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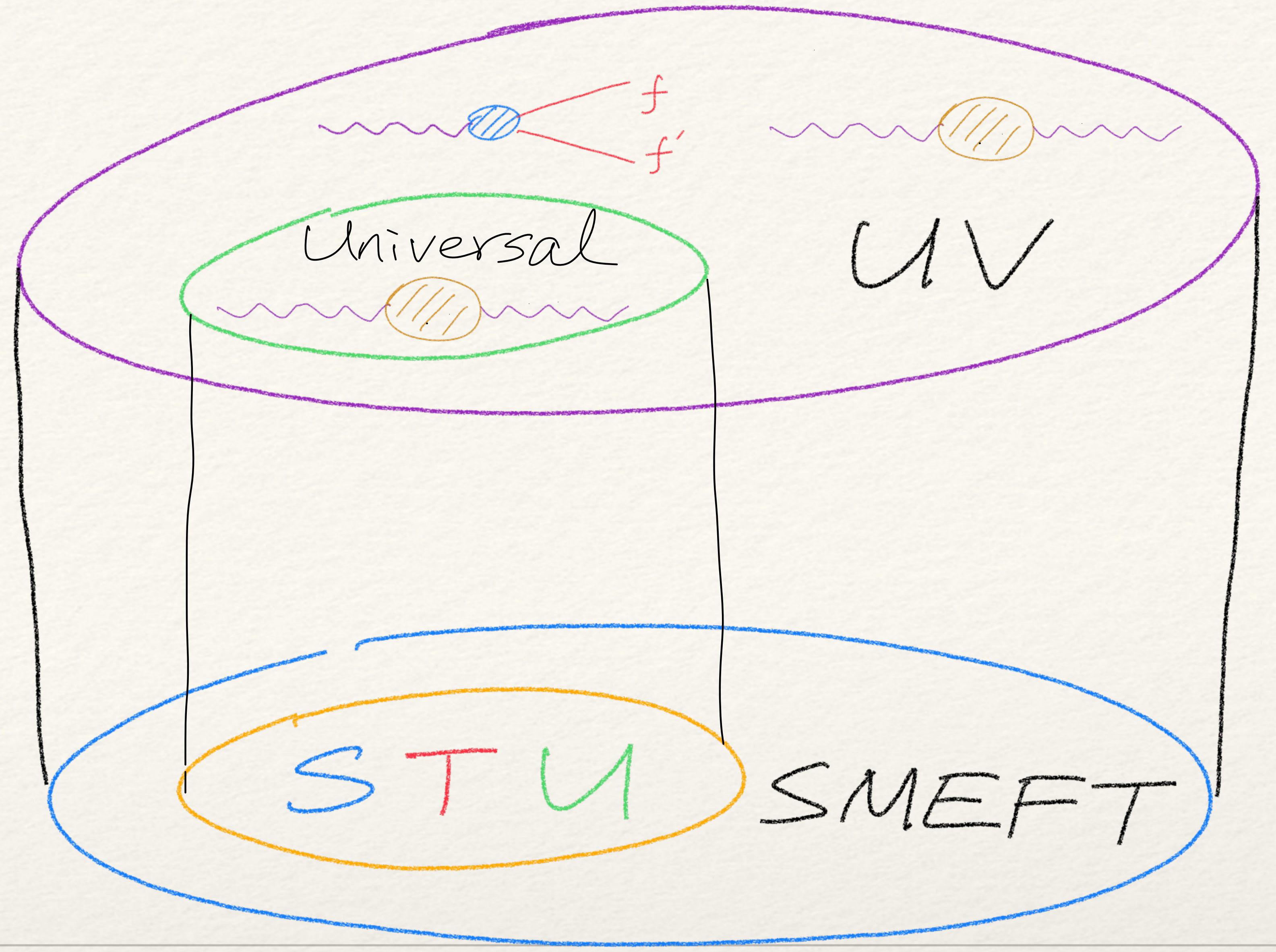
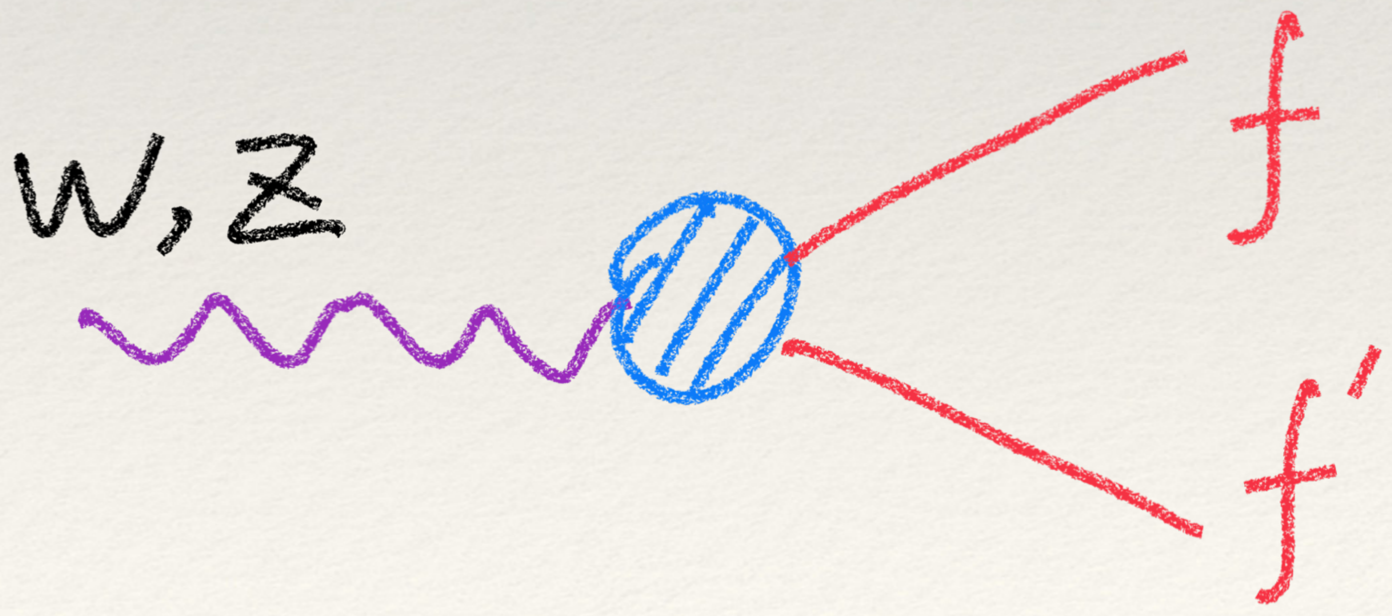


Custodial Symmetry Beyond The *Oblique*

arXiv: 2009.10725 with Graham Kribs, Xiaochuan Lu, Adam Martin



Custodial Symmetry

- The Higgs potential is invariant under

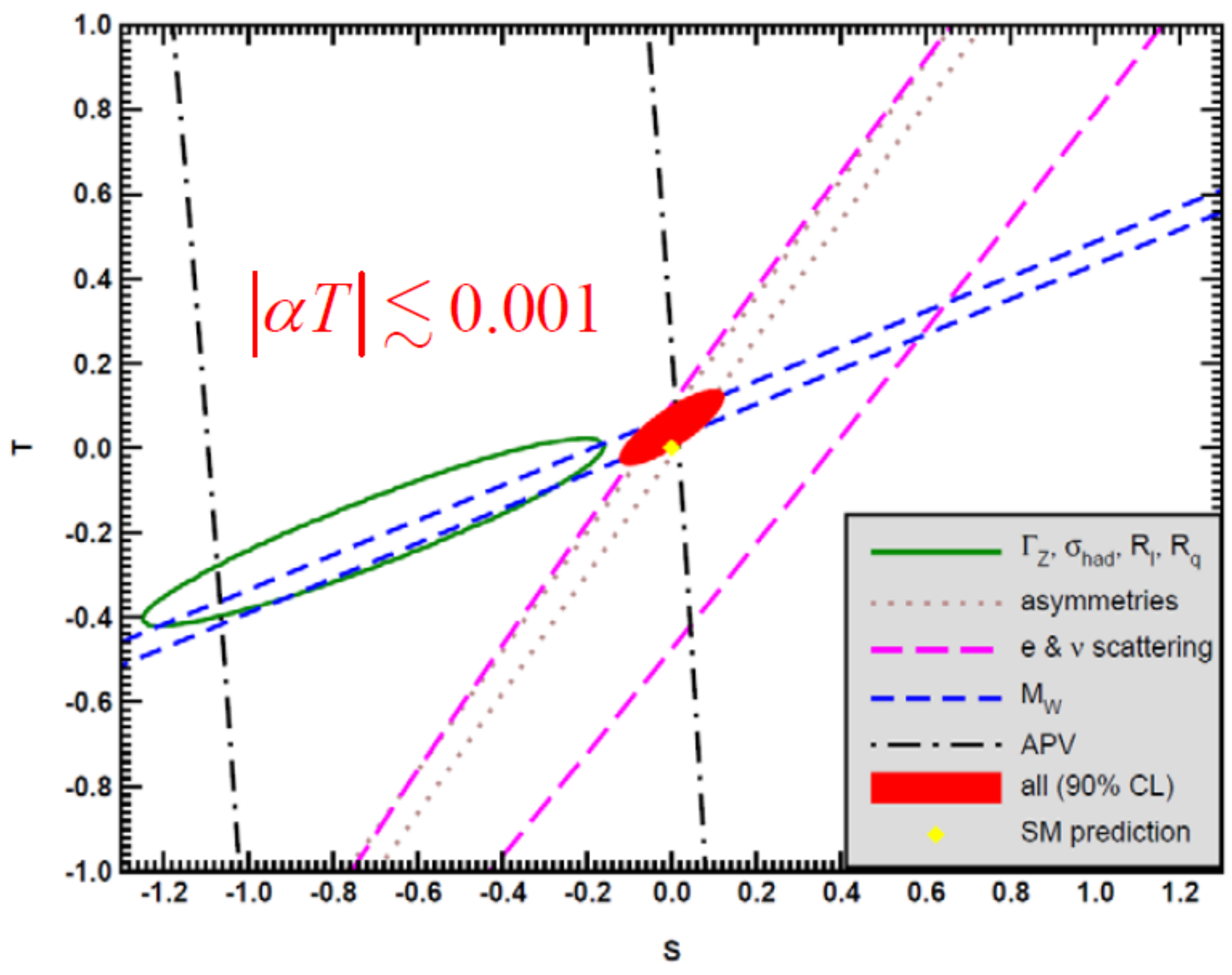
$$SO(4) \sim SU(2)_L \times SU(2)_R \longrightarrow SO(3) \sim SU(2)_V$$

- UV theories that violate custodial symmetry are generally believed to be severely constrained ($\Lambda \gtrsim 10$ TeV) by data from electroweak precision measurements.

- We are interested in the robustness of this result in the context of SMEFT @ dim-6.

Is BSM physics custodial symmetric?

$$\alpha T = \frac{\Pi_{WW}(0) - \Pi_{33}(0)}{M_W^2} \sim 0 \qquad \rho = \frac{M_W^2}{M_Z^2 \cos^2\theta_W} = 1 + \alpha T \sim 1$$



31 *10. Electroweak Model and Constraints on New Physics*

The dominant effect of many extensions of the SM can be described by the ρ_0 parameter,

$$\rho_0 \equiv \frac{M_W^2}{M_Z^2 \hat{c}_Z^2 \hat{\rho}}, \tag{10.66}$$

which describes new sources of SU(2) breaking that cannot be accounted for by the SM Higgs doublet or by m_t effects. $\hat{\rho}$ is calculated as in Eq. (10.18) assuming the validity of the SM. In the presence of $\rho_0 \neq 1$, Eq. (10.66) generalizes the second Eq. (10.18) while the first remains unchanged. Provided that the new physics which yields $\rho_0 \neq 1$ is a small perturbation which does not significantly affect other radiative corrections, ρ_0 can be regarded as a phenomenological parameter which multiplies G_F in Eqs. (10.21) and (10.41), as well as Γ_Z in Eq. (10.60c). There are enough data to determine ρ_0, M_H, m_t, and α_s, simultaneously. From the global fit,

$$\rho_0 = 1.00038 \pm 0.00020, \tag{10.67a}$$

$$\alpha_s(M_Z) = 0.1188 \pm 0.0017, \tag{10.67b}$$

A heavy non-degenerate multiplet of fermions or scalars contributes positively to T as

$$\rho_0 - 1 = \frac{1}{1 - \hat{\alpha}(M_Z)T} - 1 \approx \hat{\alpha}(M_Z)T, \tag{10.74}$$

Particle Data Group Collaboration, P. Zyla et al., "Review of Particle Physics," *PETP* 2020 (2020) no. 8, 083C01.

Custodial Symmetry: Peskin-Takeuchi

- As Peskin and Takeuchi had correctly pointed out, there are two different ρs.

$$\rho = \frac{M_W^2}{M_Z^2 \cos^2\theta_W}$$ is called the Veltman ρ

$$\rho = 1 + \frac{\alpha}{\cos 2\theta}\left(-\frac{1}{2}S + \cos^2\theta\, T + \frac{\cos 2\theta}{4\sin^2\theta}U\right)$$

T parameter is defined by $\rho_*(0)$

$$\rho_*(0) = 1 + \alpha T$$

where $\rho_*(0) = \dfrac{\mathit{Charged\ Current}}{\mathit{Neutral\ Current}}$

in the zero-momentum limit.

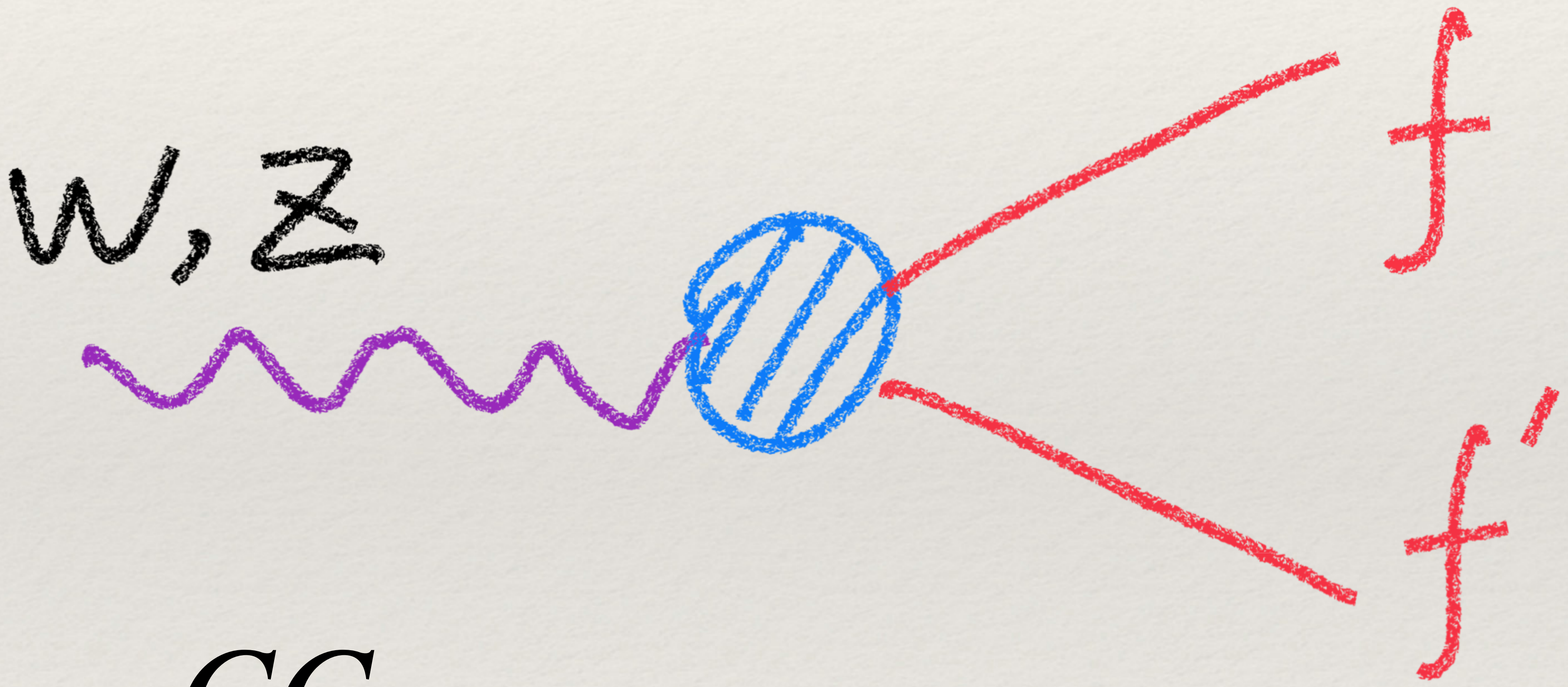
Custodial Symmetry: Universal Theories

- The electroweak precision parameters *S, T, U* work properly only under the ***oblique assumption***: all the corrections from heavy new physics are in the gauge boson 2-point functions.

- Those UV theories following the ***oblique assumption*** are called Universal Theories.

Custodial Symmetry: Non-Universal Theories

- Non-Universal Theories do not follow the ***oblique assumption.***
- They have **vertex corrections** from heavy new physics, which means that S, T, U are incomplete and problematic.



- Specifically, $\rho_*(0) = \frac{CC}{NC}$ is no longer uniquely defined in a Non-Universal Theory. It depends on the fermion species.

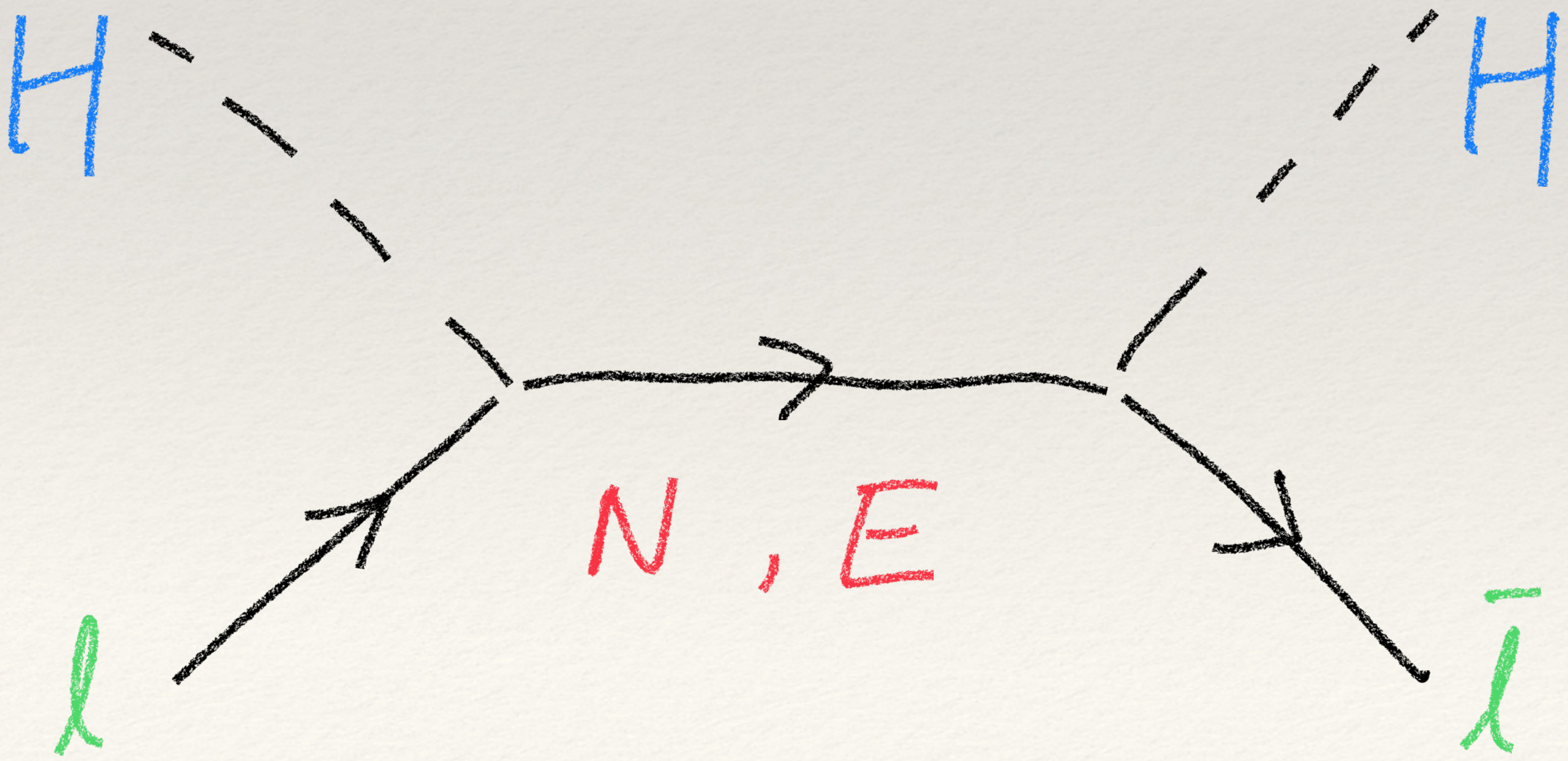
Example: Vector-like Fermions (Non-Universal)

$$\mathcal{L}_{\rm UV} = \mathcal{L}_{\rm SM} + \bar{N}(i\not{D} - M)N + \bar{E}(i\not{D} - M)E - \left(Y_N\, \bar{l}\tilde{H}N + Y_E\, \bar{l}HE + \text{h.c.}\right) .$$

- Matching at the leading order, this theory generates

$$\mathscr{L}_{SMEFT} \supset \left(H^\dagger i D_\mu H\right)\left(\bar{l}\gamma^\mu l\right)$$

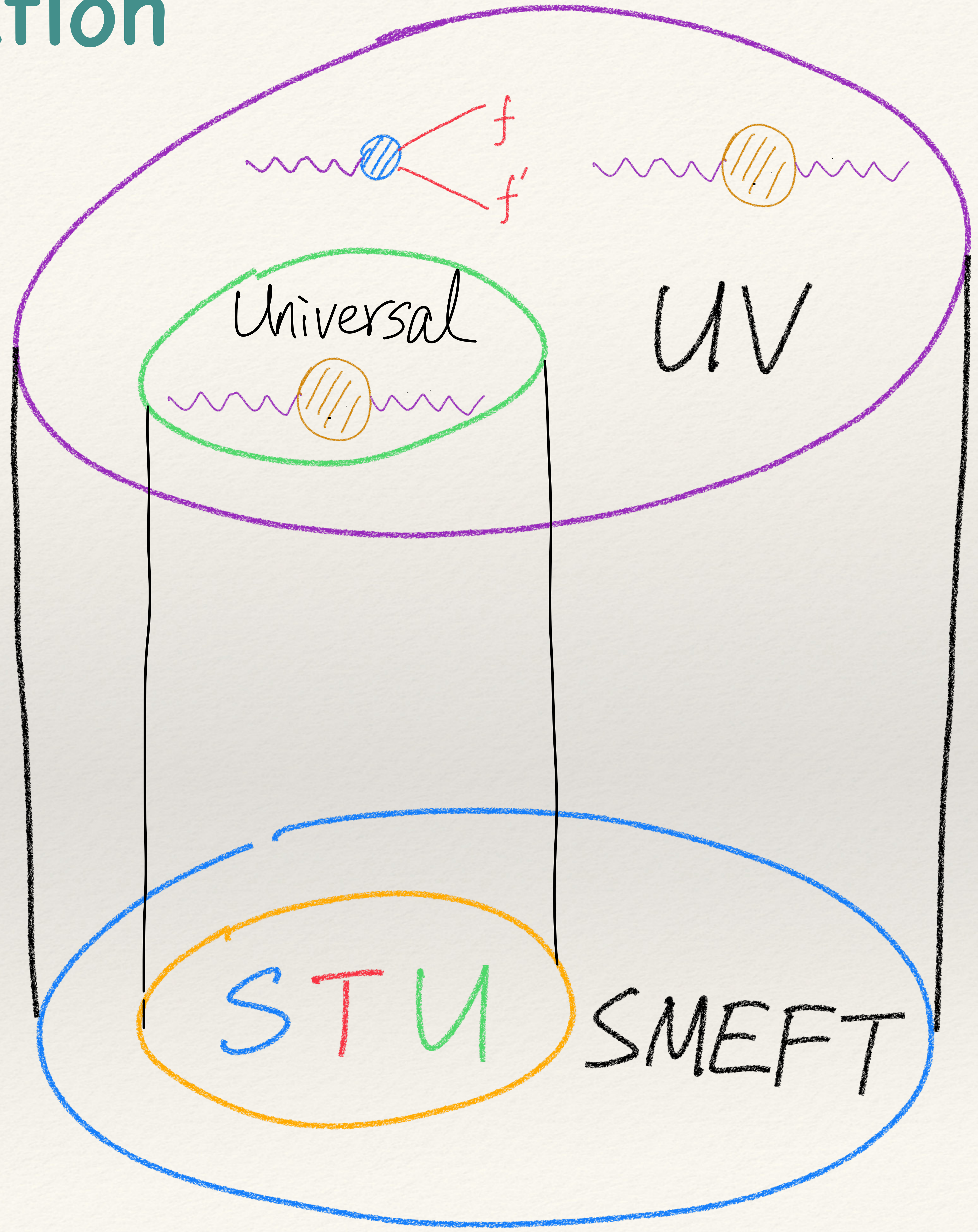
Custodial Violating !



$$\alpha T = \frac{\Pi_{WW}(0) - \Pi_{33}(0)}{M_W^2} = 0$$

Our approach toward a resolution

- Define custodial symmetry in the UV
- Custodial Basis of SMEFT @ dim-6
- Map onto observables @ tree level
- Find the correlations between them when custodial symmetry is imposed
- Construct a generalization to the *T*



Custodial Symmetry in the UV

- UV physics is **custodial symmetric** when there is a global $SU(2)_R$ symmetry preserved, in the limit $g_1 \to 0$, by all UV interactions with the Higgs sector of the SM.

- The breakings of **custodial** $SU(2)_R$ by UV interactions:
  1. "Soft": **vanish** in the limit $g_1 \to 0$
  2. "Hard": **persist** in the limit $g_1 \to 0$

Custodial Basis of νSMEFT

- Warsaw Basis of dim-6 SMEFT, with right-handed neutrinos included, extended to manifest $SU(2)_L \times SU(2)_R$ symmetry.
- Writing $\Sigma = \begin{pmatrix} \tilde{H} & H \end{pmatrix}$, the Higgs $(2,2)$ bifundamental scalar.
- Example: Two operators with hard custodial breaking (τ_R^3).

$$C_{HD}\, Q_{HD} \longrightarrow a_{HD}\, O_{HD} = a_{HD} \left[Tr \left(\Sigma^\dagger i D_\mu \Sigma \tau_R^3 \right) \right]^2$$

$$C_{Hl}^{(1)}\, Q_{Hl}^{(1)} \longrightarrow a_{Hl}^{(1)}\, O_{Hl}^{(1)} = a_{Hl}^{(1)} \left[Tr \left(\Sigma^\dagger i D_\mu \Sigma \tau_R^3 \right) \left(\bar{l} \gamma^\mu l \right) \right]$$

Custodial Basis of νSMEFT

- Based on the Warsaw Basis of dim-6 SMEFT
- Includes right-handed neutrinos (νSMEFT)
- The red operators violate custodial symmetry with hard breakings
- The operators circled by purple are relevant to us

$1: X^3$	
O_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$
$O_{\widetilde{G}}$	$f^{ABC} \widetilde{G}_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$
O_W	$\epsilon^{abc} W_\mu^{a\nu} W_\nu^{b\rho} W_\rho^{c\mu}$
$O_{\widetilde{W}}$	$\epsilon^{abc} \widetilde{W}_\mu^{a\nu} W_\nu^{b\rho} W_\rho^{c\mu}$

$2: H^6$	
O_H	$\left[\mathrm{tr}\,(\Sigma^\dagger\Sigma)\right]^3$

$3: H^4 D^2$	
$O_{H\Box}$	$\left[\mathrm{tr}\,(\Sigma^\dagger iD_\mu\Sigma)\right]^2$
O_{HD}	$\left[\mathrm{tr}\,(\Sigma^\dagger iD_\mu\Sigma\tau_R^3)\right]^2$

$5: \bar\psi\psi H^3 + \mathrm{h.c.}$	
$O_{lH}^{\pm}$	$\mathrm{tr}(\Sigma^\dagger\Sigma)\left(\bar{l}\Sigma P_\pm l_R\right)$
$O_{qH}^{\pm}$	$\mathrm{tr}(\Sigma^\dagger\Sigma)\left(\bar{q}\Sigma P_\pm q_R\right)$

$4: X^2 H^2$	
O_{HG}	$\mathrm{tr}\,(\Sigma^\dagger\Sigma)\, G_{\mu\nu}^A G^{A\mu\nu}$
$O_{H\widetilde{G}}$	$\mathrm{tr}\,(\Sigma^\dagger\Sigma)\, \widetilde{G}_{\mu\nu}^A G^{A\mu\nu}$
O_{HW}	$\mathrm{tr}\,(\Sigma^\dagger\Sigma)\, W_{\mu\nu}^a W^{a\mu\nu}$
$O_{H\widetilde{W}}$	$\mathrm{tr}\,(\Sigma^\dagger\Sigma)\, \widetilde{W}_{\mu\nu}^a W^{a\mu\nu}$
O_{HB}	$\mathrm{tr}\,(\Sigma^\dagger\Sigma)\, B_{\mu\nu} B^{\mu\nu}$
$O_{H\widetilde{B}}$	$\mathrm{tr}\,(\Sigma^\dagger\Sigma)\, \widetilde{B}_{\mu\nu} B^{\mu\nu}$
O_{HWB}	$\mathrm{tr}\,(\Sigma^\dagger\tau^a\Sigma\tau_R^3)\, W_{\mu\nu}^a B^{\mu\nu}$
$O_{H\widetilde{W}B}$	$\mathrm{tr}\,(\Sigma^\dagger\tau^a\Sigma\tau_R^3)\, \widetilde{W}_{\mu\nu}^a B^{\mu\nu}$

$6: \bar\psi\psi XH + \mathrm{h.c.}$	
$O_{lW}^{\pm}$	$(\bar{l}\sigma^{\mu\nu}\tau^a\Sigma P_\pm l_R)W_{\mu\nu}^a$
$O_{lB}^{\pm}$	$(\bar{l}\sigma^{\mu\nu}\Sigma P_\mp l_R)B_{\mu\nu}$
$O_{qG}^{\pm}$	$(\bar{q}\sigma^{\mu\nu}T^A\Sigma P_\pm q_R)G_{\mu\nu}^A$
$O_{qW}^{\pm}$	$(\bar{q}\sigma^{\mu\nu}\tau^a\Sigma P_\pm q_R)W_{\mu\nu}^a$
$O_{qB}^{\pm}$	$(\bar{q}\sigma^{\mu\nu}\Sigma P_\mp q_R)B_{\mu\nu}$

$7: \bar\psi\psi H^2 D$	
$O_{Hl}^{(1)}$	$\mathrm{tr}\,(\Sigma^\dagger iD_\mu\Sigma\tau_R^3)\,(\bar{l}\gamma^\mu l)$
$O_{Hl}^{(3)}$	$\mathrm{tr}\,(\Sigma^\dagger\tau^a iD_\mu\Sigma)\,(\bar{l}\gamma^\mu\tau^a l)$
$O_{Hq}^{(1)}$	$\mathrm{tr}\,(\Sigma^\dagger iD_\mu\Sigma\tau_R^3)\,(\bar{q}\gamma^\mu q)$
$O_{Hq}^{(3)}$	$\mathrm{tr}\,(\Sigma^\dagger\tau^a iD_\mu\Sigma)\,(\bar{q}\gamma^\mu\tau^a q)$
$O_{Hl_R}^{(1)\pm}$	$\mathrm{tr}\,(\Sigma^\dagger iD_\mu\Sigma\tau_R^3)\left(\bar{l}_R\gamma^\mu P_\pm l_R\right)$
$O_{Hl_R}^{(3)\pm}$	$\mathrm{tr}\,(\Sigma^\dagger iD_\mu\Sigma\tau_R^a)\left(\bar{l}_R\gamma^\mu\tau_R^a P_\pm l_R\right)$
$O_{Hq_R}^{(1)\pm}$	$\mathrm{tr}\,(\Sigma^\dagger iD_\mu\Sigma\tau_R^3)\left(\bar{q}_R\gamma^\mu P_\pm q_R\right)$
$O_{Hq_R}^{(3)\pm}$	$\mathrm{tr}\,(\Sigma^\dagger iD_\mu\Sigma\tau_R^a)\left(\bar{q}_R\gamma^\mu\tau_R^a P_\pm q_R\right)$

$8: (\bar{L}L)(\bar{L}L)$	
O_{ll}	$(\bar{l}\gamma_\mu l)(\bar{l}\gamma^\mu l)$
$O_{qq}^{(1)}$	$(\bar{q}\gamma_\mu q)(\bar{q}\gamma^\mu q)$
$O_{qq}^{(3)}$	$(\bar{q}\gamma_\mu\tau^a q)(\bar{q}\gamma^\mu\tau^a q)$
$O_{lq}^{(1)}$	$(\bar{l}\gamma_\mu l)(\bar{q}\gamma^\mu q)$
$O_{lq}^{(3)}$	$(\bar{l}\gamma_\mu\tau^a l)(\bar{q}\gamma^\mu\tau^a q)$

$8: (\bar{R}R)(\bar{R}R)$	
$O_{l_R l_R}^{\pm\pm}$	$(\bar{l}_R\gamma_\mu P_\pm l_R)(\bar{l}_R\gamma^\mu P_\pm l_R)$
$O_{l_R l_R}^{+-}$	$(\bar{l}_R\gamma_\mu P_+ l_R)(\bar{l}_R\gamma^\mu P_- l_R)$
$O_{q_R q_R}^{(1)\pm\pm}$	$(\bar{q}_R\gamma_\mu P_\pm q_R)(\bar{q}_R\gamma^\mu P_\pm q_R)$
$O_{q_R q_R}^{(1)+-}$	$(\bar{q}_R\gamma_\mu P_+ q_R)(\bar{q}_R\gamma^\mu P_- q_R)$
$O_{q_R q_R}^{(3)++}$	$(\bar{q}_R\gamma_\mu\tau_R^a q_R)(\bar{q}_R\gamma^\mu\tau_R^a q_R)$
$O_{l_R q_R}^{(1)\pm\pm}$	$(\bar{l}_R\gamma_\mu P_\pm l_R)(\bar{q}_R\gamma^\mu P_\pm q_R)$
$O_{l_R q_R}^{(1)\pm\mp}$	$(\bar{l}_R\gamma_\mu P_\pm l_R)(\bar{q}_R\gamma^\mu P_\mp q_R)$
$O_{l_R q_R}^{(3)+\pm}$	$(\bar{l}_R\gamma_\mu\tau_R^a l_R)(\bar{q}_R\gamma^\mu\tau_R^a P_\pm q_R)$

$8: (\bar{L}L)(\bar{R}R)$	
$O_{ll_R}^{\pm}$	$(\bar{l}\gamma_\mu l)(\bar{l}_R\gamma^\mu P_\pm l_R)$
$O_{lq_R}^{\pm}$	$(\bar{l}\gamma_\mu l)(\bar{q}_R\gamma^\mu P_\pm q_R)$
$O_{ql_R}^{\pm}$	$(\bar{q}\gamma_\mu q)(\bar{l}_R\gamma^\mu P_\pm l_R)$
$O_{qq_R}^{(1)\pm}$	$(\bar{q}\gamma_\mu q)(\bar{q}_R\gamma^\mu P_\pm q_R)$
$O_{qq_R}^{(8)\pm}$	$(\bar{q}\gamma_\mu T^A q)(\bar{q}_R\gamma^\mu T^A P_\pm q_R)$

Observables

$$\left\{\hat{\alpha},\ \hat{G}_F,\ \hat{M}_Z^2\right\}$$

- Taken as our SM inputs
- Use them to calculate other observables

$$\left\{\hat{M}_W^2,\ \hat{\Gamma}_{Z\nu_L\bar{\nu}_L},\ \hat{\Gamma}_{Ze_L\bar{e}_L},\ \hat{\Gamma}_{Ze\bar{e}}\right\}$$

- Predicted observables by the inputs
- Calculated in SMEFT @ tree level
- Compare the predictions to experiments

How are these observables measured?

$$\left\{ \hat{\alpha},\ \hat{G}_F,\ \hat{M}_Z^2 \right\}$$

- $\hat{\alpha}$ — electron $g-2$
- $\hat{G}_F$ — muon lifetime
- $\hat{M}_Z^2$ — LEP

$$\left\{ \hat{M}_W^2,\ \hat{\Gamma}_{Z\nu_L\bar{\nu}_L},\ \hat{\Gamma}_{Ze_L\bar{e}_L},\ \hat{\Gamma}_{Ze\bar{e}} \right\}$$

- $\hat{M}_W^2$ — LHC
- $3\,\hat{\Gamma}_{Z\nu_L\bar{\nu}_L} = \hat{\Gamma}_Z - \hat{\Gamma}_{Zll} - \hat{\Gamma}_{Zqq}$
- $\hat{\Gamma}_{Ze_L\bar{e}_L}$ and $\hat{\Gamma}_{Ze\bar{e}}$

  — $\left(\hat{\Gamma}_{Ze_L\bar{e}_L} + \hat{\Gamma}_{Ze\bar{e}}\right)$ and $\hat{A}_{FB}^{0,e}$

Mapping SMEFT onto the observables

We swap out $\hat{M}_W^2$ for the Veltman $\hat{\rho}$

$$\hat{r}_{Zf\bar{f}} = \frac{\Gamma_{SMEFT}}{\Gamma_{SM}}$$

$$\hat{\rho} = 1 + \frac{v^2}{c_{2\theta}}\left[2s_\theta^2\left(\frac{2c_\theta}{s_\theta}\,a_{HWB} - a_{Hl}^{(3)}\right) + \frac{1}{2}s_\theta^2\,a_{12} - 2c_\theta^2\,a_{HD}\right],$$

$$\hat{r}_{Z\nu_L\bar{\nu}_L} = 1 + v^2\left[\frac{1}{2}\,a_{12} - 2\,a_{HD} + 2\,a_{Hl}^{(1)}\right],$$

$$\hat{r}_{Ze_L\bar{e}_L} = 1 + \frac{v^2}{c_{2\theta}^2}\left[4s_\theta^2\left(\frac{2c_\theta}{s_\theta}\,a_{HWB} - a_{Hl}^{(3)}\right) + \frac{1}{2}\,a_{12} - 2\,a_{HD} - 2c_{2\theta}\,a_{Hl}^{(1)}\right],$$

$$\hat{r}_{Ze\bar{e}} = 1 + \frac{v^2}{c_{2\theta}}\left[-2\left(\frac{2c_\theta}{s_\theta}\,a_{HWB} - a_{Hl}^{(3)}\right) - \frac{1}{2}\,a_{12} + 2\,a_{HD} + \frac{c_{2\theta}}{s_\theta^2}\left(a_{Hl_R}^{(1)+} - a_{Hl_R}^{(1)-} - a_{Hl_R}^{(3)+} + a_{Hl_R}^{(3)-}\right)\right].$$

Constructing $\mathcal{T}_\ell$ to replace the T parameter

- UV theories with **custodial symmetry** have a *correlation* among these observables:

$$(\hat{\rho} - 1) + \frac{1}{2}(\hat{r}_{Z\nu_L\bar{\nu}_L} - 1) - \frac{1}{2}c_{2\theta}(\hat{r}_{Ze_L\bar{e}_L} - 1) = 0$$

Constructing $\mathcal{T}_\ell$ to replace the T parameter

- UV theories violate **custodial symmetry** yield an *expression* with these observables:

$$(\hat{\rho} - 1) + \frac{1}{2}(\hat{r}_{Z\nu_L\bar{\nu}_L} - 1) - \frac{1}{2}c_{2\theta}(\hat{r}_{Ze_L\bar{e}_L} - 1) = -2v^2\left[a_{HD} - a_{Hl}^{(1)}\right]$$

$$= \alpha\,\mathcal{T}_\ell$$

- Eventually, from these *correlated observables* we constructed our generalizaion to the Peskin-Takeuchi *T* parameter.

- $\mathcal{T}_\ell$ captures hard CV from both ***oblique*** and **vertex corrections.**

Example 1: Real Triplet Scalar (Universal)

$$\mathcal{L}_{\rm UV} = \mathcal{L}_{\rm SM} + \frac{1}{2}\left(D^\mu\phi^a\right)\left(D_\mu\phi^a\right) - \frac{1}{2}M^2\phi^a\phi^a - AH^\dagger t^a H\phi^a - \kappa|H|^2\phi^a\phi^a - \lambda_\phi(\phi^a\phi^a)^2\,.$$

- Matching @ the leading order, this theory generates

$$a_{HD}\,O_{HD} = a_{HD}\left[Tr\left(\Sigma^\dagger iD_\mu\Sigma\tau_R^3\right)\right]^2$$

$$\alpha T = -\frac{1}{2}v^2 C_{HD} = -2v^2\,a_{HD}$$

$$\alpha\,\mathcal{T}_\ell = -2v^2\left[a_{HD} - a_{Hl}^{(1)}\right] \quad (a_{Hl}^{(1)} = 0)$$

$$= -2v^2\,a_{HD} = \alpha T$$

- $\mathcal{T}_\ell$ works equivalently to the T parameter for Universal Theories.

Example 2: Vector-like Fermions (Non-Universal)

$$\mathcal{L}_{\rm UV} = \mathcal{L}_{\rm SM} + \bar{N}(i\not{D} - M)N + \bar{E}(i\not{D} - M)E - \left(Y_N\, \bar{l}\tilde{H}N + Y_E\, \bar{l}HE + \text{h.c.}\right) .$$

- Matching @ the leading order, this theory generates

$$a_{Hl}^{(1)}\, O_{Hl}^{(1)} = a_{Hl}^{(1)} \left[Tr\left(\Sigma^\dagger i D_\mu \Sigma \tau_R^3\right)\left(\bar{l}\gamma^\mu l\right)\right], \quad \text{while} \quad a_{HD} = 0$$

$$\alpha T = -\,2v^2\, a_{HD} = 0$$

$$\alpha \mathcal{T}_\ell = -\,2v^2 \left[a_{HD} - a_{Hl}^{(1)} \right] = 2v^2\, a_{Hl}^{(1)} \neq 0 \;!$$

- $\mathcal{T}_\ell$ works with Non-Universal Theories while T fails.

Constraints on custodial violating UV physics

- Constraints depend on the *largest uncertainty* with respect to the measurements of the observables.

$$\alpha \mathcal{T}_\ell = (\hat{\rho} - 1) + \frac{1}{2}(\hat{r}_{Z\nu_L\bar{\nu}_L} - 1) - \frac{1}{2}c_{2\theta}(\hat{r}_{Ze_L\bar{e}_L} - 1)$$

- Due to the *uncertainty* on the Z boson partial decay width to left-handed electrons, the constraints on custodial violating UV physics is expected to be different.

Take Home Messages

- Veltman ρ is NOT an indicator of custodial violation.
- Peskin-Takeuchi T parameter works as an indicator of custodial violation only when the BSM physics is *oblique.*
- We have generalized the T parameter into

$$\alpha \mathcal{T}_{\ell} = -2v^2 \left[a_{HD} - a_{Hl}^{(1)} \right] = -\frac{1}{2} v^2 \left[C_{HD} + 4C_{Hl}^{(1)} \right]$$

which is constructed from well-measured **observables.**

- At tree level, it captures custodial violation of both Universal and Non-Universal Theories.

Thanks!