





Neutrino masses in the Standard Model effective field theory

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Higgs and Effective Field Theory — HEFT 2021 15 April 2021



- Motivation
- One-loop renormalisation of the operators contributing to the neutrino mass matrix at $d \leq 7$
- Leading-log correction to the neutrino mass matrix
- Some implications

Motivation

Tiny neutrino masses suggest that the new physics scale $\Lambda \gg v \approx 246 \; \text{GeV}$

This justifies using the Standard Model effective field theory (SMEFT) to describe physics at low energies

At $d \leq 7$, the only operators contributing to the neutrino mass matrix M_{ν} at tree level are

 $\mathcal{O}_{LH}^{(5)} = \epsilon_{ij} \epsilon_{mn} \left(L^i C L^m \right) H^j H^n \qquad \text{Weinberg, PRL 43 (1979) 1566}$

 $\mathcal{O}_{LH}^{(7)} = \epsilon_{ij} \epsilon_{mn} \left(L^{i} C L^{m} \right) H^{j} H^{n} \left(H^{\dagger} H \right)$

$$M_{\nu}^{\text{tree}} = -\frac{\mathrm{v}^2}{\Lambda} \left(\alpha_{LH}^{(5)} + \alpha_{LH}^{(7)} \frac{\mathrm{v}^2}{2\Lambda^2} \right)$$

Motivation

At one loop, M_{ν} gets corrections due to

- ▶ dim-4 SM interactions (gauge couplings, λ_H , Yukawas)
- dim-6 interactions
- dim-7 interactions

$$M_\nu = M_\nu^{\rm tree} + \delta M_\nu$$

Aim: compute δM_{ν} in the leading-log (LL) approximation to order v^3/Λ^3

- ▶ Important if e.g. $\alpha_{LH}^{(7)} v^2 / \Lambda^2$ partially cancels $\alpha_{LH}^{(5)}$ in M_{ν}^{tree}
- Potential new probe of dim-6 operators
- One step forward towards the one-loop renormalisation of the SMEFT to order v^3/Λ^3

Technical details

Approximations:

- vanishing down-type quark and charged lepton Yukawas
- up-type quark Yukawa matrix Y_u real and diagonal

EFT operator bases:

- ▶ d = 6, independent on shell Grzadkowski, Iskrzynski, Misiak, Rosiek, 1008.4884
- ▶ d = 6, independent off shell Gherardi, Marzocca, Venturini, 2003.12525
- ▶ d = 7, independent on shell Lehman, 1410.4193; Liao and Ma, 1607.07309
- ▶ d = 7, independent off shell (can be inferred from Lehman, 1410.4193)

To fix the divergences of the relevant Wilson coefficients (WCs), we compute all necessary 1PI diagrams off shell with the help of FeynRules + FeynArts + Formcalc

Operators potentially affecting δM_{ν}

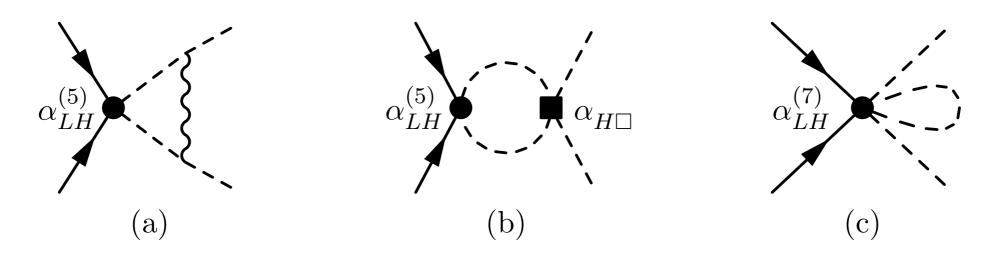
d = 6		d = 7		
$egin{array}{c} \mathcal{O}_H & & \ \mathcal{O}_{H\Box} & & \ \mathcal{O}_{HD} & & \ \mathcal{O}_{HB} & & \ \end{array}$	$(H^{\dagger}H)^{3}$ $(H^{\dagger}H)\Box(H^{\dagger}H)$ $(H^{\dagger}D^{\mu}H)^{*}(H^{\dagger}D_{\mu}H)$ $(H^{\dagger}H)B_{\mu\nu}B^{\mu\nu}$	$\begin{array}{ccc} \mathcal{O}_{LHD1} & \epsilon_{ij}\epsilon_{mn} \left(L^{i}CD^{\mu}L^{j}\right)H^{m} \left(D_{\mu}H^{n}\right) \\ \mathcal{O}_{LHD2} & \epsilon_{im}\epsilon_{jn} \left(L^{i}CD^{\mu}L^{j}\right)H^{m} \left(D_{\mu}H^{n}\right) \\ \mathcal{O}_{LHW} & \epsilon_{ij} (\epsilon\sigma)_{mn} \left(L^{i}C\sigma_{\mu\nu}L^{m}\right)H^{j}H^{n}W^{I\mu\nu} \\ \mathcal{O}_{QuLLH} & \epsilon_{ij} \left(\overline{Q}u\right) \left(LCL^{i}\right)H^{j} \end{array}$		
\mathcal{O}_{HW} \mathcal{O}_{HWB} \mathcal{O}_{3W}	$ \begin{array}{l} (H^{\dagger}H)W^{I}_{\mu\nu}W^{I\mu\nu},\\ (H^{\dagger}\sigma^{I}H)W^{I}_{\mu\nu}B^{\mu\nu}\\ \epsilon^{IJK}W^{I\nu}_{\mu}W^{J\rho}_{\nu}W^{K\mu}_{\rho} \end{array} $	$\mathcal{O}_{LH}^{(7)} \qquad \qquad \epsilon_{ij} \epsilon_{mn} \left(L^i C L^m \right) H^j H^n \left(H^{\dagger} H \right)$		
$\mathcal{O}_{H\widetilde{B}} \ \mathcal{O}_{H\widetilde{W}} \ \mathcal{O}_{H\widetilde{W}B}$	$(H^{\dagger}H)\widetilde{B}_{\mu\nu}B^{\mu\nu}, (H^{\dagger}H)\widetilde{W}^{I}_{\mu\nu}W^{I\mu\nu} (H^{\dagger}\sigma^{I}H)\widetilde{W}^{I}_{\mu\nu}B^{\mu\nu} (\overline{H}^{\dagger}\sigma^{I}H)\widetilde{W}^{I}_{\mu\nu}B^{\mu\nu}$	5 lepton number violating (LNV) dim-7 operators		
$egin{array}{lll} \mathcal{O}_{uH}^{(1)} \ \mathcal{O}_{HL}^{(3)} \ \mathcal{O}_{HL}^{(3)} \end{array}$	$ \begin{array}{c} (\overline{Q}\widetilde{H}u)(H^{\dagger}H) \\ (H^{\dagger}i\overleftrightarrow{D}_{\mu}H)(\overline{L}\gamma^{\mu}L) \\ (H^{\dagger}i\overleftrightarrow{D}_{\mu}^{I}H)(\overline{L}\gamma^{\mu}\sigma^{I}L) \end{array} $	will contribute to $\delta M_{ u}$		
$egin{array}{llllllllllllllllllllllllllllllllllll$	$(H^{\dagger}i\overleftrightarrow{D}_{\mu}H)(\overline{e}\gamma^{\mu}e) \\ (H^{\dagger}i\overleftrightarrow{D}_{\mu}H)(\overline{Q}\gamma^{\mu}Q) \\ (H^{\dagger}i\overleftrightarrow{D}_{\mu}^{I}H)(\overline{Q}\gamma^{\mu}\sigma^{I}Q)$			
$egin{array}{lll} \mathcal{O}_{Hu} \ \mathcal{O}_{Hd} \ \mathcal{O}_{LL} \end{array}$	$ \begin{array}{c} \left(H^{\dagger}i\overleftrightarrow{D}_{\mu}H\right)\left(\overline{u}\gamma^{\mu}u\right) \\ \left(H^{\dagger}i\overleftarrow{D}_{\mu}H\right)\left(\overline{d}\gamma^{\mu}d\right) \\ \left(\overline{L}\gamma_{\mu}L\right)\left(\overline{L}\gamma^{\mu}L\right) \end{array} $			

11 dim-6 operators will contribute to δM_{ν}

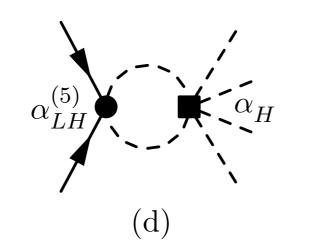
(operators in grey won't contribute)

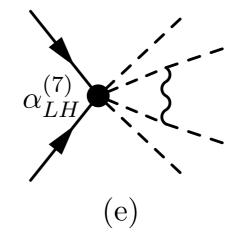
Direct renormalisation of the Weinberg operators

Direct renormalisation of $\alpha_{LH}^{(5)}$



Direct renormalisation of $\alpha_{LH}^{(7)}$





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Direct renormalisation of the Weinberg operators

Result of the direct renormalisation:

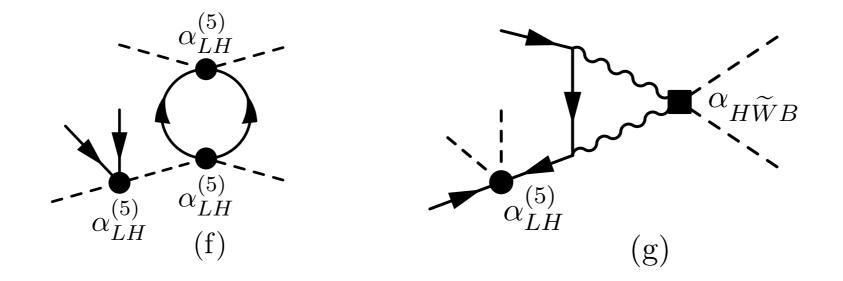
$$\left(\tilde{\alpha}_{LH}^{(5)} \right)^{pq} = -\frac{1}{64\pi^2 \epsilon} \left\{ \left(g_1^2 - 3g_2^2 + 4\lambda_H \right) \left(\alpha_{LH}^{(5)} \right)^{pq} - 16 \left(\alpha_{LH}^{(5)} \right)^{pq} \alpha_{H\Box} \frac{\mu_H^2}{\Lambda^2} - 8 \left[\left(\alpha_{LH}^{(5)} \right)^{pr} \left(\alpha_{HL}^{(1)} \right)^{rq} - 2 \left(\alpha_{LH}^{(5)} \right)^{pr} \left(\alpha_{HL}^{(3)} \right)^{rq} + p \leftrightarrow q \right] \frac{\mu_H^2}{\Lambda^2} - 16 \left(\alpha_{LH}^{(7)} \right)^{pq} \frac{\mu_H^2}{\Lambda^2} \right\},$$

$$\begin{split} \left(\widetilde{\alpha}_{LH}^{(7)}\right)^{pq} &= \frac{1}{256\pi^{2}\epsilon} \Biggl\{ 16 \bigl(\alpha_{LH}^{(5)}\bigr)^{pq} \left[6\alpha_{H} - 12\lambda_{H}\alpha_{H\Box} + 2\lambda_{H}\alpha_{HD} - 3g_{1}^{2}\alpha_{HB} - 3g_{2}^{2}\alpha_{HW} - 3g_{1}g_{2}\alpha_{HWB} \right] \\ &\quad + 24 \left(g_{1}^{2} + g_{2}^{2} \right) \left[\bigl(\alpha_{LH}^{(5)}\bigr)^{pr} \bigl(\alpha_{HL}^{(1)}\bigr)^{rq} - \bigl(\alpha_{LH}^{(5)}\bigr)^{pr} \bigl(\alpha_{HL}^{(3)}\bigr)^{rq} + p \leftrightarrow q \right] \\ &\quad - 32\lambda_{H} \left[\bigl(\alpha_{LH}^{(5)}\bigr)^{pr} \bigl(\alpha_{HL}^{(1)}\bigr)^{rq} - 2\bigl(\alpha_{LH}^{(5)}\bigr)^{pr} \bigl(\alpha_{HL}^{(3)}\bigr)^{rq} + p \leftrightarrow q \right] \\ &\quad + 8 \left(3g_{2}^{2} - 20\lambda_{H} \right) \bigl(\alpha_{LH}^{(7)}\bigr)^{pq} + 6g_{2}^{3} \left[4\alpha_{LHW}^{pq} + g_{2}\alpha_{LHD1}^{pq} + p \leftrightarrow q \right] \\ &\quad + 3 \left(g_{1}^{4} + 3g_{2}^{4} + 2g_{1}^{2}g_{2}^{2} \right) \left[\alpha_{LHD2}^{pq} + p \leftrightarrow q \right] + 48Y_{u}^{rt}Y_{u}^{us}Y_{u}^{ut} \left[\alpha_{QuLLH}^{rspq} + p \leftrightarrow q \right] \Biggr\}, \end{split}$$

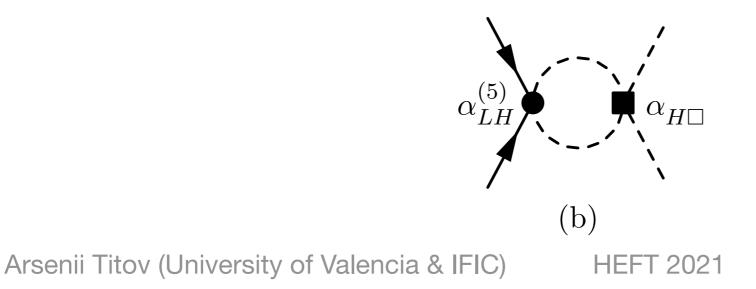
In the second equation, the terms proportional to dim-7 operators have been cross-checked against Liao and Ma, 1901.10302

Indirect renormalisation of the Weinberg operators

Renormalisation of redundant dim-6 interactions that on shell contribute to the Weinberg operators



Renormalisation of redundant dim-7 interactions that on shell contribute to the Weinberg operators



Relevant redundant operators

	\mathcal{O}_{2B}	$-\frac{1}{2} \left(\partial_{\mu} B^{\mu\nu} \right) \left(\partial^{\rho} B_{\rho\nu} \right)$
	\mathcal{O}_{2W}	$-\frac{1}{2} \left(D_{\mu} W^{I \mu \nu} \right) \left(D^{\rho} W^{I}_{\rho \nu} \right)$
	\mathcal{O}_{BDH}	$\partial_{\nu}B^{\mu u}\left(H^{\dagger}i\overleftarrow{D}_{\mu}H\right)$
	\mathcal{O}_{WDH}	$D_{\nu}W^{I\mu\nu}\left(H^{\dagger}i\overleftarrow{D}_{\mu}^{I}H\right)$
	\mathcal{O}_{DH}	$\left(D_{\mu}D^{\mu}H\right)^{\dagger}\left(D_{\nu}D^{\nu}H\right)$
	$\mathcal{O}_{HD}^{\prime}$	$\left(H^{\dagger}H\right)\left(D_{\mu}H\right)^{\dagger}\left(D^{\mu}H\right)$
d = 6	${\cal O}_{HD}^{\prime\prime}$	$(H^{\dagger}H)D_{\mu}(H^{\dagger}i\overleftrightarrow{D}^{\mu}H)$
	\mathcal{O}_{LD}	$\frac{i}{2}\overline{L}\left\{D_{\mu}D^{\mu},D^{\mu}\right\}L$
	${\cal O}_{HL}^{\prime(1)}$	$(H^{\dagger}H)(\overline{L}i\overleftrightarrow{D}L)$
	${\cal O}_{HL}^{\prime\prime(1)}$	$\partial_{\mu} \left(H^{\dagger} H ight) \left(\overline{L} \gamma^{\mu} L ight)$
	${\cal O}_{HL}^{\prime(3)}$	$(H^{\dagger}\sigma^{I}H)(\overline{L}i\overleftrightarrow{D}^{I}L)$
	$\mathcal{O}_{HL}^{\prime\prime(3)}$	$D_{\mu}(H^{\dagger}\sigma^{I}H)(\overline{L}\gamma^{\mu}\sigma^{I}L)$
d = 7	$\mathcal{O}_{LHD}^{(R)}$	$\epsilon_{ij}\epsilon_{mn}L^iCL^mH^j\Box H^n$

On-shell projections onto the Weinberg operators

Upon using the SMEFT equations of motion to $\mathcal{O}(1/\Lambda)$, in particular, $D^{2}H^{i} = \mu_{H}^{2}H^{i} - \lambda_{H}(H^{\dagger}H)H^{i} + Y_{u}^{pq}\epsilon^{ij}\overline{Q_{p}^{j}}u_{q} - \frac{\left(\alpha_{LH}^{(5)}\right)_{pq}^{*}}{\Lambda}\epsilon^{ij}\left[\overline{L_{q}^{j}}(\widetilde{H}^{T}L_{p}^{c}) + (\overline{L_{q}}\widetilde{H})L_{p}^{c,j}\right],$ $i \not \!\!D L_{q}^{i} = -2 \frac{\left(\alpha_{LH}^{(5)}\right)_{pq}^{*}}{\Lambda} \widetilde{H}^{i}(\widetilde{H}^{T}L_{p}^{c}),$ Barzinji, Trott, Vasudevan, 1806.06354

we get the following on-shell projections onto the Weinberg operators:

$$\mathcal{O}_{2W} \supset \frac{g_2^2}{2\Lambda} (\alpha_{LH}^{(5)})^{pq} (\mathcal{O}_{LH}^{(7)})^{pq} + \text{h.c.}, \qquad \qquad \mathcal{O}_{HD}^{\prime\prime} \supset -\frac{2i}{\Lambda} (\alpha_{LH}^{(5)})^{pq} (\mathcal{O}_{LH}^{(7)})^{pq} + \text{h.c.}, \qquad \qquad \mathcal{O}_{HD}^{\prime\prime} \supset -\frac{2g_2}{\Lambda} (\alpha_{LH}^{(5)})^{pq} (\mathcal{O}_{LH}^{(7)})^{pq} + \text{h.c.}, \qquad \qquad \delta^{pq} (\mathcal{O}_{HL}^{\prime\prime(1)})^{pq} \supset \frac{2i}{\Lambda} (\alpha_{LH}^{(5)})^{rp} (\mathcal{O}_{LH}^{(7)})^{rp} + \text{h.c.}, \qquad \qquad \delta^{pq} (\mathcal{O}_{HL}^{\prime\prime(3)})^{pq} \supset -\frac{2i}{\Lambda} (\alpha_{LH}^{(5)})^{rp} (\mathcal{O}_{LH}^{(7)})^{rp} + \text{h.c.}, \qquad \qquad \delta^{pq} (\mathcal{O}_{HL}^{\prime\prime(3)})^{pq} \supset -\frac{2i}{\Lambda} (\alpha_{LH}^{(5)})^{rp} (\mathcal{O}_{LH}^{(7)})^{rp} + \text{h.c.}, \qquad \qquad \delta^{pq} (\mathcal{O}_{HL}^{\prime\prime(3)})^{pq} \supset -\frac{2i}{\Lambda} (\alpha_{LH}^{(5)})^{rp} (\mathcal{O}_{LH}^{(7)})^{rp} + \text{h.c.}, \qquad \qquad \delta^{pq} (\mathcal{O}_{HL}^{\prime\prime(3)})^{pq} \supset -\frac{2i}{\Lambda} (\alpha_{LH}^{(5)})^{rp} (\mathcal{O}_{LH}^{(7)})^{rp} + \text{h.c.}, \qquad \qquad \delta^{pq} (\mathcal{O}_{HL}^{\prime\prime(3)})^{pq} \supset -\frac{2i}{\Lambda} (\alpha_{LH}^{(5)})^{rp} (\mathcal{O}_{LH}^{(7)})^{rp} + \text{h.c.}, \qquad \qquad \delta^{pq} (\mathcal{O}_{HL}^{\prime\prime(3)})^{pq} \supset -\frac{2i}{\Lambda} (\alpha_{LH}^{(5)})^{rp} (\mathcal{O}_{LH}^{(7)})^{rp} + \text{h.c.}, \qquad \qquad \delta^{pq} (\mathcal{O}_{HL}^{\prime\prime(3)})^{pq} \supset -\frac{2i}{\Lambda} (\alpha_{LH}^{(5)})^{rp} (\mathcal{O}_{LH}^{(7)})^{rp} + \text{h.c.}, \qquad \qquad \delta^{pq} (\mathcal{O}_{HL}^{\prime\prime(3)})^{pq} \supset -\frac{2i}{\Lambda} (\alpha_{LH}^{(5)})^{rp} (\mathcal{O}_{LH}^{\prime\prime(7)})^{rp} + \text{h.c.}, \qquad \qquad \delta^{pq} (\mathcal{O}_{HL}^{\prime\prime(3)})^{pq} \supset -\frac{2i}{\Lambda} (\alpha_{LH}^{(5)})^{rp} (\mathcal{O}_{LH}^{\prime\prime(7)})^{rp} + \text{h.c.}, \qquad \qquad \delta^{pq} (\mathcal{O}_{HL}^{\prime\prime(3)})^{pq} \supset -\frac{2i}{\Lambda} (\alpha_{LH}^{\prime\prime(5)})^{rp} (\mathcal{O}_{LH}^{\prime\prime(7)})^{rp} + \text{h.c.}, \qquad \qquad \delta^{pq} (\mathcal{O}_{HL}^{\prime\prime(3)})^{pq} \supset -\frac{2i}{\Lambda} (\alpha_{LH}^{\prime\prime(5)})^{rp} (\mathcal{O}_{LH}^{\prime\prime(7)})^{rp} (\mathcal{O}_{LH}^{\prime\prime(7)})^{rp} + \text{h.c.}, \qquad \qquad \delta^{pq} (\mathcal{O}_{HL}^{\prime\prime(5)})^{rp} \supset -\frac{2i}{\Lambda} (\alpha_{LH}^{\prime\prime(5)})^{rp} (\mathcal{O}_{LH}^{\prime\prime(7)})^{rp} (\mathcal{O}_{LH}^{\prime$$

$$\left(\mathcal{O}_{LHD}^{(R)}\right)^{pq} \supset \mu_H^2 \left(\mathcal{O}_{LH}^{(5)}\right)^{pq} - \lambda_H \left(\mathcal{O}_{LH}^{(7)}\right)^{pq}$$

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Renormalisation of the redundant operators

The divergencies of the redundant dim-6 operators:

$$\begin{split} \widetilde{\alpha}_{2W} &= -\frac{3g_2}{4\pi^2 \epsilon} \alpha_{3W} \,, \\ \widetilde{\alpha}_{WDH} &= -\frac{g_2}{96\pi^2 \epsilon} \left[\alpha_{H\Box} + 18g_2 \alpha_{3W} + 4 \left(\alpha_{HL}^{(3)} \right)^{pp} + 12 \left(\alpha_{HQ}^{(3)} \right)^{pp} \right] , \\ \widetilde{\alpha}_{HD}' &= -\frac{1}{32\pi^2 \epsilon} \left[6 \left(g_1^2 \alpha_{HB} + g_1 g_2 \alpha_{HWB} + 3g_2^2 \alpha_{HW} \right) + \left(3g_1^2 - 3g_2^2 + 2\lambda_H \right) \alpha_{HD} + \left(6g_2^2 - 4\lambda_H \right) \alpha_{H\Box} \right. \\ &\quad + 6Y_u^{pq} \left(\alpha_{uH}^{pq} + \alpha_{uH}^{pq*} \right) - 24Y_u^{pr} Y_u^{qr} \left(\alpha_{HQ}^{(3)} \right)^{pq} - 8 \left(\alpha_{LH}^{(5)} \right)^{pq} \left(\alpha_{LH}^{(5)} \right)_{pq}^* \right] , \\ \widetilde{\alpha}_{HD}'' &= -\frac{3i}{32\pi^2 \epsilon} Y_u^{pq} \left(\alpha_{uH}^{pq} - \alpha_{uH}^{pq*} \right) \,, \\ \widetilde{\alpha}_{HL}''^{(1)} \right)^{pq} &= \frac{3}{32\pi^2 \epsilon} \left(g_1^2 \alpha_{H\widetilde{B}} + 3g_2^2 \alpha_{H\widetilde{W}} \right) \delta^{pq} \,, \\ \widetilde{\alpha}_{HL}''^{(3)} \right)^{pq} &= -\frac{3}{32\pi^2 \epsilon} g_1 g_2 \alpha_{H\widetilde{W}B} \delta^{pq} \,. \end{split}$$

The divergency of the redundant dim-7 operator:

$$\begin{split} \left(\widetilde{\alpha}_{LHD}^{(R)}\right)^{pq} &= \frac{1}{128\pi^{2}\epsilon} \left\{ 16 \left(\alpha_{LH}^{(5)}\right)^{pq} \alpha_{H\Box} - 16 \left[\left(\alpha_{LH}^{(5)}\right)^{pr} \left(\alpha_{HL}^{(1)}\right)^{rq} - 2 \left(\alpha_{LH}^{(5)}\right)^{pr} \left(\alpha_{HL}^{(3)}\right)^{rq} + p \leftrightarrow q \right] \right. \\ &+ 3 \left[2g_{2}^{2} \alpha_{LHD1}^{pq} + g_{2}^{2} \alpha_{LHD2}^{pq} + 4Y_{u}^{rs} \alpha_{QuLLH}^{rspq} + p \leftrightarrow q \right] \right\}, \end{split}$$

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Renormalisation group equation

Divergencies of the kinetic terms (wave function renormalisation):

$$\begin{split} K_H &= -\frac{1}{32\pi^2 \epsilon} \left[g_1^2 + 3g_2^2 - 6 \operatorname{Tr} \left(Y_u^2 \right) + 2 \left(2\alpha_{H\Box} - \alpha_{HD} \right) \frac{\mu_H^2}{\Lambda^2} \right], \\ K_L &= \frac{1}{64\pi^2 \epsilon} \left(g_1^2 + 3g_2^2 \right) \,. \end{split}$$

Canonical normalisation of the kinetic terms:

$$H \to (1 - K_H/2) H$$
 and $L \to (1 - K_L/2) L$

Bare and renormalised quantities:

$$(\alpha_{LH}^{(5)})^0 = \mu^{2\epsilon} Z_{LH}^{(5)} \alpha_{LH}^{(5)}$$
 with $Z_{LH}^{(5)} = 1 - \tilde{\alpha}_{LH}^{(5)} / \alpha_{LH}^{(5)}$

RGE:

$$\mu \frac{\mathrm{d}(\alpha_{LH}^{(5)})^{0}}{\mathrm{d}\mu} = 0 \qquad \Rightarrow \qquad \mu \frac{\mathrm{d}\alpha_{LH}^{(5)}}{\mathrm{d}\mu} = \epsilon \,\alpha_{LH}^{(5)} \frac{\partial Z_{LH}^{(5)}}{\partial g_k} n_k g_k$$

 g_k are all Lagrangian couplings, renormalisable and not n_k are tree-level anomalous dimensions of g_k

LL correction to neutrino mass matrix

Solving the RGEs to LL approximation, and upon EWSB, we obtain

$$\begin{split} \delta M_{\nu}^{pq} &= -\frac{v^2}{16\pi^2\Lambda} \log \frac{\Lambda}{v} \bigg\{ \left[3g_2^2 - 2\lambda_H - 6\operatorname{Tr}\left(Y_u^2\right) \right] \left(\alpha_{LH}^{(5)}\right)^{pq} \\ &+ \frac{v^2}{\Lambda^2} \left(\alpha_{LH}^{(5)}\right)^{pq} \bigg[-4\left(\alpha_{LH}^{(5)}\right)^{st} \left(\alpha_{LH}^{(5)}\right)_{st}^* + 6\alpha_H + \frac{2}{3} \left(5g_2^2 - 12\lambda_H\right) \alpha_{H\Box} + \frac{1}{2} \left(3g_1^2 - 3g_2^2 + 4\lambda_H\right) \alpha_{HD} + 6g_2^2 \alpha_{HW} \\ &+ 3ig_1^2 \alpha_{H\widetilde{B}} + 9ig_2^2 \alpha_{H\widetilde{W}} + 3ig_1g_2 \alpha_{H\widetilde{W}B} + 6Y_u^{st} \alpha_{uH}^{st*} + \frac{4}{3}g_2^2 \left(\alpha_{HL}^{(3)}\right)^{rr} + 4g_2^2 \left(\alpha_{HQ}^{(3)}\right)^{rr} - 12Y_u^{su}Y_u^{tu} \left(\alpha_{HQ}^{(3)}\right)^{st} \bigg] \\ &+ \frac{v^2}{\Lambda^2} \frac{3}{2} \left(g_1^2 + g_2^2\right) \left[\left(\alpha_{LH}^{(5)}\right)^{pr} \left(\alpha_{HL}^{(1)}\right)^{rq} - \left(\alpha_{LH}^{(5)}\right)^{pr} \left(\alpha_{HL}^{(3)}\right)^{rq} + p \leftrightarrow q \right] + \frac{v^2}{\Lambda^2} \frac{3}{4} \bigg[g_1^2 + 5g_2^2 - 8\lambda_H - 8\operatorname{Tr}\left(Y_u^2\right) \bigg] \left(\alpha_{LH}^{(7)}\right)^{pq} \\ &+ \frac{v^2}{\Lambda^2} \bigg[\frac{3}{8}g_2^4 \alpha_{LHD1}^{pq} + \frac{3}{16} \left(g_1^4 + 2g_1^2g_2^2 + 3g_2^4\right) \alpha_{LHD2}^{pq} + \frac{3}{2}g_2^3 \alpha_{LHW}^{pq} + 3Y_u^{rt}Y_u^{us}Y_u^{ut} \alpha_{QuLLH}^{rspq} + p \leftrightarrow q \bigg] \bigg\} \end{split}$$

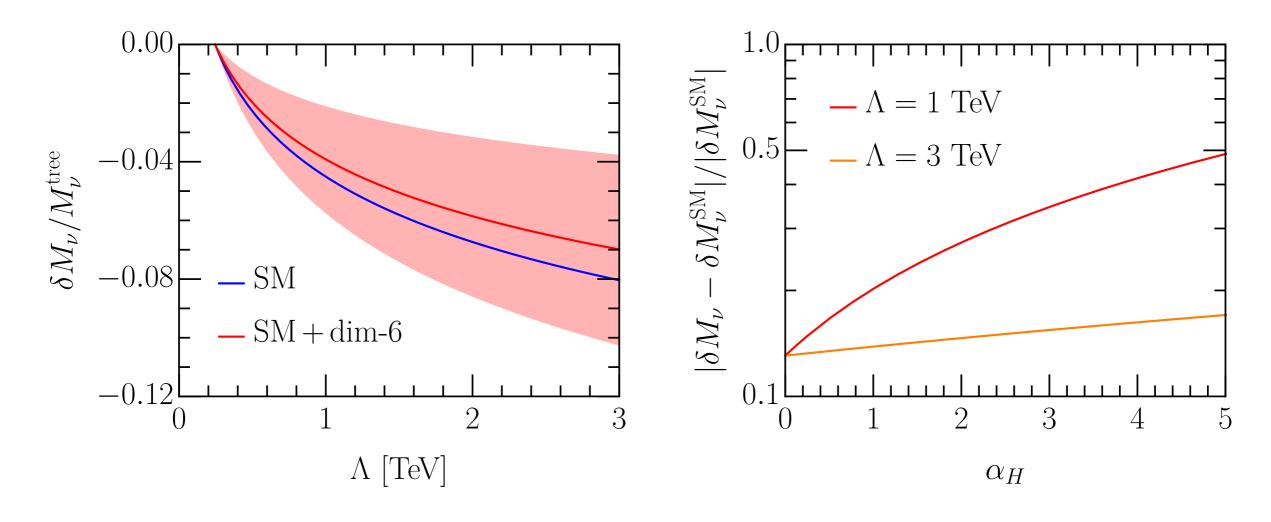
1 dim-5, 11 dim-6 and 5 dim-7 operators contribute to δM_{ν}

The first line (renormalisation of $\alpha_{LH}^{(5)}$ by the SM interactions) agrees with the previous well-known result Chankowski and Pluciennik, hep-ph/9306333 Babu, Leung, Pantaleone, hep-ph/9309223 Antusch et al., hep-ph/0108005 Arsenii Titov (University of Valencia & IFIC) HEFT 2021 14

Implications

- ► $\alpha_{LH}^{(5)} = \alpha_{LH}^{(7)} = 0 \implies M_{\nu}^{\text{tree}} = 0 \implies$ neutrino masses are induced by the dim-7 operators other than $\mathcal{O}_{LH}^{(7)}$ at $\Lambda \sim 10^3$ TeV (for order 1 WCs)
- ▷ $\alpha_{LH}^{(7)} v^2 / \Lambda^2$ partially cancels $\alpha_{LH}^{(5)}$ in M_{ν}^{tree}
- Certain combinations of the operators lead to $\delta M_{\nu} = 0$, e.g. $g_2^2 O_H - O_{HW}$ or $g_2 O_{LHW} - 4 O_{LHD1}$
- ▶ If the LNV and dim-6 operators are suppressed by different scales, $\Lambda_{LNV} \gg \Lambda_{LNC}$, then the $v^3/(\Lambda_{LNV}\Lambda_{LNC}^2)$ provides the first correction to the renormalisation of the SMEFT obtained in Jenkins, Manohar, Trott, 1308.2627, 1310.4838 and Alonso, Jenkins, Manohar, Trott, 1312.2014

Impact of dim-6 operators



 $\begin{aligned} &\alpha_{LH}^{(5)} \text{ is such that } M_{\nu}^{\text{tree}} = 0.01 \text{ eV} \\ &\delta M_{\nu}^{\text{SM}} : \text{ WCs of all dim-6 and dim-7 operators set to zero} \\ &\delta M_{\nu} : \text{WCs of dim-6 operators are fixed according to} \\ &\text{ the marginalised fit of Ellis et al., 2012.02779;} \\ &\text{ other dim-6 and dim-7 WCs are set to zero} \end{aligned}$



SM Lagrangian and conventions

$$\mathcal{L}_{\rm ren} = -\frac{1}{4} G^A_{\mu\nu} G^{A\mu\nu} - \frac{1}{4} W^I_{\mu\nu} W^{I\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + (D_{\mu}H)^{\dagger} (D^{\mu}H) + \mu_H^2 H^{\dagger}H - \frac{1}{2} \lambda_H (H^{\dagger}H)^2 + i \left(\overline{Q} \not{D} Q + \overline{u} \not{D} u + \overline{d} \not{D} d + \overline{L} \not{D} L + \overline{e} \not{D} e \right) - \left(\overline{Q} Y_d H d + \overline{Q} Y_u \ddot{H} u + \overline{L} Y_e H e + \text{h.c.} \right).$$
(3)

$$\tilde{H} = \epsilon H^*$$

$$D_{\mu} = \partial_{\mu} - ig_1 Y B_{\mu} - ig_2 \frac{\sigma^I}{2} W^I_{\mu} - ig_s \frac{\lambda^A}{2} G^A_{\mu} ,$$

$$\begin{split} B_{\mu\nu} &= \partial_{\mu}B_{\nu} - \partial_{\nu}B_{\mu} \,, \\ W^{I}_{\mu\nu} &= \partial_{\mu}W^{I}_{\nu} - \partial_{\nu}W^{I}_{\mu} + g_{2}\varepsilon^{IJK}W^{J}_{\mu}W^{K}_{\nu} \,, \\ G^{A}_{\mu\nu} &= \partial_{\mu}G^{A}_{\nu} - \partial_{\nu}G^{A}_{\mu} + g_{s}f^{ABC}G^{B}_{\mu}G^{C}_{\nu} \,, \end{split}$$

Results of the global fit

Ellis et al., 2012.02779

	Individual			Marginalised		
SMEFT	Best fit	95% CL	Scale	Best fit	95% CL	Scale
Coeff.	$[\Lambda=1~{\rm TeV}]$	range	$\frac{\Lambda}{\sqrt{C}}$ [TeV]	$[\Lambda=1~{\rm TeV}]$	range	$\frac{\Lambda}{\sqrt{C}}$ [TeV]
$C_{H\Box}$	-0.27	[-1, +0.47]	1.2	-0.9	[-3, +1.2]	0.69
C_{HD}	-0.01	[-0.023, +0.0027]	8.8	-0.39	[-1.6, +0.81]	0.91
C_{HW}	0.00	[-0.012, +0.006]	11.0	0.12	[-0.38, +0.62]	1.4
C_{tH}	-0.09	[-1, +0.84]	1.0	1.5	[-2.8, +5.7]	0.48
$C_{Hl}^{(1)}$	0.00	[-0.0044, +0.013]	11.0	0.11	[-0.19, +0.41]	1.8
$C_{Hl}^{(3)}$	0.00	[-0.01, +0.003]	12.0	-0.03	[-0.13, +0.055]	3.3
$C_{HQ}^{(3)}$	0.01	[-0.032, +0.048]	5.0	-0.1	[-0.67, +0.46]	1.3

The WCs of dim-6 operators contributing to δM_{ν} and not constrained by the fit:

 $\alpha_H\,,\quad \alpha_{H\widetilde{B}}\,,\quad \alpha_{H\widetilde{W}}\,,\quad \alpha_{H\widetilde{W}B}$