Institute of High Energy Physics

Positivity in Multi-Field EFTs

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Apr. 16 Higgs and Effective Field Theory - HEFT 2021

base on 2101.01191 with C. Yang, H. Xu, C. Zhang, and S.-Y. Zhou

Motivation: Positivity Bounds

All possible
Ultraviolet(UV) physics

↓

EFT:

$$\mathcal{L}_{EFT} = \mathcal{L}_{SM} + \sum_i \frac{C_i^{(6)} O_i^{(6)}}{\Lambda^2} + \sum_i \frac{C_i^{(8)} O_i^{(8)}}{\Lambda^4} + \cdots$$

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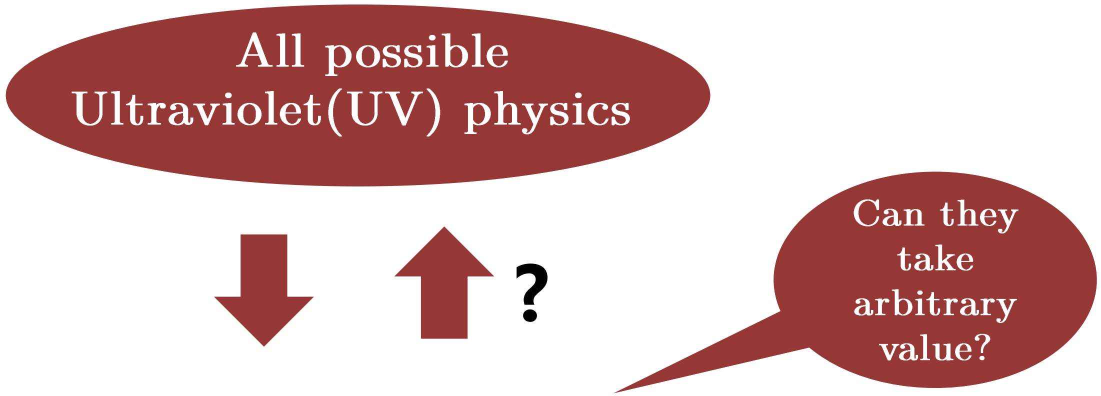

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If UV physics satisfied causality, unitarity,
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$$\begin{cases} \sum_i a_i C_i \geq 0 \\ \vdots \end{cases}$$

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Relevant literature: [N. Arkani-Hamed, et al. 2012.15849], [B. Bellazzini. et al. 2011.00037], [A. Tolley et al., 2011.02400], [T. Trott, 2011.10058], [S. C-Huot. et al. 2011.02957] See talk by Trott, Tolley ...

Motivation: Positivity Bounds

$$\mathcal{L}_{EFT} = \mathcal{L}_{SM} + \sum_i \frac{C_i^{(6)} O_i^{(6)}}{\Lambda^2} + \sum_i \frac{C_i^{(8)} O_i^{(8)}}{\Lambda^4} + \cdots$$

2-to-2 forward amplitude (spin-0): $A(s,0) = c_0 + c_2 s^2 + c_4 s^4 + \cdots$

Dim-8 have leading energy dependence only, s2.

To extract dim-8 effect, we consider:

$$\frac{d^2}{ds^2} A(s,0)$$

See talk by Trott

EFTs can involve more than one particles. (e.g. SMEFT operators; or those involving multiplet particles, chiral PT, spin-2 EFTs, ...)

Outline

1 Framework

2 A Scalar EFT

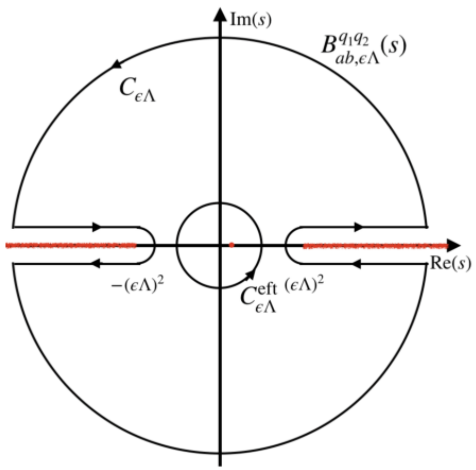
3 Numerical approach

4 Examples

Framework

Positivity Bounds

For 2-to-2 forward scattering($t \approx 0$): $$f = \frac{1}{2\pi i}\oint ds \frac{A(s,0)}{(s-\mu^2)^3}$$



i,j,k,l : particle index

$M^2 = m_i^2 + m_j^2 + m_k^2 + m_l^2$

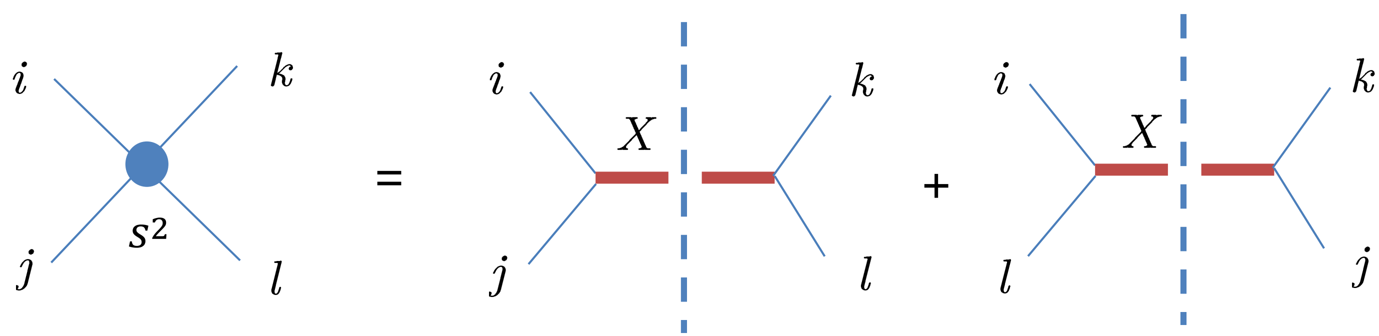
$$\frac{d^2}{ds^2} M_{ij\to kl}\left(s=\frac{1}{2}M^2, t=0\right)$$

$$= \sum_X \int_{(\epsilon\Lambda)^2}^{\infty} ds \frac{M_{ij\to X}(s,\Pi_X) M^*_{kl\to X}(s,\Pi_X)}{\pi\left(s-\frac{1}{2}M^2\right)^3} + (j \leftrightarrow l)$$

$X =$ BSM states

$\epsilon \leq 1$

$s \leftrightarrow u$ crossing



Master formula

Define: $$M^{ijkl} \equiv \frac{d^2}{ds^2} M_{ij\to kl}\left(\frac{1}{2}M^2\right)$$ Take massless limit

$$m_X^{ij} \equiv M_{ij\to X}(\mu, \Pi_X)$$

$$M^{ijkl} = \sum_X \int_{(\epsilon\Lambda)^2}^{\infty} \frac{d\mu}{\pi} \frac{m_X{}^{ij} m_X{}^{kl}}{(\mu - M^2/2)^3} + (j \leftrightarrow l)$$

Forward scattering amp, at low energy (calculable in EFT), represented by Wilson coef.

Amplitude of SM $\to X$

Elastic Positivity Bounds

$$M^{ijkl} = \sum_X \int_{(\epsilon\Lambda)^2}^{\infty} \frac{d\mu}{\pi} \frac{{m_X}^{ij}{m_X}^{kl}}{(\mu - M^2/2)^3} + (j \leftrightarrow l)$$

Elastic: When $i = k, j = l$ $(ij \to ij)$, RHS $\to \mathrm{Tr}\left(mm^T\right) \geq 0$

$$\Rightarrow \quad M^{ijij} \geq 0$$

For more general:

Superposition elastic: $M(|u\rangle + |v\rangle \to |u\rangle + |v\rangle) = u^i v^j u^{k*} v^{l*} \cdot M^{ijkl}$

with $|u\rangle = u^i |i\rangle, |v\rangle = v^j |j\rangle$

RHS $\to$ $|u \cdot m_X \cdot v|^2 + |u \cdot m_X \cdot v^*|^2 \geq 0$

$$\Rightarrow \quad u^i v^j u^{k*} v^{l*} M^{ijkl} \geq 0$$

Elastic Positivity Bounds

$$M^{ijkl} = \sum_X \int_{(\epsilon\Lambda)^2}^{\infty} \frac{d\mu}{\pi} \frac{{m_X}^{ij} {m_X}^{kl}}{(\mu - M^2/2)^3} + (j \leftrightarrow l)$$

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$$\Rightarrow \quad u^i v^j u^{k*} v^{l*} M^{ijkl} \geq 0$$

<u>However, the elastic bounds are not the optimal !</u>

Convex cone nature

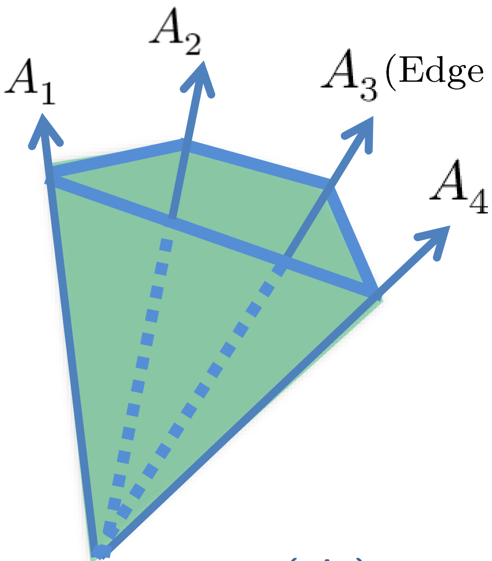
[CZ and S.-Y. Zhou, PRL 125, 201601]

Two observation:

$$M^{ijkl} = \sum_X \int_{(\epsilon\Lambda)^2}^{\infty} \frac{d\mu}{\pi} \frac{{m_X}^{ij}{m_X}^{kl}}{(\mu - M^2/2)^3} + (j \leftrightarrow l)$$

M^{ijkl} is positive linear combinations of ${m_X}^{ij}{m_X}^{kl} + {m_X}^{il}{m_X}^{kj}$

→ **1. M^{ijkl} is a convex cone**



cone(Ai)

Any vector inside cone can always be written as positive linear combinations of Ai

So we conclude:

$$M^{ijkl} = \text{cone}(\{{m_X}^{ij}{m_X}^{kl} + {m_X}^{il}{m_X}^{kj}, m \in \mathbb{R}^{n^2}\})$$

Positivity bounds arise as boundary of cone!

Projectors as ER

[CZ and S.-Y. Zhou, PRL 125, 201601]

2. X couple to i and j , X belong to the direct product space of i and j

$$\mathbf{r}_i \otimes \mathbf{r}_j = \mathbf{X}_1 \oplus \mathbf{X}_2 \oplus \ldots$$

$$M^{ijkl} = \sum_X \int_{(\epsilon\Lambda)^2}^{\infty} \frac{d\mu}{\pi} \frac{m_X{}^{ij} m_X{}^{kl}}{(\mu - M^2/2)^3} + (j \leftrightarrow l)$$

$$M(ij \to X^\alpha) = \langle X|\mathcal{M}|\mathbf{X}_r\rangle C_{i,j}^{r,\alpha}$$

C is the CG coefficients for the direct sum decomposition of $\mathbf{r}_i \otimes \mathbf{r}_j$

$$M^{ijkl} = \int_{(\epsilon\Lambda)^2}^{\infty} d\mu \sum_{X \ in \ \mathbf{X}_r} \frac{|\langle X|\mathcal{M}|\mathbf{X}_r\rangle|^2}{\pi(\mu - M^2/2)^3} P_r^{i(j|k|l)}$$

Projector: $P_r^{i(j|k|l)} \equiv \Sigma_\alpha C_{i,j}^{r,\alpha} \left(C_{k,l}^{r,\alpha}\right)^*$

M^{ijkl} is a convex cone: $\text{cone}\left(\left\{P_r^{i(j|k|l)}\right\}\right)$

EFT with symmetry

[CZ and S.-Y. Zhou, 2005.03047]

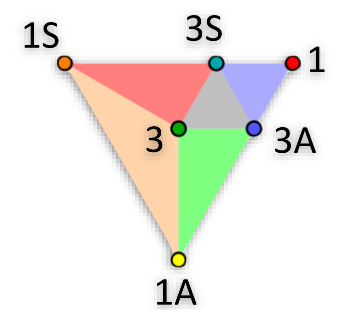
4-Higgs operators $\mathbf{2} \otimes \mathbf{2} = \mathbf{1} \oplus \mathbf{3}$

$$O_{S,0} = [(D_\mu \Phi)^\dagger D_\nu \Phi] \times [(D^\mu \Phi)^\dagger D^\nu \Phi],$$
$$O_{S,1} = [(D_\mu \Phi)^\dagger D^\mu \Phi] \times [(D_\nu \Phi)^\dagger D^\nu \Phi],$$
$$O_{S,2} = [(D_\mu \Phi)^\dagger D_\nu \Phi] \times [(D^\nu \Phi)^\dagger D^\mu \Phi].$$

$\Rightarrow$

$$F_{S,0} \geq 0\,,$$
$$F_{S,0} + F_{S,2} \geq 0\,,$$
$$F_{S,0} + F_{S,1} + F_{S,2} \geq 0\,.$$

Triangular cone



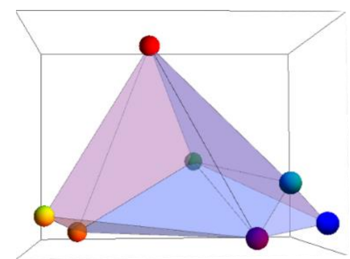
4-W operators $\mathbf{3} \otimes \mathbf{3} = \mathbf{1} \oplus \mathbf{3} \oplus \mathbf{5}$

$$O_{T,0} = \mathrm{Tr}[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}]\mathrm{Tr}[\hat{W}_{\alpha\beta}\hat{W}^{\alpha\beta}]$$
$$O_{T,2} = \mathrm{Tr}[\hat{W}_{\alpha\mu}\hat{W}^{\mu\beta}]\mathrm{Tr}[\hat{W}_{\beta\nu}\hat{W}^{\nu\alpha}]$$
$$O_{T,1} = \mathrm{Tr}[\hat{W}_{\alpha\nu}\hat{W}^{\mu\beta}]\mathrm{Tr}[\hat{W}_{\mu\beta}\hat{W}^{\alpha\nu}]$$
$$O_{T,10} = \mathrm{Tr}[\hat{W}_{\mu\nu}\tilde{W}^{\mu\nu}]\mathrm{Tr}[\hat{W}_{\alpha\beta}\tilde{W}^{\alpha\beta}]$$

$\Rightarrow$

$$F_{T,2} \geq 0,$$
$$4F_{T,1} + F_{T,2} \geq 0,$$
$$F_{T,2} + 8F_{T,10} \geq 0,$$
$$8F_{T,0} + 4F_{T,1} + 3F_{T,2} \geq 0,$$
$$12F_{T,0} + 4F_{T,1} + 5F_{T,2} + 4F_{T,10} \geq 0,$$
$$4F_{T,0} + 4F_{T,1} + 3F_{T,2} + 12F_{T,10} \geq 0.$$

6-facet 4D cone



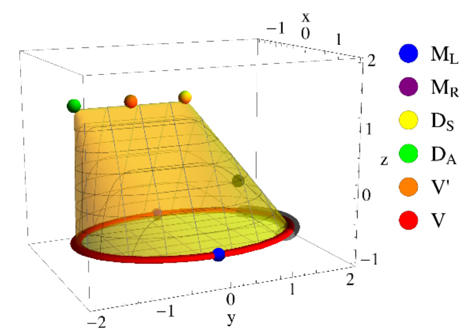
4-electron operators

$$O_1 = \partial^\alpha(\bar{e}\gamma^\mu e)\partial_\alpha(\bar{e}\gamma_\mu e)\,,$$
$$O_2 = \partial^\alpha(\bar{e}\gamma^\mu e)\partial_\alpha(\bar{l}\gamma_\mu l)\,,$$
$$O_3 = D^\alpha(\bar{e}l)\;D_\alpha(\bar{l}e),$$
$$O_4 = \partial^\alpha(\bar{l}\gamma^\mu l)\;\partial_\alpha(\bar{l}\gamma_\mu l)\,,$$

$\Rightarrow$

$$C_1 \leq 0, C_3 \geq 0, C_4 \leq 0,$$
$$2\sqrt{C_1 C_4} \geq C_2$$
$$2\sqrt{C_1 C_4} \geq -(C_2 + C_3)$$

4D "circular cone"



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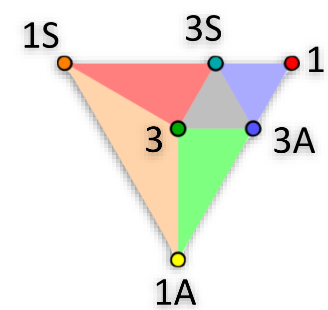
Triangular cone



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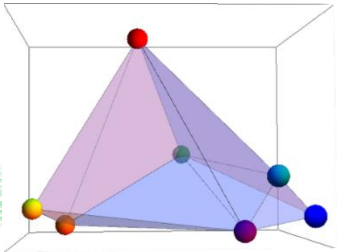
Beyond elastic positivity!

6-facet 4D cone



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$$O_3=D^\alpha(\bar e l)\;D_\alpha(\bar l e),$$
$$O_4=\partial^\alpha(\bar l\gamma^\mu l)\;\partial_\alpha(\bar l\gamma_\mu l)\,,$$

$$C_1\leq 0, C_3\geq 0, C_4\leq 0,$$
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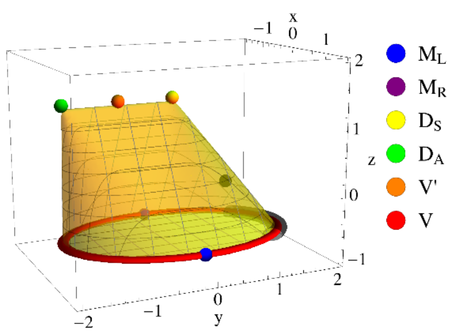

EFT without symmetry?

[X. Li, et al, 2101.01191]

The approach is valid so far, however…

Q: What if there is no symmetries? How to characterize bounds?

Solution : use the dual property of cone

Dual cone is defined as

$$\mathbf{C}^{n^4 *} = \{\mathcal{Q} | \mathcal{Q} \cdot \mathcal{M} \geq 0, \forall \mathcal{M} \in \mathbf{C}^{n^4}\}$$

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1. C* is a cone.
2. C* is a set that contain all possible linear bounds
3. Hyperplane separation theorem→(C*)*=C
——it is enough to carve out exactly the C

These properties make sure our bounds are complete

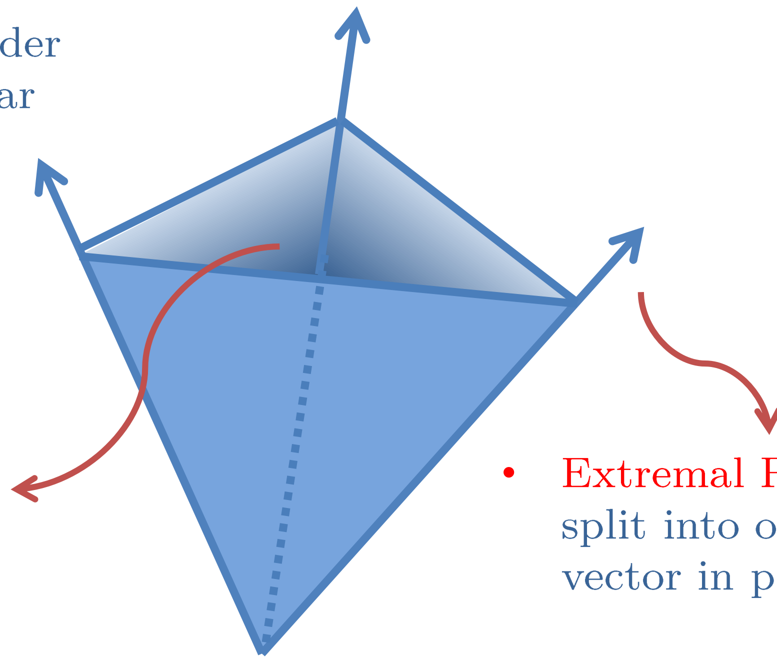
Dual cone

posi. bounds
= vectors in dual cone

Independent possible bounds ⟷ **Extremal Rays of cone C***

- A convex cone is closed under additions and positive scalar multiplications

$\vec{n} \cdot \vec{C} \geq 0$
but if $\vec{n} = \sum_i a_i \vec{n}_i^{ex}$
then $\vec{n}$ is not independent,
because $a_i \geq 0$, $\vec{n}_i^{ex} \cdot \vec{C} \geq 0$



- Extremal Ray (ER) : $\vec{n}_i^{ex}$ cannot be split into other vectors (like an edge vector in polyhedral cone)

dual cone: C*

Dual cone

How to find the ERs in dual cone? …

Index symmetries of M^{ijkl}

$i \leftrightarrow k$ or $j \leftrightarrow l$ ⟺ Crossing symmetry: $s \leftrightarrow u$

$i \leftrightarrow j + k \leftrightarrow l$ ⟺ Rotation symmetry (Pi around y-axis)

Defined a subspace of M : $\mathcal{M} \in \vec{\mathbf{S}}^{n^4}$ $(\mathcal{M}^{ijkl} = \mathcal{M}^{jilk} = \mathcal{M}^{klij} = \mathcal{M}^{ilkj})$

$\mathcal{Q} \in \vec{\mathbf{S}}^{n^4}$ cross-antisymmetric one will vanish $\mathcal{Q} \cdot \mathcal{M}$

$$\mathcal{Q} \cdot \mathcal{M} \geq 0$$

$$\Rightarrow \mathcal{Q}^{ijkl} \sum_\alpha (m_\alpha^{ij} m_\alpha^{kl} + m_\alpha^{il} m_\alpha^{kj}) = 2 \sum_\alpha m_\alpha^{ij} \mathcal{Q}^{ijkl} m_\alpha^{kl}$$

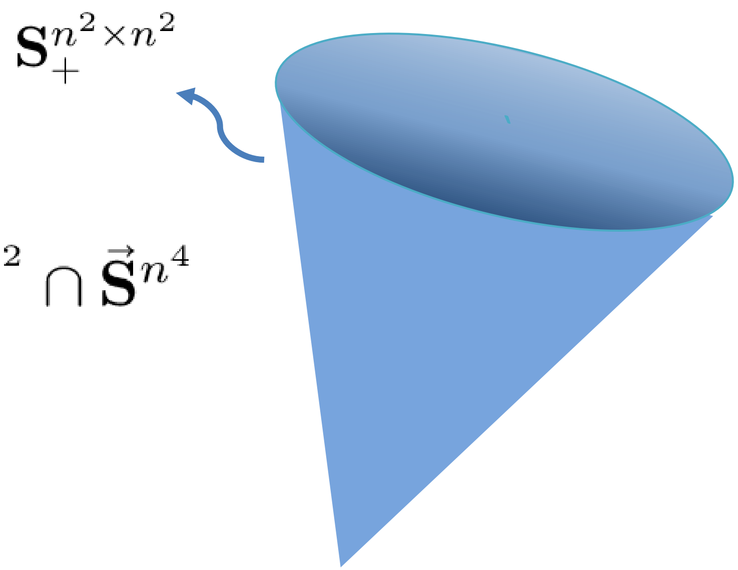
$$\Rightarrow \mathcal{Q}^{(ij),(kl)} \succcurlyeq 0 \Rightarrow \mathcal{Q} \in \mathbf{S}_+^{n^2 \times n^2}$$

➡ $\mathbf{Q}^{n^4} = \mathbf{S}_+^{n^2 \times n^2} \cap \vec{\mathbf{S}}^{n^4}$

Semi-definite matrices

Dual cone---Spectrahedron

$\mathbf{S}_{+}^{n^2\times n^2}$: the set of n × n positive semi-definite matrices forms a convex cone



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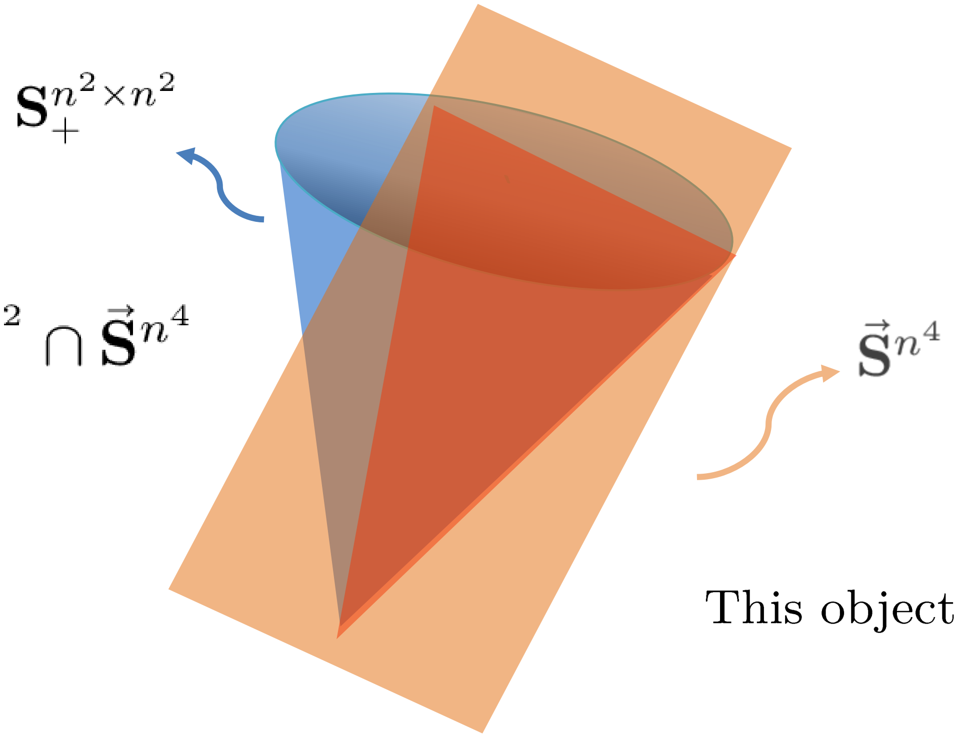

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This object is a spectrahedron !

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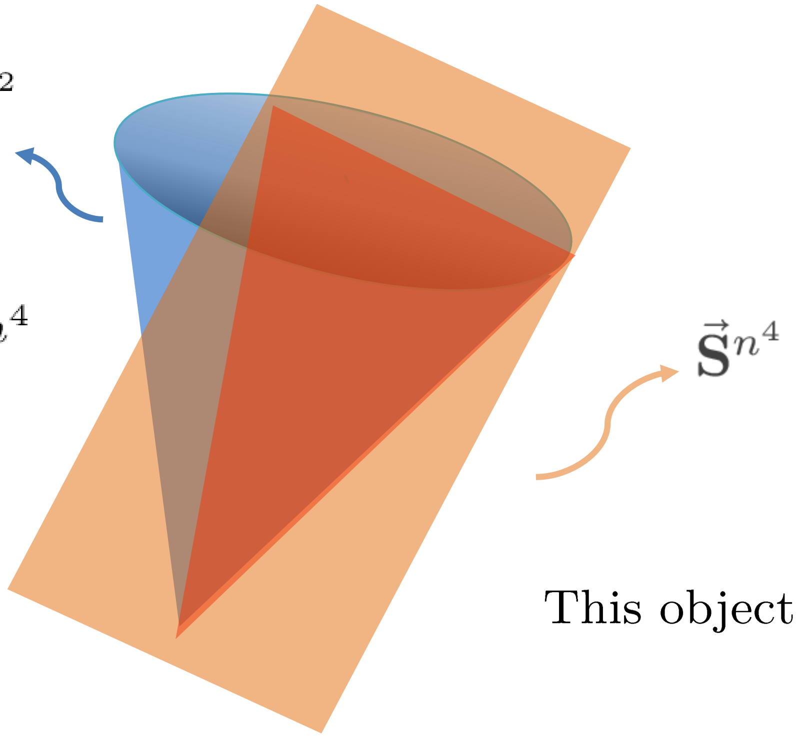

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Spectrahedron: the intersection of a cone with a linear (affine) subspace

is well-defined in math

Dual cone---Spectrahedron

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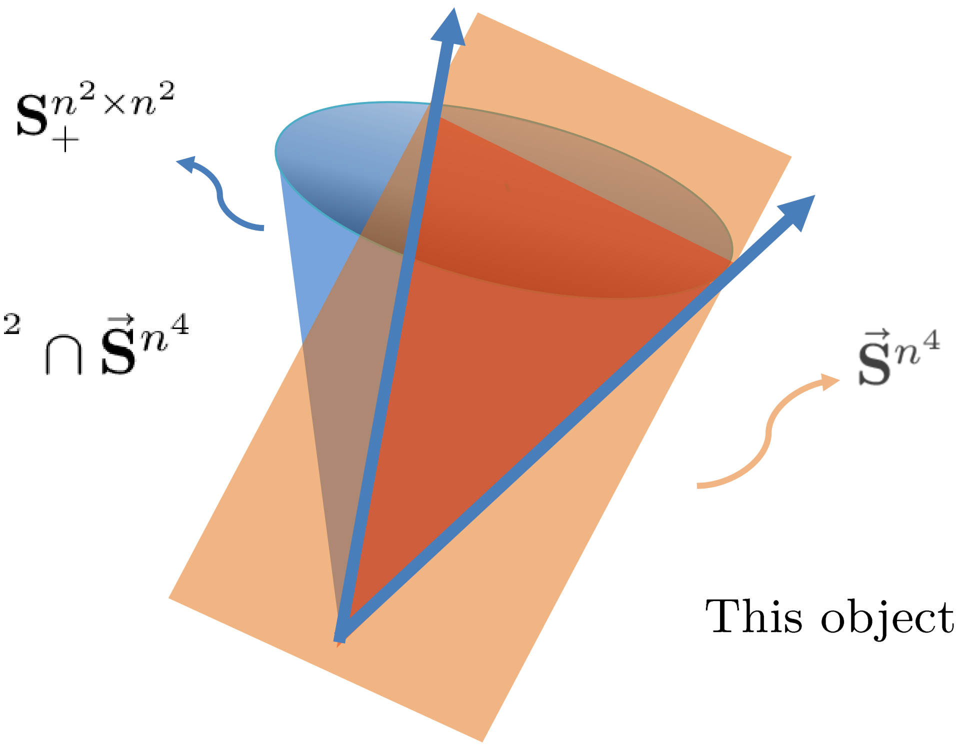


This object is a spectrahedron !

Spectrahedron: the intersection of a cone with a linear (affine) subspace is well-defined in math

Ultimate goal : finding ERs of Spectrahedron!

Spectrahedron

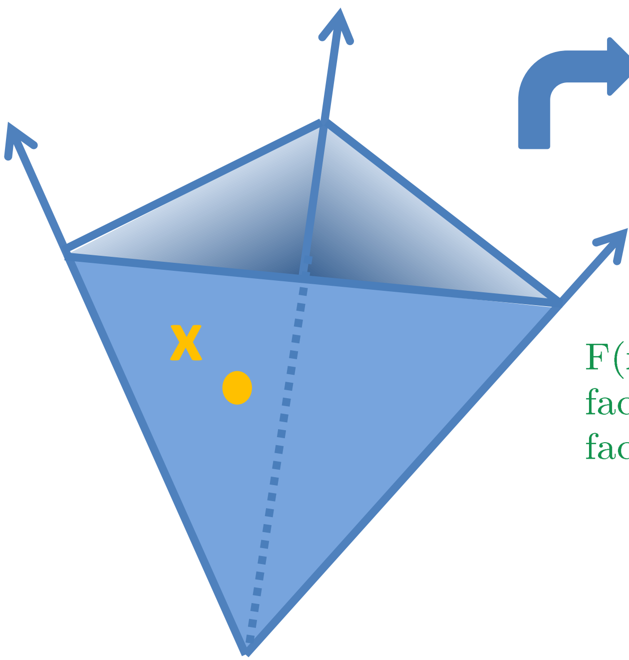
[Ramana & Goldman 1995]

– Let $Q_i, i = 0, \dots, m$ be the basis matrices of the space

$$Q(x) = Q_0 + x_i Q_i$$

– The spectrahedron: $G = \{x | Q(x) \succcurlyeq 0\}$

question: whether a vector x is at a ER? → iff the rank of B is m-1 (or dimension of F(x) is 1)



$\{u_i\}$ be basis of Null(Q(x))

Null(): space span by the independent null vectors

$$B = \begin{bmatrix} Q_1 u_1 & \cdots & Q_m u_1 \\ \vdots & \ddots & \vdots \\ Q_1 u_k & \cdots & Q_m u_k \end{bmatrix}$$

F(x) is the lowest unique face that contains x (the face is k-face)

Spectrahedron

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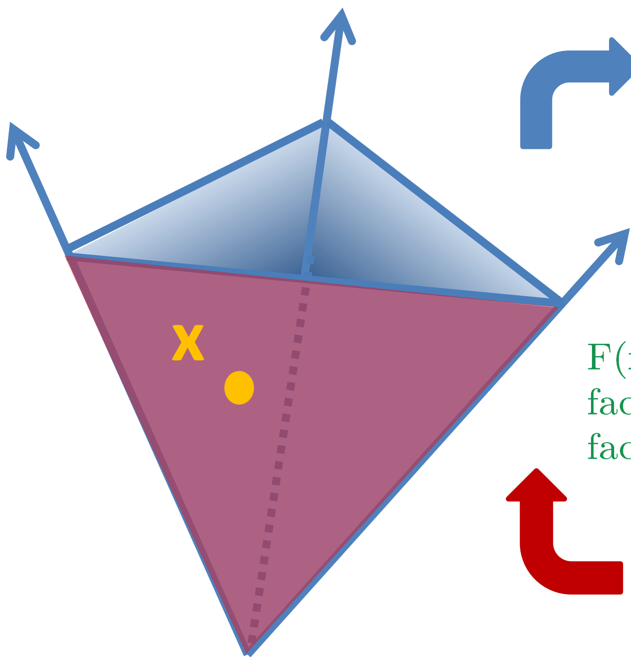
- The spectrahedron: $G = \{x | Q(x) \succcurlyeq 0\}$

question: whether a vector x is at a ER? → iff the rank of B is m-1 (or dimension of F(x) is 1)



$\{u_i\}$ be basis of Null(Q(x))

Null(): space span by the independent null vectors

$$B = \begin{bmatrix} Q_1 u_1 & \cdots & Q_m u_1 \\ \vdots & \ddots & \vdots \\ Q_1 u_k & \cdots & Q_m u_k \end{bmatrix}$$

F(x) is the lowest unique face that contains x (the face is k-face)

Null(B) is the linear span of F(x)

A Scalar EFT

General 2-scalar case

With Z2 symmetry $\phi_i \to -\phi_i$

$$\mathcal{L} \supset \frac{1}{\Lambda^4} C_{ijkl} O_{ijkl}, \quad O_{ijkl} = \partial_\mu \phi_i \partial^\mu \phi_j \partial_\nu \phi_k \partial^\nu \phi_l$$

$$\mathcal{M}_{\text{scalar}} = \begin{array}{c} \\ ij{=}11 \\ 22 \\ 12 \\ 21 \end{array} \begin{array}{c} \begin{array}{cccc} kl{=}11 & 22 & 12 & 21 \end{array} \\ \begin{bmatrix} 4C_{1111} & C'_{1122} & C_{1112} & C_{1112} \\ C'_{1122} & 4C_{2222} & C_{1222} & C_{1222} \\ C_{1112} & C_{1222} & C_{1212} & C'_{1122} \\ C_{1112} & C_{1222} & C'_{1122} & C_{1212} \end{bmatrix} \end{array}$$

$$\mathbf{Q}^{2^4} \ni \mathcal{Q} = \begin{pmatrix} a & b & e & e \\ b & c & f & f \\ e & f & d & b \\ e & f & b & d \end{pmatrix}$$

$a \geq 0, c \geq 0,$
$ac \geq b^2, d \geq |b|$

$$\{Q_i, 1 \leq i \leq 6\} = \left\{ \begin{pmatrix} 1&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0 \end{pmatrix}, \begin{pmatrix} 0&1&0&0\\1&0&0&0\\0&0&0&1\\0&0&1&0 \end{pmatrix}, \begin{pmatrix} 0&0&0&0\\0&1&0&0\\0&0&0&0\\0&0&0&0 \end{pmatrix}, \begin{pmatrix} 0&0&0&0\\0&0&0&0\\0&0&1&0\\0&0&0&1 \end{pmatrix}, \begin{pmatrix} 0&0&1&1\\0&0&0&0\\1&0&0&0\\1&0&0&0 \end{pmatrix}, \begin{pmatrix} 0&0&0&0\\0&0&1&1\\0&1&0&0\\0&1&0&0 \end{pmatrix} \right\}$$

$$Q(\boldsymbol{x}) = aQ_1 + bQ_2 + cQ_3 + dQ_4 \succcurlyeq 0$$

Two kinds of ER: $Q_{\text{ex1}}(r) = \begin{bmatrix} 1 & r & 0 & 0 \\ r & r^2 & 0 & 0 \\ 0 & 0 & |r| & r \\ 0 & 0 & r & |r| \end{bmatrix}, \quad Q^{12}_{\text{ex2}} = \begin{bmatrix} 0&0&0&0\\0&0&0&0\\0&0&1&0\\0&0&0&1 \end{bmatrix}$

Bounds:
$Q_{\text{ex}} \cdot \mathcal{M} \geq 0$

Visualize the cones

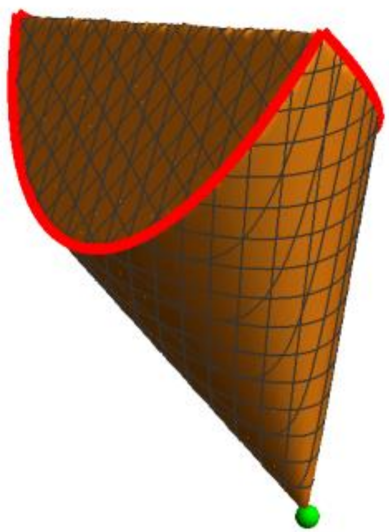
3D "cross section" of 4D cones

Dual space (spectrahedron)

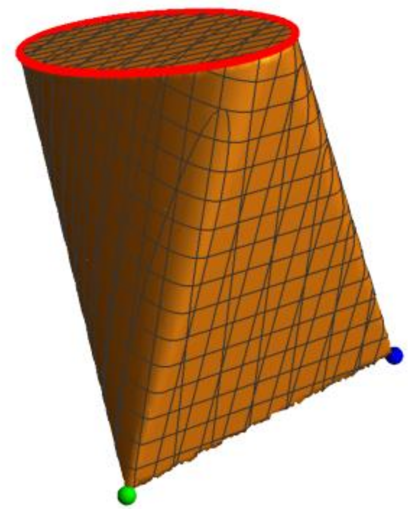
Amplitude space

$a \geq 0, c \geq 0,$

$ac \geq b^2, d \geq |b|$



ERs = posi. bounds



Bounds

$$C_{1111} \geq 0,\ C_{2222} \geq 0,\ C_{1212} \geq 0$$
$$4\sqrt{C_{1111}C_{2222}} \geq \pm(2C_{1122} + C_{1212}) - C_{1212}$$

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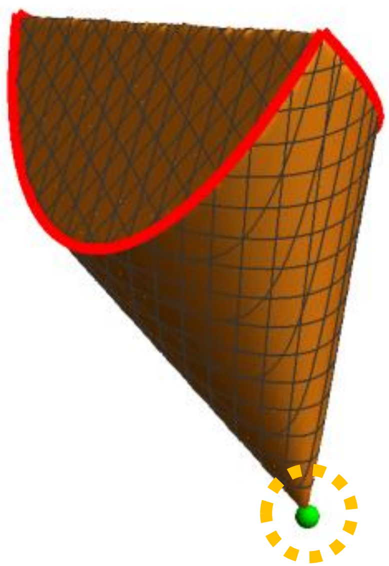
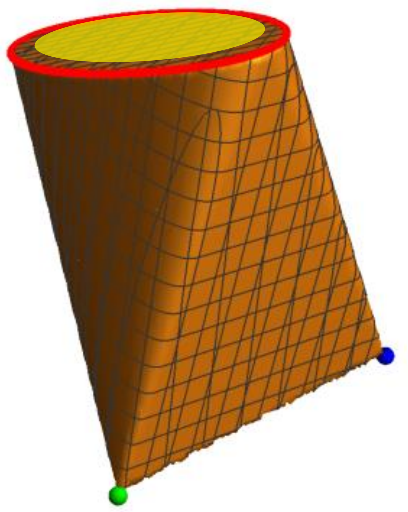


ERs = posi. bounds



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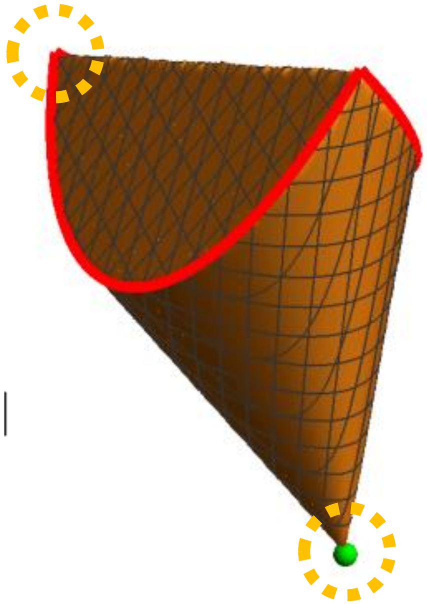

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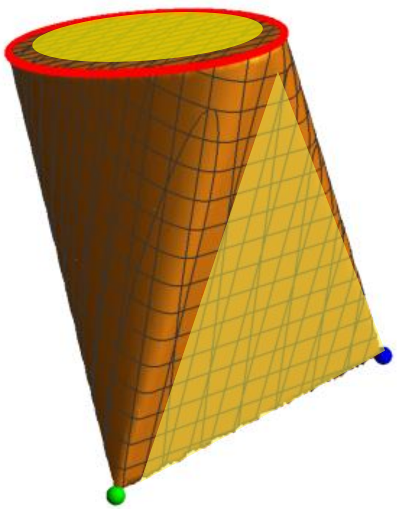

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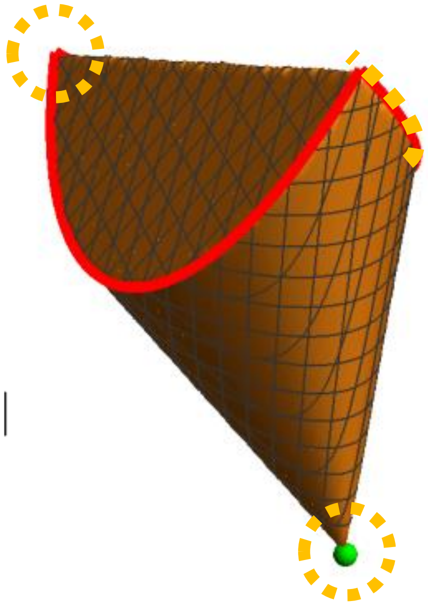

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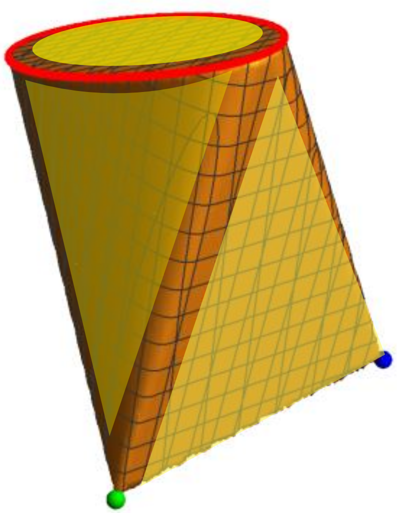

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Bounds

$$C_{1111} \geq 0,\ C_{2222} \geq 0,\ C_{1212} \geq 0$$
$$4\sqrt{C_{1111}C_{2222}} \geq \pm(2C_{1122} + C_{1212}) - C_{1212}$$

General 2-scalar case

Without Z2 symmetry

$$\mathcal{L} \supset \frac{1}{\Lambda^4} C_{ijkl} O_{ijkl}, \quad O_{ijkl} = \partial_\mu \phi_i \partial^\mu \phi_j \partial_\nu \phi_k \partial^\nu \phi_l$$

$$\mathcal{M}_{\text{scalar}} = \begin{bmatrix} 4C_{1111} & C'_{1122} & C_{1112} & C_{1112} \\ C'_{1122} & 4C_{2222} & C_{1222} & C_{1222} \\ C_{1112} & C_{1222} & C_{1212} & C'_{1122} \\ C_{1112} & C_{1222} & C'_{1122} & C_{1212} \end{bmatrix} \qquad \mathbf{Q}^{2^4} \ni \mathcal{Q} = \begin{pmatrix} a & b & e & e \\ b & c & f & f \\ e & f & d & b \\ e & f & b & d \end{pmatrix}$$

$$kl = 11 \quad 22 \quad 12 \quad 21$$

$$\text{ERs } \mathcal{Q}_{\text{ex}} \to \begin{bmatrix} a^2 & ab & ac & ac \\ ab & b^2 & bc & bc \\ ac & bc & 2c^2 - ab & ab \\ ac & bc & ab & 2c^2 - ab \end{bmatrix} \begin{matrix} ij = 11 \\ 22 \\ 12 \\ 21 \end{matrix}$$

With $c^2 \geq ab$

variable substitution

$$\mathcal{Q}_{\text{ex}} \cdot \mathcal{M} \equiv \begin{bmatrix} w^2 & \frac{rw+sw}{2} & rs \end{bmatrix} \cdot D \cdot \begin{bmatrix} w^2 & \frac{rw+sw}{2} & rs \end{bmatrix}^T$$

$$\geq 0 \quad \forall r, s, w \in \mathbb{R},$$

It is quartic !

$$D = \begin{bmatrix} 2C_{1111} & C_{1112} & C_{1122} \\ C_{1112} & 2C_{1212} & C_{1222} \\ C_{1122} & C_{1222} & 2C_{2222} \end{bmatrix}$$

Positivity bounds for general 2-scalar EFTs

Finally get bounds !

$$
\begin{aligned}
& C_{1111} \geq 0 \quad \text{and} \quad 4C_{1111}C_{1212} - C_{1112}^2 \geq 0 \\
& \text{and} \quad \Big\{ C_{1112}C_{1122}C_{1222} - C_{1111}C_{1222}^2 - C_{1112}^2 C_{2222} + C_{1212}\left(-C_{1122}^2 + 4C_{1111}C_{2222}\right) \geq 0 \\
& \text{or} \quad \Big[\Delta \equiv 3\left(4C_{1111}C_{2222} - C_{1112}C_{1222}\right) + \left(C_{1122} + C_{1212}\right)^2 \geq 0 \\
& \text{and} \quad \frac{3C_{1112}^2}{4C_{1111}} - 2\left(C_{1122} + C_{1212}\right) \leq \sqrt{\Delta} \leq C_{1212} - 2C_{1122} \\
& \text{and} \quad 2\Delta^{3/2} \geq 27\left(C_{1111}C_{1222}^2 + C_{1112}^2 C_{2222}\right) - 9\left(C_{1122} + C_{1212}\right)\left(8C_{1111}C_{2222} + C_{1112}C_{1222}\right) \\
& \qquad + 2\left(C_{1122} + C_{1212}\right)^3 \Big] \Big\}
\end{aligned}
$$

What if $n > 2$?

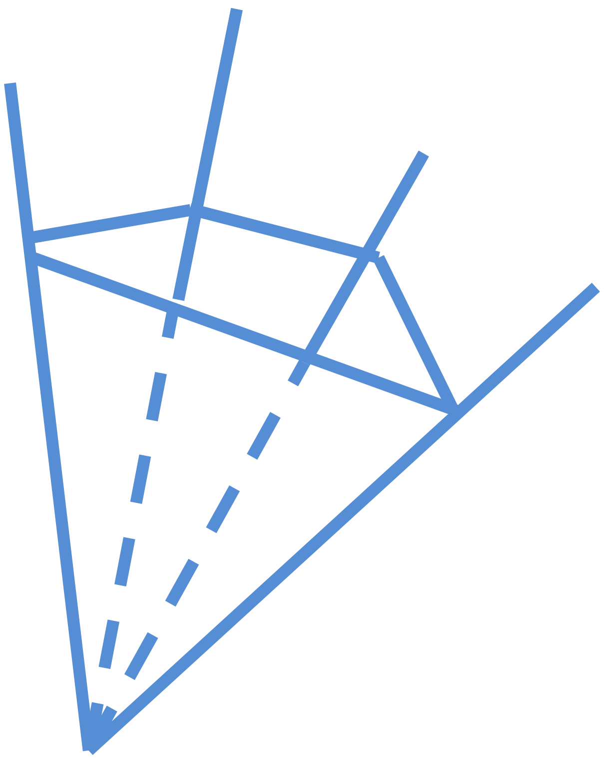
——resort to the numerical approach

(Base on semi-definite programming (SDP)).

Numerical approach

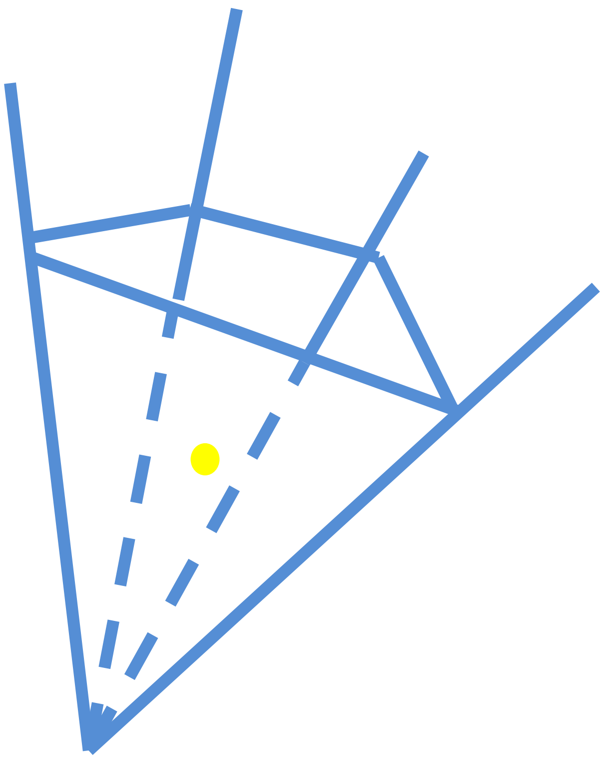
The "MC" approach

Randomly search ERs

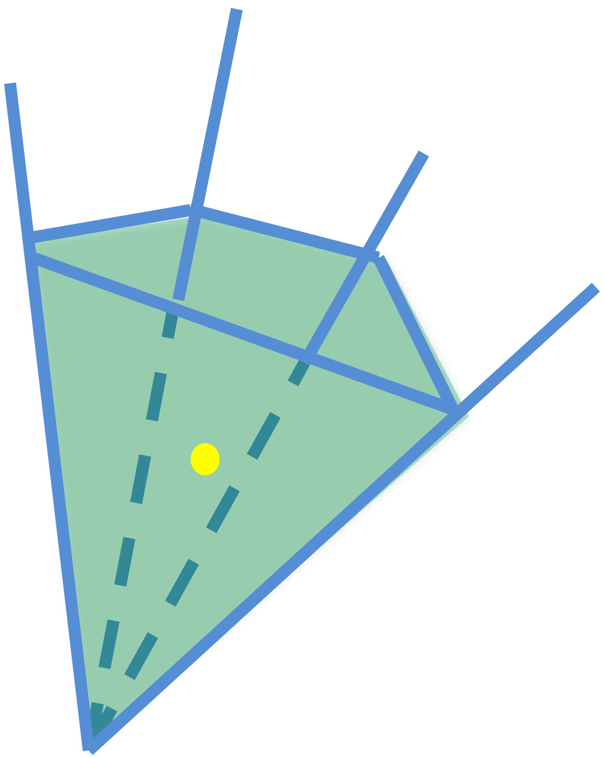


Randomly search ERs

☛ Start with a **random point x**



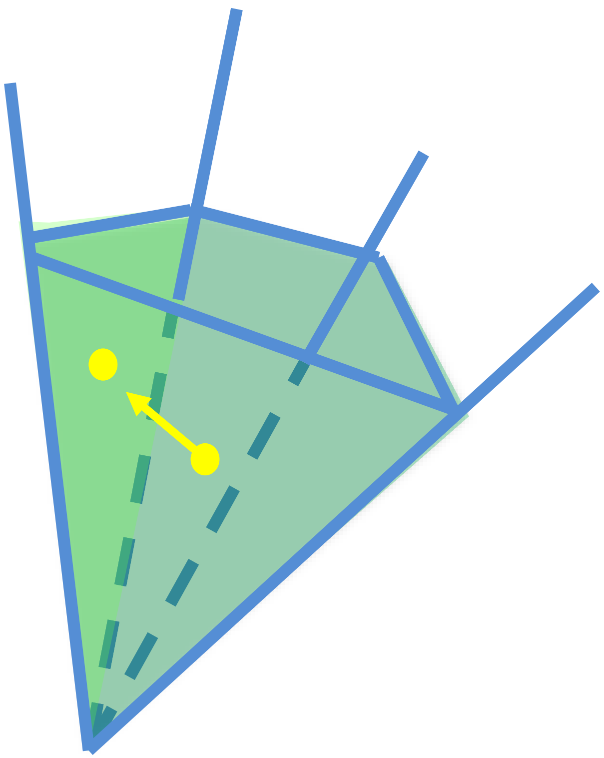
Randomly search ERs



- Start with a **random point x**
- Find the (k-)face F(x)

The "MC" approach

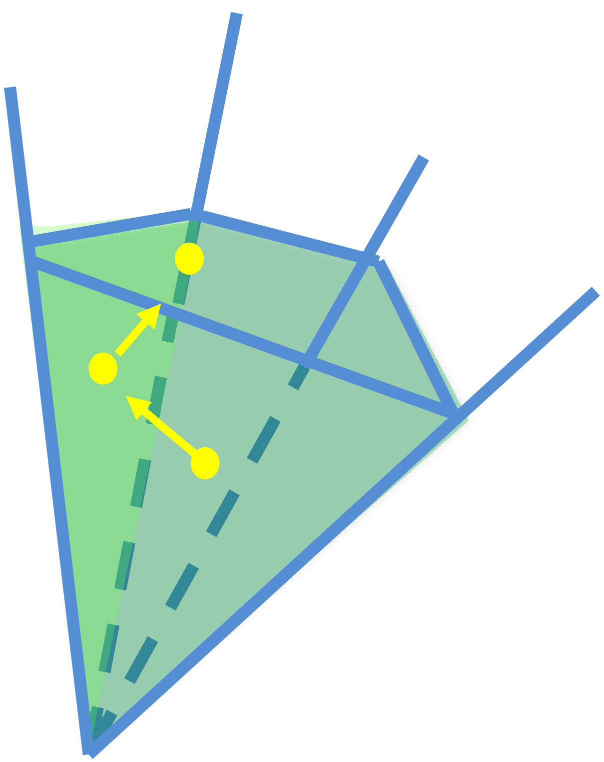
Randomly search ERs



- Start with a **random point x**
- Find the (k-)face F(x)
- Take a random straight-line in F(x) that crosses x. Find its **intersection** with the boundary of the cone (this is a SDP).

The "MC" approach

Randomly search ERs



- Start with a **random point x**
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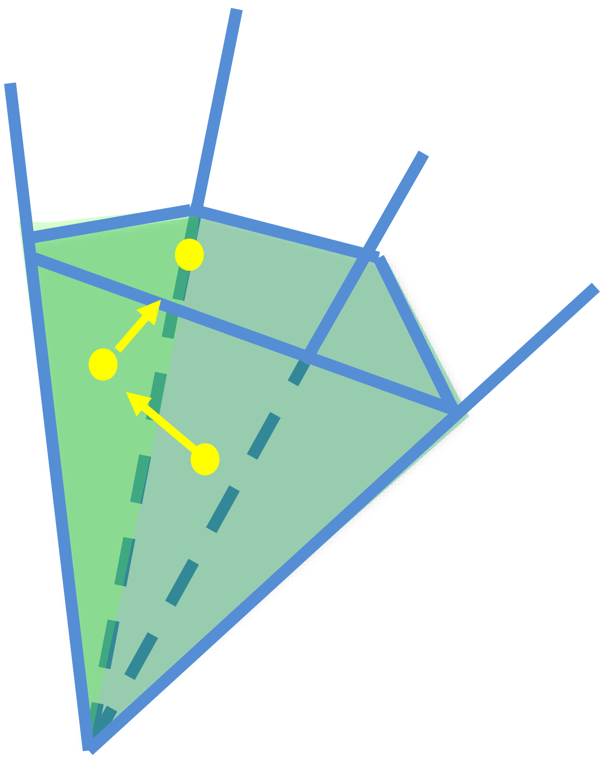
- Take x to be the intersection point and **iterate**, if F(x) is not **dimension 1**

The "MC" approach

Randomly search ERs



- Start with a **random point x**
- Find the (k-)face F(x)
- Take a random straight-line in F(x) that crosses x. Find its **intersection** with the boundary of the cone (this is a SDP).
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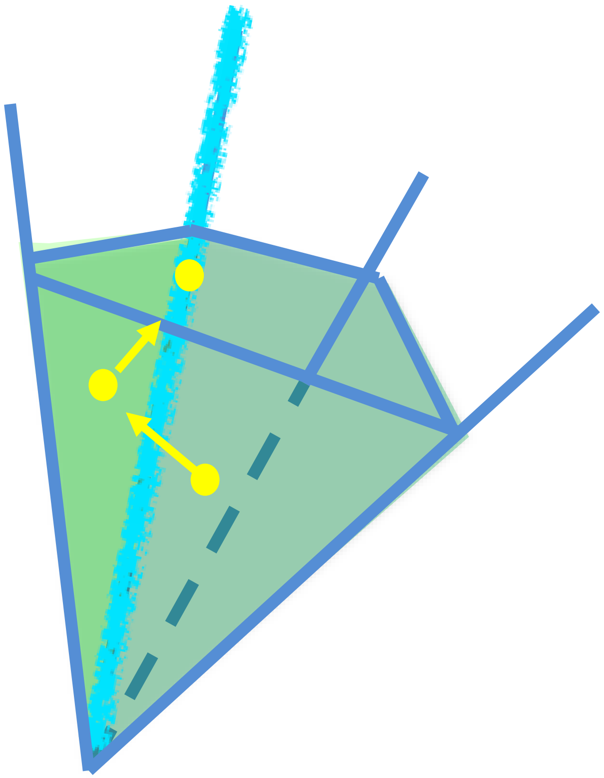
Randomly search ERs



- Start with a **random point x**
- Find the (k-)face F(x)
- Take a random straight-line in F(x) that crosses x. Find its **intersection** with the boundary of the cone (this is a SDP).
- Take x to be the intersection point and **iterate**, if F(x) is not **dimension 1**
- If F(x) is **dimension 1**, An ER is found.

The SDP approach

The semi-definite programming (SDP) approach:

$$\min \quad \mathcal{Q} \cdot \mathcal{M}$$

Given a M

$$\text{subject to} \quad \mathcal{Q} \in \text{spectrahedron}$$

If the minimum is not negative, then M is allowed by positivity.

Advantage

1. Solvable within polynomial complexity.
   (in contrast to elastic approach, which is NP-hard.)

2. Guarantee bounds are accurate

Examples

4-gluon case

$i, j, k, l = g$ (n =16 fields)

EFT operators:

$Q^{(1)}_{G^4}$	$(G^A_{\mu\nu}G^{A\mu\nu})(G^B_{\rho\sigma}G^{B\rho\sigma})$	$Q^{(7)}_{G^4}$	$d^{ABE}d^{CDE}(G^A_{\mu\nu}G^{B\mu\nu})(G^C_{\rho\sigma}G^{D\rho\sigma})$
$Q^{(2)}_{G^4}$	$(G^A_{\mu\nu}\widetilde{G}^{A\mu\nu})(G^B_{\rho\sigma}\widetilde{G}^{B\rho\sigma})$	$Q^{(8)}_{G^4}$	$d^{ABE}d^{CDE}(G^A_{\mu\nu}\widetilde{G}^{B\mu\nu})(G^C_{\rho\sigma}\widetilde{G}^{D\rho\sigma})$
$Q^{(3)}_{G^4}$	$(G^A_{\mu\nu}G^{B\mu\nu})(G^A_{\rho\sigma}G^{B\rho\sigma})$		
$Q^{(4)}_{G^4}$	$(G^A_{\mu\nu}\widetilde{G}^{B\mu\nu})(G^A_{\rho\sigma}\widetilde{G}^{B\rho\sigma})$		

Plus a $(D6)^2$ term: $f^{abc}G^{a\nu}_{\mu}G^{b\rho}_{\nu}G^{c\mu}_{\rho}$

$\vec{n}\cdot\vec{C} \geq 0 \ \rightarrow$ n given by

[0, 0, 0, 1, 0, 0, 0]	[0, 0, 6, 3, 7, 2, 0]	[24, 0, 12, 21, 15, 14, 0]	[0, 0, 96, 24, 64, 40, −81]
[0, 0, 1, 1, 1, 0, 0]	[8, 6, 1, 6, 0, 2, 0]	[24, 32, 24, 4, 8, 0, −27]	[40, 32, 80, 4, 0, 0, −189]
[2, 0, 1, 0, 0, 0, 0]	[0, 6, 3, 12, 5, 0, 0]	[48, 36, 21, 27, 25, 0, 0]	[0, 0, 24, 120, 40, 104, −81]
[0, 2, 0, 1, 0, 0, 0]	[8, 6, 1, 12, 0, 0, 0]	[32, 40, 4, 80, 0, 0, −27]	[0, 0, 120, 24, 104, 40, −81]
[0, 0, 3, 0, 2, 0, 0]	[0, 6, 6, 9, 10, 4, 0]	[0, 48, 0, 48, 0, 40, −81]	[96, 0, 144, 24, 64, 40, −81]
[0, 0, 0, 3, 0, 2, 0]	[0, 12, 0, 14, 0, 0, −9]	[24, 0, 36, 24, 16, 40, −81]	[48, 0, 96, 24, 0, 40, −243]
[1, 1, 2, 2, 0, 0, 0]	[0, 0, 8, 8, 0, 8, −27]	[0, 0, 48, 24, 32, 40, −81]	[0, 192, 168, 96, 112, 120, −405]
[6, 0, 3, 0, 2, 0, 0]	[12, 0, 14, 0, 0, 0, −27]	[0, 0, 24, 48, 16, 56, −81]	[168, 480, 168, 156, 56, 160, −729]
[4, 2, 2, 1, 2, 0, 0]	[6, 8, 12, 1, 0, 0, −27]	[88, 32, 56, 4, 40, 0, −27]	[264, 384, 156, 168, 16, 200, −729]
[0, 0, 4, 0, 0, 0, −9]	[8, 16, 4, 8, 0, 8, −27]	[96, 42, 27, 84, 25, 0, 0]	[288, 384, 216, 168, 0, 200, −891]
[6, 0, 6, 0, 5, 0, 0]	[0, 24, 0, 12, 0, 8, −27]	[96, 66, 42, 39, 50, 4, 0]	[480, 384, 480, 168, 160, 200, −729]
[0, 0, 3, 6, 5, 4, 0]	[8, 22, 1, 14, 0, 10, −27]	[120, 42, 39, 42, 40, 14, 0]	[336, 768, 672, 216, 0, 200, −2187]

We can prove only a few of them can obtained by elastic

4-gluon case

$i, j, k, l = g$ (n =16 fields)

EFT operators:

$Q_{G^4}^{(1)}$	$(G^A_{\mu\nu}G^{A\mu\nu})(G^B_{\rho\sigma}G^{B\rho\sigma})$
$Q_{G^4}^{(2)}$	$(G^A_{\mu\nu}\widetilde{G}^{A\mu\nu})(G^B_{\rho\sigma}\widetilde{G}^{B\rho\sigma})$
$Q_{G^4}^{(3)}$	$(G^A_{\mu\nu}G^{B\mu\nu})(G^A_{\rho\sigma}G^{B\rho\sigma})$
$Q_{G^4}^{(4)}$	$(G^A_{\mu\nu}\widetilde{G}^{B\mu\nu})(G^A_{\rho\sigma}\widetilde{G}^{B\rho\sigma})$

$Q_{G^4}^{(7)}$	$d^{ABE}d^{CDE}(G^A_{\mu\nu}G^{B\mu\nu})(G^C_{\rho\sigma}G^{D\rho\sigma})$
$Q_{G^4}^{(8)}$	$d^{ABE}d^{CDE}(G^A_{\mu\nu}\widetilde{G}^{B\mu\nu})(G^C_{\rho\sigma}\widetilde{G}^{D\rho\sigma})$

Plus a (D6)^2 term: $f^{abc}G^{a\nu}_{\mu}G^{b\rho}_{\nu}G^{c\mu}_{\rho}$

$\vec{n}\cdot\vec{C} \geq 0 \;\rightarrow\;$ n given by

7D polyhedral cone with 48 facets!

$[0,0,0,1,0,0,0]$	$[0,0,6,3,7,2,0]$	$[24,0,12,21,15,14,0]$	$[0,0,96,24,64,40,-81]$
$[0,0,1,1,1,0,0]$	$[8,6,1,6,0,2,0]$	$[24,32,24,4,8,0,-27]$	$[40,32,80,4,0,0,-189]$
$[2,0,1,0,0,0,0]$	$[0,6,3,12,5,0,0]$	$[48,36,21,27,25,0,0]$	$[0,0,24,120,40,104,-81]$
$[0,2,0,1,0,0,0]$	$[8,6,1,12,0,0,0]$	$[32,40,4,80,0,0,-27]$	$[0,0,120,24,104,40,-81]$
$[0,0,3,0,2,0,0]$	$[0,6,6,9,10,4,0]$	$[0,48,0,48,0,40,-81]$	$[96,0,144,24,64,40,-81]$
$[0,0,0,3,0,2,0]$	$[0,12,0,14,0,0,-9]$	$[24,0,36,24,16,40,-81]$	$[48,0,96,24,0,40,-243]$
$[1,1,2,2,0,0,0]$	$[0,0,8,8,0,8,-27]$	$[0,0,48,24,32,40,-81]$	$[0,192,168,96,112,120,-405]$
$[6,0,3,0,2,0,0]$	$[12,0,14,0,0,0,-27]$	$[0,0,24,48,16,56,-81]$	$[168,480,168,156,56,160,-729]$
$[4,2,2,1,2,0,0]$	$[6,8,12,1,0,0,-27]$	$[88,32,56,4,40,0,-27]$	$[264,384,156,168,16,200,-729]$
$[0,0,4,0,0,0,-9]$	$[8,16,4,8,0,8,-27]$	$[96,42,27,84,25,0,0]$	$[288,384,216,168,0,200,-891]$
$[6,0,6,0,5,0,0]$	$[0,24,0,12,0,8,-27]$	$[96,66,42,39,50,4,0]$	$[480,384,480,168,160,200,-729]$
$[0,0,3,6,5,4,0]$	$[8,22,1,14,0,10,-27]$	$[120,42,39,42,40,14,0]$	$[336,768,672,216,0,200,-2187]$

We can prove only a few of them can obtained by elastic

Example: SM flavor sector

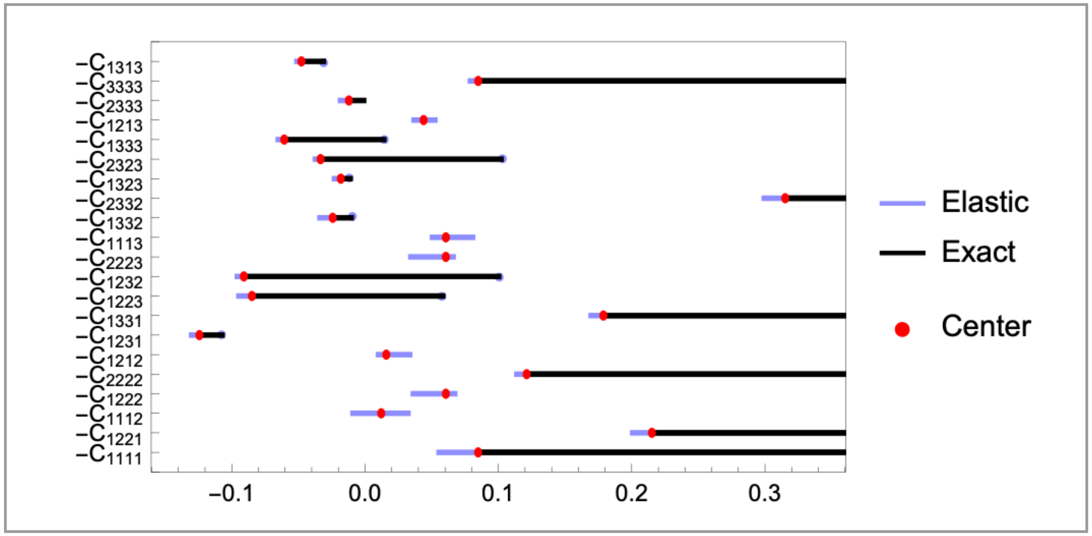
$i, j, k, l = e_R, \mu_R, \tau_R$

- SM flavor sector (n=3 fields): [2004.02885, Remmen & Rodd]

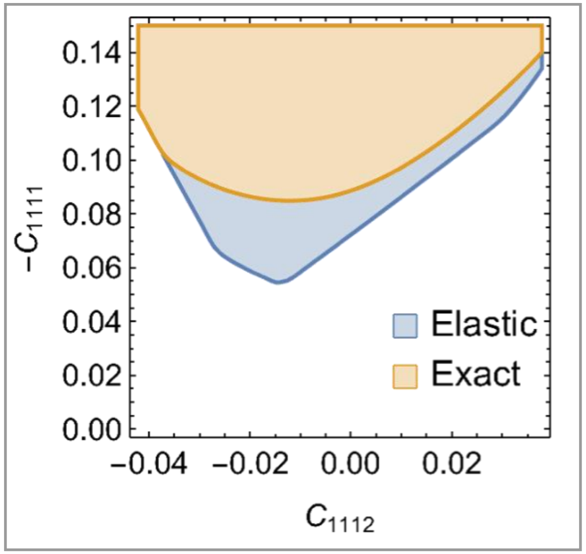
4-fermion operator in dim-8:

$$O_{ijkl} = \partial_\mu(\bar{f}_i \gamma_\nu f_j)\partial^\mu(\bar{f}_k \gamma^\nu f_l)$$

Elastic: from elastic scattering
Exact: from SDP approach



$e^+e^- \to e^+e^-$



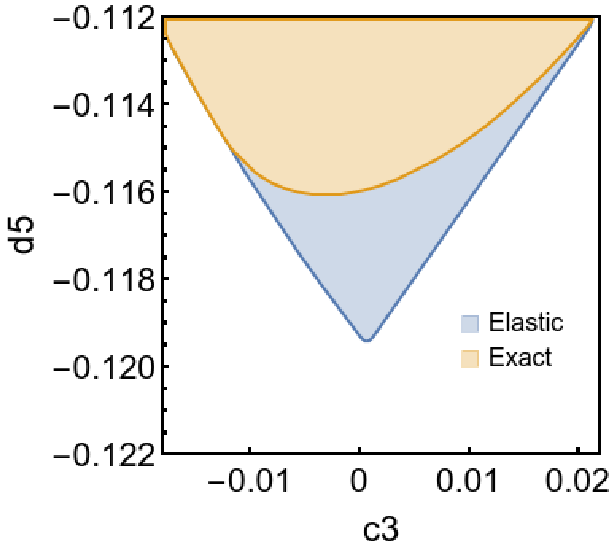
$\mu \to 3e$

SDP always give stronger bounds

Example: Spin-2 EFT

- dRGT massive gravity (n=5) —— (c3, d5):
  [PRL.106(2011) 231101, C. de Rham, et, al]

Elastic: elastic approach(superposed)
[JHEP 04 (2016) 002. C. Cheung and G. Remmen]



- Exact: SDP approach:

  improves slightly the minimum value of d5.

Summary

- Positive structures arise at the dim-8 level in EFT coefficient space, as a consequence of **axiomatic QFT principles**.

- Realistic problems often involve multi-field EFTs, in which a **convex geometric perspective** helps to understand these structures.

- We convert the problem of finding bounds to a geometric problem: finding the ERs of a spectrahedron.

  - For small n, can be solved **analytically**.

  - For large n, can be solved as a **semi-definite programming** problem.

- Improved some previous results, and gave some new results.

Institute of High Energy Physics

Thank You!

Xu Li
Institute of High Energy Physics

Apr. 14 Higgs and Effective Field Theory - HEFT 2021

base on 2101.01191 with C. Yang, H. Xu, C. Zhang, and S.-Y. Zhou

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Backup

Non elastic bounds for n=3

$$\mathcal{Q}_{ex} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 2 & 2 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 2 & 3 & 0 & 1 & 1 & 1 & 1 & 2 \\ 0 & 0 & 0 & 2 & 0 & 1 & 2 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 2 & 1 & 1 & 3 & 1 & 2 \\ 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 2 & 1 & 1 & 1 & 2 & 1 & 3 \end{bmatrix}$$

Which can be apply in SM flavor sector (n=3 fields)

This is a rank-4 matrix, so it cannot be written as uvu^*v^* form, which is at most rank-2 by definition

ERs for without Z2 symmetry

Hilbert 16th problem: if the variables are less than 3, then the quartic polynomial can be always written as a sum of squares.

$$f(r,s,w) \equiv \left[w^2 \;\; \tfrac{rw+sw}{2} \;\; rs\right] \cdot D \cdot \left[w^2 \;\; \tfrac{rw+sw}{2} \;\; rs\right]^T$$
$$= \sum_\alpha \left(x_\alpha \cdot \left[w^2 \;\; rs \;\; rw \;\; sw\right]\right)^2 = \sum_{i,j} X_{ij} W_{ij}$$

$$W = \begin{bmatrix} w^4 & rsw^2 & rw^3 & sw^3 \\ rsw^2 & r^2s^2 & r^2sw & rs^2w \\ rw^3 & r^2sw & r^2w^2 & rsw^2 \\ sw^3 & rs^2w & rsw^2 & s^2w^2 \end{bmatrix}$$

$$X = \sum_\alpha x_\alpha x_\alpha^T \in \mathbf{S}_+^{4\times4} \cap \overleftrightarrow{\mathbf{S}}^{n^4}$$

$$x_\alpha = \left[x_\alpha^1 \;\; x_\alpha^2 \;\; x_\alpha^3 \;\; x_\alpha^4\right]$$

$\overrightarrow{\mathbf{S}}^{n^4}$ $\quad \mathcal{T}^{ijkl} = \mathcal{T}^{ilkj} = \mathcal{T}^{kjil} = \mathcal{T}^{jilk}$

$\overleftrightarrow{\mathbf{S}}^{n^4}$ $\quad \mathcal{T}^{ijkl} = \mathcal{T}^{ilkj} = \mathcal{T}^{kjil} = \mathcal{T}^{jilk}$

$\mathbf{S}_+^{4\times4} \cap \overrightarrow{\mathbf{S}}^{n^4}$

$$X = \frac{1}{2}\mathcal{M}_{\text{scalar}} + d \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 4C_{1111} & C'_{1122}+2d & C_{1112} & C_{1112} \\ C'_{1122}+2d & 4C_{2222} & C_{1222} & C_{1222} \\ C_{1112} & C_{1222} & C_{1212} & C'_{1122}-2d \\ C_{1112} & C_{1222} & C'_{1122}-2d & C_{1212} \end{bmatrix} \succcurlyeq 0$$

ERs for without Z2 symmetry

$$\begin{bmatrix} 4C_{1111} & C'_{1122}+2d & C_{1112} & C_{1112} \\ C'_{1122}+2d & 4C_{2222} & C_{1222} & C_{1222} \\ C_{1112} & C_{1222} & C_{1212} & C'_{1122}-2d \\ C_{1112} & C_{1222} & C'_{1122}-2d & C_{1212} \end{bmatrix} \succcurlyeq 0$$

Sylvester's criterion → The determinants of all principal minors are larger than zero

$$|4C_{1111}| \geq 0 \quad \Rightarrow \quad C_{1111} \geq 0$$

$$\begin{vmatrix} 4C_{1111} & C_{1112} \\ C_{1112} & C_{1212} \end{vmatrix} \geq 0 \quad \Rightarrow \quad 4C_{1111}C_{1212} - C_{1112}^2 \geq 0$$

$$\begin{vmatrix} 4C_{1111} & C'_{1122}+2d & C_{1112} \\ C'_{1122}+2d & 4C_{2222} & C_{1222} \\ C_{1112} & C_{1222} & C_{1212} \end{vmatrix} \geq 0 \quad \Rightarrow$$

$$\text{and} \quad \Big\{C_{1112}C_{1122}C_{1222} - C_{1111}C_{1222}^2 - C_{1112}^2C_{2222} + C_{1212}\left(-C_{1122}^2 + 4C_{1111}C_{2222}\right) \geq 0$$

$$\text{or} \quad \Big[\Delta \equiv 3\left(4C_{1111}C_{2222} - C_{1112}C_{1222}\right) + \left(C_{1122} + C_{1212}\right)^2 \geq 0$$

$$\text{and} \quad \frac{3C_{1112}^2}{4C_{1111}} - 2\left(C_{1122} + C_{1212}\right) \leq \sqrt{\Delta} \leq C_{1212} - 2C_{1122}$$

$$\text{and} \quad 2\Delta^{3/2} \geq 27\left(C_{1111}C_{1222}^2 + C_{1112}^2C_{2222}\right) - 9\left(C_{1122} + C_{1212}\right)\left(8C_{1111}C_{2222} + C_{1112}C_{1222}\right) + 2\left(C_{1122} + C_{1212}\right)^3\Big]\Big\}$$

$+$

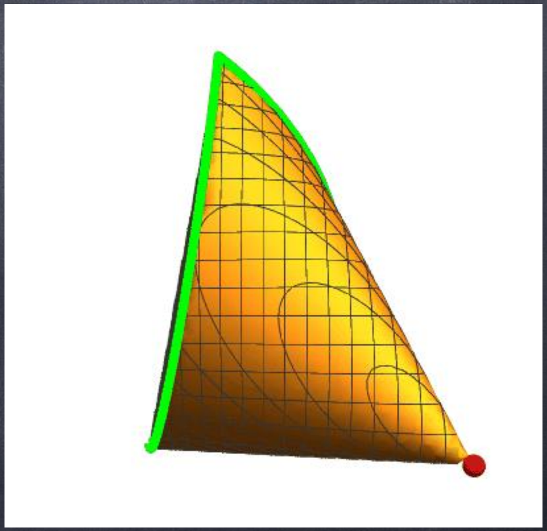
…

- At least for simple cases, the ext(G) can be found by inspection.
- E.g. simplest case:
  n=2, with some Z2 symmetry, e=f=0, T ->

$$\begin{pmatrix} a & b & 0 & 0 \\ b & c & 0 & 0 \\ 0 & 0 & d & b \\ 0 & 0 & b & d \end{pmatrix}$$

- There are two kinds of ERs
  - ER1: a=b=c=0, d=1
  - ER2: $ac=b^2$, d=|b|, a,c>0

A 3D cross section
of the 4D cone (a,b,c,d)

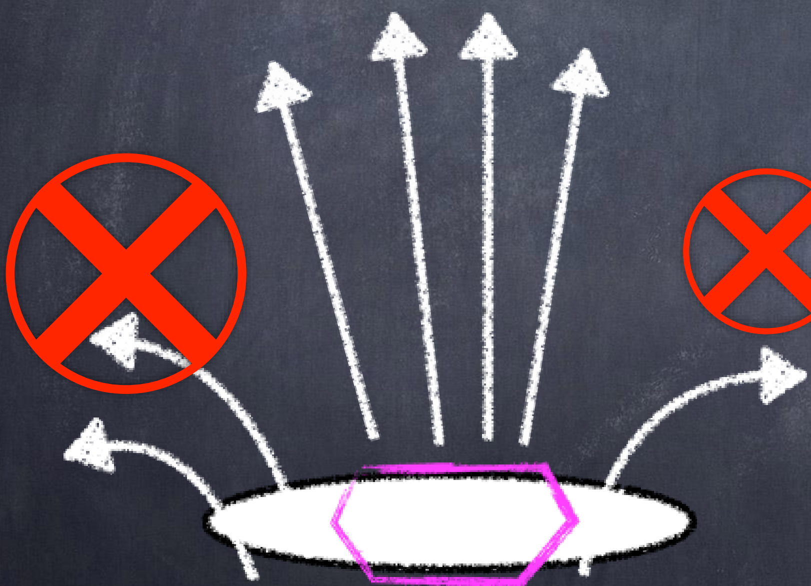


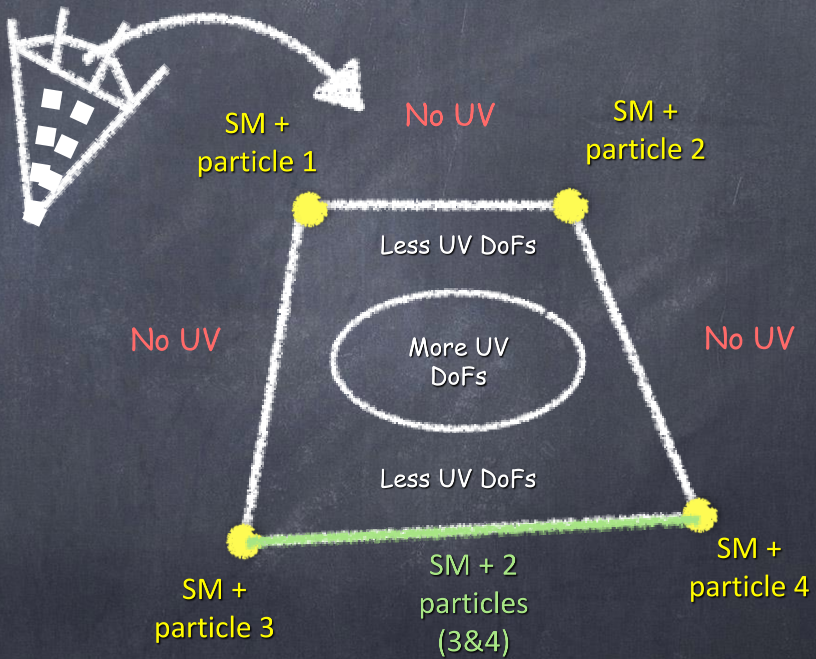
$$M^{ijkl} = \begin{pmatrix} C_1 & C_2 & 0 & 0 \\ C_2 & C_3 & 0 & 0 \\ 0 & 0 & C_4 & C_2 \\ 0 & 0 & C_2 & C_4 \end{pmatrix}$$

$$C_1, C_3, C_4 \geq 0 \text{ and } \sqrt{C_1 C_3} \geq \pm 2C_2 - C_4$$

- Infer UV model from EFT measurements

Inverse problem: Given the measured values of the operator coefficients around the electroweak scale, to what extend can we possibly determine the nature of the new physics beyond the SM? [Gu, Wang, 2008.07551]
see also [S. Dawson et al. 2007.01296] [N. Arkani-Hamed et al. hep-ph/0512190]





[CZ and S.-Y. Zhou 2005.03047]
[2009.02212 B. Fuks, Y. Liu, CZ, S.-Y. Zhou]