EFT calculations from Amplitude methods

Alex Pomarol, IFAE & UAB (Barcelona)

P. Baratella, C. Fernandez, AP 2005.07129

B. vonHarling, P. Baratella, C. Fernandez, AP 2010.13809

Outline

- **Some motivations for on-shell amplitude methods**
- **EFT (EFfective Theories) from amplitudes, instead of Lagrangians**
- **Renormalization of EFT using on-shell methods:**

  Loops from tree-level on-shell amplitudes

  ☛ **Simple**, elegant, and **efficient**

  ☛ **Selection rules** can explain many **non-renormalizations**

  ☛ Clean **relations** between different anomalous dimensions

  ☛ Easy **recycling**: Every calculation can be re-used!

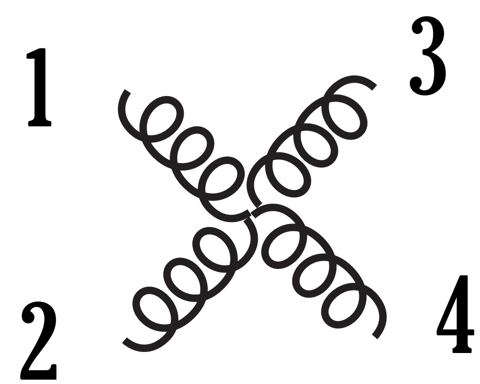
1. Some motivation

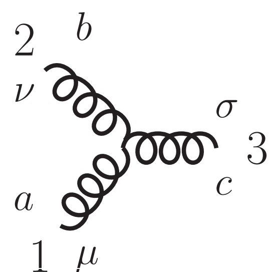
Amplitude methods

Extremely useful for simplifying calculations

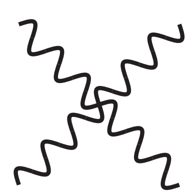
Example I: graviton + graviton → graviton + graviton

à la Feynman !





$$
\begin{aligned}
& i\mathrm{Sym}\Big[-\frac{1}{2}P_3(p_1\cdot p_2\eta_{\mu\rho}\eta_{\nu\lambda}\eta_{\sigma\tau})-\frac{1}{2}P_6(p_{1\nu}p_{1\lambda}\eta_{\mu\rho}\eta_{\sigma\tau})+\frac{1}{2}P_3(p_1\cdot p_2\eta_{\mu\nu}\eta_{\rho\lambda}\eta_{\sigma\tau}) \\
&\quad + P_6(p_1\cdot p_2\eta_{\mu\rho}\eta_{\nu\sigma}\eta_{\lambda\tau})+2P_3(p_{1\nu}p_{1\tau}\eta_{\mu\rho}\eta_{\lambda\sigma})-P_3(p_{1\lambda}p_{2\mu}\eta_{\rho\nu}\eta_{\sigma\tau}) \\
&\quad + P_3(p_{1\sigma}p_{2\tau}\eta_{\mu\nu}\eta_{\rho\lambda})+P_6(p_{1\sigma}p_{1\tau}\eta_{\mu\nu}\eta_{\rho\lambda})+2P_6(p_{1\nu}p_{2\tau}\eta_{\lambda\mu}\eta_{\rho\sigma}) \\
&\quad + 2P_3(p_{1\nu}p_{2\mu}\eta_{\lambda\sigma}\eta_{\tau\rho})-2P_3(p_1\cdot p_2\eta_{\rho\nu}\eta_{\lambda\sigma}\eta_{\tau\mu})\Big],
\end{aligned}
$$



$$
\begin{aligned}
& \mathrm{Sym}[-\tfrac{1}{8}P_6(p\cdot p'\eta^{\mu\nu}\eta^{\sigma\tau}\eta^{\rho\lambda}\eta^{\iota\kappa})-\tfrac{1}{8}P_{12}(p^\sigma p^\tau\eta^{\mu\nu}\eta^{\rho\lambda}\eta^{\iota\kappa})-\tfrac{1}{4}P_6(p^\sigma p'^\mu\eta^{\nu\tau}\eta^{\rho\lambda}\eta^{\iota\kappa})+\tfrac{1}{8}P_6(p\cdot p'\eta^{\mu\sigma}\eta^{\nu\tau}\eta^{\rho\lambda}\eta^{\iota\kappa}) \\
& +\tfrac{1}{4}P_6(p\cdot p'\eta^{\mu\nu}\eta^{\sigma\tau}\eta^{\rho\iota}\eta^{\lambda\kappa})+\tfrac{1}{4}P_{12}(p^\sigma p^\tau\eta^{\mu\nu}\eta^{\rho\iota}\eta^{\lambda\kappa})+\tfrac{1}{2}P_6(p^\sigma p'^\mu\eta^{\nu\tau}\eta^{\rho\iota}\eta^{\lambda\kappa})-\tfrac{1}{4}P_6(p\cdot p'\eta^{\mu\sigma}\eta^{\nu\tau}\eta^{\rho\iota}\eta^{\lambda\kappa}) \\
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& -\tfrac{1}{2}P_{12}(p\cdot p'\eta^{\nu\sigma}\eta^{\tau\rho}\eta^{\lambda\mu}\eta^{\iota\kappa})-\tfrac{1}{2}P_{12}(p^\sigma p'^\mu\eta^{\tau\rho}\eta^{\lambda\nu}\eta^{\iota\kappa})+\tfrac{1}{2}P_{12}(p^\sigma p^\rho\eta^{\tau\lambda}\eta^{\mu\nu}\eta^{\iota\kappa})-\tfrac{1}{2}P_{24}(p\cdot p'\eta^{\mu\nu}\eta^{\tau\rho}\eta^{\lambda\iota}\eta^{\kappa\sigma}) \\
& -P_{12}(p^\sigma p^\tau\eta^{\nu\rho}\eta^{\lambda\iota}\eta^{\kappa\mu})-P_{12}(p^\rho p'^\lambda\eta^{\nu\iota}\eta^{\kappa\sigma}\eta^{\tau\mu})-P_{24}(p_\sigma p'^\rho\eta^{\tau\iota}\eta^{\kappa\mu}\eta^{\nu\lambda})-P_{12}(p^\rho p'^\iota\eta^{\lambda\sigma}\eta^{\tau\mu}\eta^{\nu\kappa}) \\
& +P_6(p\cdot p'\eta^{\nu\rho}\eta^{\lambda\sigma}\eta^{\tau\iota}\eta^{\kappa\mu})-P_{12}(p^\sigma p^\rho\eta^{\mu\nu}\eta^{\tau\iota}\eta^{\kappa\lambda})-\tfrac{1}{2}P_{12}(p\cdot p'\eta^{\mu\rho}\eta^{\nu\lambda}\eta^{\sigma\iota}\eta^{\tau\kappa})-P_{12}(p^\sigma p^\rho\eta^{\tau\lambda}\eta^{\mu\iota}\eta^{\nu\kappa}) \\
& \qquad -P_6(p^\rho p'^\iota\eta^{\lambda\kappa}\eta^{\mu\sigma}\eta^{\nu\tau})-P_{24}(p^\sigma p'^\rho\eta^{\tau\mu}\eta^{\nu\iota}\eta^{\kappa\lambda})-P_{12}(p^\sigma p'^\mu\eta^{\tau\rho}\eta^{\lambda\iota}\eta^{\kappa\nu})+2P_6(p\cdot p'\eta^{\nu\sigma}\eta^{\tau\rho}\eta^{\lambda\iota}\eta^{\kappa\mu})]
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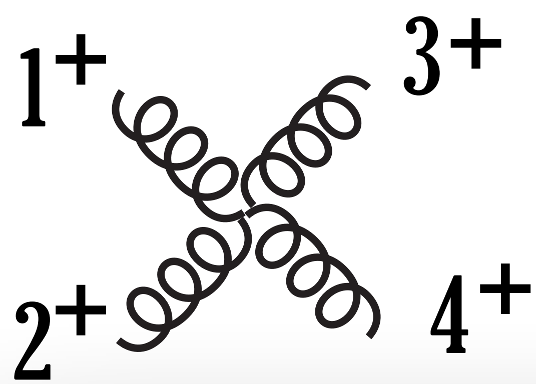
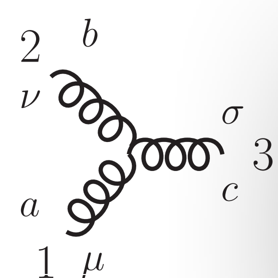
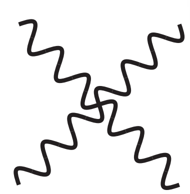


± ☛ h =±2



$$\mathcal{A}(1^+2^+3^+4^+) = 0$$



$$\begin{aligned}\mathrm{Sym}\Big[&-\tfrac{1}{8}P_6(p\cdot p'\eta^{\mu\nu}\eta^{\sigma\tau}\eta^{\rho\lambda}\eta^{\iota\kappa})-\tfrac{1}{8}P_{12}(p^\sigma p^\tau\eta^{\mu\nu}\eta^{\rho\lambda}\eta^{\iota\kappa})-\tfrac{1}{4}P_6(p^\sigma p'^\mu\eta^{\nu\tau}\eta^{\rho\lambda}\eta^{\iota\kappa})+\tfrac{1}{8}P_6(p\cdot p'\eta^{\mu\sigma}\eta^{\nu\tau}\eta^{\rho\lambda}\eta^{\iota\kappa})\\
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&-P_{12}(p^\sigma p^\tau\eta^{\nu\rho}\eta^{\lambda\iota}\eta^{\kappa\mu})-P_{12}(p^\rho p'^\lambda\eta^{\nu\iota}\eta^{\kappa\sigma}\eta^{\tau\mu})-P_{24}(p_\sigma p'^\rho\eta^{\tau\iota}\eta^{\kappa\mu}\eta^{\nu\lambda})-P_{12}(p^\rho p'^\iota\eta^{\lambda\sigma}\eta^{\tau\mu}\eta^{\nu\kappa})\\
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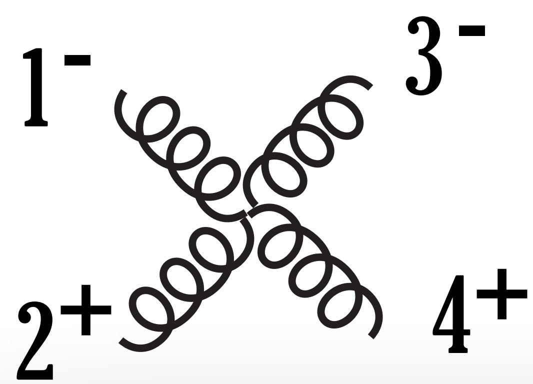
Amplitude methods

Extremely useful for simplifying calculations

Example I: graviton + graviton → graviton + graviton

à la Feynman !



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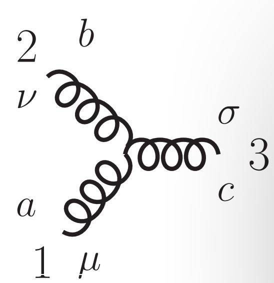
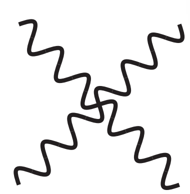


$$\mathcal{A}(1^-2^+3^-4^+) = \frac{\langle 13\rangle^4[24]^4}{stu}$$



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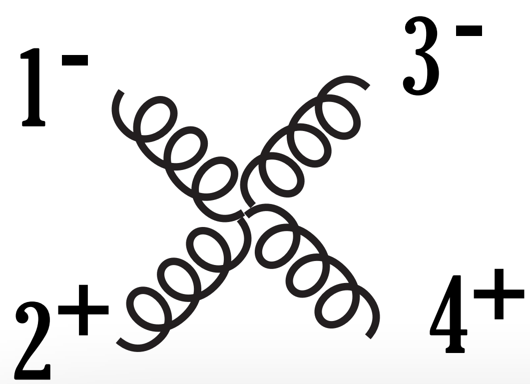
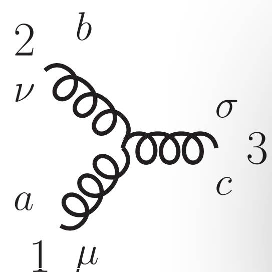
...end of Feynman realm?



$\pm$ ☛ h =$\pm$2



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$\sim$

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$\rho\eta^{\lambda\iota}\eta^{\kappa\sigma})$

$\eta^{\nu\kappa})$

$$-P_6(p^\rho p'^\iota\eta^{\lambda\kappa}\eta^{\mu\sigma}\eta^{\nu\tau})-P_{24}(p^\sigma p'^\rho\eta^{\tau\mu}\eta^{\nu\iota}\eta^{\kappa\lambda})-P_{12}(p^\sigma p'^\mu\eta^{\tau\rho}\eta^{\lambda\iota}\eta^{\kappa\nu})+2P_6(p\cdot p'\eta^{\nu\sigma}\eta^{\tau\rho}\eta^{\lambda\iota}\eta^{\kappa\mu})]$$

Trivial to see by **on-shell amplitude methods**

Example II: SM as an EFT

(assuming lepton & baryon number)

$$\mathcal{L}_{\rm eff} = \frac{\Lambda^4}{g_*^2}\mathcal{L}\left(\frac{D_\mu}{\Lambda}\,,\ \frac{g_H H}{\Lambda}\,,\ \frac{g_{f_{L,R}} f_{L,R}}{\Lambda^{3/2}}\,,\ \frac{g F_{\mu\nu}}{\Lambda^2}\right) \simeq \mathcal{L}_4 + \mathcal{L}_6 + \cdots$$

SM

leading deviations from the SM

Λ —— $\mathcal{O}_i, \mathcal{O}_j, \ldots$

m_W —— $\mathcal{O}_i$

m_e ——

One-loop operator mixing important:

(tells us how BSM enter in observables)

$$\boxed{\gamma_{c_i} = \frac{dc_i}{d\log\mu} = \gamma_{c_i}(c_j)}$$

c_i = Wilson coefficient

Example II: **SM as an EFT**

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leading deviations from the SM

Λ —— $\mathcal{O}_i, \mathcal{O}_j, \ldots$

m_W ——

m_e ——

One-loop operator mixi... portant:

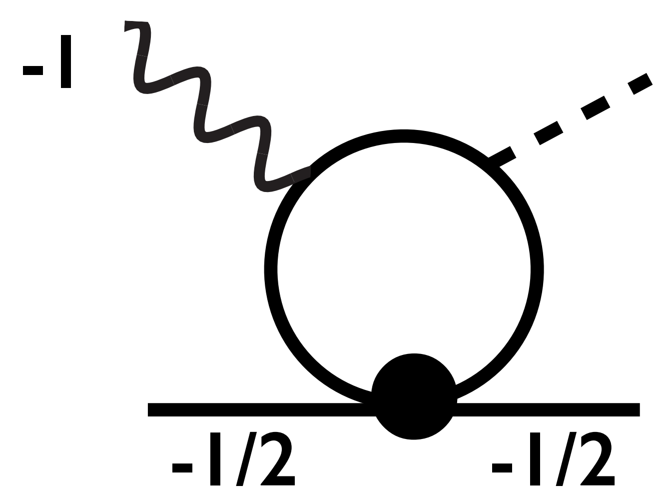
$$\frac{\ \ }{d\log\mu} = \gamma_{c_i}(c_j)$$

Many non-trivial zeros in γ_i found from explicit calculations !

c_i = Wilson coefficient

I. No 4-fermion $(\bar{\psi}\gamma^\mu\psi)^2$ corrections to dipoles

$$F^{\mu\nu}\bar{\psi}\sigma_{\mu\nu}\psi\, H$$

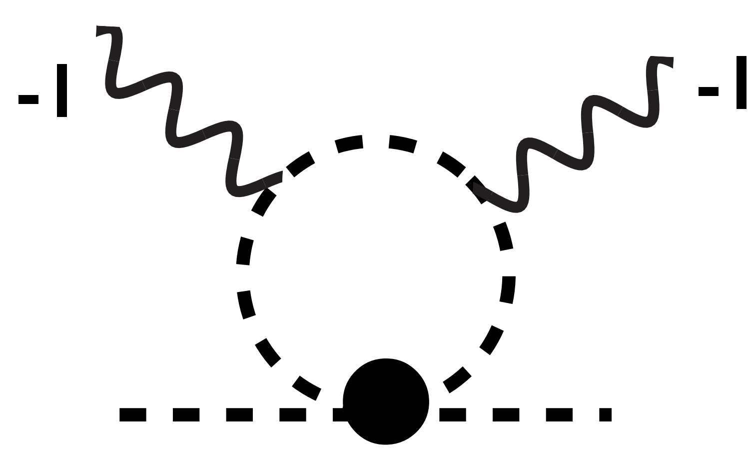


II. No p^2H^4 corrections to $H\gamma\gamma$

$$F_{\alpha\beta}F^{\alpha\beta}h^2$$

e.g.

$$(H^\dagger D_\mu H)^2$$



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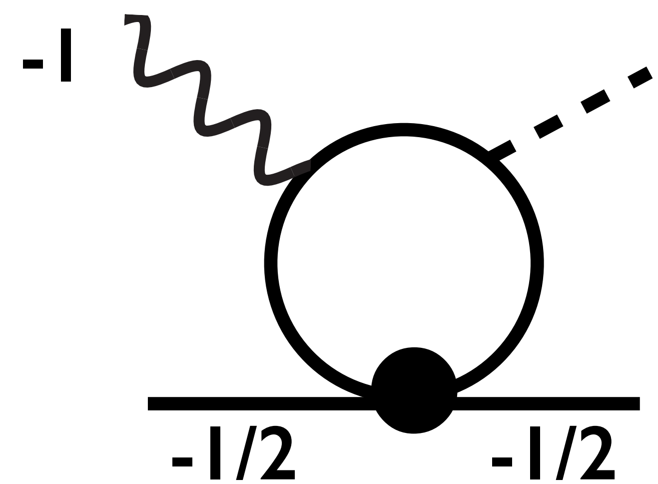

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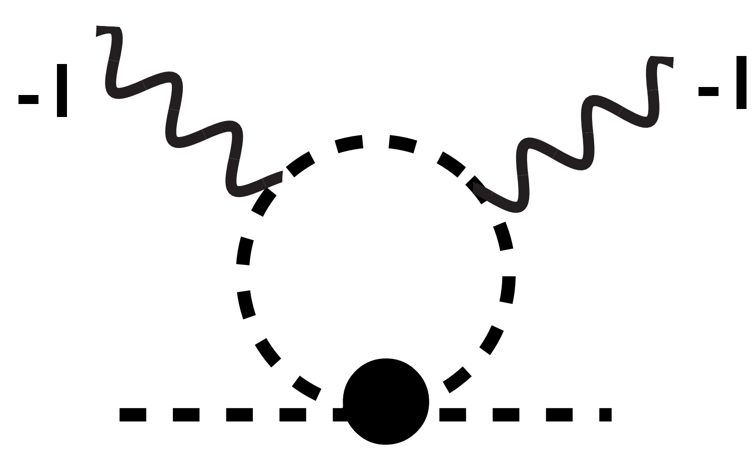
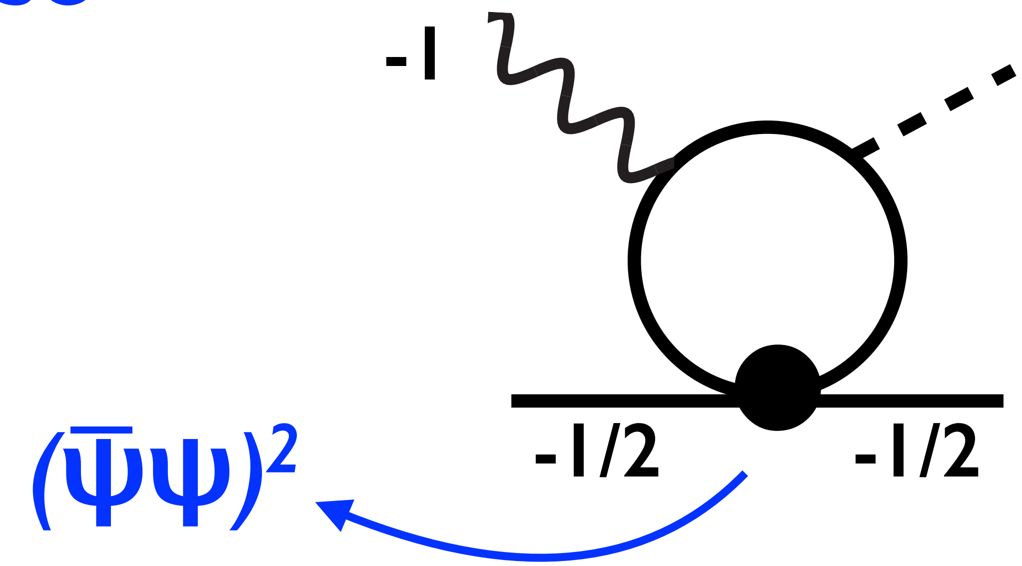
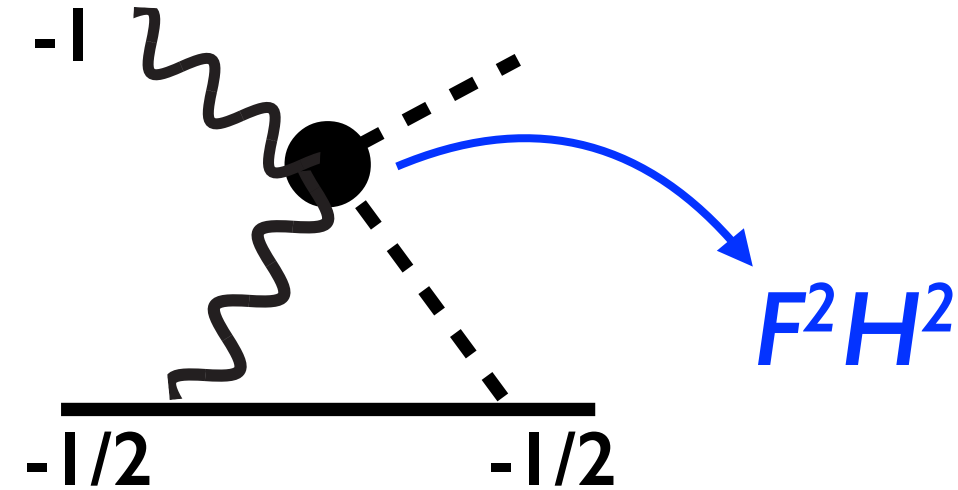


Zeros **not seen** from Feyman diagrams!

Also





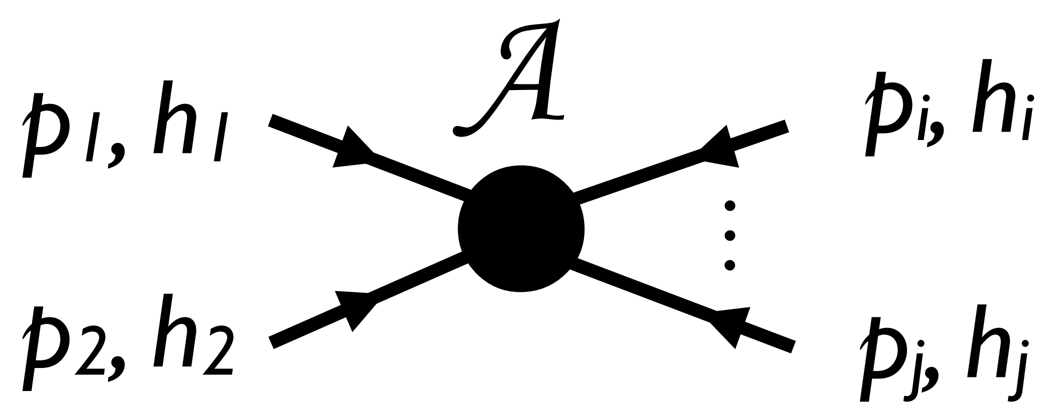
very **different** contributions
from Feynman diagrams
give the **same result** !

(up to color factors)

**On-shell amplitude methods
can explain straightforwardly these results!**

II. EFT (EFfective Theories) from amplitudes

convention: all particles incoming

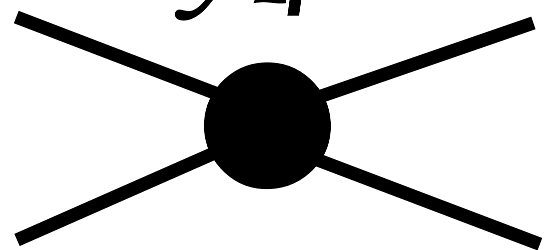


h_i **= helicity of the amplitude**

An important gain in simplicity:

the power of being on-shell !

$\mathcal{O}_i \longrightarrow \mathcal{A}_i$



Ghosts, Golstones,…
($p^2 \neq 0$)

One must eliminate redundancies

Many missed in original papers (Buchmuller,Wyler,…)!

only physical states ($p^2=0$)

definite helicity

Spinor-helicity formalism

Lorentz $\longrightarrow$ SL(2,C)

$$p^\mu \longrightarrow p_{\alpha\dot\alpha} = |p\rangle_\alpha [p|_{\dot\alpha}$$

(1/2,1/2) =(1/2,0)⊗(0,1/2)

“angle” spinor

“squared” spinor

Momenta & helicities as a function of spinors:

$$u_\mp(p) = P_\mp \begin{pmatrix} |p\rangle_\alpha \\ |p]^{\dot\alpha} \end{pmatrix}, \qquad \bar v_\mp(p) = \big(\langle p|^\alpha \;\, [p|_{\dot\alpha}\big) P_\mp\,, \qquad P_\mp = (1 \pm \gamma_5)/2$$

$$\epsilon^+_\mu = \frac{\langle q|\sigma_\mu|p]}{\sqrt{2}\langle qp\rangle}\,, \qquad \epsilon^-_\mu = -\frac{\langle p|\sigma_\mu|q]}{\sqrt{2}[qp]}$$

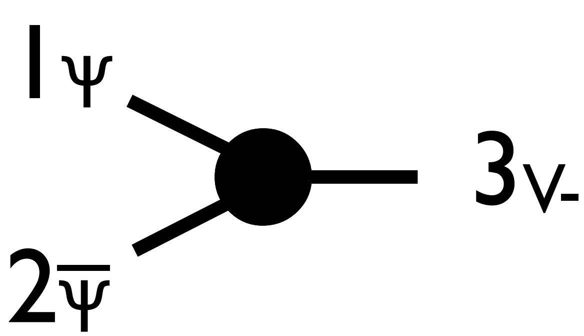
$$2\,p_i \cdot p_j = \langle ij\rangle[ji]$$

keep complex momenta!

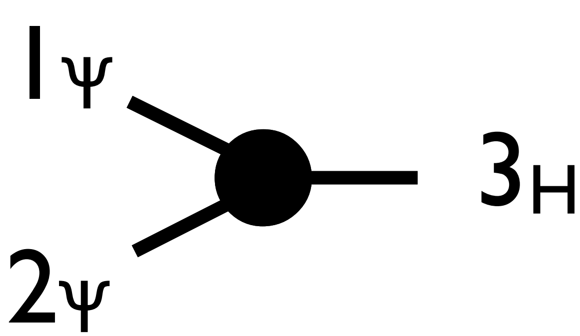
The SM as an EFT = *EFfective Theory*

Expansion: $\langle pq \rangle / \Lambda^2$

SM "Building-blocks":



$$= \frac{\langle 13 \rangle^2}{\langle 12 \rangle}$$



$$= \langle 12 \rangle$$

...

At O(E²/Λ²):

n = number of external states
h = helicity of the amplitude

$$\mathcal{A}_{F^3}(1_{V_-}, 2_{V_-}, 3_{V_-}) = \frac{C_{F^3}}{\Lambda^2}\langle 12\rangle\langle 23\rangle\langle 31\rangle$$

n=3
h=-3

$$\begin{aligned}
\mathcal{A}_{F^2\phi^2}(1_{V_-}, 2_{V_-}, 3_\phi, 4_\phi) &= \frac{C_{F^2\phi^2}}{\Lambda^2}\langle 12\rangle^2\,, \\
\mathcal{A}_{F\psi^2\phi}(1_{V_-}, 2_\psi, 3_\psi, 4_\phi) &= \frac{C_{F\psi^2\phi}}{\Lambda^2}\langle 12\rangle\langle 13\rangle\,, \\
\mathcal{A}_{\psi^4}(1_\psi, 2_\psi, 3_\psi, 4_\psi) &= \left(C_{\psi^4}\langle 12\rangle\langle 34\rangle + C'_{\psi^4}\langle 13\rangle\langle 24\rangle\right)\frac{1}{\Lambda^2}
\end{aligned}$$

n=4
h=-2

$$\begin{aligned}
\mathcal{A}_{\Box\phi^4}(1_\phi, 2_\phi, 3_\phi, 4_\phi) &= \left(C_{\Box\phi^4}\langle 12\rangle[12] + C'_{\Box\phi^4}\langle 13\rangle[13]\right)\frac{1}{\Lambda^2} \\
\mathcal{A}_{\psi\bar{\psi}\phi^2}(1_\psi, 2_{\bar{\psi}}, 3_\phi, 4_\phi) &= \frac{C_{\psi\bar{\psi}\phi^2}}{\Lambda^2}\langle 13\rangle[23]\,, \\
\mathcal{A}_{\psi^2\bar{\psi}^2}(1_\psi, 2_\psi, 3_{\bar{\psi}}, 4_{\bar{\psi}}) &= \frac{C_{\psi^2\bar{\psi}^2}}{\Lambda^2}\langle 12\rangle[34]\,.
\end{aligned}$$

n=4
h=0

$$\mathcal{A}_{\psi^2\phi^3}(1_\psi, 2_\psi, 3_\phi, 4_\phi, 5_\phi) = \frac{C_{\psi^2\phi^3}}{\Lambda^2}\langle 12\rangle$$

n=5
h=-1

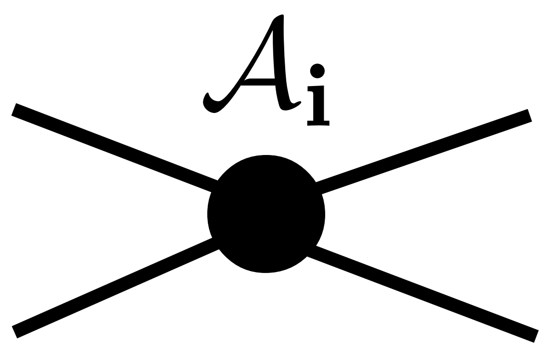
$$\mathcal{A}_{\phi^6}(1_\phi, 2_\phi, 3_\phi, 4_\phi, 5_\phi, 6_\phi) = \frac{C_{\phi^6}}{\Lambda^2}$$

n=6
h=0

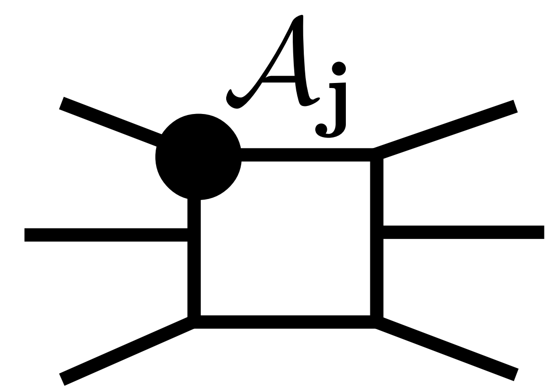
III. One-loop renormalization from amplitude methods

One-loop mixing (anomalous dimensions)

$\mathcal{A}_{\mathbf{i}}^{\mathbf{1-loop}}$ =

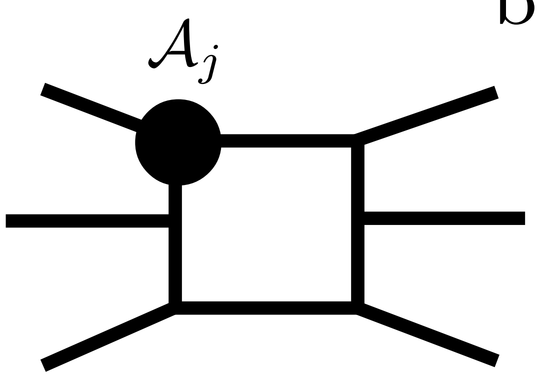


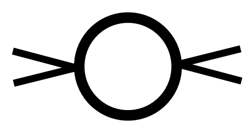
+

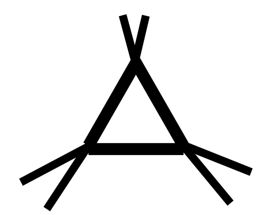


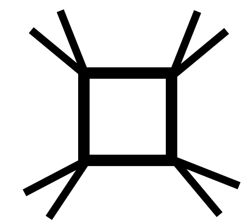
After one-loop reduction to Passarino-Veltman integrals

$$\mathcal{A}_{\mathbf{i}} = \sum_{\text{bubble}} \mathbf{c_2 I_2} + \sum_{\text{triangle}} \mathbf{c_3 I_3} + \sum_{\text{box}} \mathbf{c_4 I_4} + \mathbf{rational}$$





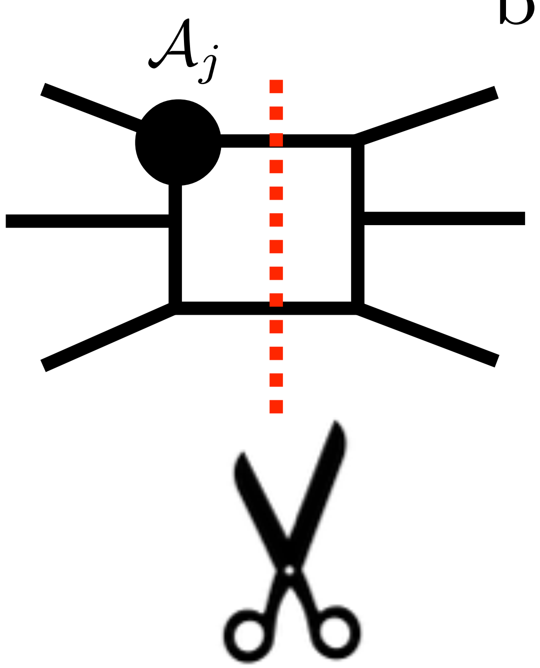


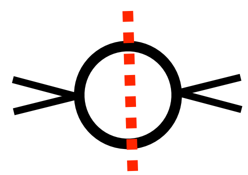


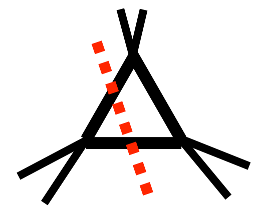
divergent ☛ c_2 = anomalous dimensions

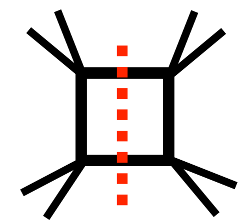
After one-loop reduction to Passarino-Veltman integrals

$$\mathcal{A}_{\mathbf{i}} = \sum_{\text{bubble}} \mathbf{c_2 I_2} + \sum_{\text{triangle}} \mathbf{c_3 I_3} + \sum_{\text{box}} \mathbf{c_4 I_4} + \mathbf{rational}$$





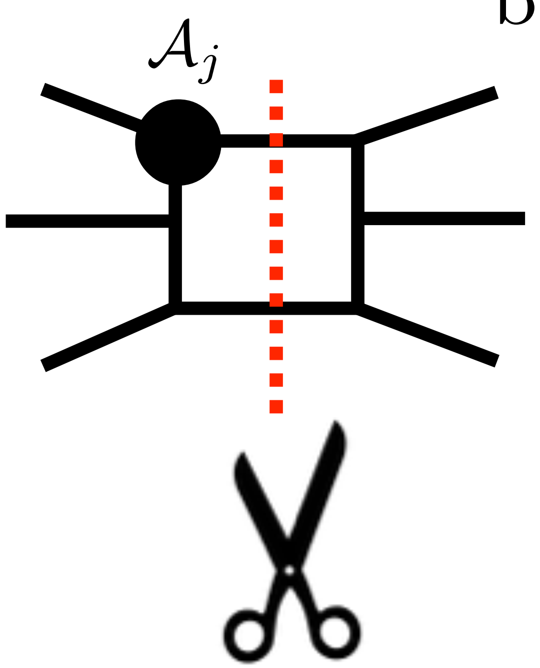




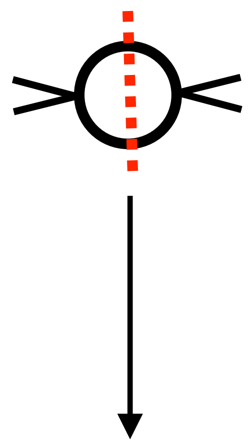
double cut

After one-loop reduction to Passarino-Veltman integrals

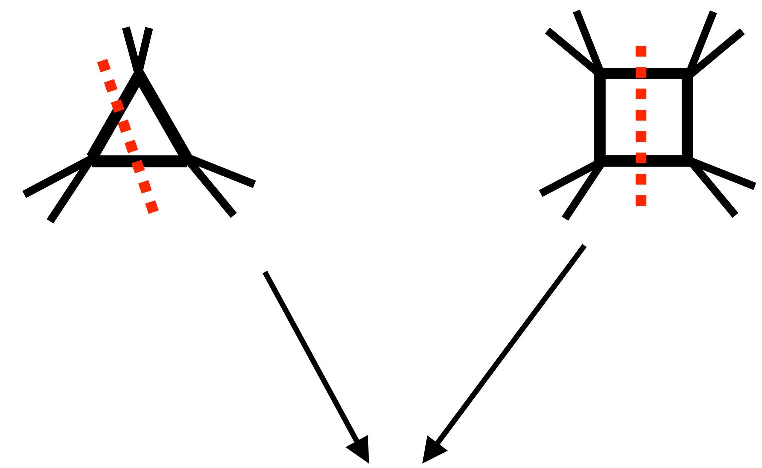
$$\mathcal{A}_\mathbf{i} = \sum_{\text{bubble}} \mathbf{c_2 I_2} + \sum_{\text{triangle}} \mathbf{c_3 I_3} + \sum_{\text{box}} \mathbf{c_4 I_4} + \textbf{rational}$$



double cut



c_2



Zero contribution

(after extracting IR-div)

P. Baratella, C. Fernandez, AP   2005.07129

After one-loop reduction to Passarino-Veltman integrals

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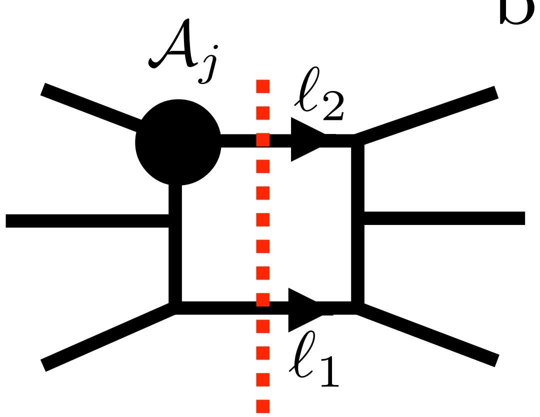
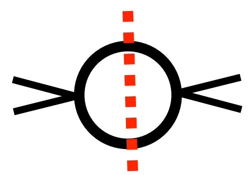
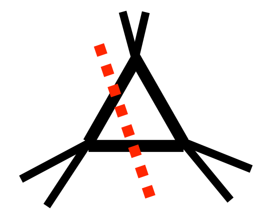
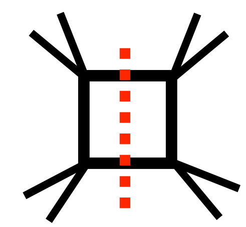


$\widehat{\mathcal{A}}_j$ $\mathcal{A}_{\mathrm{SM}}$



$\mathbf{c}_2$





Zero contribution
(after extracting IR-div)

P. Baratella, C. Fernandez, AP 2005.07129

$$\gamma_{ij}\,\mathcal{A}_i(1,2,\ldots,n) = -\frac{1}{4\pi^3}\frac{C_i}{C_j}\int d\mathrm{LIPS} \sum_{\substack{\text{ext. legs}\\ \text{distrib.}}} \sum_{\ell_1,\ell_2} \widehat{\mathcal{A}}_j(\ldots,-\ell_1,-\ell_2) \times \mathcal{A}_{\mathrm{SM}}(\ell_2,\ell_1,\ldots)$$

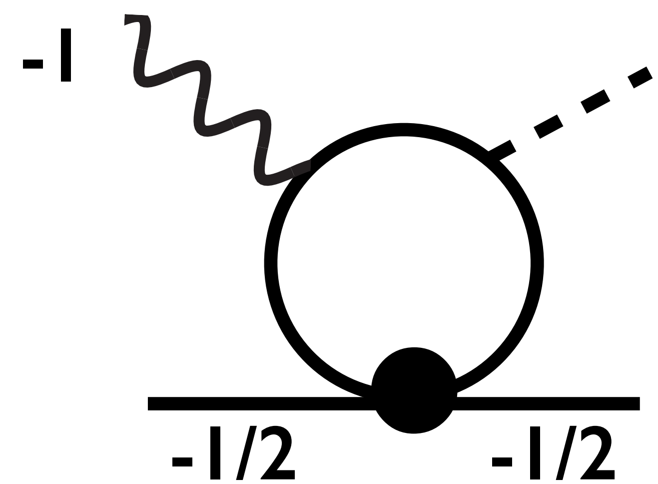
phase-space integration & sum over internal states

also explained by susy techniques: arXiv:1412.7151

I. No 4-fermion $(\bar{\psi}\gamma^{\mu}\psi)^2$ corrections to dipoles

$$F^{\mu\nu}\bar{\psi}\sigma_{\mu\nu}\psi\, H$$

$$\mathcal{A}(1_e, 2_{l_j}, 3_{W^a_-}, 4_{H_i^\dagger})$$



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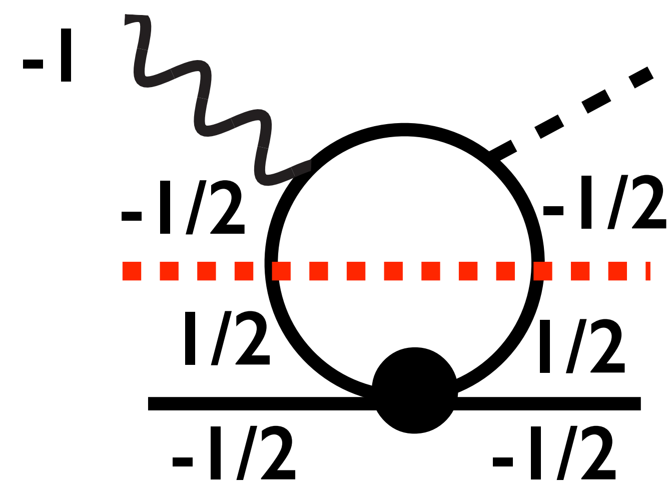
I. No 4-fermion $(\bar{\psi}\gamma^\mu\psi)^2$ corrections to dipoles

$$F^{\mu\nu}\bar{\psi}\sigma_{\mu\nu}\psi\, H$$

$\mathcal{A}(1_e, 2_{l_j}, 3_{W^a_-}, 4_{H^\dagger_i})$



$$\gamma\mathcal{A}_{WHle} = -\frac{1}{4\pi^3}\int d\text{LIPS}\,\mathcal{A}_{luqe}(1_e, 2_l, 3'_{\bar{e}}, 4'_{\bar{l}}) \times \mathcal{A}_{\text{SM}}(4'_e, 3'_l, 3_{W^a_-}, 4_{H^\dagger})$$

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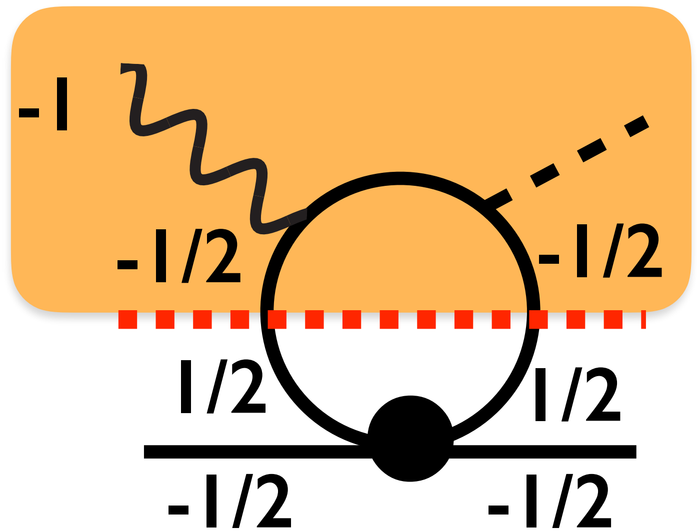


h_{total}=-2

Absent in the SM

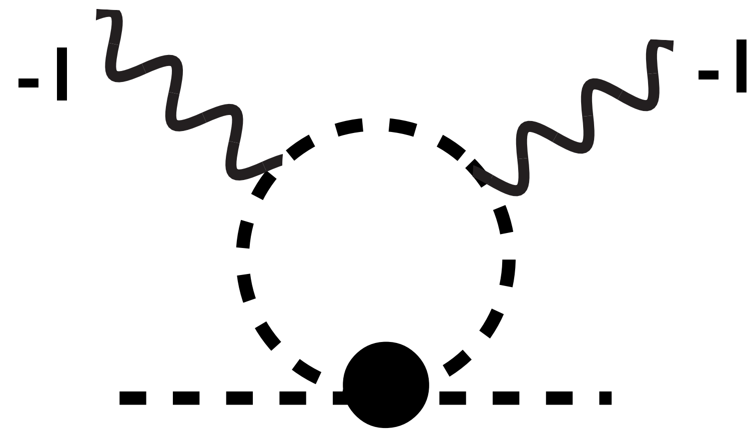
$$\gamma\mathcal{A}_{WHle} = -\frac{1}{4\pi^3}\int d\text{LIPS}\,\mathcal{A}_{luqe}(1_e, 2_l, 3'_{\bar{e}}, 4'_{\bar{l}}) \times \mathcal{A}_{\text{SM}}(4'_e, 3'_l, 3_{W_-^a}, 4_{H^\dagger})$$

II. No p^2H^4 corrections to $H\gamma\gamma$

e.g.

$$(H^\dagger D_\mu H)^2$$

$$F_{\alpha\beta}F^{\alpha\beta}h^2$$



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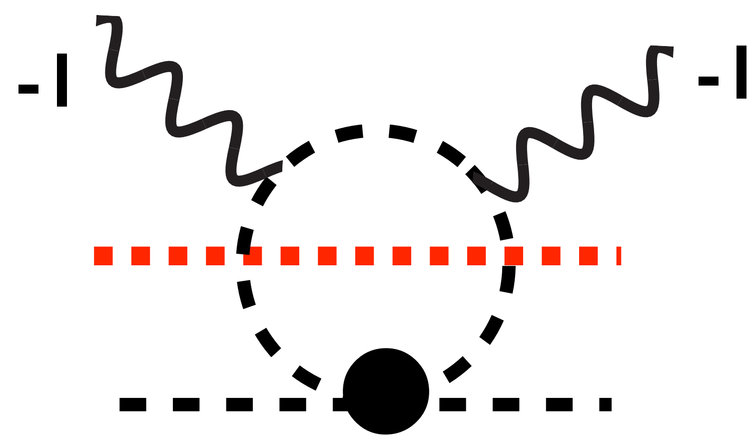

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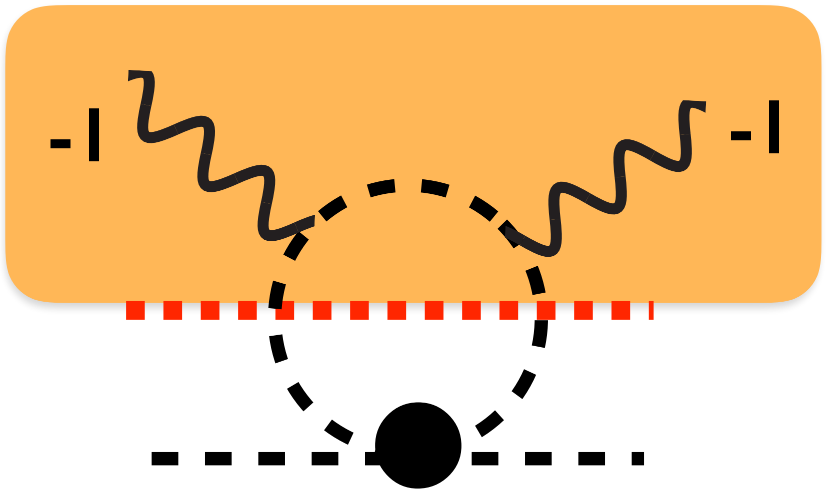
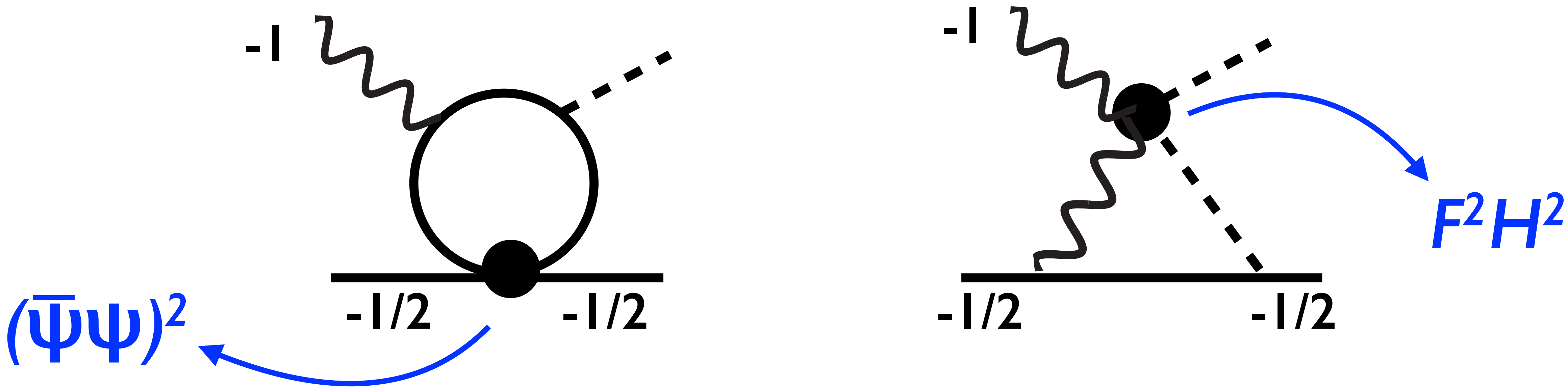


h_{total}=-2

Absent in the SM

But the **on-shell methods** also tell us about the non-zero result

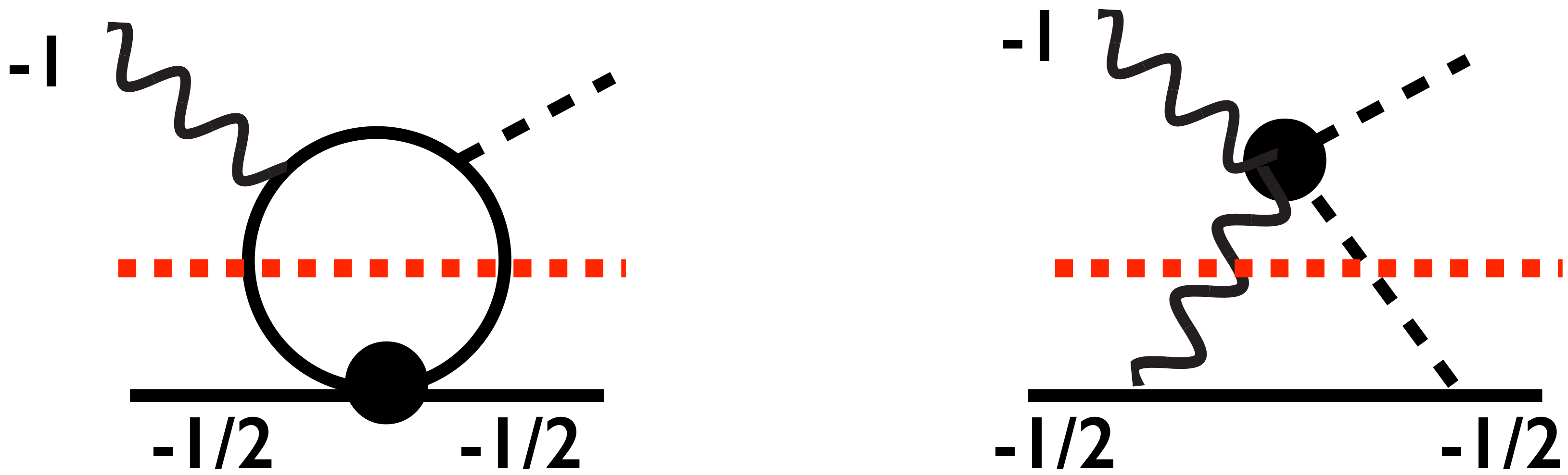
Contributions to **dipoles** from **Feynman** approach:



very **different** contributions

But the **on-shell methods** also tell us about the non-zero result

From **on-shell** approach: $\gamma \mathcal{A}_{\mathbf{i}} \sim \sum\!\!\!\!\!\!\int \mathcal{A}_{\mathbf{j}} \mathcal{A}_{\mathbf{SM}}$



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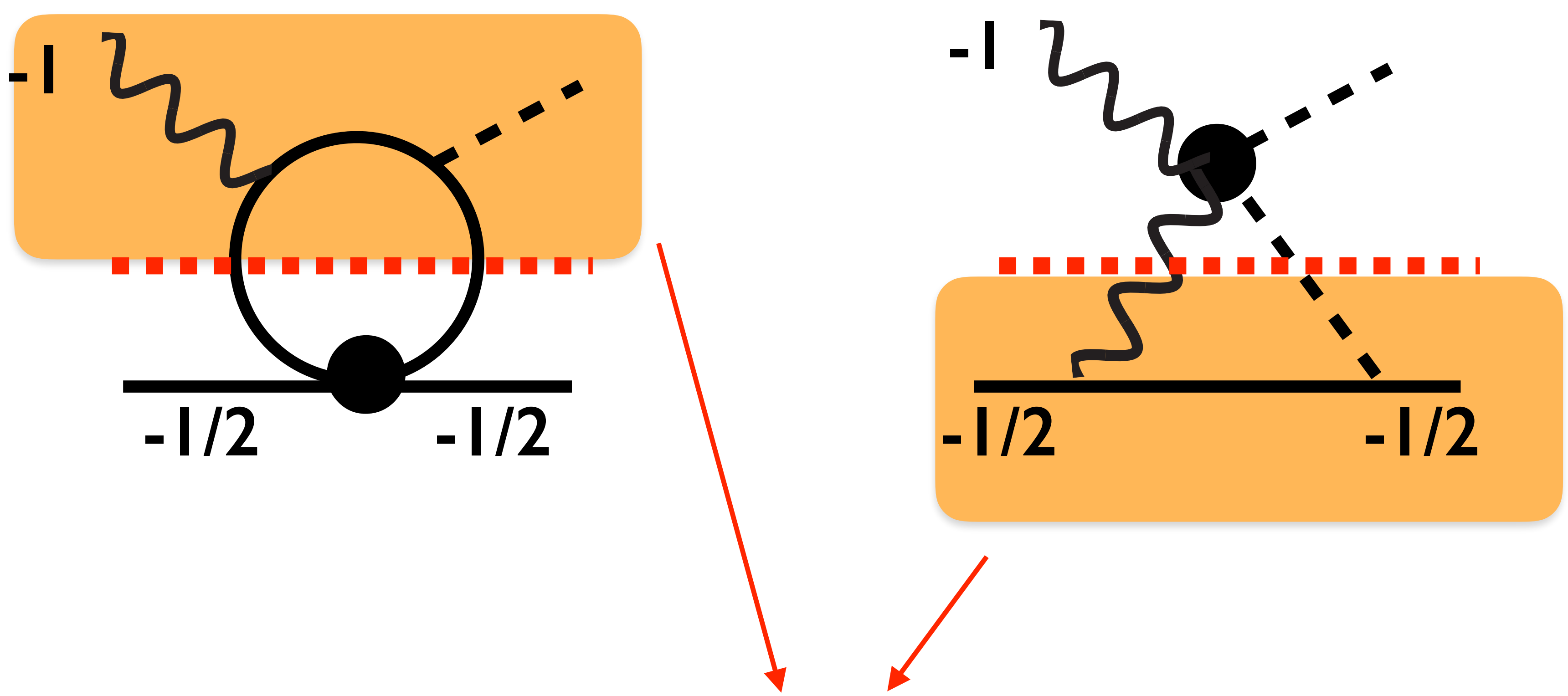
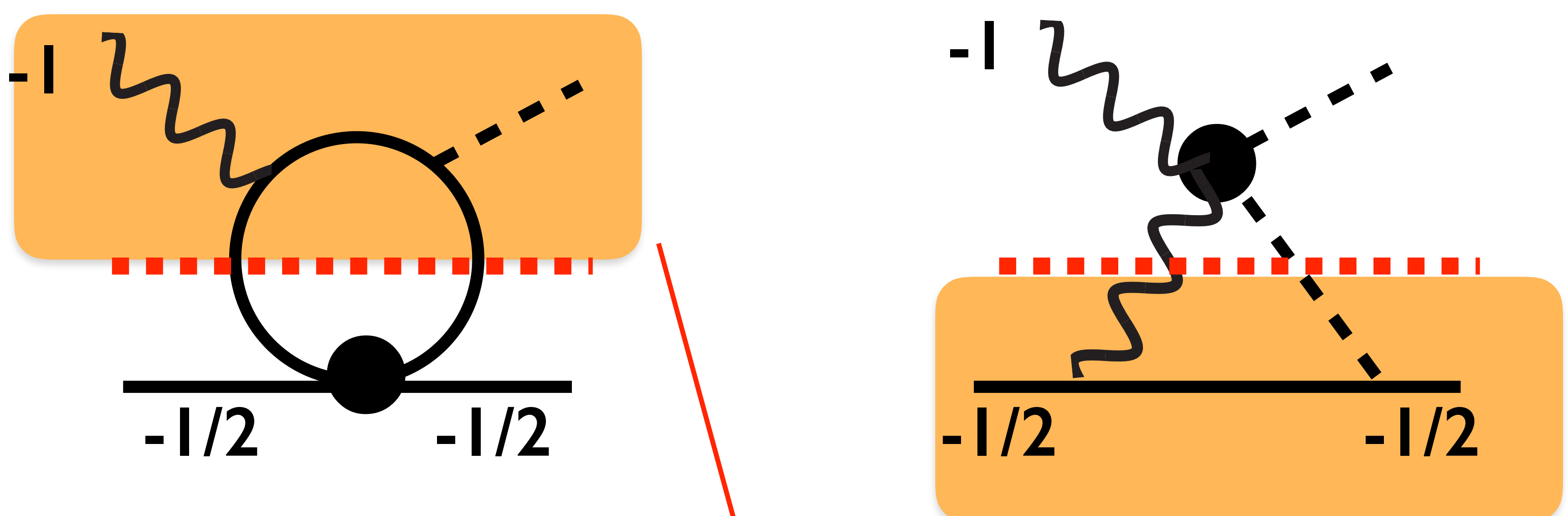


$$\mathcal{A}_{\mathrm{SM}}(1_{\bar{\psi}}, 2_{\bar{\psi}}, 3_{V_-}, 4_{H^\dagger})$$

from the same SM amplitude!

But the **on-shell methods** also tell us about the non-zero result

From **on-shell** approach: $\gamma \mathcal{A}_{\mathbf{i}} \sim \sum\!\!\!\!\!\!\!\int \mathcal{A}_{\mathbf{j}} \mathcal{A}_{\mathrm{SM}}$



$$\mathcal{A}_{\mathrm{SM}}(1_{\bar{\psi}}, 2_{\bar{\psi}}, 3_{V_-}, 4_{H^\dagger})$$

No calculation wasted in the on-shell method

from the same SM amplitude!

At O(E²/Λ²):

$$\mathcal{A}_{F^3}(1_{V_-},2_{V_-},3_{V_-}) = \frac{C_{F^3}}{\Lambda^2}\langle 12\rangle\langle 23\rangle\langle 31\rangle$$

$$\mathcal{A}_{SM}(1_{\bar\psi},2_{\bar\psi},3_{V_-},4_{H^\dagger})$$

$$\begin{aligned}
\mathcal{A}_{F^2\phi^2}(1_{V_-},2_{V_-},3_\phi,4_\phi) &= \frac{C_{F^2\phi^2}}{\Lambda^2}\langle 12\rangle^2\,,\\
\mathcal{A}_{F\psi^2\phi}(1_{V_-},2_\psi,3_\psi,4_\phi) &= \frac{C_{F\psi^2\phi}}{\Lambda^2}\langle 12\rangle\langle 13\rangle\,,\\
\mathcal{A}_{\psi^4}(1_\psi,2_\psi,3_\psi,4_\psi) &= \left(C_{\psi^4}\langle 12\rangle\langle 34\rangle + C'_{\psi^4}\langle 13\rangle\langle 24\rangle\right)\frac{1}{\Lambda^2}
\end{aligned}$$

n=4
h=-2

$$\begin{aligned}
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\mathcal{A}_{\psi\bar\psi\phi^2}(1_\psi,2_{\bar\psi},3_\phi,4_\phi) &= \frac{C_{\psi\bar\psi\phi^2}}{\Lambda^2}\langle 13\rangle[23]\,,\\
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\end{aligned}$$

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\end{aligned}$$

$$\mathcal{A}_{SM}(1_{\bar\psi}, 2_\psi, 3_H, 4_{H^\dagger})$$

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\end{aligned}$$

n=4
h=0

number of states

helicity	3	4	5	6
0		$\bar{\psi}^2\psi^2$ $p^2\lvert HH^\dagger\rvert^2$ $p\lvert H\rvert^2\bar{\psi}\psi$		$\lvert H\rvert^6$
1			$H\lvert H\rvert^2\psi\psi$	
2		$\lvert H\rvert^2F^2$ ψ^4 $HF\psi\psi$		
3	F^3			

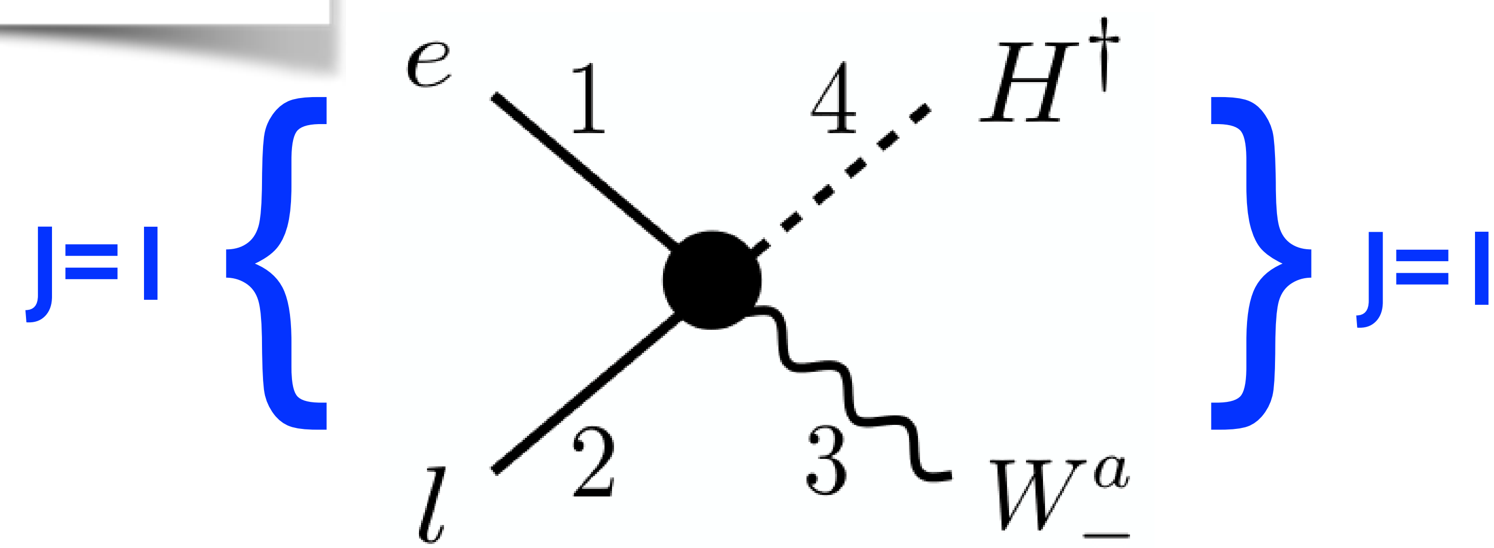
$\mathcal{A}_{\rm SM}(1_{\bar{\psi}}, 2_{\psi}, 3_H, 4_{H^\dagger})$

$\mathcal{A}_{\rm SM}(1_{\bar{\psi}}, 2_{\bar{\psi}}, 3_{V_-}, 4_{H^\dagger})$

But there is more to say by **angular-momentum decomposition (partial-waves)**

$$\mathcal{A}(1_{h_1}, 2_{h_2}, 3_{h_3}, 4_{h_4}) = e^{i\phi(h_{12}-h_{43})} \left(\frac{\sqrt{s}}{\Lambda}\right)^{w} \sum_J n_J d^J_{h_{12}h_{43}}(\theta) a^J$$

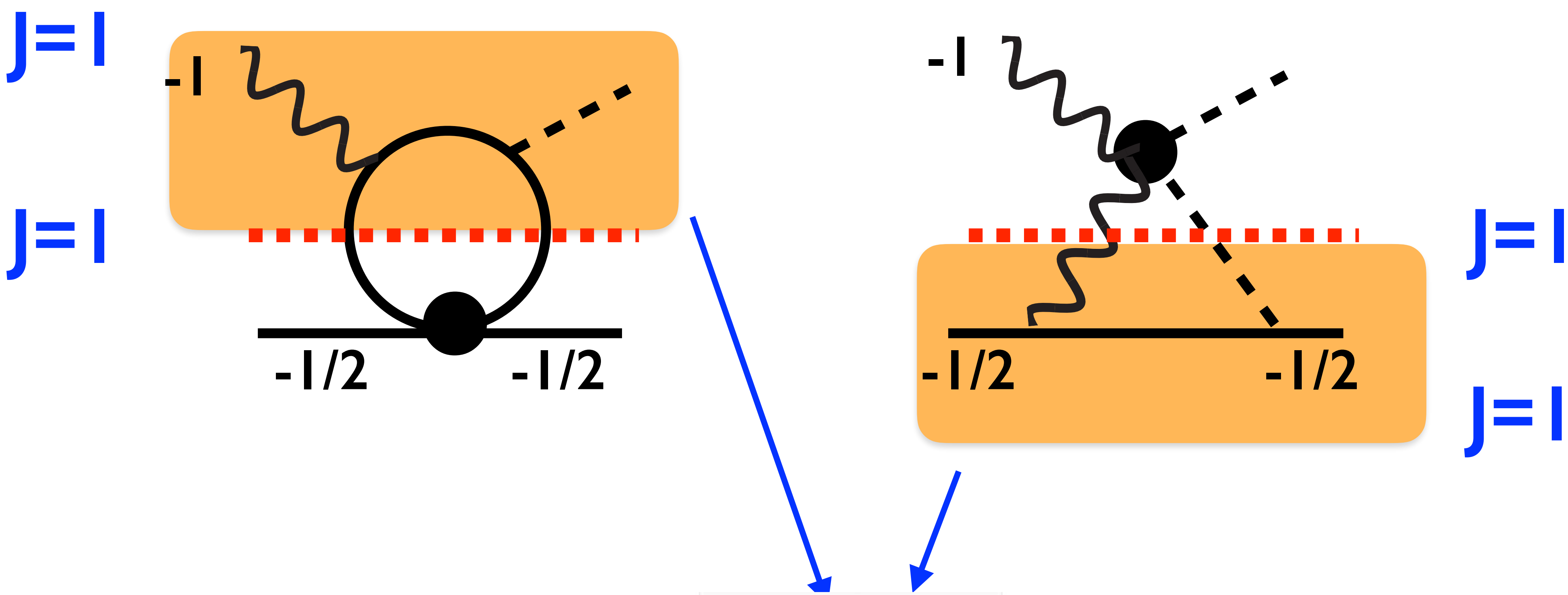
Example of dipoles:



$$\mathcal{A}(1_e, 2_l, 3_{W_-}, 4_{H^\dagger}) = 3e^{-i\phi} d^{J=1}_{01}(\theta)\, a^{J=1}$$

only one partial-wave!

But there is more to say by **angular-momentum decomposition (partial-waves)**



Not needed the full SM amplitude, only: $a_{\mathrm{SM}}^{J=1}$

☛ angular-momentum selection rules:

Amplitudes with J≠1 cannot contribute to dipoles

see also arXiv:2001.04481

Anomalous Dimensions as a product of partial waves

$$\gamma_i \sim a^J_{\mathrm{SM}} a^J_{\mathrm{BSM}}$$

$1/\Lambda^2$ amplitude

$$\mathcal{A}_{WHle}(1_e, 2_{l_j}, 3_{W_-^a}, 4_{H_i^\dagger}) = \frac{C_{WHle}}{\Lambda^2}\langle 31\rangle\langle 32\rangle (T^a)_{ij}\,,$$

$$\mathcal{A}_{eluq,0}(1_e, 2_{l_i}, 3_u, 4_{q_j}) = \frac{C_{eluq,0}}{\Lambda^2}\langle 12\rangle\langle 34\rangle\, \epsilon_{ij}\,,$$

$$\mathcal{A}_{eluq,1}(1_e, 2_{l_i}, 3_u, 4_{q_j}) = \frac{C_{eluq,1}}{\Lambda^2}\frac{1}{2}\left(\langle 23\rangle\langle 41\rangle + \langle 13\rangle\langle 42\rangle\right)\epsilon_{ij}\,,$$

$$\mathcal{A}_{W^2H^2}(1_{W_-^a}, 2_{H_i}, 3_{W_-^a}, 4_{H_i^\dagger}) = \frac{C_{W^2H^2}}{\Lambda^2}\langle 13\rangle^2\,.$$

n=4
h=-2

Anomalous dimension mixings:

$$\begin{pmatrix} \gamma_{WHle}\; C^{-1}_{WHle} a^1_{WHle} \\ \gamma_{eluq,1}\; C^{-1}_{eluq,1} a^1_{eluq,1} \\ \gamma_{W^2H^2}\; C^{-1}_{W^2H^2} a^1_{W^2H^2} \end{pmatrix} = -\frac{\widetilde{a}^{J=1}_{\rm SM}}{8\pi^2}\begin{pmatrix} \times & -N_c y_u & y_e \\ -y_u & \times & 0 \\ y_e & 0 & \times \end{pmatrix}\begin{pmatrix} a^1_{WHle} \\ a^1_{eluq,1} \\ a^1_{W^2H^2} \end{pmatrix}$$

Color factors & signs from fermions

After treating IR-div, *shocking* **simplicity** for **gravity**:

$\mathbf{a^J}$	$J=0$	$\boxed{J=2}$	$J=4$	
$\mathcal{A}_{\mathrm{GR}_{+-}}$	0	-6	$-\frac{25}{3}$	
$\mathcal{A}_{\mathrm{GR}_{-}}$	$\frac{5}{3}$	$\frac{1}{15}$	0	$\times\ \frac{r}{16\pi^2}$
$\widehat{\mathcal{A}}_{R^3}$	$\frac{1}{6}$	$-\frac{1}{30}$	0	$\times\ C_{R^3}$
$\mathcal{A}_{R^4}$	$\frac{7}{5}$	$\frac{4}{35}$	$\frac{1}{315}$	$\times\ C_{R^4}$

$$\gamma_{R^3}\widehat{\mathcal{A}}_{R^3} = -\frac{C_{R^3}}{8\pi^2}\left(\frac{s}{M_P^2}\right)^3 \sum_J n_J\, a^J_{\mathrm{GR}_-}\, a^J_{\mathrm{GR}_{+-}}|_{\mathrm{reg}}\, P_J\left(\frac{t-u}{s}\right) + \text{crossing}$$

$$\gamma_{R^4}\mathcal{A}_{R^4} = -C_{R^4}\frac{5}{8\pi^2}\left(\frac{s}{M_P^2}\right)^4 a^{J=2}_{R^3}\, a^{J=2}_{\mathrm{GR}_{+-}}|_{\mathrm{reg}}\, P_2\left(\frac{t-u}{s}\right) + \text{crossing}$$

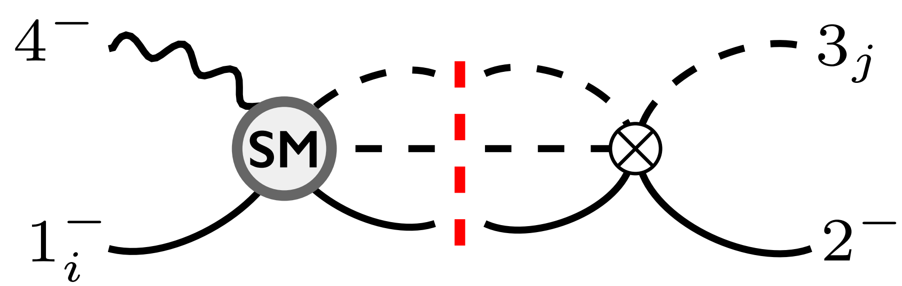
General formula for the anomalous dimensions for **all terms** of a non-linear sigma model:

$$\Delta\gamma_r = -\frac{C_{w_L r_L} C_{w_R r_R}}{16\pi^2}\left(\frac{N}{n_r}\delta_{r_L r}\,\delta_{r_R r} + 2\,\delta_{r_L r}\,\kappa^r_{w_R r_R} + 2\,\delta_{r_R r}\,\kappa^r_{w_L r_L} + 4\sum_{J=0}^{\min(\frac{w_L}{2},\frac{w_R}{2})} n_J n_r\,\kappa^J_{w_L r_L}\kappa^J_{w_R r_R}\kappa^r_{wJ}\right)$$

$$\kappa^J_{wr} = \sum_{k=0}^{r} \frac{(-1)^{w/2+J-k}(r+k)!\,[(w/2-k)!]^2}{[k!]^2(r-k)!\,(w/2+J+1-k)!\,(w/2-J-k)!}$$

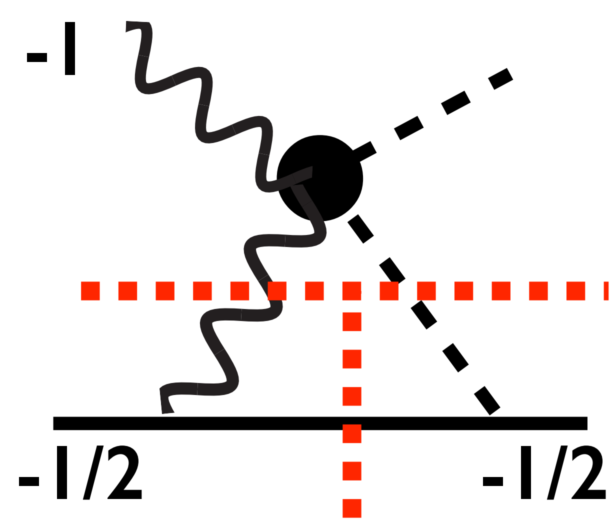
Bright future ahead

two-loop:



2005.06983
2005.12917

finite terms:



0806.4600

from 3-cut with massive internal particles

Conclusions

Amplitude methods seems quite **suited for** dealing with EFTs for **BSM**

- Allows to construct **BSM without Lagrangians:**
- Calculation of anomalous dimensions:

Simpler with easy recycling

☛ many "emergent" **selection rules**
☛ many relations **between anomalous dimensions**

where Feynman approach is quite obscure

A lot to do! Stay Tuned!