EFT of Dark Matter Direct Detection With Collective Excitations

Zhengkang "Kevin" Zhang (Caltech)

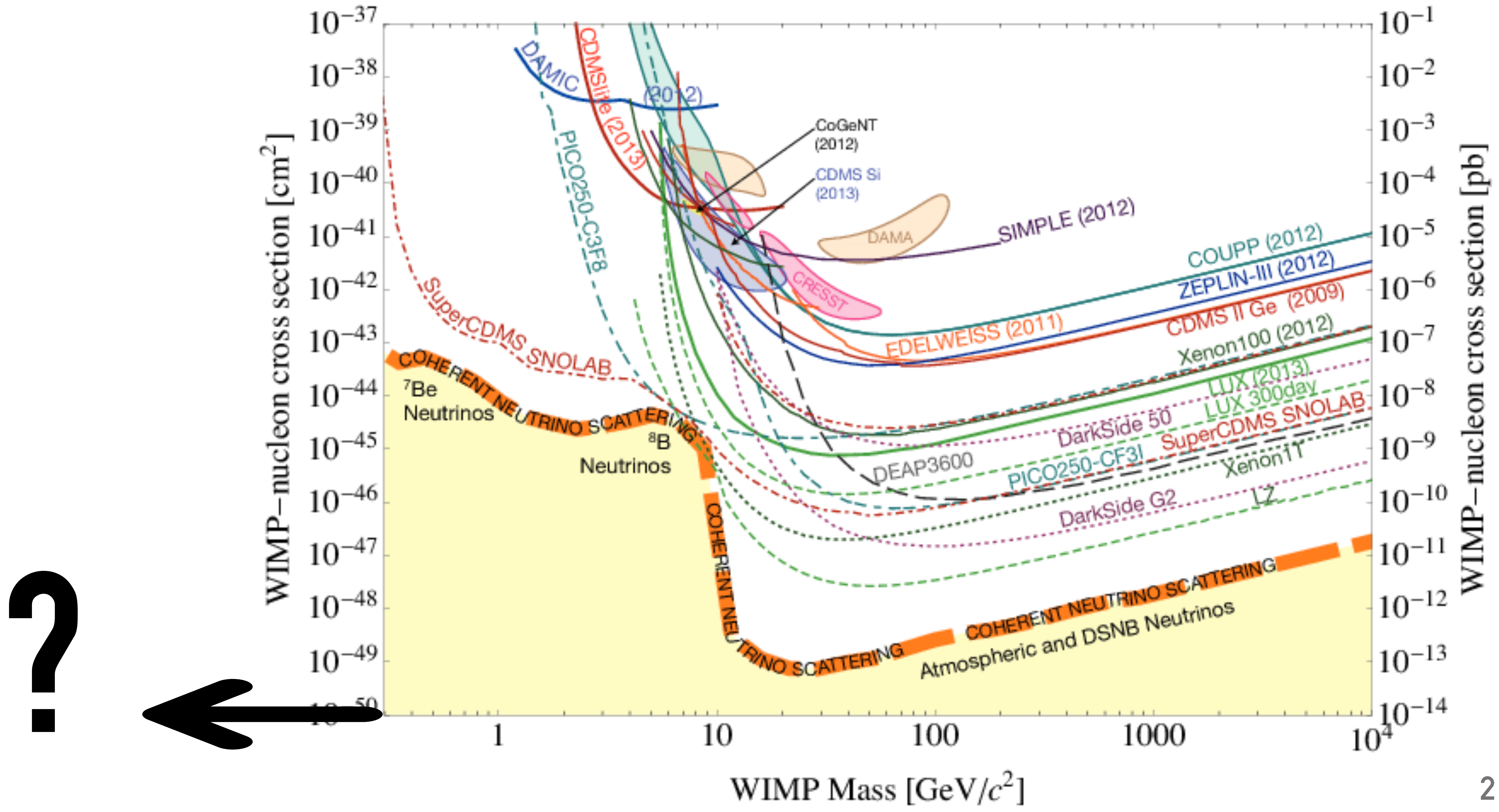
BURKE INSTITUTE

Caltech

Based on 2009.13534 (w/ Tanner Trickle, Kathryn Zurek)

Toward lighter DM in direct detection

- Conventional WIMP searches.
  - Nuclear recoils.
  - Lose sensitivity below DM mass ~GeV.



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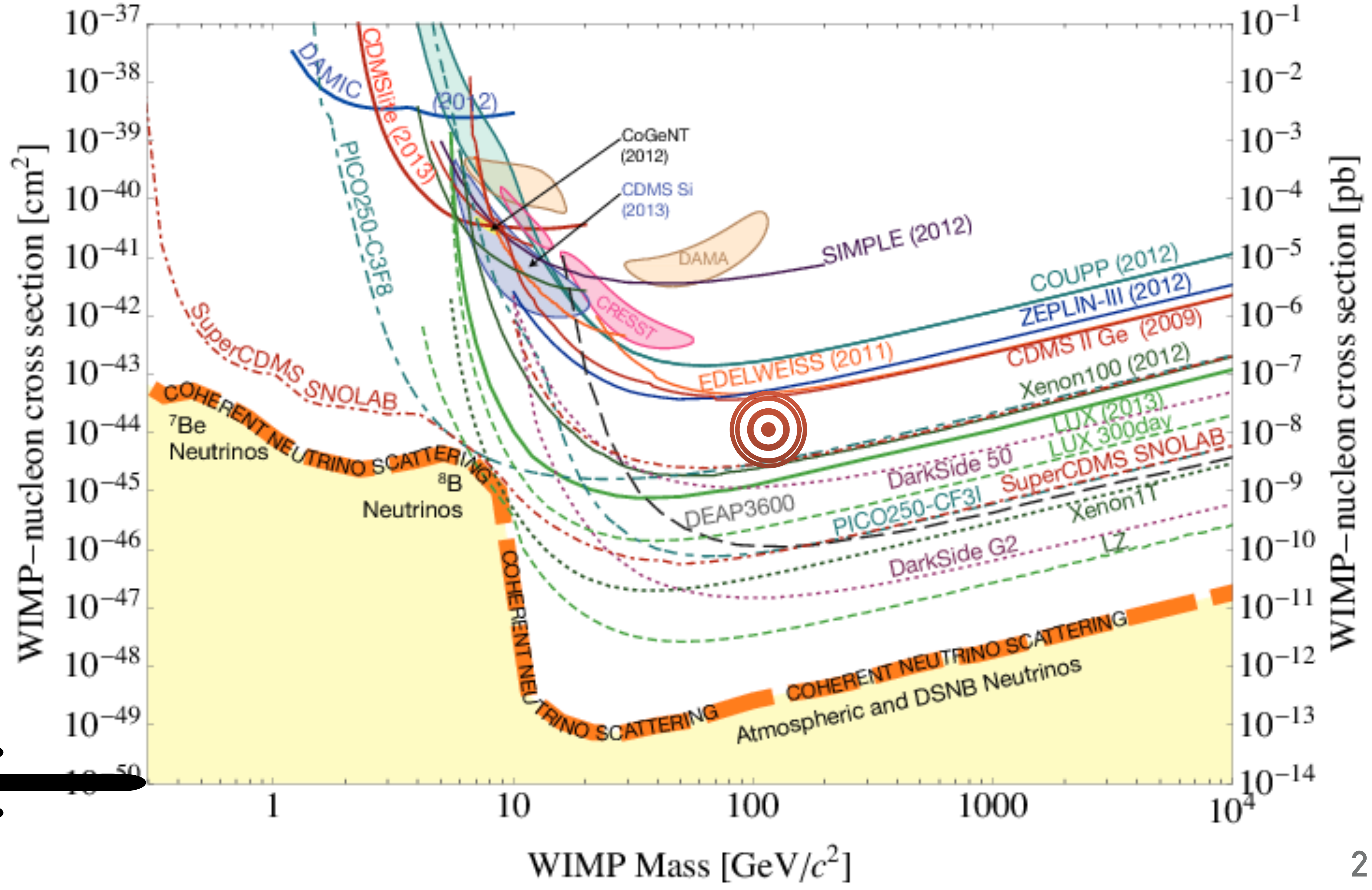

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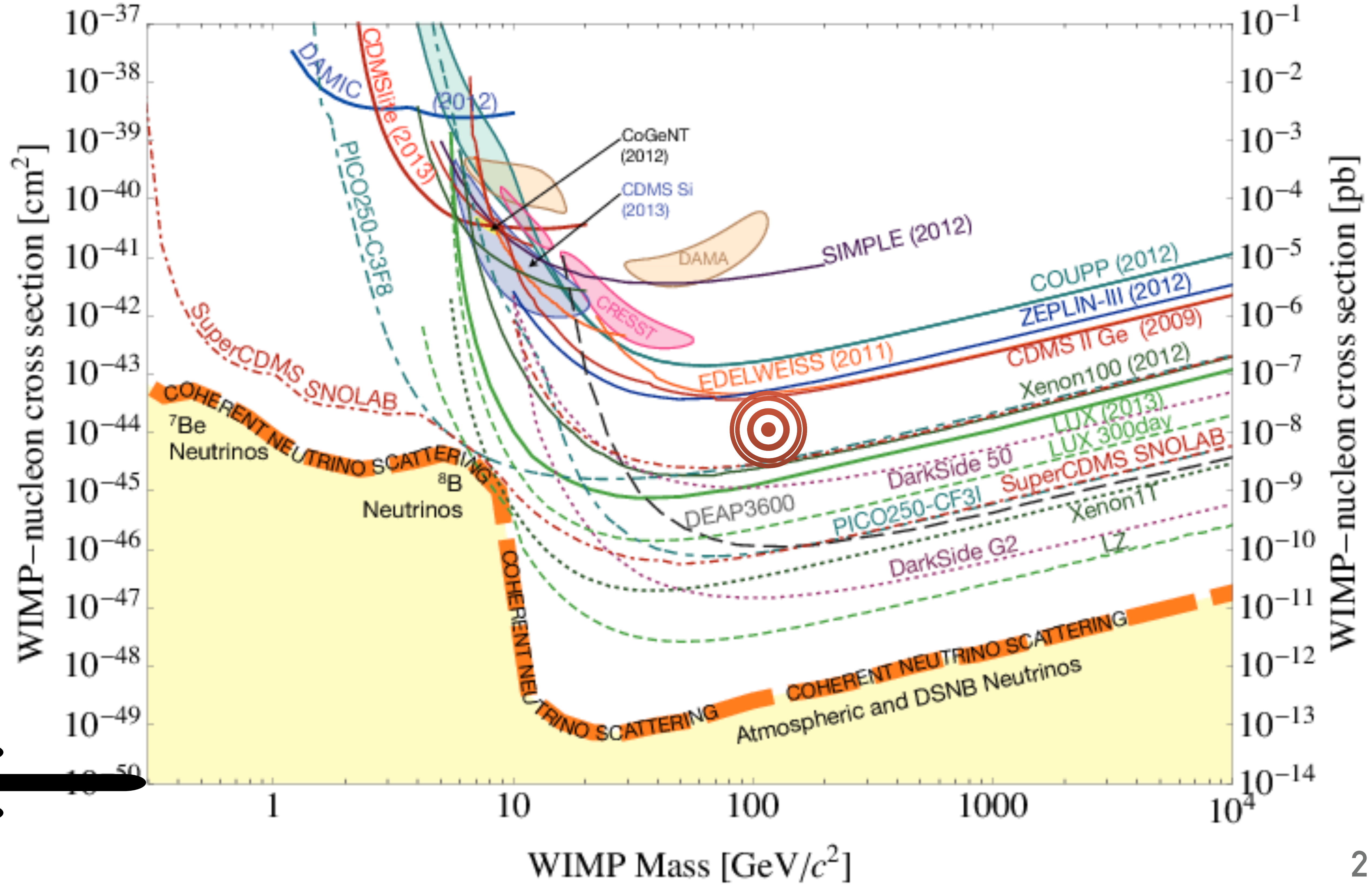
DM

Xe

?



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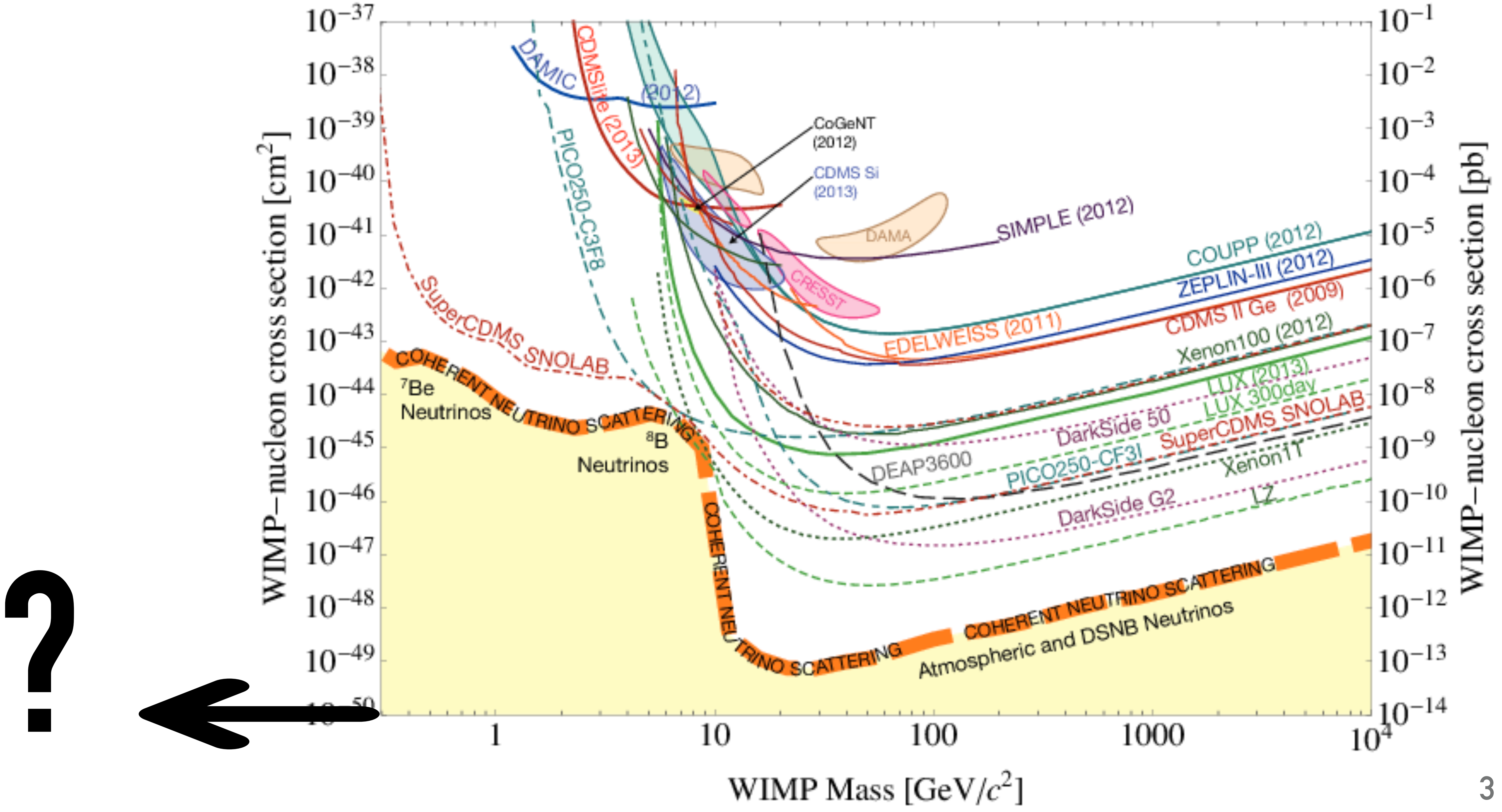

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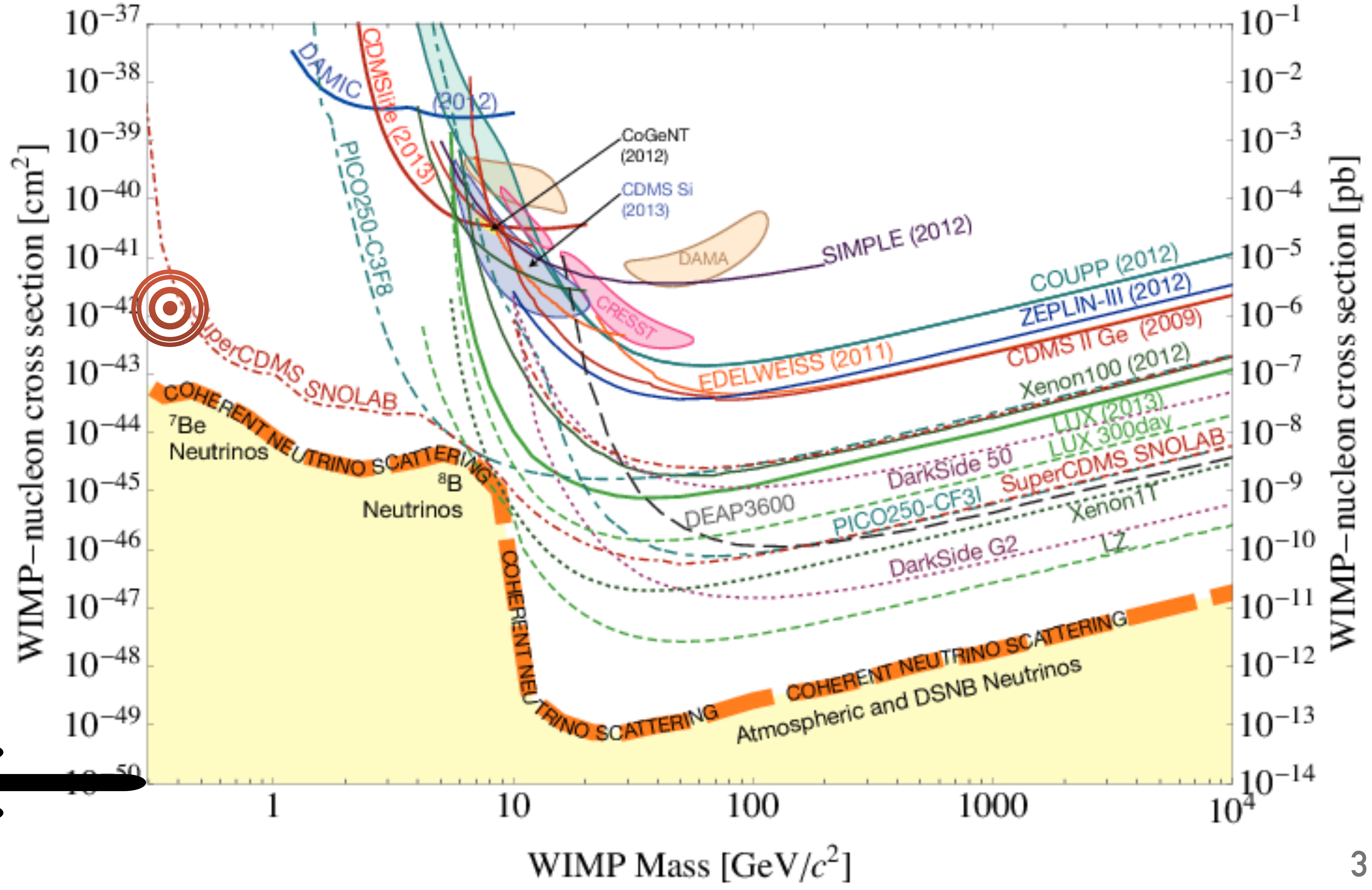

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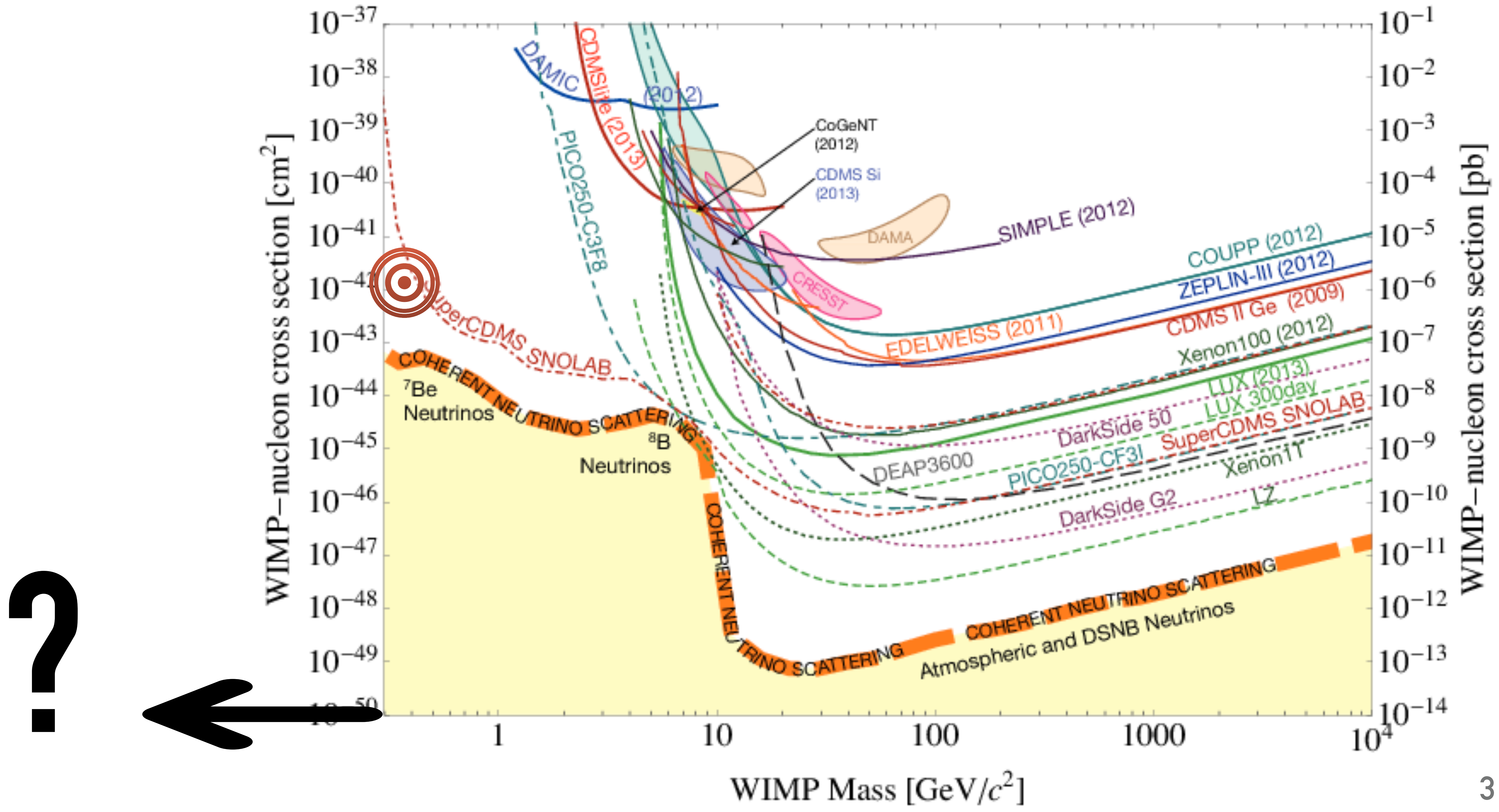

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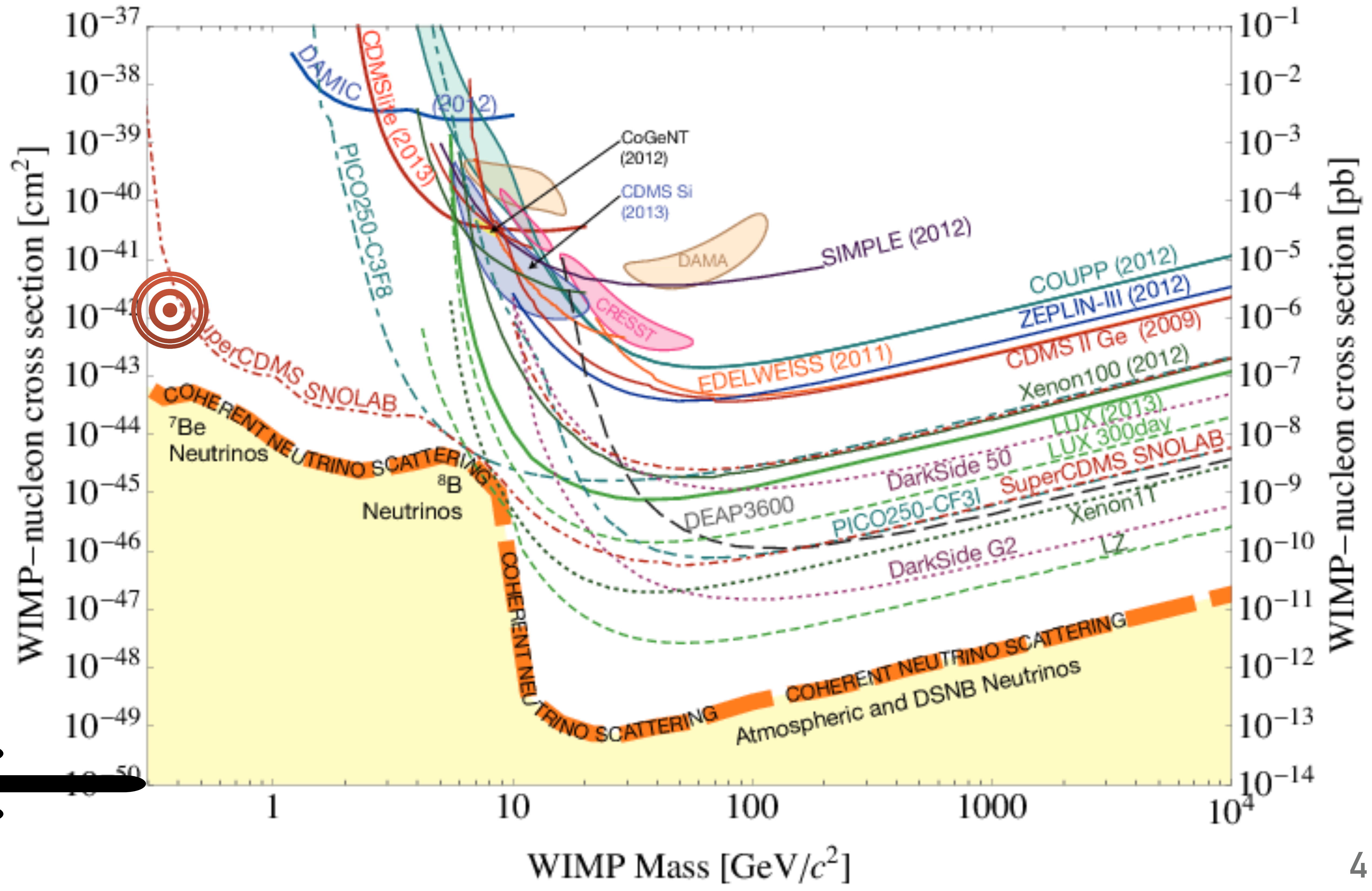

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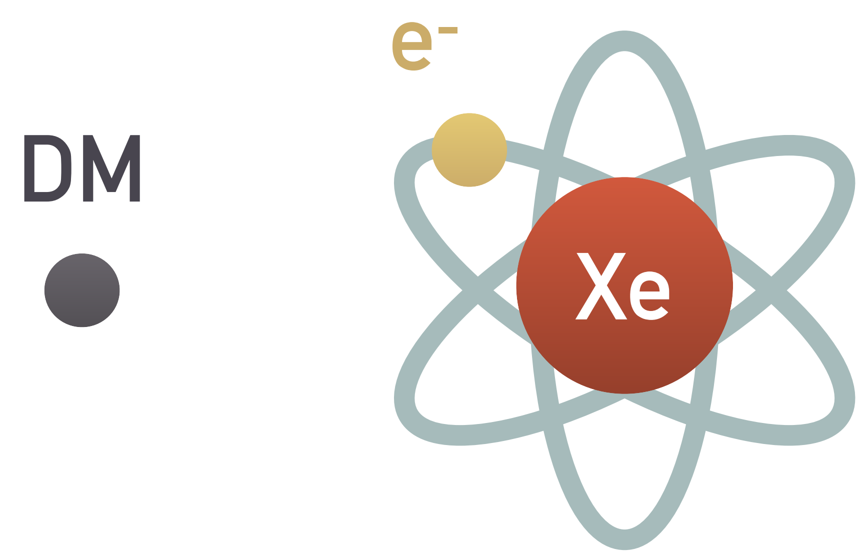

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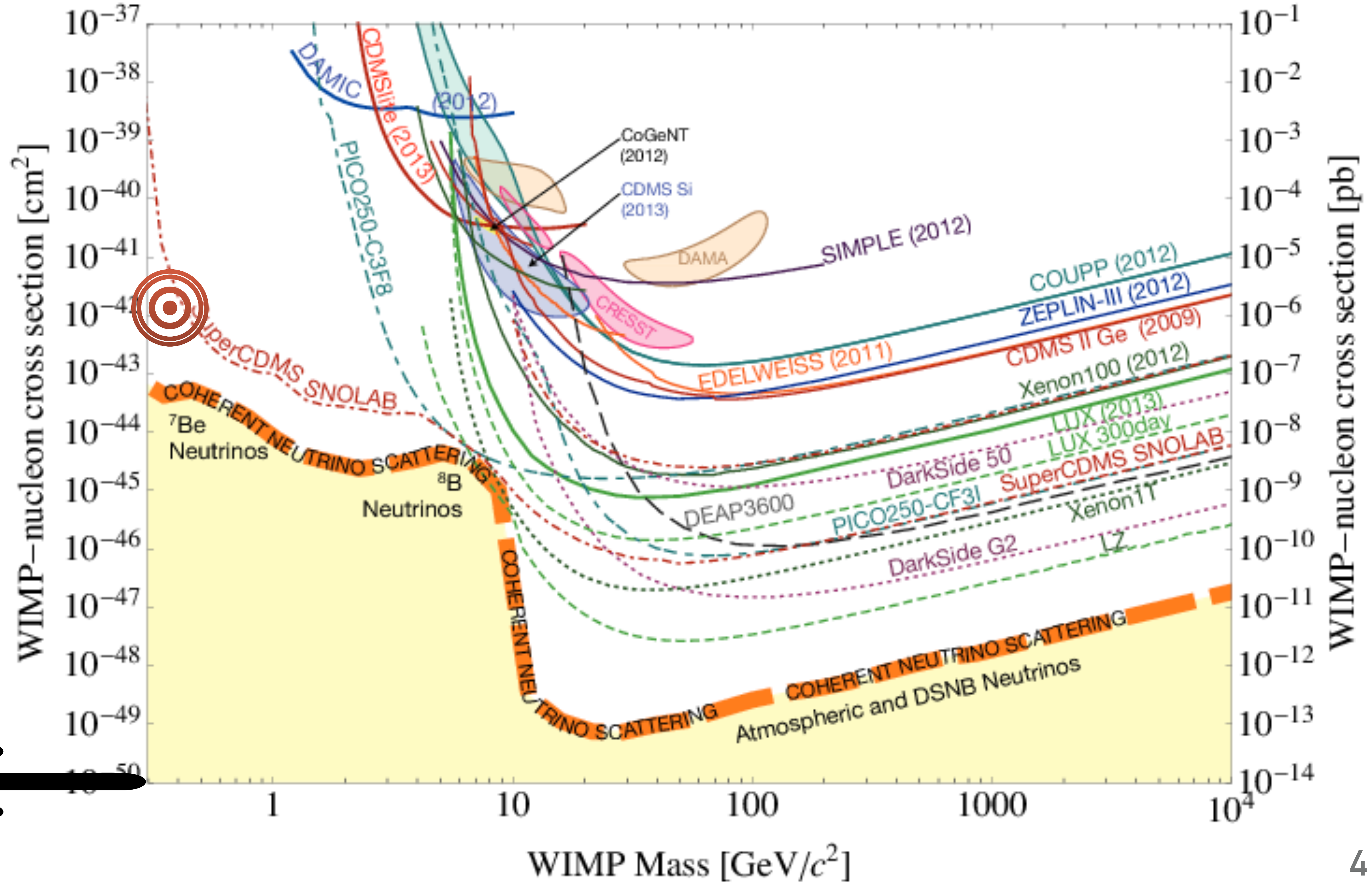

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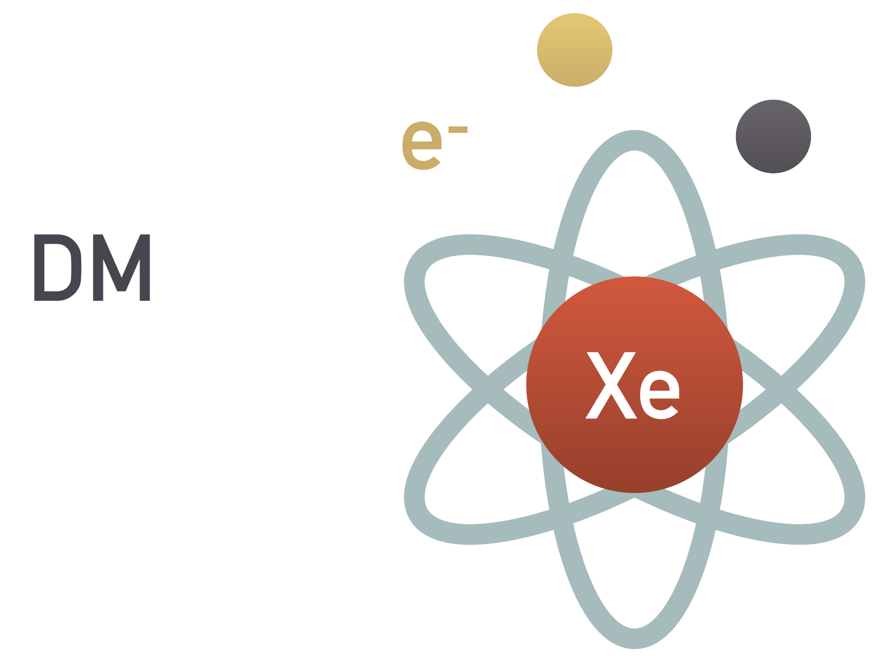

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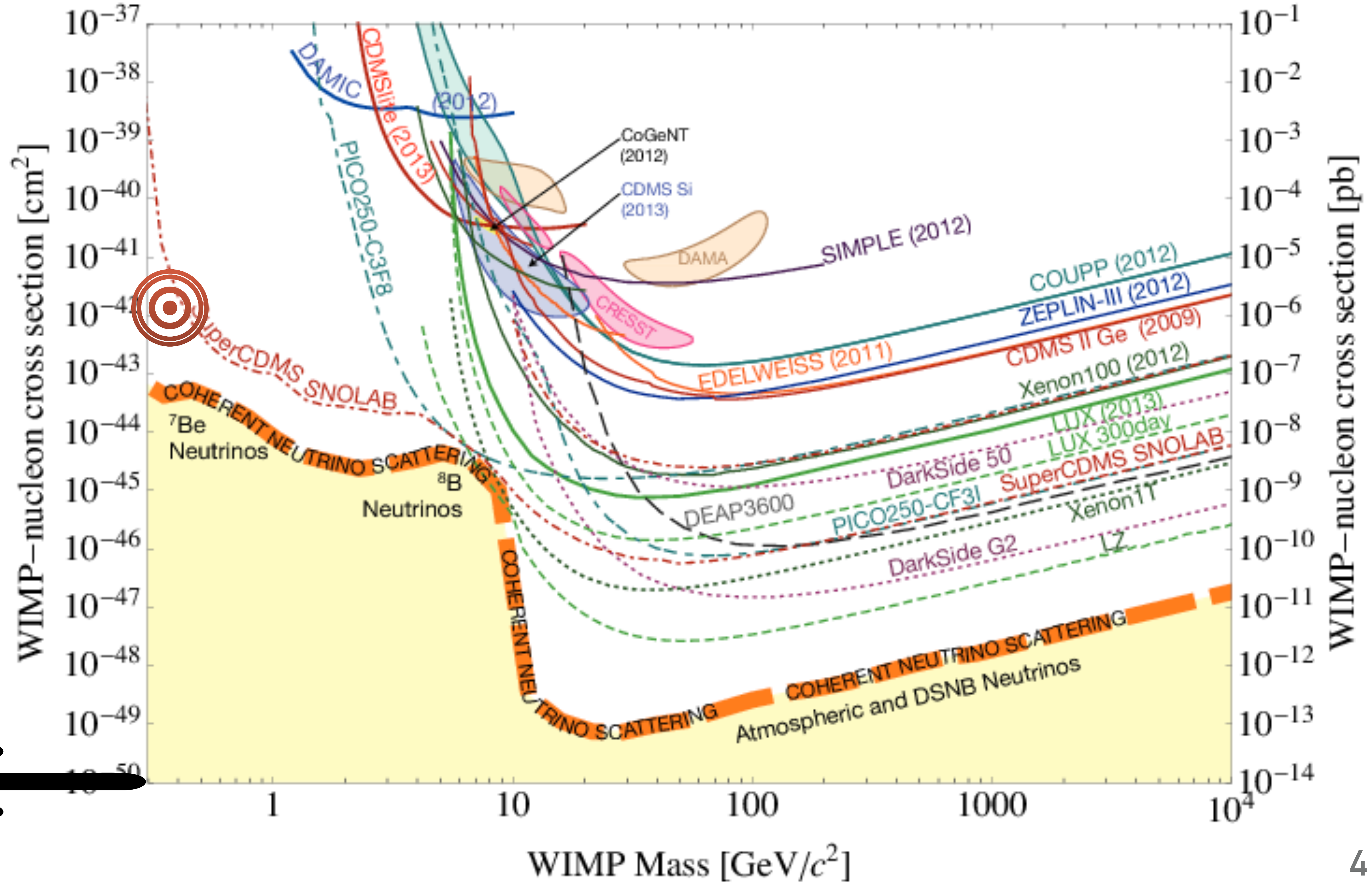

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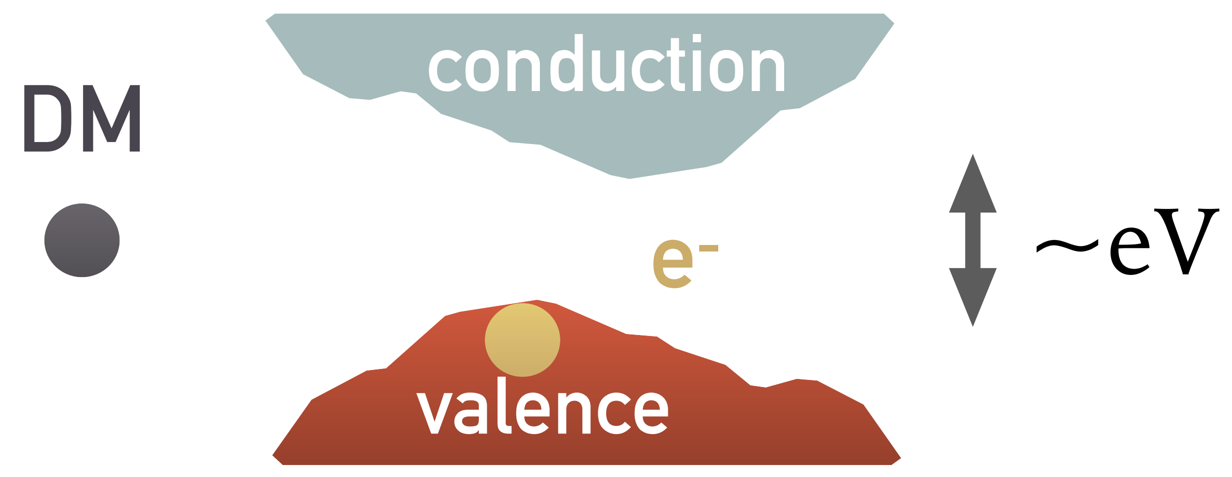

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Other similar proposals
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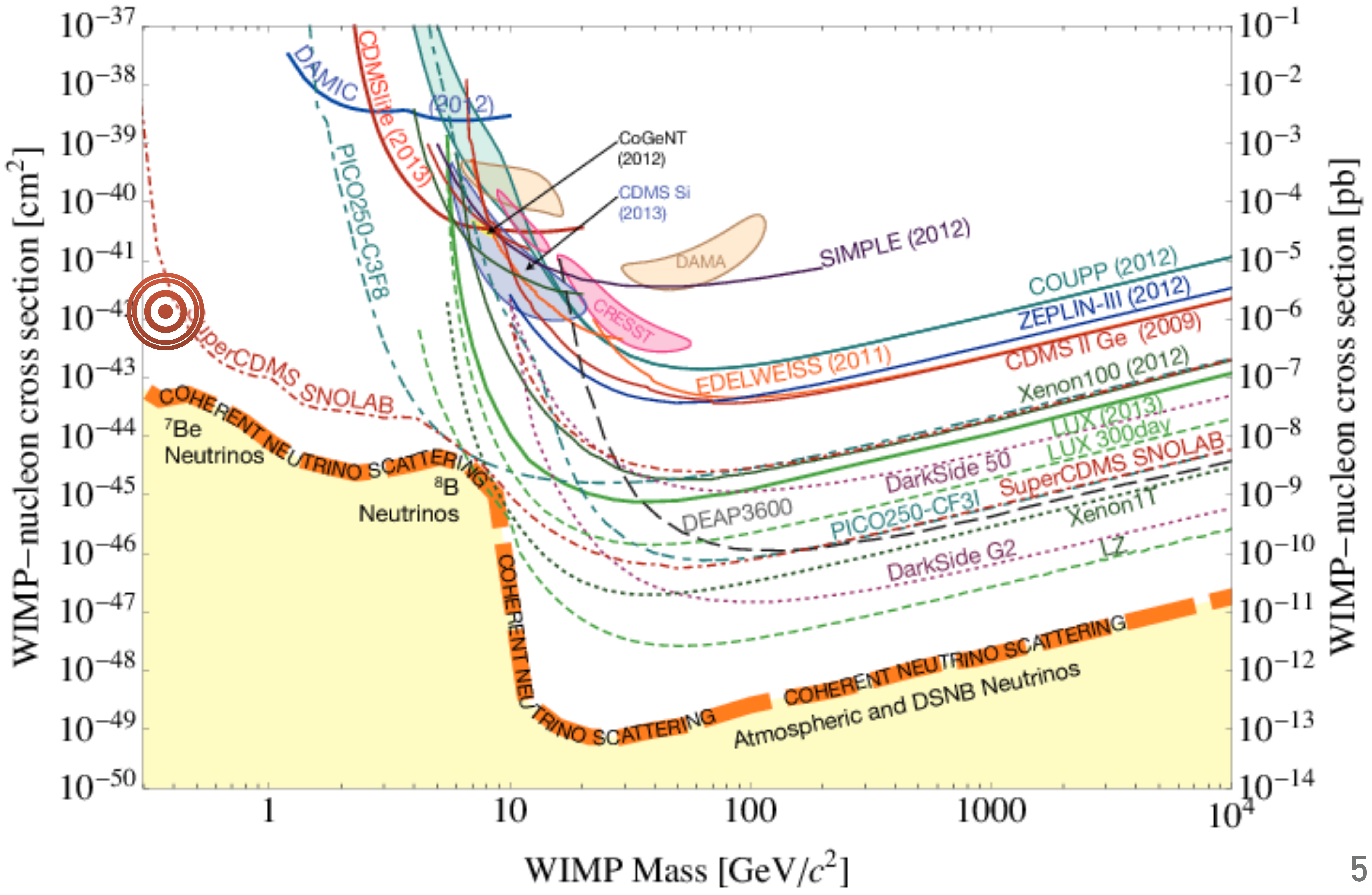

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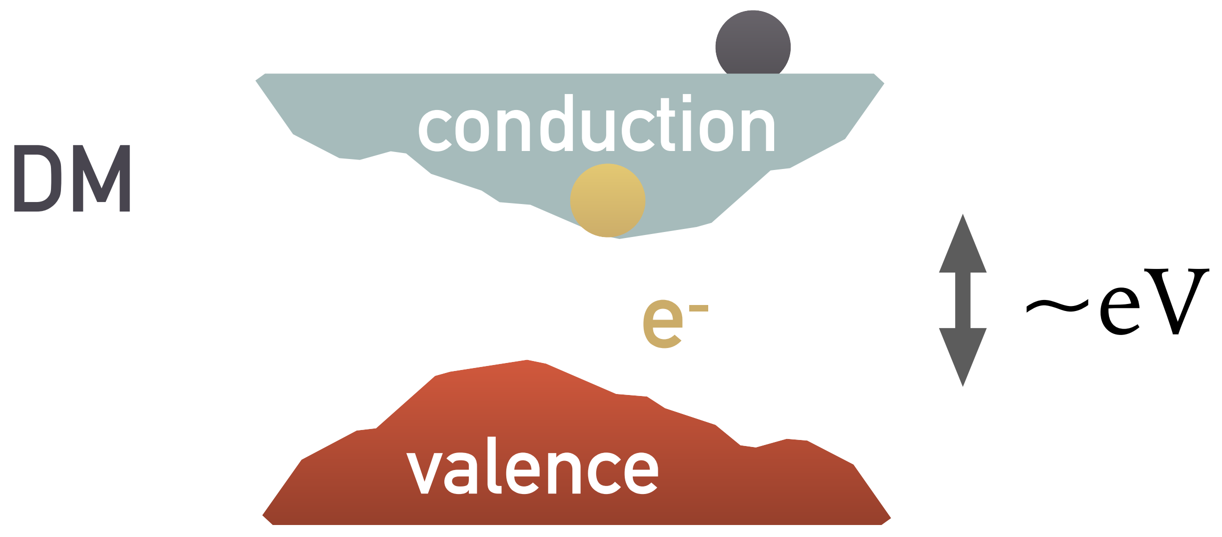

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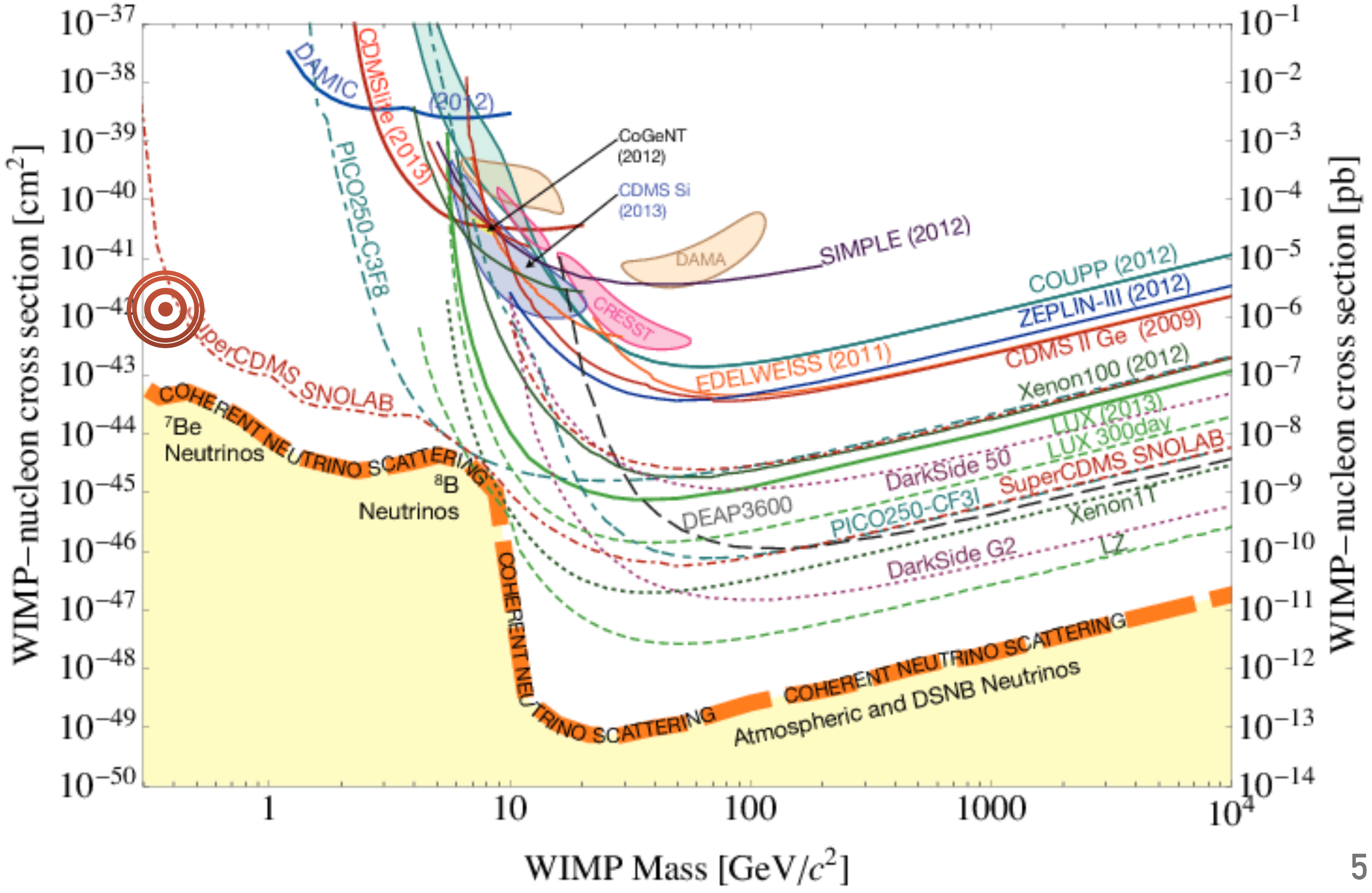

Toward lighter DM in direct detection

- Electron excitations.
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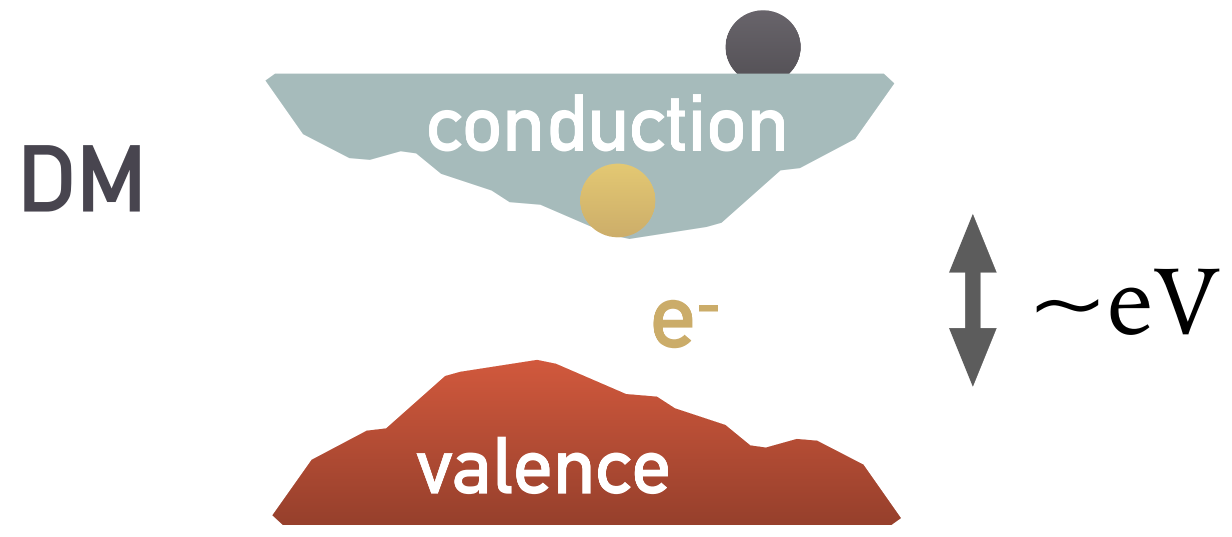
  - Reach down to DM mass~MeV.
    - Recall $E \sim mv^2 \sim 10^{-6}\, m$.



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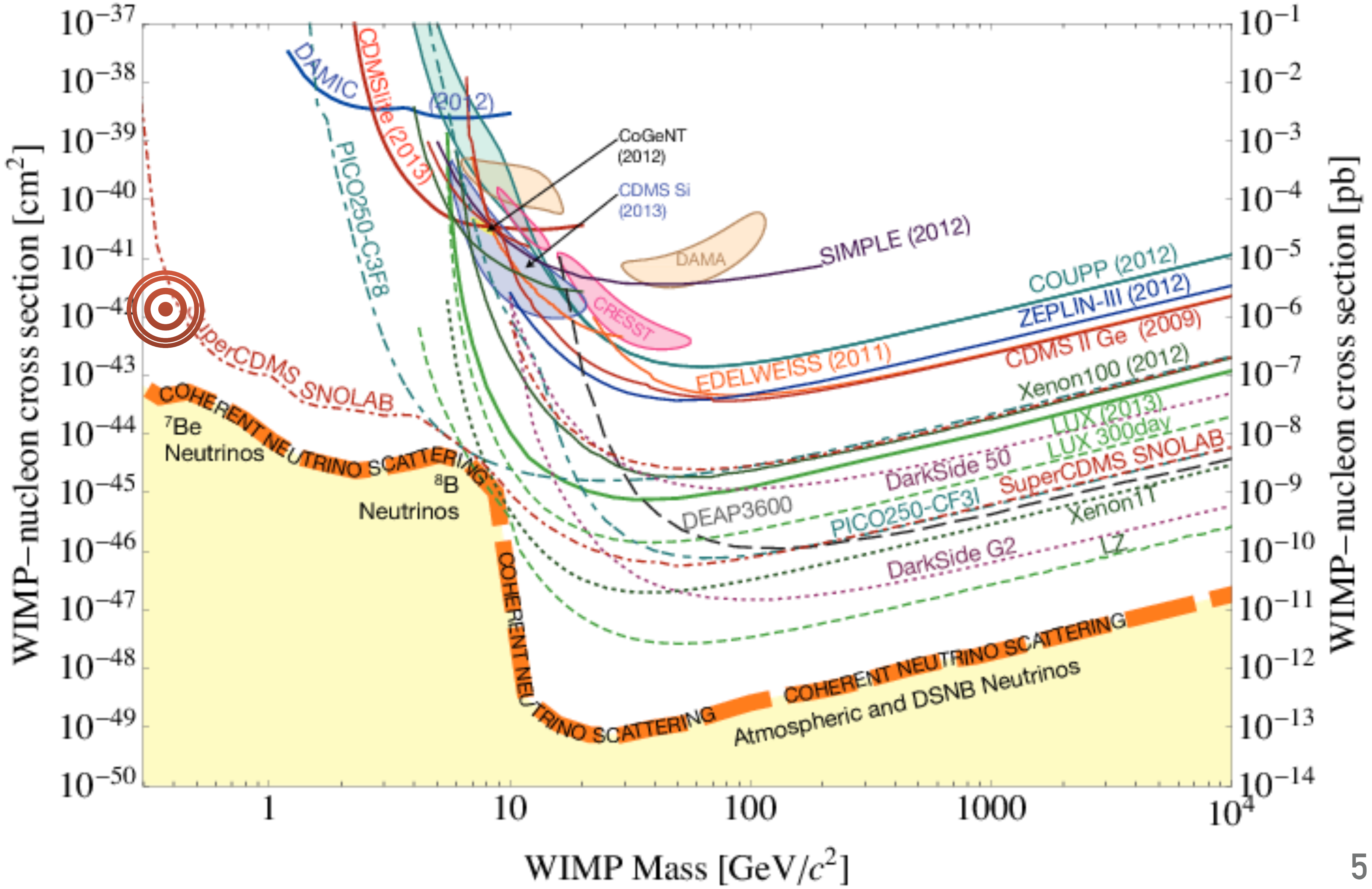

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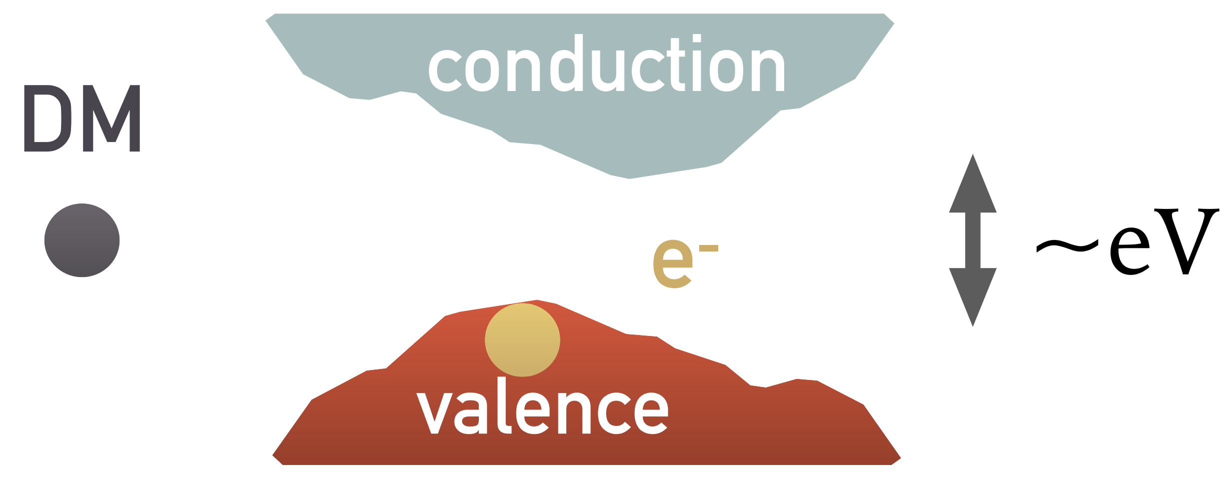

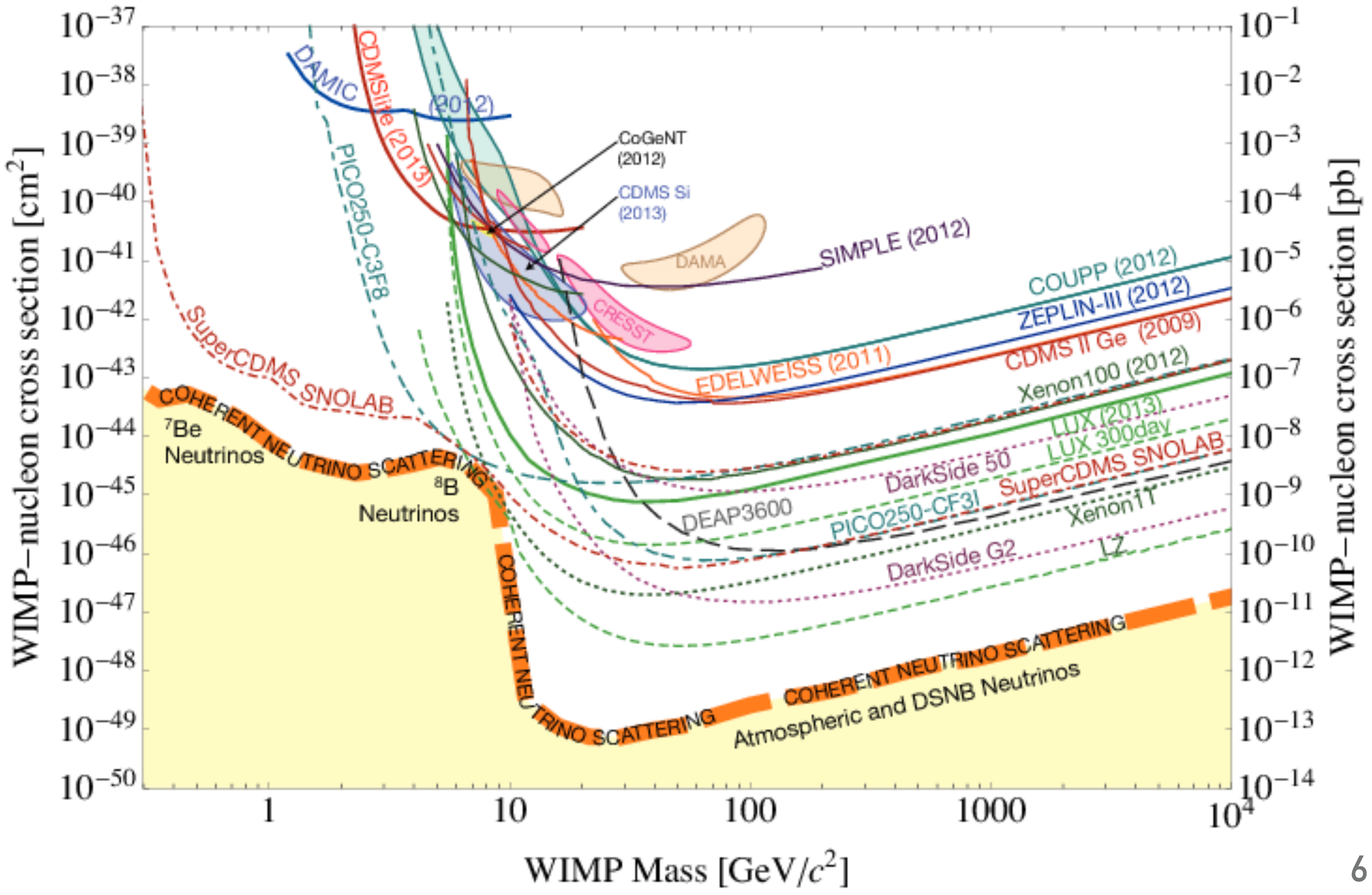

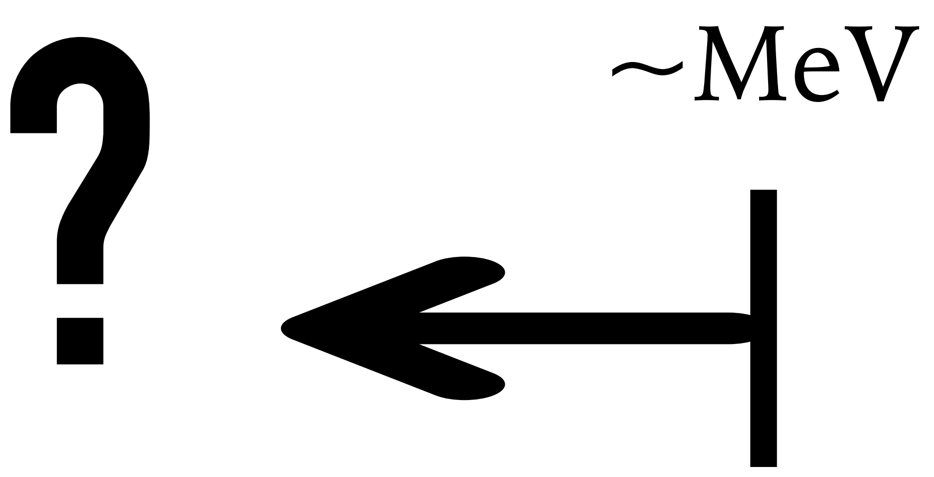

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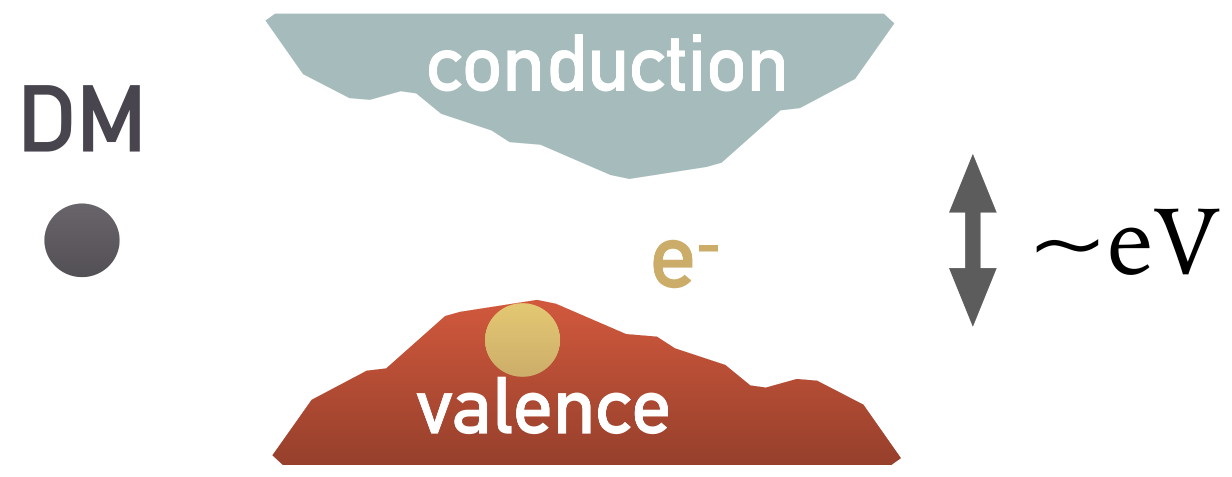

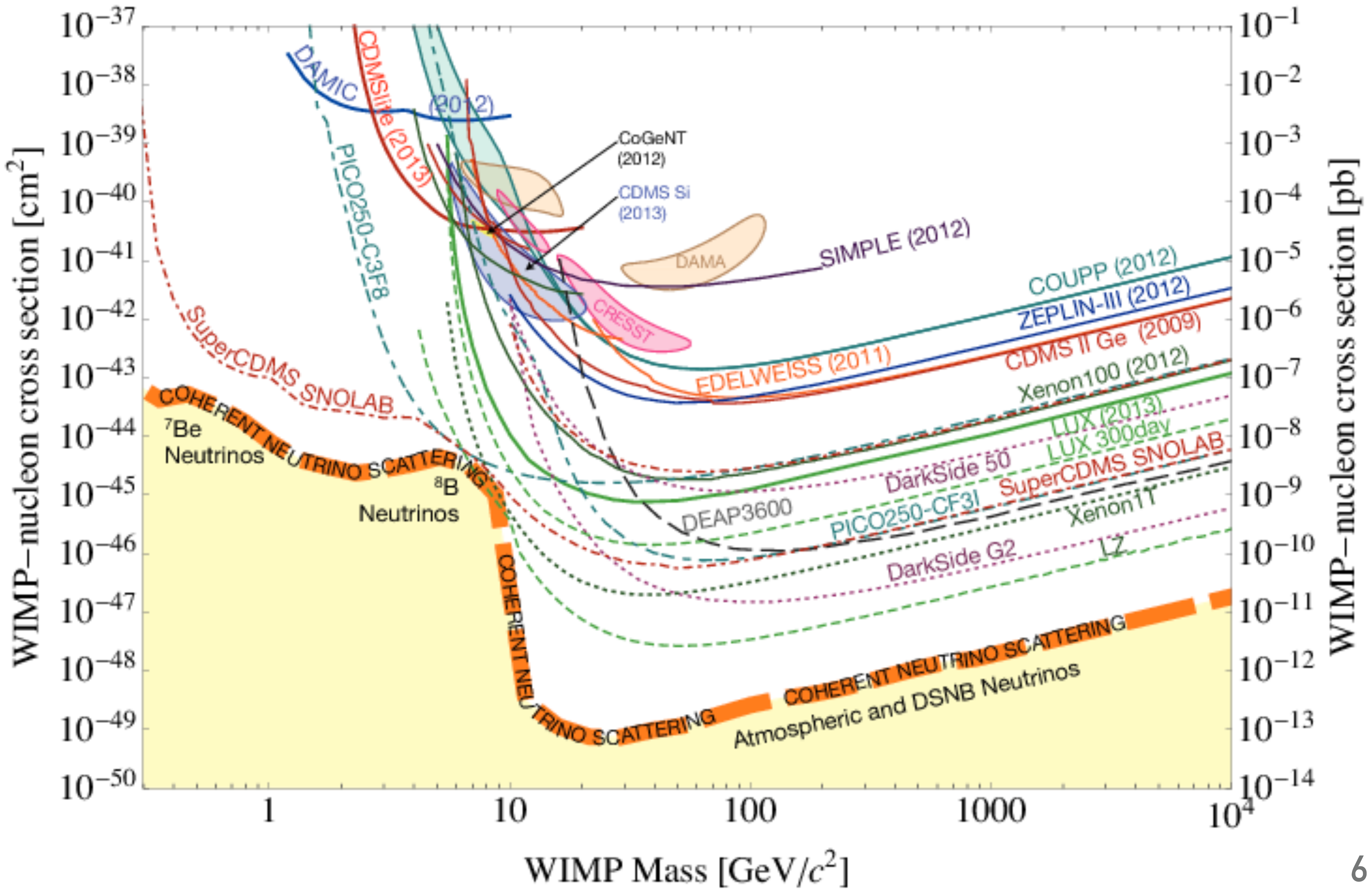

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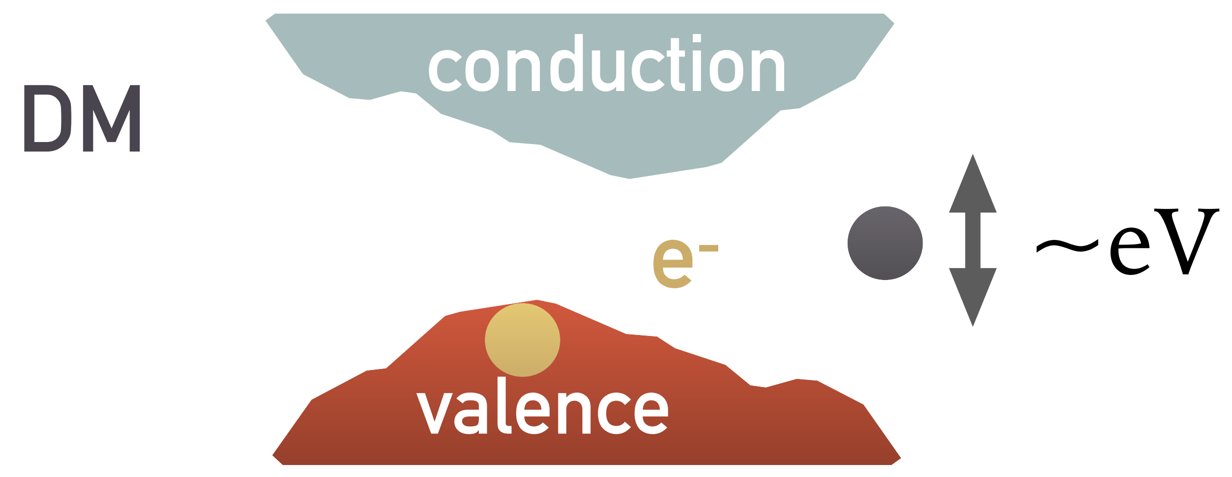

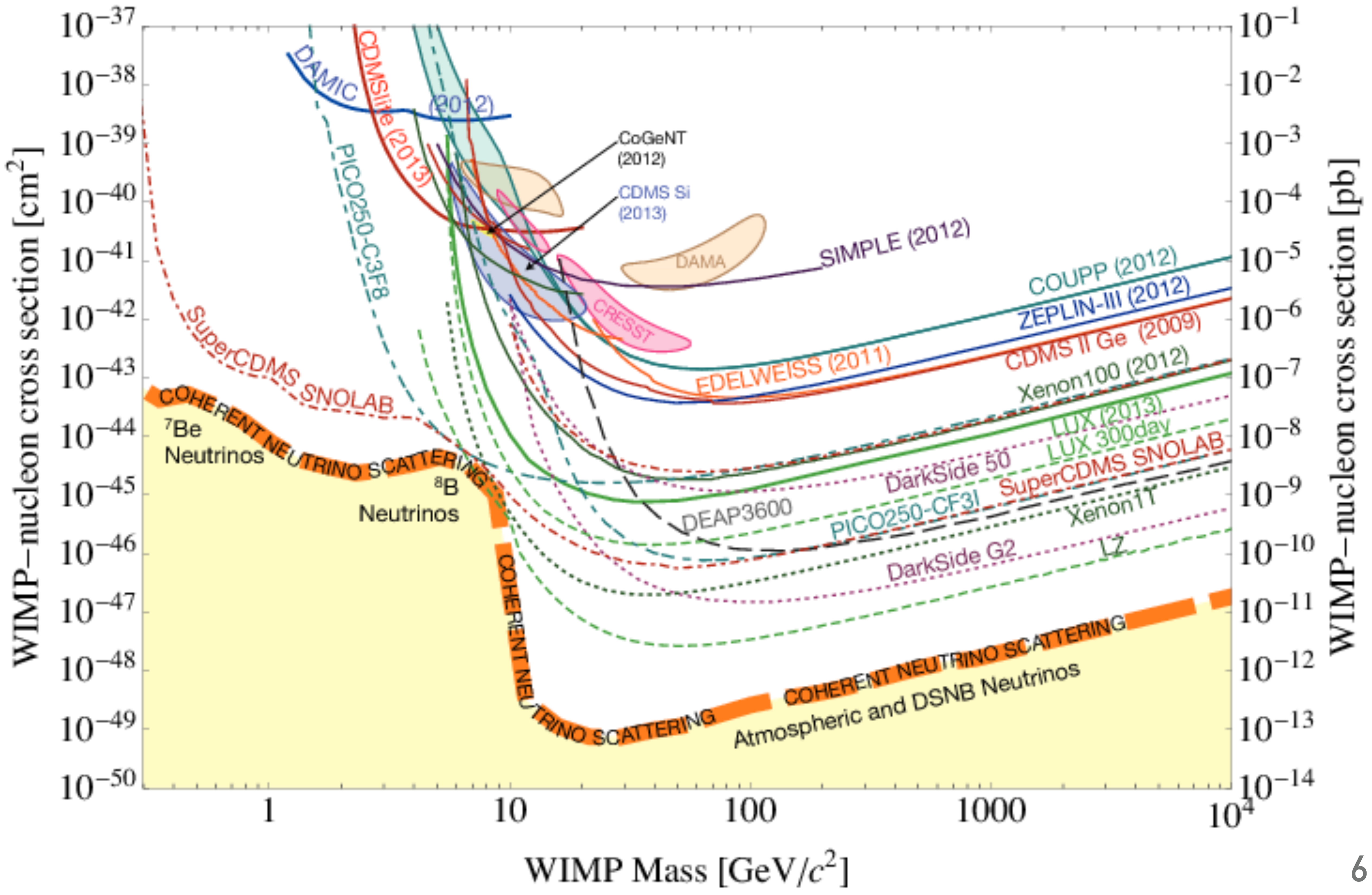

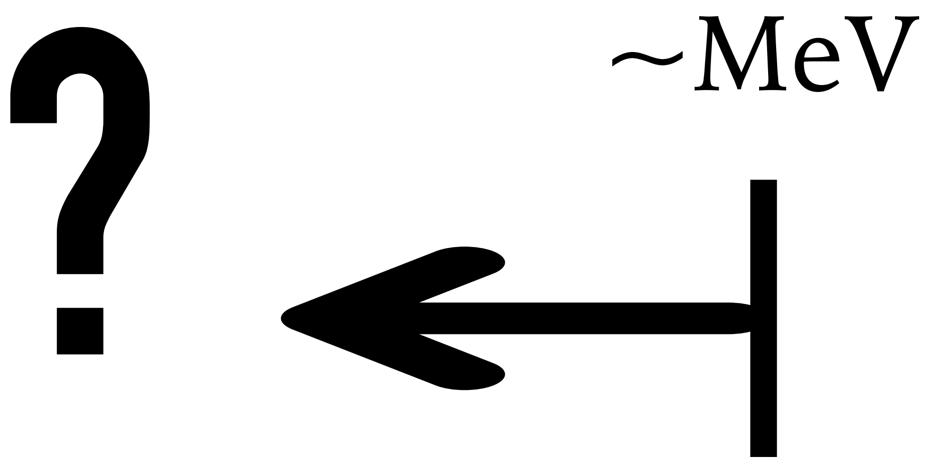

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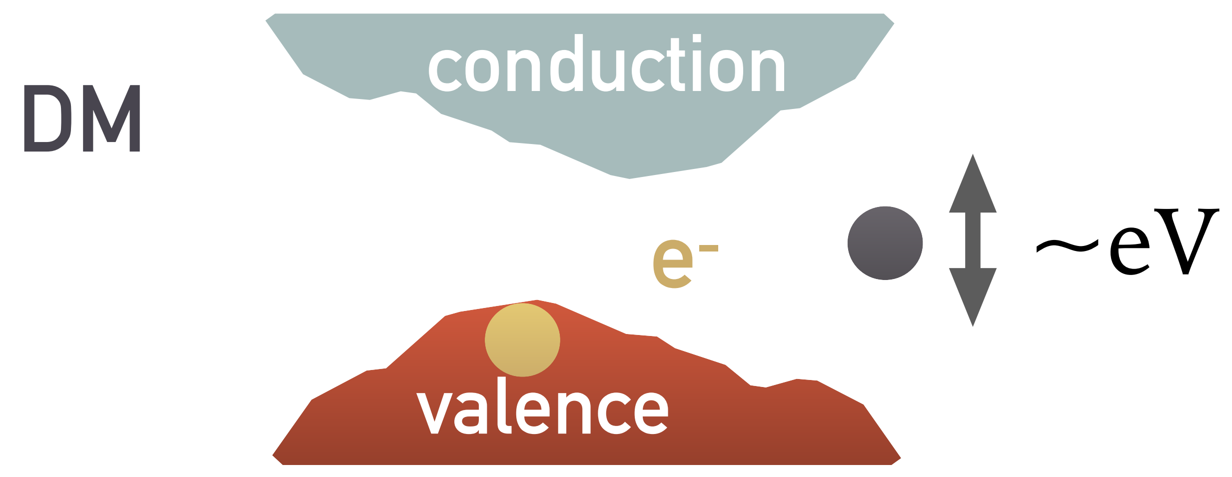

<u>Proposed meV-gap targets (somewhat futuristic)</u>
[Superconductors]
Hochberg, Zhao, Zurek, 1504.07237.
Hochberg, Pyle, Zhao, Zurek, 1512.04533.
[Dirac materials]
Hochberg et al, 1708.08929.
Geilhufe, Kahlhoefer, Winkler, 1910.02091.
Coskuner, Mitridate, Olivares, Zurek, 1909.09170.

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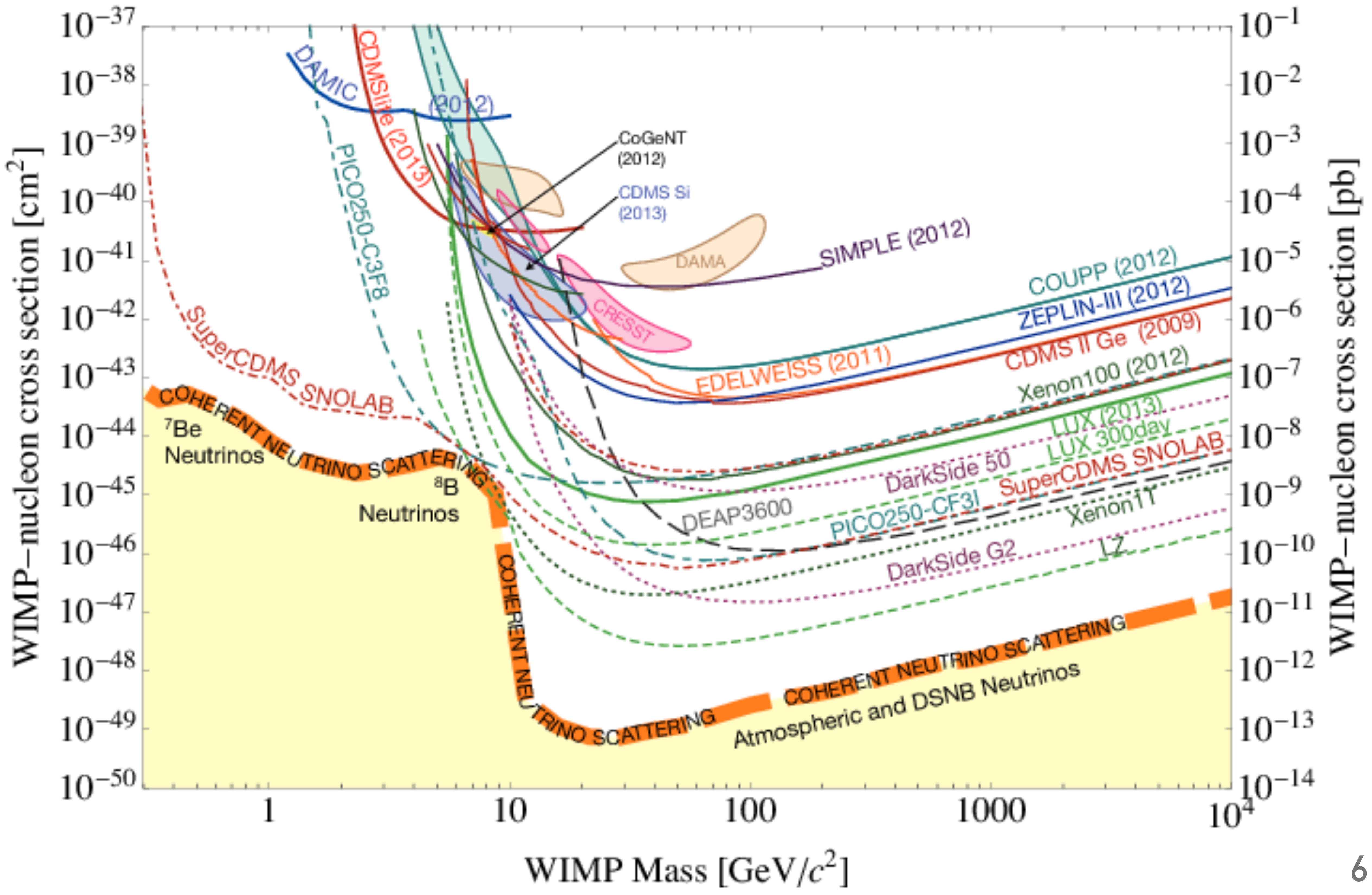

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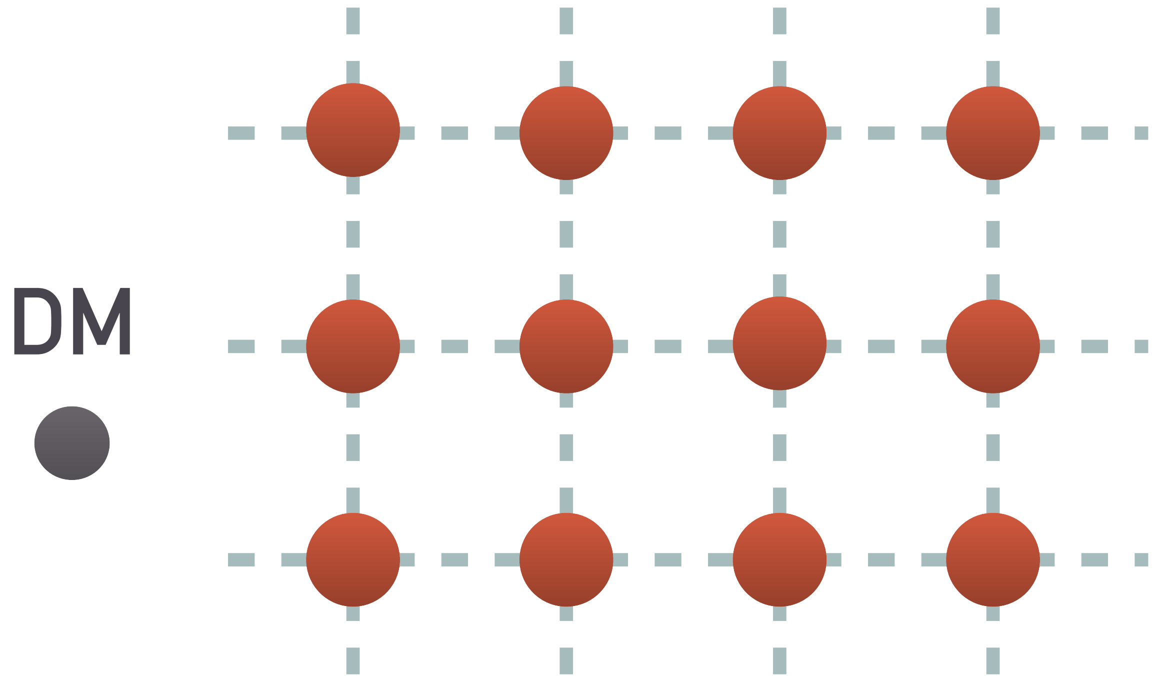

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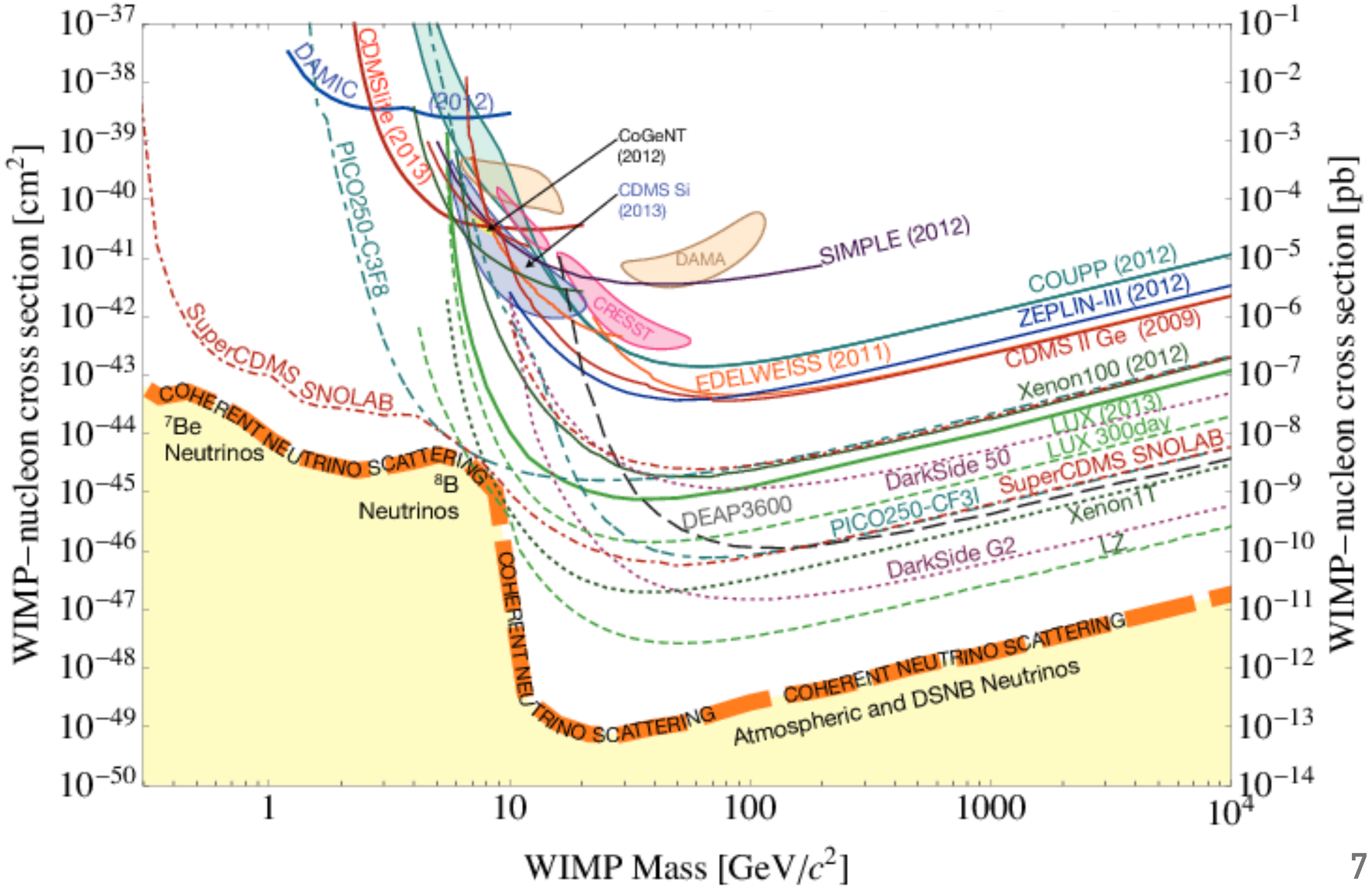
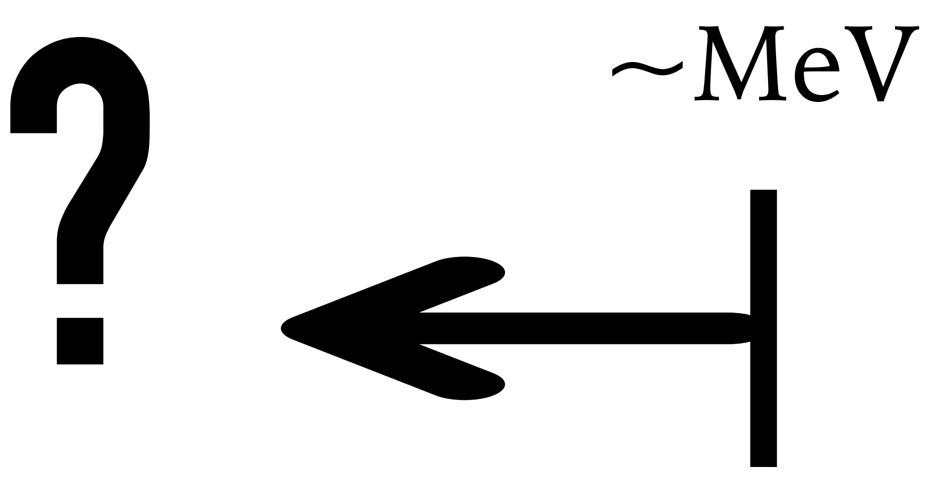
Coskuner, Trickle, ZZ, Zurek, 2102.09567.





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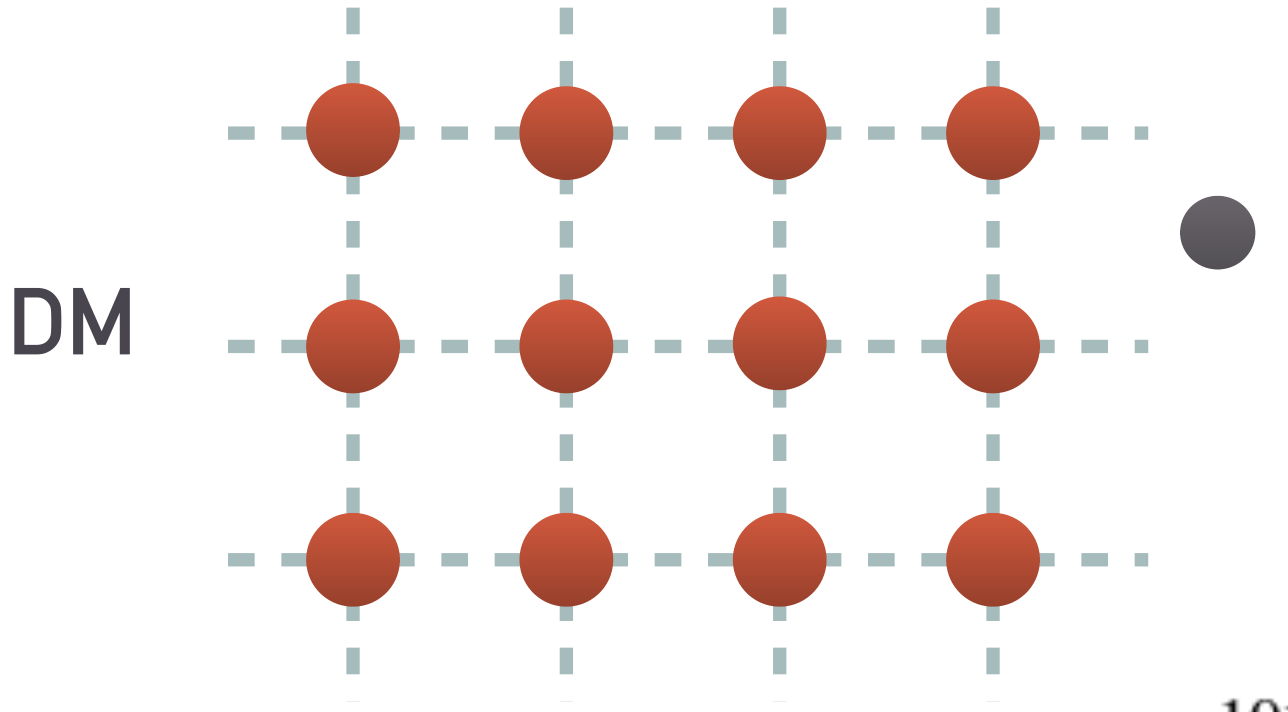

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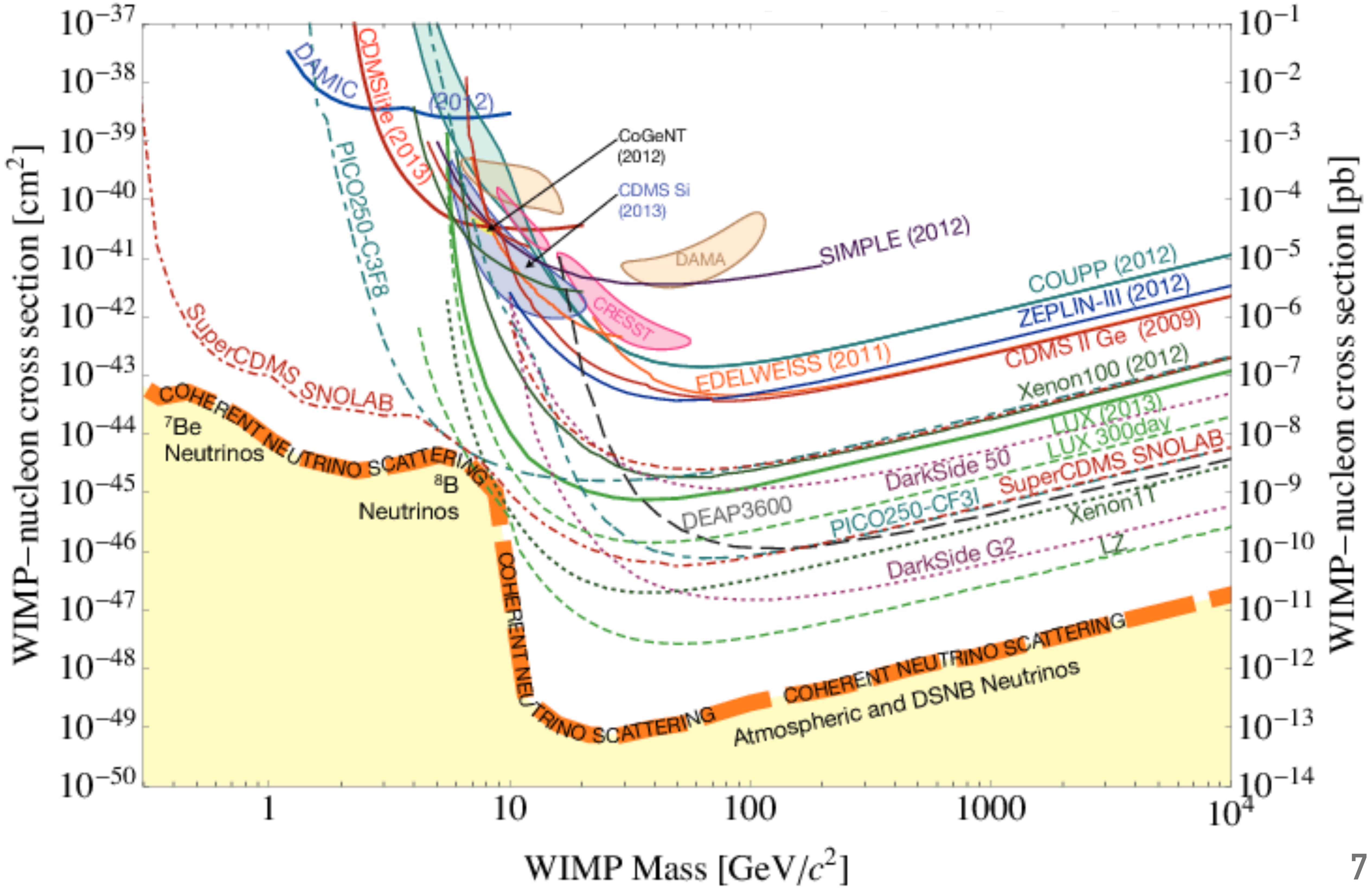

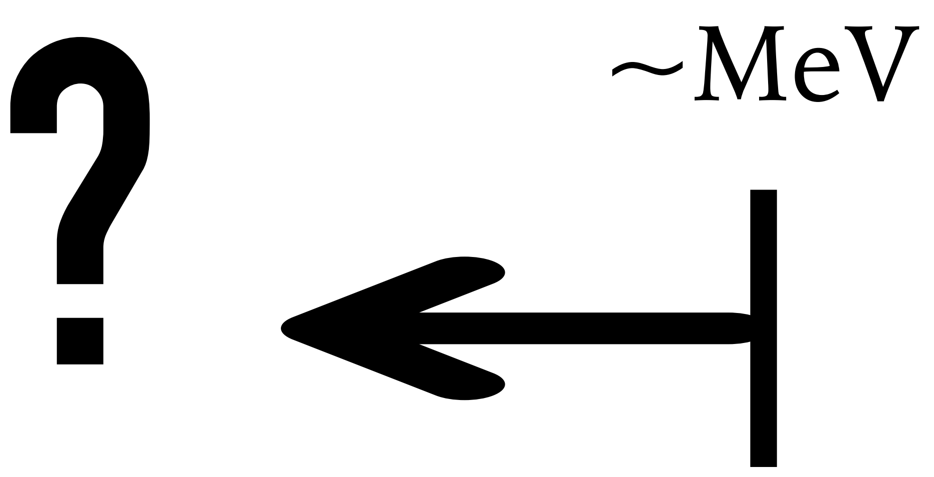

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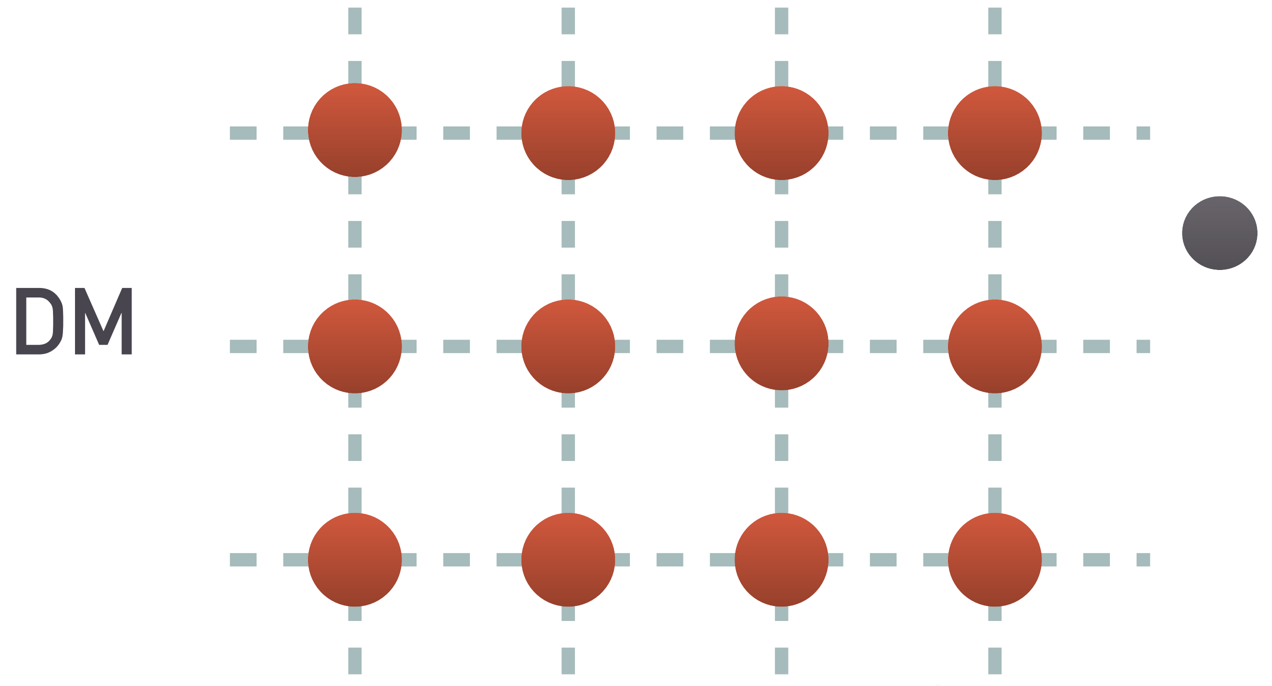

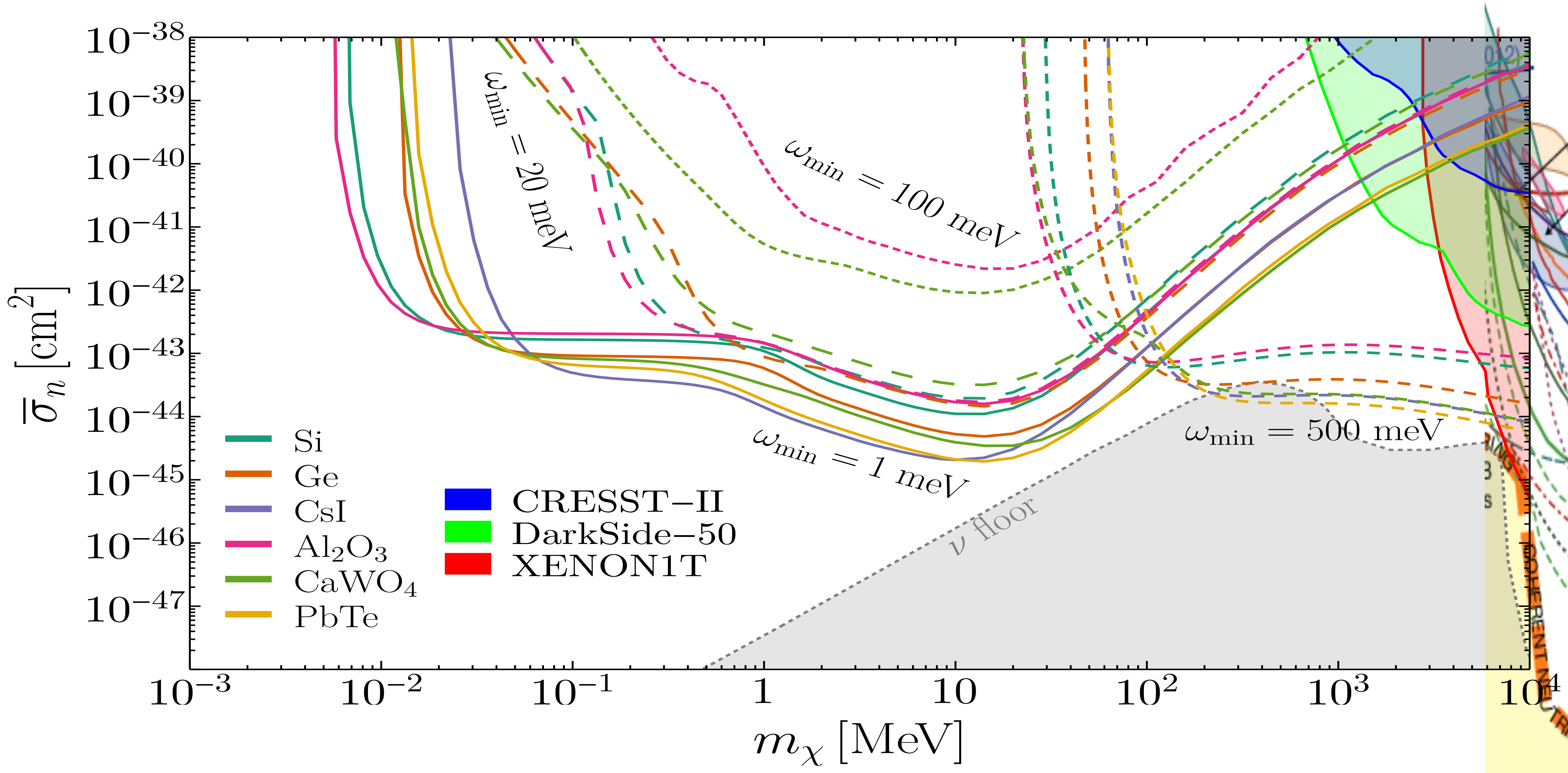

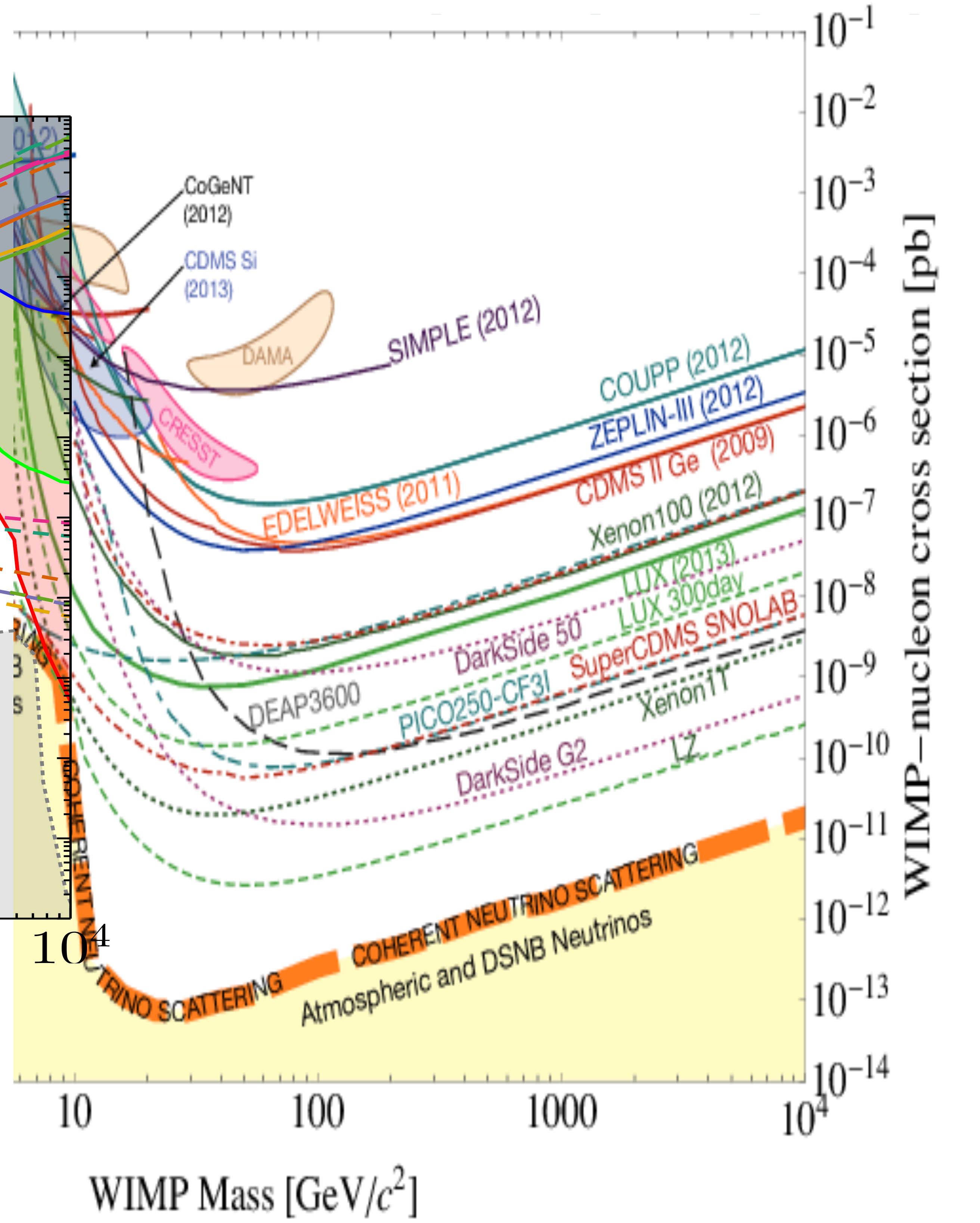

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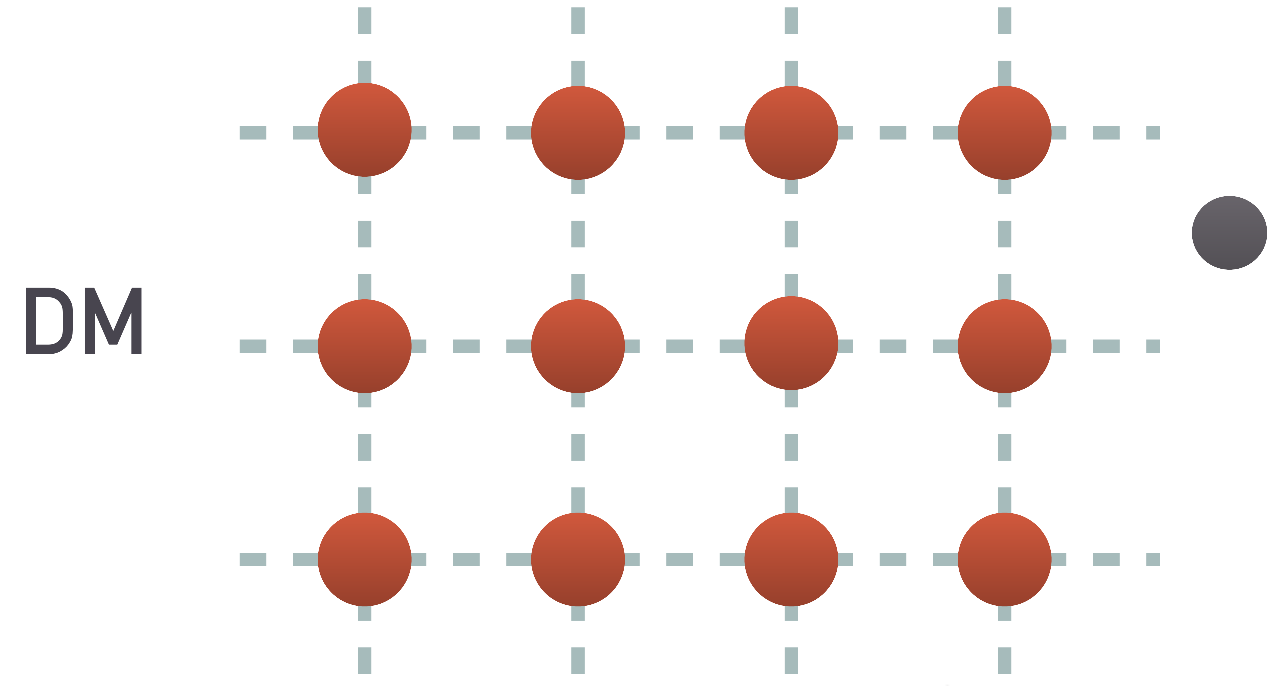


Snowmass2021 - Letter of Interest

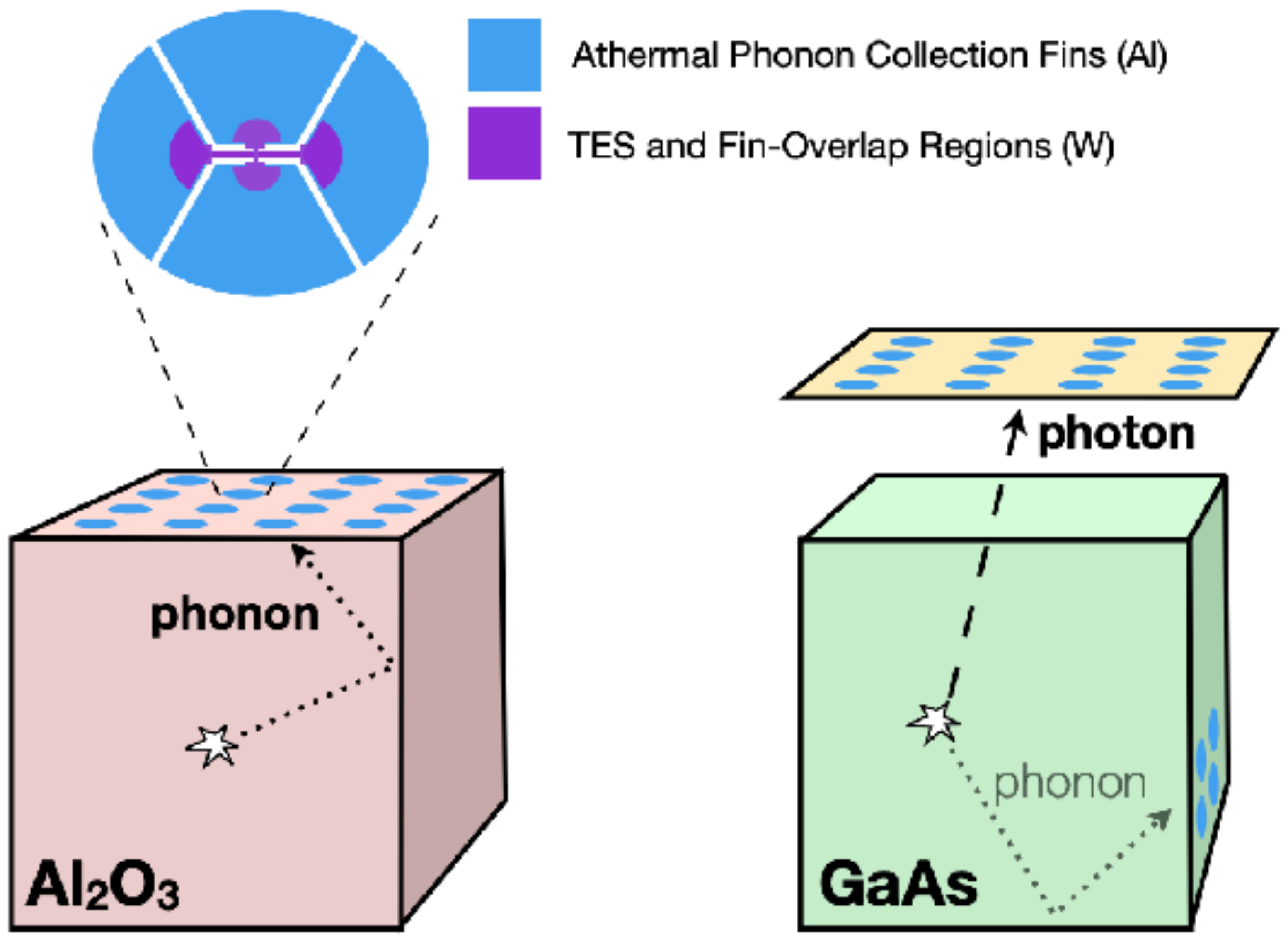
The TESSERACT Dark Matter Project

Thematic Areas:

- IF1 Quantum Sensors
- IF8 Noble Elements
- CF1 Dark Matter: Particle-like
- CF2 Dark Matter: Wavelike

Contact Information:
Dan McKinsey (LBNL and UC Berkeley) [daniel.mckinsey@berkeley.edu]:
TESSERACT Collaboration

Authors:
C. Chang (ANL), S. Derenzo (LBNL), Y. Efremenko (ANL), W. Guo (Florida State University), S. Hertel (University of Massachusetts), M. Garcia-Sciveres, R. Mahapatra (Texas A&M University), D. N. McKinsey (LBNL and UC Berkeley), B. Penning (University of Michigan), M. Pyle (LBNL and UC Berkeley), P. Sorensen (LBNL), A. Suzuki (LBNL), G. Wang (ANL), K. Zurek (Caltech)



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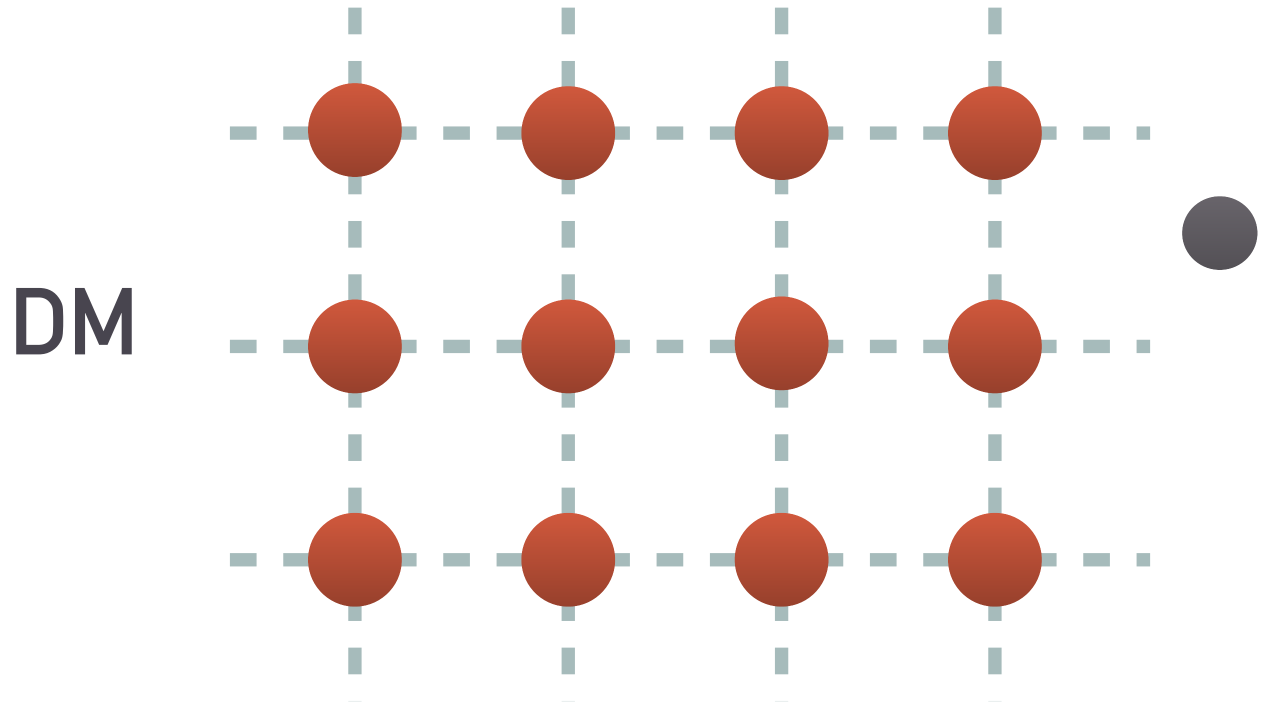

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Also discussed for axion detection.
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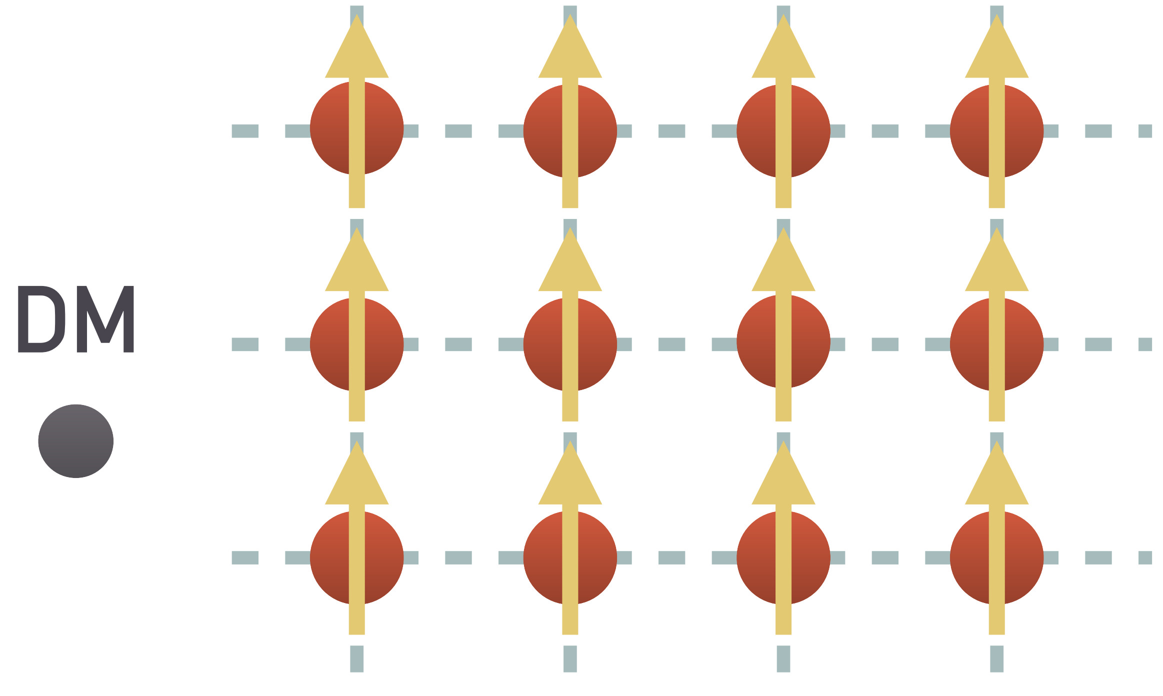

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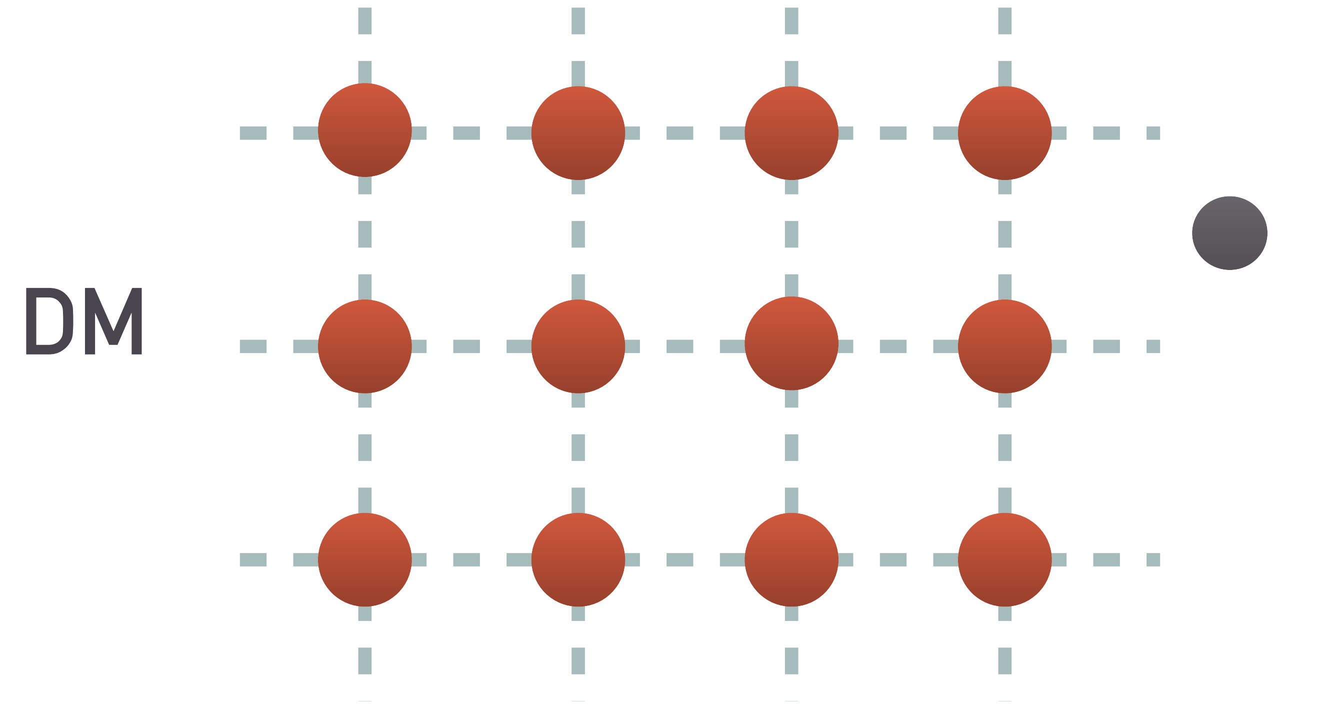

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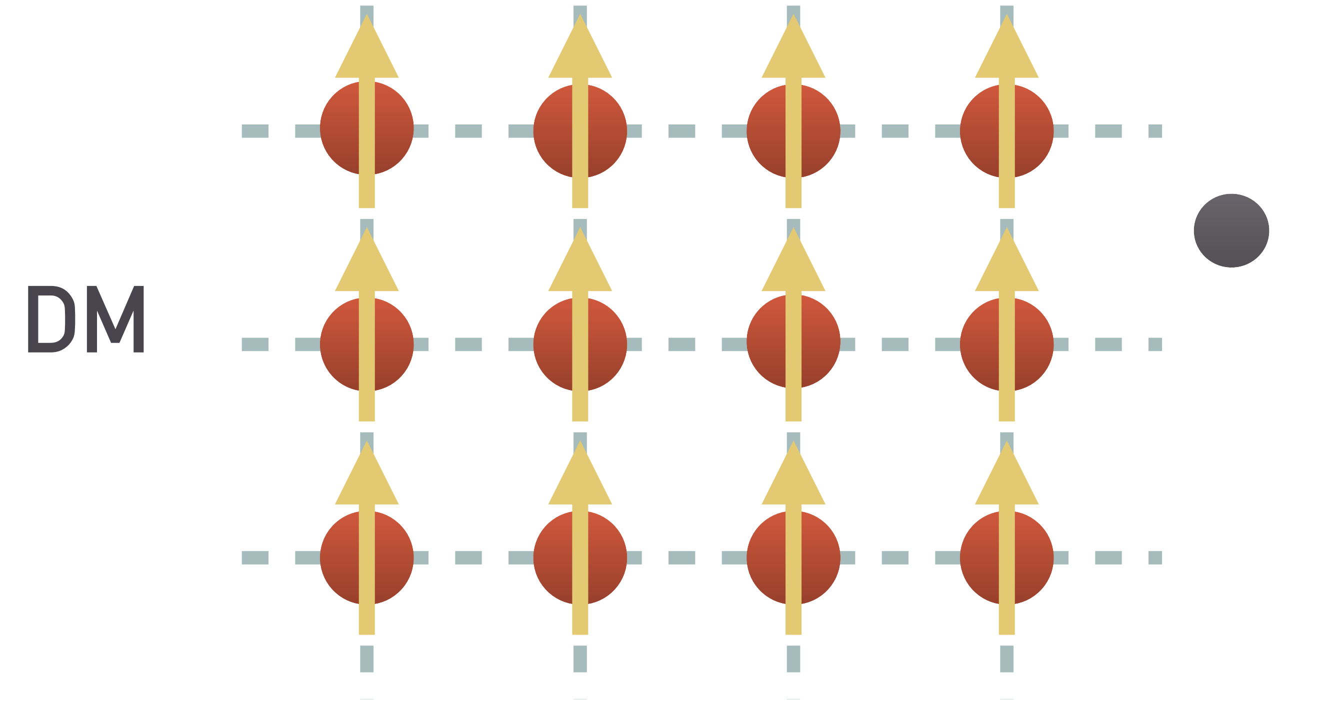
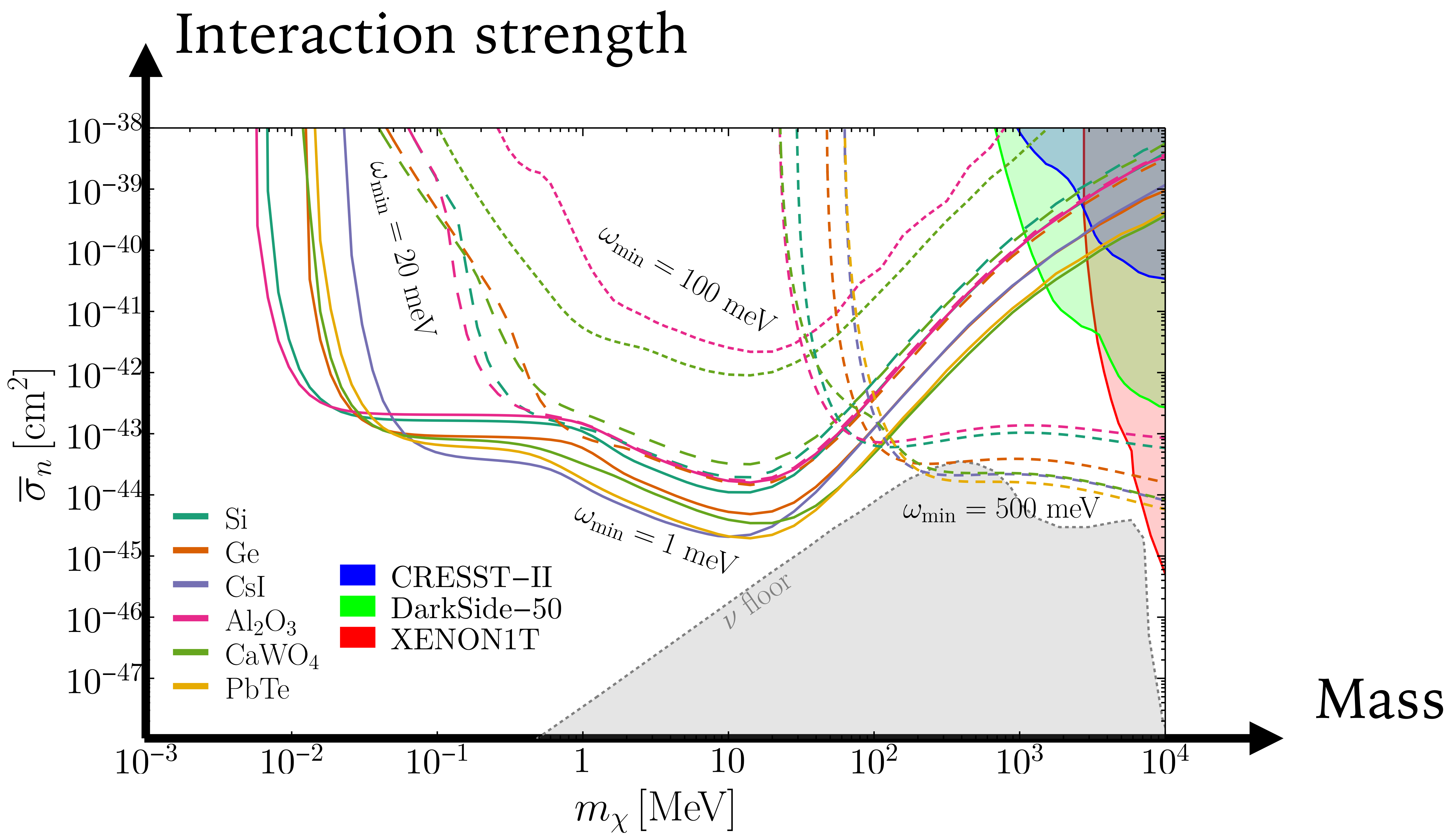


A 3rd axis of DM's parameter space



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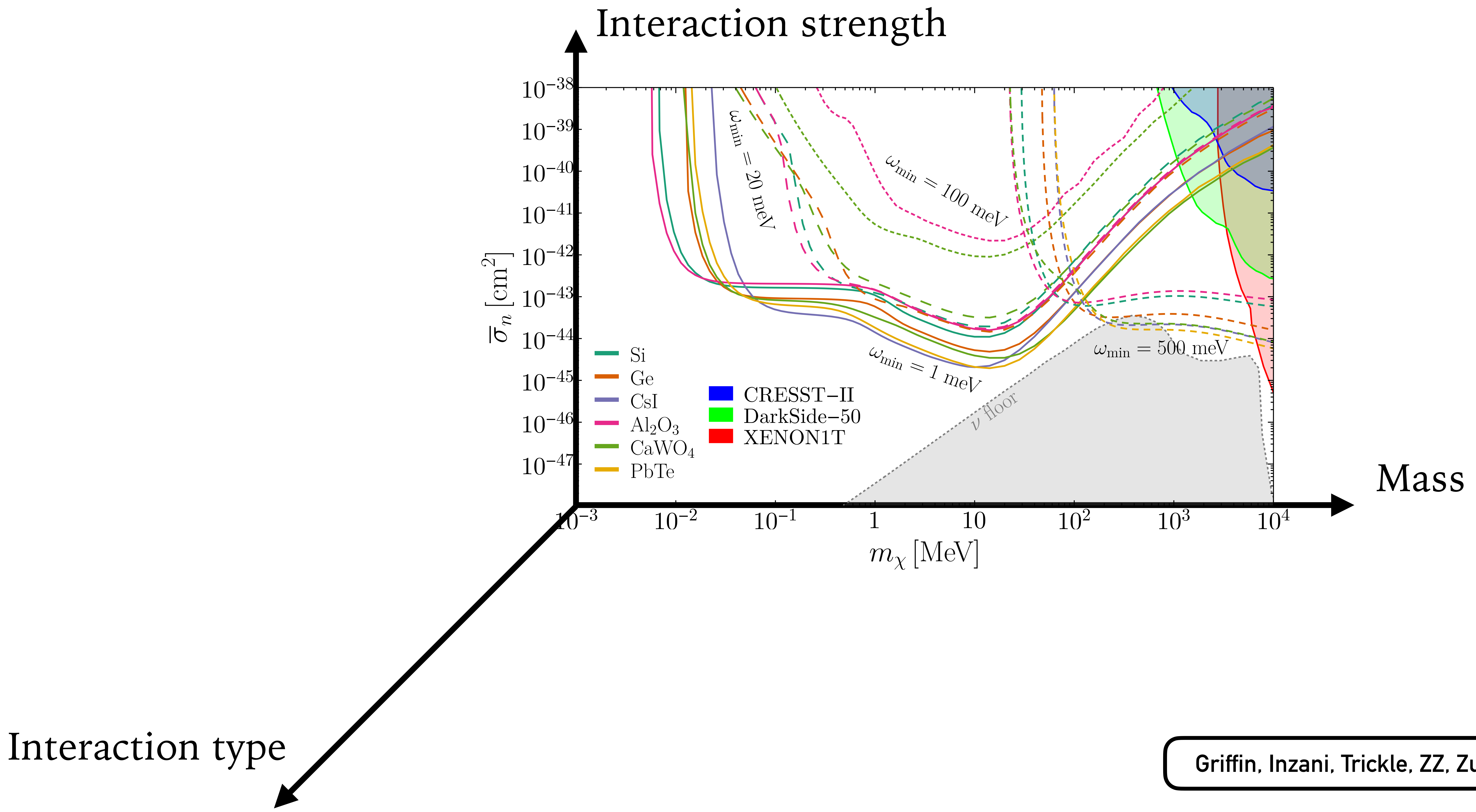

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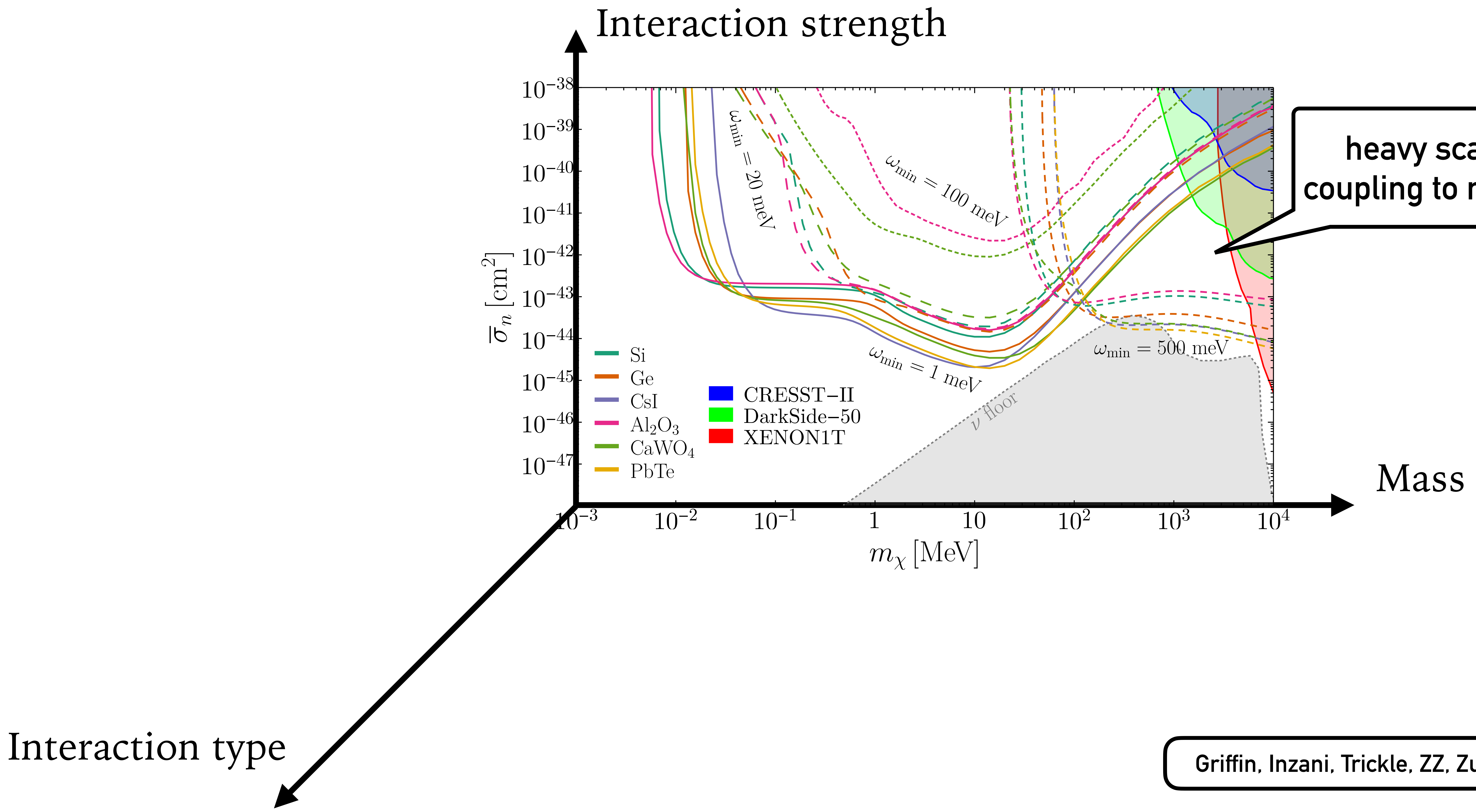

heavy scalar mediator coupling to nucleon number

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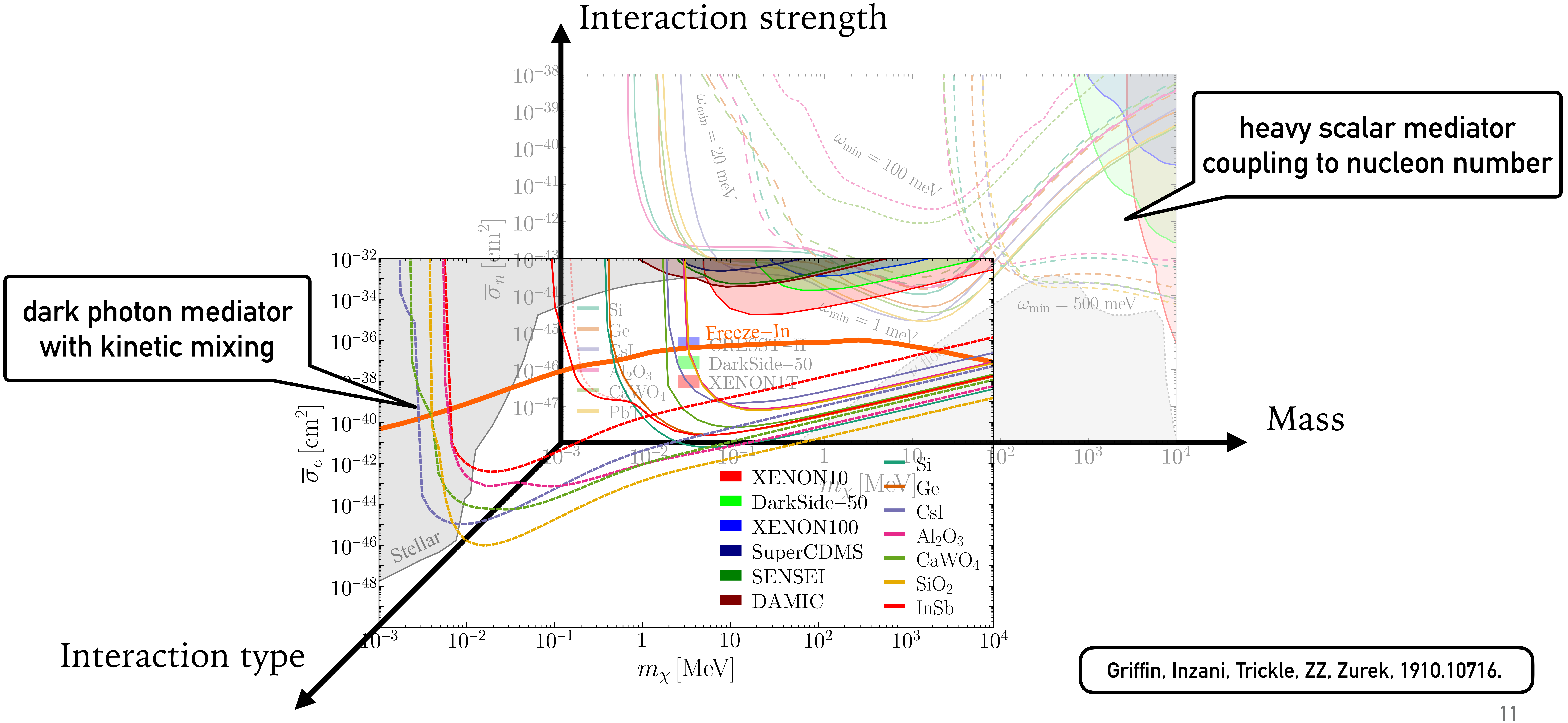
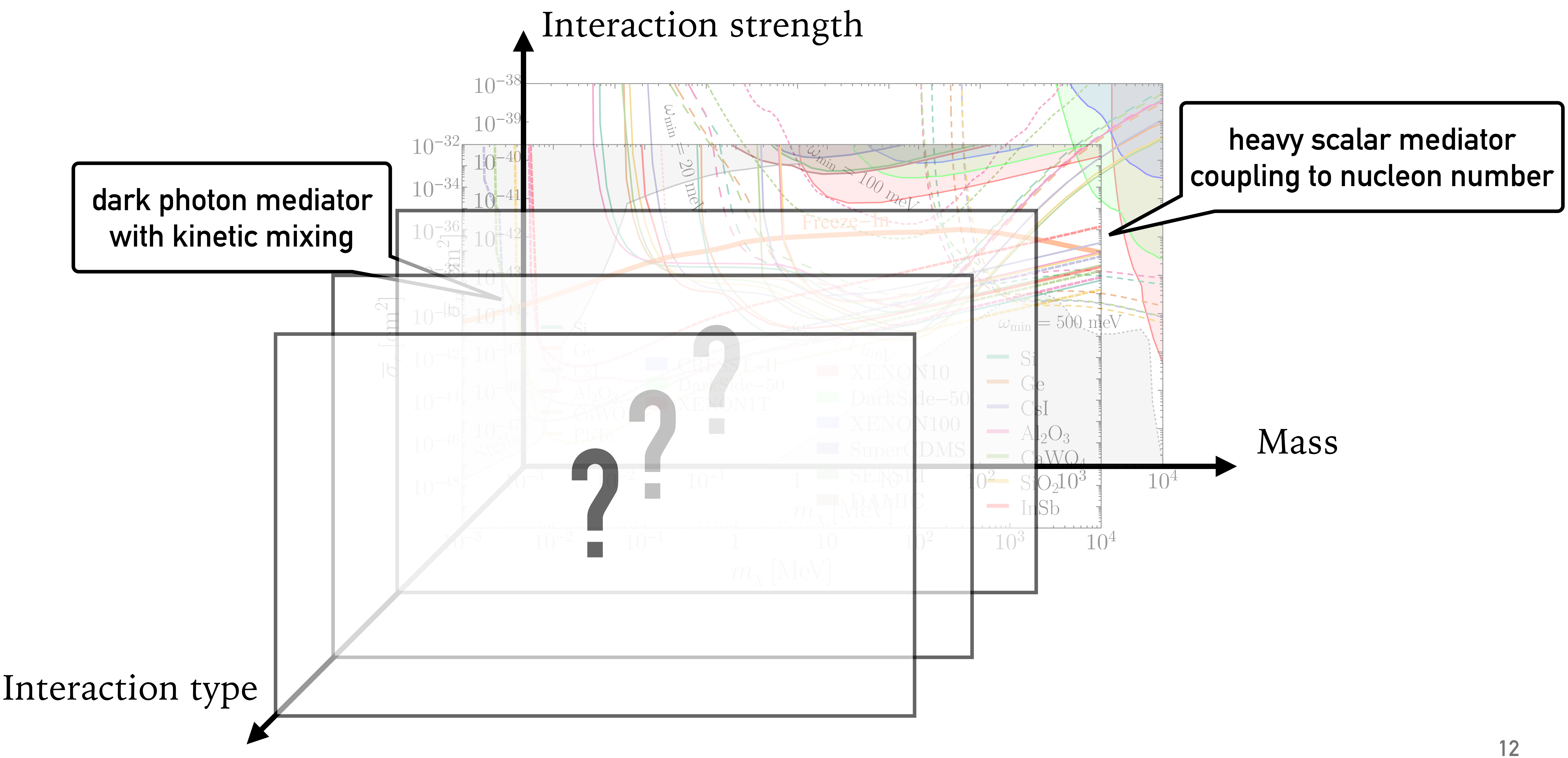
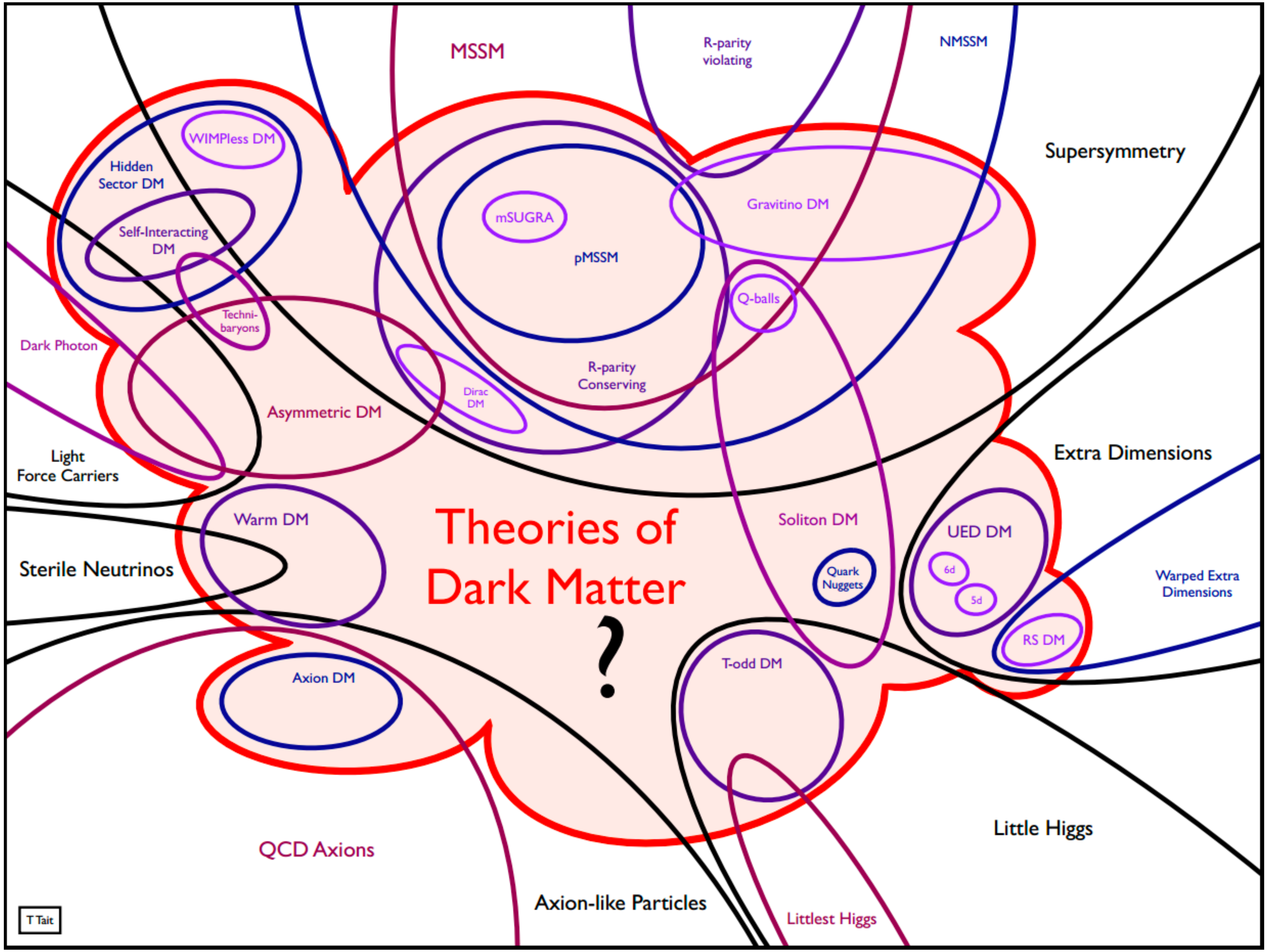


Griffin, Inzani, Trickle, ZZ, Zurek, 1910.10716.

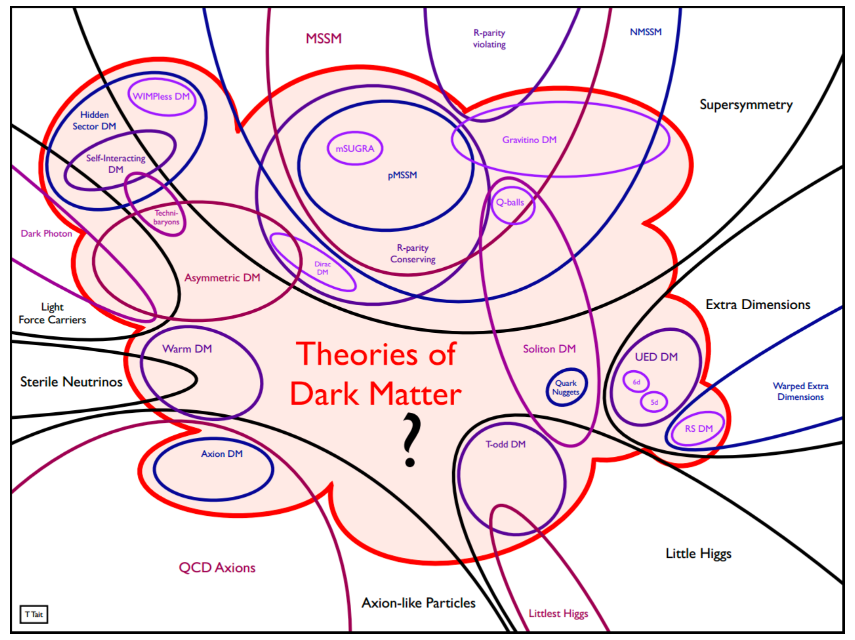
A 3rd axis of DM's parameter space





MSSM
R-parity violating
NMSSM
Supersymmetry
WIMPless DM
Hidden Sector DM
Self-Interacting DM
Techni-baryons
mSUGRA
pMSSM
Gravitino DM
Q-balls
Dark Photon
Dirac DM
R-parity Conserving
Asymmetric DM
Light Force Carriers
Extra Dimensions
Warm DM
Theories of Dark Matter
?
Soliton DM
UED DM
Sterile Neutrinos
Quark Nuggets
6d
5d
Warped Extra Dimensions
RS DM
Axion DM
T-odd DM
Little Higgs
QCD Axions
Axion-like Particles
Littlest Higgs
T Tait

A common description at low energy



Nonrelativistic (NR) EFT of DM-SM interactions

$$\mathcal{O}_1^{(\psi)} = \mathbb{1}$$
$$\mathcal{O}_{11}^{(\psi)} = \boldsymbol{S}_\chi \cdot \frac{i\boldsymbol{q}}{m_\psi}$$
$$\mathcal{O}_5^{(\psi)} = \boldsymbol{S}_\chi \cdot \left(\frac{i\boldsymbol{q}}{m_\psi} \times \boldsymbol{v}^\perp\right)$$
$$\mathcal{O}_8^{(\psi)} = \boldsymbol{S}_\chi \cdot \boldsymbol{v}^\perp$$

$$\mathcal{O}_3^{(\psi)} = \boldsymbol{S}_\psi \cdot \left(\frac{i\boldsymbol{q}}{m_\psi} \times \boldsymbol{v}^\perp\right)$$
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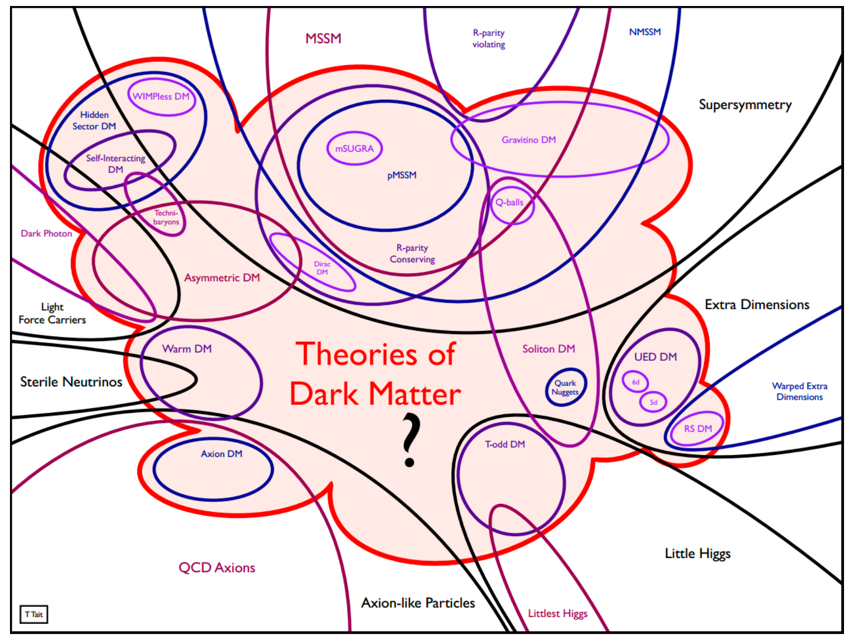
EFT of DM direct detection w/ phonons & magnons

Trickle, ZZ, Zurek, 2009.13534.



Nonrelativistic (NR) EFT of DM-SM interactions

Crystal responses

DM couplings to lattice d.o.f.

$\boldsymbol{N}$ (particle number)	$\boldsymbol{S}$ (spin)
$\boldsymbol{L}$ (orbital angular momentum)	$\boldsymbol{L}\otimes\boldsymbol{S}$ (spin-orbit coupling)

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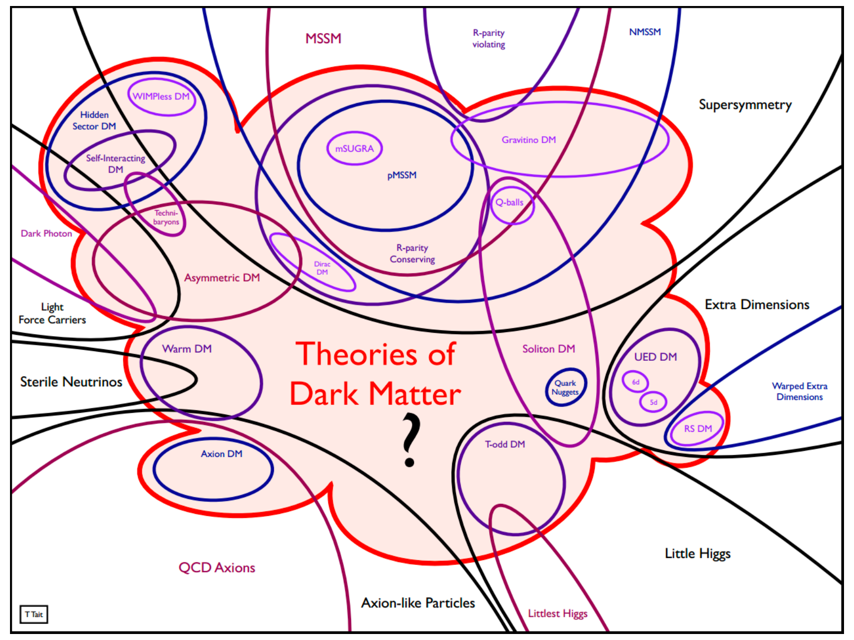
Trickle, ZZ, Zurek, 2009.13534.

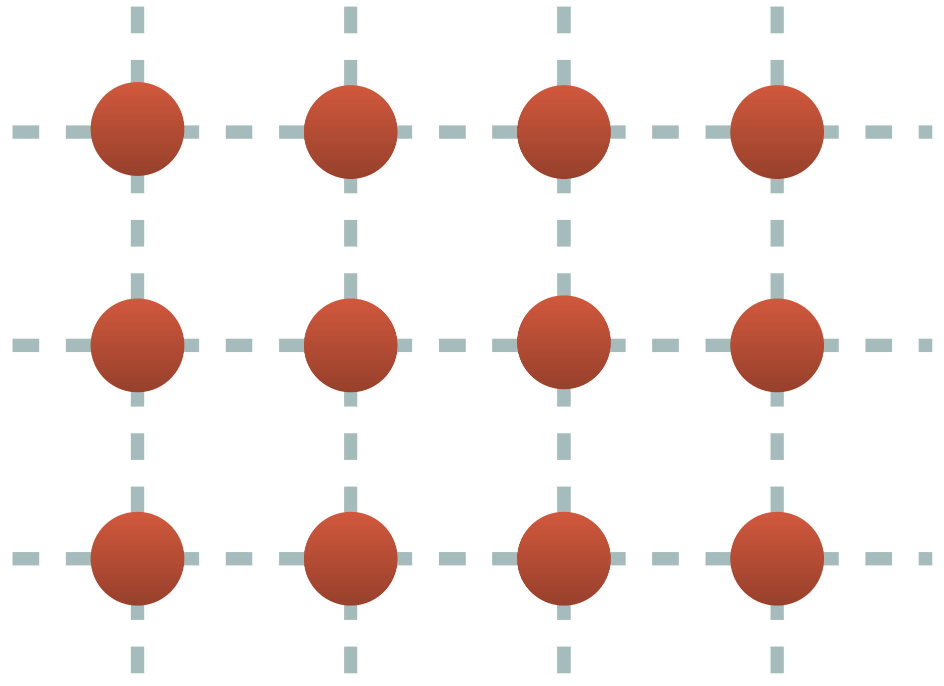


Nonrelativistic (NR) EFT of DM-SM interactions

Crystal responses

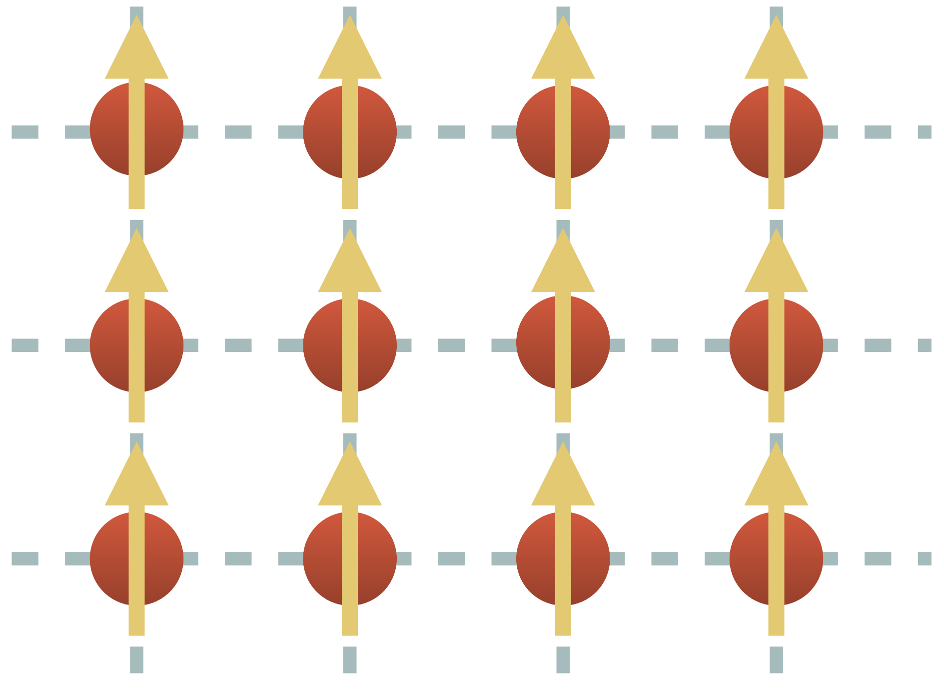
Phonon & magnon excitation rates

DM



(via N, S, L, $L \otimes S$)

DM



(via S_e, L_e)

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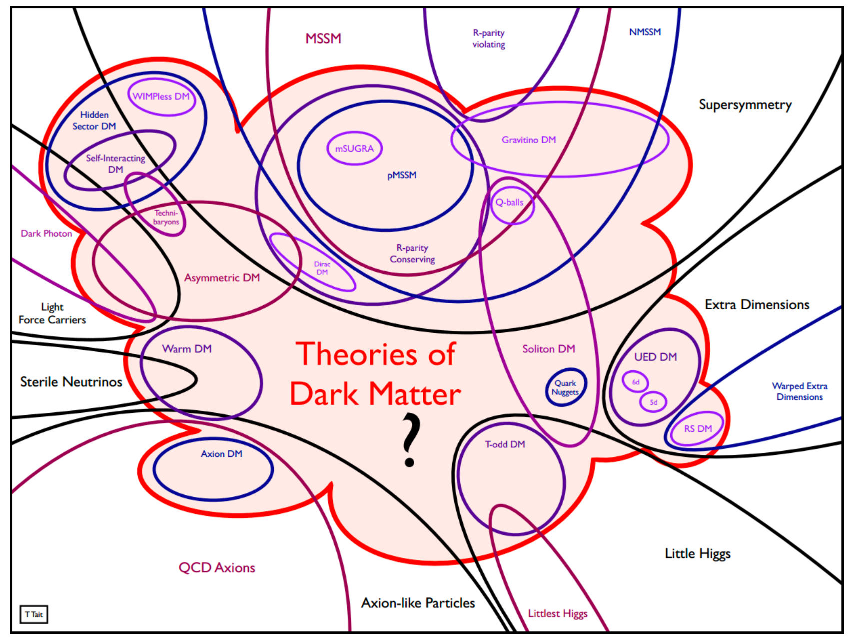

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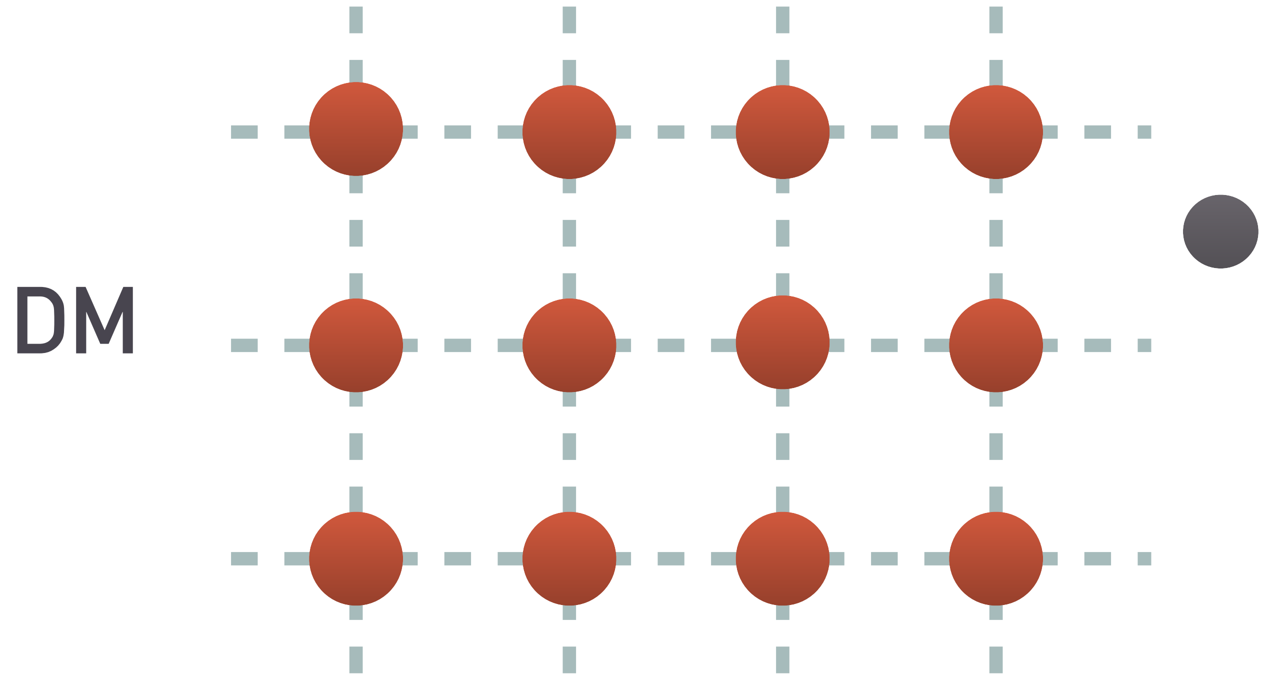

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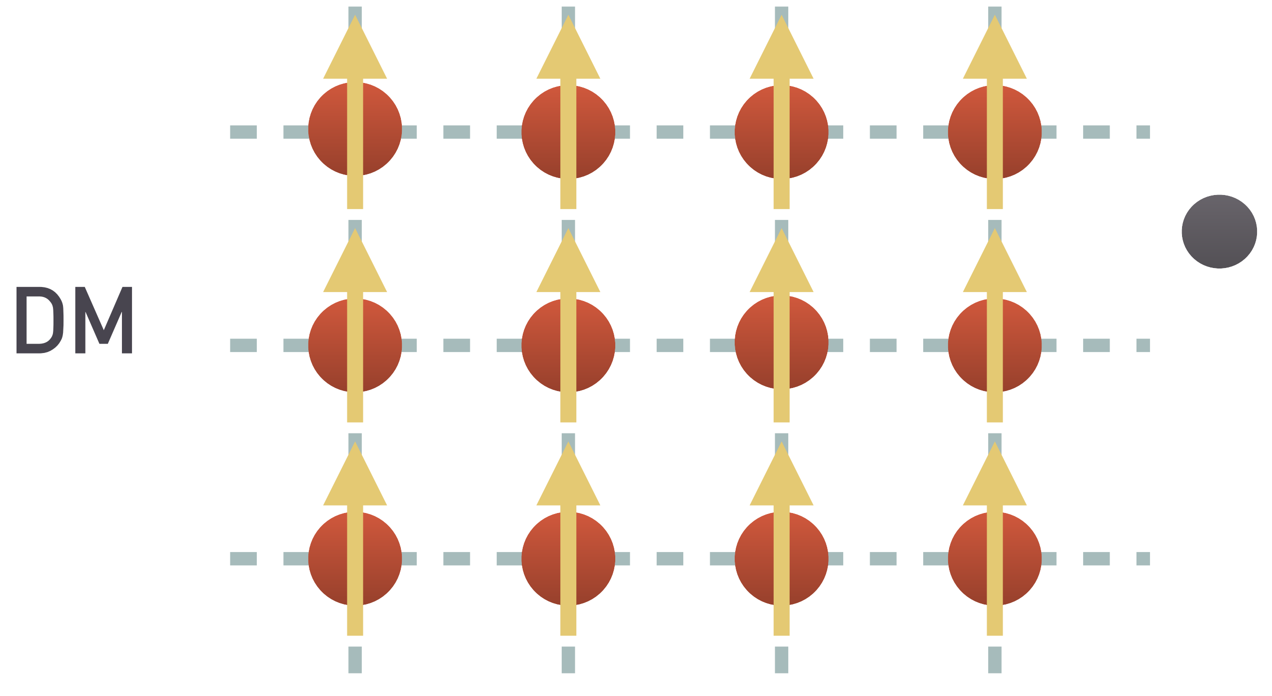

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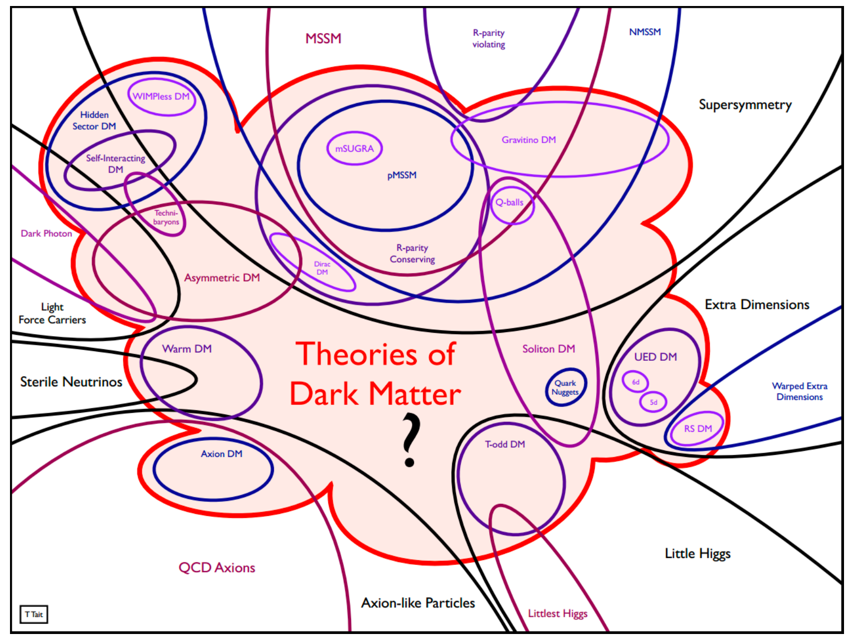
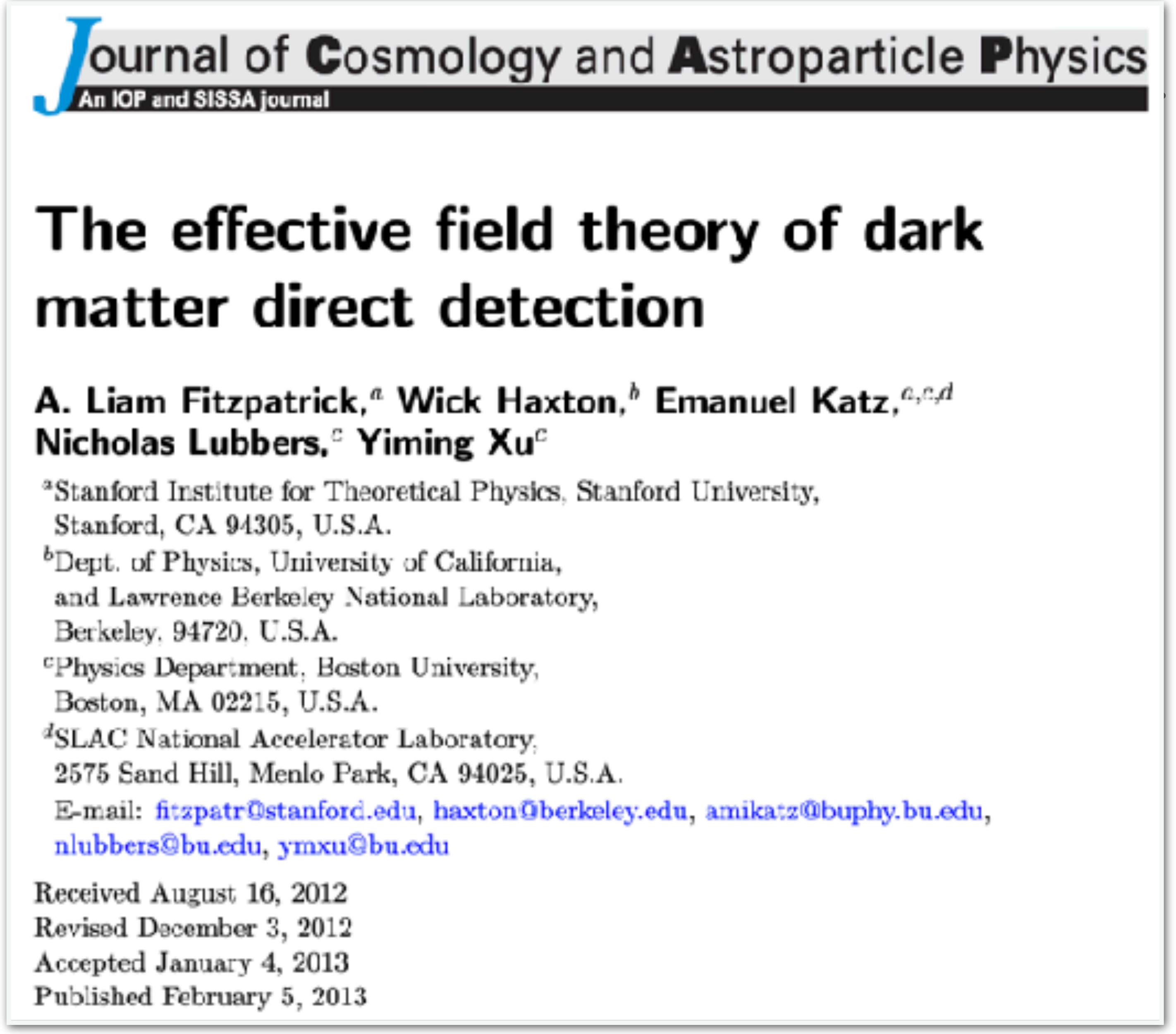


Similar situation in nuclear recoil calculations.

- At first, just spin-independent (SI) and spin-dependent (SD) benchmarks.
- Later on, extended to EFT.
- UV model ⇒ EFT ⇒ nuclear responses ⇒ rates.



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The effective field theory of dark matter direct detection

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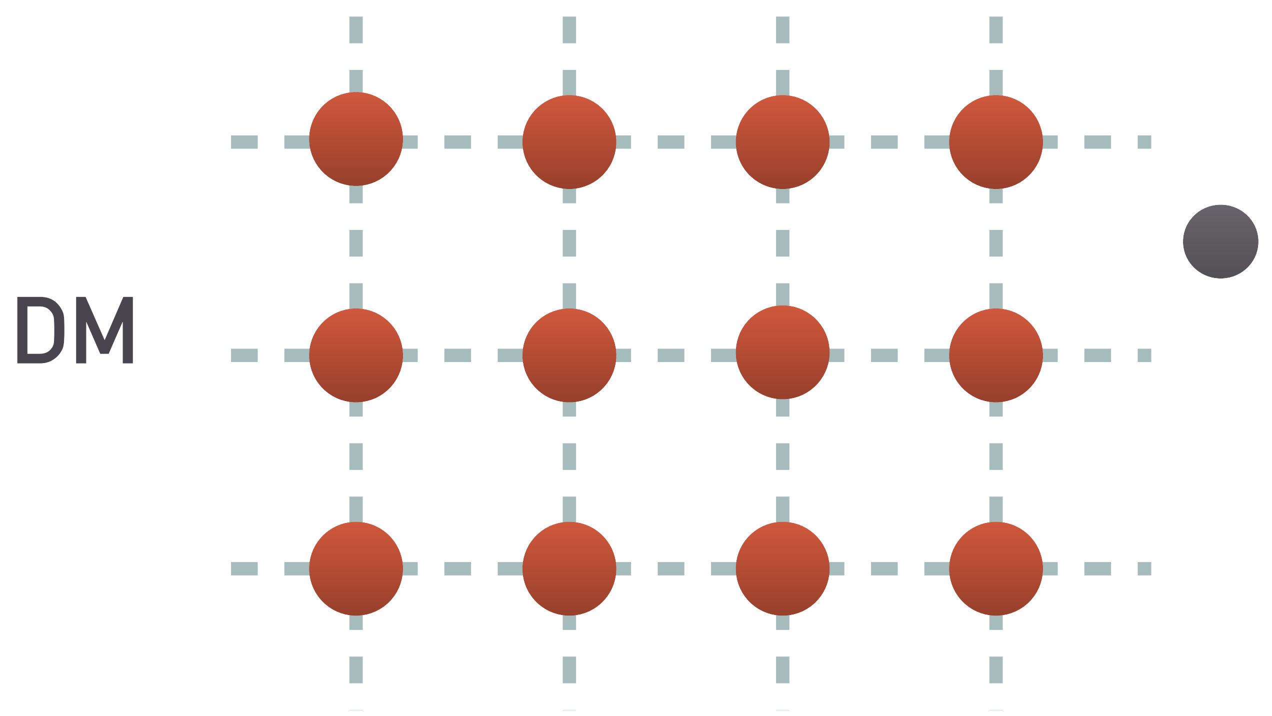
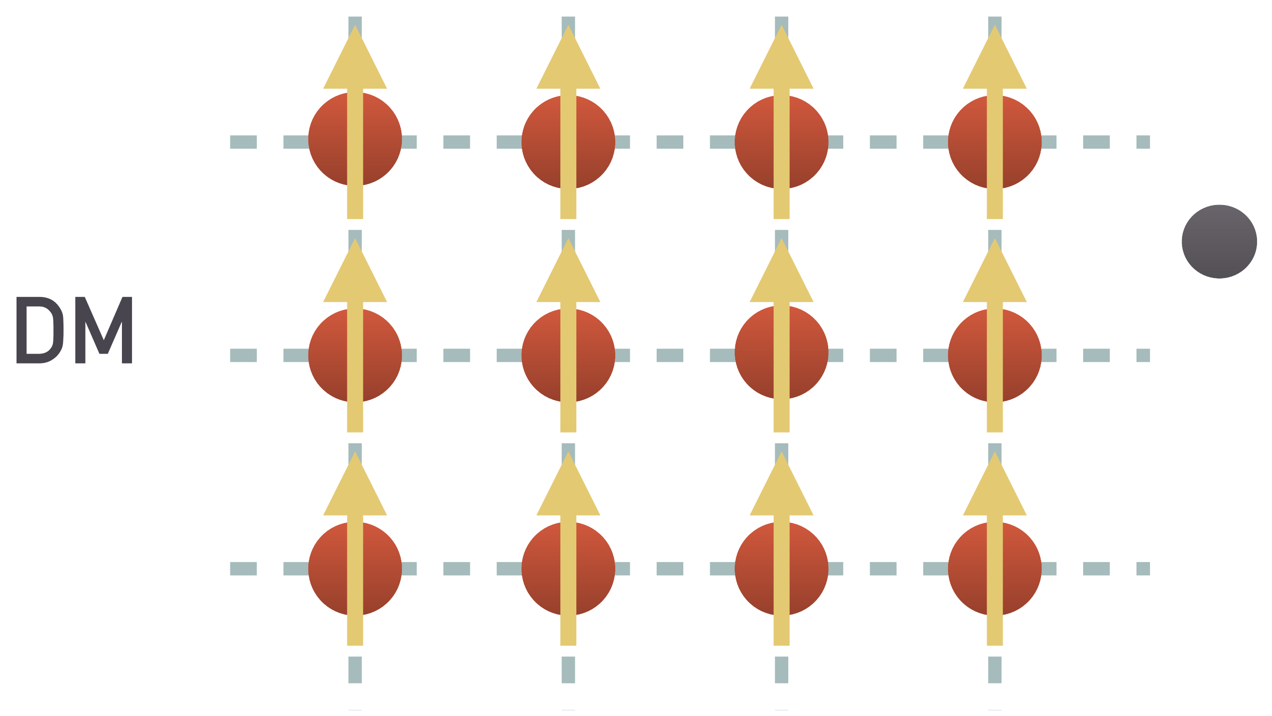


(via N, S, L, $L \otimes S$)



(via S_e, L_e)

See also:
Cirelli, Del Nobile, Panci, 1307.5955.
Anand, Fitzpatrick, Haxton, 1308.6288 + 1405.6690.
Gresham, Zurek, 1401.3739.
Del Nobile, 1806.01291.

Similar calculation for electron excitations in atoms:
Catena, Emken, Spaldin, Tarantino, 1912.08204.

Example

Trickle, ZZ, Zurek, 2009.13534.

- Dark photon mediator models.

$$\mathcal{L} \supset -g_e V_\mu J^\mu_{\text{EM}} + \dots$$

  - Several possibilities on how the DM couples.

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Crystal responses

$$\begin{aligned}
\tilde{\mathcal{V}}_{lj}(-\boldsymbol{q},\boldsymbol{v}) = \sum_{\psi=p,n,e} & c_1^{(\psi)}\langle N_\psi\rangle_{lj} \\
&+ c_3^{(\psi)}\left[-\frac{iq}{m_\psi}\boldsymbol{v}'\cdot\left(\hat{\boldsymbol{q}}\times\langle\boldsymbol{S}_\psi\rangle_{lj}\right)+\frac{q^2}{2m_\psi^2}\left(\delta^{ik}-\hat{q}^i\hat{q}^k\right)\left(\langle\boldsymbol{L}_\psi\otimes\boldsymbol{S}_\psi\rangle_{lj}\right)^{ik}\right] \\
&+ c_4^{(\psi)}\boldsymbol{S}_\chi\cdot\langle\boldsymbol{S}_\psi\rangle_{lj} \\
&+ c_5^{(\psi)}\left[\frac{i\boldsymbol{q}}{m_\psi}\cdot\left(\boldsymbol{v}'\times\boldsymbol{S}_\chi\right)\langle N_\psi\rangle_{lj}+\frac{q^2}{2m_\psi^2}\boldsymbol{S}_\chi\cdot\left(\mathbb{1}-\hat{\boldsymbol{q}}\hat{\boldsymbol{q}}\right)\cdot\langle\boldsymbol{L}_\psi\rangle_{lj}\right] \\
&+ c_6^{(\psi)}\frac{q^2}{m_\psi^2}\left(\hat{\boldsymbol{q}}\cdot\boldsymbol{S}_\chi\right)\left(\hat{\boldsymbol{q}}\cdot\langle\boldsymbol{S}_\psi\rangle_{lj}\right) \\
&+ c_7^{(\psi)}\left[\boldsymbol{v}'\cdot\langle\boldsymbol{S}_\psi\rangle_{lj}+\epsilon^{ikk'}\frac{iq^{k'}}{2m_\chi}\left(\langle\boldsymbol{L}_\psi\otimes\boldsymbol{S}_\psi\rangle_{lj}\right)^{ik}\right] \\
&+ c_8^{(\psi)}\left[\left(\boldsymbol{v}'\cdot\boldsymbol{S}_\chi\right)\langle N_\psi\rangle_{lj}+\frac{iq}{2m_\psi}\boldsymbol{S}_\chi\cdot\left(\hat{\boldsymbol{q}}\times\langle\boldsymbol{L}_\psi\rangle_{lj}\right)\right] \\
&+ c_9^{(\psi)}\frac{iq}{m_\psi}\boldsymbol{S}_\chi\cdot\left(\langle\boldsymbol{S}_\psi\rangle_{lj}\times\hat{\boldsymbol{q}}\right) \\
&+ c_{10}^{(\psi)}\frac{i\boldsymbol{q}}{m_\psi}\cdot\langle\boldsymbol{S}_\psi\rangle_{lj} \\
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&+ c_{13}^{(\psi)}\left[\frac{iq}{m_\psi}\left(\boldsymbol{v}'\cdot\boldsymbol{S}_\chi\right)\left(\hat{\boldsymbol{q}}\cdot\langle\boldsymbol{S}_\psi\rangle_{lj}\right)+\frac{q^2}{2m_\psi^2}\left(\hat{\boldsymbol{q}}\times\boldsymbol{S}_\chi\right)\cdot\langle\boldsymbol{L}_\psi\otimes\boldsymbol{S}_\psi\rangle_{lj}\cdot\hat{\boldsymbol{q}}\right] \\
&+ c_{14}^{(\psi)}\left[\frac{iq}{m_\psi}\left(\hat{\boldsymbol{q}}\cdot\boldsymbol{S}_\chi\right)\left(\boldsymbol{v}'\cdot\langle\boldsymbol{S}_\psi\rangle_{lj}\right)-\epsilon^{ikk'}\frac{q^2}{2m_\psi^2}\hat{q}^{k'}\left(\hat{\boldsymbol{q}}\cdot\boldsymbol{S}_\chi\right)\left(\langle\boldsymbol{L}_\psi\otimes\boldsymbol{S}_\psi\rangle_{lj}\right)^{ik}\right] \\
&+ c_{15}^{(\psi)}\left[-\frac{q^2}{m_\psi^2}\left(\hat{\boldsymbol{q}}\cdot\left(\boldsymbol{v}'\times\boldsymbol{S}_\chi\right)\right)\left(\hat{\boldsymbol{q}}\cdot\langle\boldsymbol{S}_\psi\rangle_{lj}\right)\right. \\
&\qquad\left.+\frac{iq^3}{2m_\psi^3}\boldsymbol{S}_\chi\cdot\left(\mathbb{1}-\hat{\boldsymbol{q}}\hat{\boldsymbol{q}}\right)\cdot\langle\boldsymbol{L}_\psi\otimes\boldsymbol{S}_\psi\rangle_{lj}\cdot\hat{\boldsymbol{q}}\right],
\end{aligned}$$

Example

Trickle, ZZ, Zurek, 2009.13534.

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$$c_5^{(\psi)} = \frac{m_\psi}{m_\chi}\frac{g_\chi g_\psi^{\text{eff}}}{\boldsymbol{q}^2 + m_V^2}$$

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Anapole DM

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Crystal responses

$$\begin{aligned}
\widetilde{\mathcal{V}}_{lj}(-\boldsymbol{q}, \boldsymbol{v}) = \sum_{\psi=p,n,e} & \boxed{c_1^{(\psi)} \langle N_\psi \rangle_{lj}} \\
& + c_3^{(\psi)}\left[-\frac{iq}{m_\psi}\boldsymbol{v}' \cdot \left(\hat{\boldsymbol{q}} \times \langle \boldsymbol{S}_\psi \rangle_{lj}\right) + \frac{q^2}{2m_\psi^2}\left(\delta^{ik} - \hat{q}^i\hat{q}^k\right)\left(\langle \boldsymbol{L}_\psi \otimes \boldsymbol{S}_\psi \rangle_{lj}\right)^{ik}\right] \\
& + c_4^{(\psi)} \boldsymbol{S}_\chi \cdot \langle \boldsymbol{S}_\psi \rangle_{lj} \\
& + c_5^{(\psi)}\left[\frac{i\boldsymbol{q}}{m_\psi} \cdot \left(\boldsymbol{v}' \times \boldsymbol{S}_\chi\right)\langle N_\psi \rangle_{lj} + \frac{q^2}{2m_\psi^2}\boldsymbol{S}_\chi \cdot \left(\mathbb{1} - \hat{\boldsymbol{q}}\hat{\boldsymbol{q}}\right) \cdot \langle \boldsymbol{L}_\psi \rangle_{lj}\right] \\
& + c_6^{(\psi)}\frac{q^2}{m_\psi^2}\left(\hat{\boldsymbol{q}} \cdot \boldsymbol{S}_\chi\right)\left(\hat{\boldsymbol{q}} \cdot \langle \boldsymbol{S}_\psi \rangle_{lj}\right) \\
& + c_7^{(\psi)}\left[\boldsymbol{v}' \cdot \langle \boldsymbol{S}_\psi \rangle_{lj} + \epsilon^{ikk'}\frac{iq^{k'}}{2m_\chi}\left(\langle \boldsymbol{L}_\psi \otimes \boldsymbol{S}_\psi \rangle_{lj}\right)^{ik}\right] \\
& + c_8^{(\psi)}\left[\left(\boldsymbol{v}' \cdot \boldsymbol{S}_\chi\right)\langle N_\psi \rangle_{lj} + \frac{iq}{2m_\psi}\boldsymbol{S}_\chi \cdot \left(\hat{\boldsymbol{q}} \times \langle \boldsymbol{L}_\psi \rangle_{lj}\right)\right] \\
& + c_9^{(\psi)}\frac{iq}{m_\psi}\boldsymbol{S}_\chi \cdot \left(\langle \boldsymbol{S}_\psi \rangle_{lj} \times \hat{\boldsymbol{q}}\right) \\
& + c_{10}^{(\psi)}\frac{i\boldsymbol{q}}{m_\psi} \cdot \langle \boldsymbol{S}_\psi \rangle_{lj} \\
& \boxed{+ c_{11}^{(\psi)}\frac{i\boldsymbol{q}}{m_\psi} \cdot \boldsymbol{S}_\chi \langle N_\psi \rangle_{lj}} \\
& + c_{12}^{(\psi)}\left[\left(\boldsymbol{v}' \times \boldsymbol{S}_\chi\right) \cdot \langle \boldsymbol{S}_\psi \rangle_{lj} + \frac{iq}{2m_\psi}\left(\left(\hat{\boldsymbol{q}} \cdot \boldsymbol{S}_\chi\right)\delta^{ik} - \hat{q}^k S_\chi^i\right)\left(\langle \boldsymbol{L}_\psi \otimes \boldsymbol{S}_\psi \rangle_{lj}\right)^{ik}\right] \\
& + c_{13}^{(\psi)}\left[\frac{iq}{m_\psi}\left(\boldsymbol{v}' \cdot \boldsymbol{S}_\chi\right)\left(\hat{\boldsymbol{q}} \cdot \langle \boldsymbol{S}_\psi \rangle_{lj}\right) + \frac{q^2}{2m_\psi^2}\left(\hat{\boldsymbol{q}} \times \boldsymbol{S}_\chi\right) \cdot \langle \boldsymbol{L}_\psi \otimes \boldsymbol{S}_\psi \rangle_{lj} \cdot \hat{\boldsymbol{q}}\right] \\
& + c_{14}^{(\psi)}\left[\frac{iq}{m_\psi}\left(\hat{\boldsymbol{q}} \cdot \boldsymbol{S}_\chi\right)\left(\boldsymbol{v}' \cdot \langle \boldsymbol{S}_\psi \rangle_{lj}\right) - \epsilon^{ikk'}\frac{q^2}{2m_\psi^2}\hat{q}^{k'}\left(\hat{\boldsymbol{q}} \cdot \boldsymbol{S}_\chi\right)\left(\langle \boldsymbol{L}_\psi \otimes \boldsymbol{S}_\psi \rangle_{lj}\right)^{ik}\right] \\
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& + c_{14}^{(\psi)}\left[\frac{iq}{m_\psi}\left(\hat{\boldsymbol{q}}\cdot\boldsymbol{S}_\chi\right)\left(\boldsymbol{v}'\cdot\langle\boldsymbol{S}_\psi\rangle_{lj}\right) - \epsilon^{ikk'}\frac{q^2}{2m_\psi^2}\hat{q}^{k'}\left(\hat{\boldsymbol{q}}\cdot\boldsymbol{S}_\chi\right)\left(\langle\boldsymbol{L}_\psi\otimes\boldsymbol{S}_\psi\rangle_{lj}\right)^{ik}\right] \\
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N response $\Rightarrow$ phonons.

$\boldsymbol{S}, \boldsymbol{L}$ responses $\Rightarrow$ magnons.

Crystal responses

$$
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& + c_6^{(\psi)}\frac{q^2}{m_\psi^2}\left(\hat{\boldsymbol{q}}\cdot\boldsymbol{S}_\chi\right)\left(\hat{\boldsymbol{q}}\cdot\langle\boldsymbol{S}_\psi\rangle_{lj}\right) \\
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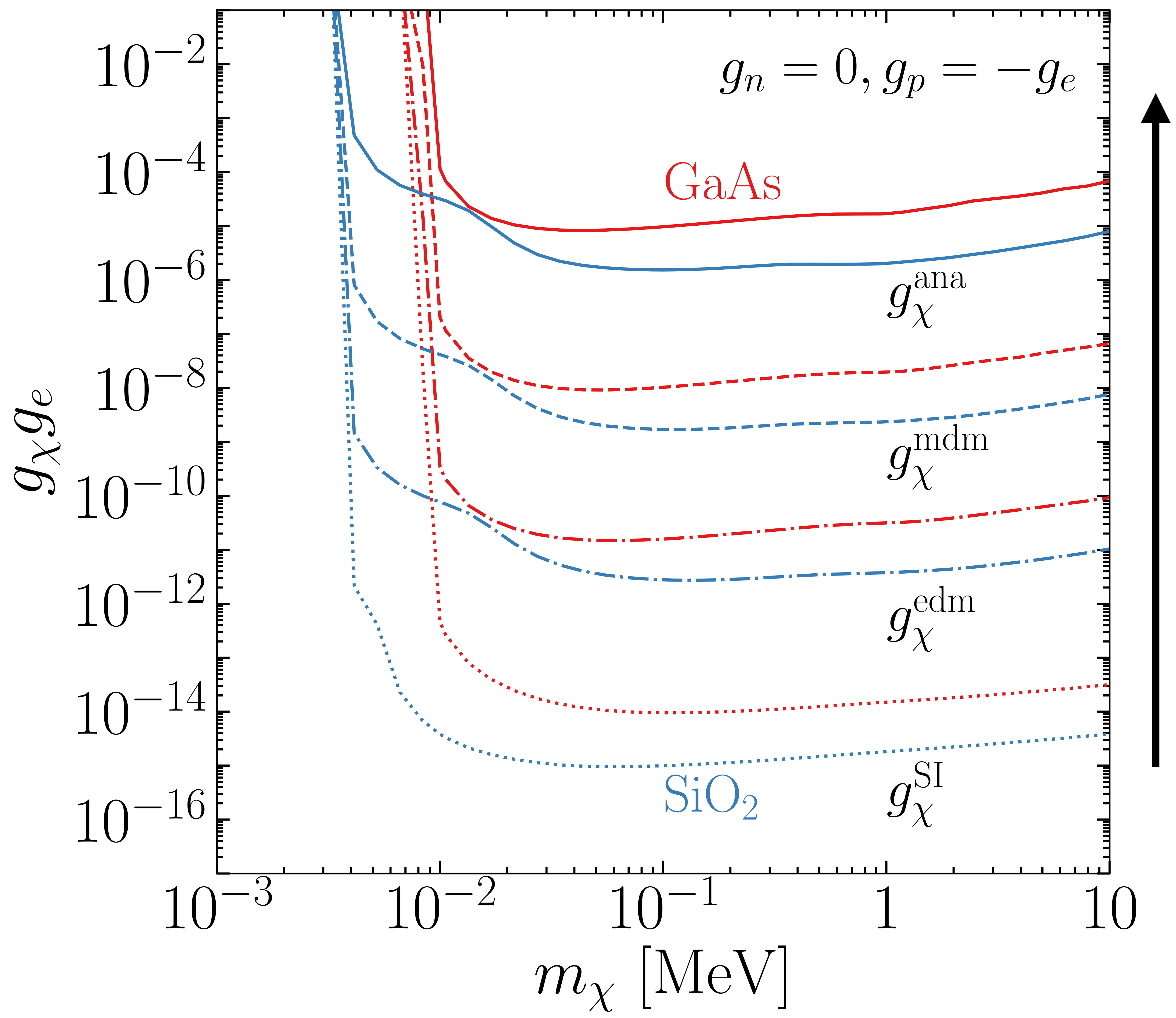
Example

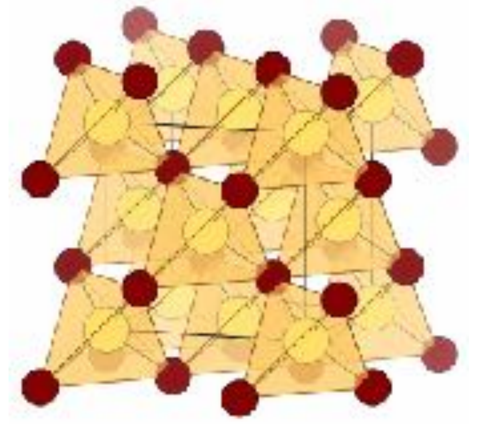
Trickle, ZZ, Zurek, 2009.13534.

➤ Phonon reach for kg-yr exposure, assuming background-free.



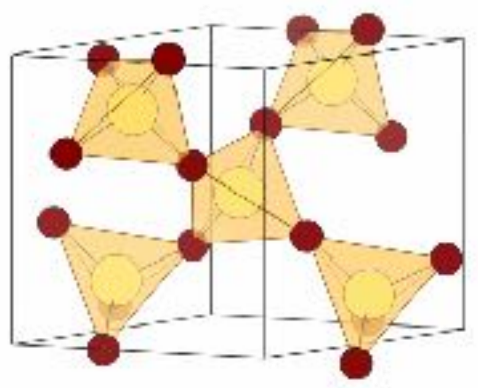
successively higher order in q

GaAs



Used in the SPICE experiment.

SiO$_2$



“Optimal” phonon target.

Example

Trickle, ZZ, Zurek, 2009.13534.

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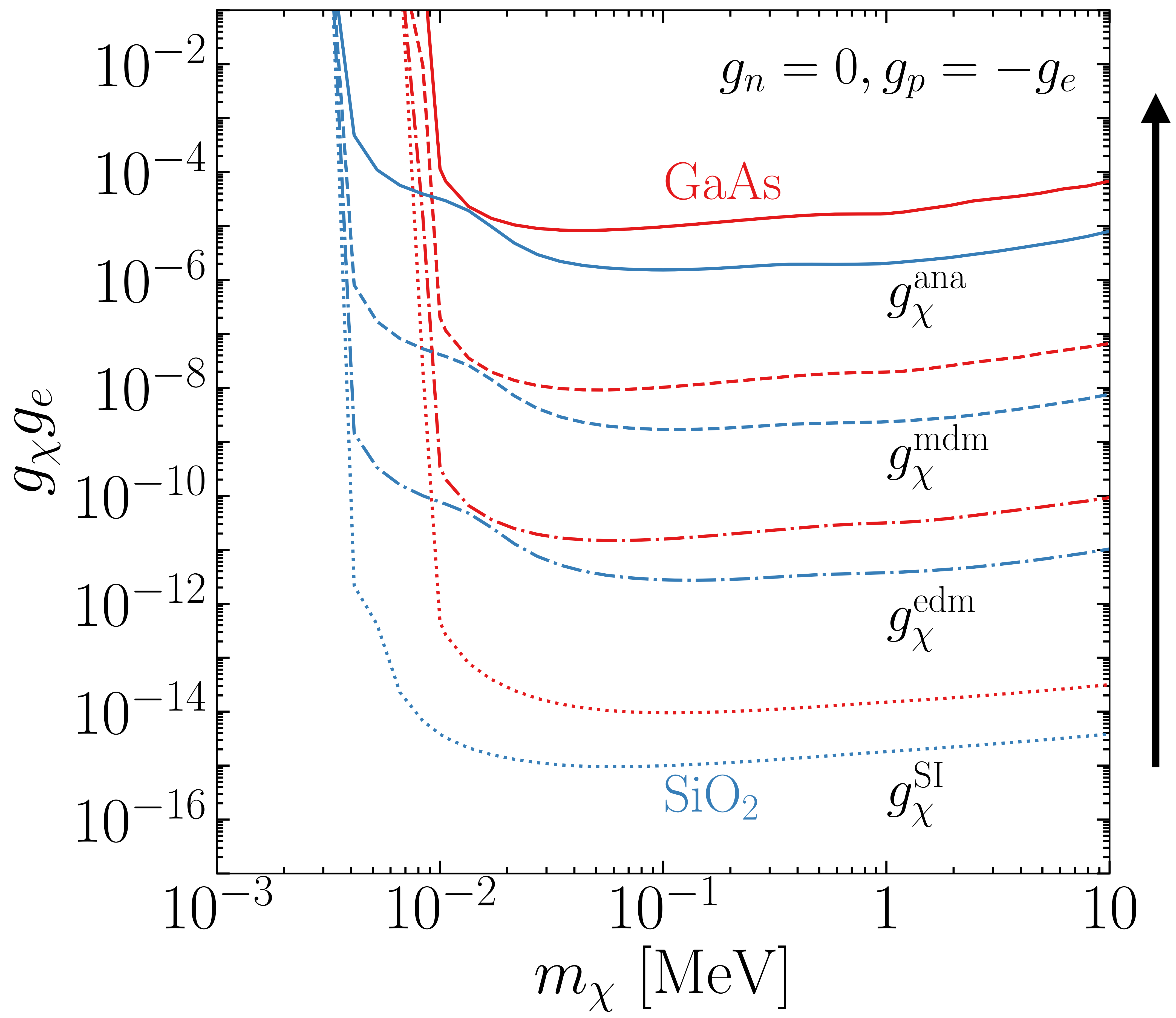

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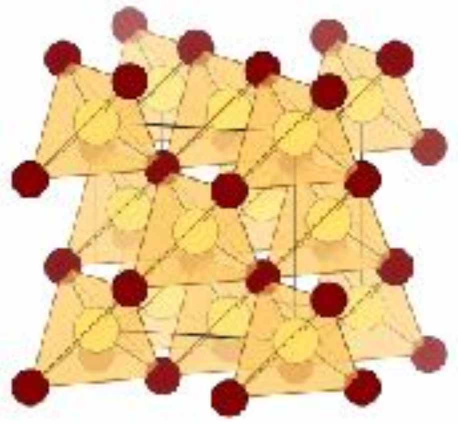

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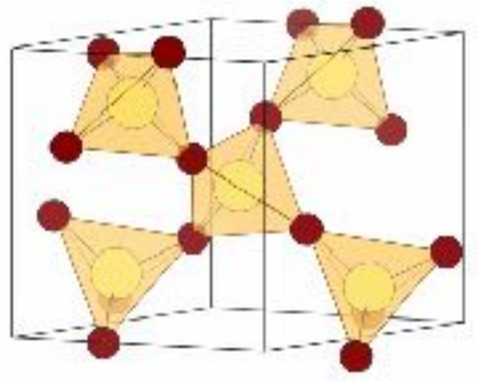

"Optimal" phonon target.

These targets do not have S or L order.

$\Rightarrow$ Phonon excitations only via the N response

(DM couplings to particle numbers).

Example

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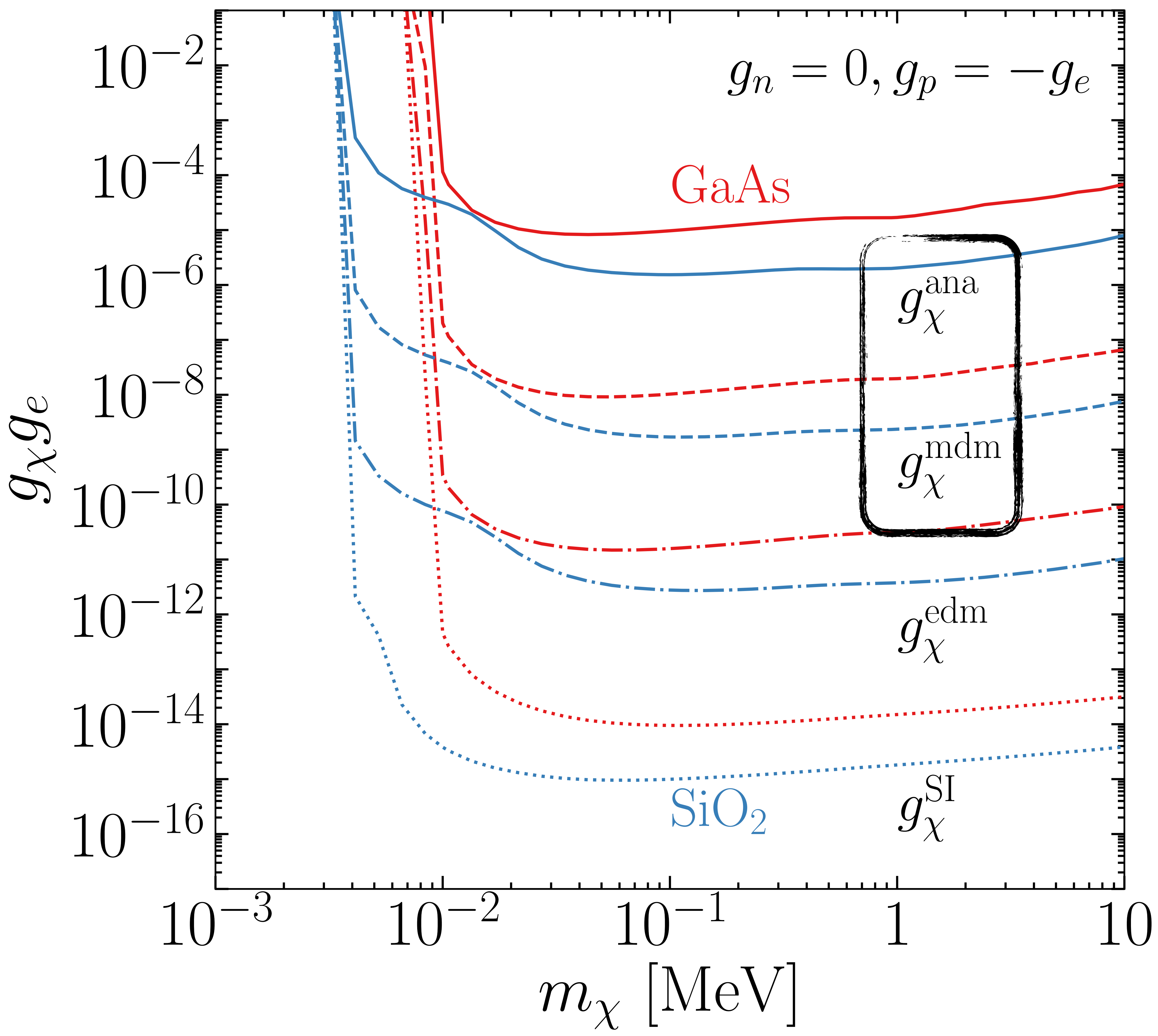


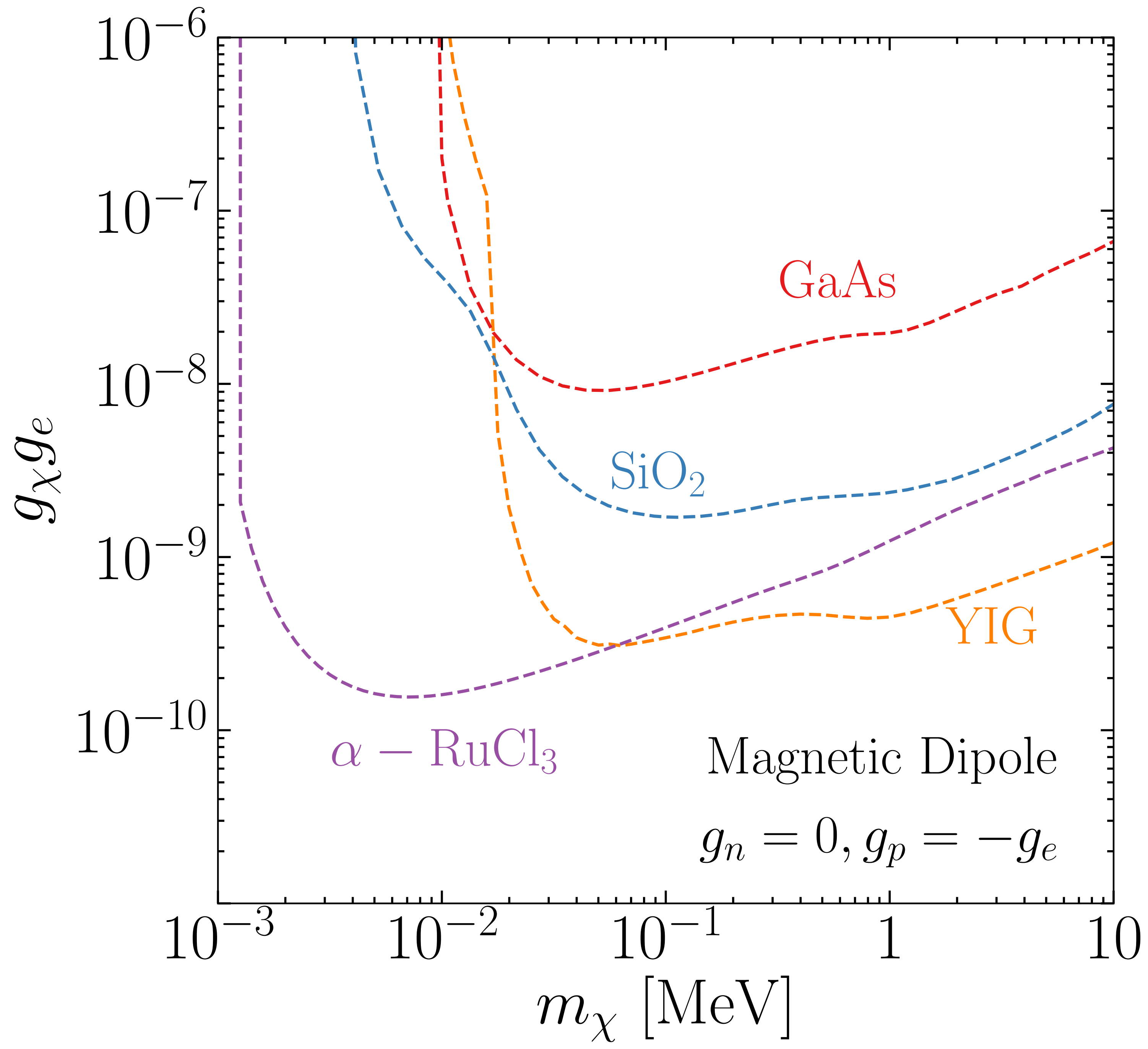
These models also generate couplings to $\boldsymbol{S}$ and $\boldsymbol{L}$.

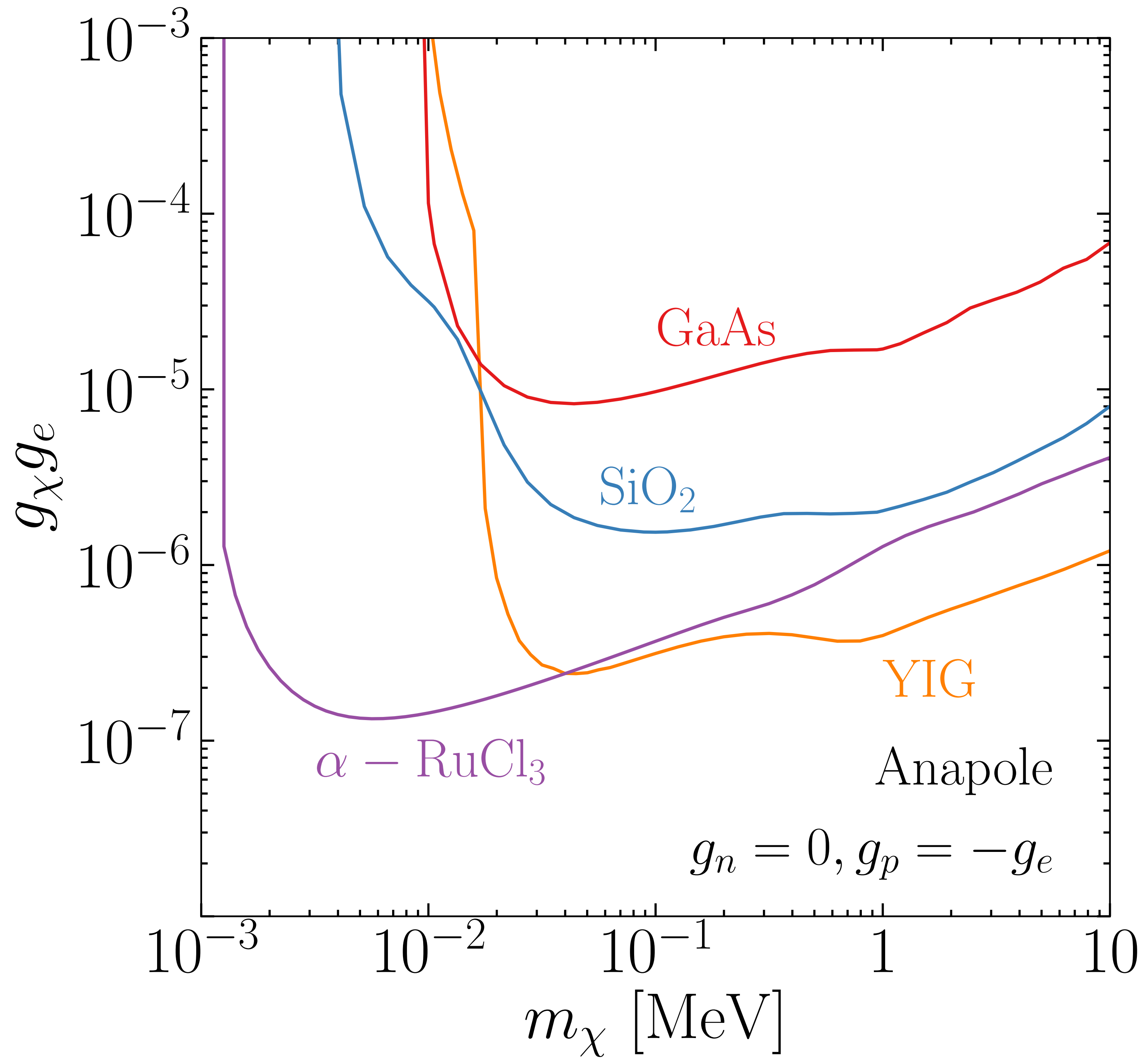
$\Rightarrow$ Best probed by magnons.

Example

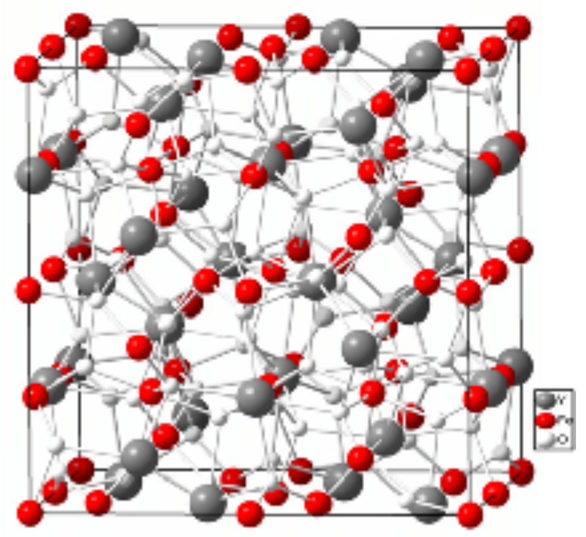
Trickle, ZZ, Zurek, 2009.13534.

- Zoom in on these two models.
- Compare phonon reach (from previous plot) vs. magnon reach.





YIG

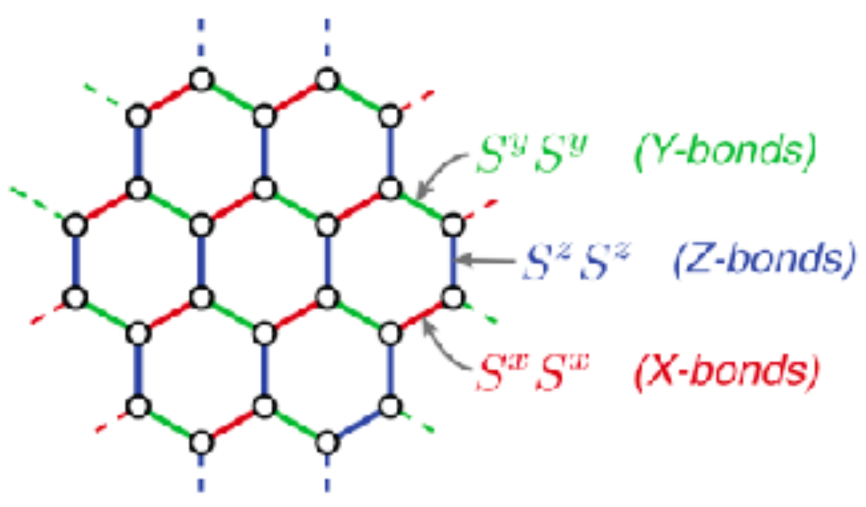


($Y_3Fe_5O_{12}$)

S order.

Used in QUAX.

α-$RuCl_3$



S & ***L*** order.

Features low energy gapped magnons.

Example

Trickle, ZZ, Zurek, 2009.13534.

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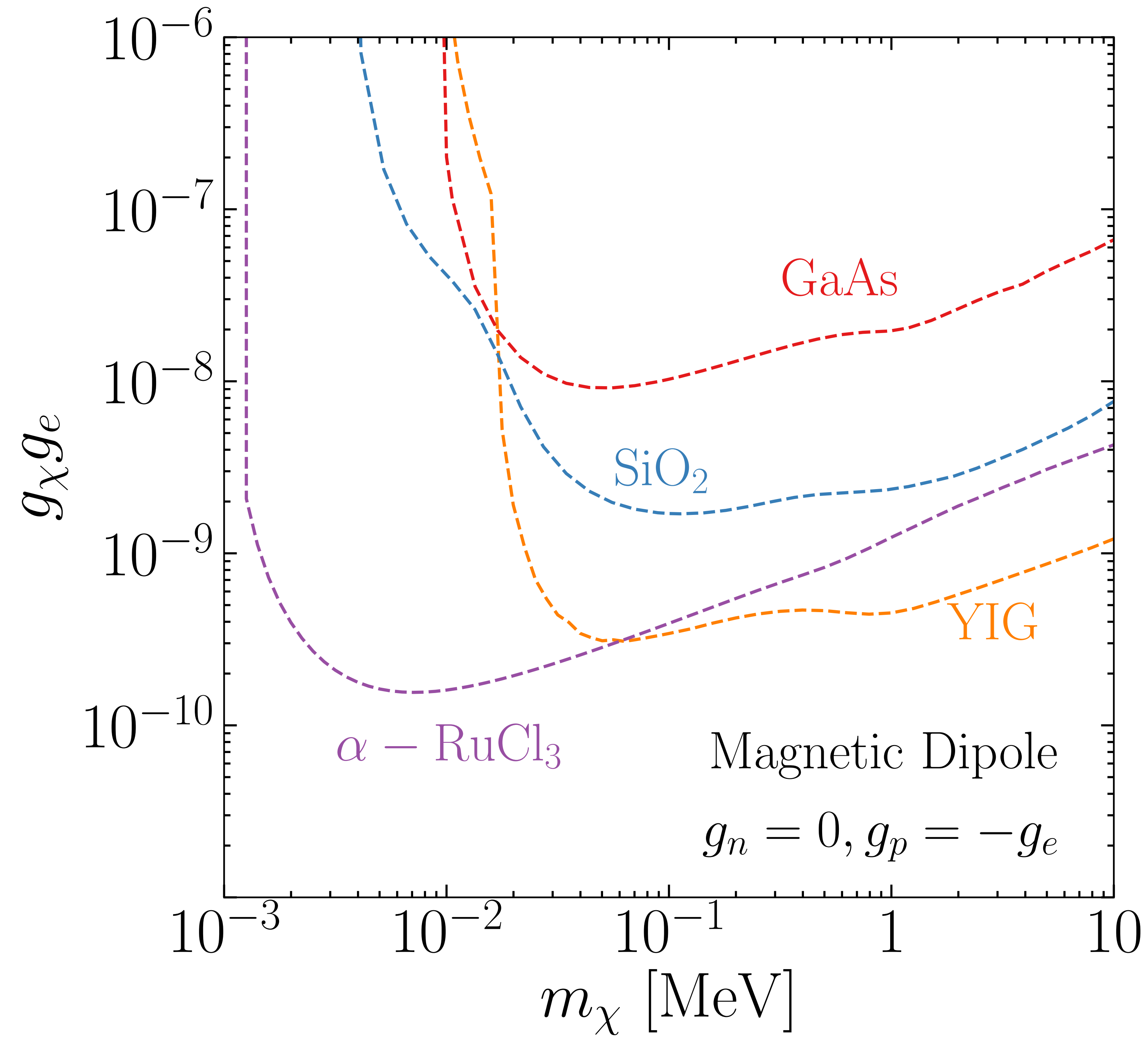
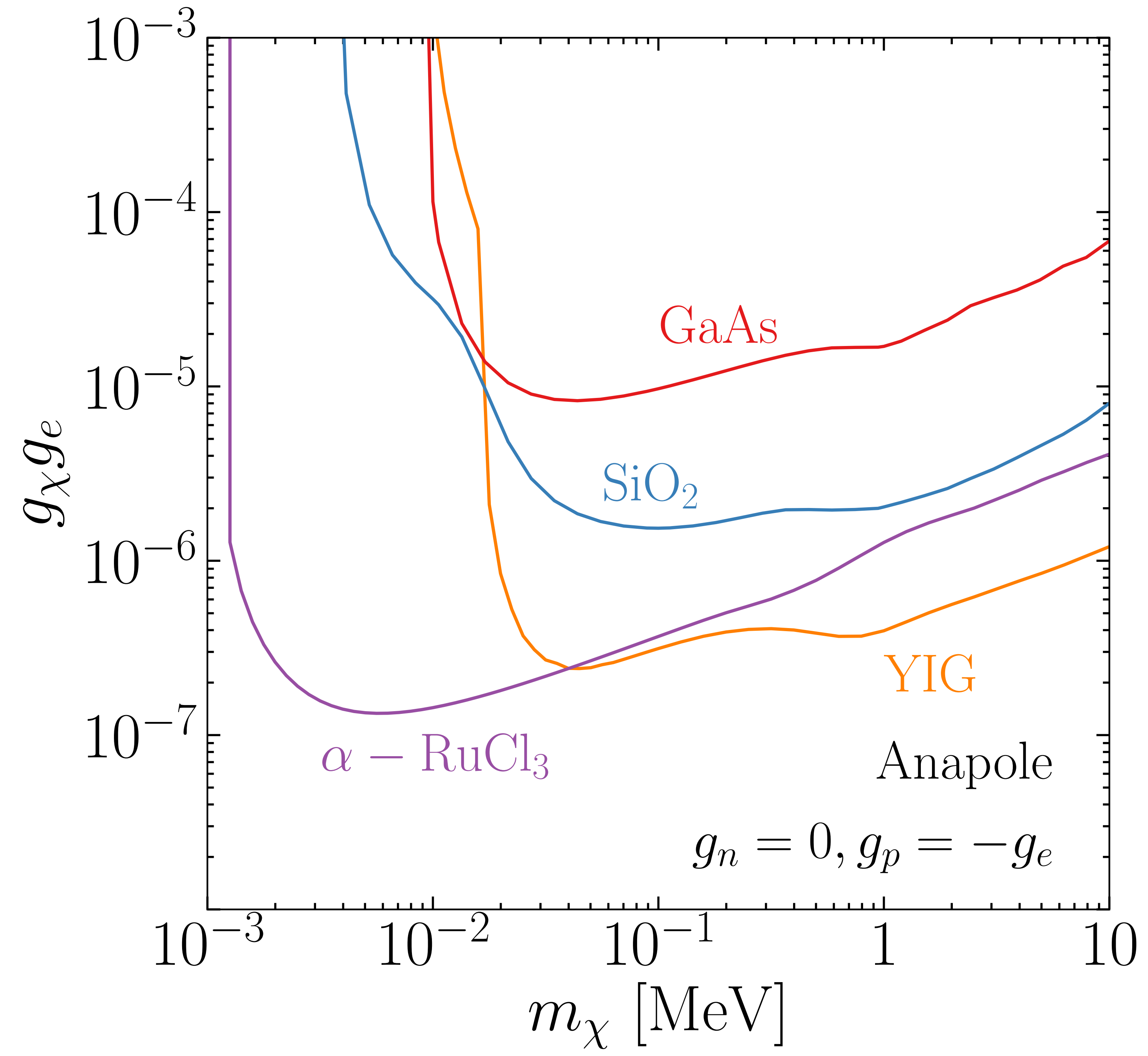
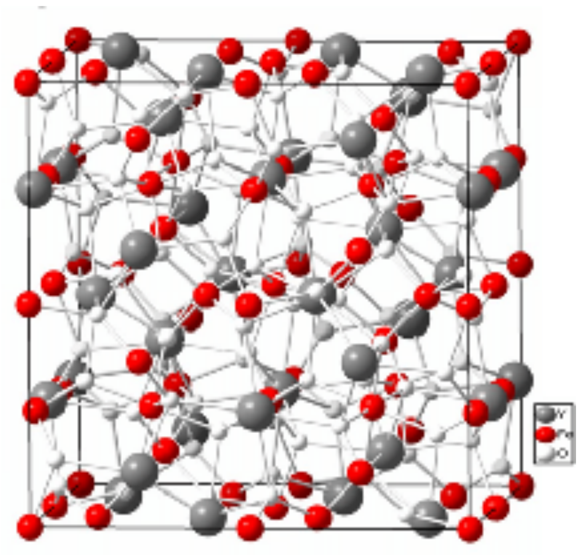




YIG



($Y_3Fe_5O_{12}$)

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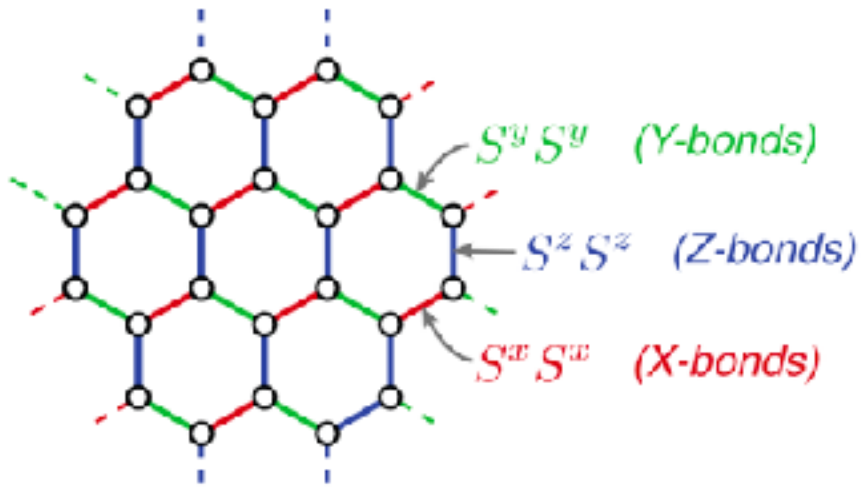
α-$RuCl_3$



S & ***L*** order.

Features low energy gapped magnons.

- Magnon reach is parametrically better, but SiO_2 (optimal phonon target) is not too far behind.
- Encouraging for the technically more mature phonon experiments.

Take-home messages

Collective excitations such as phonons and magnons offer a novel path to detect light DM.

New experiments such as SPICE (TESSERACT) are moving forward.

We have developed the EFT tools for computing detection rates for general DM models.

THANK YOU