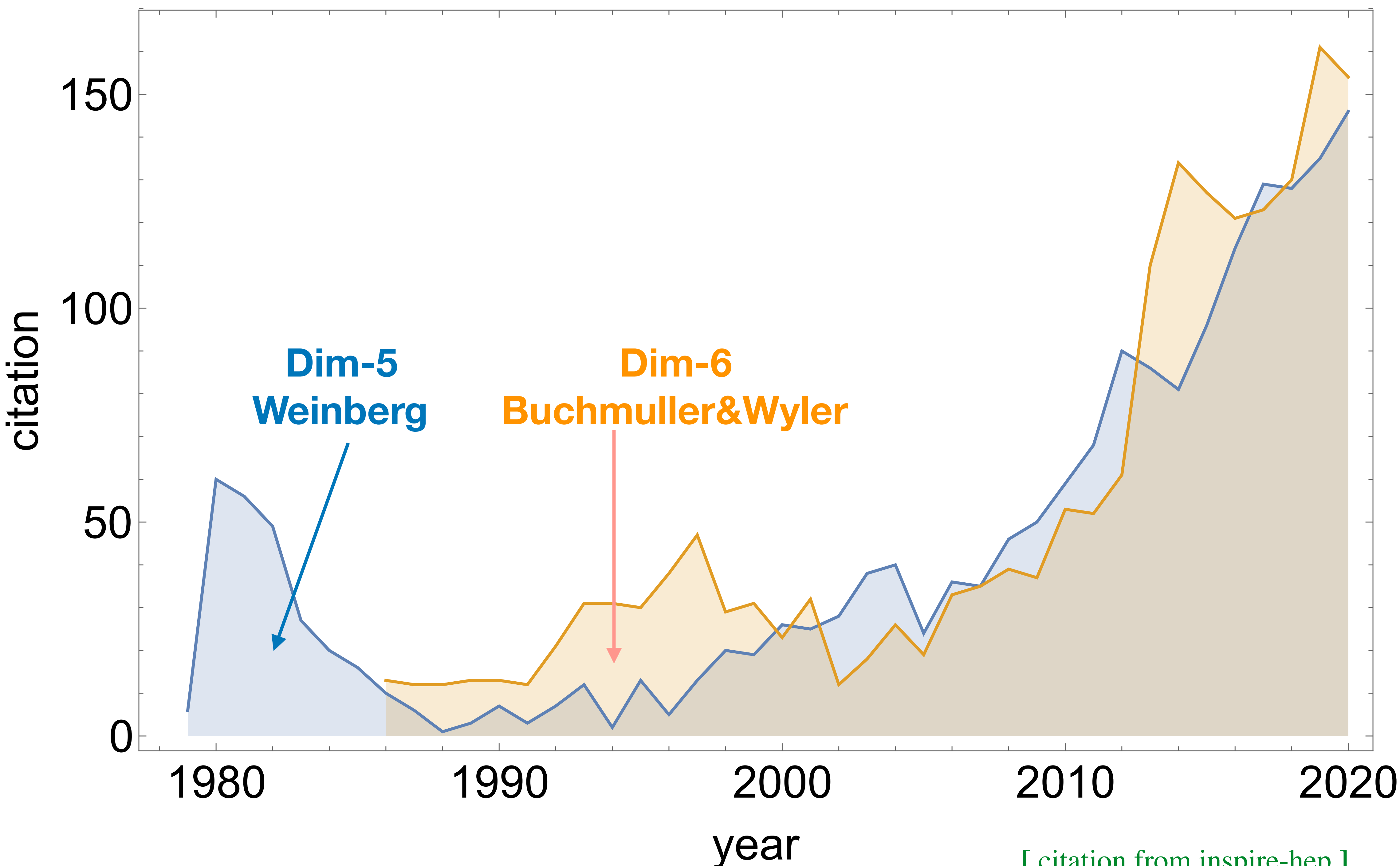


$$\mathscr{L}_{\rm EFT} = \mathscr{L}_{\mathscr{D}\leq 4} + \frac{\mathscr{L}_5}{\Lambda} + \frac{\mathscr{L}_6}{\Lambda^2} + \dots$$

SMEFT provides systematical parametrization of

… all possible new physics!

Post-LHC Run II Era

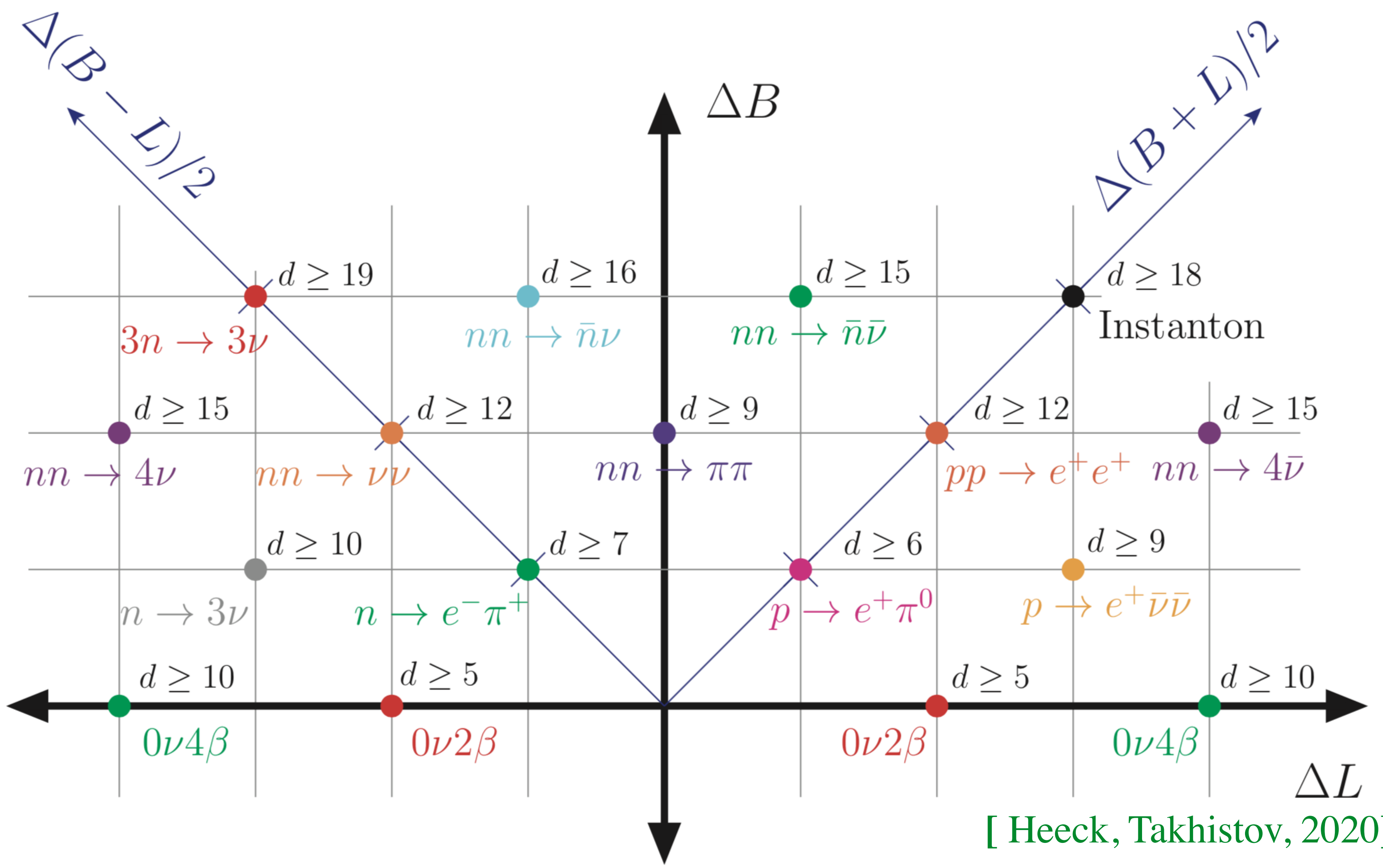


[citation from inspire-hep]

Higher Dim Operators?

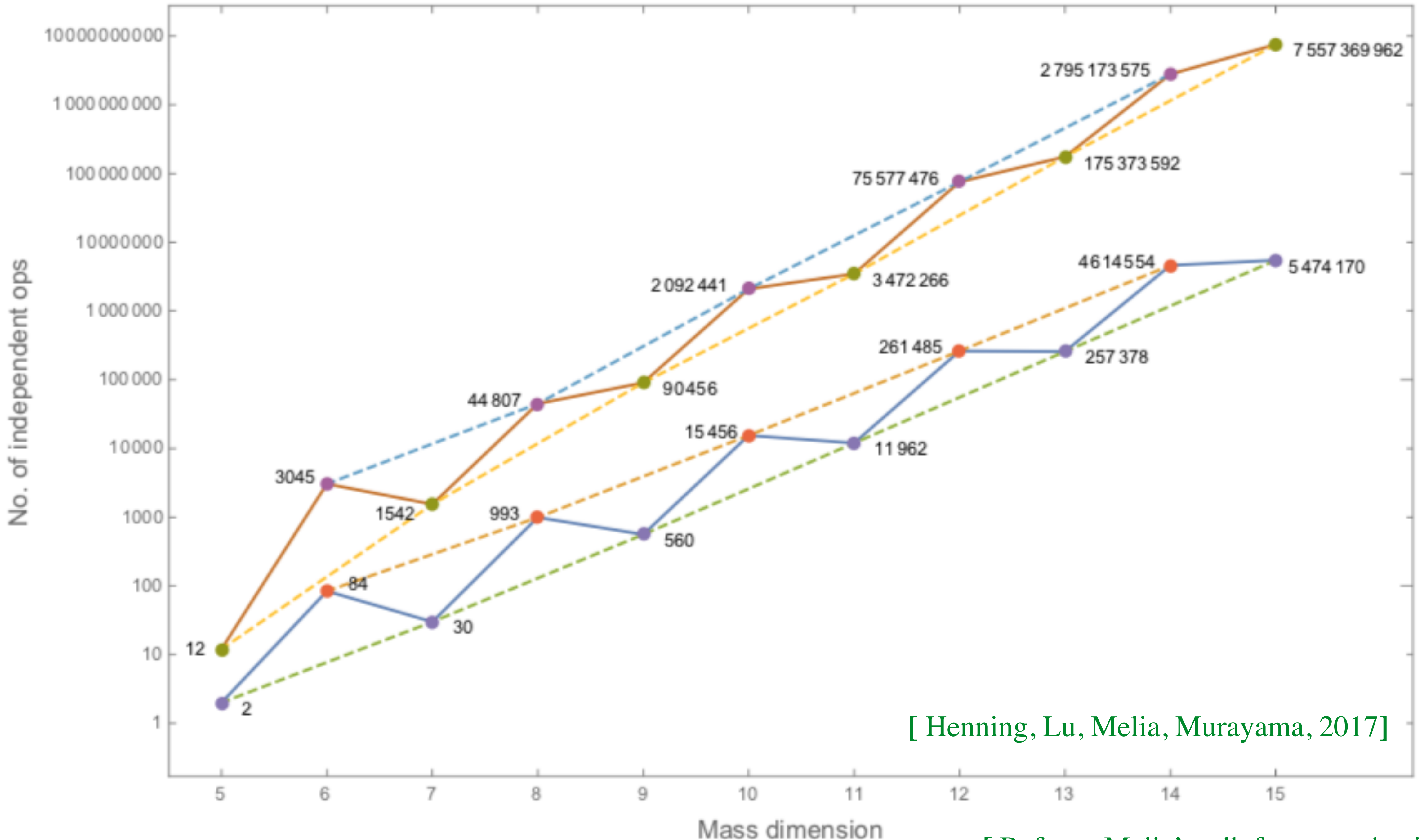
Evidences of new physics: neutrino masses and baryon asymmetry

B and L violation



[Heeck, Takhistov, 2020]

Hilbert Series Counting



[Refer to Melia's talk for more details]

Higher Dim Operators!?

Given numbers of independent operators

does not mean we know explicit form of operators!

Derivatives

$BWHH^\dagger D^2$

2

$$(D^2H^\dagger)HB_{L\mu\nu}W_L^{\mu\nu},\ (D^\mu D_\nu H^\dagger)HB_{L\mu\rho}W_L^{\nu\rho},\ (D_\nu D^\mu H^\dagger)HB_{L\mu\rho}W_L^{\nu\rho},\ (D_\mu H^\dagger)(D^\mu H)B_{L\nu\rho}W_L^{\nu\rho},$$
$$(D_\mu H^\dagger)(D^\nu H)B_{L\nu\rho}W_L^{\mu\rho},\ (D^\nu H^\dagger)(D_\mu H)B_{L\nu\rho}W_L^{\mu\rho},\ (D_\mu H^\dagger)H(D^\mu B_{L\nu\rho})W_L^{\nu\rho},\ (D_\mu H^\dagger)H(D^\nu B_{L\nu\rho})W_L^{\mu\rho},$$
$$(D^\nu H^\dagger)H(D_\mu B_{L\nu\rho})W_L^{\mu\rho},\ (D_\mu H^\dagger)HB_{L\nu\rho}(D^\mu W_L^{\nu\rho}),\ (D_\mu H^\dagger)HB_{L\nu\rho}(D^\nu W_L^{\mu\rho}),\ (D^\nu H^\dagger)HB_{L\nu\rho}(D_\mu W_L^{\mu\rho}),$$
$$H^\dagger(D^2H)B_{L\mu\nu}W_L^{\mu\nu},\ H^\dagger(D^\mu D_\nu H)B_{L\mu\rho}W_L^{\nu\rho},\ H^\dagger(D_\nu D^\mu H)B_{L\mu\rho}W_L^{\nu\rho},\ H^\dagger(D^\mu H)(D_\mu B_{L\nu\rho})W_L^{\nu\rho},$$
$$H^\dagger(D^\nu H)(D_\mu B_{L\nu\rho})W_L^{\mu\rho},\ H^\dagger(D_\mu H)(D^\nu B_{L\nu\rho})W_L^{\mu\rho},\ H^\dagger(D^\mu H)B_{L\nu\rho}(D_\mu W_L^{\nu\rho}),\ H^\dagger(D^\nu H)B_{L\nu\rho}(D_\mu W_L^{\mu\rho}),$$
$$H^\dagger(D_\mu H)B_{L\nu\rho}(D^\nu W_L^{\mu\rho}),\ H^\dagger H(D^2B_{L\mu\nu})W_L^{\mu\nu},\ H^\dagger H(D^\mu D_\nu B_{L\mu\rho})W_L^{\nu\rho},\ H^\dagger H(D_\nu D^\mu B_{L\mu\rho})W_L^{\nu\rho},$$
$$H^\dagger H(D^\mu B_{L\nu\rho})(D_\mu W_L^{\nu\rho}),\ H^\dagger H(D^\nu B_{L\nu\rho})(D_\mu W_L^{\mu\rho}),\ H^\dagger H(D_\mu B_{L\nu\rho})(D^\nu W_L^{\mu\rho}),\ H^\dagger HB_{L\mu\nu}(D^2W_L^{\mu\nu}),$$
$$H^\dagger HB_{L\mu\rho}(D^\mu D_\nu W_L^{\nu\rho}),\ H^\dagger HB_{L\mu\rho}(D_\nu D^\mu W_L^{\nu\rho}). \tag{14}$$

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Which 2 should be picked up?

Repeated fields

$QQQL$

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$$Q^{qqq\ell}_{prst} = C^{prst} \begin{matrix} \epsilon^{abc}\epsilon^{ik}\epsilon^{jl}(L_{pi}Q_{raj})(Q_{sbk}Q_{tcl}) \\ \epsilon^{abc}\epsilon^{ik}\epsilon^{jl}(L_{pi}Q_{sbk})(Q_{raj}Q_{tcl}) \\ \epsilon^{abc}\epsilon^{ij}\epsilon^{kl}(L_{pi}Q_{sbk})(Q_{raj}Q_{tcl}) \\ \epsilon^{abc}\epsilon^{ij}\epsilon^{kl}(L_{pi}Q_{raj})(Q_{sbk}Q_{tcl}) \end{matrix} \qquad p,r,s,t = 1,2,3$$

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What flavor relations should be imposed?

Operator as Group Invariant

Start from the effective operator contributing to muon g-2:

Gauge invariance: gauge factor

$$2 \otimes 1 \otimes 2 \otimes 3 = 1 \oplus 3 \oplus 3 \oplus 5$$

$$(\bar{l}_p \sigma^{\mu\nu} e_r) \tau^I \varphi W^I_{\mu\nu} \qquad (\bar{l}_p \sigma^{\mu\nu} e_r) \varphi B_{\mu\nu}$$

Lorentz invariance: Lorentz indices contracted in pair

$$\left(\frac{1}{2},0\right) \otimes \left(\frac{1}{2},0\right) \otimes (0,0) \otimes [(1,0) \oplus (0,1)] = (0,0) \oplus (1,0) \oplus (2,0) \oplus (1,1) \oplus (1,0) \oplus (0,1)$$

Redundancies: equation of motion, integration by part, covariant derivative commutator

$$X^{\mu\nu} \bar{\psi} \gamma_\mu D_\nu \psi \qquad \varphi \bar{\psi} \sigma^{\mu\nu} D_\mu D_\nu \psi \qquad (D^\mu \varphi) \bar{\psi} D_\mu \psi$$

$$\varphi \bar{l}_p \sigma^{\mu\nu} D_\mu D_\nu e_r \qquad (\bar{l}_p D_\mu e_r) D^\mu \varphi$$

Operator as On-shell Amplitude

EFT operator = Contact amplitude = Group invariant + little group scaling

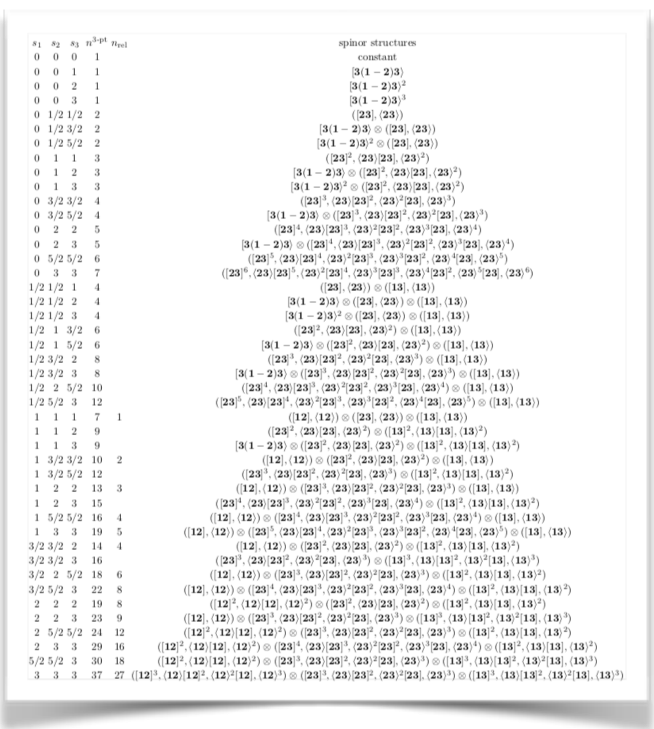
ψ_α	λ_α
$\psi^\dagger_{\dot\alpha}$	$\tilde\lambda_{\dot\alpha}$
$F^-_{\alpha\beta} = F_{\mu\nu}\sigma^{\mu\nu}_{\alpha\beta}$	$\lambda_\alpha\lambda_\beta$
$F^+_{\dot\alpha\dot\beta} = F_{\mu\nu}\bar\sigma^{\mu\nu}_{\dot\alpha\dot\beta}$	$\tilde\lambda_\alpha\tilde\lambda_\beta$

$$\langle ij\rangle = \tilde\lambda_i \epsilon \tilde\lambda_j$$

$$[ij] = \lambda_i \epsilon \lambda_j$$

$$(\bar l_p \sigma^{\mu\nu} e_r)\tau^I \varphi W^I_{\mu\nu} \longrightarrow \lambda_{1\alpha}\lambda_{2\beta}\lambda_4^\alpha\lambda_4^\beta\tau^I \longrightarrow [14][24]\tau^I$$

[Shadmi, Weiss, 2018]
[Ma, Shu, Xiao, 2019]
[Durieux, Kitahara, Shadmi, Weiss, 2019]
[Falkowski, Machado, 2019]
[Durieux, Kitahara, Machado, Shadmi, Weiss, 2020]



Stripped contact term bases for all 4-point amplitudes

[Refer to Machado's talk for more details]

All 3-particle massless amplitudes except F^3 vanish at on-shell

~~$X^{\mu\nu}\bar\psi\gamma_\mu D_\nu\psi$~~ ~~$\varphi\bar\psi\sigma^{\mu\nu}D_\mu D_\nu\psi$~~ ~~$(D^\mu\varphi)\bar\psi D_\mu\psi$~~

Operator as On-shell Amplitude

Currently hard to systematically construct more than 4-particle amplitude

[Durieux, Kitahara, Machado, Shadmi, Weiss, 2020]

Dim-8 SMEFT

$N+h$ \ $N-h$	0	2	4	6	8
0					F^4
2				$F^2\bar\psi\psi D$, $\psi^4 D^2$, $F\psi^2\phi D^2$, $F^2\phi^2 D^2$	$F\psi^4$, $F^2\psi^2\phi$, $F^3\phi^2$
4			$\bar F^2 F^2$, $\bar F F\bar\psi\psi D$, $\bar\psi^2\psi^2 D^2$, $\bar F\psi^2\phi D^2$, $\bar F F\phi^2 D^2$, $\phi^4 D^4$, $\bar\psi\psi\phi^2 D^3$	$F\bar\psi^2\psi^2$, $F^2\bar\psi^2\phi$, $\bar\psi\psi^3\phi D$, $F\bar\psi\psi\phi^2 D$, $\psi^2\phi^3 D^2$, $F\phi^4 D^2$	$\psi^4\phi^2$, $F\psi^2\phi^3$, $F^2\phi^4$
6		$\bar F^2\bar\psi\psi D$, $\bar\psi^4 D^2$, $\bar F\bar\psi^2\phi D^2$, $\bar F^2\phi^2 D^2$	$\bar F\bar\psi^2\psi^2$, $\bar F^2\psi^2\phi$, $\bar\psi^3\psi\phi D$, $\bar F\bar\psi\psi\phi^2 D$, $\bar\psi^2\phi^3 D^2$, $\bar F\phi^4 D^2$	$\bar\psi^2\psi^2\phi^2$, $\bar\psi\psi\phi^4 D$, $\phi^6 D^2$	$\psi^2\phi^5$
8	$\bar F^4$	$\bar F\bar\psi^4$, $\bar F^2\bar\psi^2\phi$, $\bar F^3\phi^2$	$\bar\psi^4\phi^2$, $\bar F\bar\psi^2\phi^3$, $\bar F^2\phi^4$	$\bar\psi^2\phi^5$	ϕ^8

N = 4

N = 5

N = 6

N = 7

N = 8

g-2 dim-8: $(\bar l_p\sigma^{\mu\nu}e_r)\tau^I\varphi W^I_{\mu\nu}$

Add $H^\dagger H$ → $\left(\tau^I\right)^k_j W^I_{\mu\nu}\left(e_{\mathbb{C}p}\sigma^{\mu\nu}L_{rk}\right)H^{\dagger j}\left(H^\dagger H\right)$

Add D^2 → $\left(\tau^I\right)^i_j W^I_{\mu\lambda}\left(e_{\mathbb{C}p}\sigma^{\nu\lambda}L_{ri}\right)D^\mu D_\nu H^{\dagger j}$

Operator as Spinor Tensor

Consider dim-8 g-2 operator with derivatives

$$(\bar{l}_p \sigma^{\mu\nu} e_r)\tau^I \varphi W^I_{\mu\nu}$$

Ways to add D^2 →

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$$\begin{gathered}
D^\mu D^\nu W_{\mu\nu}(e_{cp}L_r)H^\dagger,\ D^\mu W_{\mu\nu}(D^\nu e_{cp}L_r)H^\dagger,\ D^\mu W_{\mu\nu}(e_{cp}D^\nu L_r)H^\dagger,\ D^\mu W_{\mu\nu}(e_{cp}L_r)D^\nu H^\dagger,\\
W_{\mu\nu}(D^\mu D^\nu e_{cp}L_r)H^\dagger,\ W_{\mu\nu}(D^\mu e_{cp}D^\nu L_r)H^\dagger,\ W_{\mu\nu}(D^\mu e_{cp}L_r)D^\nu H^\dagger,\ W_{\mu\nu}(e_{cp}D^\mu D^\nu L_r)H^\dagger,\\
W_{\mu\nu}(e_{cp}D^\mu L_r)D^\nu H^\dagger,\ W_{\mu\nu}(e_{cp}L_r)D^\mu D^\nu H^\dagger,\ D^\mu D_\nu W_{\mu\lambda}(e_{cp}\sigma^{\nu\lambda}L_r)H^\dagger,\ D_\nu D^\mu W_{\mu\lambda}(e_{cp}\sigma^{\nu\lambda}L_r)H^\dagger,\\
D^\mu W_{\mu\lambda}(D_\nu e_{cp}\sigma^{\nu\lambda}L_r)H^\dagger,\ D_\nu W_{\mu\lambda}(D^\mu e_{cp}\sigma^{\nu\lambda}L_r)H^\dagger,\ D^\mu W_{\mu\lambda}(e_{cp}\sigma^{\nu\lambda}D_\nu L_r)H^\dagger,\ D_\nu W_{\mu\lambda}(e_{cp}\sigma^{\nu\lambda}D^\mu L_r)H^\dagger,\\
D^\mu W_{\mu\lambda}(e_{cp}\sigma^{\nu\lambda}L_r)D_\nu H^\dagger,\ D_\nu W_{\mu\lambda}(e_{cp}\sigma^{\nu\lambda}L_r)D^\mu H^\dagger,\ W_{\mu\lambda}(D^\mu D_\nu e_{cp}\sigma^{\nu\lambda}L_r)H^\dagger,\ W_{\mu\lambda}(D_\nu D^\mu e_{cp}\sigma^{\nu\lambda}L_r)H^\dagger,\\
W_{\mu\lambda}(D^\mu e_{cp}\sigma^{\nu\lambda}D_\nu L_r)H^\dagger,\ W_{\mu\lambda}(D_\nu e_{cp}\sigma^{\nu\lambda}D^\mu L_r)H^\dagger,\ W_{\mu\lambda}(D^\mu e_{cp}\sigma^{\nu\lambda}L_r)D_\nu H^\dagger,\ W_{\mu\lambda}(D_\nu e_{cp}\sigma^{\nu\lambda}L_r)D^\mu H^\dagger,\\
W_{\mu\lambda}(e_{cp}\sigma^{\nu\lambda}D^\mu D_\nu L_r)H^\dagger,\ W_{\mu\lambda}(e_{cp}\sigma^{\nu\lambda}D_\nu D^\mu L_r)H^\dagger,\ W_{\mu\lambda}(e_{cp}\sigma^{\nu\lambda}D^\mu L_r)D_\nu H^\dagger,\ W_{\mu\lambda}(e_{cp}\sigma^{\nu\lambda}D_\nu L_r)D^\mu H^\dagger,\\
W_{\mu\lambda}(e_{cp}\sigma^{\nu\lambda}L_r)D^\mu D_\nu H^\dagger,\ W_{\mu\lambda}(e_{cp}\sigma^{\nu\lambda}L_r)D_\nu D^\mu H^\dagger,\ \{W\to\tilde{W}\}
\end{gathered}$$

Each field belongs to a SL(2,C) irrep

$$H_i \in (0,0) \quad \psi_\alpha \in (1/2,0) \quad F_{\mathrm{L}\alpha\beta} = \frac{i}{2}F_{\mu\nu}\sigma^{\mu\nu}_{\alpha\beta} \in (1,0) \quad D_{\alpha\dot{\alpha}} = D_\mu \sigma^\mu_{\alpha\dot{\alpha}} \in (1/2,1/2),$$

Operator with explicit spinor indices

$$\epsilon_{\alpha_1\alpha_3}\epsilon_{\alpha_1\alpha_3}\epsilon_{\alpha_2\alpha_4}\epsilon^{\dot{\alpha}_3\dot{\alpha}_4} F_1^{\alpha_1^2}\psi_2^{\alpha_2}(D\psi_3)^{\alpha_3^2}_{\dot{\alpha}_3}(D\phi_4)^{\alpha_4}_{\dot{\alpha}_4}$$

$$F_1^{\alpha\beta}\psi_2^{\gamma}(D\psi_3)_{\alpha\beta\dot{\alpha}}(D\phi_4)_\gamma{}^{\dot{\alpha}}$$

How to obtain independent operator with derivatives systematically?

Equation of Motion (EOM)

For fields with derivatives, symmetric and antisymmetric indices:

$$(D\psi)_{\alpha\beta\dot{\alpha}} = -\frac{1}{2}\epsilon_{\alpha\beta}(\not{D}\psi)_{\dot{\alpha}} + \frac{1}{2}(D\psi)_{(\alpha\beta)\dot{\alpha}}$$

$\left(\frac{1}{2},\frac{1}{2}\right) \times \left(\frac{1}{2},0\right)$ **(0,1/2)** **(1,1/2)**

$$\left(\frac{1}{2},\frac{1}{2}\right) \otimes \left(\frac{1}{2},0\right) = \left(0,\frac{1}{2}\right) \oplus \left(1,\frac{1}{2}\right)$$

$$(D^2\phi)_{\alpha\beta\dot{\alpha}\dot{\beta}} = \frac{1}{2}\epsilon_{\alpha\beta}\epsilon_{\dot{\alpha}\dot{\beta}}D^{\mu}D_{\mu}\phi - \frac{i}{4}\epsilon_{\dot{\alpha}\dot{\beta}}\sigma^{\mu\nu}_{\alpha\beta}[D_{\mu},D_{\nu}]\phi - \frac{i}{4}\epsilon_{\alpha\beta}\bar{\sigma}^{\mu\nu}_{\dot{\alpha}\dot{\beta}}[D_{\mu},D_{\nu}]\phi + \frac{1}{4}(D^2\phi)_{(\alpha\beta)(\dot{\alpha}\dot{\beta})}$$

$\left(\frac{1}{2},\frac{1}{2}\right) \times \left(\frac{1}{2},\frac{1}{2}\right)$ **(0,0)** **(1,0)** **(0,1)** **(1,1)**

Only take the symmetric indices part for field with derivatives

$$D^w\Psi \in \left(j_l + \frac{w}{2}, j_r + \frac{w}{2}\right) \oplus \text{~~lower weights~~}$$

with totally symmetric spinor indices

EOM removed by taking highest weight!

Covariant derivative commutator, Bianchi identity also removed

Operator as Spinor Tensor

Any operator can be written with totally symmetric spinor indices:

$$\mathcal{O} = (\epsilon^{\alpha_i \alpha_j})^n (\tilde{\epsilon}_{\dot{\alpha}_i \dot{\alpha}_j})^{\tilde{n}} \prod_{i=1}^{N} (D^{r_i - |h_i|} \Phi_i)^{\dot{\alpha}_i^{r_i + h_i}}_{\alpha_i^{r_i - h_i}}$$

$$\epsilon_{\alpha_1 \alpha_3} \epsilon_{\alpha_1 \alpha_3} \epsilon_{\alpha_2 \alpha_4} \epsilon^{\dot{\alpha}_3 \dot{\alpha}_4} F_1^{\alpha_1^2} \psi_2^{\alpha_2} (D\psi_3)^{\alpha_3^2}_{\dot{\alpha}_3} (D\phi_4)^{\alpha_4}_{\dot{\alpha}_4}$$

Transformation under SL(2,C) x SU(N)

$$\epsilon^{\alpha_i \alpha_j} \to \sum_{k,l} \mathcal{U}^i_k \mathcal{U}^j_l \epsilon^{\alpha_k \alpha_l}, \quad \tilde{\epsilon}_{\dot{\alpha}_i \dot{\alpha}_j} \to \sum_{k,l} \mathcal{U}^{\dagger k}_{\ i} \mathcal{U}^{\dagger l}_{\ j} \tilde{\epsilon}_{\dot{\alpha}_k \dot{\alpha}_l}.$$

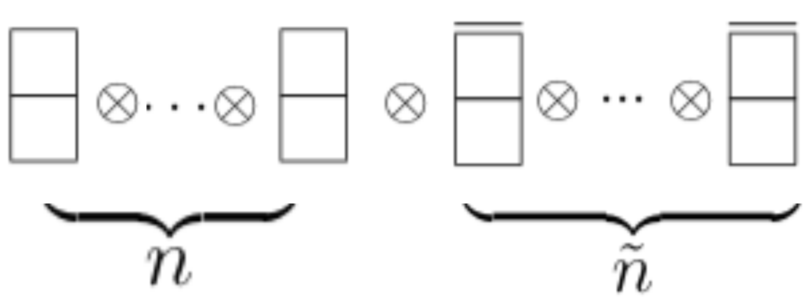
$i, j, k, l = 1 \text{ to } N$

 $= [1^2]$

 $= [1^{N-2}]$

$\epsilon^{\alpha_i \alpha_j}$

$\mathcal{E}^{ijk_1,\ldots,k_{N-2}} \tilde{\epsilon}_{\dot{\alpha}_i \dot{\alpha}_j}$



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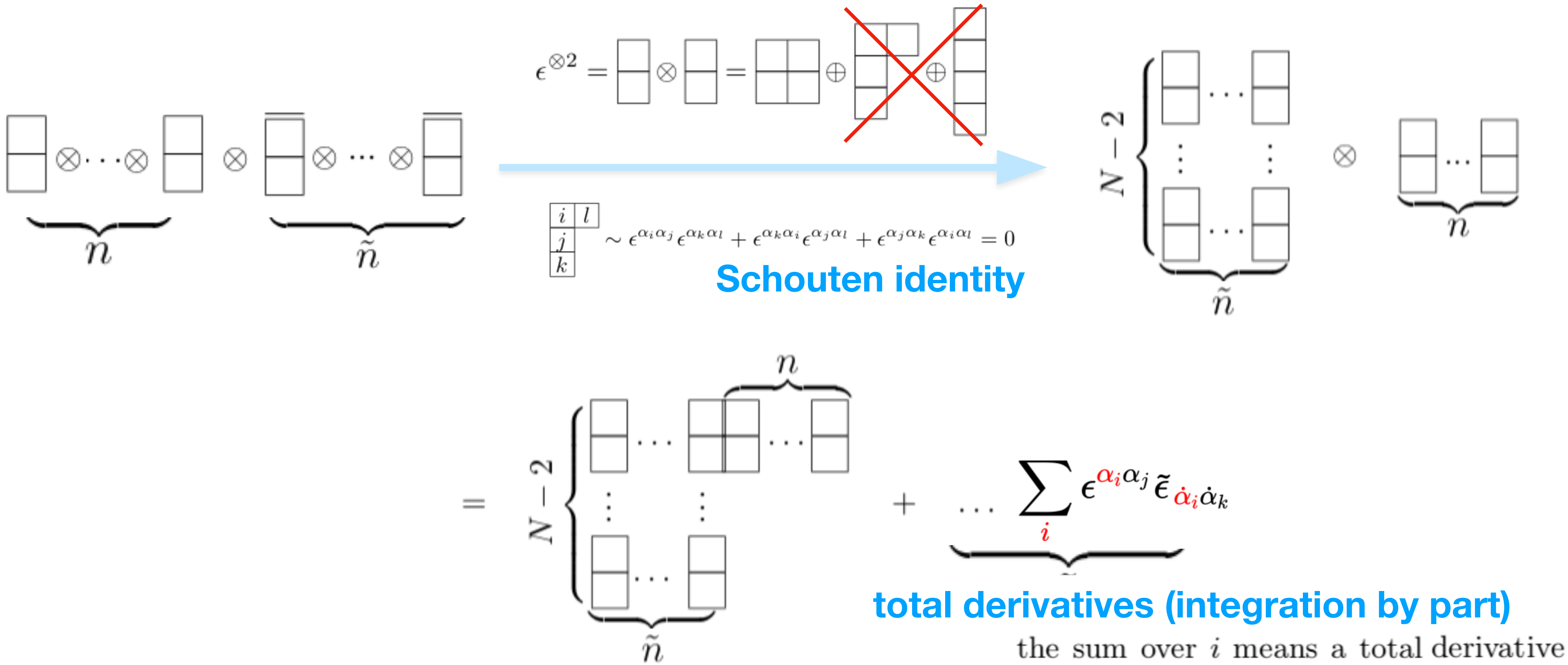
Total Derivatives



[Such Young diagram also obtained from conformal K harmonics]

[Henning, Melia, 2019]

Differently we obtain Young diagram using epsilon tensor transformation

No need conformal symmetry!

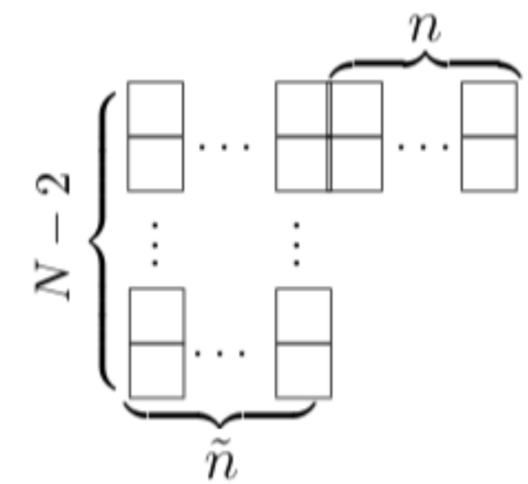
Independent Lorentz Structure

To obtain independent operator, we invent a new Young diagram filling procedure!

Filling rules on semi-standard Young tableau (SSYT) with given class

$$\{\underbrace{1,\ldots,1}_{\#1},\underbrace{2,\ldots,2}_{\#2},\ldots\} \qquad \#i = \tilde{n} - 2h_i$$

$$Y_{N,n,\tilde{n}} =$$



Fock's condition removes redundancy

$$\boxed{(\tau^I)^i_j\, W^I_{\mu\lambda}\left(e_{\mathbb{C}p}\sigma^{\nu\lambda}L_{ri}\right) D^\mu D_\nu H^{\dagger j}}$$

$$(\tilde{n}=1, n=3)$$

$$\#1 = 3, \#2 = \#3 = 2, \#4 = 1.$$

Basis
- **YT method guarantees independence!**
- **Filling all SSYT guarantees completeness!**

New filling: any operator could be converted to this basis

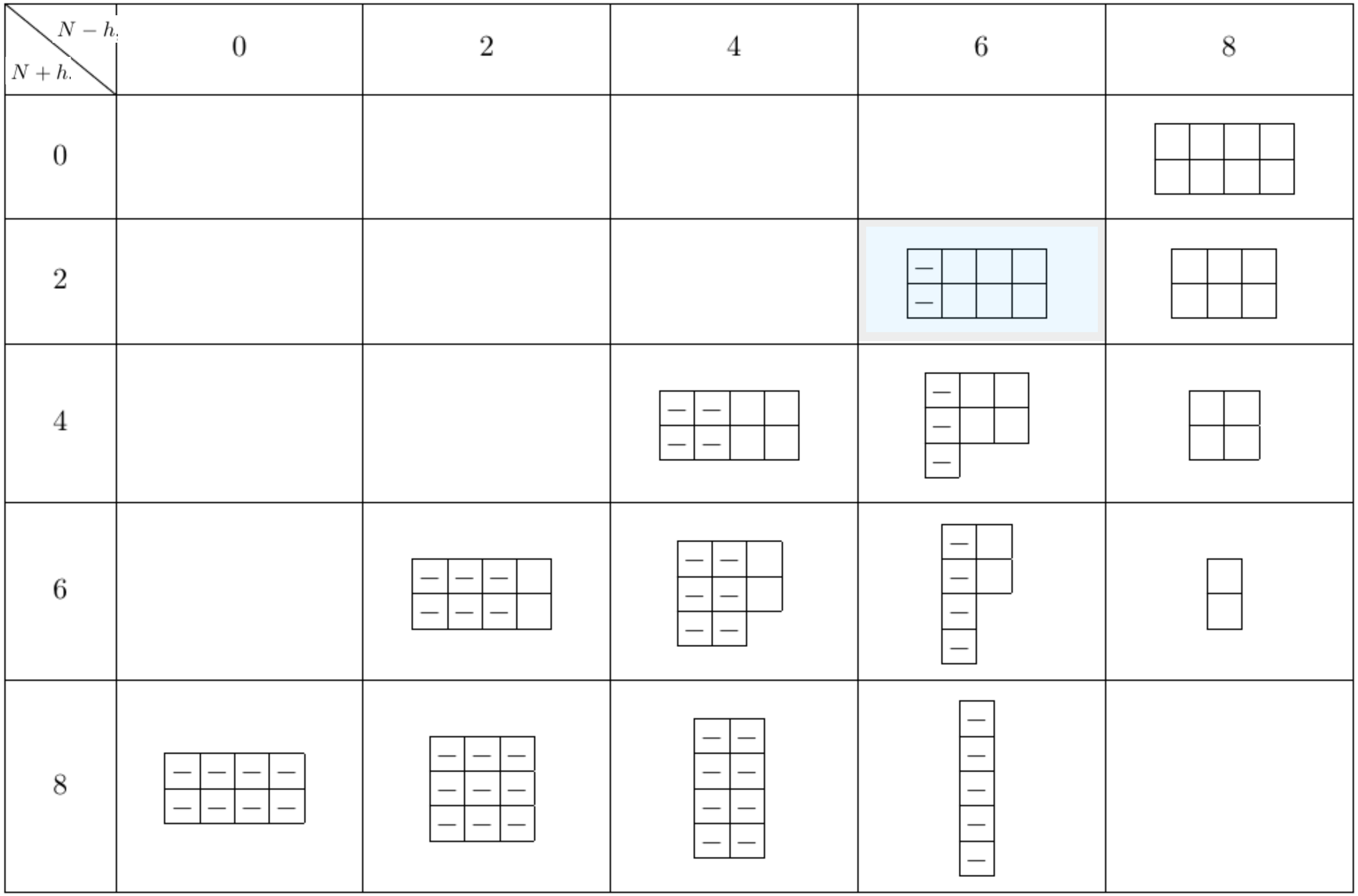
1	1	1	2
2	3	3	4

$$\epsilon_{\alpha_1\alpha_3}\epsilon_{\alpha_1\alpha_3}\epsilon_{\alpha_2\alpha_4}\epsilon^{\dot\alpha_3\dot\alpha_4} \qquad F_{\rm L1}^{\alpha\beta}\psi_2^\gamma\,(D\psi_3)_{\alpha\beta\dot\alpha}\,(D\phi_4)_\gamma{}^{\dot\alpha} \qquad \langle 13\rangle\langle 13\rangle\langle 24\rangle[34]$$

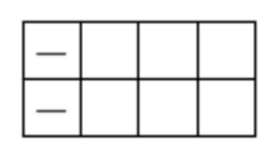
1	1	1	3
2	2	3	4

$$\epsilon_{\alpha_1\alpha_2}\epsilon_{\alpha_1\alpha_3}\epsilon_{\alpha_3\alpha_4}\epsilon^{\dot\alpha_3\dot\alpha_4} \qquad F_{\rm L1}^{\alpha\beta}\psi_{2\alpha}\,(D\psi_3)_\beta{}^\gamma{}_{\dot\alpha}\,(D\phi_4)_\gamma{}^{\dot\alpha} \qquad \langle 12\rangle\langle 13\rangle\langle 34\rangle[34]$$

Dim-8 Young Diagrams



Different filling corresponds different operator!



$F^2\bar{\psi}\psi D$, $\psi^4 D^2$, $F\psi^2\phi D^2$, $F^2\phi^2 D^2$

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Different Operator with Same YD

$(\tilde{n}=1, n=3)$

$W e_{\mathbb{C}} L H^{\dagger} D^2$ $\#1=3, \#2=\#3=2, \#4=1.$

1	1	1	2
2	3	3	4

1	1	1	3
2	2	3	4

$\epsilon_{\alpha_1\alpha_3}\epsilon_{\alpha_1\alpha_3}\epsilon_{\alpha_2\alpha_4}\epsilon^{\dot{\alpha}_3\dot{\alpha}_4}$ $\epsilon_{\alpha_1\alpha_2}\epsilon_{\alpha_1\alpha_3}\epsilon_{\alpha_3\alpha_4}\epsilon^{\dot{\alpha}_3\dot{\alpha}_4}$

$F_{\mathrm{L1}}^{\alpha\beta}\psi_2^{\gamma}\left(D\psi_3\right)_{\alpha\beta\dot{\alpha}}\left(D\phi_4\right)_{\gamma}{}^{\dot{\alpha}}$ $F_{\mathrm{L1}}^{\alpha\beta}\psi_{2\alpha}\left(D\psi_3\right)_{\beta}{}^{\gamma}{}_{\dot{\alpha}}\left(D\phi_4\right)_{\gamma}{}^{\dot{\alpha}}$

$B W H H^{\dagger} D^2$ $\#1=3, \#2=3, \#3=1, \#4=1$

1	1	1	3
2	2	2	4

1	1	1	2
2	2	3	4

$\epsilon_{\dot{\alpha}_3\dot{\alpha}_4}\epsilon^{\alpha_1\alpha_2}\epsilon^{\alpha_1\alpha_2}\epsilon^{\alpha_3\alpha_4}$ $\epsilon_{\dot{\alpha}_3\dot{\alpha}_4}\epsilon^{\alpha_1\alpha_2}\epsilon^{\alpha_1\alpha_3}\epsilon^{\alpha_2\alpha_4}$

$B_L{}^{\alpha\beta}W_{L\alpha\beta}\left(DH^{\dagger}\right)^{\gamma}{}_{\dot{\alpha}}\left(DH\right)_{\gamma}{}^{\dot{\alpha}},$ $B_L{}^{\alpha\beta}W_{L\alpha}{}^{\gamma}\left(DH^{\dagger}\right)_{\beta\dot{\alpha}}\left(DH\right)_{\gamma}{}^{\dot{\alpha}}$

Traditional vs Young Tensor

$(\tilde{n}=1, n=3)$

$W e_{\mathbb{C}} L H^\dagger D^2$ $\qquad$ $\#1=3, \#2=\#3=2, \#4=1.$

1	1	1	2
2	3	3	4

$\epsilon_{\alpha_1\alpha_3}\epsilon_{\alpha_1\alpha_3}\epsilon_{\alpha_2\alpha_4}\epsilon^{\dot\alpha_3\dot\alpha_4}$

$F_{\rm L1}^{\alpha\beta}\psi_2^{\gamma}(D\psi_3)_{\alpha\beta\dot\alpha}(D\phi_4)_{\gamma}{}^{\dot\alpha}$

1	1	1	3
2	2	3	4

$\epsilon_{\alpha_1\alpha_2}\epsilon_{\alpha_1\alpha_3}\epsilon_{\alpha_3\alpha_4}\epsilon^{\dot\alpha_3\dot\alpha_4}$

$F_{\rm L1}^{\alpha\beta}\psi_{2\alpha}(D\psi_3)_{\beta}{}^{\gamma}{}_{\dot\alpha}(D\phi_4)_{\gamma}{}^{\dot\alpha}$

$BWHH^\dagger D^2$ $\qquad$ $\#1=3, \#2=3, \#3=1, \#4=1$

1	1	1	3
2	2	2	4

$\epsilon_{\dot\alpha_3\dot\alpha_4}\epsilon^{\alpha_1\alpha_2}\epsilon^{\alpha_1\alpha_2}\epsilon^{\alpha_3\alpha_4}$

$B_L{}^{\alpha\beta}W_{L\alpha\beta}(DH^\dagger)^{\gamma}{}_{\dot\alpha}(DH)_{\gamma}{}^{\dot\alpha}$

2

1	1	1	2
2	2	3	4

$\epsilon_{\dot\alpha_3\dot\alpha_4}\epsilon^{\alpha_1\alpha_2}\epsilon^{\alpha_1\alpha_3}\epsilon^{\alpha_2\alpha_4}$

$B_L{}^{\alpha\beta}W_{L\alpha}{}^{\gamma}(DH^\dagger)_{\beta\dot\alpha}(DH)_{\gamma}{}^{\dot\alpha}$

Young tensor method (No need EoM&IBP)

Traditional method

$BWHH^\dagger D^2$

[Hays, Martin, Sanz, Setford, 2018]

$$(D^2H^\dagger)HB_{L\mu\nu}W_L^{\mu\nu},\ (D^\mu D_\nu H^\dagger)HB_{L\mu\rho}W_L^{\nu\rho},\ (D_\nu D^\mu H^\dagger)HB_{L\mu\rho}W_L^{\nu\rho},\ (D_\mu H^\dagger)(D^\mu H)B_{L\nu\rho}W_L^{\nu\rho},$$
$$(D_\mu H^\dagger)(D^\nu H)B_{L\nu\rho}W_L^{\mu\rho},\ (D^\nu H^\dagger)(D_\mu H)B_{L\nu\rho}W_L^{\mu\rho},\ (D_\mu H^\dagger)H(D^\mu B_{L\nu\rho})W_L^{\nu\rho},\ (D_\mu H^\dagger)H(D^\nu B_{L\nu\rho})W_L^{\mu\rho},$$
$$(D^\nu H^\dagger)H(D_\mu B_{L\nu\rho})W_L^{\mu\rho},\ (D_\mu H^\dagger)HB_{L\nu\rho}(D^\mu W_L^{\nu\rho}),\ (D_\mu H^\dagger)HB_{L\nu\rho}(D^\nu W_L^{\mu\rho}),\ (D^\nu H^\dagger)HB_{L\nu\rho}(D_\mu W_L^{\mu\rho}),$$
$$H^\dagger(D^2H)B_{L\mu\nu}W_L^{\mu\nu},\ H^\dagger(D^\mu D_\nu H)B_{L\mu\rho}W_L^{\nu\rho},\ H^\dagger(D_\nu D^\mu H)B_{L\mu\rho}W_L^{\nu\rho},\ H^\dagger(D^\mu H)(D_\mu B_{L\nu\rho})W_L^{\nu\rho},$$
$$H^\dagger(D^\nu H)(D_\mu B_{L\nu\rho})W_L^{\mu\rho},\ H^\dagger(D_\mu H)(D^\nu B_{L\nu\rho})W_L^{\mu\rho},\ H^\dagger(D^\mu H)B_{L\nu\rho}(D_\mu W_L^{\nu\rho}),\ H^\dagger(D^\nu H)B_{L\nu\rho}(D_\mu W_L^{\mu\rho}),$$
$$H^\dagger(D_\mu H)B_{L\nu\rho}(D^\nu W_L^{\mu\rho}),\ H^\dagger H(D^2B_{L\mu\nu})W_L^{\mu\nu},\ H^\dagger H(D^\mu D_\nu B_{L\mu\rho})W_L^{\nu\rho},\ H^\dagger H(D_\nu D^\mu B_{L\mu\rho})W_L^{\nu\rho},$$
$$H^\dagger H(D^\mu B_{L\nu\rho})(D_\mu W_L^{\nu\rho}),\ H^\dagger H(D^\nu B_{L\nu\rho})(D_\mu W_L^{\mu\rho}),\ H^\dagger H(D_\mu B_{L\nu\rho})(D^\nu W_L^{\mu\rho}),\ H^\dagger HB_{L\mu\nu}(D^2W_L^{\mu\nu}),$$
$$H^\dagger HB_{L\mu\rho}(D^\mu D_\nu W_L^{\nu\rho}),\ H^\dagger HB_{L\mu\rho}(D_\nu D^\mu W_L^{\nu\rho}). \qquad (14)$$

30

EOM

$$(DH^\dagger)_{\alpha\dot\alpha}(DH)_{\beta\dot\beta}B_{L\{\gamma\delta\}}W_{L\{\xi\eta\}}\ \epsilon^{\dot\alpha\dot\beta}\epsilon^{\alpha\beta}\epsilon^{\gamma\xi}\epsilon^{\delta\eta}$$
$$(DH^\dagger)_{\alpha\dot\alpha}(DH)_{\beta\dot\beta}B_{L\{\gamma\delta\}}W_{L\{\xi\eta\}}\ \tfrac{1}{2}\epsilon^{\dot\alpha\dot\beta}\epsilon^{\delta\xi}(\epsilon^{\alpha\gamma}\epsilon^{\beta\eta}+\epsilon^{\beta\gamma}\epsilon^{\alpha\eta})$$
$$(DH^\dagger)_{\alpha\dot\alpha}\,H\,(DB_L)_{\{\beta\gamma\delta\},\dot\beta}W_{L\{\xi\eta\}}\ \epsilon^{\dot\alpha\dot\beta}\epsilon^{\alpha\beta}\epsilon^{\gamma\xi}\epsilon^{\delta\eta}$$
$$(DH^\dagger)_{\alpha\dot\alpha}\,H\,B_{L\{\xi\eta\}}(DW_L)_{\{\beta\gamma\delta\},\dot\beta}\ \epsilon^{\dot\alpha\dot\beta}\epsilon^{\alpha\beta}\epsilon^{\gamma\xi}\epsilon^{\delta\eta}$$
$$H^\dagger\,(DH)_{\alpha\dot\alpha}\,(DB_L)_{\{\beta\gamma\delta\},\dot\beta}W_{L\{\xi\eta\}}\ \epsilon^{\dot\alpha\dot\beta}\epsilon^{\alpha\beta}\epsilon^{\gamma\xi}\epsilon^{\delta\eta}$$
$$H^\dagger\,(DH)_{\alpha\dot\alpha}\,B_{L\{\xi\eta\}}(DW_L)_{\{\beta\gamma\delta\},\dot\beta}\ \epsilon^{\dot\alpha\dot\beta}\epsilon^{\alpha\beta}\epsilon^{\gamma\xi}\epsilon^{\delta\eta}$$
$$H^\dagger H\,(DB_L)_{\{\alpha\beta\gamma\},\dot\alpha}(DW_L)_{\{\xi\eta\delta\},\dot\beta}\ \epsilon^{\dot\alpha\dot\beta}\epsilon^{\alpha\xi}\epsilon^{\beta\eta}\epsilon^{\gamma\delta}$$

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IBP

$$B_L{}^{\alpha\beta}W_{L\alpha\beta}(DH^\dagger)^{\gamma}{}_{\dot\alpha}(DH)_{\gamma}{}^{\dot\alpha}$$
$$B_L{}^{\alpha\beta}W_{L\alpha}{}^{\gamma}(DH^\dagger)_{\beta\dot\alpha}(DH)_{\gamma}{}^{\dot\alpha}$$

2

Gauge Structure

Gauge structure (internal sym) is easier than Lorentz structure (spacetime sym)

Dim-6 four fermion B-conserving operators: 25

[Grzadkowski, Iskrzynski, Misiak, Rosiek, 2010]

$(\bar{L}L)(\bar{L}L)$		$(\bar{R}R)(\bar{R}R)$		$(\bar{L}L)(\bar{R}R)$	
Q_{ll}	$(\bar{l}_p\gamma_\mu l_r)(\bar{l}_s\gamma^\mu l_t)$	Q_{ee}	$(\bar{e}_p\gamma_\mu e_r)(\bar{e}_s\gamma^\mu e_t)$	Q_{le}	$(\bar{l}_p\gamma_\mu l_r)(\bar{e}_s\gamma^\mu e_t)$
$Q_{qq}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{q}_s\gamma^\mu q_t)$	Q_{uu}	$(\bar{u}_p\gamma_\mu u_r)(\bar{u}_s\gamma^\mu u_t)$	Q_{lu}	$(\bar{l}_p\gamma_\mu l_r)(\bar{u}_s\gamma^\mu u_t)$
$Q_{qq}^{(3)}$	$(\bar{q}_p\gamma_\mu \tau^I q_r)(\bar{q}_s\gamma^\mu \tau^I q_t)$	Q_{dd}	$(\bar{d}_p\gamma_\mu d_r)(\bar{d}_s\gamma^\mu d_t)$	Q_{ld}	$(\bar{l}_p\gamma_\mu l_r)(\bar{d}_s\gamma^\mu d_t)$
$Q_{lq}^{(1)}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{q}_s\gamma^\mu q_t)$	Q_{eu}	$(\bar{e}_p\gamma_\mu e_r)(\bar{u}_s\gamma^\mu u_t)$	Q_{qe}	$(\bar{q}_p\gamma_\mu q_r)(\bar{e}_s\gamma^\mu e_t)$
$Q_{lq}^{(3)}$	$(\bar{l}_p\gamma_\mu \tau^I l_r)(\bar{q}_s\gamma^\mu \tau^I q_t)$	Q_{ed}	$(\bar{e}_p\gamma_\mu e_r)(\bar{d}_s\gamma^\mu d_t)$	$Q_{qu}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{u}_s\gamma^\mu u_t)$
		$Q_{ud}^{(1)}$	$(\bar{u}_p\gamma_\mu u_r)(\bar{d}_s\gamma^\mu d_t)$	$Q_{qu}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{u}_s\gamma^\mu T^A u_t)$
		$Q_{ud}^{(8)}$	$(\bar{u}_p\gamma_\mu T^A u_r)(\bar{d}_s\gamma^\mu T^A d_t)$	$Q_{qd}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{d}_s\gamma^\mu d_t)$
				$Q_{qd}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{d}_s\gamma^\mu T^A d_t)$

$(\bar{L}R)(\bar{R}L)$ and $(\bar{L}R)(\bar{L}R)$		B-violating	
Q_{ledq}	$(\bar{l}_p^j e_r)(\bar{d}_s q_t^j)$	Q_{duq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(q_s^{\gamma j})^T C l_t^k\right]$
$Q_{quqd}^{(1)}$	$(\bar{q}_p^j u_r)\varepsilon_{jk}(\bar{q}_s^k d_t)$	Q_{qqu}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{quqd}^{(8)}$	$(\bar{q}_p^j T^A u_r)\varepsilon_{jk}(\bar{q}_s^k T^A d_t)$	Q_{qqq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jn}\varepsilon_{km}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$
$Q_{lequ}^{(1)}$	$(\bar{l}_p^j e_r)\varepsilon_{jk}(\bar{q}_s^k u_t)$	Q_{duu}	$\varepsilon^{\alpha\beta\gamma}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{lequ}^{(3)}$	$(\bar{l}_p^j \sigma_{\mu\nu} e_r)\varepsilon_{jk}(\bar{q}_s^k \sigma^{\mu\nu} u_t)$		

Buchmuller&Wyler wrote 29: 5 redundant operators (Fierz) + 1 missing

$$(\bar{l}_p\gamma_\mu\tau^I l_r)(\bar{l}_s\tau^I\gamma^\mu l_t) = 2Q_{ll}^{ptsr} - Q_{ll}^{prst}$$

Fierz identity for SU(N): $\sum_a (T_a)_{ij}(T_a)_{kl} = \delta_{il}\delta_{kj} - \frac{1}{N}\delta_{ij}\delta_{kl}$

Gauge Structure

How to obtain independent and complete gauge structure systematically?

g-2 dim 8 operator

$W e_{\mathbb{C}} L H^\dagger D^2$

$\left(\tau^I\right)^i_j W^I_{\mu\nu} \left(e_{\mathbb{C}p} D^\mu L_{ri}\right) D^\nu H^{\dagger j}$

$\left(\tau^I\right)^i_j W^I_{\mu\lambda} \left(e_{\mathbb{C}p} \sigma^{\nu\lambda} L_{ri}\right) D^\mu D_\nu H^{\dagger j}$

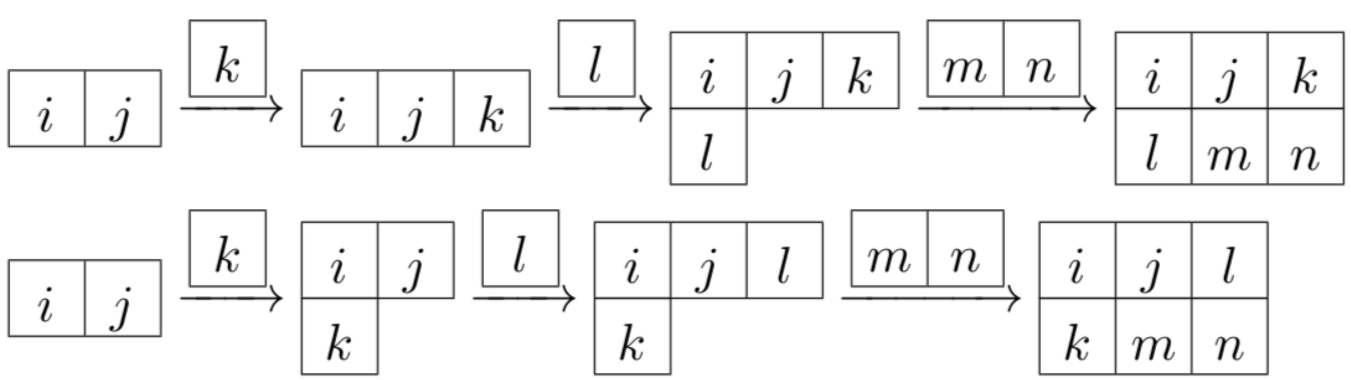
$W e_{\mathbb{C}} L H H^{\dagger 2}$

$\left(\tau^I\right)^k_j W^I_{\mu\nu} \left(e_{\mathbb{C}p} \sigma^{\mu\nu} L_{rk}\right) H^{\dagger j} \left(H^\dagger H\right)$

?

We invent Littlewood-Richardson method at Young tableau level

$\tau^I{}_{ij} W^I$: $\boxed{i\,|\,j}$, L_k: $\boxed{k}$, H_l: $\boxed{l}$, $H^\dagger_m H^\dagger_n$: $\boxed{m\,|\,n}$



$\epsilon^{il}\epsilon^{jm}\epsilon^{kn}$ $\quad W^I L_k H^{\dagger k} \left(H^\dagger \tau H\right)$

$\epsilon^{ik}\epsilon^{jm}\epsilon^{ln}$ $\quad \left(\tau^I\right)^k_j W^I L_k H^{\dagger j} \left(H^\dagger H\right)$

Find the 4-th g-2 dim 8 operator:

$W^I_{\mu\nu} \left(e_{\mathbb{C}p} \sigma^{\mu\nu} L_{ri}\right) H^{\dagger i} \left(H^\dagger \tau^I H\right)$

Operator Y-Basis

Direct product of Lorentz and gauge structures gives operator Y-basis

$W e_{\mathbb{C}} L H^\dagger D^2$

$$\left(\begin{array}{|c|c|c|c|}\hline 1&1&1&2\\\hline 2&3&3&4\\\hline\end{array} + \begin{array}{|c|c|c|c|}\hline 1&1&1&3\\\hline 2&2&3&4\\\hline\end{array}\right)\times\begin{array}{|c|c|}\hline i&j\\\hline k&l\\\hline\end{array} = \boxed{(\tau^I)^i_j W^I_{\mu\nu}\left(e_{\mathbb{C}p} D^\mu L_{ri}\right) D^\nu H^{\dagger j}} + \boxed{(\tau^I)^i_j W^I_{\mu\lambda}\left(e_{\mathbb{C}p}\sigma^{\nu\lambda} L_{ri}\right) D^\mu D_\nu H^{\dagger j}}$$

$W e_{\mathbb{C}} L H H^{\dagger 2}$

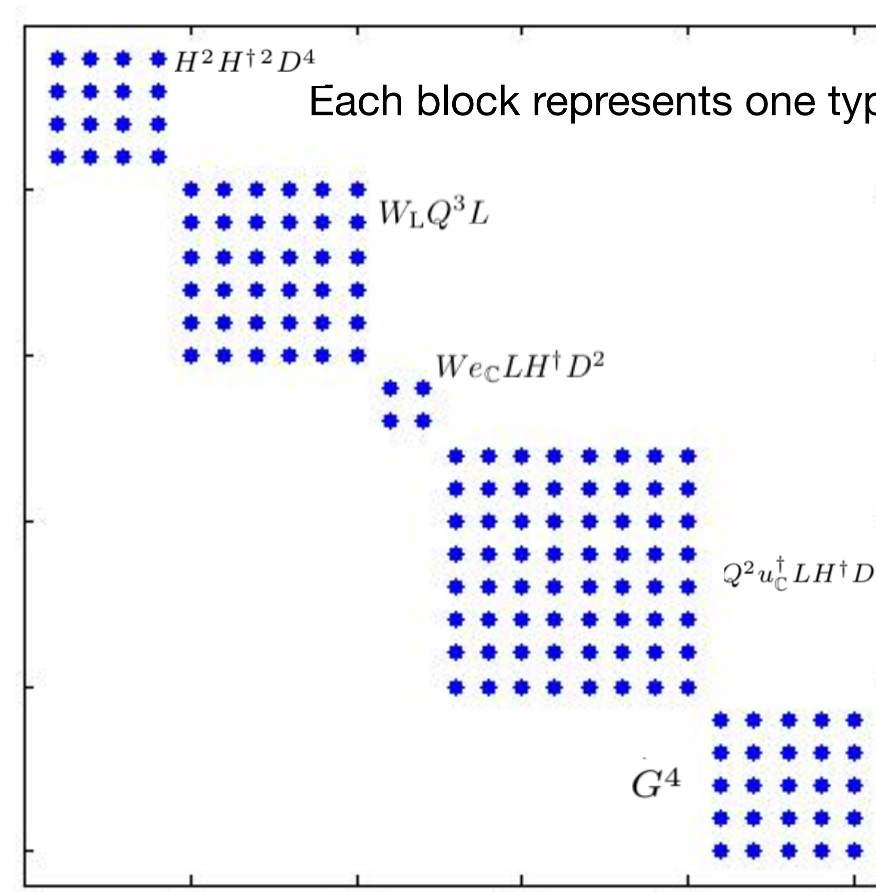
$$\begin{array}{|c|c|}\hline 1&1\\\hline 2&3\\\hline\end{array}\times\left(\begin{array}{|c|c|c|}\hline i&j&l\\\hline k&m&n\\\hline\end{array} + \begin{array}{|c|c|c|}\hline i&j&k\\\hline l&m&n\\\hline\end{array}\right) = \boxed{(\tau^I)^k_j W^I_{\mu\nu}\left(e_{\mathbb{C}p}\sigma^{\mu\nu} L_{rk}\right) H^{\dagger j}\left(H^\dagger H\right)} + \boxed{W^I_{\mu\nu}\left(e_{\mathbb{C}p}\sigma^{\mu\nu} L_{ri}\right) H^{\dagger i}\left(H^\dagger \tau^I H\right)}$$

Each type of operator forms

a linear operator space

Complete sets of dim-8 operators forms

a block-diagonal linear space

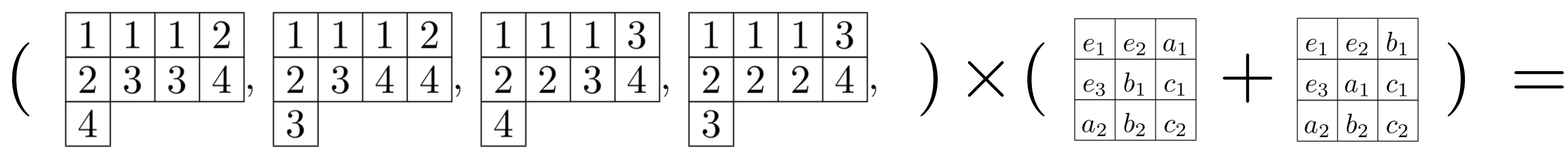


no meaning for x and y axis

Operators with Repeated Field

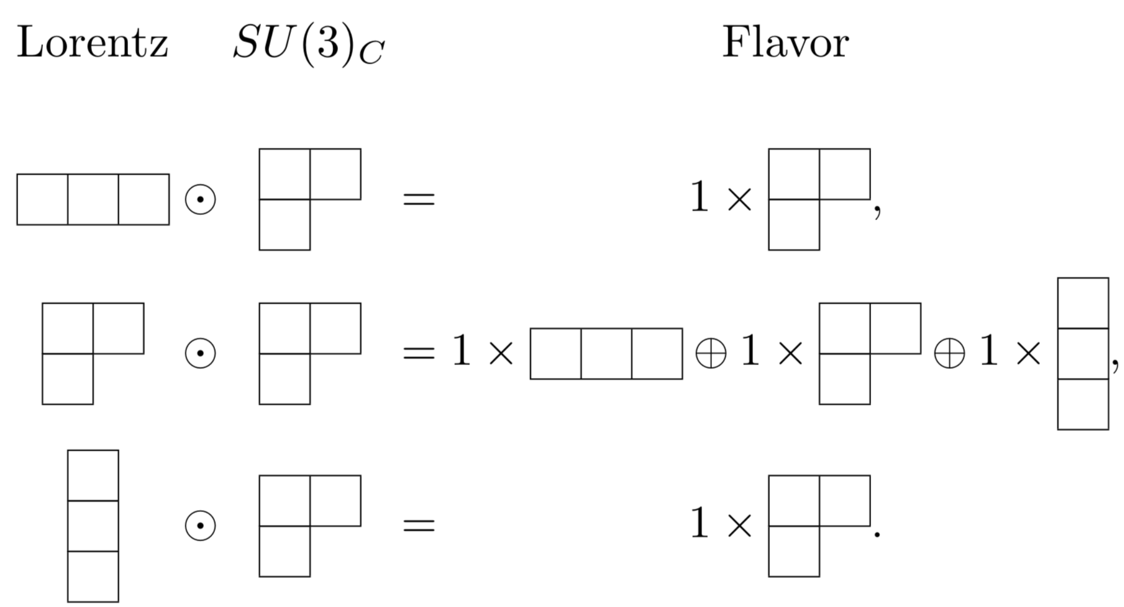
The Y-basis needs to be reorganized if repeated field in type

$G_L d_c^3 e_c^\dagger D$



$$= (\mathcal{M}_1^{\rm m} + \mathcal{M}_2^{\rm m} + \mathcal{M}_3^{\rm m} + \mathcal{M}_4^{\rm m}) \times (T^{\rm m}_{\rm SU3,1} + T^{\rm m}_{\rm SU3,2}) =$$ **8 operators**

Re-organize by symmetric group on repeated field dc

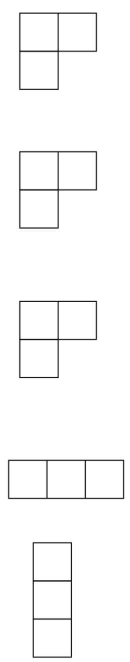


Refer to Hao-Lin Li's talk for details

Flavor Bland P-Basis

Linear transformation between Y-basis and P-basis:

$$\mathcal{O}^{\mathrm{p}} = \mathcal{K}^{\mathrm{py}}.\mathcal{O}^{\mathrm{y}}$$



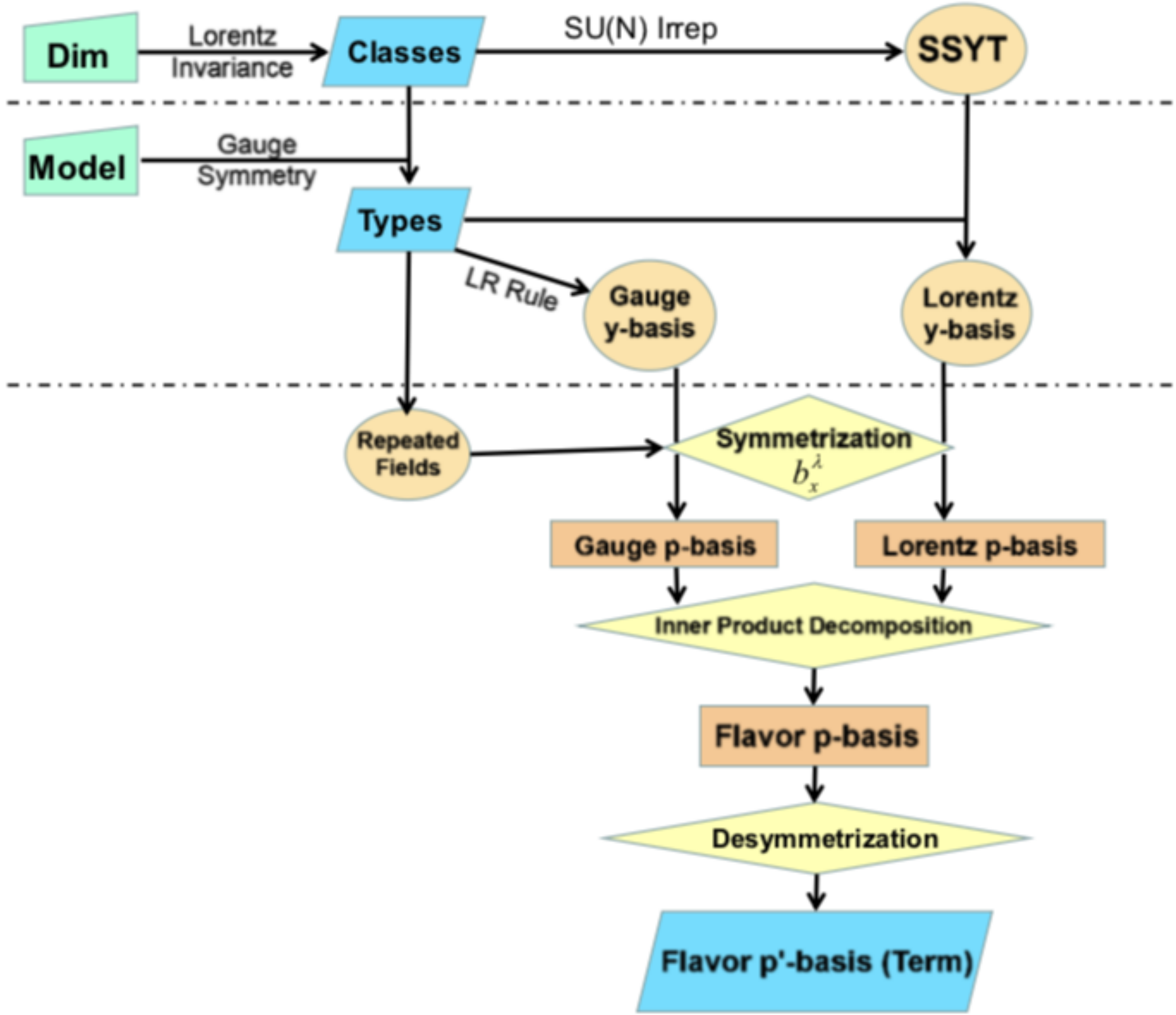
$$\begin{pmatrix} \mathcal{O}^{(\mathrm{p})}_{([2,1],1),1} \\ \mathcal{O}^{(\mathrm{p})}_{([2,1],2),1} \\ \mathcal{O}^{(\mathrm{p})}_{([2,1],1),2} \\ \mathcal{O}^{(\mathrm{p})}_{([2,1],2),2} \\ \mathcal{O}^{(\mathrm{p})}_{([2,1],1),3} \\ \mathcal{O}^{(\mathrm{p})}_{([2,1],2),3} \\ \mathcal{O}^{(\mathrm{p})}_{([3],1),1} \\ \mathcal{O}^{(\mathrm{p})}_{([1^3],1),1} \end{pmatrix} = \begin{pmatrix} 0 & \frac{4}{3} & 0 & \frac{4}{3} & 0 & \frac{4}{3} & 0 & 0 \\ -\frac{4}{3} & 0 & -\frac{4}{3} & 0 & -\frac{4}{3} & 0 & 0 & 0 \\ -\frac{4}{9} & -\frac{4}{9} & -\frac{8}{9} & \frac{4}{9} & \frac{4}{9} & -\frac{8}{9} & -\frac{4}{9} & -\frac{4}{9} \\ -\frac{4}{9} & \frac{8}{9} & \frac{4}{9} & \frac{4}{9} & -\frac{8}{9} & \frac{4}{9} & -\frac{4}{9} & \frac{8}{9} \\ -\frac{8}{3} & \frac{4}{3} & \frac{8}{3} & -\frac{4}{3} & \frac{8}{3} & -\frac{4}{3} & \frac{16}{3} & -\frac{8}{3} \\ \frac{4}{3} & -\frac{8}{3} & -\frac{4}{3} & \frac{8}{3} & -\frac{4}{3} & \frac{8}{3} & -\frac{8}{3} & \frac{16}{3} \\ \frac{2}{3} & -\frac{4}{3} & \frac{4}{3} & -\frac{2}{3} & -\frac{2}{3} & -\frac{2}{3} & \frac{2}{3} & -\frac{4}{3} \\ -\frac{4}{3} & 0 & 0 & -\frac{4}{3} & -\frac{4}{3} & \frac{4}{3} & -\frac{4}{3} & 0 \end{pmatrix} \begin{pmatrix} \mathcal{M}^{\mathrm{m}}_1 T^{\mathrm{m}}_{\mathrm{SU3},1} \\ \mathcal{M}^{\mathrm{m}}_1 T^{\mathrm{m}}_{\mathrm{SU3},2} \\ \mathcal{M}^{\mathrm{m}}_2 T^{\mathrm{m}}_{\mathrm{SU3},1} \\ \mathcal{M}^{\mathrm{m}}_2 T^{\mathrm{m}}_{\mathrm{SU3},2} \\ \mathcal{M}^{\mathrm{m}}_3 T^{\mathrm{m}}_{\mathrm{SU3},1} \\ \mathcal{M}^{\mathrm{m}}_3 T^{\mathrm{m}}_{\mathrm{SU3},2} \\ \mathcal{M}^{\mathrm{m}}_4 T^{\mathrm{m}}_{\mathrm{SU3},1} \\ \mathcal{M}^{\mathrm{m}}_4 T^{\mathrm{m}}_{\mathrm{SU3},2} \end{pmatrix}$$

Compared to dim-8 paper, dim-9 paper tackled flavor structure of operators

[Li, Ren, Xiao, **Yu**, Zheng, **2020**]

Refer to Hao-Lin Li's talk for details

Automized Procedure



Jiang-Hao Yu

SMEFT

Dimension-5

$$\epsilon_{ij}\epsilon_{mn}(L^i C L^m)H^j H^n$$

[Weinberg, 1979]

2

Dimension-6

Φ^6 and $\Phi^4 D^2$	$\psi^2\Phi^3$	X^3
$\mathcal{O}_{\Phi} = (\Phi^\dagger\Phi)^3$	$\mathcal{O}_{l\Phi} = (\Phi^\dagger\Phi)(\bar{L}_i\, l_j\Phi)$	$\mathcal{O}_G = -f^{ABC}G^{A\nu}_{\mu}G^{B\rho}_{\nu}G^{C\mu}_{\rho}$
$\mathcal{O}_{\Phi\Box} = (\Phi^\dagger\Phi)\Box(\Phi^\dagger\Phi)$	$\mathcal{O}_{u\Phi} = (\Phi^\dagger\Phi)(\bar{Q}_i\, u_j\Phi^c)$	$\mathcal{O}_{\widetilde{G}} = -f^{ABC}\widetilde{G}^{A\nu}_{\mu}G^{B\rho}_{\nu}G^{C\mu}_{\rho}$
$\mathcal{O}_{\Phi D} = (\Phi^\dagger D^\mu\Phi)^*(\Phi^\dagger D_\mu\Phi)$	$\mathcal{O}_{d\Phi} = (\Phi^\dagger\Phi)(\bar{Q}_i\, d_j\Phi)$	$\mathcal{O}_W = -\epsilon^{abc}W^{a\nu}_{\mu}W^{b\rho}_{\nu}W^{c\mu}_{\rho}$
		$\mathcal{O}_{\widetilde{W}} = -\epsilon^{abc}\widetilde{W}^{a\nu}_{\mu}W^{b\rho}_{\nu}W^{c\mu}_{\rho}$

$X^2\Phi^2$	$\psi^2 X$
$\mathcal{O}_{\Phi G} = (\Phi^\dagger\Phi)G^A_{\mu\nu}G^{A\mu\nu}$	$\mathcal{O}_{uG}$
$\mathcal{O}_{\Phi\widetilde{G}} = (\Phi^\dagger\Phi)\widetilde{G}^A_{\mu\nu}G^{A\mu\nu}$	$\mathcal{O}_{dG}$
$\mathcal{O}_{\Phi W} = (\Phi^\dagger\Phi)W^a_{\mu\nu}W^{a\mu\nu}$	$\mathcal{O}_{lW}$
$\mathcal{O}_{\Phi\widetilde{W}} = (\Phi^\dagger\Phi)\widetilde{W}^a_{\mu\nu}W^{a\mu\nu}$	$\mathcal{O}_{uW}$
$\mathcal{O}_{\Phi B} = (\Phi^\dagger\Phi)B_{\mu\nu}B^{\mu\nu}$	$\mathcal{O}_{dW}$
$\mathcal{O}_{\Phi\widetilde{B}} = (\Phi^\dagger\Phi)\widetilde{B}_{\mu\nu}B^{\mu\nu}$	$\mathcal{O}_{lB}$
$\mathcal{O}_{\Phi WB} = -(\Phi^\dagger\tau^a\Phi)W^a_{\mu\nu}B^{\mu\nu}$	$\mathcal{O}_{uB}$
$\mathcal{O}_{\Phi\widetilde{W}B} = -(\Phi^\dagger\tau^a\Phi)\widetilde{W}^a_{\mu\nu}B^{\mu\nu}$	$\mathcal{O}_{dB}$

$(\bar{L}L)(\bar{L}L)$	$(\bar{R}R)(\bar{R}R)$	$(\bar{L}L)(\bar{R}R)$
$\mathcal{O}_{LL} = (\bar{L}_i\gamma_\mu L_j)(\bar{L}_k\gamma^\mu L_l)$	$\mathcal{O}_{ll} = (\bar{l}_i\gamma_\mu l_j)(\bar{l}_k\gamma^\mu l_l)$	$\mathcal{O}_{Ll} = (\bar{L}_i\gamma_\mu L_j)(\bar{l}_k\gamma^\mu l_l)$
$\mathcal{O}^{(1)}_{QQ} = (\bar{Q}_i\gamma_\mu Q_j)(\bar{Q}_k\gamma^\mu Q_l)$	$\mathcal{O}_{uu} = (\bar{u}_i\gamma_\mu u_j)(\bar{u}_k\gamma^\mu u_l)$	$\mathcal{O}_{Lu} = (\bar{L}_i\gamma_\mu L_j)(\bar{u}_k\gamma^\mu u_l)$
$\mathcal{O}^{(3)}_{QQ} = (\bar{Q}_i\gamma_\mu\tau^a Q_j)(\bar{Q}_k\gamma^\mu\tau^a Q_l)$	$\mathcal{O}_{dd} = (\bar{d}_i\gamma_\mu d_j)(\bar{d}_k\gamma^\mu d_l)$	$\mathcal{O}_{Ld} = (\bar{L}_i\gamma_\mu L_j)(\bar{d}_k\gamma^\mu d_l)$
$\mathcal{O}^{(1)}_{LQ} = (\bar{L}_i\gamma_\mu L_j)(\bar{Q}_k\gamma^\mu Q_l)$	$\mathcal{O}_{lu} = (\bar{l}_i\gamma_\mu l_j)(\bar{u}_k\gamma^\mu u_l)$	$\mathcal{O}_{Ql} = (\bar{Q}_i\gamma_\mu Q_j)(\bar{l}_k\gamma^\mu l_l)$
$\mathcal{O}^{(3)}_{LQ} = (\bar{L}_i\gamma_\mu\tau^a L_j)(\bar{Q}_k\gamma^\mu\tau^a Q_l)$	$\mathcal{O}_{ld} = (\bar{l}_i\gamma_\mu l_j)(\bar{d}_k\gamma^\mu d_l)$	$\mathcal{O}^{(1)}_{Qu} = (\bar{Q}_i\gamma_\mu Q_j)(\bar{u}_k\gamma^\mu u_l)$
	$\mathcal{O}^{(1)}_{ud} = (\bar{u}_i\gamma_\mu u_j)(\bar{d}_k\gamma^\mu d_l)$	$\mathcal{O}^{(8)}_{Qu} = \left(\bar{Q}_i\gamma_\mu\frac{\lambda^A}{2}Q_j\right)\left(\bar{u}_k\gamma^\mu\frac{\lambda^A}{2}u_l\right)$
	$\mathcal{O}^{(8)}_{ud} = \left(\bar{u}_i\gamma_\mu\frac{\lambda^A}{2}u_j\right)\left(\bar{d}_k\gamma^\mu\frac{\lambda^A}{2}d_l\right)$	$\mathcal{O}^{(1)}_{Qd} = (\bar{Q}_i\gamma_\mu Q_j)(\bar{d}_k\gamma^\mu d_l)$
		$\mathcal{O}^{(8)}_{Qd} = \left(\bar{Q}_i\gamma_\mu\frac{\lambda^A}{2}Q_j\right)\left(\bar{d}_k\gamma^\mu\frac{\lambda^A}{2}d_l\right)$

$(\bar{L}R)(\bar{R}L)$ and $(\bar{L}R)(\bar{L}R)$	B-violating
$\mathcal{O}_{LldQ} = (\bar{L}^\sigma_i l_j)(\bar{d}_k Q^\sigma_l)$	$\mathcal{O}_{duQ} = \epsilon^{\alpha\beta\gamma}\epsilon_{\sigma\tau}\left[(d^\alpha_i)^T C u^\beta_j\right]\left[(Q^{\gamma\sigma}_k)^T C L^\tau_l\right]$
$\mathcal{O}^{(1)}_{QuQd} = (\bar{Q}^\sigma_i u_j)\epsilon_{\sigma\tau}(\bar{Q}^\tau_k d_l)$	$\mathcal{O}_{QQu} = \epsilon^{\alpha\beta\gamma}\epsilon_{\sigma\tau}\left[(Q^{\alpha\sigma}_i)^T C Q^{\beta\tau}_j\right]\left[(u^\gamma_k)^T C l_l\right]$
$\mathcal{O}^{(8)}_{QuQd} = \left(\bar{Q}^\sigma_i\frac{\lambda^A}{2}u_j\right)\epsilon_{\sigma\tau}\left(\bar{Q}^\tau_k\frac{\lambda^A}{2}d_l\right)$	$\mathcal{O}_{QQQ} = \epsilon^{\alpha\beta\gamma}\epsilon_{\sigma\tau}\epsilon_{\sigma'\tau'}\left[(Q^{\alpha\sigma}_i)^T C Q^{\beta\tau}_j\right]\left[(Q^{\gamma\sigma'}_k)^T C L^{\tau'}_l\right]$
$\mathcal{O}^{(1)}_{LlQu} = (\bar{L}^\sigma_i l_j)\epsilon_{\sigma\tau}(\bar{Q}^\tau_k u_l)$	$\mathcal{O}_{duu} = \epsilon^{\alpha\beta\gamma}\left[(d^\alpha_i)^T C u^\beta_j\right]\left[(u^\gamma_k)^T C l_l\right]$
$\mathcal{O}^{(3)}_{LlQu} = (\bar{L}^\sigma_i\sigma_{\mu\nu}l_j)\epsilon_{\sigma\tau}(\bar{Q}^\tau_k\sigma^{\mu\nu}u_l)$	

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[Buchmuller, Wyler, 1986] [Grzadkowski, Iskrzynski, Misiak, Rosiek, 2010]

Dimension-7

1 : $\psi^2 X H^2$ + h.c.

$Q_{l^2WH^2}$	$\epsilon_{mn}(\tau^I\epsilon)_{jk}(l^m_p C i\sigma^{\mu\nu}l^j_r)H^n H^k W^I_{\mu\nu}$
$Q_{l^2BH^2}$	$\epsilon_{mn}\epsilon_{jk}(l^m_p C i\sigma^{\mu\nu}l^j_r)H^n H^k B_{\mu\nu}$

2 : $\psi^2 H^4$ + h.c.

$Q_{l^2H^4}$	$\epsilon_{mn}\epsilon_{jk}(l^m_p C l^j_r)H^n H^k(H^\dagger H)$

3(B) : $\psi^4 H$ + h.c.

Q_{l^3eH}	$\epsilon_{jk}\epsilon_{mn}(\bar{e}_p l^j_r)(l^k_s C l^m_t)H^n$
Q_{leudH}	$\epsilon_{jk}(\bar{d}_p l^j_r)(u_s C e_t)H^k$
$Q^{(1)}_{l^2qdH}$	$\epsilon_{jk}\epsilon_{mn}(\bar{d}_p l^j_r)(q^k_s C l^m_t)H^n$
$Q^{(2)}_{l^2qdH}$	$\epsilon_{jm}\epsilon_{kn}(\bar{d}_p l^j_r)(q^k_s C l^m_t)H^n$
Q_{l^2quH}	$\epsilon_{jk}(\bar{q}^m_p u_r)(l_{sm} C l^j_t)H^k$

3($\cancel{B}$) : $\psi^4 H$ + h.c.

Q_{lud^2H}	$\epsilon_{\alpha\beta\gamma}(\bar{l}_p d^\alpha_r)(u^\beta_s C d^\gamma_t)\widetilde{H}$
Q_{lq^2dH}	$\epsilon_{\alpha\beta\gamma}\epsilon_{jk}(\bar{l}^m_p d^\alpha_r)(q^\beta_{sm} C q^{j\gamma}_t)\widetilde{H}^k$
Q_{ld^3H}	$\epsilon_{\alpha\beta\gamma}(\bar{l}_p d^\alpha_r)(d^\beta_s C d^\gamma_t)H$
Q_{eqd^2H}	$\epsilon_{\alpha\beta\gamma}\epsilon_{jk}(\bar{e}_p q^{j\alpha}_r)(d^\beta_s C d^\gamma_t)\widetilde{H}^k$

4 : $\psi^2 H^3 D$ + h.c.

Q_{leH^3D}	$\epsilon_{mn}\epsilon_{jk}(l^m_p C\gamma^\mu e_r)H^n H^j iD_\mu H^k$

5(B) : $\psi^4 D$ + h.c.

Q_{l^2udD}	$\epsilon_{jk}(\bar{d}_p\gamma^\mu u_r)(l^j_s C iD_\mu l^k_t)$

6 : $\psi^2 H^2 D^2$ + h.c.

$Q^{(1)}_{l^2H^2D^2}$	$\epsilon_{jk}\epsilon_{mn}(l^j_p C D^\mu l^k_r)H^m(D_\mu H^n)$
$Q^{(2)}_{l^2H^2D^2}$	$\epsilon_{jm}\epsilon_{kn}(l^j_p C D^\mu l^k_r)H^m(D_\mu H^n)$

5($\cancel{B}$) : $\psi^4 D$ + h.c.

Q_{lqd^2D}	$\epsilon_{\alpha\beta\gamma}(\bar{l}_p\gamma^\mu q^\alpha_r)(d^\beta_s C iD_\mu d^\gamma_t)$
Q_{ed^3D}	$\epsilon_{\alpha\beta\gamma}(\bar{e}_p\gamma^\mu d^\alpha_r)(d^\beta_s C iD_\mu d^\gamma_t)$

30

[Lehman, 2014]

Dimension-8

[Li, Ren, Shu, Xiao, Yu, Zheng, 2020]

[Murphy, 2020]

N	$(n,\tilde{n})$	Subclasses	$\mathcal{N}_{\text{type}}$	$\mathcal{N}_{\text{term}}$	$\mathcal{N}_{\text{operator}}$	Equations
4	(4, 0)	$F_{\rm L}^4 + h.c.$	14	26	26	(4.19)
	(3, 1)	$F_{\rm L}^2\psi\psi^\dagger D + h.c.$	22	22	$22n_f^2$	(4.51)
		$\psi^4 D^2 + h.c.$	$4\underline{+4}$	$18\underline{+14}$	$12n_f^4\underline{+n_f^3(5n_f-1)}$	(4.75, 4.78, 4.80)
		$F_{\rm L}\psi^2\phi D^2 + h.c.$	16	32	$32n_f^2$	(4.44)
		$F_{\rm L}^2\phi^2 D^2 + h.c.$	8	12	12	(4.14)
	(2, 2)	$F_{\rm L}^2 F_{\rm R}^2$	14	17	17	(4.19)
		$F_{\rm L}F_{\rm R}\psi\psi^\dagger D$	27	35	$35n_f^2$	(4.50, 4.51)
		$\psi^2\psi^{\dagger 2}D^2$	$17\underline{+4}$	$54\underline{+8}$	$\frac{1}{2}n_f^2(75n_f^2+11)\underline{+6n_f^4}$	(4.74, 4.79-4.81)
		$F_{\rm R}\psi^2\phi D^2 + h.c.$	16	16	$16n_f^2$	(4.44)
		$F_{\rm L}F_{\rm R}\phi^2 D^2$	5	6	6	(4.14)
		$\psi\psi^\dagger\phi^2 D^3$	7	16	$16n_f^2$	(4.31, 4.32)
		$\phi^4 D^4$	1	3	3	(4.8)
5	(3, 0)	$F_{\rm L}\psi^4 + h.c.$	$12\underline{+10}$	$66\underline{+54}$	$42n_f^4\underline{+2n_f^3(9n_f+1)}$	(4.86, 4.88, 4.89, 4.91)
		$F_{\rm L}^2\psi^2\phi + h.c.$	32	60	$60n_f^2$	(4.47, 4.48)
		$F_{\rm L}^3\phi^2 + h.c.$	6	6	6	(4.16)
	(2, 1)	$F_{\rm L}\psi^2\psi^{\dagger 2} + h.c.$	$84\underline{+24}$	$172\underline{+32}$	$2n_f^2(59n_f^2-2)\underline{+24n_f^4}$	(4.84-4.85), (4.88-4.92)
		$F_{\rm R}^2\psi^2\phi + h.c.$	32	36	$36n_f^2$	(4.47, 4.48)
		$\psi^3\psi^\dagger\phi D + h.c.$	$32\underline{+14}$	$180\underline{+56}$	$n_f^3(135n_f-1)\underline{+n_f^3(29n_f+3)}$	(4.66, 4.69-4.72)
		$F_{\rm L}\psi\psi^\dagger\phi^2 D + h.c.$	38	92	$92n_f^2$	(4.39, 4.40)
		$\psi^2\phi^3 D^2 + h.c.$	6	36	$36n_f^2$	(4.28)
		$F_{\rm L}\phi^4 D^2 + h.c.$	4	6	6	(4.10)
6	(2, 0)	$\psi^4\phi^2 + h.c.$	$12\underline{+4}$	$48\underline{+18}$	$5(5n_f^4+n_f^2)\underline{+\frac{2}{3}(8n_f^4+n_f^2)}$	(4.55, 4.59, 4.62, 4.64)
		$F_{\rm L}\psi^2\phi^3 + h.c.$	16	22	$22n_f^2$	(4.36)
		$F_{\rm L}^2\phi^4 + h.c.$	8	10	10	(4.12)
	(1, 1)	$\psi^2\psi^{\dagger 2}\phi^2$	$23\underline{+10}$	$57\underline{+14}$	$n_f^2(42n_f^2+n_f+2)\underline{+3n_f^3(3n_f-1)}$	(4.54, 4.55, 4.59-4.63)
		$\psi\psi^\dagger\phi^4 D$	7	13	$13n_f^2$	(4.24, 4.25)
		$\phi^6 D^2$	1	2	2	(4.8)
7	(1, 0)	$\psi^2\phi^5 + h.c.$	6	6	$6n_f^2$	(4.21)
8	(0, 0)	ϕ^8	1	1	1	(4.8)
Total		48	$471\underline{+70}$	$1070\underline{+196}$	$993(n_f=1)$, $44807(n_f=3)$	

993

Dimension-9

[Li, Ren, Xiao, Yu, Zheng, 2020]

[Liao, Ma, 2020]

N	$(n,\tilde{n})$	Classes	$\mathcal{N}_{\text{type}}$	$\mathcal{N}_{\text{term}}$	$\mathcal{N}_{\text{operator}}$	Equations
4	(3, 2)	$\psi^3\psi^\dagger D^3 + h.c.$	$0+4+2+0$	10	$\frac{2}{3}n_f^2(7n_f^2-1)$	(5.50)(5.51)
		$\psi^2\phi^2 D^4 + h.c.$	$0+0+2+0$	6	$3n_f(n_f+1)$	(5.21)
5	(3, 1)	$F_{\rm L}\psi^3\psi^\dagger D + h.c.$	$0+10+6+0$	72	$32n_f^4$	(5.59)(5.60)
		$\psi^4\phi D^2 + h.c.$	$0+4+4+0$	100	$40n_f^4$	(5.45-5.48)
		$F_{\rm L}\psi^2\phi^2 D^2 + h.c.$	$0+0+4+0$	34	$17n_f^2-n_f$	(5.28)(5.29)
	(2, 2)	$F_{\rm R}\psi^3\psi^\dagger D + h.c.$	$0+10+6+0$	54	$4n_f^3(6n_f+1)$	(5.59)(5.60)
		$\psi^2\psi^{\dagger 2}\phi D^2$	$0+4+4+0$	84	$n_f^3(49n_f+1)$	(5.45-5.48)
		$F_{\rm R}\psi^2\phi^2 D^2 + h.c.$	$0+0+4+0$	20	$2n_f(5n_f-1)$	(5.28)(5.29)
		$\psi\psi^\dagger\phi^3 D^3$	$0+0+2+0$	6	$6n_f^2$	(5.19)
6	(3, 0)	$\psi^6 + h.c.$	$2+4+6+0$	116	$\frac{1}{24}n_f^2(415n_f^4+53n_f^3+59n_f^2+139n_f+6)$	(5.63-5.70)
		$F_{\rm L}\psi^4\phi + h.c.$	$0+12+10+0$	102	$2n_f^3(21n_f+1)$	(5.54-5.56)
		$F_{\rm L}^2\psi^2\phi^2 + h.c.$	$0+0+8+0$	20	$2n_f(5n_f+2)$	(5.32)
	(2, 1)	$\psi^4\psi^{\dagger 2} + h.c.$	$4+26+20+4$	244	$\frac{1}{6}n_f^3(382n_f^3-9n_f^2+2n_f+21)$	(5.63-5.69)
		$F_{\rm L}\psi^2\psi^{\dagger 2}\phi + h.c.$	$0+24+24+0$	92	$52n_f^4$	(5.54-5.56)
		$F_{\rm L}^2\psi^{\dagger 2}\phi^2 + h.c.$	$0+0+8+0$	12	$2n_f(3n_f+2)$	(5.32)
		$\psi^3\psi^\dagger\phi^2 D + h.c.$	$0+12+18+0$	186	$\frac{2}{3}n_f^2(146n_f^2+1)$	(5.39-5.42)
		$F_{\rm L}\psi\psi^\dagger\phi^3 D + h.c.$	$0+0+8+0$	12	$12n_f^2$	(5.25)
		$\psi^2\phi^4 D^2 + h.c.$	$0+0+4+0$	24	$2n_f(6n_f+1)$	(5.17)
7	(2, 0)	$\psi^4\phi^3 + h.c.$	$0+6+6+0$	32	$\frac{4}{3}n_f^2(10n_f^2-1)$	(5.35-5.37)
		$F_{\rm L}\psi^2\phi^4 + h.c.$	$0+0+4+0$	8	$2n_f(2n_f-1)$	(5.23)
	(1, 1)	$\psi^2\psi^{\dagger 2}\phi^3$	$0+6+10+0$	24	$14n_f^4$	(5.35-5.37)
		$\psi\psi^\dagger\phi^5 D$	$0+0+2+0$	2	$2n_f^2$	(5.12)
8	(1, 0)	$\psi^2\phi^6 + h.c.$	$0+0+2+0$	2	$n_f^2+n_f$	(5.9)
Total		42	6+122+164+4	1262	$8+204+348+0\ (n_f=1)$ $2862+42234+44874+486\ (n_f=3)$	

560

Low Energy EFT

Dimension-5

Dim-5 operators

N	$(n,\tilde n)$	Classes	$\mathcal{N}_{\rm type}$
3	$(2,0)$	$F_{\rm L}\psi_{\rm L}{}^2+h.c.$	$10+0+2+0$

10

Dimension-6

Dim-6 operators

N	$(n,\tilde n)$	Classes	$\mathcal{N}_{\rm type}$	$\mathcal{N}_{\rm term}$
3	$(3,0)$	$F_{\rm L}^3+h.c.$	$2+0+0+0$	2
4	$(2,0)$	$\psi_{\rm L}^4+h.c.$	$14+12+8+2$	78
	$(1,1)$	$\psi_{\rm L}^2\psi_{\rm R}^2$	$40+20+12+0$	84
Total		5	$56+32+20+2$	164

[Jenkins, Manohar, Stoffer, 2017]

Dimension-7

Dim-7 operators

N	$(n,\tilde n)$	Classes	$\mathcal{N}_{\rm type}$	$\mathcal{N}_{\rm term}$
4	$(3,0)$	$F_{\rm L}^2\psi_{\rm L}^2+h.c.$	$16+0+4+0$	32
	$(2,1)$	$F_{\rm L}^2\psi_{\rm R}^2+h.c.$	$16+0+4+0$	24
		$\psi_{\rm L}^3\psi_{\rm R}D+h.c.$	$50+32+22+$	
Total		6	$82+32+30+$	

120

166

[Liao, Ma, Wang, 2020]

Dimension-8

[Li, Ren, Xiao, **Yu**, Zheng, 2020]

[Murphy, 2020]

N	$(n,\tilde n)$	Subclasses	$\mathcal{N}_{\rm type}$	$\mathcal{N}_{\rm term}$	$\mathcal{N}_{\rm operator}$	Equations
4	$(4,0)$	$F_{\rm L}^4+h.c.$	14	26	26	(4.19)
	$(3,1)$	$F_{\rm L}^2\psi\psi^\dagger D+h.c.$	22	22	$22n_f^2$	(4.51)
		$\psi^4D^2+h.c.$	$4\underline{+4}$	$18\underline{+14}$	$12n_f^4\underline{+n_f^3(5n_f-1)}$	(4.75, 4.78, 4.80)
		$F_{\rm L}\psi^2\phi D^2+h.c.$	16	32	$32n_f^2$	(4.44)
		$F_{\rm L}^2\phi^2D^2+h.c.$	8	12	12	(4.14)
	$(2,2)$	$F_{\rm L}^2F_{\rm R}^2$	14	17	17	(4.19)
		$F_{\rm L}F_{\rm R}\psi\psi^\dagger D$	27	35	$35n_f^2$	(4.50, 4.51)
		$\psi^2\psi^{\dagger 2}D^2$	$17\underline{+4}$	$54\underline{+8}$	$\frac{1}{2}n_f^2(75n_f^2+11)\underline{+6n_f^4}$	(4.74, 4.79-4.81)
		$F_{\rm R}\psi^2\phi D^2+h.c.$	16	16	$16n_f^2$	(4.44)
		$F_{\rm L}F_{\rm R}\phi^2D^2$	5	6	6	(4.14)
		$\psi\psi^\dagger\phi^2D^3$	7	16	$16n_f^2$	(4.31, 4.32)
		ϕ^4D^4	1	3	3	(4.8)
5	$(3,0)$	$F_{\rm L}\psi^4+h.c.$	$12\underline{+10}$	$66\underline{+54}$	$42n_f^4\underline{+2n_f^3(9n_f+1)}$	(4.86, 4.88, 4.89, 4.91)
		$F_{\rm L}^2\psi^2\phi+h.c.$	32	60	$60n_f^2$	(4.47, 4.48)
		$F_{\rm L}^3\phi^2+h.c.$	6	6	6	(4.16)
	$(2,1)$	$F_{\rm L}\psi^2\psi^{\dagger 2}+h.c.$	$84\underline{+24}$	$172\underline{+32}$	$2n_f^2(59n_f^2-2)\underline{+24n_f^4}$	(4.84-4.85), (4.88-4.92)
		$F_{\rm R}^2\psi^2\phi+h.c.$	32	36	$36n_f^2$	(4.47, 4.48)
		$\psi^3\psi^\dagger\phi D+h.c.$	$32\underline{+14}$	$180\underline{+56}$	$n_f^3(135n_f-1)\underline{+n_f^3(29n_f+3)}$	(4.66, 4.69-4.72)
		$F_{\rm L}\psi\psi^\dagger\phi^2D+h.c.$	38	92	$92n_f^2$	(4.39, 4.40)
		$\psi^2\phi^3D^2+h.c.$	6	36	$36n_f^2$	(4.28)
		$F_{\rm L}\phi^4D^2+h.c.$	4	6	6	(4.10)
6	$(2,0)$	$\psi^4\phi^2+h.c.$	$12\underline{+4}$	$48\underline{+18}$	$5(5n_f^4+n_f^2)\underline{+\frac{2}{3}(8n_f^4+n_f^2)}$	(4.55, 4.59, 4.62, 4.64)
		$F_{\rm L}\psi^2\phi^3+h.c.$	16	22	$22n_f^2$	(4.36)
		$F_{\rm L}^2\phi^4+h.c.$	8	10	10	(4.12)
	$(1,1)$	$\psi^2\psi^{\dagger 2}\phi^2$	$23\underline{+10}$	$57\underline{+14}$	$n_f^2(42n_f^2+n_f+2)\underline{+3n_f^3(3n_f-1)}$	(4.54, 4.55, 4.59-4.63)
		$\psi\psi^\dagger\phi^4D$	7	13	$13n_f^2$	(4.24, 4.25)
		ϕ^6D^2	1	2	2	(4.8)
7	$(1,0)$	$\psi^2\phi^5+h.c.$	6	6	$6n_f^2$	(4.21)
8	$(0,0)$	ϕ^8	1	1	1	(4.8)
Total		48	$471\underline{+70}$	$1070\underline{+196}$	$993(n_f=1)$, $44807(n_f=3)$	

783

Dimension-9

[Li, Ren, Xiao, **Yu**, Zheng, 2020]

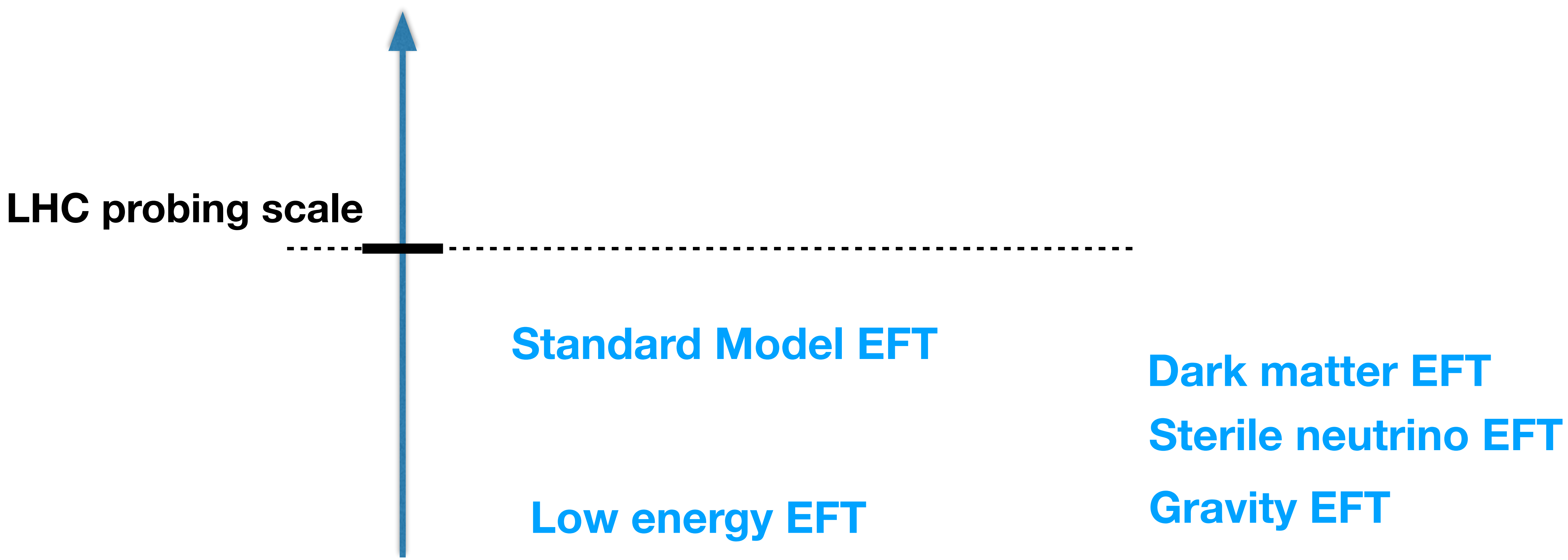
N	$(n,\tilde n)$	Classes	$\mathcal{N}_{\rm type}$	$\mathcal{N}_{\rm term}$	$\mathcal{N}_{\rm operator}$	Equations
4	$(3,2)$	$\psi^3\psi^\dagger D^3+h.c.$	$0+4+2+0$	10	$\frac{2}{3}n_f^2(7n_f^2-1)$	(5.50)(5.51)
		$\psi^2\phi^2D^4+h.c.$	$0+0+2+0$	6	$3n_f(n_f+1)$	(5.21)
5	$(3,1)$	$F_{\rm L}\psi^3\psi^\dagger D+h.c.$	$0+10+6+0$	72	$32n_f^4$	(5.59)(5.60)
		$\psi^4\phi D^2+h.c.$	$0+4+4+0$	100	$40n_f^4$	(5.45-5.48)
		$F_{\rm L}\psi^2\phi^2D^2+h.c.$	$0+0+4+0$	34	$17n_f^2-n_f$	(5.28)(5.29)
	$(2,2)$	$F_{\rm R}\psi^3\psi^\dagger D+h.c.$	$0+10+6+0$	54	$4n_f^3(6n_f+1)$	(5.59)(5.60)
		$\psi^2\psi^{\dagger 2}\phi D^2$	$0+4+4+0$	84	$n_f^3(49n_f+1)$	(5.45-5.48)
		$F_{\rm R}\psi^2\phi^2D^2+h.c.$	$0+0+4+0$	20	$2n_f(5n_f-1)$	(5.28)(5.29)
		$\psi\psi^\dagger\phi^3D^3$	$0+0+2+0$	6	$6n_f^2$	(5.19)
6	$(3,0)$	$\psi^6+h.c.$	$2+4+6+0$	116	$\frac{1}{24}n_f^2(415n_f^4+53n_f^3+59n_f^2+139n_f+6)$	(5.63-5.70)
		$F_{\rm L}\psi^4\phi+h.c.$	$0+12+10+0$	102	$2n_f^3(21n_f+1)$	(5.54-5.56)
		$F_{\rm L}^2\psi^2\phi^2+h.c.$	$0+0+8+0$	20	$2n_f(5n_f+2)$	(5.32)
	$(2,1)$	$\psi^4\psi^{\dagger 2}+h.c.$	$4+26+20+4$	244	$\frac{1}{6}n_f^3(382n_f^3-9n_f^2+2n_f+21)$	(5.63-5.69)
		$F_{\rm L}\psi^2\psi^{\dagger 2}\phi+h.c.$	$0+24+24+0$	92	$52n_f^4$	(5.54-5.56)
		$F_{\rm L}^2\psi^{\dagger 2}\phi^2+h.c.$	$0+0+8+0$	12	$2n_f(3n_f+2)$	(5.32)
		$\psi^3\psi^\dagger\phi^2D+h.c.$	$0+12+18+0$	186	$\frac{2}{3}n_f^2(146n_f^2+1)$	(5.39-5.42)
		$F_{\rm L}\psi\psi^\dagger\phi^3D+h.c.$	$0+0+8+0$	12	$12n_f^2$	(5.25)
		$\psi^2\phi^4D^2+h.c.$	$0+0+4+0$	24	$2n_f(6n_f+1)$	(5.17)
7	$(2,0)$	$\psi^4\phi^3+h.c.$	$0+6+6+0$	32	$\frac{4}{3}n_f^2(10n_f^2-1)$	(5.35-5.37)
		$F_{\rm L}\psi^2\phi^4+h.c.$	$0+0+4+0$	8	$2n_f(2n_f-1)$	(5.23)
	$(1,1)$	$\psi^2\psi^{\dagger 2}\phi^3$	$0+6+10+0$	24	$14n_f^4$	(5.35-5.37)
		$\psi\psi^\dagger\phi^5D$	$0+0+2+0$	2	$2n_f^2$	(5.12)
8	$(1,0)$	$\psi^2\phi^6+h.c.$	$0+0+2+0$	2	$n_f^2+n_f$	(5.9)
Total		42	6+122+164+4	1262	$8+204+348+0\ (n_f=1)$ $2862+42234+44874+486\ (n_f=3)$	

3774

Jiang-Hao Yu

Landscape of Generic EFTs

Any EFT with Lorentz inv. and any gauge symmetries, SU(5), LRSM, etc

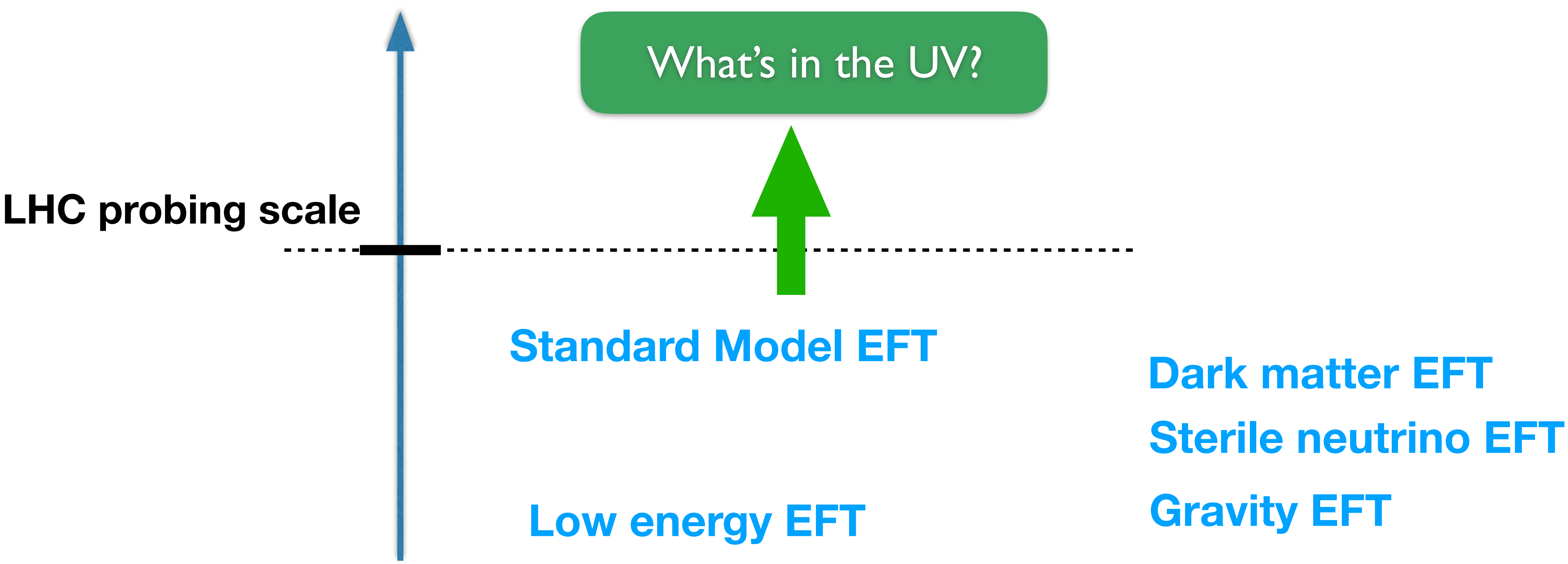


Li, Shu, Xiao, JHYu, arXiv: 2012.11615

Li, Ren, Xiao, JHYu, Zheng, in preparation

Landscape of Generic EFTs

Any EFT with Lorentz inv. and any gauge symmetries, SU(5), LRSM, etc

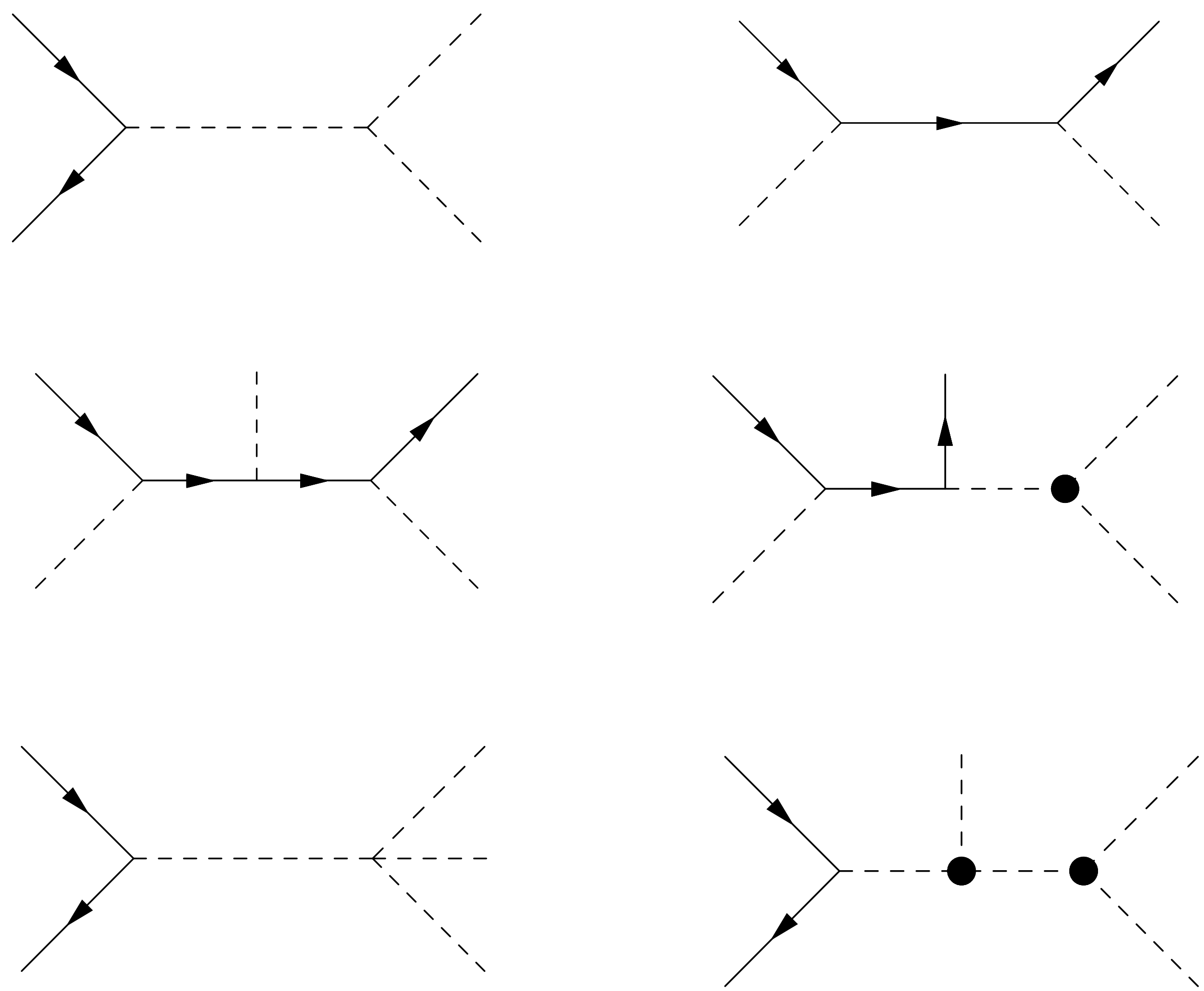


Li, Shu, Xiao, JHYu, arXiv: 2012.11615

Li, Ren, Xiao, JHYu, Zheng, in preparation

UV Origin of SMEFT

Analyze the possible UV resonances of effective operators by topologies



$\psi^2\varphi^2, \psi^2 F^2, \psi^2\varphi^2 D, \cdots$

$\psi^2\varphi^3, \psi^2\varphi^2 F, \psi^2\varphi^3 D, \cdots$

+ loops

J-Basis Operator: Partial Wave

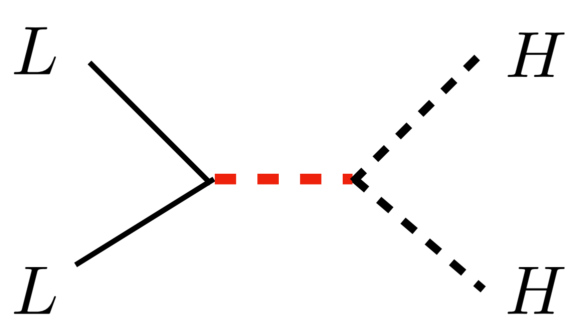
$$\mathcal{Y}\left[\boxed{p}\boxed{r}\right] \epsilon_{ik}\epsilon_{jl}\epsilon_{\alpha\beta} L_p^{\alpha i} L_r^{\beta j} H^k H^l$$

Partial wave expansion on operator

$$\mathbf{W}^2 \mathcal{B}^J = -sJ(J+1)\mathcal{B}^J$$

$LL \to HH$ channel

```
GetJBasisForType[Model, "H"^2 "L"^2, {{1, 2}, {3, 4}}]
⟨|basis → {ϵ^ik ϵ^jl H_k H_l (L_pi L_rj), ϵ^ij ϵ^kl H_k H_l (L_pi L_rj)},
 j-basis → {<|{L1, L2} → {0, {0, 0}, {2}}, {H3, H4} → {0, {0, 0}, {2}}|> → {{-1, 1/2}},
   <|{L1, L2} → {0, {0, 0}, {0}}, {H3, H4} → {0, {0, 0}, {0}}|> → {{0, -1/2}}}|⟩
```

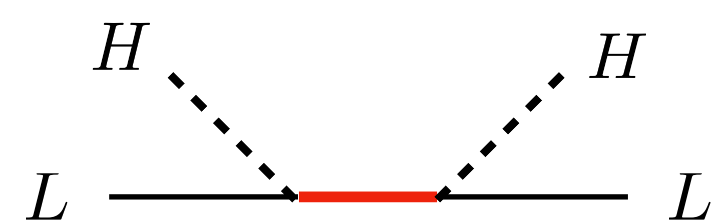


$J = 0$

Type-II: SU(2) **triplet**, or singlet (excluded by repeated field)

$LH \to LH$ channel

```
GetJBasisForType[Model, "H"^2 "L"^2, {{1, 3}, {2, 4}}]
⟨|basis → {ϵ^ik ϵ^jl H_k H_l (L_pi L_rj), ϵ^ij ϵ^kl H_k H_l (L_pi L_rj)},
 j-basis → {⟨|{L1, H3} → {1/2, {0, 0}, {2}}, {L2, H4} → {1/2, {0, 0}, {2}}|⟩ → {{1/2, -1}},
   ⟨|{L1, H3} → {1/2, {0, 0}, {0}}, {L2, H4} → {1/2, {0, 0}, {0}}|⟩ → {{-1, 0}}}|⟩
```



$J = \frac{1}{2}$

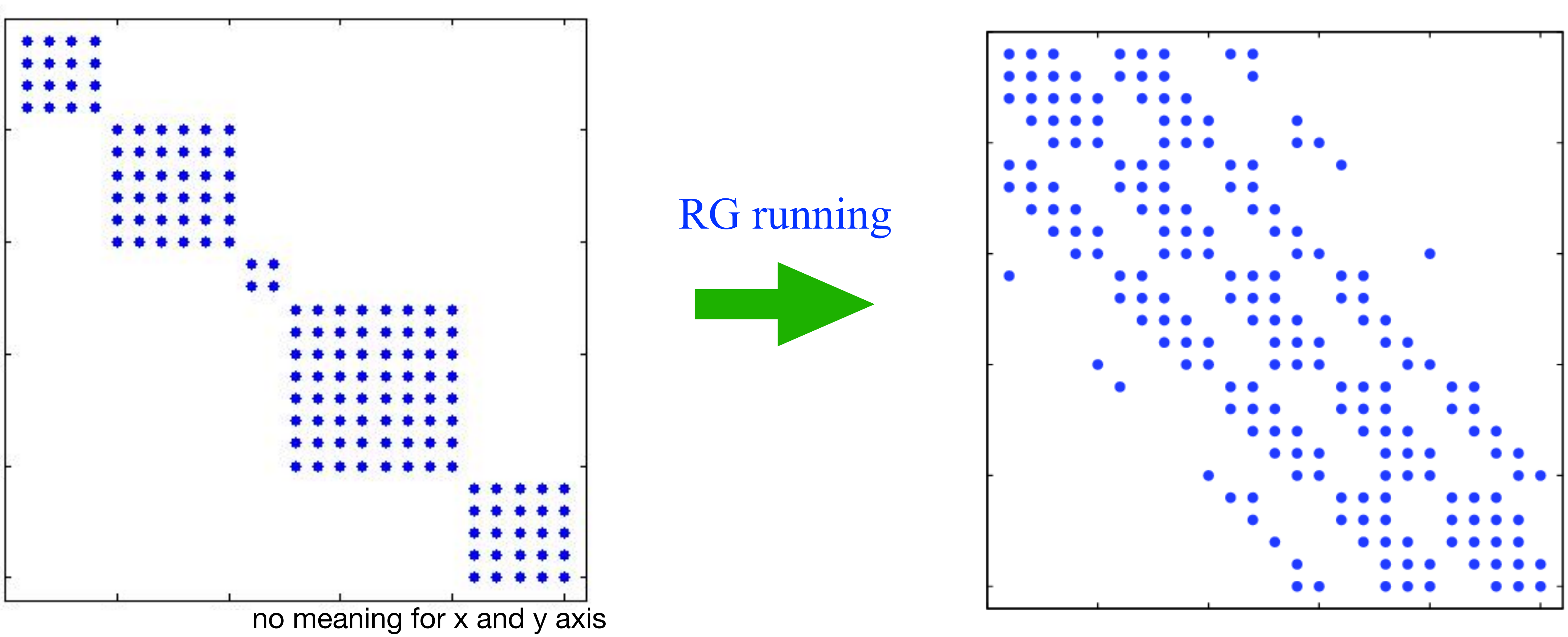
Type-I and III: SU(2) **single** and **triplet**

More complex topology done similarly

Refer to Ming-Lei Xiao's talk for details

Operator RG Flow

RG running (anomalous dilatation) mix among classes of operators



Operator space

Operator space

Young tensor basis provides a preferred basis to perform RG Running!

Refer to Ming-Lei Xiao's talk for details

Summary

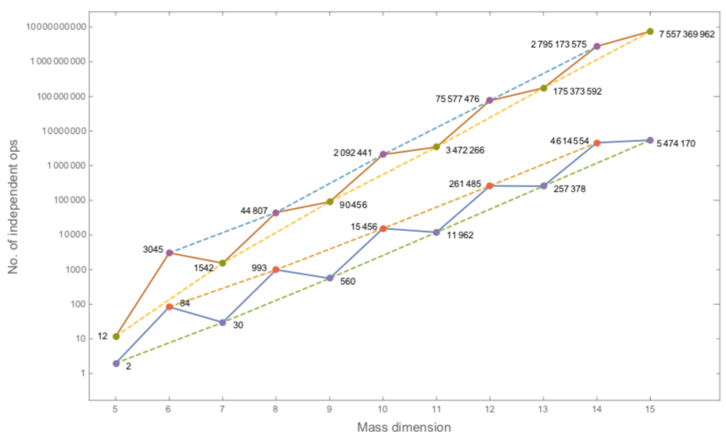
Take home message 1: From operator counting to operator writing systematically

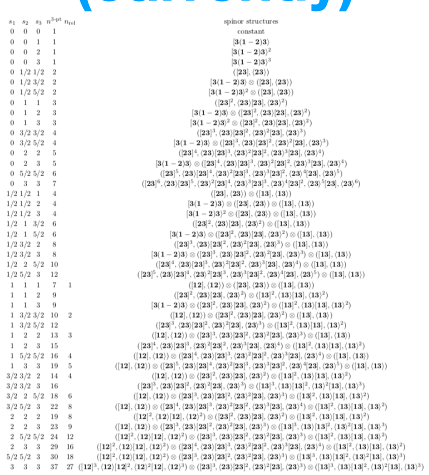
Hilbert series → On-shell EFT → Young tensor

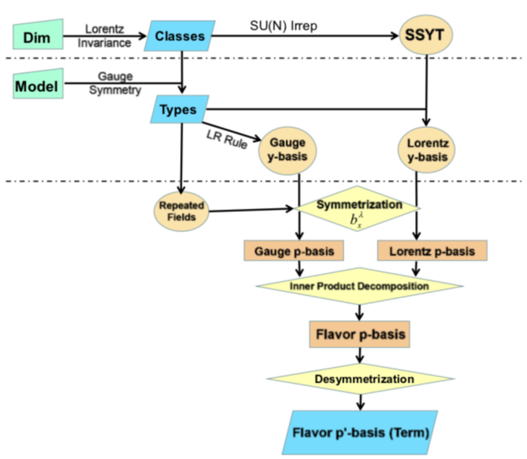
Only counting

up to 4 fields in operator (currently)

Any operator to any mass dimension







Take home message 2: Unified construction of Lorentz&gauge by Young tableau

For any generic EFT with Lorentz and any gauge symmetry

Thank you for listening!

Backup Slides

Notation in the talk

$$\psi_\alpha \in (1/2, 0), \quad \psi^\dagger_{\dot\alpha} \in (0, 1/2), \qquad H_i \in (0,0)\,, \quad H^{\dagger i} \in (0,0)\,,$$

$$F_{\mathrm{L}\alpha\beta} = \frac{i}{2} F_{\mu\nu}\sigma^{\mu\nu}_{\alpha\beta} \in (1,0), \quad F_{\mathrm{R}\dot\alpha\dot\beta} = -\frac{i}{2} F_{\mu\nu}\bar\sigma^{\mu\nu}_{\dot\alpha\dot\beta} \in (0,1). \qquad X^{\mu\nu}_{L,R} = \tfrac{1}{2}(X^{\mu\nu} \mp i\widetilde{X}^{\mu\nu})$$

$$h = j_r - j_l$$

Fields	$SU(2)_l \times SU(2)_r$	h	$SU(3)_C$	$SU(2)_W$	$U(1)_Y$	Flavor
$G^A_{\mathrm{L}\alpha\beta}$	$(1,0)$	-1	$\mathbf{8}$	$\mathbf{1}$	0	1
$W^I_{\mathrm{L}\alpha\beta}$	$(1,0)$	-1	$\mathbf{1}$	$\mathbf{3}$	0	1
$B_{\mathrm{L}\alpha\beta}$	$(1,0)$	-1	$\mathbf{1}$	$\mathbf{1}$	0	1
$L_{\alpha i}$	$(\frac{1}{2},0)$	$-1/2$	$\mathbf{1}$	$\mathbf{2}$	$-1/2$	n_f
$e_{\mathrm{c}\alpha}$	$(\frac{1}{2},0)$	$-1/2$	$\mathbf{1}$	$\mathbf{1}$	1	n_f
$Q_{\alpha a i}$	$(\frac{1}{2},0)$	$-1/2$	$\mathbf{3}$	$\mathbf{2}$	$1/6$	n_f
$u^a_{\mathrm{c}\alpha}$	$(\frac{1}{2},0)$	$-1/2$	$\overline{\mathbf{3}}$	$\mathbf{1}$	$-2/3$	n_f
$d^a_{\mathrm{c}\alpha}$	$(\frac{1}{2},0)$	$-1/2$	$\overline{\mathbf{3}}$	$\mathbf{1}$	$1/3$	n_f
H_i	$(0,0)$	0	$\mathbf{1}$	$\mathbf{2}$	$1/2$	1

Hermitian conjugate

$$(F_{\mathrm{L}\alpha\beta})^\dagger = F_{\mathrm{R}\dot\alpha\dot\beta}$$

$H^\dagger$ (and $L^\dagger, Q^\dagger$) as a 2 of $SU(2)$ $\quad \epsilon^{ij} H^\dagger_i H_j. \quad H^\dagger_{\frac{1}{2}} = \epsilon H^\dagger_2$ $\quad e_R = e^\dagger_C, u_R = u^\dagger_C, d_R = d^\dagger_C$

$$\tilde H = \epsilon H^\dagger$$

Fierz and Schouten Identities

$$\Psi = \begin{pmatrix} \xi_\alpha \\ \chi^{\dagger\dot{\alpha}} \end{pmatrix}, \quad \bar{\Psi} = \Psi^\dagger \gamma^0 = \begin{pmatrix} \chi^\alpha, & \xi^\dagger_{\dot{\alpha}} \end{pmatrix}$$

Fierz identity

$$\begin{pmatrix} \delta_{ij}\delta_{kl} \\ (\gamma^\mu)_{ij}(\gamma_\mu)_{kl} \\ \frac{1}{2}(\sigma^{\mu\nu})_{ij}(\sigma_{\mu\nu})_{kl} \\ (\gamma^\mu\gamma_5)_{ij}(\gamma_\mu\gamma_5)_{kl} \\ (\gamma_5)_{ij}(\gamma_5)_{kl} \end{pmatrix} = \begin{pmatrix} 1/4 & 1/4 & 1/4 & -1/4 & 1/4 \\ 1 & -1/2 & 0 & -1/2 & -1 \\ 3/2 & 0 & -1/2 & 0 & 3/2 \\ -1 & -1/2 & 0 & -1/2 & 1 \\ 1/4 & -1/4 & 1/4 & 1/4 & 1/4 \end{pmatrix} \begin{pmatrix} \delta_{il}\delta_{kj} \\ (\gamma^\mu)_{il}(\gamma_\mu)_{kj} \\ \frac{1}{2}(\sigma^{\mu\nu})_{il}(\sigma_{\mu\nu})_{kj} \\ (\gamma^\mu\gamma_5)_{il}(\gamma_\mu\gamma_5)_{kj} \\ (\gamma_5)_{il}(\gamma_5)_{kj} \end{pmatrix}$$

Schouten identity

$$g_{\mu\nu}\sigma^\mu_{\alpha\dot{\alpha}}\sigma^\nu_{\beta\dot{\beta}} = 2\epsilon_{\alpha\beta}\epsilon_{\dot{\alpha}\dot{\beta}},$$

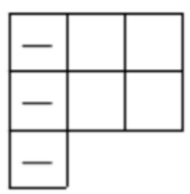
$$\epsilon^{\alpha\beta}\delta^\gamma_\kappa + \epsilon^{\beta\gamma}\delta^\alpha_\kappa + \epsilon^{\gamma\alpha}\delta^\beta_\kappa = 0.$$

$$\tilde{\epsilon}_{\dot{\alpha}\dot{\beta}}\delta^{\dot{\kappa}}_{\dot{\gamma}} + \tilde{\epsilon}_{\dot{\beta}\dot{\gamma}}\delta^{\dot{\kappa}}_{\dot{\alpha}} + \tilde{\epsilon}_{\dot{\gamma}\dot{\alpha}}\delta^{\dot{\kappa}}_{\dot{\beta}} = 0$$

$$\begin{aligned}(\bar{d}l)(\bar{l}d) =& -\frac{1}{4}(\bar{d}d)(\bar{l}l) - \frac{1}{4}(\bar{d}\gamma^\mu d)(\bar{l}\gamma_\mu l) - \frac{1}{8}(\bar{d}\sigma^{\mu\nu}d)(\bar{l}\sigma_{\mu\nu}l) + \frac{1}{4}(\bar{d}\gamma^\mu\gamma_5 d)(\bar{l}\gamma_\mu\gamma_5 l) - \frac{1}{4}(\bar{d}\gamma_5 d)(\bar{l}\gamma_5 l) \\ =& -\frac{1}{2}(\bar{d}\gamma^\mu d)(\bar{l}\gamma_\mu l)\,, \\ (\bar{l}C\bar{q})(lCq) =& -\frac{1}{4}(\bar{l}l)(\bar{q}q) + \frac{1}{4}(\bar{l}\gamma^\mu l)(\bar{q}\gamma_\mu q) + \frac{1}{8}(\bar{l}\sigma^{\mu\nu}l)(\bar{q}\sigma_{\mu\nu}q) + \frac{1}{4}(\bar{l}\gamma^\mu\gamma_5 l)(\bar{q}\gamma_\mu\gamma_5 q) - \frac{1}{4}(\bar{l}\gamma_5 l)(\bar{q}\gamma_5 q) \\ =& \frac{1}{2}(\bar{l}\gamma^\mu l)(\bar{q}\gamma_\mu q)\,. \end{aligned}$$

Different Filling for Young Diagram

[Henning, Melia, 2019]



SSYT for class I

SSYT for class II

....

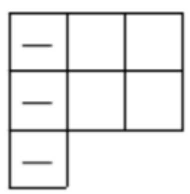
SSYT for helicity = 3/2

SSYT for helicity = 2

Further selection

Operator Lorentz structure
(cannot deal repeated field)

[Li, Ren, Xiao, Yu, Zheng, 2020]



SSYT for unique class and helicity

One-to-one correspondence
No need selection

Operator Lorentz structure
(can deal repeated field)

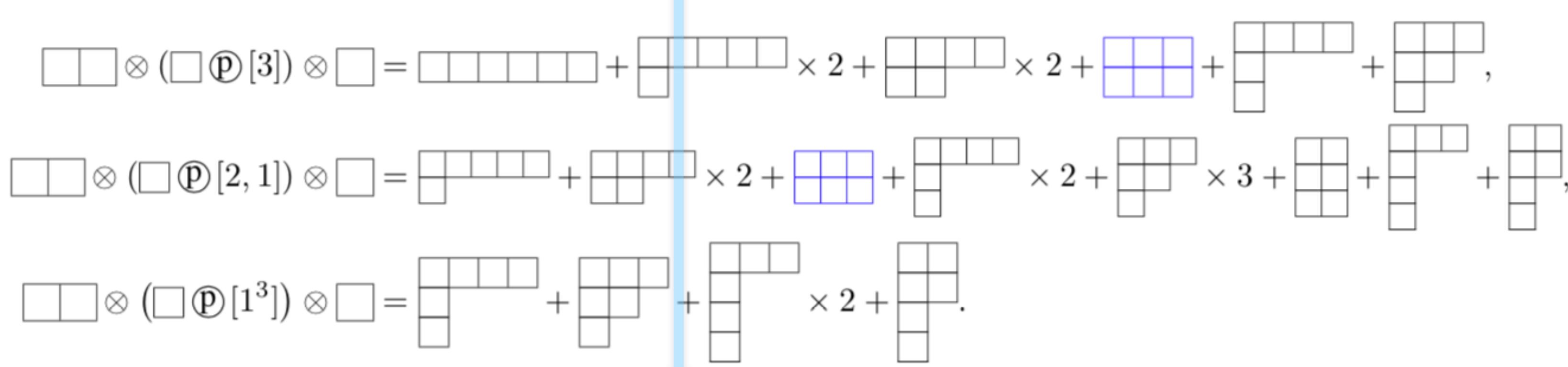
Young Tensor with Repeated Field

$W_{\mathrm{L}} Q^3 L$

$n = 3, \tilde{n} = 0. \quad \#1 = 2, \#2 = \#3 = \#4 = \#5 = 1$

$$\begin{array}{|c|c|c|}\hline 1 & 1 & 2 \\ \hline 3 & 4 & 5 \\ \hline \end{array} \qquad \begin{array}{|c|c|c|}\hline 1 & 1 & 3 \\ \hline 2 & 4 & 5 \\ \hline \end{array} \qquad \begin{array}{|c|c|c|}\hline 1 & 1 & 4 \\ \hline 2 & 3 & 5 \\ \hline \end{array}$$

$$\epsilon^{\alpha_1 \alpha_3} \epsilon^{\alpha_1 \alpha_4} \epsilon^{\alpha_2 \alpha_5} \qquad \epsilon^{\alpha_1 \alpha_2} \epsilon^{\alpha_1 \alpha_4} \epsilon^{\alpha_3 \alpha_5} \qquad \epsilon^{\alpha_1 \alpha_2} \epsilon^{\alpha_1 \alpha_3} \epsilon^{\alpha_4 \alpha_5}$$



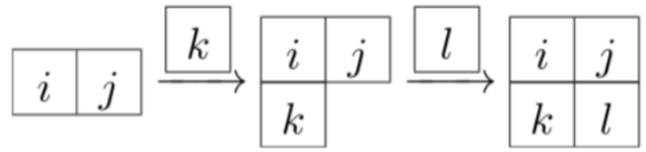
$$\mathcal{M}^{[1^3]}_{3,1} = \frac{1}{3} (\mathcal{M}_1 + \mathcal{M}_2 + \mathcal{M}_3)$$

$$\mathcal{M}^{[2,1]}_{3,x} = \left\{ \frac{1}{3} (\mathcal{M}_1 + \mathcal{M}_2 - 2\mathcal{M}_3) \ , \ \frac{1}{3} (\mathcal{M}_1 - 2\mathcal{M}_2 + \mathcal{M}_3) \right\}_x$$

Gauge Structure Details

$W e_{\mathbb{C}} L H^\dagger D^2$

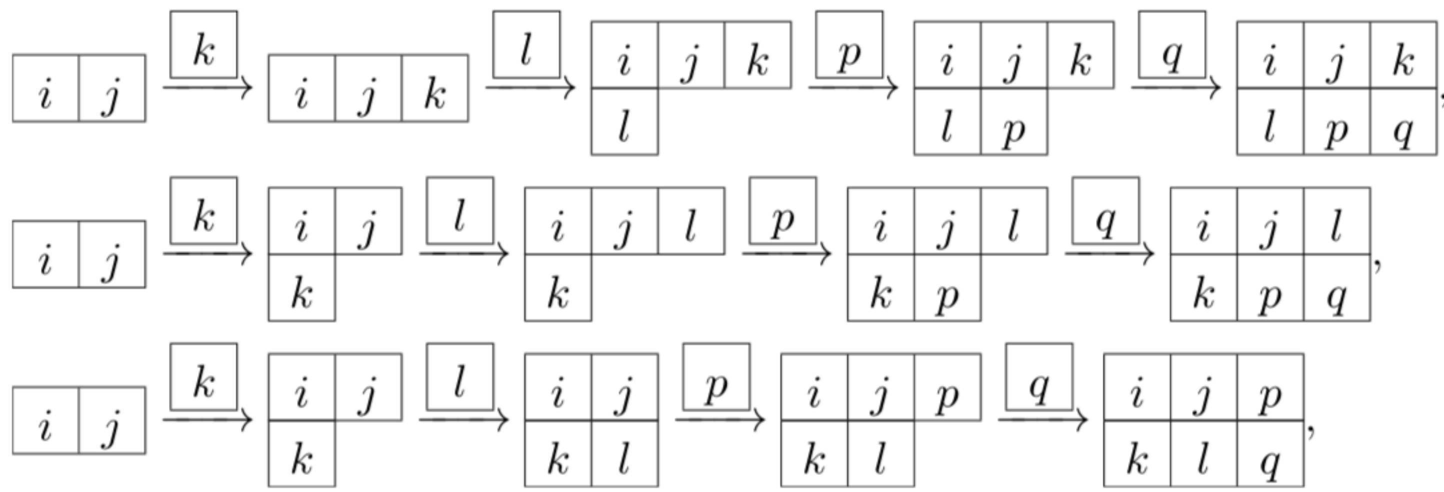
$(\tau^I)_{ij} W^I$: $\boxed{i\,|\,j}$, L_k: $\boxed{k}$, $H_l^\dagger$: $\boxed{l}$



$$\epsilon^{ik}\epsilon^{jl}\,(\tau^I)_{ij} W^I L_k H_l^\dagger \propto (\tau^I)^i_j\, W^I L_i H^{\dagger j}$$

$W e_{\mathbb{C}} L H H^{\dagger 2}$

$(\tau^I)_i^{z_1}\epsilon_{j z_1} W^I : \boxed{i\,|\,j}$, $L_k : \boxed{k}$, $H_l : \boxed{l}$, $\epsilon_{pm} H^{\dagger m} : \boxed{p}$, $\epsilon_{qn} H^{\dagger n} : \boxed{q}$,



$$\epsilon^{il}\epsilon^{jp}\epsilon^{kq} \to \delta^k_n (\tau^I)^l_m$$

$$\epsilon^{ik}\epsilon^{jp}\epsilon^{lq} \to \delta^l_n (\tau^I)^k_m$$

$2 \times \boxed{\,|\,}$

$$\delta^k_n (\tau^I)^l_m + \delta^k_m (\tau^I)^l_n$$

$$\delta^l_n (\tau^I)^k_m + \delta^l_m (\tau^I)^k_n$$

$$\epsilon^{ik}\epsilon^{jl}\epsilon^{pq} \to \delta^l_n (\tau^I)^k_m - \delta^l_m (\tau^I)^k_n$$