Is SMEFT Enough?

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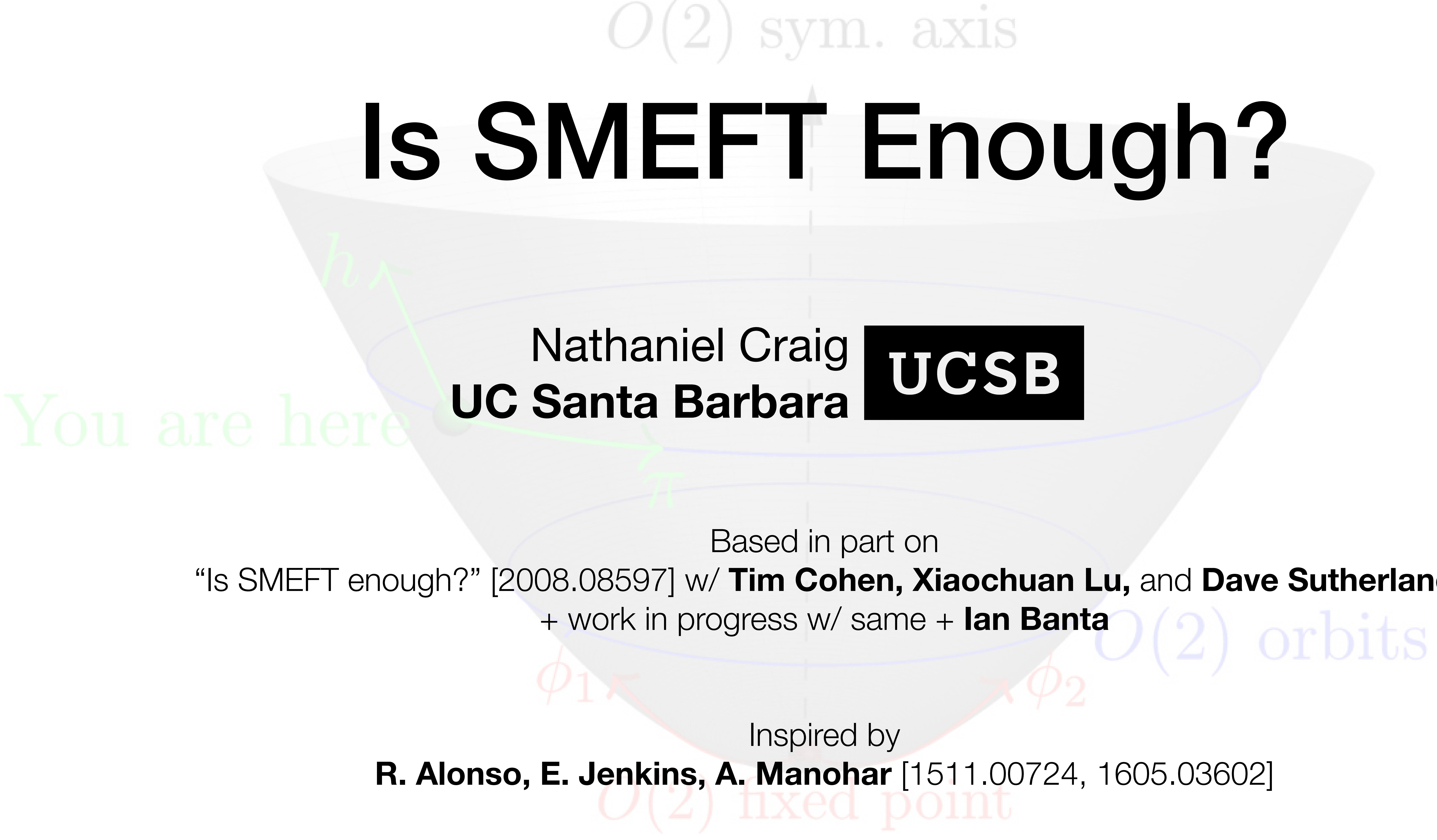
Based in part on
"Is SMEFT enough?" [2008.08597] w/ **Tim Cohen, Xiaochuan Lu,** and **Dave Sutherland**
+ work in progress w/ same + **Ian Banta**

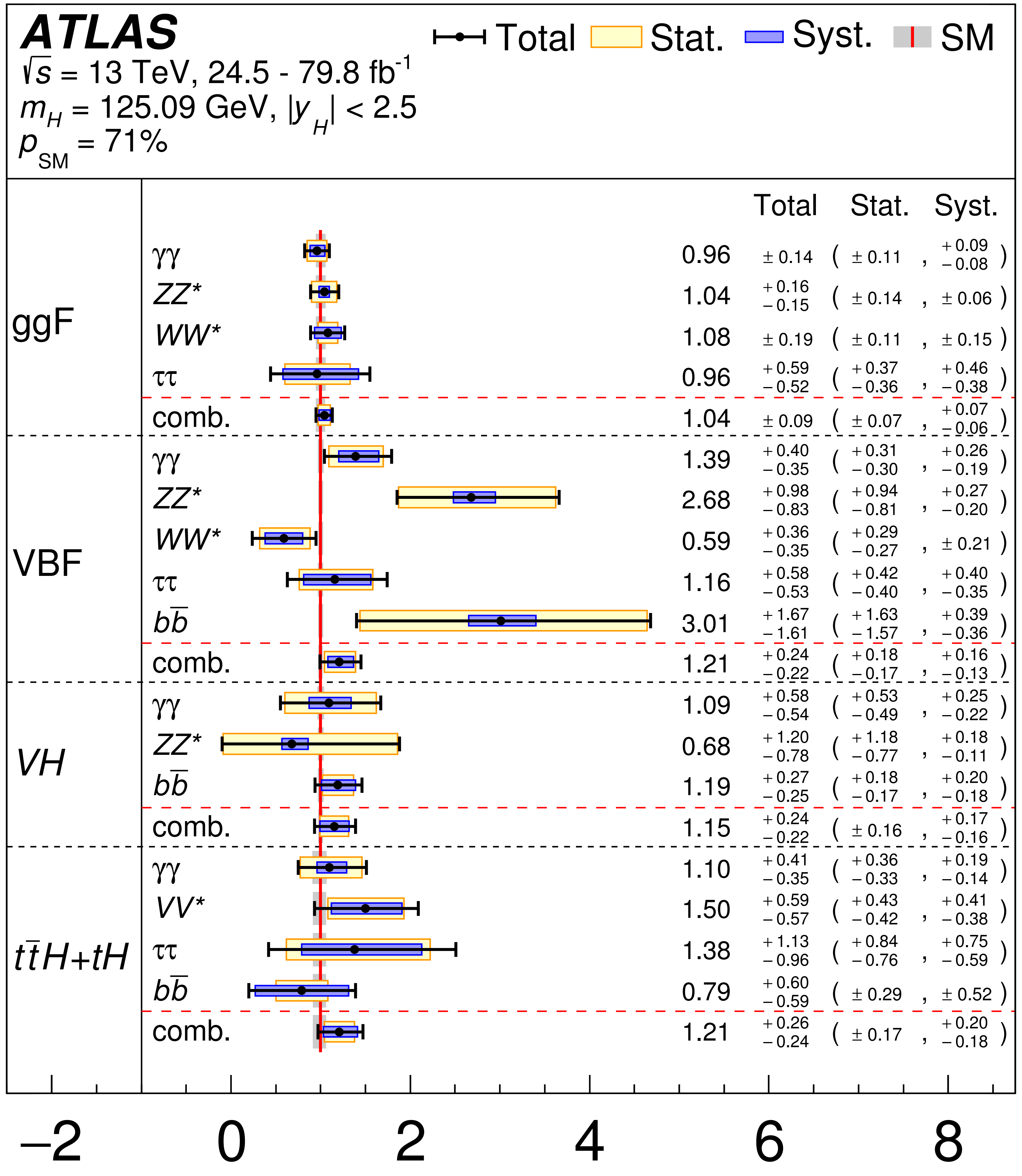
Inspired by
R. Alonso, E. Jenkins, A. Manohar [1511.00724, 1605.03602]

HEFT 2021: Hefei, China



Measurements → Meaning

[ATLAS 1909.02845]



Precision Higgs measurements a key program of LHC3/HL-LHC.

Anticipated 5-10% precision provides unprecedented tests. Future colliders to ~0.5%

Interpreting either agreement or disagreement with SM invites an EFT framework.*

Strong motivation to develop and understand Higgs EFTs!

This talk: which EFT?

*unless new physics is light.

Higgs EFTs

SM *$SU(2)_L$x$U(1)_Y$*

$$(D_\mu H)^\dagger (D^\mu H) - m^2|H|^2 - \lambda |H|^4$$

HEFT* *$U(1)_{em}$*

[Feruglio '93, Bagger et al. '93, …]

$$\frac{1}{2}(\partial_\mu h)^2 + \frac{1}{2}\left[vF(h/v)\right]^2 (\partial\vec{n})^2 - V(h) + \ldots$$

SMEFT *$SU(2)_L$x$U(1)_Y$*

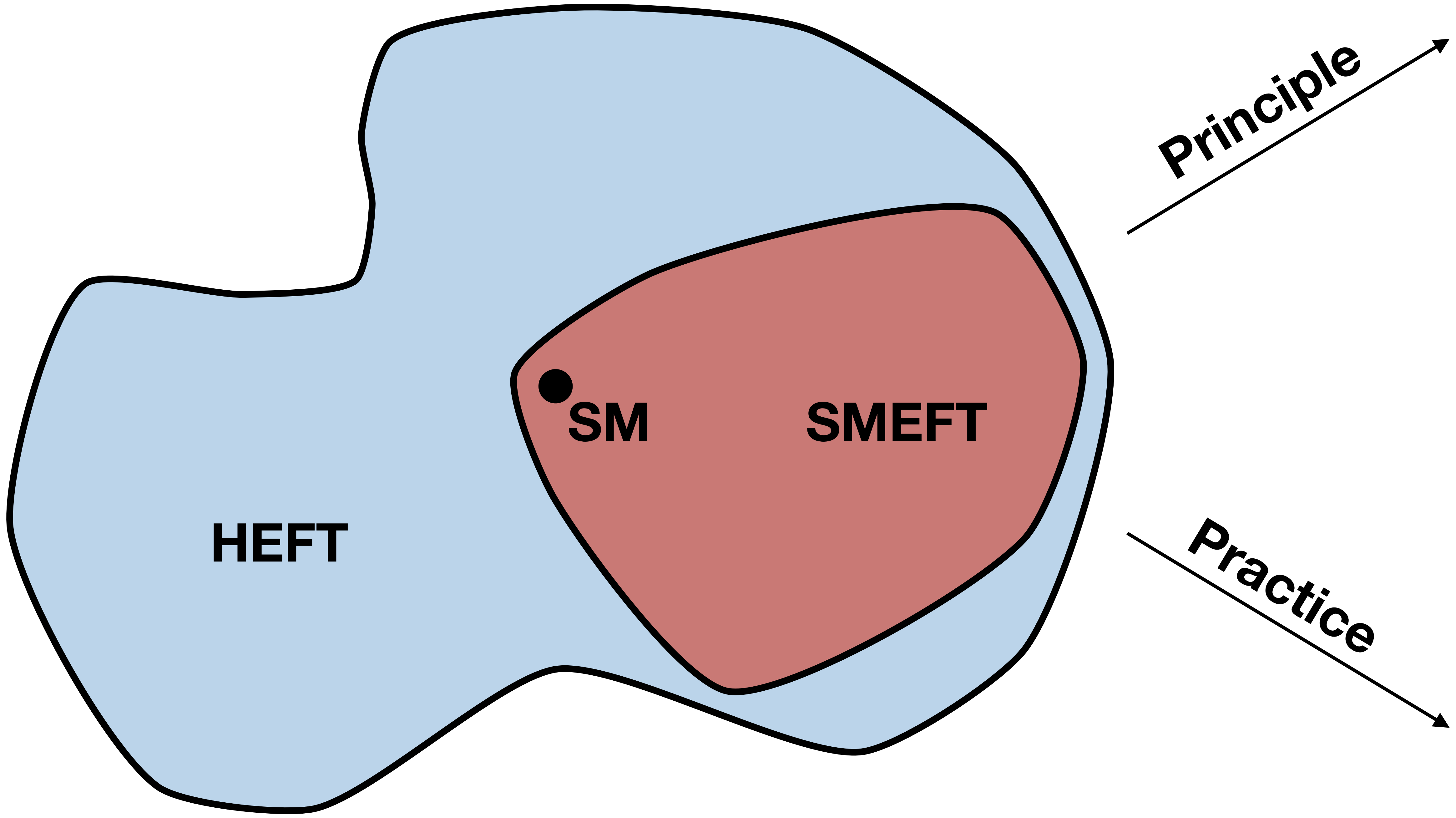
[Weinberg '79, Buchmuller, Wyler '86, …]

$$(D_\mu H)^\dagger (D^\mu H) - m^2|H|^2 - \lambda |H|^4 + \frac{c_H}{2\Lambda^2}(\partial_\mu |H|^2)^2 + \frac{c_6}{\Lambda^2}|H|^6 + \ldots$$

*Alternately, "Higgs-Electroweak Chiral Lagrangian", …

Comprehensive review: [Brivio, Trott 1703.10924]

Why bother

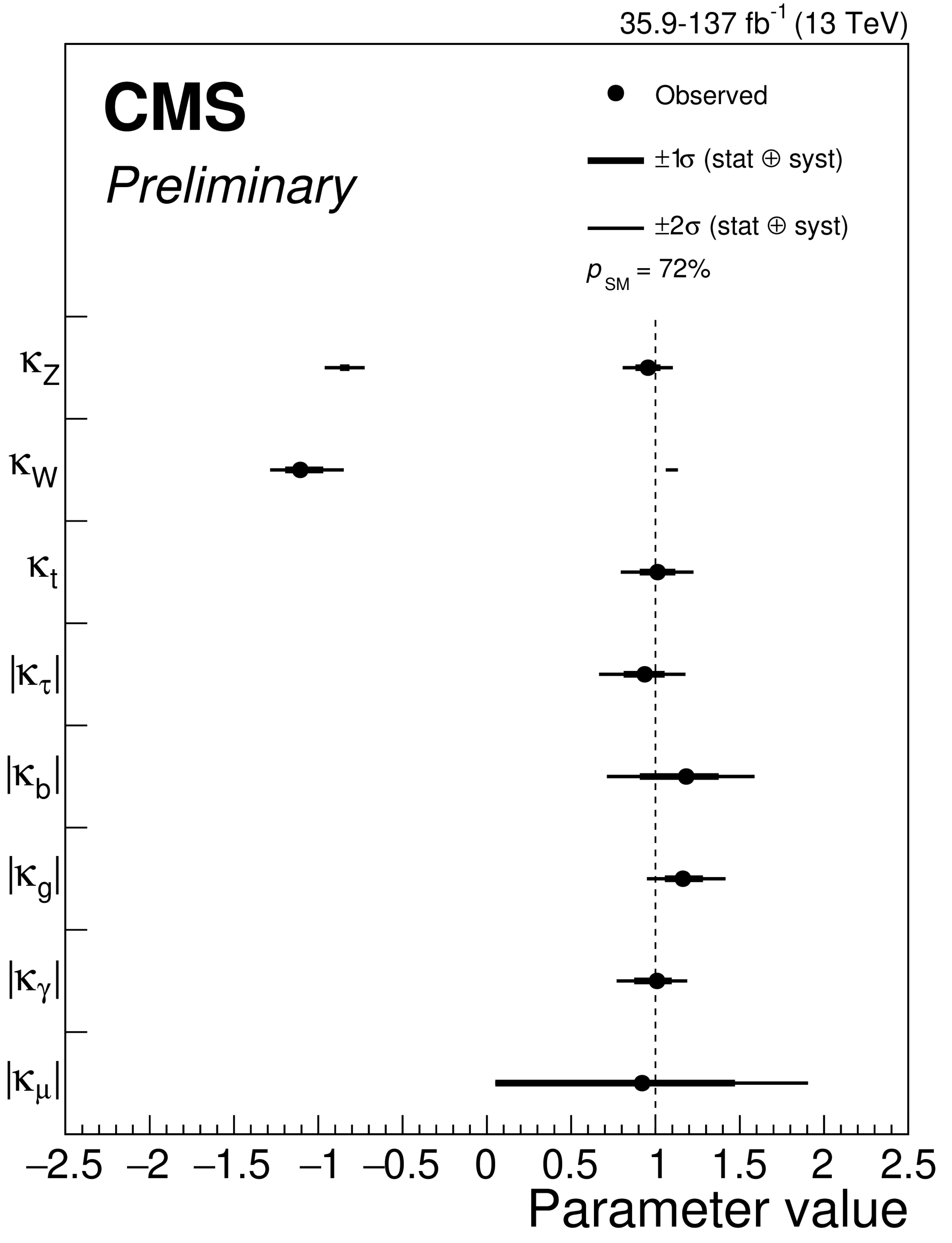


One of these is the theory of nature*, might as well study the heck out of it

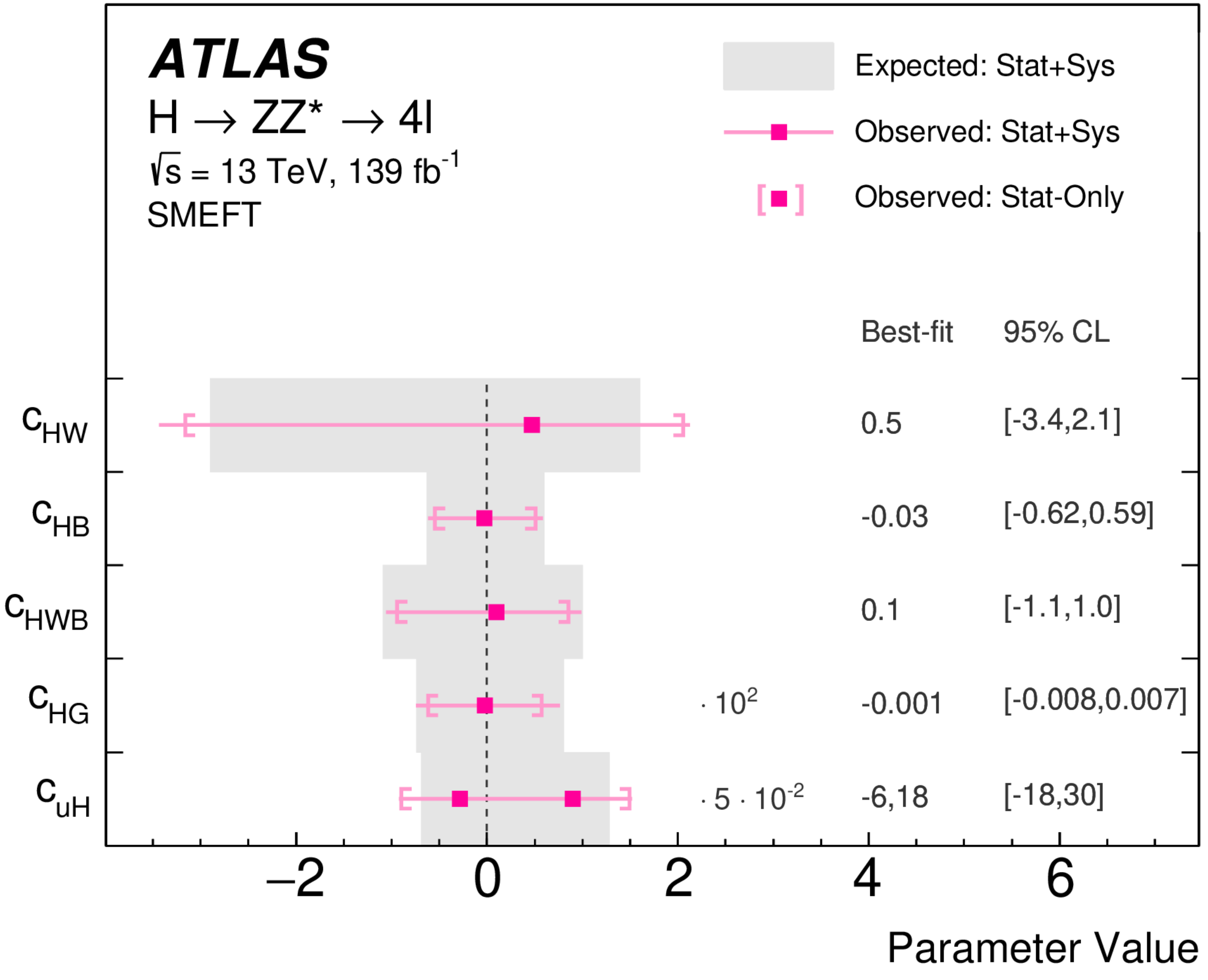
Provide meaningful framework for experimental tests of Higgs properties

*unless new physics is light.

Data!



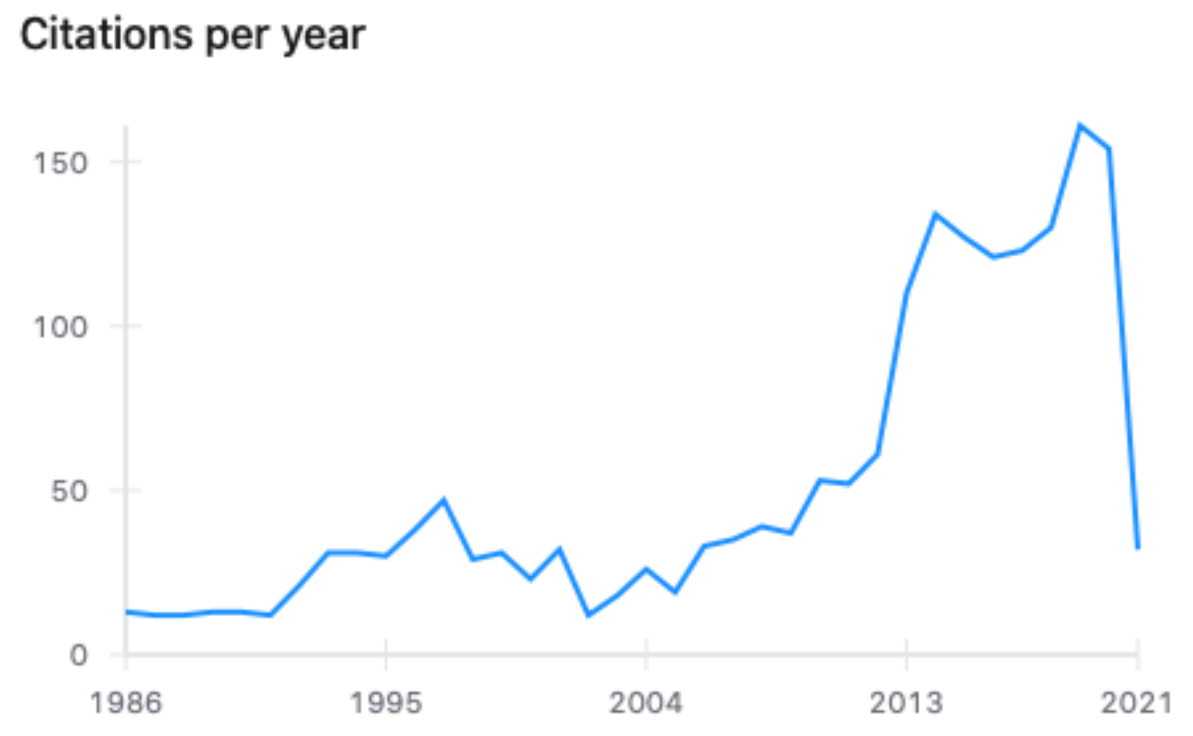
35.9-137 fb-1 (13 TeV)
CMS
Preliminary
Observed
±1σ (stat ⊕ syst)
±2σ (stat ⊕ syst)
p SM = 72%
κZ
κW
κt
|κτ|
|κb|
|κg|
|κγ|
|κμ|
−2.5 −2 −1.5 −1 −0.5 0 0.5 1 1.5 2 2.5
Parameter value



ATLAS
H → ZZ* → 4l
√s = 13 TeV, 139 fb-1
SMEFT
Expected: Stat+Sys
Observed: Stat+Sys
Observed: Stat-Only
Best-fit
95% CL
cHW
0.5
[-3.4,2.1]
cHB
-0.03
[-0.62,0.59]
cHWB
0.1
[-1.1,1.0]
cHG
· 10²
-0.001
[-0.008,0.007]
cuH
· 5 · 10⁻²
-6,18
[-18,30]
−2 0 2 4 6
Parameter Value

Which EFT?

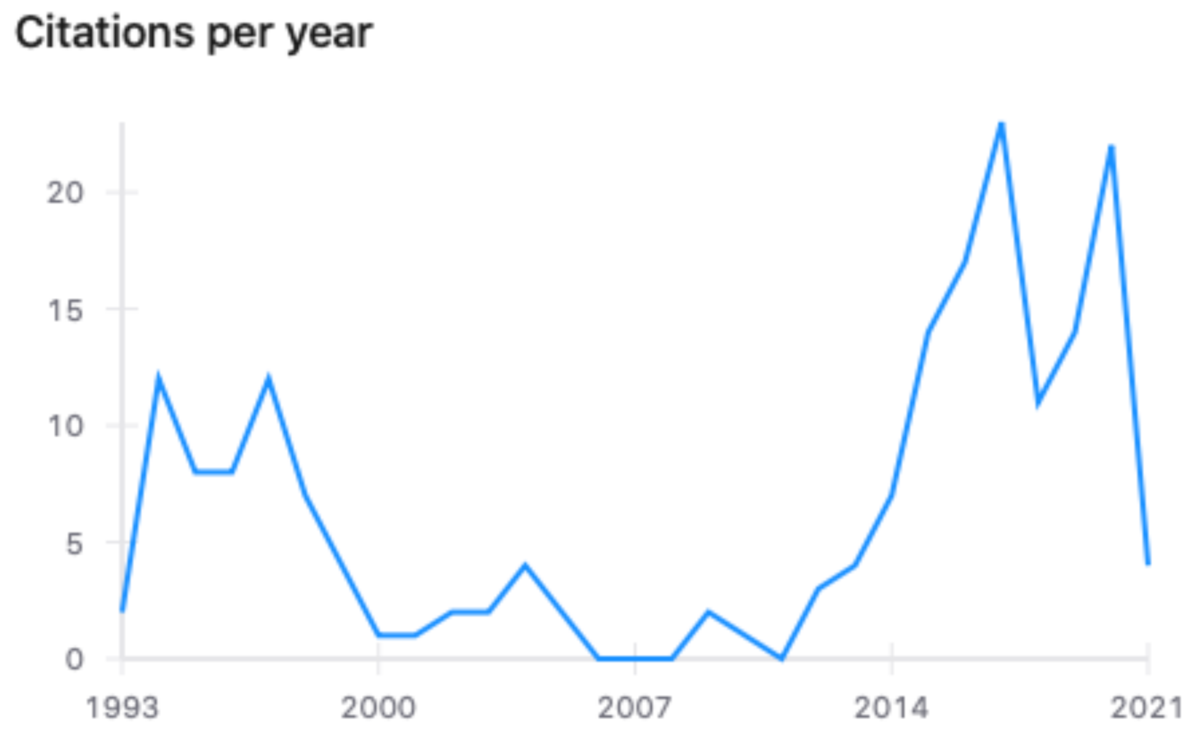
Vastly more progress in SMEFT since c. 2012 (precision, fits, projections, theorems,…)



Effective Lagrangian Analysis of New Interactions and Flavor Conservation

W. Buchmuller (CERN), D. Wyler (Zurich, ETH)

Aug, 1985



The Chiral approach to the electroweak interactions

F. Feruglio (Padua U. and INFN, Padua)

Sep, 1992

Seems justified: SU(2)xU(1) an apparently good symmetry, no O(1) deviations or custodial symmetry violation

(When) Is HEFT necessary?

See also: [Burgess, Matias, Pospelov '99; Grinstein & Trott '07; Alonso, Gavela, Merlo, Rigolin, Yepes '12; Espriu, Mescia, Yencho '13; Buchalla, Cata, Krause '13; Brivio et al. '13; Falkowski & Rattazzi ' 19]

Amplitudes perspective: [Durieux, Kitahara, Shadmi, Weiss '19]

For this talk: focus exclusively on scalar sector in the global limit, assume custodial symmetry, restrict to 2-derivative order.

The Standard Model EFT

SMEFT: EFT where 4 scalar d.o.f. are arranged into an SU(2) doublet (equivalently, O(4) fundamental; assuming custodial symmetry):

$$\vec{\phi} = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \end{pmatrix}, \qquad \vec{\phi} \to O\vec{\phi}, \qquad H = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}$$

where $O \in O(4) \supset SU(2) \times U(1)$

"Electroweak symmetry is linearly realized."

$$\mathcal{L}_{\text{SM}} = \frac{1}{2}(\partial\vec{\phi} \cdot \partial\vec{\phi}) - \frac{1}{4}\lambda(\vec{\phi} \cdot \vec{\phi} - v^2)^2$$

$$\mathcal{L}_{\text{SMEFT}} = \frac{1}{2}A(\vec{\phi} \cdot \vec{\phi})(\partial\vec{\phi} \cdot \partial\vec{\phi}) + \frac{1}{2}B\left(\vec{\phi} \cdot \vec{\phi}\right)(\vec{\phi} \cdot \partial\vec{\phi})^2 - V\left(\vec{\phi} \cdot \vec{\phi}\right) + \mathcal{O}\left(\partial^4\right)$$

Reminder: only worrying about scalars up to 2 derivatives…

The Higgs EFT

Alternately, HEFT:
construct EFT out of
singlet h and Goldstones π_i

No presumed relation between h, π

$$h \qquad \vec{n} = \begin{pmatrix} n_1 = \pi_1/v \\ n_2 = \pi_2/v \\ n_3 = \pi_3/v \\ n_4 = \sqrt{1 - n_1^2 - n_2^2 - n_3^2} \end{pmatrix}$$

$$h \to h\,, \qquad \vec{n} \to O\vec{n}\,, \qquad O \in O(4)$$

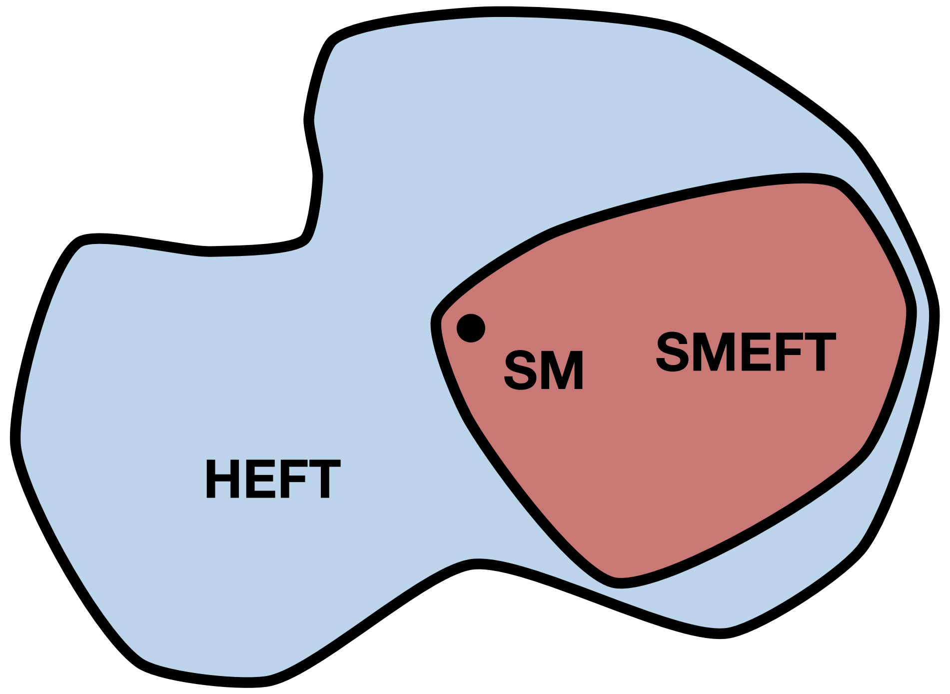
"Electroweak symmetry is nonlinearly realized."

$$\mathcal{L}_{\rm SM} = \frac{1}{2}\left(\partial h\right)^2 + \frac{1}{2}(v+h)^2\left(\partial\vec{n}\right)^2 - \frac{1}{4}\lambda(h^2+2vh)^2$$

$$\mathcal{L}_{\rm HEFT} = \frac{1}{2}[K\left(h\right)]^2(\partial h)^2 + \frac{1}{2}[vF\left(h\right)]^2(\partial\vec{n})^2 - V\left(h\right) + \mathcal{O}\left(\partial^4\right)$$

(*K(h)* redundant, conventional to redefine h to set *K(h)* = 1; retaining *K(h)* clearer for matching)

SM ⊂ SMEFT ⊂ HEFT



[R. Alonso, E. Jenkins, A. Manohar 1511.00724 & 1605.03602]

$$\vec{\phi} = (v+h)\,\vec{n}\,(\pi)\,; \quad \vec{\phi}\cdot\vec{\phi} = (v+h)^2$$

SMEFT can always be written as HEFT:

$$\begin{aligned}\mathcal{L} &= \frac{1}{2}A(\vec{\phi}\cdot\vec{\phi})(\partial\vec{\phi}\cdot\partial\vec{\phi}) + \frac{1}{2}B\left(\vec{\phi}\cdot\vec{\phi}\right)(\vec{\phi}\cdot\partial\vec{\phi})^2 - V\left(\vec{\phi}\cdot\vec{\phi}\right)\\ &= \frac{1}{2}\left[A + (v+h)^2\,B\right](\partial h)^2 + \frac{1}{2}(v+h)^2\,A(\partial\vec{n})^2 - V\end{aligned}$$

Correlations at every order between h, v

HEFT cannot always be written as SMEFT:

$$\begin{aligned}\mathcal{L} &= \frac{1}{2}[K(h)]^2(\partial h)^2 + \frac{1}{2}[vF(h)]^2(\partial\vec{n})^2 - V(h)\\ &= \frac{1}{2}\frac{v^2F}{\vec{\phi}\cdot\vec{\phi}}(\partial\vec{\phi})^2 + \frac{1}{2}(\vec{\phi}\cdot\partial\vec{\phi})^2\frac{1}{\vec{\phi}\cdot\vec{\phi}}\left(K^2 - \frac{v^2F^2}{\vec{\phi}\cdot\vec{\phi}}\right) - \tilde{V}(\vec{\phi}\cdot\vec{\phi})\end{aligned}$$

Generically non-analytic at the origin

HEFT or SMEFT?

When can a theory be written as HEFT but not SMEFT?

Maybe you can always just tell by eye…

$$\mathcal{L} = \frac{1}{2}\left(1+\frac{h}{2v}\right)^2 (\partial h)^2 + \frac{1}{2}(v+h)^2\left(\frac{3}{4}+\frac{h}{4v}\right)^2 (\partial \vec{n})^2 - V$$

Definitely HEFT, right?

HEFT or SMEFT?

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$$\mathcal{L} = \frac{1}{2}\left(1 + \frac{h}{2v}\right)^2 (\partial h)^2 + \frac{1}{2}(v+h)^2\left(\frac{3}{4} + \frac{h}{4v}\right)^2 (\partial \vec{n})^2 - V$$

Definitely HEFT, right?

But now let's perform the field redefinition

$$h \to \tilde{h} \equiv h + \frac{1}{4v}h^2$$

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Definitely HEFT, right?

But now let's perform the field redefinition

$$h \to \tilde{h} \equiv h + \frac{1}{4v}h^2$$

$$\mathcal{L} = \frac{1}{2}(\partial\tilde{h})^2 + \frac{1}{2}(\tilde{v}+\tilde{h})^2(\partial\vec{n})^2 + \cdots = |\partial\tilde{H}|^2 + \dots$$

Actually the SM

Field redefinitions readily obscure the distinction at the level of the Lagrangian.

A Geometric Perspective

***Instead*: classify EFTs based on geometry.**

Two-derivative terms define a metric on the scalar field manifold

$$\mathcal{L} = \frac{1}{2} g_{ij}(\phi)\partial\phi^i \partial\phi^j - V(\phi)$$

Field space corresponds to a (possibly curved) manifold with functions (e.g. V) defined on it; the field parameterization corresponds to charts on the manifold. Use geometric invariants to classify EFTs.

Long history (primarily) applied to nonlinear sigma models, e.g.
[Honerkamp ’72; Tataru ’75; Alvarez-Gaume, Freedman, Mukhi ’81, …]

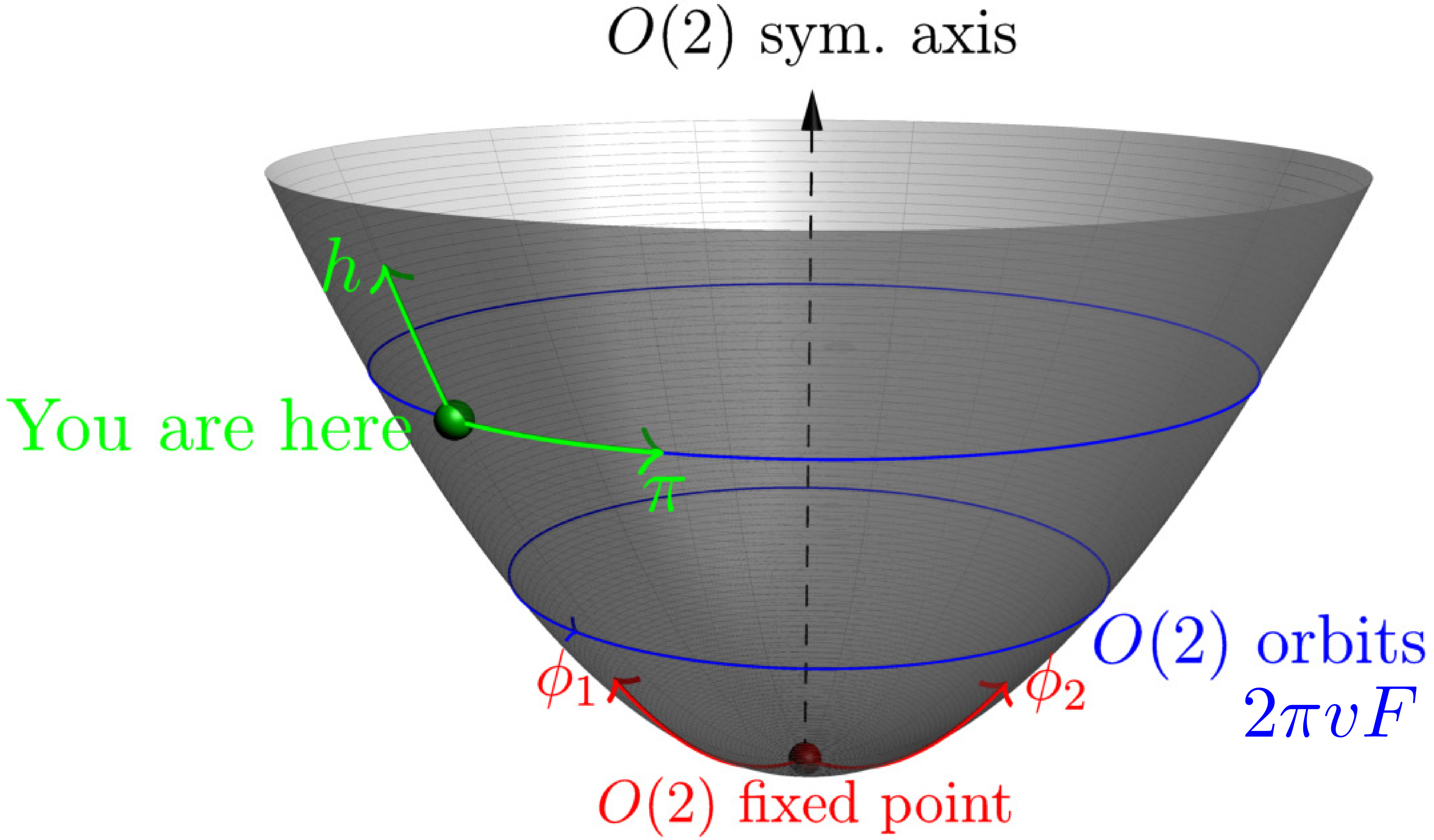
Application to HEFT: [Alonso, Jenkins, Manohar 1511.00724 & 1605.03602]
(Applied to SMEFT: [Helset, Martin, Trott 2001.01453])

SM: flat manifold **HEFT:** curved manifold

SMEFT: curved manifold w/ O(4) invariant point

A Geometric Perspective

(Think O(4), but O(2) is easier to illustrate)



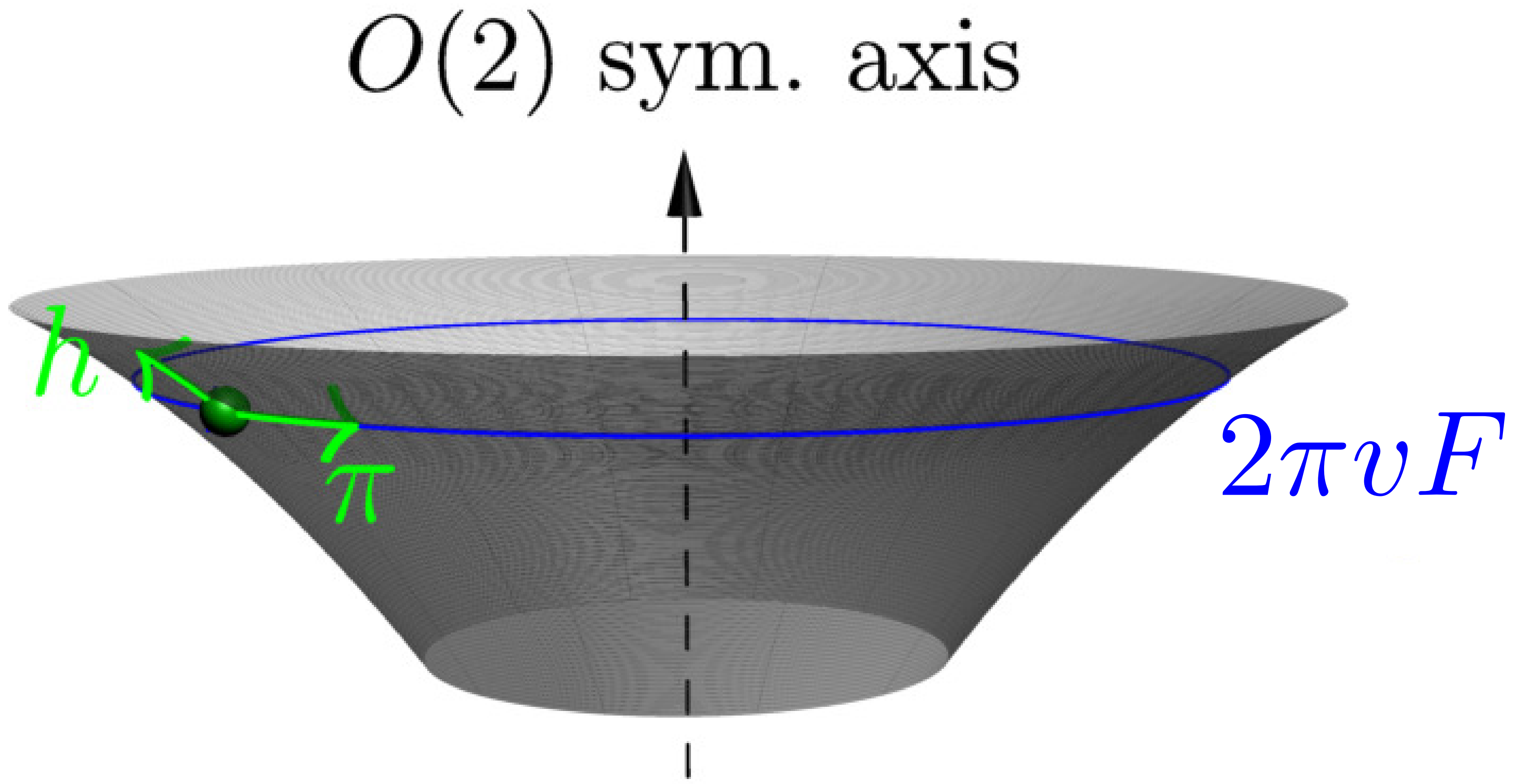
$$\mathcal{L}_{\mathrm{HEFT}} = \frac{1}{2}(\partial h)^2 + \frac{1}{2}[vF(h)]^2(\partial \vec{n})^2 - V(h) + \mathcal{O}\left(\partial^4\right)$$

SMEFT if O(4) fixed point on manifold → F(h) = 0 somewhere (say, *h* = -*v*)

HEFT not SMEFT: Case I

[Alonso, Jenkins, Manohar 1605.03602]

When there's a hole s.t. h = -v is not on the manifold
(no O(4) fixed point about which to expand in SMEFT coordinates)



$$\mathcal{L}_{\mathrm{HEFT}} = \frac{1}{2}(\partial h)^2 + \frac{1}{2}[vF(h)]^2(\partial \vec{n})^2 - V(h) + \mathcal{O}(\partial^4)$$

Corresponds to $F(h) \neq 0$ **everywhere**

HEFT not SMEFT: Case I

How does this arise? *When UV physics also breaks the symmetry.*

A toy example: 2AHM, i.e. two Higgses charged under a U(1) gauge symmetry

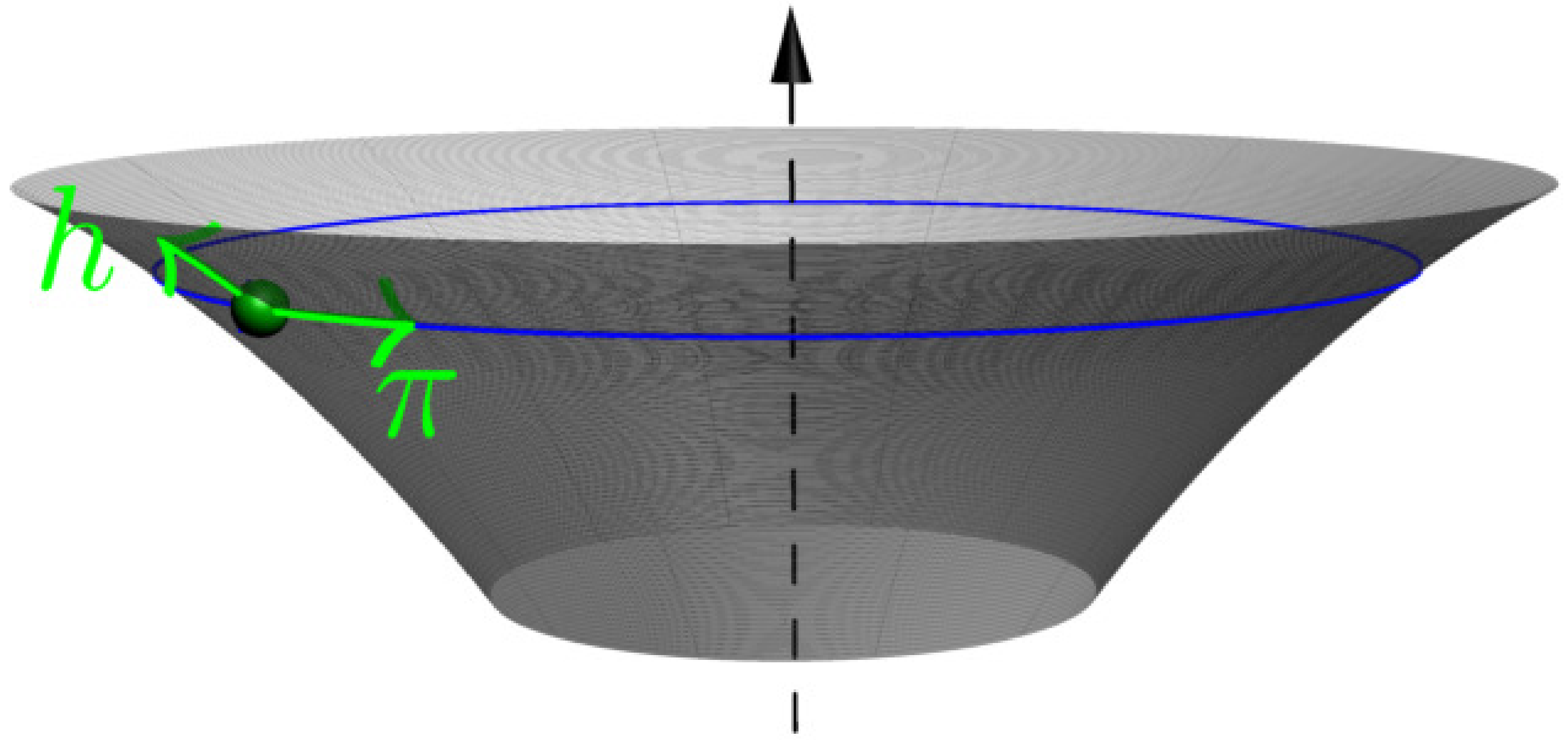
Acquire vevs s.t. $v^2 \equiv 4v_1^2 + v_2^2$

Field	Q
H_1	$+2$
H_2	$+1$

Spectrum: light Higgs *h*, goldstone *π*, heavy fields *H, Π*

Integrate out *H, Π* to obtain EFT of *h, π*

$$K(h) = 1, \qquad F(h) = \frac{1}{v}\sqrt{4(v_1 + c_\alpha h)^2 + (v_2 + s_\alpha h)^2}$$



Generically F(h) ≠ 0 everywhere for nonzero v_1, v_2

HEFT not SMEFT: Case II

When there's a cone or cusp at *h*=-*v*

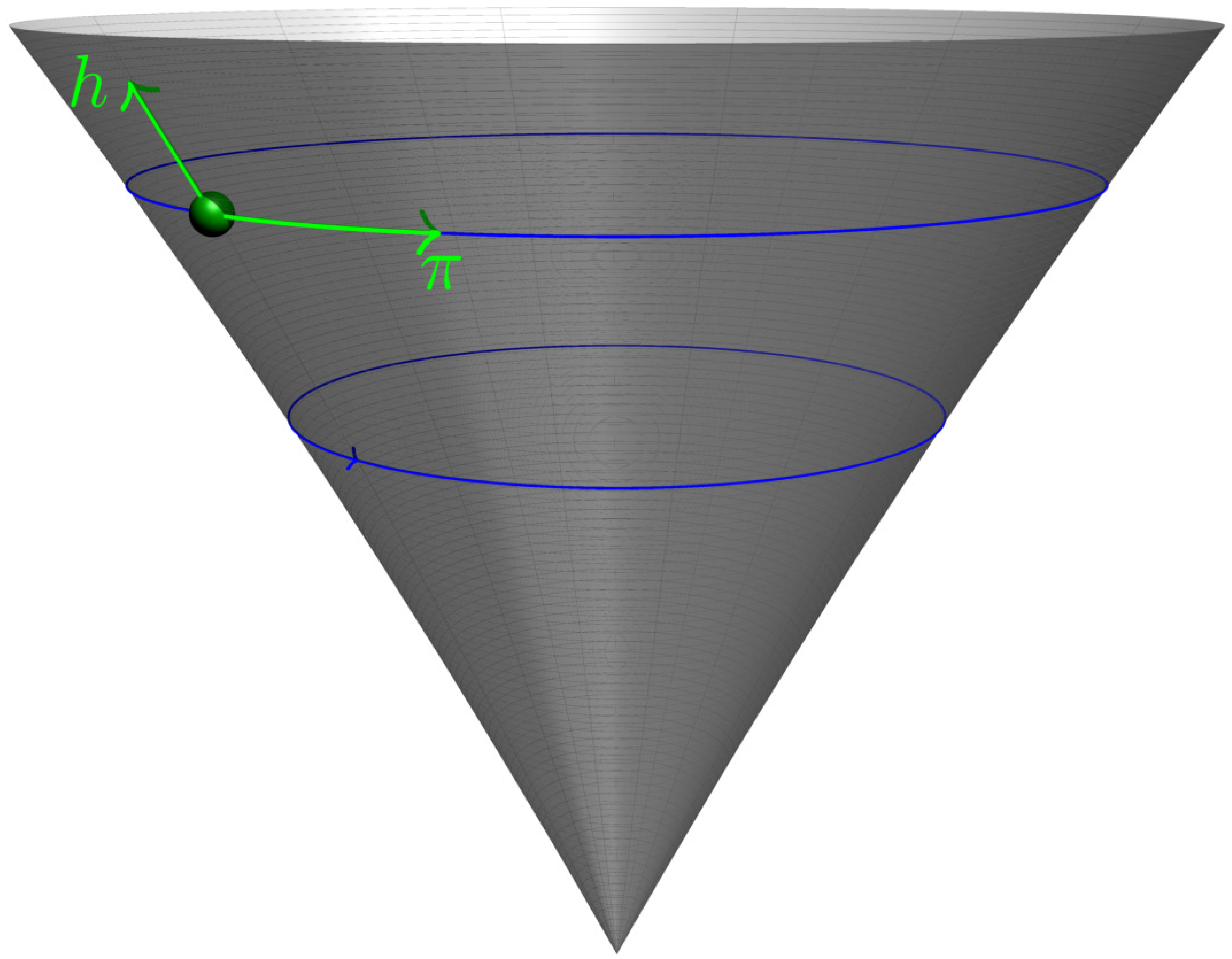
Can often tell by inspecting F(h), V(h) for non-analyticities, but this does not always work.

$$\mathcal{L}_{\rm HEFT} = \frac{1}{2}(\partial h)^2 + \frac{1}{2}[vF(h)]^2(\partial\vec{n})^2 - V(h)$$

But can diagnose singularities as in GR:

If $(\nabla^2)^n R$ and $(\nabla^2)^{n+1} V$

are finite at *h*=-*v*, then can write HEFT as SMEFT (gives the requisite infinite set of conditions!)



Otherwise, there is a cone/cusp and HEFT is required.

HEFT not SMEFT: Case II

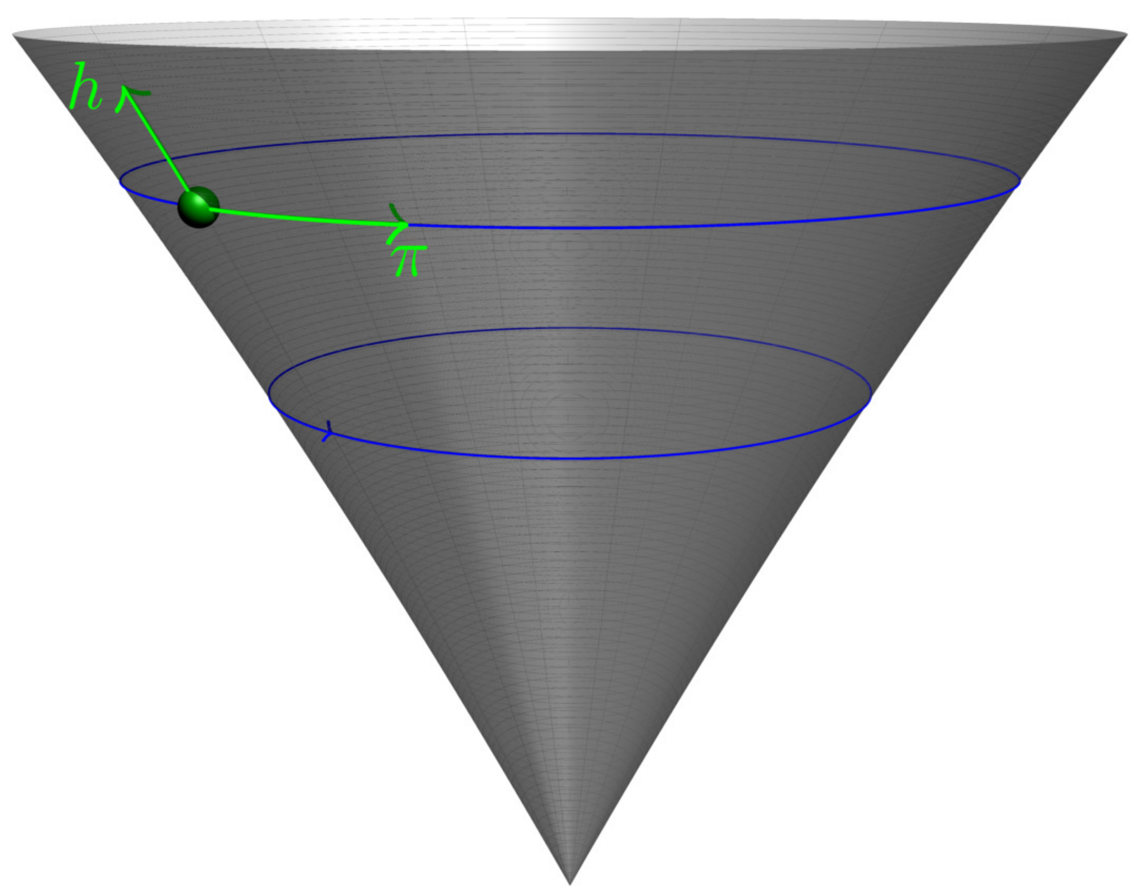
How does this arise? *When a field becomes massless.*

An example: integrating out anything that acquires all of its mass from EWSB, e.g. *M=0* limit of

$$\mathcal{L} \supset \bar{\psi}_1(i\not{\partial} - M)\psi_1 + \bar{\psi}_2(i\not{\partial} - M)\psi_2 - y\bar{\psi}_1 H\psi_2 + \text{h.c.}$$

F(h=-v) = 0, so okay according to Case I

Compute Ricci scalar: $$R(h=-v) \propto \frac{|y|^4}{16\pi^2}\frac{1}{M^2}$$



When *M ≠ 0*, curvature finite and SMEFT is consistent

For *M=0*, curvature blows up.
K, *F*, and *V* all non-analytic at *h=-v* due to *log(v+h)*

HEFT as SMEFT IFF

$$\mathcal{L}_{\mathrm{HEFT}} = \frac{1}{2}(\partial h)^2 + \frac{1}{2}[vF(h)]^2(\partial\vec{n})^2 - V(h) + \mathcal{O}\left(\partial^4\right)$$

1. ***F(h*) = 0*** at some h=h* (candidate O(4) f.p.)

2. ***Metric is analytic*** at h=h*: F(h), K(h) admit convergent Taylor expansions here, and curvature invariants ~$\nabla^n R$ are finite for n≥0.

3. ***Potential is analytic*** at h=h*: V(h) admits convergent Taylor expansion here, and invariants ~$\nabla^n V$ are finite for n≥0.

Satisfying these conditions ensures the theory admits a SMEFT expansion around the O(4) fixed point. However, a further consideration: *that expansion should converge at our vacuum (h=0).*

SMEFT Convergence

Even when SMEFT exists, the SMEFT expansion may not converge at our vacuum.

Clear example: for SMEFT with $\Lambda < v$, $\mathcal{L} \supset \sum_{n=1}^{\infty} c_n \frac{|H|^{4+2n}}{\Lambda^{2n}}$ diverges, w/out optimal truncation

To make this more concrete…

Consider a singlet scalar with nonzero bare mass,

$$\mathcal{L}_{\rm UV} = |\partial H|^2 + \mu_h^2 |H|^2 - \frac{1}{2}\lambda_h |H|^4 + \frac{1}{2} S \left(-\partial^2 - m^2 - \kappa |H|^2\right) S$$

Integrating out the scalar gives 0- & 2-derivative effective lagrangian for H:

$$\delta\mathcal{L}_{\rm Eff}^{(0)} = \frac{1}{(4\pi)^2}\frac{1}{4}\left(m^2 + \kappa|H|^2\right)^2 \left(\ln\frac{\mu^2}{m^2 + \kappa|H|^2} + \frac{3}{2}\right)$$

$$\delta\mathcal{L}_{\rm Eff}^{(2)} = \frac{1}{(4\pi)^2}\frac{1}{4}\frac{1}{6}\frac{\kappa^2}{m^2 + \kappa|H|^2}\left(\partial|H|^2\right)^2$$

SMEFT Convergence

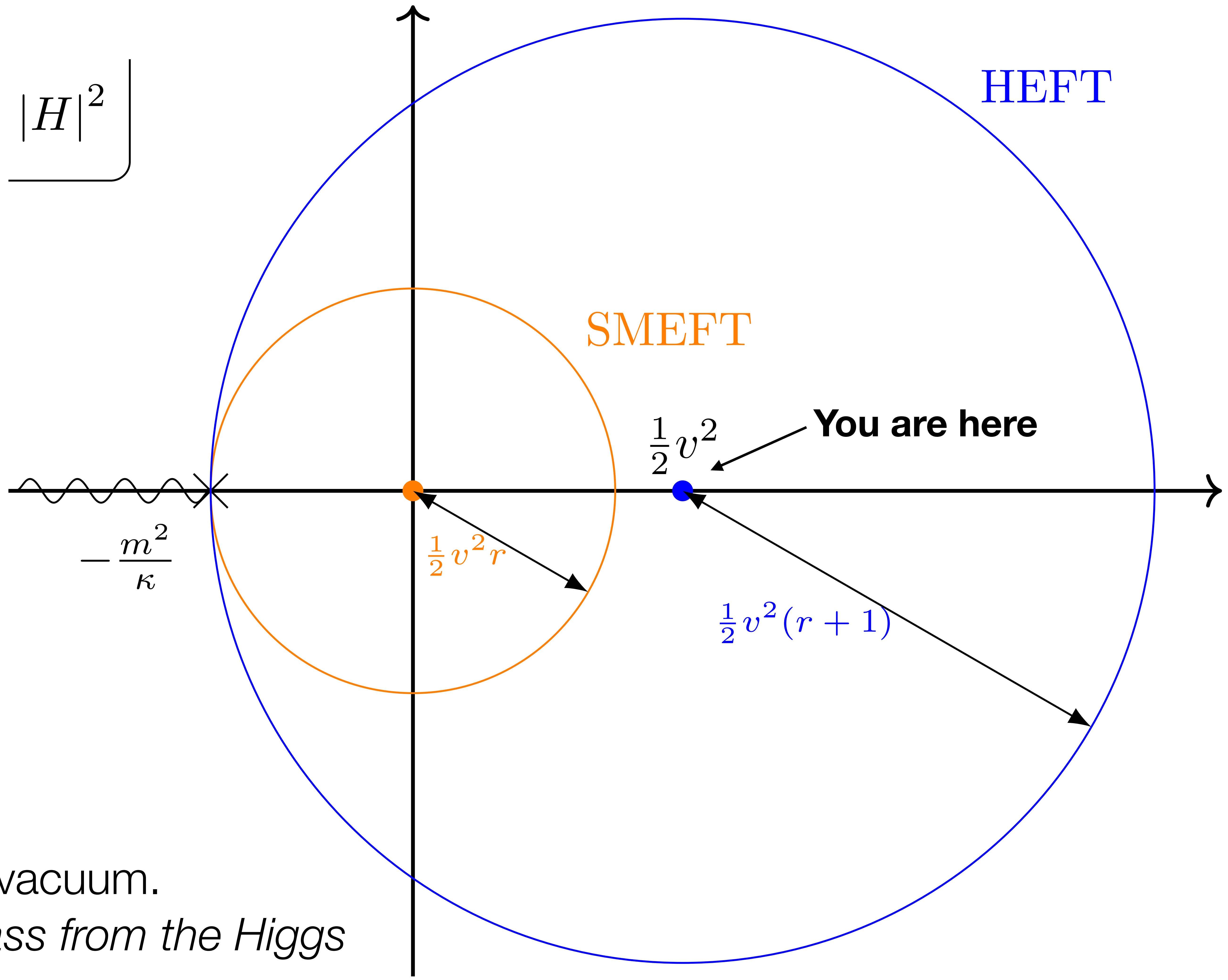
Consider analytic structure of the effective Lagrangian in the complex $|H|^2$ plane

$$r \equiv \frac{\text{bare mass}^2}{\text{mass}^2 \text{ from Higgs}} = \frac{m^2}{\frac{1}{2}\kappa v^2}$$

Branch cut at $|H|^2 = -\frac{m^2}{\kappa} \Rightarrow$

SMEFT radius of convergence is $v^2 r/2$

HEFT radius of convergence is $v^2(r+1)/2$



$r < 1$: SMEFT expansion does not converge at our vacuum.

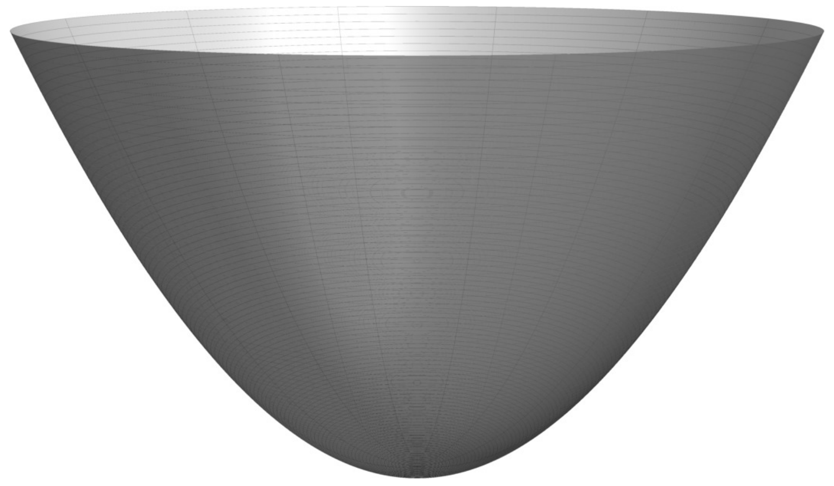
HEFT required by states w/ more than half of their mass from the Higgs

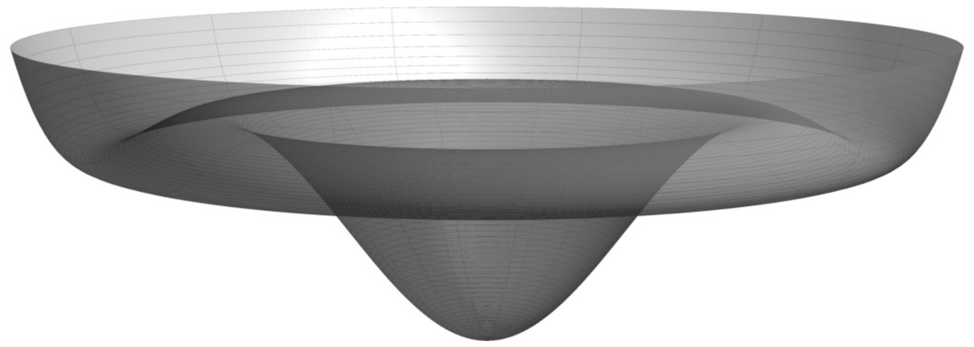
SMEFT Convergence

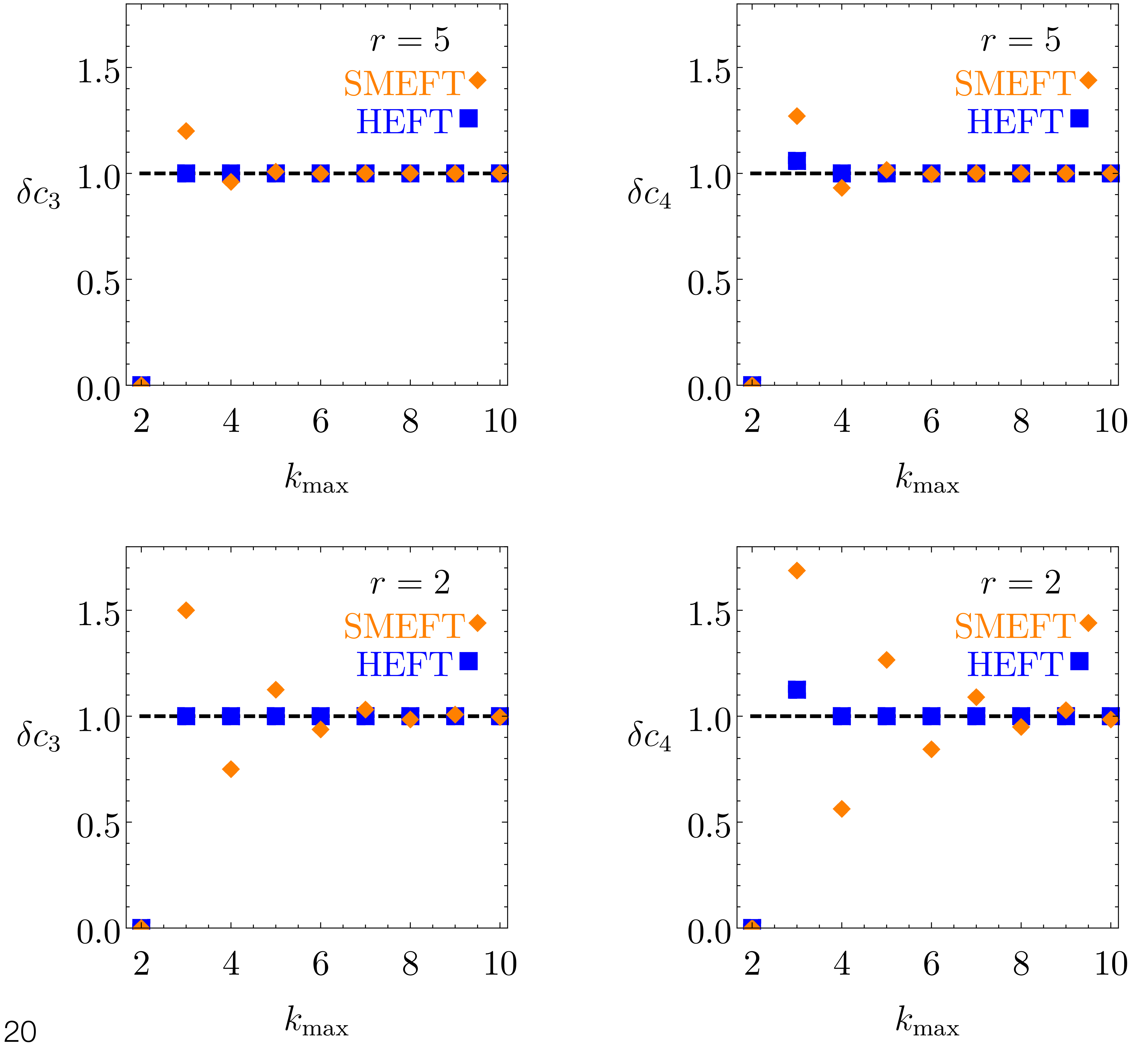
Even for $r \gtrsim 1$, HEFT can capture true corrections to SM using fewer terms in the relevant expansion than SMEFT.

[Englert et al. 1403.7191; Brehmer et al. 1510.03443] Improve agreement between truncation of SMEFT and true corrections by defining matching scale as physical mass of new particles in the broken phase ("v-improved matching").

Practically amounts to matching in HEFT, converting to SMEFT coordinates.







Loryons*

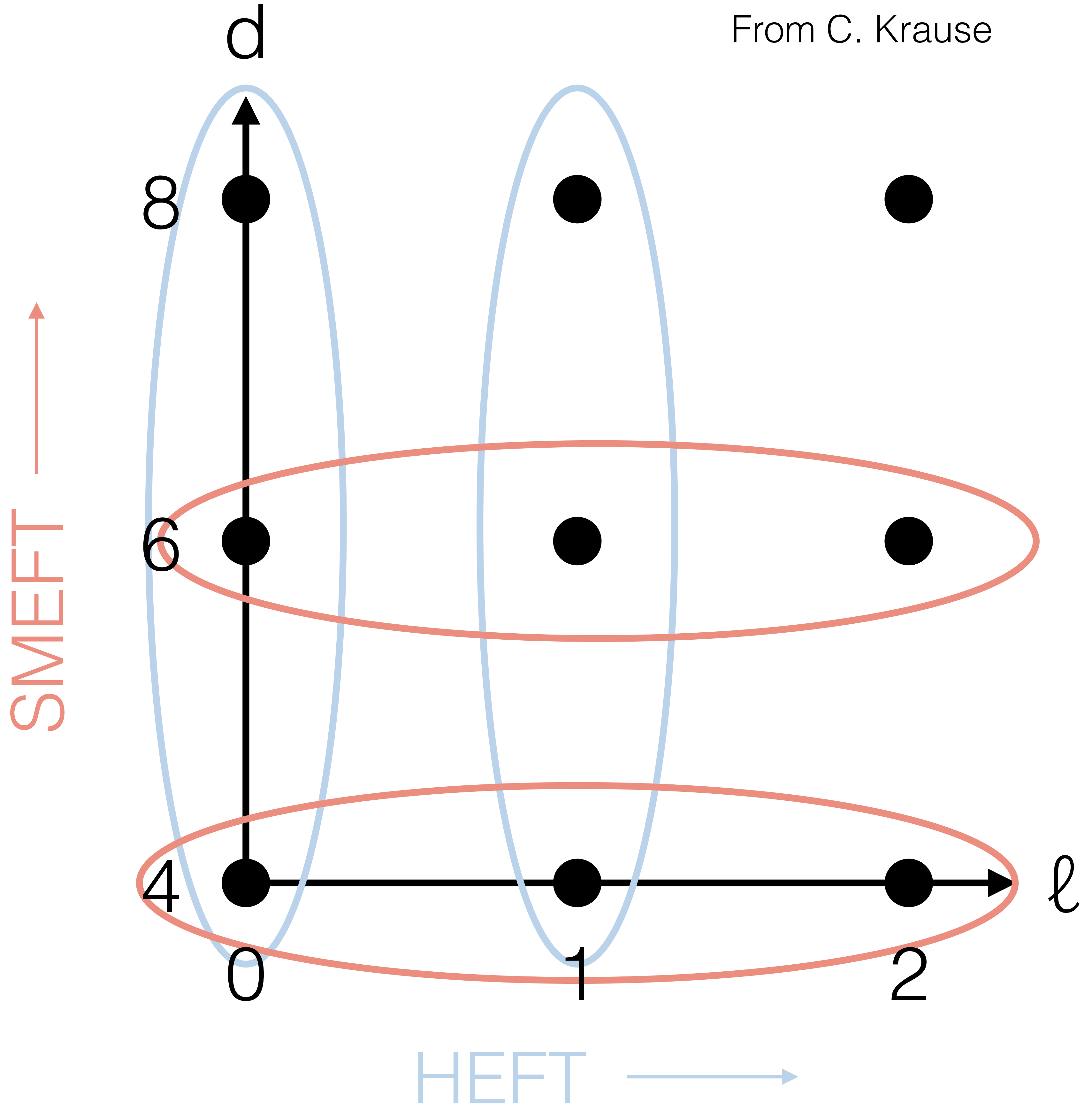
*Following Gell-Mann, from *Finnegan's Wake*: "with Pa's new heft...see Loryon the comaleon."

HEFT required whenever a new particle ("Loryon") acquires more than half of its mass from the Higgs.

Many such Loryons viable, consistent with all existing data (see also [Bonnefoy et al. 2011.10025])

HEFT Surprises?

HEFT 1-loop anomalous dimensions [Buchalla et al. 2004.11348]



One loop divergences due to some SMEFT operators reproduced in HEFT, e.g.

$$\mathcal{O}_{\phi\Box} = \phi^\dagger\phi\Box\phi^\dagger\phi$$

Such operators give “surprising zeroes” in SMEFT matrix of anomalous dimensions; should also manifest in HEFT.

More generally, expect to discover rich structure of 1- and 2-loop surprises mirroring that of SMEFT (e.g. [Bern, Parra-Martinez, Sawyer 2005.12917])…

Conclusions

- Universal geometric criteria for HEFT vs. SMEFT:

SMEFT **HEFT** **HEFT** **~HEFT**

- *Many* ways to get $U(1)_{em}$ Higgs EFT starting from SU(2)xU(1) symmetry in the UV, consistent w/ data. Perhaps premature to focus heavily on SMEFT interpretations.

- HEFT can be the preferred EFT for data even when both HEFT & SMEFT expansions valid.

- Interesting connections between geometric picture & scattering amplitudes, e.g. [Alonso, Jenkins, Manohar ’16; Nagai, Tanabashi, Tsumura, Uchida ’19; Cohen, NC, Lu, Sutherland *to appear*]

- Motivates giving HEFT more thorough attention, both “in principle” and “in practice.” Plethora of structural questions currently being explored in SMEFT can also be addressed in HEFT…

Thank you!