STrEAMlining EFT Matching

Higgs and Effective Field Theory, 2021

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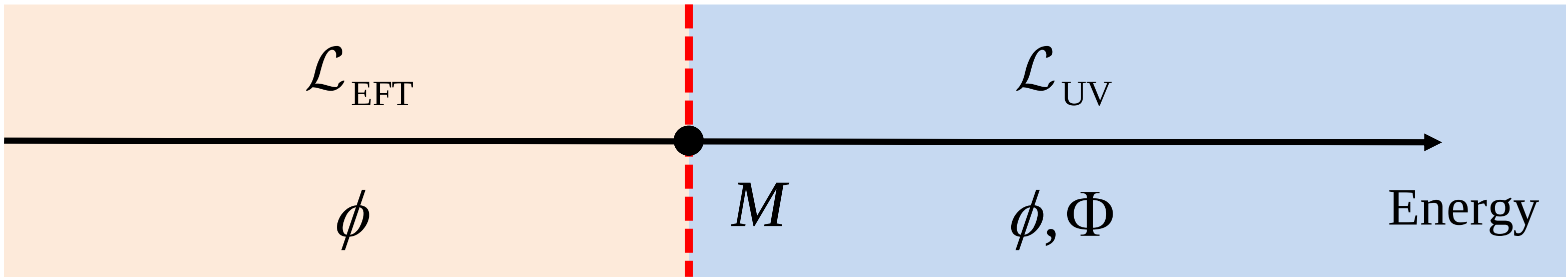
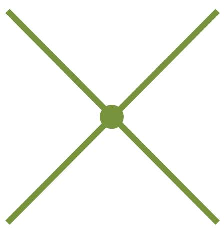
arXiv: 2011.02484, 2012.07851
with Tim Cohen and Zhengkang (Kevin) Zhang

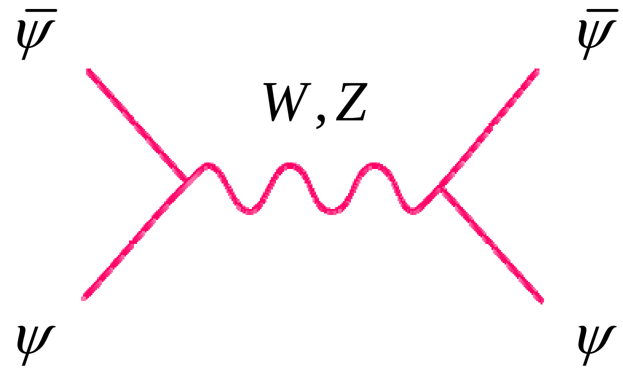
Outline

- What is EFT matching? Why do we need it?
- How to perform EFT matching calculation?
  --- Amplitude approach vs functional methods
- Functional supertraces
  --- General: Log-type and Power-type
  --- SuperTrace Evaluation Automated for Matching
- Prescription summary and application to SM + Singlet

Prototype of EFT matching







$$-\frac{4G_F}{\sqrt{2}}\left[J_{+\mu}J_-^\mu + \left(J_3^\mu - s_\theta^2 J_{\text{EM}}^\mu\right)^2\right]$$

$$\mathcal{L}_{\text{SM}} \supset \frac{g_2}{\sqrt{2}}\left(W_\mu^+ J_+^\mu + W_\mu^- J_-^\mu\right) + \frac{g_2}{c_\theta} Z_\mu \left(J_3^\mu - s_\theta^2 J_{\text{EM}}^\mu\right)$$

Modern examples of EFT matching

$\mathcal{L}_{\text{EFT}}$ | M | $\mathcal{L}_{\text{UV}}$ → Energy

- **SMEFT (or HEFT)**

  EFT: $\phi_{\text{SM}} = \{G_\mu, W_\mu, B_\mu, \psi, H\}$ | UV: $\phi_{\text{SM}} \;,\; \Phi_{\text{BSM}}$

- **Low-energy Effective Field Theory (LEFT)**

  EFT: $\phi = \phi_{\text{SM}} / \{t_L, t_R, W^\pm, Z, h\}$ | UV: $\phi \;,\; \Phi = \{t_L, t_R, W^\pm, Z, h, \Phi_{\text{BSM}}\}$

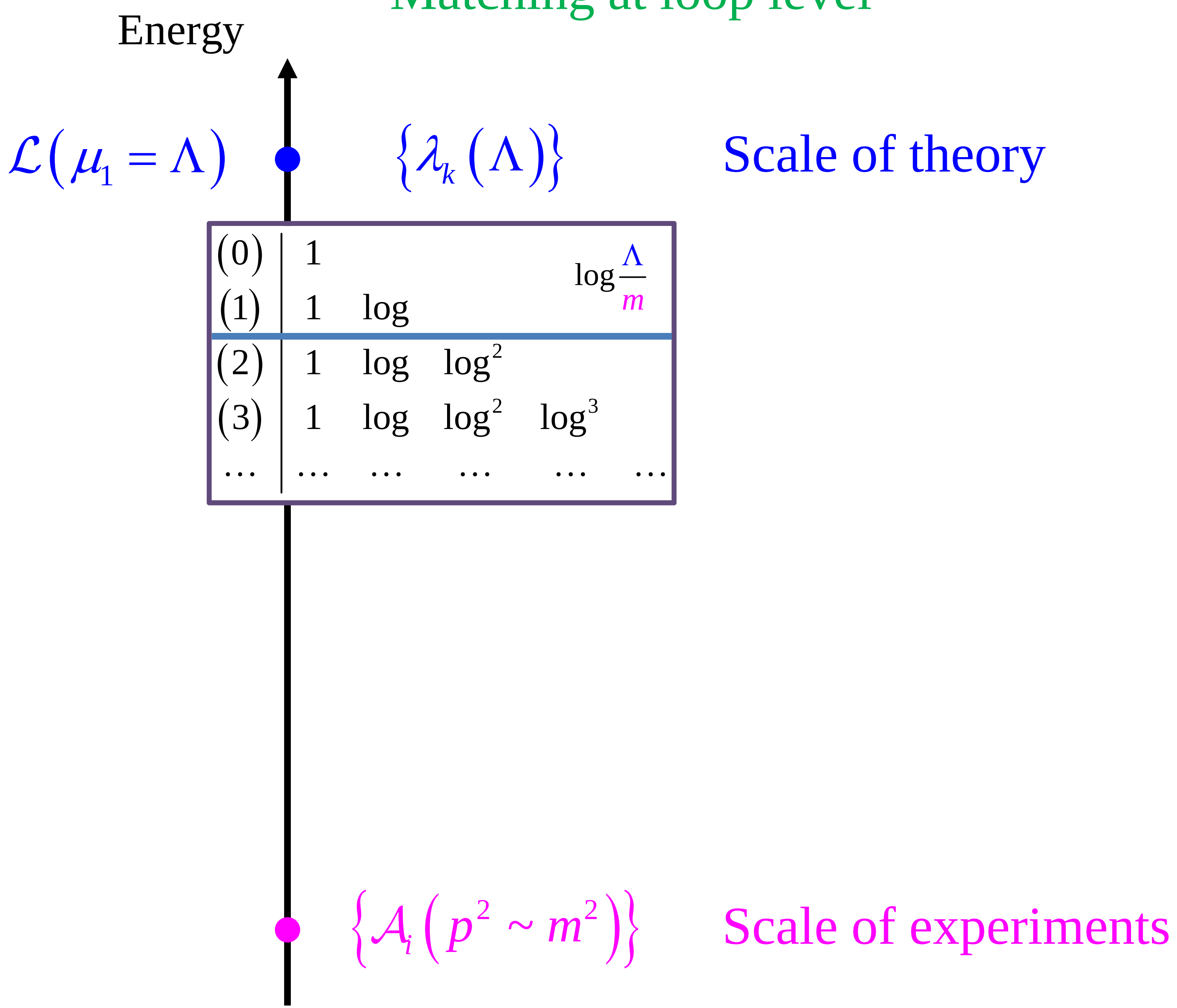
- **Heavy Quark Effective Theory (HQET)**

  EFT: ϕ_v | UV: $\phi_v = e^{im_Q v\cdot x}\dfrac{1+\not{v}}{2}Q \;,\; \Phi_v = e^{im_Q v\cdot x}\dfrac{1-\not{v}}{2}Q$

- **Mesonic QCD Chiral Lagrangian**

  EFT: $\phi = \pi$ | UV: $\Phi = \{G_\mu, q, \bar{q}\}$

Matching at loop level



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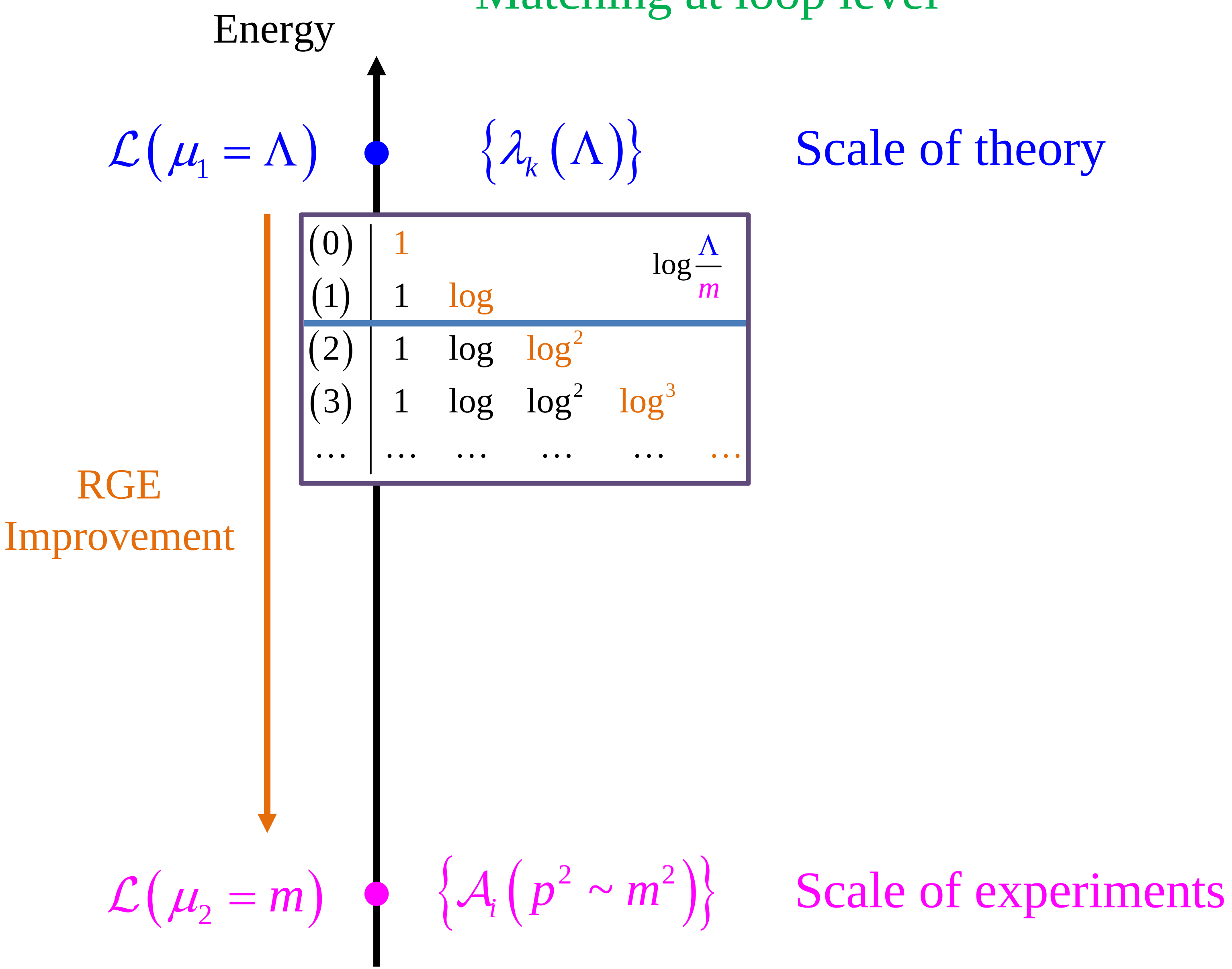

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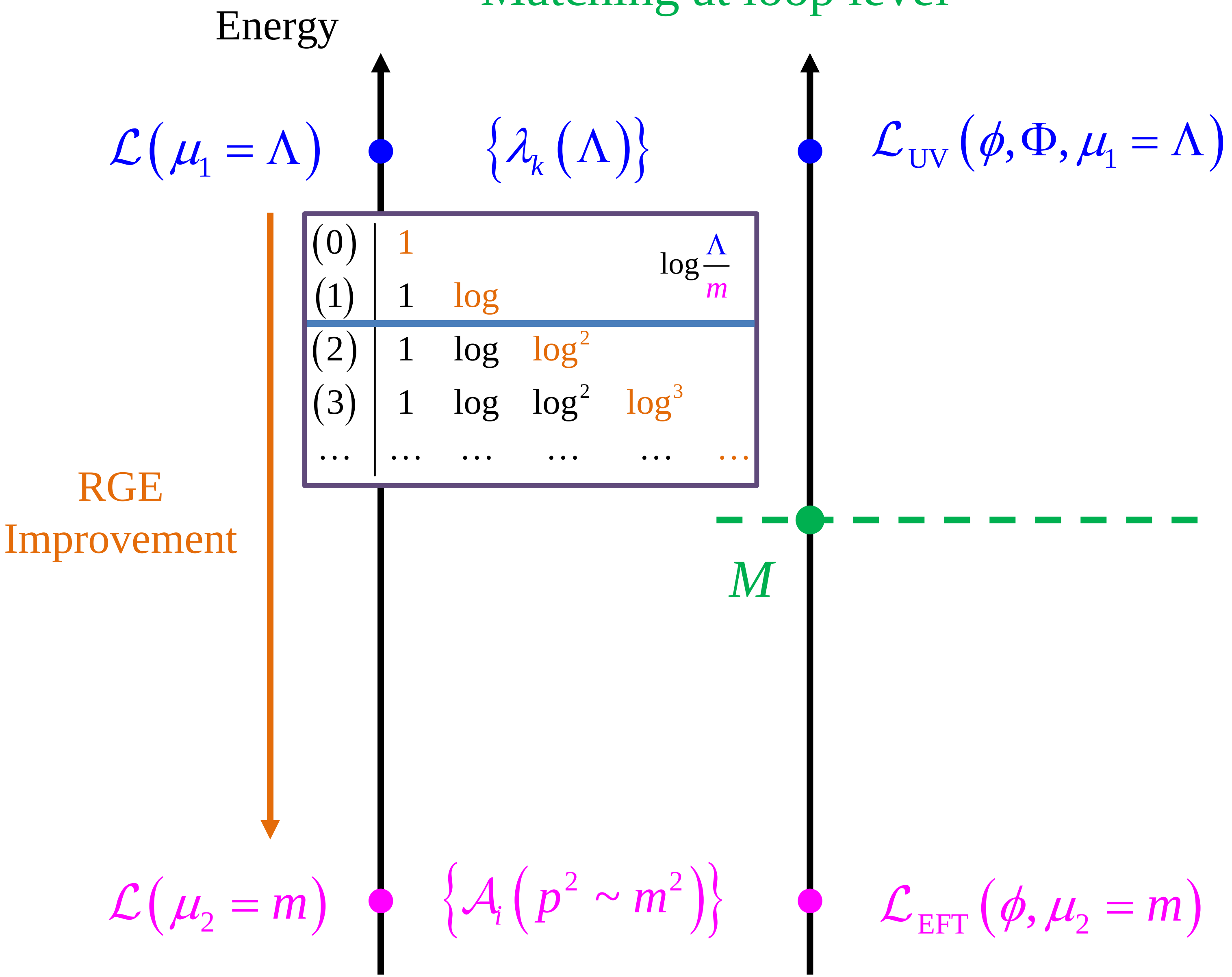

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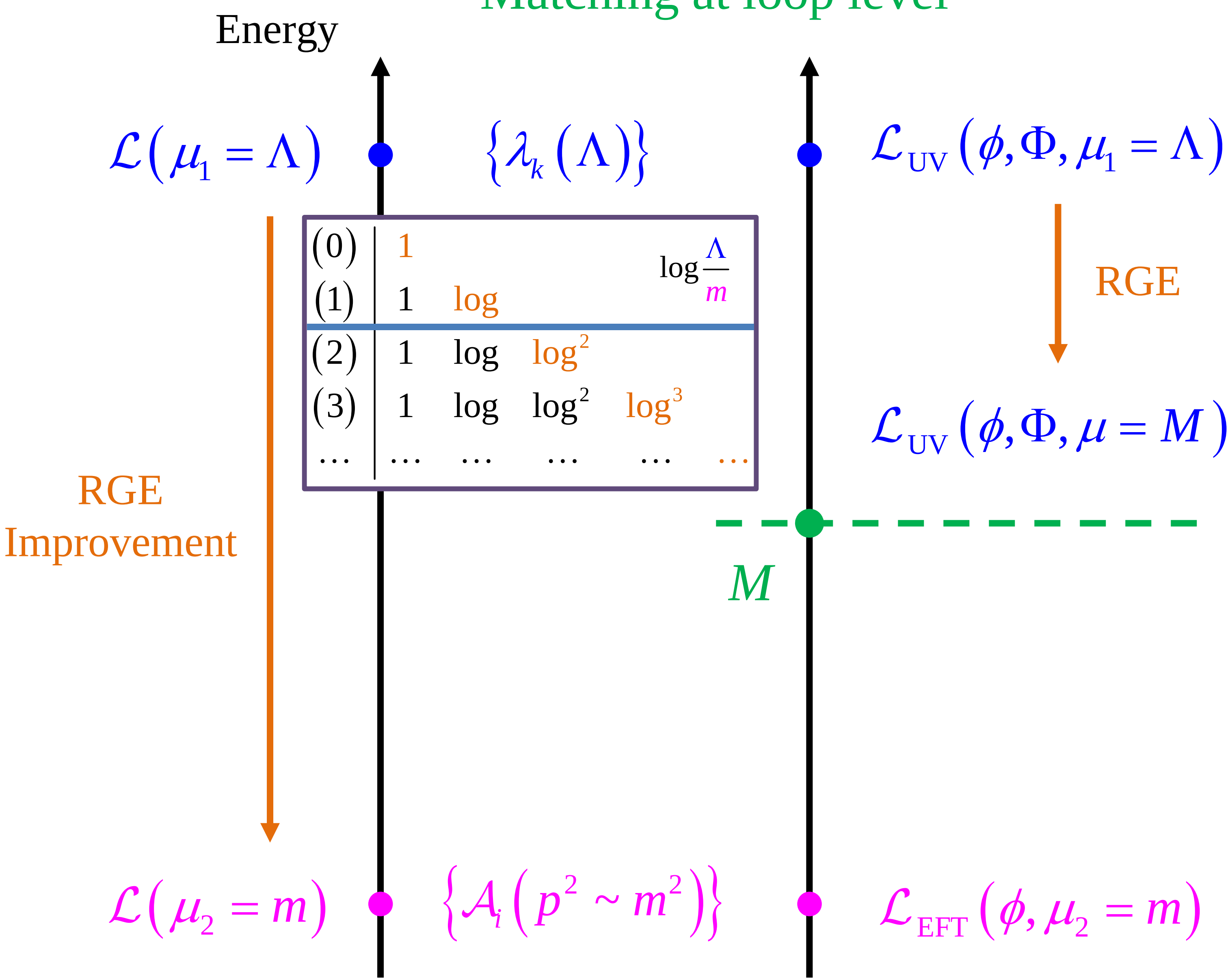

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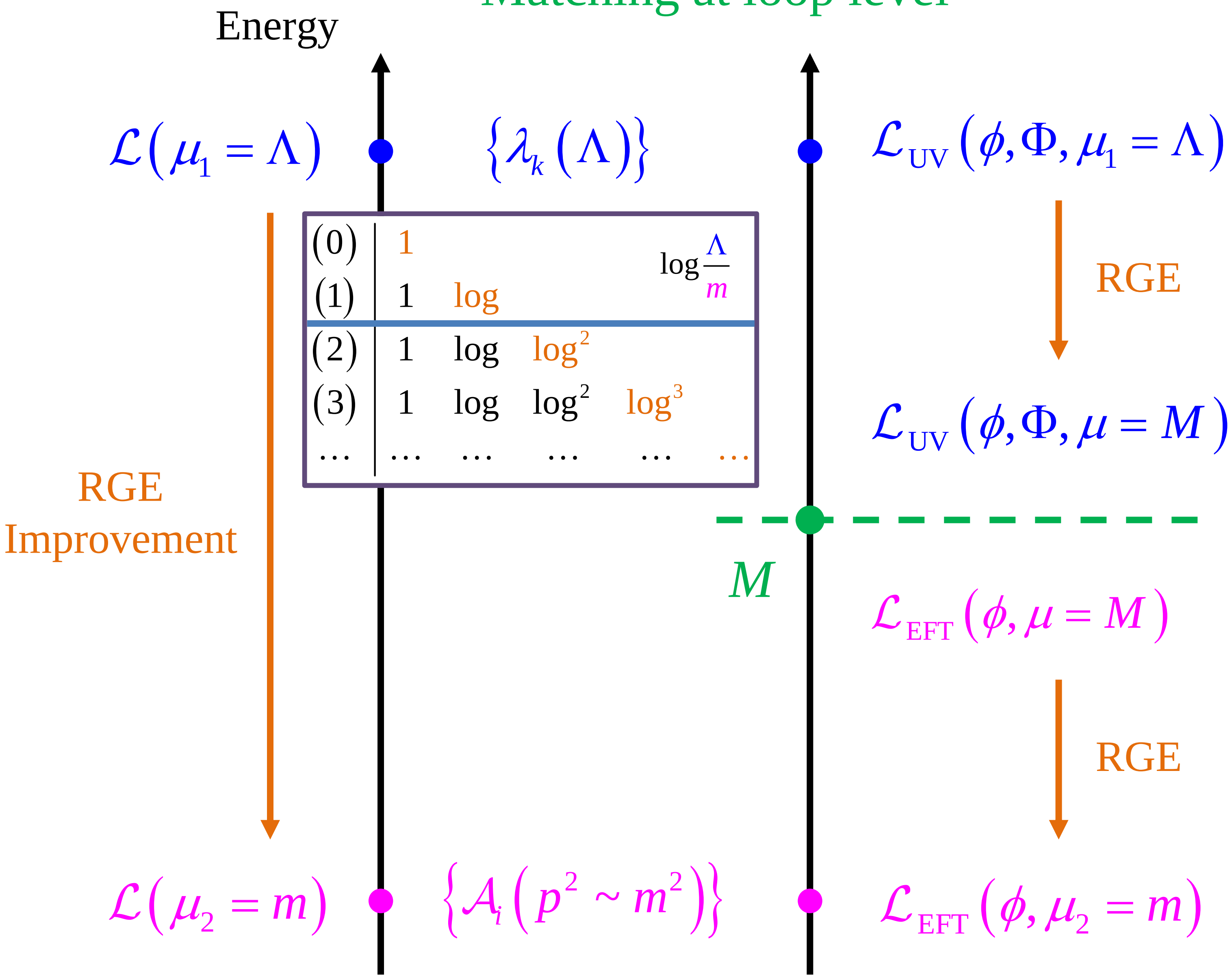

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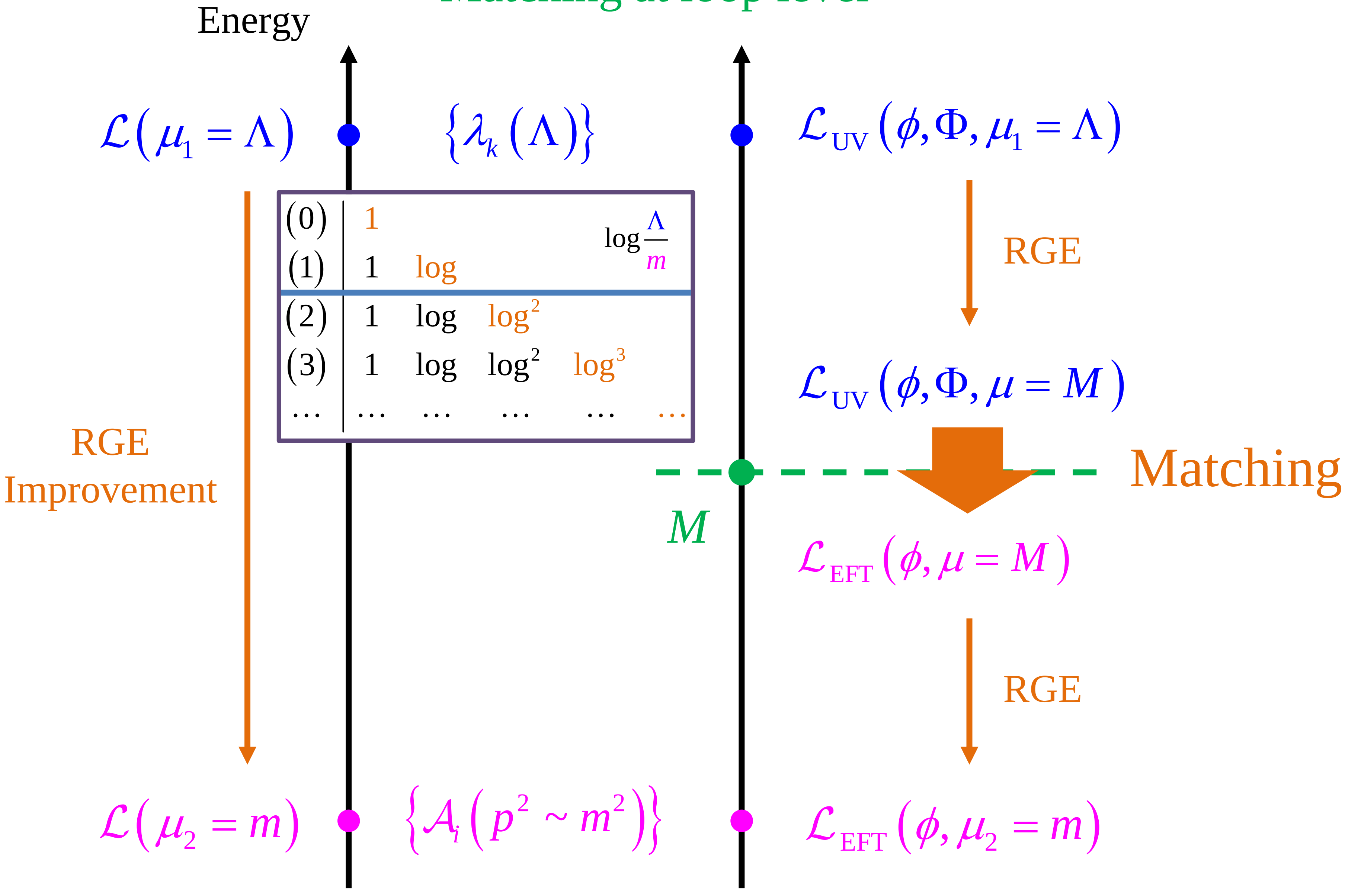
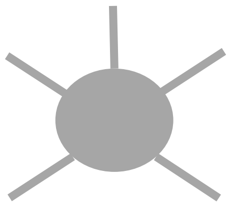


Summary of Lessons

- Effective Field Theories (EFTs) describe physics with only accessible degrees of freedom, i.e. (modes of) fields
- At tree level, one can obtain the EFT by "shrinking" heavy propagators
- At loop level, calculating with a theory defined at a UV scale typically results in large logs
- Running the theory down to the scale of experiments resums leading large logs, and helps with precision
- A mass threshold in between the UV scale and the scale of experiments blocks a smooth RGE, and matching is needed

Matching by Amplitudes

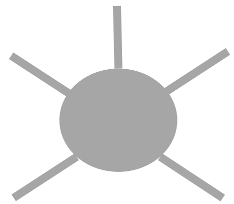
$$\mathcal{L}_{\text{UV}} = \mathcal{L}_{\text{SM}} + \mathcal{L}_{\text{BSM}}\left(\phi_{\text{SM}}, \Phi_{\text{BSM}}\right)$$



$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \sum_i C_i\, \mathcal{O}_i\left(\phi_{\text{SM}}\right)$$

Matching by Amplitudes

$$\mathcal{L}_{\text{UV}} = \mathcal{L}_{\text{SM}} + \mathcal{L}_{\text{BSM}}\left(\phi_{\text{SM}}, \Phi_{\text{BSM}}\right) \Rightarrow \left\{\mathcal{A}_{\text{UV}}\left(\lambda_{\text{UV}}\right)\right\}$$



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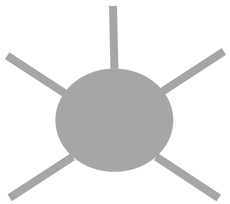

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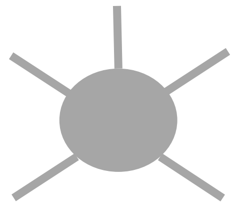

$$\Updownarrow \Rightarrow C_i\left(\lambda_{\text{UV}}\right)$$

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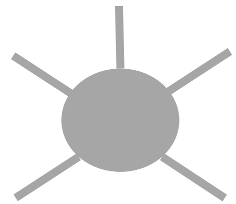

$$C_i\left(\lambda_{\text{UV}}\right)$$

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Challenges in amplitude approach:

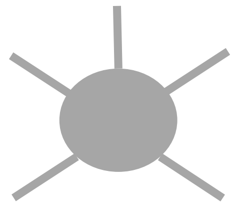
1. Need to know the effective operators $\mathcal{O}_i\left(\phi\right)$ in advance
2. Need to figure out the set of amplitudes $\left\{\mathcal{A}_i\right\}$ to compute

   --- often complicated by linear redundancies among $\mathcal{O}_i\left(\phi\right)$
3. Computationally expensive

Matching by Amplitudes

$$\mathcal{L}_{\rm UV} = \mathcal{L}_{\rm SM} + \mathcal{L}_{\rm BSM}\left(\phi_{\rm SM}, \Phi_{\rm BSM}\right) \rightarrow \left\{\mathcal{A}_{\rm UV}\left(\lambda_{\rm UV}\right)\right\}$$

Ideally $\downarrow$ $\mathcal{L}_{\rm EFT}\left(\mathcal{L}_{\rm UV}\right)$



$\updownarrow \rightarrow C_i\left(\lambda_{\rm UV}\right)$

$$\mathcal{L}_{\rm SMEFT} = \mathcal{L}_{\rm SM} + \sum_i C_i \mathcal{O}_i\left(\phi_{\rm SM}\right) \rightarrow \left\{\mathcal{A}_{\rm SMEFT}\left(C_i\right)\right\}$$

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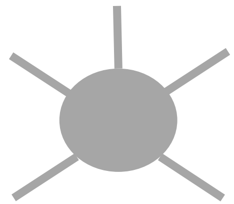
Ideally ⬇ $\mathcal{L}_{\text{EFT}}\left(\mathcal{L}_{\text{UV}}\right)$



$\Rightarrow C_i\left(\lambda_{\text{UV}}\right)$ ⬍ Functional Matching

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \sum_i C_i \mathcal{O}_i\left(\phi_{\text{SM}}\right) \Rightarrow \left\{\mathcal{A}_{\text{SMEFT}}\left(C_i\right)\right\} \qquad \Gamma_{\text{EFT, 1PI}}[\phi]$$

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$$\Gamma_{\text{EFT, 1PI}}[\phi] = \Gamma_{\text{UV, 1LPI}}[\phi] \quad \Rightarrow \quad \begin{cases} \mathcal{L}_{\text{EFT}}^{(\text{tree})} = \mathcal{L}_{\text{UV}}\left(\phi, \Phi = \Phi_c[\phi]\right) \quad , \quad \left.\dfrac{\delta S_{\text{UV}}}{\delta \Phi}\right|_{\Phi=\Phi_c[\phi]} = 0 \\ \displaystyle\int d^4x\, \mathcal{L}_{\text{EFT}}^{(\text{1-loop})} = \frac{i}{2} \text{STr} \log \left[\left. -\frac{\delta^2 S_{\text{UV}}}{\delta\left(\phi, \Phi\right)^2} \right|_{\Phi=\Phi_c} \right] \Bigg|_{\text{hard}} \end{cases}$$

- B. Henning, XL, H. Murayama, "One-loop Matching and Running with Covariant Derivative Expansion," arXiv: 1604.01019
- S. A. R. Ellis, J. Quevillon, T. You, and Z. Zhang, "Mixed heavy-light matching in the Universal One-Loop Effective Action," arXiv: 1604.02445
- J. Fuentes-Martin, J. Portoles, and P. Ruiz-Femenia, "Integrating out heavy particles with functional methods: a simplified framework," arXiv: 1607.02142
- Z. Zhang, "Covariant diagrams for one-loop matching," arXiv: 1610.00710

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Method of regions

M. Beneke and V. A. Smirnov, "Asymptotic expansion of Feynman integrals near threshold," *Nucl. Phys.* **B522** (1998) 321–344, arXiv:hep-ph/9711391 [hep-ph].

V. A. Smirnov, "Applied asymptotic expansions in momenta and masses," *Springer Tracts Mod. Phys.* **177** (2002) 1–262.

$$-i\,\mathrm{STr}\left(\frac{1}{P^2 - M^2} U_1 \frac{1}{P^2 - m^2} U_2\right) = -i\int \mathrm{d}^d x \int \frac{\mathrm{d}^d q}{(2\pi)^d} \langle q|x\rangle \left\langle x \left| \mathrm{tr}\left(\frac{1}{P^2 - M^2} U_1 \frac{1}{P^2 - m^2} U_2\right)\right| q\right\rangle$$

$$= -i\int \mathrm{d}^d x \int \frac{\mathrm{d}^d q}{(2\pi)^d}\,\mathrm{tr}\left[\frac{1}{(P_\mu - q_\mu)^2 - M^2} U_1 \frac{1}{(P_\mu - q_\mu)^2 - m^2} U_2\right]$$

$$\supset -i\int \mathrm{d}^d x \int \frac{\mathrm{d}^d q}{(2\pi)^d}\,\mathrm{tr}\left[\frac{1}{q^2 - M^2} U_1 \frac{1}{q^2 - m^2} U_2\right] + \mathcal{O}(P_\mu)$$

$$|q| \sim M \gg m \quad \Rightarrow \quad \frac{1}{q^2 - m^2} = \frac{1}{q^2} + \frac{m^2}{q^4} + \frac{m^4}{q^6} + \cdots$$

$$\Gamma_{\text{EFT, 1PI}}[\phi] = \Gamma_{\text{UV, 1LPI}}[\phi] \quad \Rightarrow \quad \begin{cases} \mathcal{L}_{\text{EFT}}^{(\text{tree})} = \mathcal{L}_{\text{UV}}\left(\phi, \Phi = \Phi_c[\phi]\right) \quad , \quad \left.\dfrac{\delta S_{\text{UV}}}{\delta \Phi}\right|_{\Phi=\Phi_c[\phi]} = 0 \\ \displaystyle\int \mathrm{d}^4 x\, \mathcal{L}_{\text{EFT}}^{(\text{1-loop})} = \frac{i}{2} \operatorname{STr} \log \left[\left. -\frac{\delta^2 S_{\text{UV}}}{\delta(\phi, \Phi)^2} \right|_{\Phi=\Phi_c} \right] \Bigg|_{\text{hard}} \end{cases}$$

Applicability:

- Any spin: scalars, fermions, vector bosons
- Contributions from heavy-light loops
- Derivative interactions in UV
- Non-renormalizable interactions in UV
- Non-relativistic EFT matching, e.g. HQET

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How to compute this functional SuperTrace?

(Cohen, XL, Zhang, arXiv: 2011.02484)

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K and X are matrices on $\varphi \equiv \begin{pmatrix} \phi \\ \Phi \end{pmatrix}$

$$= \frac{i}{2}\,\text{STr}\log(K - X)\big|_{\text{hard}}$$

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Focusing on relativistic EFTs:

$$K_i = \begin{cases} P^2 - m_i^2 & (\text{spin - }0) \\ \not{P} - m_i & (\text{spin - }1/2) \\ -\eta^{\mu\nu}\left(P^2 - m_i^2\right) & (\text{spin - }1) \end{cases}, \qquad X_{ij} = U_{ij} + P_\mu Z_{ij}^\mu + \bar{Z}_{ij}^\mu P_\mu + \mathcal{O}\left(P_\mu^2\right)$$

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$$\int d^4x\, \mathcal{L}_{\rm EFT}^{(1\text{-loop})} = \frac{i}{2}\,{\rm STr}\log\left[\left. -\frac{\delta^2 S_{\rm UV}}{\delta(\phi,\Phi)^2}\right|_{\Phi=\Phi_c}\right]\Bigg|_{\rm hard}$$

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$$= \frac{i}{2}\,{\rm STr}\log K\Big|_{\rm hard} - \frac{i}{2}\sum_{n=1}^{\infty}\frac{1}{n}\,{\rm STr}\left[\left(\frac{1}{K}X\right)^n\right]\Bigg|_{\rm hard}$$

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positive operator dimension

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Log-type

Power-type

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$$= \int \mathrm{d}^4 x \frac{1}{16\pi^2}\,\mathrm{tr}\left[\left(\log\frac{M^2}{\mu^2}\right)\frac{1}{12}F_{\mu\nu}F^{\mu\nu} + \frac{1}{M^2}\frac{1}{60}\left(P^\mu F_{\mu\nu}\right)\left(P_\rho F^{\rho\nu}\right) - \frac{1}{M^2}\frac{1}{90}iF_\mu{}^\nu F_\nu{}^\rho F_\rho{}^\mu + \cdots\right]$$

$$i\,\mathrm{STr}\log\left(\not{P}-M\right)\Big|_{\text{hard}} = i\int \mathrm{d}^d x \int \frac{\mathrm{d}^d q}{(2\pi)^d}\,\mathrm{tr}\log\left[\left(\not{P}-\not{q}\right)-M\right]$$

$$= \int \mathrm{d}^4 x \frac{1}{16\pi^2}\,\mathrm{tr}\left[\left(\log\frac{M^2}{\mu^2}\right)\frac{1}{12}F_{\mu\nu}F^{\mu\nu} + \frac{1}{M^2}\frac{1}{30}\left(P^\mu F_{\mu\nu}\right)\left(P_\rho F^{\rho\nu}\right) + \frac{1}{M^2}\frac{1}{180}iF_\mu{}^\nu F_\nu{}^\rho F_\rho{}^\mu + \cdots\right]$$

Universal across UV theories

$$= \frac{i}{2}\mathrm{STr}\log K\Big|_{\text{hard}} - \frac{i}{2}\sum_{n=1}^{\infty}\frac{1}{n}\mathrm{STr}\left[\left(\frac{1}{K}X\right)^n\right]\Bigg|_{\text{hard}}$$

Log-type

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$$-i\,\mathrm{STr}\left[\frac{1}{P^2-M^2}U_1\frac{1}{P^2-m^2}P_\mu Z^\mu\frac{1}{P^2}U_3\frac{1}{P^2-m^2}U_4\right]\Bigg|_{\text{hard}}$$

$$=-i\int \mathrm{d}^d x\int\frac{\mathrm{d}^d q}{(2\pi)^d}\,\mathrm{tr}\left[\frac{1}{(P-q)^2-M^2}U_1\frac{1}{(P-q)^2-m^2}\left(P_\mu-q_\mu\right)Z^\mu\frac{1}{(P-q)^2}U_3\frac{1}{(P-q)^2-m^2}U_4\right]$$

$$=\int \mathrm{d}^4 x\frac{1}{16\pi^2}\,\mathrm{tr}\left\{\begin{array}{l}\frac{1}{2M^4}\left(\log\frac{\mu^2}{M^2}+\frac{3}{2}\right)\left[2U_1\left(P_\mu Z^\mu\right)U_3U_4+U_1Z^\mu\left(P_\mu U_3\right)U_4\right]\\+\frac{1}{2M^4}\left(\log\frac{\mu^2}{M^2}+\frac{5}{2}\right)\left(P_\mu U_1\right)Z^\mu U_3U_4+\mathcal{O}\left(U_1Z^\mu U_3U_4P_\mu^3\right)\end{array}\right\}$$

Depend on UV theories

$$=\frac{i}{2}\mathrm{STr}\log K\Big|_{\text{hard}}-\frac{i}{2}\sum_{n=1}^{\infty}\frac{1}{n}\mathrm{STr}\left[\left(\frac{1}{K}X\right)^n\right]\Bigg|_{\text{hard}}$$

Focusing on relativistic EFTs:

Power-type

$$K_i=\begin{cases}P^2-m_i^2 & (\text{spin - 0})\\ \not{P}-m_i & (\text{spin - 1/2})\\ -\eta^{\mu\nu}\left(P^2-m_i^2\right) & (\text{spin - 1})\end{cases},\qquad X_{ij}=U_{ij}+P_\mu Z_{ij}^\mu+\bar{Z}_{ij}^\mu P_\mu+\mathcal{O}\left(P_\mu^2\right)$$

$$-i\,\mathrm{STr}\left[\frac{1}{P^2-M^2}U_1\frac{1}{P^2-m^2}P_\mu Z^\mu\frac{1}{P^2}U_3\frac{1}{P^2-m^2}U_4\right]\Bigg|_{\text{hard}}$$

$$=-i\int \mathrm{d}^d x\int\frac{\mathrm{d}^d q}{(2\pi)^d}\,\mathrm{tr}\left[\frac{1}{(P-q)^2-M^2}U_1\frac{1}{(P-q)^2-m^2}(P_\mu-q_\mu)Z^\mu\frac{1}{(P-q)^2}U_3\frac{1}{(P-q)^2-m^2}U_4\right]$$

$$=\int \mathrm{d}^4 x\frac{1}{16\pi^2}\,\mathrm{tr}\left\{\begin{aligned}&\frac{1}{2M^4}\left(\log\frac{\mu^2}{M^2}+\frac{3}{2}\right)\left[2U_1(P_\mu Z^\mu)U_3U_4+U_1Z^\mu(P_\mu U_3)U_4\right]\\&+\frac{1}{2M^4}\left(\log\frac{\mu^2}{M^2}+\frac{5}{2}\right)(P_\mu U_1)Z^\mu U_3U_4+\mathcal{O}\left(U_1Z^\mu U_3U_4P_\mu^3\right)\end{aligned}\right\}$$

Depend on UV theories

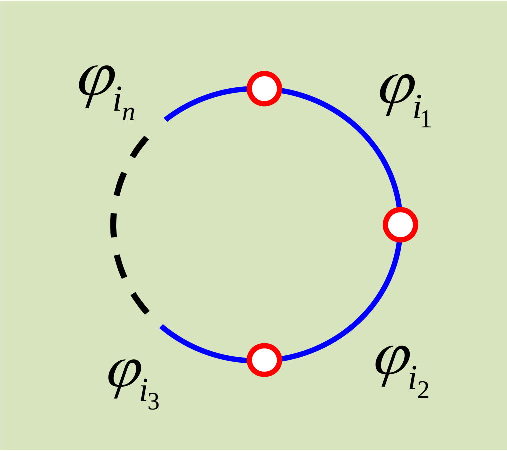
How do we find all of them?

$$=\frac{i}{2}\mathrm{STr}\log K\Big|_{\text{hard}}-\frac{i}{2}\sum_{n=1}^{\infty}\frac{1}{n}\mathrm{STr}\left[\left(\frac{1}{K}X\right)^n\right]\Bigg|_{\text{hard}}$$

Power-type

Focusing on relativistic EFTs:

$$K_i=\begin{cases}P^2-m_i^2 & (\text{spin - }0)\\ \not{P}-m_i & (\text{spin - }1/2)\\ -\eta^{\mu\nu}\left(P^2-m_i^2\right) & (\text{spin - }1)\end{cases},\qquad X_{ij}=U_{ij}+P_\mu Z_{ij}^\mu+\bar{Z}_{ij}^\mu P_\mu+\mathcal{O}\left(P_\mu^2\right)$$



Use "covariant graphs" to enumerate

$$\left. -i\,\mathrm{STr}\left(\frac{1}{K_{i_1}} X_{i_1 i_2} \frac{1}{K_{i_2}} X_{i_2 i_3} \cdots \frac{1}{K_{i_n}} X_{i_n i_1} \right) \right|_{\mathrm{hard}}$$

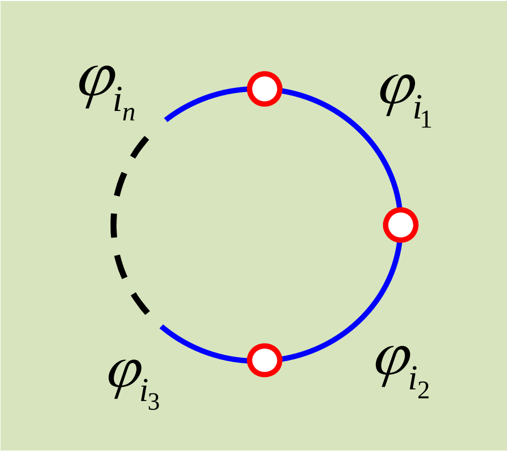
How do we find all of them?

$$= \frac{i}{2}\,\mathrm{STr}\log K\Big|_{\mathrm{hard}} - \frac{i}{2}\sum_{n=1}^{\infty}\frac{1}{n}\,\mathrm{STr}\left[\left(\frac{1}{K}X\right)^n\right]\Bigg|_{\mathrm{hard}}$$

Power-type

Focusing on relativistic EFTs:

$$K_i = \begin{cases} P^2 - m_i^2 & (\text{spin - 0}) \\ \not{P} - m_i & (\text{spin - 1/2}) \\ -\eta^{\mu\nu}\left(P^2 - m_i^2\right) & (\text{spin - 1}) \end{cases}, \qquad X_{ij} = U_{ij} + P_\mu Z_{ij}^\mu + \bar{Z}_{ij}^\mu P_\mu + \mathcal{O}\left(P_\mu^2\right)$$



Use “covariant graphs” to enumerate

- All distinct sets of propagator sequences
- At least one heavy propagator
- Truncate according to the desired operator dim

$$-i\,\mathrm{STr}\left(\frac{1}{K_{i_1}}X_{i_1 i_2}\frac{1}{K_{i_2}}X_{i_2 i_3}\cdots\frac{1}{K_{i_n}}X_{i_n i_1}\right)\Bigg|_{\mathrm{hard}}$$

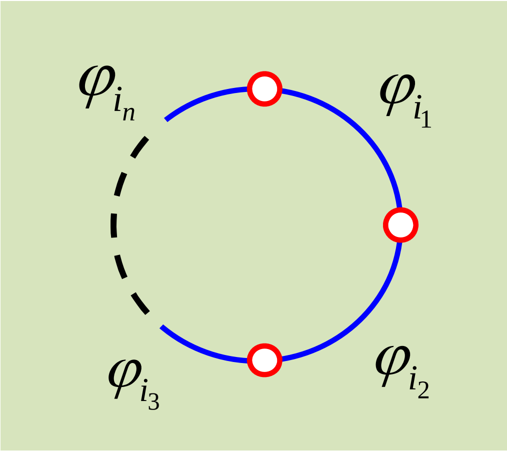
How do we find all of them?

$$=\frac{i}{2}\,\mathrm{STr}\log K\Big|_{\mathrm{hard}}-\frac{i}{2}\sum_{n=1}^{\infty}\frac{1}{n}\,\mathrm{STr}\left[\left(\frac{1}{K}X\right)^{n}\right]\Bigg|_{\mathrm{hard}}$$

Power-type

Focusing on relativistic EFTs:

$$K_i=\begin{cases} P^2-m_i^2 & (\text{spin - 0})\\ \not{P}-m_i & (\text{spin - 1/2})\\ -\eta^{\mu\nu}\left(P^2-m_i^2\right) & (\text{spin - 1})\end{cases}\quad,\qquad X_{ij}=U_{ij}+P_\mu Z_{ij}^{\mu}+\bar{Z}_{ij}^{\mu}P_\mu+\mathcal{O}\left(P_\mu^2\right)$$



Use "covariant graphs" to enumerate

- All distinct sets of propagator sequences
- At least one heavy propagator
- Truncate according to the desired operator dim

$$-i\,\mathrm{STr}\left(\frac{1}{K_{i_1}}X_{i_1 i_2}\frac{1}{K_{i_2}}X_{i_2 i_3}\cdots\frac{1}{K_{i_n}}X_{i_n i_1}\right)\Bigg|_{\text{hard}}$$

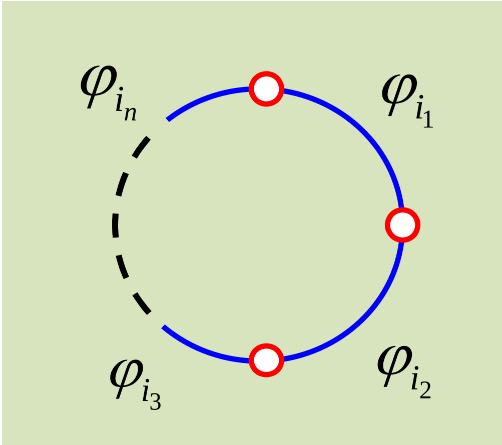
Evaluate them all by hand?

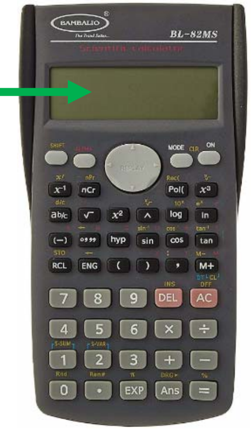
$$=\frac{i}{2}\mathrm{STr}\log K\Big|_{\text{hard}}-\frac{i}{2}\sum_{n=1}^{\infty}\frac{1}{n}\mathrm{STr}\left[\left(\frac{1}{K}X\right)^{n}\right]\Bigg|_{\text{hard}}$$

Power-type

Focusing on relativistic EFTs:

$$K_i=\begin{cases}P^2-m_i^2 & (\text{spin - }0)\\ \not{P}-m_i & (\text{spin - }1/2)\\ -\eta^{\mu\nu}\left(P^2-m_i^2\right) & (\text{spin - }1)\end{cases}\quad,\qquad X_{ij}=U_{ij}+P_\mu Z_{ij}^{\mu}+\bar{Z}_{ij}^{\mu}P_\mu+\mathcal{O}\left(P_\mu^2\right)$$





$$-i\,\mathrm{STr}\left(\frac{1}{K_{i_1}}X_{i_1 i_2}\frac{1}{K_{i_2}}X_{i_2 i_3}\cdots\frac{1}{K_{i_n}}X_{i_n i_1}\right)\Big|_{\text{hard}}$$

Evaluate them all by hand?

$$=\frac{i}{2}\mathrm{STr}\log K\Big|_{\text{hard}}-\frac{i}{2}\sum_{n=1}^{\infty}\frac{1}{n}\mathrm{STr}\left[\left(\frac{1}{K}X\right)^{n}\right]\Big|_{\text{hard}}$$

Power-type

Focusing on relativistic EFTs:

$$K_i=\begin{cases}P^2-m_i^2 & (\text{spin - }0)\\ \not{P}-m_i & (\text{spin - }1/2)\\ -\eta^{\mu\nu}\left(P^2-m_i^2\right) & (\text{spin - }1)\end{cases}\quad,\qquad X_{ij}=U_{ij}+P_\mu Z_{ij}^{\mu}+\bar{Z}_{ij}^{\mu}P_\mu+\mathcal{O}\left(P_\mu^2\right)$$

General form of power-type supertraces:

$$-i\,\mathrm{STr}\left[f\right]\Big|_{\text{hard}} \quad , \quad f = \left[\cdots\left(P_{\mu_1}\cdots P_{\mu_n}\right)\left(\Delta_i \text{ or } \Lambda_i\right)\left(P_{\nu_1}\cdots P_{\nu_m}\right)U_k\cdots\right]$$

$$= \frac{i}{2}\mathrm{STr}\log K\Big|_{\text{hard}} - \frac{i}{2}\sum_{n=1}^{\infty}\frac{1}{n}\mathrm{STr}\left[\left(\frac{1}{K}X\right)^n\right]\Big|_{\text{hard}}$$

Power-type

Focusing on relativistic EFTs:

$$K_i = \begin{cases} P^2 - m_i^2 \\ \not{P} - m_i \\ -\eta^{\mu\nu}\left(P^2 - m_i^2\right) \end{cases} \qquad \frac{1}{K_i} \sim \begin{cases} \dfrac{1}{P^2 - m_i^2} \equiv \Delta_i \\ \dfrac{1}{\not{P} - m_i} \equiv \Lambda_i \end{cases},$$

$$X_{ij} = U_{ij} + P_\mu Z_{ij}^\mu + \bar{Z}_{ij}^\mu P_\mu + \mathcal{O}\left(P_\mu^2\right)$$

$$X_{ij} \sim \left(P_{\mu_1}\cdots P_{\mu_n}\right)U_k\left(P_{\nu_1}\cdots P_{\nu_m}\right)$$

General form of power-type supertraces:

$$-i\,\mathrm{STr}\left[f\right]\Big|_{\text{hard}} \quad , \quad f = \left[\,\cdots\left(P_{\mu_1}\cdots P_{\mu_n}\right)\left(\Delta_i \text{ or } \Lambda_i\right)\left(P_{\nu_1}\cdots P_{\nu_m}\right)U_k\cdots\,\right]$$

Log-type can also be converted into power-type:

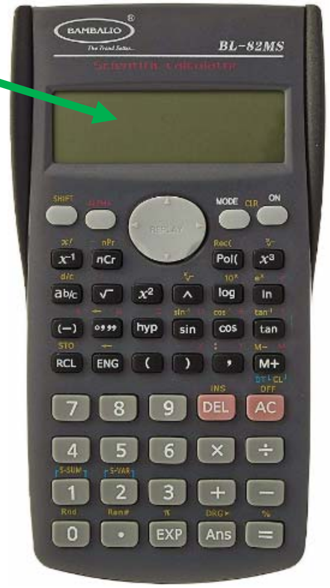
$$\frac{\partial}{\partial M^2}\left[i\,\mathrm{STr}\log\left(P^2-M^2\right)\right] = -i\,\mathrm{STr}\left(\frac{1}{P^2-M^2}\right) \quad , \quad \frac{\partial}{\partial M}\left[i\,\mathrm{STr}\log\left(\not{P}-M\right)\right] = -i\,\mathrm{STr}\left(\frac{1}{\not{P}-M}\right)$$

STrEAM.m and STrEAM_examples.nb
https://github.com/EFTMatching/STrEAM

$$-i\,\mathrm{STr}\left[f\right]\Big|_{\mathrm{hard}} \quad , \quad f = \left[\cdots\left(P_{\mu_1}\cdots P_{\mu_n}\right)\left(\Delta_i \text{ or } \Lambda_i\right)\left(P_{\nu_1}\cdots P_{\nu_m}\right)U_k\cdots\right]$$

STrEAM

Cohen, XL, Zhang, arXiv: 2012.07851



STrEAM.m and STrEAM_examples.nb
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$$-i\,\mathrm{STr}\left[f\right]\Big|_{\mathrm{hard}} \quad , \quad f=\left[\cdots\left(P_{\mu_1}\cdots P_{\mu_n}\right)\left(\Delta_i \text{ or } \Lambda_i\right)\left(P_{\nu_1}\cdots P_{\nu_m}\right)U_k\cdots\right]$$

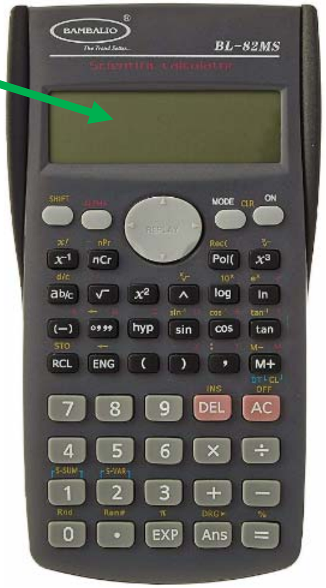
A different calculator:

SUPER TRACER

J. Fuentes-Martín, M. König, J. Pagès, A. E. Thomsen, and F. Wilsch, "SuperTracer: A calculator of functional supertraces for one-loop EFT matching," arXiv: 2012.08506

STrEAM

Cohen, XL, Zhang, arXiv: 2012.07851



STrEAM.m and STrEAM_examples.nb
https://github.com/EFTMatching/STrEAM

$$-i\,\mathrm{STr}\left[\frac{1}{P^2-m_1^2}U_1^{[2]}\right]\Bigg|_{\mathrm{hard}}=\int \mathrm{d}^4x\,\frac{1}{16\pi^2}\,\mathrm{tr}\left[m_1^2\left(1-\log\frac{m_1^2}{\mu^2}\right)U_1+\frac{1}{12m_1^2}F_{\mu\nu}F^{\mu\nu}U_1+\mathcal{O}\left(\text{dim-8}\right)\right]$$

STrEAM.m and STrEAM_examples.nb
https://github.com/EFTMatching/STrEAM

$$-i\,\mathrm{STr}\left[\frac{1}{P^2-m_1^2}U_1^{[2]}\right]\Bigg|_{\text{hard}} = \int d^4x\,\frac{1}{16\pi^2}\,\mathrm{tr}\left[m_1^2\left(1-\log\frac{m_1^2}{\mu^2}\right)U_1 + \frac{1}{12m_1^2}F_{\mu\nu}F^{\mu\nu}U_1 + \mathcal{O}(\text{dim-8})\right]$$

```
In[2]:= SuperTrace[6, {Δ1, U1}, Udimlist → {2}, display → True]
```

$$-i\mathrm{STr}\left[\frac{1}{P^2-m_1^2}U_1\right]\Big|_{\text{hard}} = \int d^4x\ \frac{1}{16\,\pi^2}\ \mathrm{tr}\{$$

$-\left(-1+\mathrm{Log}\left[\frac{m_1^2}{\mu^2}\right]\right)m_1^2$	(U_1)	(dim-2)
$\frac{1}{12\,m_1^2}$	$(F_{\mu_1,\mu_2})\,(F_{\mu_1,\mu_2})\,(U_1)$	(dim-6)

$\}$

Out[2]= $\left\{\left\{\left\{\left\{-\left(-1+\mathrm{Log}\left[\frac{m_1^2}{\mu^2}\right]\right)m_1^2\right\}\right\},\ \{\{U_1\}\},\ 2\right\},\ \left\{\left\{\left\{\frac{1}{12\,m_1^2}\right\}\right\},\ \{\{F_{\mu_1,\mu_2}\},\ \{F_{\mu_1,\mu_2}\},\ \{U_1\}\},\ 6\right\}\right\}$

STrEAM.m and STrEAM_examples.nb
https://github.com/EFTMatching/STrEAM

$$-i\,\mathrm{STr}\left[\frac{1}{P^2-m_1^2}U_1^{[2]}\right]\Bigg|_{\text{hard}} = \int d^4x\,\frac{1}{16\pi^2}\,\mathrm{tr}\left[m_1^2\left(1-\log\frac{m_1^2}{\mu^2}\right)U_1+\frac{1}{12m_1^2}F_{\mu\nu}F^{\mu\nu}U_1+\mathcal{O}(\text{dim-8})\right]$$

dimension truncation

In[2]:= SuperTrace[6, {Δ1, U1}, Udimlist → {2}, display → True]

$$-i\mathrm{STr}\left[\frac{1}{P^2-m_1^2}U_1\right]\Big|_{\text{hard}} = \int d^4x\ \frac{1}{16\,\pi^2}\ \mathrm{tr}\{$$

$-\left(-1+\mathrm{Log}\left[\frac{m_1^2}{\mu^2}\right]\right)m_1^2 \qquad (U_1) \qquad (\text{dim}-2)$

$\frac{1}{12\,m_1^2} \qquad (F_{\mu_1,\mu_2})\,(F_{\mu_1,\mu_2})\,(U_1) \qquad (\text{dim}-6)$

}

Out[2]= $\left\{\left\{\left\{\left\{-\left(-1+\mathrm{Log}\left[\frac{m_1^2}{\mu^2}\right]\right)m_1^2\right\}\right\},\{\{U_1\}\},2\right\},\left\{\left\{\left\{\frac{1}{12\,m_1^2}\right\}\right\},\{\{F_{\mu_1,\mu_2}\},\{F_{\mu_1,\mu_2}\},\{U_1\}\},6\right\}\right\}$

derivative interaction   massless (covariant) propagator

$$-i\,\mathrm{STr}\left[\frac{1}{P^2-m_1^2}U_1^{[1]}\frac{1}{P^2-m_2^2}P_\mu Z^{\mu[1]}\frac{1}{P^2}U_3^{[2]}\frac{1}{P^2-m_2^2}U_4^{[1]}\right]\Bigg|_{\text{hard}}$$

```
In[7]:= SuperTrace[6, {Δ1, U1, Δ2, Pν, Zν, Δ0, U3, Δ2, U4}, Udimlist → {1, 1, 2, 1}, display → True];
```

$$-i\mathrm{STr}\left[\frac{1}{P^2-m_1^2}U_1\frac{1}{P^2-m_2^2}P_\nu Z_\nu\frac{1}{P^2}U_3\frac{1}{P^2-m_2^2}U_4\right]\Big|_{\text{hard}} = \int d^4x\ \frac{1}{16\pi^2}\ \mathrm{tr}\{$$

$$\frac{3-2\,\mathrm{Log}\left[\frac{m_1^2}{\mu^2}\right]}{4\,m_1^4}\quad (U_1)\,(Z_{\mu_1})\,(P_{\mu_1}U_3)\,(U_4)\quad (\text{dim}-6)$$

$$\frac{3-2\,\mathrm{Log}\left[\frac{m_1^2}{\mu^2}\right]}{2\,m_1^4}\quad (U_1)\,(P_{\mu_1}Z_{\mu_1})\,(U_3)\,(U_4)\quad (\text{dim}-6)$$

$$\frac{5-2\,\mathrm{Log}\left[\frac{m_1^2}{\mu^2}\right]}{4\,m_1^4}\quad (P_{\mu_1}U_1)\,(Z_{\mu_1})\,(U_3)\,(U_4)\quad (\text{dim}-6)$$

}

fermionic (covariant) propagator

$$-i\,\mathrm{STr}\left[\frac{1}{P^2-m_1^2}U_1^{[1]}\frac{1}{P^2-m_2^2}U_2^{[3/2]}\frac{1}{\not{P}}U_3^{[3/2]}\frac{1}{P^2-m_2^2}U_4^{[1]}\right]\Bigg|_{\mathrm{hard}}$$

```
In[8]:= SuperTrace[6, {Δ1, U1, Δ2, U2, Δ0, U3, Δ2, U4}, Udimlist → {1, 3/2, 3/2, 1}, display → True];
```

$$-i\mathrm{STr}\left[\frac{1}{P^2-m_1^2}U_1\frac{1}{P^2-m_2^2}U_2\frac{1}{\mathrm{Pslash}}U_3\frac{1}{P^2-m_2^2}U_4\right]\Big|_{\mathrm{hard}} = \int d^4x\,\frac{1}{16\pi^2}\,\mathrm{tr}\{$$

$$\frac{3-2\,\mathrm{Log}\left[\frac{m_1^2}{\mu^2}\right]}{4\,m_1^4}\quad (U_1)\,(U_2)\,(P_{\mu_1}\gamma_{\mu_1}U_3)\,(U_4)\quad (\mathrm{dim}-6)$$

$$\frac{1}{2\,m_1^4}\quad (U_1)\,(P_{\mu_1}U_2)\,(\gamma_{\mu_1}U_3)\,(U_4)\quad (\mathrm{dim}-6)$$

$$\frac{5-2\,\mathrm{Log}\left[\frac{m_1^2}{\mu^2}\right]}{4\,m_1^4}\quad (P_{\mu_1}U_1)\,(U_2)\,(\gamma_{\mu_1}U_3)\,(U_4)\quad (\mathrm{dim}-6)$$

$$\}$$

Log-type

$$\frac{\partial}{\partial m_1^2}\left[i\,\mathrm{STr}\log\left(P^2-m_1^2\right)\right]=-i\,\mathrm{STr}\left(\frac{1}{P^2-m_1^2}\right)\qquad \frac{\partial}{\partial m_1}\left[i\,\mathrm{STr}\log\left(\not{P}-m_1\right)\right]=-i\,\mathrm{STr}\left(\frac{1}{\not{P}-m_1}\right)$$

In[3]:= **SuperTrace[6, {Δ1}, display → True];**

$$-i\mathrm{STr}\left[\frac{1}{P^2-m_1^2}\right]\Big|_{\mathrm{hard}} = \int d^4x\,\frac{1}{16\pi^2}\,\mathrm{tr}\Big\{$$

$\frac{1}{12\,m_1^2}$	$(F_{\mu_1,\mu_2})\,(F_{\mu_1,\mu_2})$	(dim-4)
$\frac{i}{90\,m_1^4}$	$(F_{\mu_1,\mu_2})\,(F_{\mu_1,\mu_3})\,(F_{\mu_2,\mu_3})$	(dim-6)
$\frac{1}{60\,m_1^4}$	$(P_{\mu_1}P_{\mu_2}F_{\mu_2,\mu_3})\,(F_{\mu_1,\mu_3})$	(dim-6)

$\Big\}$

In[4]:= **SuperTrace[6, {Λ1}, NoγinU → True, display → True];**

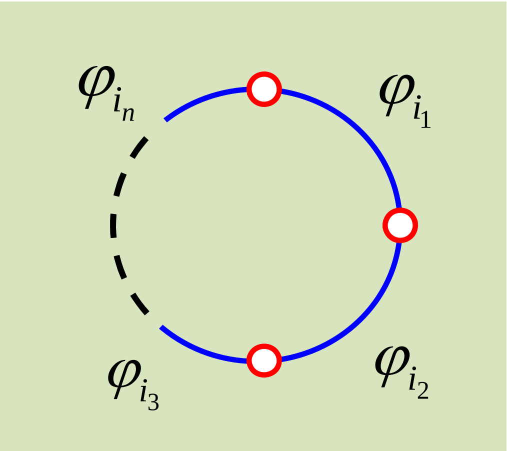
$$-i\mathrm{STr}\left[\frac{1}{\mathrm{Pslash}-m_1}\right]\Big|_{\mathrm{hard}} = \int d^4x\,\frac{1}{16\pi^2}\,\mathrm{tr}\Big\{$$

$\frac{1}{6\,m_1}$	$(F_{\mu_1,\mu_2})\,(F_{\mu_1,\mu_2})$	(dim-4)
$-\frac{i}{90\,m_1^3}$	$(F_{\mu_1,\mu_2})\,(F_{\mu_1,\mu_3})\,(F_{\mu_2,\mu_3})$	(dim-6)
$\frac{1}{15\,m_1^3}$	$(P_{\mu_1}P_{\mu_2}F_{\mu_2,\mu_3})\,(F_{\mu_1,\mu_3})$	(dim-6)

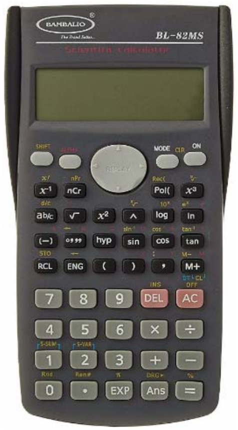
$\Big\}$

$$\Gamma_{\text{EFT, 1PI}}[\phi] = \Gamma_{\text{UV, 1LPI}}[\phi] \quad \Rightarrow \quad \begin{cases} \mathcal{L}_{\text{EFT}}^{(\text{tree})} = \mathcal{L}_{\text{UV}}\left(\phi, \Phi = \Phi_c[\phi]\right) \quad , \quad \left.\dfrac{\delta S_{\text{UV}}}{\delta \Phi}\right|_{\Phi=\Phi_c[\phi]} = 0 \\ \int d^4x\, \mathcal{L}_{\text{EFT}}^{(\text{1-loop})} = \dfrac{i}{2} \operatorname{STr} \log \left[-\left.\dfrac{\delta^2 S_{\text{UV}}}{\delta(\phi,\Phi)^2}\right|_{\Phi=\Phi_c} \right]\Bigg|_{\text{hard}} \end{cases}$$

Covariant graphs



STrEAM



How to compute this functional SuperTrace?

$$\Gamma_{\text{EFT, 1PI}}[\phi] = \Gamma_{\text{UV, 1LPI}}[\phi] \quad \Rightarrow \quad \begin{cases} \mathcal{L}_{\text{EFT}}^{(\text{tree})} = \mathcal{L}_{\text{UV}}\left(\phi, \Phi = \Phi_{\text{c}}[\phi]\right) \quad , \quad \left.\dfrac{\delta S_{\text{UV}}}{\delta \Phi}\right|_{\Phi=\Phi_{\text{c}}[\phi]} = 0 \\ \displaystyle\int d^4x\, \mathcal{L}_{\text{EFT}}^{(\text{1-loop})} = \frac{i}{2}\,\text{STr}\log\left[-\left.\frac{\delta^2 S_{\text{UV}}}{\delta(\phi,\Phi)^2}\right|_{\Phi=\Phi_{\text{c}}} \right]\Bigg|_{\text{hard}} \end{cases}$$

How to compute this functional SuperTrace?

Prescription for Functional Matching:

1. Derive heavy EOM(s) and $\mathcal{L}_{\text{EFT}}^{(\text{tree})}$

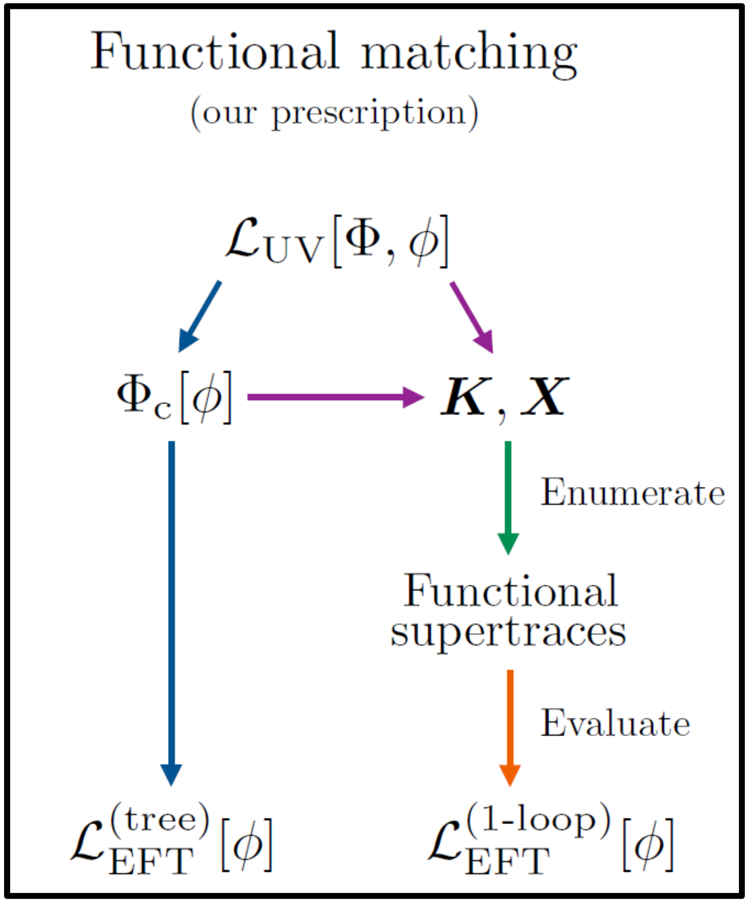
2. Derive K and X matrices

3. Enumerate supertraces

   Covariant graphs

4. Evaluate supertraces to obtain $\mathcal{L}_{\text{EFT}}^{(\text{1-loop})}$

   Mathematica package STrEAM.m
   (Cohen, XL, Zhang, arXiv: 2012.07851)



Cohen, XL, Zhang, arXiv: 2011.02484

A Real Example: SM + Singlet

$$\mathcal{L}_{\text{UV}} = \mathcal{L}_{\text{SM}} + \frac{1}{2}\left(\partial_\mu S\right)^2 - \frac{1}{2}M^2S^2 - A|H|^2 S - \frac{1}{2}\kappa|H|^2 S^2 - \frac{1}{6}\mu_S S^3 - \frac{1}{24}\lambda_S S^4 \quad \boxed{\Rightarrow \quad \mathcal{L}_{\text{SMEFT}}}$$

$$\mathcal{L}_{\text{SM}} = |D_\mu H|^2 + \sum_{\psi=q,u,d,l,e} \bar{\psi} i \not{D} \psi - \frac{1}{4}G^A_{\mu\nu}G^{A\mu\nu} - \frac{1}{4}W^I_{\mu\nu}W^{I\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} - m^2|H|^2 - \frac{1}{2}\lambda_H|H|^4 - \left(\bar{q}\, y_u u \tilde{H} + \bar{q}\, y_d\, d H + \bar{l}\, y_e e H + \text{h.c.}\right)$$

Amplitude approach:

- M. Jiang, N. Craig, Y.-Y. Li, and D. Sutherland, arXiv: 1811.08878
- U. Haisch, M. Ruhdorfer, E. Salvioni, E. Venturini, and A. Weiler, arXiv: 2003.05936

Functional approach: Cohen, XL, Zhang, arXiv: 2011.02484

A Real Example: SM + Singlet

$$\mathcal{L}_{\text{UV}} = \mathcal{L}_{\text{SM}} + \frac{1}{2}\left(\partial_\mu S\right)^2 - \frac{1}{2}M^2 S^2 - A\left|H\right|^2 S - \frac{1}{2}\kappa\left|H\right|^2 S^2 - \frac{1}{6}\mu_S S^3 - \frac{1}{24}\lambda_S S^4 \quad \Rightarrow \quad \mathcal{L}_{\text{SMEFT}}$$

$$\mathcal{L}_{\text{SM}} = \left|D_\mu H\right|^2 + \sum_{\psi=q,u,d,l,e} \bar{\psi} i \not{D} \psi - \frac{1}{4} G^A_{\mu\nu} G^{A\mu\nu} - \frac{1}{4} W^I_{\mu\nu} W^{I\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} - m^2 \left|H\right|^2 - \frac{1}{2}\lambda_H \left|H\right|^4 - \left(\bar{q}\, y_u u \tilde{H} + \bar{q}\, y_d d H + \bar{l}\, y_e e H + \text{h.c.}\right)$$

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Functional approach: Cohen, XL, Zhang, arXiv: 2011.02484

1. Derive heavy EOM(s) and $\mathcal{L}_{\text{EFT}}^{(\text{tree})}$ $\quad \phi = \phi_{\text{SM}} = \{H, q, u, d, l, e, G, W, B\} \quad , \quad \Phi = S$

$$-A\left|H\right|^2 + \left(-\partial^2 - M^2 - \kappa\left|H\right|^2\right) S_{\text{c}} - \frac{1}{2}\mu_S S_{\text{c}}^2 - \frac{1}{6}\lambda_S S_{\text{c}}^3 = 0$$

$$S_{\text{c}} = S_{\text{c}}^{(2)} + S_{\text{c}}^{(4)} + S_{\text{c}}^{(6)} + \cdots$$

A Real Example: SM + Singlet

$$\mathcal{L}_{\text{UV}} = \mathcal{L}_{\text{SM}} + \frac{1}{2}\left(\partial_\mu S\right)^2 - \frac{1}{2}M^2 S^2 - A|H|^2 S - \frac{1}{2}\kappa |H|^2 S^2 - \frac{1}{6}\mu_S S^3 - \frac{1}{24}\lambda_S S^4 \quad \boxed{\Rightarrow \quad \mathcal{L}_{\text{SMEFT}}}$$

$$\mathcal{L}_{\text{SM}} = \left|D_\mu H\right|^2 + \sum_{\psi=q,u,d,l,e} \bar{\psi} i \not{D} \psi - \frac{1}{4}G^A_{\mu\nu}G^{A\mu\nu} - \frac{1}{4}W^I_{\mu\nu}W^{I\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} - m^2|H|^2 - \frac{1}{2}\lambda_H |H|^4 - \left(\bar{q}\, y_u u \tilde{H} + \bar{q}\, y_d d H + \bar{l}\, y_e e H + \text{h.c.}\right)$$

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Functional approach: Cohen, XL, Zhang, arXiv: 2011.02484

1. Derive heavy EOM(s) and $\mathcal{L}_{\text{EFT}}^{(\text{tree})}$ $\qquad \phi = \phi_{\text{SM}} = \{H, q, u, d, l, e, G, W, B\} \quad , \quad \Phi = S$

$$-A|H|^2 + \left(-\partial^2 - M^2 - \kappa |H|^2\right) S_c - \frac{1}{2}\mu_S S_c^2 - \frac{1}{6}\lambda_S S_c^3 = 0$$

$$S_c = S_c^{(2)} + S_c^{(4)} + S_c^{(6)} + \cdots$$

$$\begin{cases} S_c^{(2)} = -\dfrac{A}{M^2}|H|^2 \\ S_c^{(4)} = \dfrac{A}{M^4}\left[\partial^2|H|^2 + \left(\kappa - \dfrac{\mu_S A}{2M^2}\right)|H|^4\right] \\ S_c^{(6)} = -\dfrac{A}{M^6}\left\{\left(\kappa - \dfrac{\mu_S A}{M^2}\right)|H|^2 \partial^2 |H|^2 + \left[\left(\kappa - \dfrac{\mu_S A}{M^2}\right)\left(\kappa - \dfrac{\mu_S A}{2M^2}\right) - \dfrac{\lambda_S A^2}{6M^2}\right]|H|^6 + \partial^2\left[\partial^2|H|^2 + \left(\kappa - \dfrac{\mu_S A}{2M^2}\right)|H|^4\right]\right\} \end{cases}$$

$$\mathcal{L}_{\text{EFT}}^{(\text{tree})} = \mathcal{L}_{\text{SM}} + \frac{A^2}{2M^2}|H|^4 - \frac{A^2}{2M^4}|H|^2 \partial^2 |H|^2 - \frac{A^2}{2M^4}\left(\kappa - \frac{\mu_S A}{3M^2}\right)|H|^6$$

A Real Example: SM + Singlet

2. Derive K and X matrices

$$\left.\frac{\delta^2 S_{\mathrm{UV}}}{\delta\varphi^2}\right|_{\Phi=\Phi_{\mathrm{c}}} = K - X$$

$$K_i = \begin{cases} P^2 - m_i^2 & (\text{spin - 0}) \\ \not{P} - m_i & (\text{spin - 1/2}) \\ -\eta^{\mu\nu}\left(P^2 - m_i^2\right) & (\text{spin - 1}) \end{cases}$$

$$X_{ij} = U_{ij} + P_\mu Z_{ij}^\mu + \bar{Z}_{ij}^\mu P_\mu + \mathcal{O}\left(P_\mu^2\right)$$

$\dim(\boldsymbol{X}) \geq$

	S	H	q	u	d	l	e	G	W	B
S	2	1								
H	1	2	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{3}{2}$		1	1
q		$\frac{3}{2}$		1	1			$\frac{3}{2}$	$\frac{3}{2}$	$\frac{3}{2}$
u		$\frac{3}{2}$	1					$\frac{3}{2}$		$\frac{3}{2}$
d		$\frac{3}{2}$	1					$\frac{3}{2}$		$\frac{3}{2}$
l		$\frac{3}{2}$					1		$\frac{3}{2}$	$\frac{3}{2}$
e		$\frac{3}{2}$				1				$\frac{3}{2}$
G			$\frac{3}{2}$	$\frac{3}{2}$	$\frac{3}{2}$			2		
W		1	$\frac{3}{2}$			$\frac{3}{2}$			2	2
B		1	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{3}{2}$		2	2

A Real Example: SM + Singlet

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$$\left.\frac{\delta^2 S_{\text{UV}}}{\delta\varphi^2}\right|_{\Phi=\Phi_c} = K - X$$

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	S	H	q	u	d	l	e	G	W	B
S	2	1								
H	1	2	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{3}{2}$		1	1
q		$\frac{3}{2}$		1	1			$\frac{3}{2}$	$\frac{3}{2}$	$\frac{3}{2}$
u		$\frac{3}{2}$	1					$\frac{3}{2}$		$\frac{3}{2}$
d		$\frac{3}{2}$	1					$\frac{3}{2}$		$\frac{3}{2}$
l		$\frac{3}{2}$					1		$\frac{3}{2}$	$\frac{3}{2}$
e		$\frac{3}{2}$				1				$\frac{3}{2}$
G			$\frac{3}{2}$	$\frac{3}{2}$	$\frac{3}{2}$			2		
W		1	$\frac{3}{2}$			$\frac{3}{2}$			2	2
B		1	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{3}{2}$		2	2

$$U_{SS} = \kappa\,|H|^2 + \mu_S\,S_c + \frac{1}{2}\lambda_S\,S_c^2 \qquad U_{HS} = (A + \kappa\,S_c)\begin{pmatrix} H \\ H^* \end{pmatrix}$$

$$U_{HW}^{\nu J} = \frac{i g_2}{2}\begin{pmatrix} -\sigma^J (D^\nu H) \\ \sigma^{J*}(D^\nu H)^* \end{pmatrix} \qquad Z_{HW}^{\rho\nu J} = \eta^{\rho\nu}\,\frac{g_2}{2}\begin{pmatrix} -\sigma^J H \\ \sigma^{J*} H^* \end{pmatrix}$$

$$U_{HH} = \left(A\,S_c + \frac{1}{2}\kappa\,S_c^2\right)\begin{pmatrix} \mathbb{1} & 0 \\ 0 & \mathbb{1} \end{pmatrix} + \lambda_H\begin{pmatrix} |H|^2\mathbb{1} + HH^\dagger & HH^T \\ H^*H^\dagger & |H|^2\mathbb{1} + H^*H^T \end{pmatrix}$$

$$U_{WW}^{\mu I,\nu J} = 2\,g_2\,\epsilon^{IJK}W^{K\mu\nu} - \frac{g_2^2}{2}\,\eta^{\mu\nu}\,\delta^{IJ}\,|H|^2$$

$$U_{qu} = \begin{pmatrix} \mathbb{1}\,\boldsymbol{y}_u\,\frac{1+\gamma^5}{2}\,\widetilde{H} & 0 \\ 0 & \mathbb{1}\,\boldsymbol{y}_u^*\,\frac{1-\gamma^5}{2}\,\widetilde{H}^* \end{pmatrix} \qquad U_{qG}^{\nu B} = \frac{g_3}{2}\begin{pmatrix} -\gamma^\nu\lambda^B q \\ \gamma^\nu\lambda^{B*}q^c \end{pmatrix}$$

$$U_{Hq} = \begin{pmatrix} \mathbb{1}\,\bar{d}\,\boldsymbol{y}_d^\dagger\,\frac{1-\gamma^5}{2} & -\epsilon\,\bar{u}^c\,\boldsymbol{y}_u^T\,\frac{1+\gamma^5}{2} \\ -\epsilon\,\bar{u}\,\boldsymbol{y}_u^\dagger\,\frac{1-\gamma^5}{2} & \mathbb{1}\,\bar{d}^c\,\boldsymbol{y}_d^T\,\frac{1+\gamma^5}{2} \end{pmatrix}$$

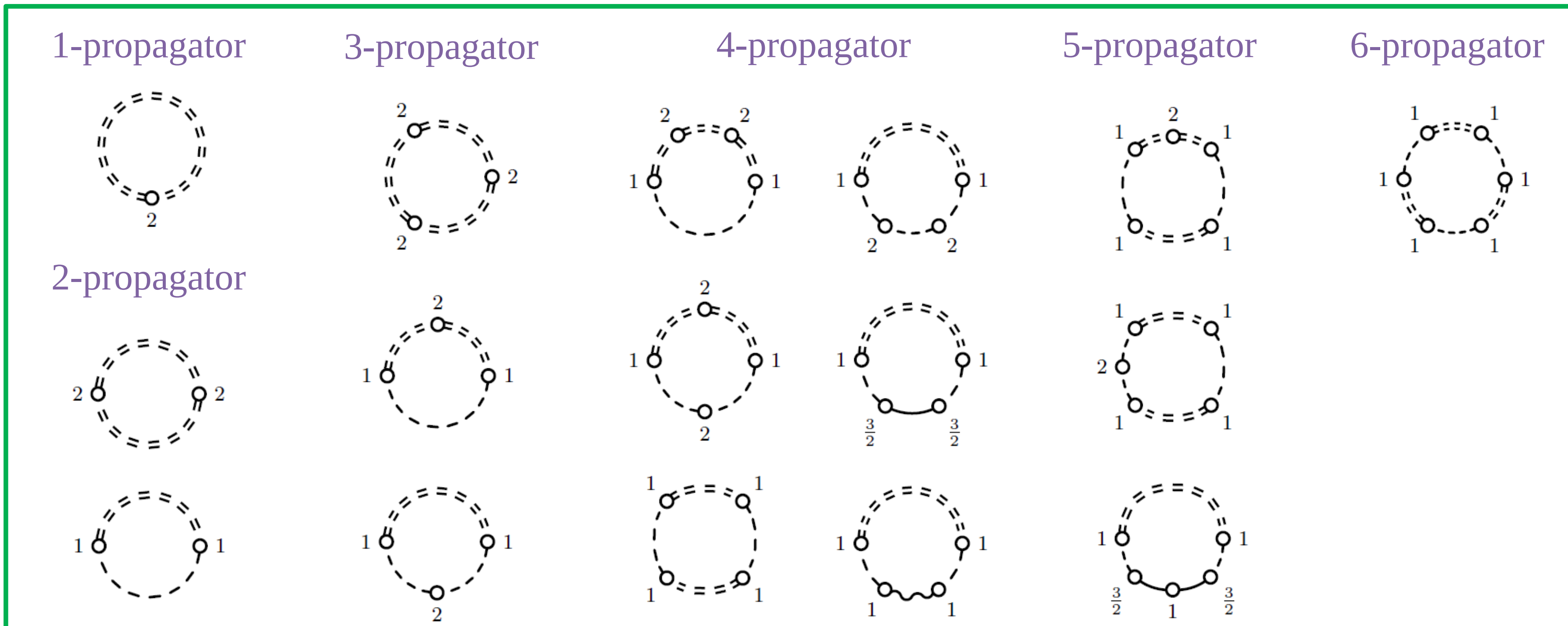
See the full list in App. B of arXiv: 2011.02484

A Real Example: SM + Singlet

3. Enumerate supertraces

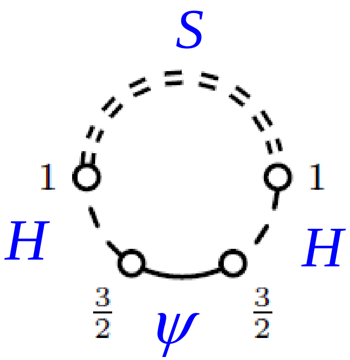
Log-type: None

Power-type: 16 covariant graphs

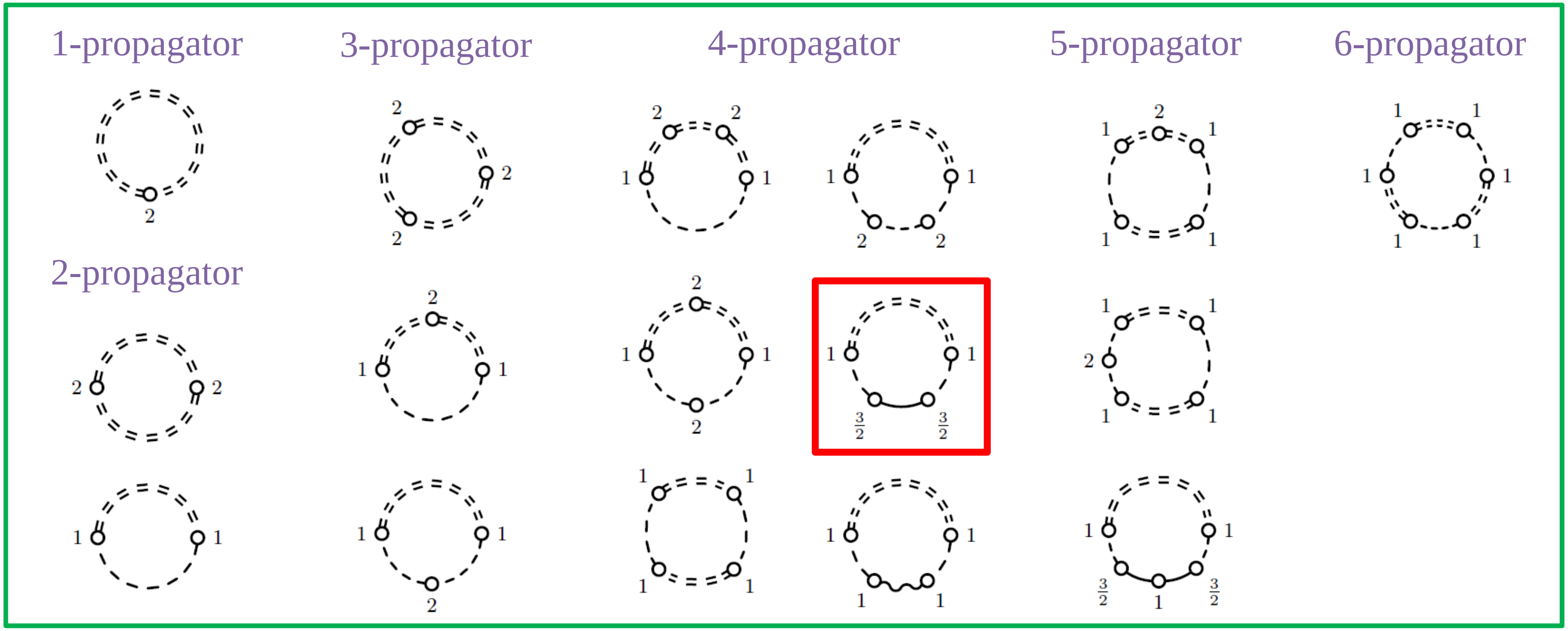


A Real Example: SM + Singlet

3. Enumerate supertraces

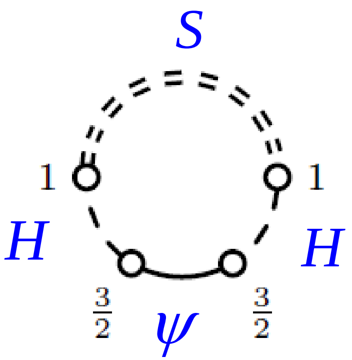


$$= -\frac{i}{2}\mathrm{STr}\left[\frac{1}{P^2-M^2}U^{[1]}_{SH}\frac{1}{P^2-m^2}U^{[3/2]}_{H\psi}\frac{1}{\not{P}}U^{[3/2]}_{\psi H}\frac{1}{P^2-m^2}U^{[1]}_{HS}\right]\Bigg|_{\text{hard}}$$



A Real Example: SM + Singlet

3. Enumerate supertraces
4. Evaluate supertraces to obtain $\mathcal{L}_{\text{EFT}}^{(\text{1-loop})}$



$$= -\frac{i}{2}\,\mathrm{STr}\left[\frac{1}{P^2-M^2}U^{[1]}_{SH}\frac{1}{P^2-m^2}U^{[3/2]}_{H\psi}\frac{1}{\not{P}}U^{[3/2]}_{\psi H}\frac{1}{P^2-m^2}U^{[1]}_{HS}\right]\Bigg|_{\text{hard}}$$

$$= \int d^4x \frac{1}{16\pi^2}\frac{A^2}{4M^4}\,\mathrm{tr}\left\{\left(\ln\frac{\mu^2}{M^2}+\frac{5}{2}\right)\left[\begin{aligned}&\frac{1}{2}\left(H^\dagger i\overleftrightarrow{D}_\mu H\right)\left(\bar{u}y_u^\dagger y_u\gamma^\mu u-\bar{d}y_d^\dagger y_d\gamma^\mu d-\bar{e}y_e^\dagger y_e\gamma^\mu e\right)\\&-2\tilde{H}^\dagger\left(iD_\mu H\right)\left(\bar{u}y_u^\dagger y_d\gamma^\mu d\right)+\tilde{H}^\dagger_\alpha\left(\bar{q}_\beta y_u y_u^\dagger\gamma^\mu q_\alpha\right)\left(iD_\mu\tilde{H}\right)_\beta\\&+H^\dagger_\alpha\left(\bar{q}_\beta y_d y_d^\dagger\gamma^\mu q_\alpha+\bar{l}_\beta y_e y_e^\dagger\gamma^\mu l_\alpha\right)\left(iD_\mu H\right)_\beta+\text{h.c.}\end{aligned}\right]\right.$$
$$\left.+\left(\ln\frac{\mu^2}{M^2}+\frac{1}{2}\right)\left[\begin{aligned}&\tilde{H}^\dagger_\alpha\left(\bar{q}_\beta y_u y_u^\dagger i\overleftrightarrow{\not{D}}q_\alpha\right)\tilde{H}_\beta+H^\dagger_\alpha\left(\bar{q}_\beta y_d y_d^\dagger i\overleftrightarrow{\not{D}}q_\alpha+\bar{l}_\beta y_e y_e^\dagger i\overleftrightarrow{\not{D}}l_\alpha\right)H_\beta\\&+|H|^2\left(\bar{u}y_u^\dagger y_u i\overleftrightarrow{\not{D}}u+\bar{d}y_d^\dagger y_d i\overleftrightarrow{\not{D}}d+\bar{e}y_e^\dagger y_e i\overleftrightarrow{\not{D}}e\right)\end{aligned}\right]\right\}$$

Final Matching Results:

Operator	Coefficient $\times 16\pi^2$
$ig_2 (D^\mu H)^\dagger \sigma^I (D^\nu H) W^I_{\mu\nu}$	$-\frac{A^2}{12M^4}$
$ig_1 (D^\mu H)^\dagger (D^\nu H) B_{\mu\nu}$	$-\frac{A^2}{12M^4}$
$\frac{ig_2}{2}(H^\dagger \sigma^I \overleftrightarrow{D}^\mu H)(D^\nu W^I_{\mu\nu})$	$-\frac{A^2}{6M^4}\left(\frac{7}{3} - \log\frac{M^2}{\mu^2}\right)$
$\frac{ig_1}{2}(H^\dagger \overleftrightarrow{D}^\mu H)(\partial^\nu B_{\mu\nu})$	$-\frac{A^2}{6M^4}\left(\frac{7}{3} - \log\frac{M^2}{\mu^2}\right)$
$\lvert H\rvert^2 W^I_{\mu\nu} W^{I\mu\nu}$	$\frac{g_2^2 A^2}{16M^4}$
$\lvert H\rvert^2 B_{\mu\nu} B^{\mu\nu}$	$\frac{g_1^2 A^2}{16M^4}$
$H^\dagger \sigma^I H W^I_{\mu\nu} B^{\mu\nu}$	$\frac{g_1 g_2 A^2}{8M^4}$

Operator	Coefficient $\times 16\pi^2$
$\lvert H\rvert^2$	$\left[\frac{1}{2}(\kappa M^2 - \mu_S A) + A^2\left(1 + \frac{m^2}{M^2} + \frac{m^4}{M^4}\right)\right]\left(1 - \log\frac{M^2}{\mu^2}\right)$
$\lvert H\rvert^4$	$\frac{\kappa^2}{4}\left(-\log\frac{M^2}{\mu^2}\right) + \frac{\mu_S A}{M^2}\left(\frac{\kappa}{2} - \frac{\mu_S A}{4M^2} + \frac{A^2}{M^2}\right)$ $+\frac{A^2}{M^2}\left[\left(\frac{\lambda_S}{4} + 3\lambda_H\right)\left(1 - \log\frac{M^2}{\mu^2}\right) - 2\left(\kappa + \frac{A^2}{M^2}\right)\left(\frac{3}{2} - \log\frac{M^2}{\mu^2}\right)\right]$ $+\frac{m^2}{M^2}\frac{A^2}{M^2}\left[6\lambda_H\left(1 - \log\frac{M^2}{\mu^2}\right) - 3\left(\kappa + \frac{2A^2}{M^2}\right)\left(\frac{4}{3} - \log\frac{M^2}{\mu^2}\right) + \frac{\mu_S A}{M^2}\left(2 - \log\frac{M^2}{\mu^2}\right)\right]$
$\lvert D_\mu H\rvert^2$	$\frac{A^2}{2M^2} + \frac{A^2 m^2}{M^4}\left(\frac{5}{2} - \log\frac{M^2}{\mu^2}\right)$

Operator	Coefficient $\times 16\pi^2$
$\lvert H\rvert^6$	$\frac{1}{M^2}\left(-\frac{\kappa^3}{12} - \frac{\kappa^2\mu_S A}{4M^2} + \frac{\kappa\mu_S^2 A^2}{2M^4} - \frac{\lambda_S A^4}{2M^4} - \frac{\mu_S^3 A^3}{6M^6} + \frac{\mu_S^2 A^4}{M^6}\right)$ $+\frac{\kappa A^2}{M^4}\left[3\kappa\left(\frac{11}{6} - \log\frac{M^2}{\mu^2}\right) - \frac{\lambda_S}{4}\left(2 - \log\frac{M^2}{\mu^2}\right)\right]$ $+\frac{9\lambda_H A^2}{M^4}\left[-\kappa\left(\frac{4}{3} - \log\frac{M^2}{\mu^2}\right) + \lambda_H\left(1 - \log\frac{M^2}{\mu^2}\right)\right]$ $+\frac{\mu_S A^3}{M^6}\left[-\kappa\left(5 - \log\frac{M^2}{\mu^2}\right) + \frac{\lambda_S}{12}\left(4 - \log\frac{M^2}{\mu^2}\right) + 3\lambda_H\left(2 - \log\frac{M^2}{\mu^2}\right)\right]$ $+\frac{A^4}{M^6}\left[\frac{21\kappa}{2}\left(\frac{37}{21} - \log\frac{M^2}{\mu^2}\right) - 18\lambda_H\left(\frac{4}{3} - \log\frac{M^2}{\mu^2}\right)\right]$ $-\frac{7\mu_S A^5}{2M^8}\left(\frac{15}{7} - \log\frac{M^2}{\mu^2}\right) + \frac{9A^6}{M^8}\left(\frac{43}{27} - \log\frac{M^2}{\mu^2}\right)$
$\lvert H\rvert^2(\partial^2\lvert H\rvert^2)$	$-\frac{\kappa^2}{24M^2} - \frac{5\kappa\mu_S A}{12M^4}$ $+\frac{A^2}{M^4}\left[2\kappa\left(\frac{17}{12} - \log\frac{M^2}{\mu^2}\right) - \frac{\lambda_S}{2}\left(1 - \log\frac{M^2}{\mu^2}\right) - \frac{\lambda_H}{2}\left(\frac{9}{2} - \log\frac{M^2}{\mu^2}\right)\right]$ $+\frac{11\mu_S^2 A^2}{24M^6} - \frac{4\mu_S A^3}{3M^6} + \frac{3A^4}{2M^6}\left(\frac{20}{9} - \log\frac{M^2}{\mu^2}\right) - \frac{3g_2^2 A^2}{8M^4}\left(\frac{5}{6} - \log\frac{M^2}{\mu^2}\right)$
$\lvert H\rvert^2\lvert D_\mu H\rvert^2$	$\frac{A^2}{M^4}\left[\left(\lambda_H - \frac{A^2}{M^2}\right)\left(\frac{9}{2} - \log\frac{M^2}{\mu^2}\right) - \frac{3\kappa}{2} + \frac{\mu_S A}{2M^2}\right] - \frac{3g_2^2 A^2}{2M^4}\left(\frac{5}{6} - \log\frac{M^2}{\mu^2}\right)$
$\frac{1}{2}(H^\dagger \overleftrightarrow{D}^\mu H)^2$	$\frac{3g_1^2 A^2}{4M^4}\left(\frac{5}{6} - \log\frac{M^2}{\mu^2}\right)$
$\lvert D^2 H\rvert^2$	$\frac{A^2}{6M^4}$

Final Matching Results:

Operator	Coefficient $\times 16\pi^2$
$ig_2(D^\mu H)^\dagger\sigma^I(D^\nu H)\,W^I_{\mu\nu}$	$-\frac{A^2}{12M^4}$
$ig_1(D^\mu H)^\dagger(D^\nu H)\,B_{\mu\nu}$	$-\frac{A^2}{12M^4}$
$\frac{ig_2}{2}(H^\dagger\sigma^I\overleftrightarrow{D}^\mu H)(D^\nu W^I_{\mu\nu})$	$-\frac{A^2}{6M^4}\left(\frac{7}{3}-\log\frac{M^2}{\mu^2}\right)$
$\frac{ig_1}{2}(H^\dagger\overleftrightarrow{D}^\mu H)(\partial^\nu B_{\mu\nu})$	$-\frac{A^2}{6M^4}\left(\frac{7}{3}-\log\frac{M^2}{\mu^2}\right)$
$\lvert H\rvert^2 W^I_{\mu\nu}W^{I\mu\nu}$	$\frac{g_2^2A^2}{16M^4}$
$\lvert H\rvert^2 B_{\mu\nu}B^{\mu\nu}$	$\frac{g_1^2A^2}{16M^4}$
$H^\dagger\sigma^I H W^I_{\mu\nu}B^{\mu\nu}$	$\frac{g_1g_2A^2}{8M^4}$

Operator	Coefficient $\times 16\pi^2$
$(H^\dagger\sigma^I i\overleftrightarrow{D}_\mu H)(\bar q\sigma^I\gamma^\mu q)$	$\frac{A^2}{8M^4}(y_uy_u^\dagger+y_dy_d^\dagger)\left(\frac{5}{2}-\log\frac{M^2}{\mu^2}\right)$
$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar q\gamma^\mu q)$	$-\frac{A^2}{8M^4}(y_uy_u^\dagger-y_dy_d^\dagger)\left(\frac{5}{2}-\log\frac{M^2}{\mu^2}\right)$
$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar u\gamma^\mu u)$	$\frac{A^2}{4M^4}y_u^\dagger y_u\left(\frac{5}{2}-\log\frac{M^2}{\mu^2}\right)$
$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar d\gamma^\mu d)$	$-\frac{A^2}{4M^4}y_d^\dagger y_d\left(\frac{5}{2}-\log\frac{M^2}{\mu^2}\right)$
$(H^\dagger\sigma^I i\overleftrightarrow{D}_\mu H)(\bar l\sigma^I\gamma^\mu l)$	$\frac{A^2}{8M^4}y_ey_e^\dagger\left(\frac{5}{2}-\log\frac{M^2}{\mu^2}\right)$
$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar l\gamma^\mu l)$	$\frac{A^2}{8M^4}y_ey_e^\dagger\left(\frac{5}{2}-\log\frac{M^2}{\mu^2}\right)$
$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar e\gamma^\mu e)$	$-\frac{A^2}{4M^4}y_e^\dagger y_e\left(\frac{5}{2}-\log\frac{M^2}{\mu^2}\right)$
$i(\widetilde{H}^\dagger(D_\mu H))(\bar u\gamma^\mu d)$ (+h.c.)	$-\frac{A^2}{2M^4}y_u^\dagger y_d\left(\frac{5}{2}-\log\frac{M^2}{\mu^2}\right)$
$(H^\dagger\sigma^I H)(\bar q\,\sigma^I i\overleftrightarrow{\not{D}}\,q)$	$-\frac{A^2}{8M^4}(y_uy_u^\dagger-y_dy_d^\dagger)\left(\frac{1}{2}-\log\frac{M^2}{\mu^2}\right)$
$\lvert H\rvert^2(\bar q\,i\overleftrightarrow{\not{D}}\,q)$	$\frac{A^2}{8M^4}(y_uy_u^\dagger+y_dy_d^\dagger)\left(\frac{1}{2}-\log\frac{M^2}{\mu^2}\right)$
$\lvert H\rvert^2(\bar u\,i\overleftrightarrow{\not{D}}\,u)$	$\frac{A^2}{4M^4}y_u^\dagger y_u\left(\frac{1}{2}-\log\frac{M^2}{\mu^2}\right)$
$\lvert H\rvert^2(\bar d\,i\overleftrightarrow{\not{D}}\,d)$	$\frac{A^2}{4M^4}y_d^\dagger y_d\left(\frac{1}{2}-\log\frac{M^2}{\mu^2}\right)$
$(H^\dagger\sigma^I H)(\bar l\,\sigma^I i\overleftrightarrow{\not{D}}\,l)$	$\frac{A^2}{8M^4}y_ey_e^\dagger\left(\frac{1}{2}-\log\frac{M^2}{\mu^2}\right)$
$\lvert H\rvert^2(\bar l\,i\overleftrightarrow{\not{D}}\,l)$	$\frac{A^2}{8M^4}y_ey_e^\dagger\left(\frac{1}{2}-\log\frac{M^2}{\mu^2}\right)$
$\lvert H\rvert^2(\bar e\,i\overleftrightarrow{\not{D}}\,e)$	$\frac{A^2}{4M^4}y_e^\dagger y_e\left(\frac{1}{2}-\log\frac{M^2}{\mu^2}\right)$
$\lvert H\rvert^2\,\bar q\,u\,\widetilde{H}$ (+h.c.)	$\frac{A^2}{M^4}y_uy_u^\dagger y_u\left(1-\log\frac{M^2}{\mu^2}\right)$
$\lvert H\rvert^2\,\bar q\,d\,H$ (+h.c.)	$\frac{A^2}{M^4}y_dy_d^\dagger y_d\left(1-\log\frac{M^2}{\mu^2}\right)$
$\lvert H\rvert^2\,\bar l\,e\,H$ (+h.c.)	$\frac{A^2}{M^4}y_ey_e^\dagger y_e\left(1-\log\frac{M^2}{\mu^2}\right)$

$-\log\frac{M^2}{\mu^2})$

$\frac{A^2}{M^2})$

$(\frac{3}{2}-\log\frac{M^2}{\mu^2})]$

$) + \frac{\mu_S A}{M^2}\left(2-\log\frac{M^2}{\mu^2}\right)]$

$\frac{A^3}{M^6} + \frac{\mu_S^2A^4}{M^6})$

$\log\frac{M^2}{\mu^2})]$

$\log\frac{M^2}{\mu^2})]$

$3\lambda_H\left(2-\log\frac{M^2}{\mu^2}\right)]$

$\log\frac{M^2}{\mu^2})]$

$\log\frac{M^2}{\mu^2})$

$\frac{\lambda_H}{2}\left(\frac{9}{2}-\log\frac{M^2}{\mu^2}\right)]$

$\frac{A^2}{M^4}\left(\frac{5}{6}-\log\frac{M^2}{\mu^2}\right)$

$\frac{3g_2^2A^2}{2M^4}\left(\frac{5}{6}-\log\frac{M^2}{\mu^2}\right)$

Summary

- EFT matching has been systematically solved using functional methods up to one-loop level
- We derived a streamlined prescription for functional matching

  --- arXiv: 2011.02484
- Supertrace evaluation and a Mathematica package STrEAM

  --- Manual in arXiv: 2012.07851

  --- STrEAM.m at https://github.com/EFTMatching/STrEAM
- Pedagogical example: SM + Singlet matched onto SMEFT

Thank you

Prescription for Functional Matching:

1. Derive heavy EOM(s) and $\mathcal{L}_{\text{EFT}}^{(\text{tree})}$

2. Derive K and X matrices

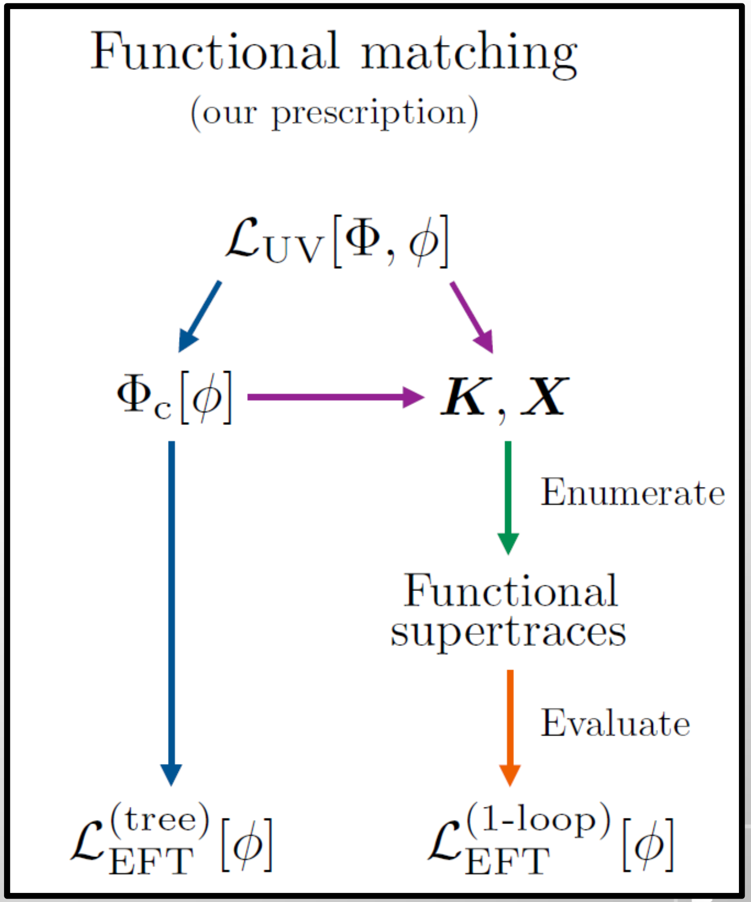
3. Enumerate supertraces

   Covariant graphs

4. Evaluate supertraces to obtain $\mathcal{L}_{\text{EFT}}^{(\text{1-loop})}$

   Mathematica package STrEAM.m

   (Cohen, XL, Zhang, arXiv: 2012.07851)



Cohen, XL, Zhang, arXiv: 2011.02484