Diagrammatic one-loop renormalization within the EChL in the R_ξ gauges and applications to scattering and decays

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Outline

- The Electroweak Chiral Lagrangian (EChL)
- Diagrammatic computational procedure
- Renormalization program
- Application to $W_L^{\pm} Z_L \to W_L^{\pm} Z_L$
- Application to $H \to \gamma\gamma$ and $H \to \gamma Z$
- Conclusions

The non-linear EChL or HEFT

- Symmetries: Lorentz, EW gauge $SU(2)_L \times U(1)_Y$ and EW Chiral $SU(2)_L \times SU(2)_R$ (based on ChPT of QCD)
- Light degrees of freedom:
  Higgs boson is a $SU(2)$ **singlet**, in contrast to the (linear) SM!
  EW gauge bosons $\Rightarrow \hat{W}_\mu = g W_\mu^a \tau^a/2,\ \hat{B}_\mu = g' B_\mu \tau^3/2,\ \hat{W}_{\mu\nu},\ \hat{B}_{\mu\nu}$.
  EW GBs π^a **transform non-linearly** under the EW Chiral symmetry
  but $U = \exp\left(\frac{i\pi^a\tau^a}{v}\right)$ that transforms linearly $U \to g_L U g_R^\dagger$
  $\Rightarrow D_\mu U = \partial_\mu U + i\hat{W}_\mu U - iU\hat{B}_\mu$
  Our assumptions: fermionic interactions as in SM.
- Based on a **derivative expansion** $\leftrightarrow$ Chiral expansion (powers of p).
  Derivatives and masses are soft scales of the EFT with power counting $\mathcal{O}(p) \Rightarrow \mathcal{L}$ organized in terms of operators $\mathcal{O}(p^2)$, $\mathcal{O}(p^4)$, ...

  $$\text{bosonic sector } \mathcal{L}_{EChL} = \mathcal{L}_2 + \mathcal{L}_4 + \ldots$$

Computational procedure beyond Tree level in the EChL

Example: computation of $T = T(WZ \to WZ)$ at $\mathcal{O}(\hbar) \to$ 1-loop

	LO ($T = T_0$)	NLO ($T = T_0 + T_1$)
EChL	$T_0 = T^{\mathcal{L}_2^{Tree}}$	$T_1 = T^{\mathcal{L}_4^{Tree}} + T^{\mathcal{L}_2^{Loop}} + T^{CT \to from\ \mathcal{L}_2^{Tree}}$ ($\mathcal{L}_4^{Tree}$ also act as CT)
SM	$T_0 = T^{\mathcal{L}_{SM}^{Tree}}$	$T_1 = T^{\mathcal{L}_{SM}^{Loop}} + T^{CT \to from\ \mathcal{L}_{SM}^{Tree}}$
SMEFT	$T_0 = T^{\mathcal{L}_{SMEFT}^{Tree}}$	$T_1 = T^{\mathcal{L}_{SMEFT}^{Loop}} + T^{CT \to from\ \mathcal{L}_{SMEFT}^{Tree}}$

New in this work

$\Rightarrow$ We include ALL Bosonic loops in T_1 (GB, gauge, mix and ghost) using the R_ξ gauges
$\Rightarrow$ We renormalize with a diagrammatic procedure all involved Γ_{1PI} (off-shell)
$\Rightarrow$ Differences with respect to previous computations within the EChL:

- Dobado et al. [PLB 1991]; Espriu et al. [1307.2400] $\to$ massless GB loops in scattering.
- Gavela et al. [1409.1571] $\to$ Renormalization of pure scalar theory with massless GB.
- Buchalla et al. [2004.11348] $\to$ Renormalization of $\mathcal{L}_{EChL}$ using path integral and BFM.

$\mathcal{L}_2$ Lagrangian with gauge-fixing in the R_ξ

Multiple GB interactions due to the non-linearity of this EFT
Also polynomial Higgs interactions since it is a singlet.

$$\mathcal{L}_2 = \frac{v^2}{4}\left(1 + 2a\frac{H}{v} + b\left(\frac{H}{v}\right)^2 + \dots\right)\langle D_\mu U^\dagger D^\mu U\rangle + \frac{1}{2}\partial_\mu H \partial^\mu H - V(H)$$
$$- \frac{1}{2g'}\langle \hat{B}_{\mu\nu}\hat{B}^{\mu\nu}\rangle - \frac{1}{2g}\langle \hat{W}_{\mu\nu}\hat{W}^{\mu\nu}\rangle + \mathcal{L}_{GF} + \mathcal{L}_{FP}$$

with $V(H) = -\mu^2\left(\frac{v+H}{\sqrt{2}}\right)^2 + \lambda\left(\frac{v+H}{\sqrt{2}}\right)^4$; $v = 246$ GeV

$$\mathcal{L}_{GF} = -\frac{1}{\xi}\Big(\underbrace{\partial^\mu W^+_\mu - \xi M_{\mathrm{W}}\pi^+}_{F_+}\Big)\Big(\underbrace{\partial^\mu W^-_\mu - \xi M_{\mathrm{W}}\pi^-}_{F_-}\Big)$$
$$- \frac{1}{\xi}\Big(\underbrace{\partial^\mu Z_\mu - \xi M_{\mathrm{Z}}\pi^3}_{F_Z}\Big)^2 - \frac{1}{\xi}\Big(\underbrace{\partial^\mu A_\mu}_{F_A}\Big)^2$$

$$\mathcal{L}_{FP} = \sum_{i=\pm,Z,A} \underbrace{\bar{c}_+\frac{\delta F_+}{\delta\alpha_+}c_i + \bar{c}_-\frac{\delta F_-}{\delta\alpha_-}c_i + \bar{c}_Z\frac{\delta F_Z}{\delta\alpha_+}c_i + \bar{c}_A\frac{\delta F_A}{\delta\alpha_+}c_i}_{\text{linear } \pi\bar{c}c \text{ as SM but multiple } \pi^{n\geq 2}\bar{c}c \text{ and NO } H\bar{c}c \text{ interactions!}}$$

$\mathcal{L}_4$ Lagrangian

Complete set $\mathcal{O}(p^4)$ operators with dimensionless EW Chiral coefficients a_i

Operators with EW gauge and GB (Custodial preserving and breaking):

$$
\begin{aligned}
\mathcal{L}_4 = a_0 \left(M_{\rm Z}^2 - M_{\rm W}^2\right) \mathrm{Tr}\left[U\tau^3 U^\dagger \mathcal{V}_\mu\right] \mathrm{Tr}\left[U\tau^3 U^\dagger \mathcal{V}^\mu\right] \\
+ a_1 \mathrm{Tr}\left[U\hat{B}_{\mu\nu} U^\dagger \hat{W}^{\mu\nu}\right] + i a_2 \mathrm{Tr}\left[U\hat{B}_{\mu\nu} U^\dagger [\mathcal{V}^\mu, \mathcal{V}^\nu]\right] - i a_3 \mathrm{Tr}\left[\hat{W}_{\mu\nu}[\mathcal{V}^\mu, \mathcal{V}^\nu]\right] \\
+ a_4 \mathrm{Tr}\left[\mathcal{V}_\mu \mathcal{V}_\nu\right] \mathrm{Tr}\left[\mathcal{V}^\mu \mathcal{V}^\nu\right] + a_5 \mathrm{Tr}\left[\mathcal{V}_\mu \mathcal{V}^\mu\right] \mathrm{Tr}\left[\mathcal{V}_\nu \mathcal{V}^\nu\right] \\
+ a_6 \mathrm{Tr}\left[\mathcal{V}_\mu \mathcal{V}_\nu\right] \mathrm{Tr}\left[U\tau^3 U^\dagger \mathcal{V}^\mu\right] \mathrm{Tr}\left[U\tau^3 U^\dagger \mathcal{V}^\nu\right] \\
+ a_7 \mathrm{Tr}\left[\mathcal{V}_\mu \mathcal{V}^\mu\right] \mathrm{Tr}\left[U\tau^3 U^\dagger \mathcal{V}^\nu\right] \mathrm{Tr}\left[U\tau^3 U^\dagger \mathcal{V}_\nu\right] \\
- \frac{a_8}{4} \mathrm{Tr}\left[U\tau^3 U^\dagger \hat{W}_{\mu\nu}\right] \mathrm{Tr}\left[U\tau^3 U^\dagger \hat{W}^{\mu\nu}\right] - i \frac{a_9}{2} \mathrm{Tr}\left[U\tau^3 U^\dagger \hat{W}_{\mu\nu}\right] \mathrm{Tr}\left[U\tau^3 U^\dagger [\mathcal{V}^\mu, \mathcal{V}^\nu]\right] \\
+ a_{10} \mathrm{Tr}\left[U\tau^3 U^\dagger \mathcal{V}^\mu\right] \mathrm{Tr}\left[U\tau^3 U^\dagger \mathcal{V}_\mu\right] \mathrm{Tr}\left[U\tau^3 U^\dagger \mathcal{V}^\nu\right] \mathrm{Tr}\left[U\tau^3 U^\dagger \mathcal{V}_\nu\right] \\
+ a_{11} \mathrm{Tr}\left[\mathcal{D}_\mu \mathcal{V}^\mu \mathcal{D}_\nu \mathcal{V}^\nu\right] + a_{12} \mathrm{Tr}\left[U\tau^3 U^\dagger \mathcal{D}_\mu \mathcal{D}_\nu \mathcal{V}^\nu\right] \mathrm{Tr}\left[U\tau^3 U^\dagger \mathcal{V}^\mu\right] \\
+ \frac{a_{13}}{2} \mathrm{Tr}\left[U\tau^3 U^\dagger \mathcal{D}_\mu \mathcal{V}_\nu\right] \mathrm{Tr}\left[U\tau^3 U^\dagger \mathcal{D}^\mu \mathcal{V}^\nu\right] \\
+ \ldots
\end{aligned}
$$

with $\mathcal{V}_\mu = (D_\mu U)U^\dagger$, $\mathcal{D}_\mu O = \partial_\mu O + i[\hat{W}_\mu, O]$

[Longhitano; Appelquist and Bernard (PRD 1980)]

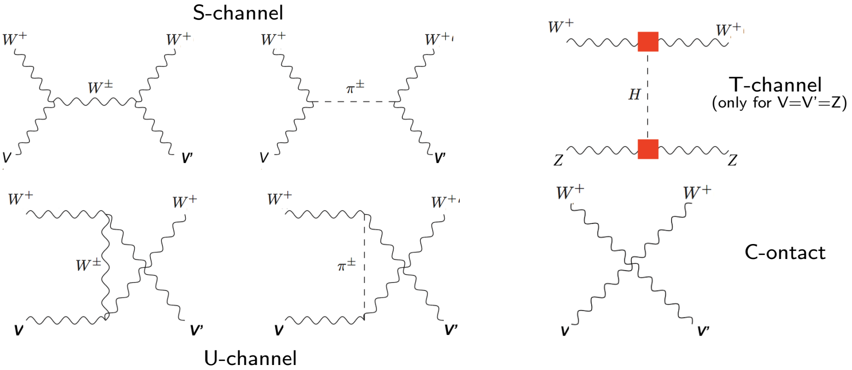
Additional terms in $\mathcal{L}_4$ with Higgs

$$
\begin{aligned}
&\ldots + a_{\Box\Box}\frac{\Box H\,\Box H}{v^2} - a_{HBB}\frac{H}{v}\mathrm{Tr}\left[\hat{B}_{\mu\nu}\hat{B}^{\mu\nu}\right] - a_{HWW}\frac{H}{v}\mathrm{Tr}\left[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}\right] + a_{\Box\mathcal{V}\mathcal{V}}\frac{\Box H}{v}\mathrm{Tr}\left[\mathcal{V}_\mu\mathcal{V}^\mu\right] \\
&+ a_{H0}\frac{H}{v}\left(M_{\mathrm{Z}}^2 - M_{\mathrm{W}}^2\right)\mathrm{Tr}\left[U\tau^3U^\dagger\mathcal{V}_\mu\right]\mathrm{Tr}\left[U\tau^3U^\dagger\mathcal{V}^\mu\right] + a_{H1}\frac{H}{v}\mathrm{Tr}\left[U\hat{B}_{\mu\nu}U^\dagger\hat{W}^{\mu\nu}\right] \\
&- \frac{a_{H8}}{4}\frac{H}{v}\mathrm{Tr}\left[U\tau^3U^\dagger\hat{W}_{\mu\nu}\right]\mathrm{Tr}\left[U\tau^3U^\dagger\hat{W}^{\mu\nu}\right] + a_{H11}\frac{H}{v}\mathrm{Tr}\left[\mathcal{D}_\mu\mathcal{V}^\mu\mathcal{D}_\nu\mathcal{V}^\nu\right] \\
&+ \frac{a_{H13}}{2}\frac{H}{v}\mathrm{Tr}\left[U\tau^3U^\dagger\mathcal{D}_\mu\mathcal{V}_\nu\right]\mathrm{Tr}\left[U\tau^3U^\dagger\mathcal{D}^\mu\mathcal{V}^\nu\right] \\
&+ ia_{d1}\frac{\partial^\nu H}{v}\mathrm{Tr}\left[U\hat{B}_{\mu\nu}U^\dagger\mathcal{V}^\mu\right] + ia_{d2}\frac{\partial^\nu H}{v}\mathrm{Tr}\left[\hat{W}_{\mu\nu}\mathcal{V}^\mu\right] + a_{d3}\frac{\partial^\nu H}{v}\mathrm{Tr}\left[\mathcal{V}_\nu\mathcal{D}_\mu\mathcal{V}^\mu\right] \\
&+ ia_{d4}\frac{\partial^\nu H}{v}\mathrm{Tr}\left[U\tau^3U^\dagger\hat{W}_{\mu\nu}\right]\mathrm{Tr}\left[U\tau^3U^\dagger\mathcal{V}^\mu\right] + a_{d5}\frac{\partial^\nu H}{v}\mathrm{Tr}\left[U\tau^3U^\dagger\mathcal{D}_\mu\mathcal{V}^\mu\right]\mathrm{Tr}\left[U\tau^3U^\dagger\mathcal{V}_\nu\right] + \ldots
\end{aligned}
$$

- Operators list given in the literature (see for instance [Brivio et al. (1311.1823)])
- We focus on these a_i's, relevant for Higgs decays and VBS processes.
- No use of e.o.m (since off-shell 1PI) and no Higgs field redefinition.

$W^+V \to W^+V'$ @LO ($T = T_0$ results)

T_0 is ξ-independent: cancellation in the unphysical charged sector.
Same procedure for $\{V, V'\} = \{\gamma, \gamma\}$, $\{\gamma, Z\}$, $\{Z, Z\}$

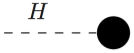


enters in red boxes

T_0 is gauge-invariant and New Physics for $a \neq 1$

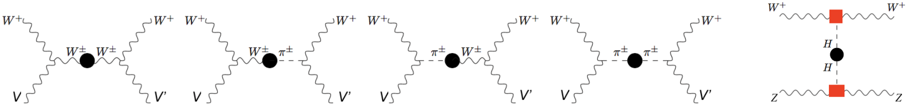
$$T_1 = \mathcal{M}_{1-leg} + \mathcal{M}_{2-legs} + \mathcal{M}_{3-legs} + \mathcal{M}_{4-legs} + \mathcal{M}_{wfr}$$

contributions from renormalized 1PI n-legs Green functions $\hat{\Gamma}_{n-legs} = \Gamma^{\rm Tree} + \Gamma^{\rm Loop} + \Gamma^{\rm CT}$

$\mathcal{M}_{1-leg} \Rightarrow \hat{\Gamma}_{Tadpole}$ 

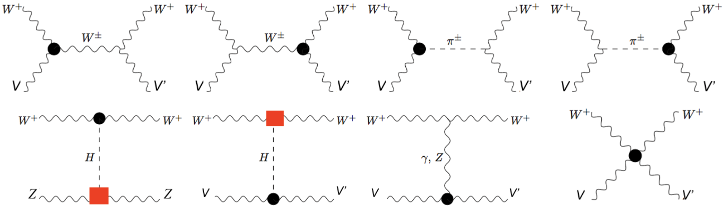
in many places (fix vanishing Tadpole very convenient!)

$\mathcal{M}_{2-legs} \Rightarrow$



(and $S \leftrightarrow U$)

$\mathcal{M}_{3-legs}$ and $\mathcal{M}_{4-legs} \Rightarrow$



(and $S \leftrightarrow U$)

$\mathcal{M}_{wfr} = T_0 \times$ "possible contributions from residues of external states"

(Diagrammatic) Renormalization Program

SM predictions recovered for $a = b = 1$ and $a_i = 0$.

(SM computation made for comparison)

$\mathcal{L}_2$ parameters (usual multiplicative renormalization)

$$B_{b\,\mu} = \sqrt{\mathcal{Z}_B} B_{R\,\mu}\,,\; W^{\pm,3}_{b\,\mu} = \sqrt{\mathcal{Z}_W} W^{\pm,3}_{R\,\mu}\,,\; H_b = \sqrt{\mathcal{Z}_H} H_R\,,\; \pi^{\pm,3}_b = \sqrt{\mathcal{Z}_\pi} \pi^{\pm,3}_R$$
$$g'_b = \mathcal{Z}_B^{-1/2}(g'_R + \delta g')\,,\; g_b = \mathcal{Z}_W^{-1/2}(g_R + \delta g)\,,\; \xi^b_{1,2} = \xi_R(1 + \delta\xi_{1,2})$$
$$v_b = \sqrt{\mathcal{Z}_\pi}(v_R + \delta v)\,,\; M^{b\,2}_{\rm H} = M^{R\,2}_{\rm H} + \delta M^2_{\rm H}\,,\; \lambda_b = \mathcal{Z}_H^{-2}(\lambda_R + \delta\lambda) \qquad (\mathcal{Z}_i = 1 + \delta\mathcal{Z}_i)$$

a_i's also act as CT in this non-linear EFT.

EChL coefficients

$$a^b = a^R + \delta a\,, \quad b^b = b^R + \delta b\,, \quad a^b_i = a^R_i + \delta a_i$$

Diagrammatic procedure dealing with Bosonic Loops contributing to all 1PI Green functions **off-shell** (relevant for $W^+V \to W^+V'$)

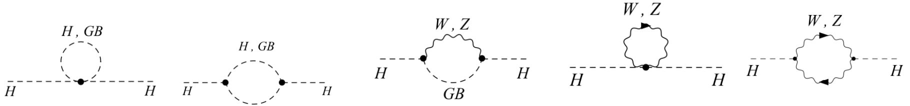
- 1-leg and 2-legs 1PI: renormalization $\mathcal{L}_2$ params and some of the a_i's.
- 3- and 4-legs 1PI: renormalization of the remaining EChL coefficients.

Computing CT's divergent part from Loop diagrams

Consider $\hat{\Gamma}_{n-legs} = \Gamma^{\rm Tree}_{n-legs} + \Gamma^{\rm Loop}_{n-legs} + \Gamma^{\rm CT}_{n-legs}$

with $\Gamma^{\rm (Bosonic)\,Loop}_{n-legs} = \Gamma^{\rm GB}_{n-legs} + \Gamma^{\rm mix}_{n-legs} + \Gamma^{\rm gauge}_{n-legs} + \Gamma^{\rm ghost}_{n-legs}$ [FeynRules+FeynArts+FormCalc,FeynCalc,Package-X] $\sim$500 diags for $WZ \to WZ$!

Starting: Higgs sector



$(\Delta_\epsilon = \frac{2}{4-D} - \gamma_{\rm E} + Log(4\pi))$

$-i\hat{\Sigma}_{HH}(q^2) = -i\Sigma^{\rm Loop}_{HH}(q^2)$ $\leftarrow \mathcal{L}_2^{Loop}$

$+i\left(\delta\mathcal{Z}_H(q^2 - M_{\rm H}^2) - \delta M_{\rm H}^2\right) + i\frac{2a_{\Box\Box}}{v^2}q^4$ $\leftarrow \mathcal{L}_2^{CT} + \mathcal{L}_4^{Tree}$

Cancelling divergencies for each power of q^2 sequentially

$\Rightarrow \quad \delta_\epsilon a_{\Box\Box} = -\frac{\Delta_\epsilon}{16\pi^2}\frac{3a^2}{4}, \quad \delta_\epsilon\mathcal{Z}_H = \frac{\Delta_\epsilon}{16\pi^2}\frac{3a^2}{v^2}(2M_{\rm W}^2 + M_{\rm Z}^2)$

and $\delta_\epsilon M_{\rm H}^2 = \frac{\Delta_\epsilon}{16\pi^2}\frac{3}{2v^2}(4M_{\rm H}^4 - 2a^2M_{\rm H}^2(2M_{\rm W}^2 + M_{\rm Z}^2) + (4a^2 + 2b)(2M_{\rm W}^4 + M_{\rm Z}^4))$

$\Rightarrow$ **they are ξ-independent!** (in contrast to the SM)

Fix the $\delta T = (-\delta M_{\rm H}^2/2 + 2\lambda v^2(\delta v/v + \delta\mathcal{Z}_\pi/2) + \delta\lambda v^2 - 2\lambda v^2\delta\mathcal{Z}_H)v$

demanding vanishing renormalized Tadpole:

$i\hat{\Gamma}_{Tadpole} = i\frac{1}{32\pi^2 v}(3M_{\rm H}^2A_0(M_{\rm H}^2) + 2a(6M_{\rm W}^2A_0(M_{\rm W}^2) - 4M_{\rm W}^4 + 3M_{\rm Z}^2A_0(M_{\rm Z}^2) - 2M_{\rm Z}^4)) - i\delta T$

Renormalized self-energies with gauge and GB

Gauge physical sector

$$i\hat{\Sigma}^{T}_{WW}(q^2) = i\Sigma^{T\,\mathrm{Loop}}_{WW}(q^2) - i\left(\left(q^2 - M_{\mathrm{W}}^{R\,2}\right)\delta\mathcal{Z}_W - \delta M_{\mathrm{W}}^2\right)$$

$$\begin{aligned} i\hat{\Sigma}^{T}_{AA}(q^2) &= i\Sigma^{T\,\mathrm{Loop}}_{AA}(q^2) - i\delta\mathcal{Z}_A q^2 \\ &- ig^2 s_{\mathrm{w}}^2(a_8 - 2a_1)q^2 \end{aligned}$$

$$\begin{aligned} i\hat{\Sigma}^{T}_{ZZ}(q^2) &= i\Sigma^{T\,\mathrm{Loop}}_{ZZ}(q^2) - i\left(\left(q^2 - M_{\mathrm{Z}}^{R\,2}\right)\delta\mathcal{Z}_Z - \delta M_{\mathrm{Z}}^2\right) \\ &- i\left(2g'^2 M_{\mathrm{Z}}^2 a_0 + (2g^2 s_{\mathrm{w}}^2 a_1 + g^2 c_{\mathrm{w}}^2 a_8 + (g^2 + g'^2)a_{13})q^2\right) \end{aligned}$$

$$\begin{aligned} i\hat{\Sigma}^{T}_{ZA}(q^2) &= i\Sigma^{T\,\mathrm{Loop}}_{ZA}(q^2) - i\left(\delta\mathcal{Z}_{ZA} q^2 - M_{\mathrm{Z}}^{R\,2} s_{\mathrm{w}} c_{\mathrm{w}}\left(g/g_R - \delta g'/g'_R\right)\right) \\ &- iq^2\left(g^2 s_{\mathrm{w}} c_{\mathrm{w}} a_8 - gg'(c_{\mathrm{w}}^2 - s_{\mathrm{w}}^2)a_1\right) \end{aligned}$$

Unphysical charged sector

$$i\hat{\Sigma}^{L}_{WW}(q^2) = i\Sigma^{L\,\mathrm{Loop}}_{WW}(q^2) + i\left(-\frac{1}{\xi}\left(q^2 - \xi M_{\mathrm{W}}^2\right)\delta\mathcal{Z}_W + \delta M_{\mathrm{W}}^2 + q^2\frac{\delta\xi_1}{\xi} - q^2 g^2 a_{11}\right)$$

$$\hat{\Sigma}_{W\pi}(q^2) = \Sigma^{\mathrm{Loop}}_{W\pi}(q^2) + \frac{\delta\xi_2 - \delta\xi_1}{2} M_{\mathrm{W}}^2 + q^2 g^2 a_{11}$$

$$-i\hat{\Sigma}_{\pi\pi}(q^2) = -\,i\Sigma^{\mathrm{Loop}}_{\pi\pi}(q^2) + i\left(\left(q^2 - \xi M_{\mathrm{W}}^2\right)\delta\mathcal{Z}_\pi - \xi\delta M_{\mathrm{W}}^2 - \xi M_{\mathrm{W}}^2\delta\xi_2\right) - i\frac{g^2}{M_{\mathrm{W}}^2} q^4 a_{11}$$

Renormalized 3- and 4-legs 1PI

$$\hat{\Gamma}^{\mu\nu}_{HAA} = \Gamma^{\text{Loop}\,\mu\nu}_{HAA} + \frac{g^2 s_{\rm w}^2}{v}(a_{HBB} + a_{HWW} - a_{H1} + a_{H8}/2)(g^{\mu\nu}(q^2 - k_1^2 - k_2^2) - 2k_2^\mu k_1^\nu)$$

$$\hat{\Gamma}^{\mu\nu}_{HAZ} = \Gamma^{\text{Loop}\,\mu\nu}_{HAZ} + \frac{a g^2 s_{\rm w} v}{2 c_{\rm w}}\left(\frac{\delta g}{g} - \frac{\delta g'}{g'}\right) g^{\mu\nu} + \text{`}a_{HBB,HWW,H1,H8,d1,d2,d4}\text{'}$$

$$\hat{\Gamma}^{\mu\nu}_{HZZ} = \Gamma^{\text{Loop}\,\mu\nu}_{HZZ} + \frac{a g^2 v}{2 c_{\rm w}^2}\left(\frac{\delta a}{a} + \frac{2\delta g}{g} c_{\rm w}^2 + \frac{2\delta g'}{g'} s_{\rm w}^2 + \frac{\delta v}{v} + \frac{\delta \mathcal{Z}_H}{2} + \frac{\delta \mathcal{Z}_\pi}{2}\right) g^{\mu\nu}$$
$$+ \text{`}a_{H0,HWW,HBB,H1,H8,H13,d1,d2,d3,d4,d5,\square 0,\square\mathcal{V}\mathcal{V},H11}\text{'}$$

$$\hat{\Gamma}^{\mu\nu}_{HWW} = \Gamma^{\text{Loop}\,\mu\nu}_{HWW} + \frac{a g^2 v}{2}\left(\frac{\delta a}{a} + \frac{2\delta g}{g} + \frac{\delta v}{v} + \frac{\delta \mathcal{Z}_H}{2} + \frac{\delta \mathcal{Z}_\pi}{2}\right) g^{\mu\nu} + \text{`}a_{HWW,d2,d3,\square\mathcal{V}\mathcal{V},H11}\text{'}$$

$$\hat{\Gamma}^{\mu\nu}_{\pi WV} = \Gamma^{\text{Loop}\,\mu\nu}_{\pi WV} + i\frac{g g' v x_V}{2}\left(\frac{\delta g}{g} + \frac{\delta g'}{g'} + \frac{\delta v}{v} + \delta\mathcal{Z}_\pi\right) g^{\mu\nu}$$
$$(x_A = c_{\rm w},\ x_Z = s_{\rm w})$$
$$+ \text{`}a_{0,1,2,3,,8,9,11,12,13}\text{'}$$

$$\hat{\Gamma}^{\mu\nu\rho}_{WWV} = \Gamma^{\text{Loop}\,\mu\nu\rho}_{WWV} + g\, y_V\left(\frac{\delta g}{g} + \delta\mathcal{Z}_W\right)(g^{\mu\nu}(-k_1^\rho + k_2^\rho) + g^{\nu\rho}(-k_1^\mu - 2k_2^\mu)$$
$$(y_A = s_{\rm w},\ y_Z = c_{\rm w})$$
$$+ g^{\rho\mu}(2k_1^\nu + k_2^\nu)) + \text{`}a_{1,2,3,8,9,11,12,13}\text{'}$$

$$\hat{\Gamma}^{\mu\nu\rho\sigma}_{VV'WW} = \Gamma^{\text{Loop}\,\mu\nu\rho\sigma}_{VV'WW} - g^2 x_{VV'}\left(2\frac{\delta g}{g} + \delta\mathcal{Z}_W\right)(2g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\rho}))$$
$$(x_{AA} = s_{\rm w}^2,\ x_{AZ} = s_{\rm w} c_{\rm w},\ x_{ZZ} = c_{\rm w}^2)$$
$$+ \text{`}a_{3,4,5,6,7,11,12}\text{'}$$

Renormalization of the $\mathcal{L}_2$ parameters (Results)

Cancelling divergencies of all 1PI in each power of q^2 sequentially ('couple system')

$$\delta_\epsilon \mathcal{Z}_H = \frac{\Delta_\epsilon}{16\pi^2}\frac{3a^2}{v^2}(2M_{\rm W}^2 + M_{\rm Z}^2) \qquad \delta_\epsilon T = \frac{\Delta_\epsilon}{16\pi^2}\frac{3}{2v}\left(M_{\rm H}^4 + 2a\left(2M_{\rm W}^4 + M_{\rm Z}^4\right)\right)$$

$$\delta_\epsilon M_{\rm H}^2 = \frac{\Delta_\epsilon}{16\pi^2}\frac{3}{2v^2}(4M_{\rm H}^4 - 2a^2 M_{\rm H}^2(2M_{\rm W}^2 + M_{\rm Z}^2) + (4a^2 + 2b)(2M_{\rm W}^4 + M_{\rm Z}^4))$$

$$\delta_\epsilon \mathcal{Z}_B = -\frac{\Delta_\epsilon}{16\pi^2}\frac{g'^2}{12}(1 + a^2) \qquad \delta_\epsilon \mathcal{Z}_W = \frac{\Delta_\epsilon}{16\pi^2}\frac{g^2}{12}(51 - a^2 - 12\xi)$$

$$\delta_\epsilon M_{\rm W}^2 = -\frac{\Delta_\epsilon}{16\pi^2}\frac{g^2}{12}\left(3(a^2 - b)M_{\rm H}^2 + (78 - 10a^2)M_{\rm W}^2 - 9M_{\rm Z}^2\right)$$

$$\delta_\epsilon M_{\rm Z}^2 = \frac{\Delta_\epsilon}{16\pi^2}\frac{g^2}{12c_{\rm w}^2}\left(-3(a^2 - b)M_{\rm H}^2 + (7(1 + a^2) + 2(-43 + a^2)c_{\rm w}^2)M_{\rm W}^2 + (10 + a^2)M_{\rm Z}^2\right)$$

$$\delta_\epsilon g'/g' = 0 \qquad \delta_\epsilon g/g = -\frac{\Delta_\epsilon}{16\pi^2}\frac{g^2}{2}(3 + \xi)$$

$$\delta_\epsilon \xi_1 = \frac{\Delta_\epsilon}{16\pi^2}\frac{g^2}{12}(51 - a^2 - 12\xi)$$

$$\delta_\epsilon \xi_2 = \frac{\Delta_\epsilon}{16\pi^2}\frac{1}{v^2}(2(a^2 - b)M_{\rm H}^2 + (23 - 19a^2/3 + 4\xi/3)M_{\rm W}^2 - (6 - 4\xi/3)M_{\rm Z}^2)$$

$$\delta_\epsilon \mathcal{Z}_\pi = -\frac{\Delta_\epsilon}{16\pi^2}\frac{1}{v^2}((a^2 - b)M_{\rm H}^2 - (3 + 3a^2 - 4\xi/3)M_{\rm W}^2 - (3 - 4\xi/3)M_{\rm Z}^2)$$

$$\delta_\epsilon v/v = \frac{\Delta_\epsilon}{16\pi^2}\frac{2(M_{\rm W}^2 + M_{\rm Z}^2)}{3v^2}\xi \qquad \left(\Delta_\epsilon = \frac{2}{4-D} - \gamma_{\rm E} + Log(4\pi)\right)$$

- New Physics for $a \neq 1$, $b \neq 1$.
- ξ-dependence in $\delta_\epsilon g$, $\delta_\epsilon \mathcal{Z}_W$ and unphysical sector
- δT and $\delta_\epsilon M_{H,W,Z}^2$ are ξ-independent separately (not only the $\overline{\delta M^2}$ combination as in the SM)
  Higgs boson is a singlet in this EFT!

Renormalization of the EChL coefficients (Results)

$$\delta_\epsilon a_0 = \frac{\Delta_\epsilon}{16\pi^2}\frac{3}{8}(1-a^2) \qquad \delta_\epsilon a_1 = \frac{\Delta_\epsilon}{16\pi^2}\frac{1}{12}(1-a^2) \qquad \delta_\epsilon a_8 = 0$$

$$\delta_\epsilon a_{\Box\Box} = -\frac{\Delta_\epsilon}{16\pi^2}\frac{3a^2}{4} \qquad \delta_\epsilon a_{11} = \frac{\Delta_\epsilon}{16\pi^2}\frac{a^2}{4}$$

$$\delta_\epsilon a = \frac{\Delta_\epsilon}{16\pi^2}\frac{3(a^2-b)(a-\kappa_3)M_{\rm H}^2+3a\Big((1-3a^2+2b)M_{\rm W}^2+(1-a^2)M_{\rm Z}^2\Big)}{2v^2}$$

$$\delta_\epsilon a_{HBB} = \delta_\epsilon a_{HWW} = \frac{\Delta_\epsilon}{16\pi^2}\frac{a(a^2-b)}{12} \qquad \delta_\epsilon a_{\Box\mathcal{V}\mathcal{V}} = -\frac{\Delta_\epsilon}{16\pi^2}\frac{a(2+a^2)}{4}$$

$$\delta_\epsilon a_{H0} = \frac{\Delta_\epsilon}{16\pi^2}\frac{3a(1-b)}{4} \qquad \delta_\epsilon a_{H1} = \frac{\Delta_\epsilon}{16\pi^2}\frac{a(a^2-b)}{6} \qquad \delta_\epsilon a_{H11} = -\frac{\Delta_\epsilon}{16\pi^2}\frac{a(a^2-b)}{2}$$

$$\delta_\epsilon a_{d1} = \delta_\epsilon a_{d2} = -\frac{\Delta_\epsilon}{16\pi^2}\frac{a(a^2-b)}{6} \qquad \delta_\epsilon a_{d3} = \frac{\Delta_\epsilon}{16\pi^2}\frac{a(a^2+b)}{2}$$

$$\delta_\epsilon a_{H8} = \delta_\epsilon a_{H13} = 0 \qquad \delta_\epsilon a_{d4} = \delta_\epsilon a_{d5} = 0$$

$$\delta_\epsilon a_6 = \delta_\epsilon a_7 = \delta_\epsilon a_9 = \delta_\epsilon a_{10} = \delta_\epsilon a_{12} = \delta_\epsilon a_{13} = 0$$

$$\delta_\epsilon a_4 = -\frac{\Delta_\epsilon}{16\pi^2}\frac{(1-a^2)^2}{12} \qquad \delta_\epsilon a_5 = -\frac{\Delta_\epsilon}{16\pi^2}\frac{a^4+4a^2-3a^2b+\frac{3}{2}b^2+1}{24}$$

Some comments about these results

⇒ **All a_i's are ξ-independent!**

⇒ By the first time, we diagrammatically derive the **RGE** for all EChL coefficients using the R_ξ gauges

$$a_i^R(\mu) = a_i^R(\mu_0) + \frac{1}{16\pi^2}\gamma_{a_i} Log\left(\frac{\mu^2}{\mu_0^2}\right) \qquad \text{(with } \delta_\epsilon a_i = \frac{\Delta_\epsilon}{16\pi^2}\gamma_{a_i}\text{)}$$

partially compatible with
[Espriu et al. (1307.2400); Dobado et al. (1311.5993); Gavela et al. (1409.1571); Buchalla et al. (2004.11348)]

CT's finite parts of the $\mathcal{L}_2$ parameters

We demand the On-Shell prescription in the physical sectors:

- vanishing Tadpole.
- physical masses $M_{\rm H}^F = M_{\rm H}^R$, $M_{\rm W}^F = M_{\rm W}^R$ and $M_{\rm Z}^F = M_{\rm Z}^R$.
- unit residues for Higgs and photon.
- no Z-A mixing for on-shell photon.
- electric charge as in QED.

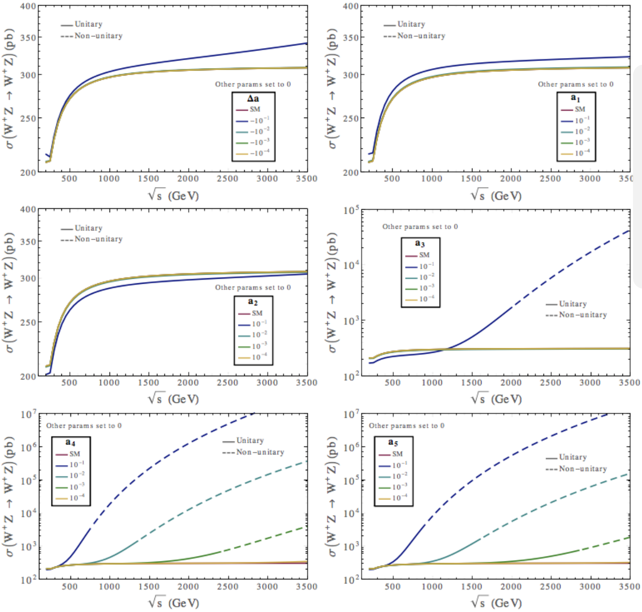
For the unphysical charged sector in the R_ξ gauges, we demand:

- poles of the renormalized propagators at $q^2 = \xi_R M_{\rm W}^{R\,2}$
- renormalized Slavnov-Taylor identiy valid for arbitrary q^2
  $q^2\hat{\Sigma}^L_{WW}(q^2) + 2q^2\hat{\Sigma}_{W\pi}(q^2) - M_{\rm W}^{R\,2}\hat{\Sigma}_{\pi\pi}(q^2) = 0$

$\Rightarrow$ All the CT are written in terms of the unrenormalized (Loop) 1PI

Tree level EChL predictions to $WZ \to WZ$

[Phys. Rev. D 100 no. 9, 096003 (2019)]



a_4 and a_5 are the most relevant EChL coefficients for this process!

Values $\mathcal{O}(10^{-4})$ allowed by current experimental data and unitarity.

$$a_{4(5)} = \frac{v^4}{16}\frac{f_{S0(S1)}}{\Lambda^4}$$

Next, consider the complete $\mathcal{O}(\hbar)$ corrections

Preliminar numerical results for $W_L^{\pm} Z_L \to W_L^{\pm} Z_L$ @NLO

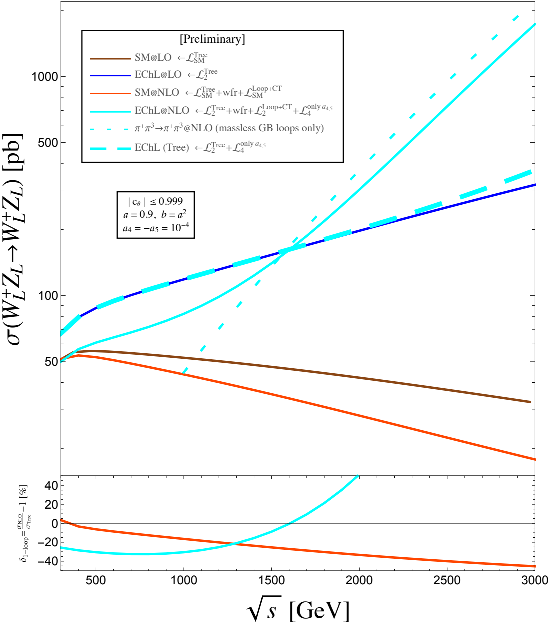
Numerical evaluation of the bosonic contributions in the Feynman gauge ($\xi = 1$)

Only a_4 and a_5 effects of $\mathcal{L}_4$
(now $a_4 = -a_5 = 10^{-4}$)

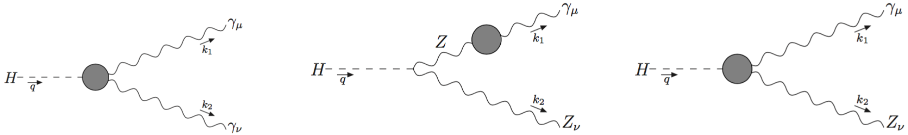
Size of the correction $\mathcal{O}(\hbar)$:
$\delta_{1-loop} = \frac{\sigma_{NLO} - \sigma_{Tree}}{\sigma_{Tree}}$

$\to \delta^{SM}_{1-loop}(1\,TeV) \sim -16\%$
in agreement with [Denner et al. (1904.00882)]

$\to \delta^{EChL}_{1-loop}(1\,TeV) \sim -30\%$



$T_0 = 0$ (NO $\mathcal{L}_2^{Tree}$) $\Rightarrow$ $\mathcal{M}_{w.f.r.} = 0$. Also $\mathcal{M}_{1-leg} = 0$ ($\hat{\Gamma}_{Tadpole} = 0$)
Then NLO contribution comes from $\mathcal{L}_2^{\mathrm{Loop+CT}}$ and $\mathcal{L}_4^{\mathrm{Tree}}$: $T_1 = \mathcal{M}^{\mu\nu}_{\gamma V}\epsilon_\mu(k_1)\epsilon_\nu(k_2)$



$$\mathcal{M}^{\mathrm{(Bosonic)\,Loop}}_{\gamma V} = \mathcal{M}^{\mathrm{GB}}_{\gamma V} + \mathcal{M}^{\mathrm{mix}}_{\gamma V} + \mathcal{M}^{\mathrm{gauge}}_{\gamma V} + \mathcal{M}^{\mathrm{ghost}}_{\gamma V}$$

$$\mathcal{M}^{\mathrm{CT}\,\mu\nu}_{\gamma\gamma} = 0 \qquad \mathcal{M}^{\mathrm{CT}\,\mu\nu}_{\gamma Z} = \frac{ag}{M_{\mathrm{W}}} M_{\mathrm{Z}}^2 s_{\mathrm{w}} c_{\mathrm{w}} \left(\frac{\delta g}{g} - \frac{\delta g'}{g'}\right) g^{\mu\nu}$$

$$\mathcal{M}^{\mathrm{Tree}\,\mu\nu}_{\gamma\gamma} = \frac{e^2 g}{M_{\mathrm{W}}} c_{H\gamma\gamma}\left(k_1 \cdot k_2 g^{\mu\nu} - k_2^\mu k_1^\nu\right) \qquad \mathcal{M}^{\mathrm{Tree}\,\mu\nu}_{\gamma Z} = \frac{eg^2 c_{\mathrm{w}}}{M_{\mathrm{W}}} c_{H\gamma Z}\left(k_1 \cdot k_2 g^{\mu\nu} - k_2^\mu k_1^\nu\right)$$

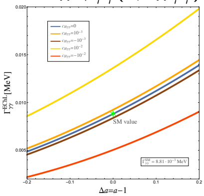
Renormalization Group Invariant combinations

$$\delta_\epsilon c_{H\gamma\gamma} = \delta_\epsilon a_{HBB} + \delta_\epsilon a_{HWW} - \delta_\epsilon a_{H1} + \delta_\epsilon a_{H8}/2 = 0$$
$$\delta_\epsilon c_{H\gamma Z} = -s_{\mathrm{w}}^2 \delta_\epsilon a_{HBB} + c_{\mathrm{w}}^2 \delta_\epsilon a_{HWW} + (1/2 - c_{\mathrm{w}}^2)\delta_\epsilon a_{H1} + c_{\mathrm{w}}^2 \delta_\epsilon a_{H8}/2 = 0$$

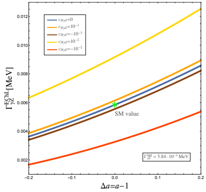
Next, consider the Higgs boson off-shell (arbitrary q^2)

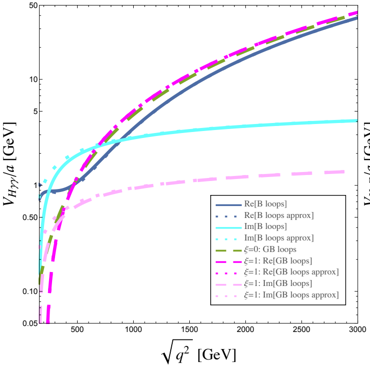
Analytic computation finite and ξ-independent $\forall q^2$! (It is not longer true in the SM)
SM recovered only for $a = 1$, $c_i = 0$ and $q^2 = M_{\rm H}^2$.
GB loops are dominant at large energies!

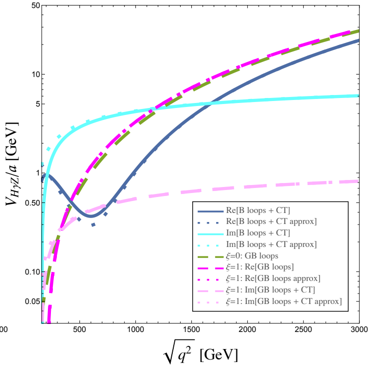
$\Gamma_{H\to\gamma\gamma}(a, c_{H\gamma\gamma})$



$\Gamma_{H\to\gamma Z}(a, c_{H\gamma Z})$







Analytic effective vertex derivation in the R_ξ gauges

$$\mathcal{M}^{\mu\nu}_{\gamma V} = V_{H\gamma V}(q, k_1, k_2)\left(g^{\mu\nu} - \frac{k_2^\mu k_1^\nu}{k_1 \cdot k_2}\right)$$

Conclusions

We perform by the first time a complete diagrammatic EChL's one-loop renormalization program in the covariant R_ξ gauges

- We compute all the bosonic loops contributing to the relevant 1PI for the VBS processes: many divergencies arise here.
- We fix the CT's divergencies of the $\mathcal{L}_2$ parameters and $\mathcal{L}_4$ coefficients leading to all the relevant 1PI Green functions be finite off-shell.
- We present the results of the complete set of the EChL coefficients and show that they are manifestly ξ-independent.
- We derive the corresponding RGE of the a_i's in the R_ξ gauges.
- The finite parts of the CTs are determined by the 'On-shell' and '$\overline{\text{MS}}$' schemes.
- Many applications to these results:
  Higgs decays ($H \to \gamma\gamma$, γZ)
  preliminar analysis in $W_L Z_L \to W_L Z_L$
  Future works on $H \to ZZ^*, WW^*$ and $WZ \to W\gamma$, $W\gamma \to W\gamma$, etc

Backup slides

Transformations under global $SU(2)_L \times SU(2)_R$

The rotations under $SU(2)_L$ and $SU(2)_R$ correspond to

$$g_L = e^{i\vec{\tau}\cdot\vec{\alpha}_L/2} \quad \text{and} \quad g_R = e^{i\vec{\tau}\cdot\vec{\alpha}_R/2}$$

Then building blocks transform under the global $SU(2)_L \times SU(2)_R$ as

$$U \mapsto U' = g_L\, U\, g_R^\dagger \qquad \text{with chiral dim.} = 0$$

$$\hat{B}_\mu \mapsto \hat{B}'_\mu = \hat{B}_\mu \qquad \text{with chiral dim.} = 1$$

$$\hat{W}_\mu \mapsto \hat{W}'_\mu = g_L\, \hat{W}_\mu\, g_L^\dagger \qquad \text{with chiral dim.} = 1$$

$$D_\mu U \mapsto (D_\mu U)' = g_L\, D_\mu U\, g_R^\dagger \qquad \text{with chiral dim.} = 1$$

$$\hat{B}_{\mu\nu} \mapsto \hat{B}'_{\mu\nu} = \hat{B}_{\mu\nu} \qquad \text{with chiral dim.} = 2$$

$$\hat{W}_{\mu\nu} \mapsto \hat{W}'_{\mu\nu} = g_L\, \hat{W}_{\mu\nu}\, g_L^\dagger \qquad \text{with chiral dim.} = 2$$

For the EW gauge symmetry $SU(2)_L \times U(1)_Y \subset SU(2)_L \times SU(2)_R$, the association of the generator of $U(1)_Y$ as the third one of the $SU(2)_R$ and the generator of $U(1)_{EM}$ as the third one of the $SU(2)_{L+R}$:

$$Y \leftrightarrow X_R^3 \;\text{ and }\; Q \leftrightarrow X_{L+R}^3 = T^3 + Y$$

Transformations under local $SU(2)_L \times U(1)_Y$

Promoting to $g_L = e^{ig\vec{\tau}\cdot\vec{\alpha}_L(x)/2}$ and $g_R = e^{ig'\tau^3\alpha_Y(x)/2}$

$\Rightarrow \delta F_\pm = \partial^\mu(\delta W^\pm_\mu) - \xi M_{\rm W}\delta\pi^\pm,\ \delta F_Z = \partial^\mu(\delta Z_\mu) - \xi M_{\rm W}\delta\pi^3,\ \delta F_A = \partial^\mu(\delta A_\mu)$

The corresponding gauge field's transformations as in the SM

$$\begin{aligned}
\delta W^\pm_\mu &= -\,\partial_\mu\alpha^\pm \mp ig(\alpha^\pm(c_{\rm w}Z_\mu + s_{\rm w}A_\mu) - (c_{\rm w}\alpha_Z + s_{\rm w}\alpha_A)W^\pm_\mu)\\
\delta Z_\mu &= -\,\partial_\mu\alpha_Z - igc_{\rm w}(\alpha^- W^+_\mu - \alpha^+ W^-_\mu)\\
\delta A_\mu &= -\,\partial_\mu\alpha_A - igs_{\rm w}(\alpha^- W^+_\mu - \alpha^+ W^-_\mu)
\end{aligned}$$

'New' GB's transformations in the EChL:

$$\begin{aligned}
\delta\pi^\pm &= M_{\rm W}\alpha^\pm \mp \frac{ig}{2}\alpha^\pm\pi^3 \pm \frac{g(c_{\rm w}^2 - s_{\rm w}^2)}{2c_{\rm w}}\alpha_Z\pi^\pm \pm igs_{\rm w}\alpha_A\pi^\pm\\
&\quad + \frac{M_{\rm W}}{3v^2}(-\alpha^\pm\pi^+\pi^- + \alpha^\mp\pi^\pm\pi^\pm - \alpha^\pm\pi^3\pi^3) + \frac{M_{\rm Z}}{3v^2}\alpha_Z\pi^3\pi^\pm + \ldots\\
\delta\pi^3 &= M_{\rm Z}\alpha_Z - \frac{ig}{2}(\alpha^-\pi^+ - \alpha^+\pi^-)\\
&\quad + \frac{M_{\rm W}}{3v^2}(\alpha^+\pi^-\pi^3 + \alpha^-\pi^+\pi^3) - \frac{2M_{\rm Z}}{3v^2}\alpha_Z\pi^+\pi^- + \ldots
\end{aligned}$$

$\Rightarrow$ linear $\pi\bar{c}c$ as in SM but multiple $\pi^{n\geq 2}\bar{c}c$ (non-linear)

and NO $H\bar{c}c$ (H is a singlet $\delta H = 0$)

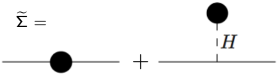
$$\mathcal{L}_{GF}^{\text{charged}} = -\left(\frac{1}{\sqrt{\xi_1^b}}\,\partial^\mu W_{b\,\mu}^+ - \sqrt{\xi_2^b}\,\frac{g_b v_b}{2}\,\pi_b^+\right)\left(\frac{1}{\sqrt{\xi_1^b}}\,\partial^\mu W_{b\,\mu}^- - \sqrt{\xi_2^b}\,\frac{g_b v_b}{2}\,\pi_b^-\right)$$

$$\langle T\,\mathcal{L}_{GF\,b}^{\text{charged}}\rangle = i \implies q^2\Delta_L^{WW}(q^2) + 2\sqrt{\xi_1^b\xi_2^b}\,q^2\Delta^{W\pi}(q^2) - \xi_1^b\xi_2^b M_{\text{W}}^{R\,2}\Delta^{\pi\pi}(q^2) = \xi_1^b$$

See [Espriu and Matias, hep-ph/9501279]

Using the Dyson form of the propagators:

$$\left(M_{\text{W}}^{R\,2} + \widetilde{\Sigma}_{WW}^{L\,\text{Loop}}\right)\left(q^2 - \widetilde{\Sigma}_{\pi\pi}^{\text{Loop}}\right) = \frac{q^2}{M_{\text{W}}^{R\,2}}\left(M_{\text{W}}^{R\,2} - \widetilde{\Sigma}_{W\pi}^{\text{Loop}}\right)^2$$

$\widetilde{\Sigma} =$ 

At 1-loop: $q^2\widetilde{\Sigma}_{WW}^{L\,\text{Loop}} + 2q^2\widetilde{\Sigma}_{W\pi}^{\text{Loop}} - M_{\text{W}}^{R\,2}\widetilde{\Sigma}_{\pi\pi}^{\text{Loop}} =$

$$q^2\left(\Sigma_{WW}^{L\,\text{Loop}} + a g_R M_{\text{W}}^R\,\frac{T^{\text{Loop}}}{M_{\text{H}}^{R\,2}}\right) + 2q^2\left(\Sigma_{W\pi}^{\text{Loop}} - a g_R M_{\text{W}}^R\,\frac{T^{\text{Loop}}}{M_{\text{H}}^{R\,2}}\right) - M_{\text{W}}^{R\,2}\left(\Sigma_{\pi\pi}^{\text{Loop}} - a\,\frac{g_R\,q^2}{M_{\text{W}}^R}\,\frac{T^{\text{Loop}}}{M_{\text{H}}^{R\,2}}\right) =$$

$$= q^2\Sigma_{WW}^{L\,\text{Loop}}(q^2) + 2q^2\Sigma_{W\pi}^{\text{Loop}}(q^2) - M_{\text{W}}^{R\,2}\Sigma_{\pi\pi}^{\text{Loop}}(q^2) = 0$$

Then the Loop contribution vanishes! Summing the CT contribution:

$$\Sigma_{WW}^{L\,\text{CT}}(q^2) = -\frac{1}{\xi_R}\left(q^2 - \xi_R M_{\text{W}}^{R\,2}\right)\delta\mathcal{Z}_W + \delta M_{\text{W}}^2 + q^2\,\frac{\delta\xi_1^W}{\xi_R} - q^2 g_R^2 a_{11} \qquad \Sigma_{W\pi}^{\text{CT}}(q^2) = \frac{\delta\xi_2^W - \delta\xi_1^W}{2}\,M_{\text{W}}^{R\,2} + q^2 g_R^2 a_{11}$$

$$\Sigma_{\pi\pi}^{\text{CT}}(q^2) = -\left(q^2 - \xi_R M_{\text{W}}^{R\,2}\right)\delta\mathcal{Z}_\pi + \xi_R\delta M_{\text{W}}^2 + \xi_R M_{\text{W}}^{R\,2}\delta\xi_2^W + \frac{g_R^2}{M_{\text{W}}^{R\,2}}\,q^4 a_{11}$$

$$\Rightarrow q^2\hat{\Sigma}_{WW}^L(q^2) + 2q^2\hat{\Sigma}_{W\pi}(q^2) - M_{\text{W}}^{R\,2}\hat{\Sigma}_{\pi\pi}(q^2) =$$

$$= \frac{q^2 - \xi_R M_{\text{W}}^{R\,2}}{\xi_R}\underbrace{\left(-\left(\delta\mathcal{Z}_W - \delta\xi_1\right)q^2 + \xi_R\delta M_{\text{W}}^2 + \xi_R M_{\text{W}}^{R\,2}\left(\delta\mathcal{Z}_\pi + \delta\xi_2\right)\right)}_{f_{ST}^{\pm}(q^2)}$$

Tadpole's contributions in the SM and EChL

Main difference: EChL-Tadpole is ξ-independent!

$T^{\rm Loop} = \frac{1}{32\pi^2 v}(3\kappa_3 M_{\rm H}^2 A_0(M_{\rm H}^2) + 2a(6M_{\rm W}^2 A_0(M_{\rm W}^2) - 4M_{\rm W}^4 + 3M_{\rm Z}^2 A_0(M_{\rm Z}^2) - 2M_{\rm Z}^4))$

The ξ-dependence in the SM-Tadpole is the key for the gauge-invariant definition of $\overline{\delta M_{\rm H}^2} = \delta M_{\rm H}^2 - 3\delta T/v$ (and similar for $\overline{\delta M_{\rm W}^2}$, $\overline{\delta M_{\rm Z}^2}$).

In the EChL, $\delta M_{\rm H}^2$, $\delta M_{\rm W}^2$ and $\delta M_{\rm Z}^2$ are already ξ-independent.

Within the SM: Higgs and GB in same doublet $\Phi = \begin{pmatrix} i\pi^+ \\ \frac{v+H-i\pi_3}{\sqrt{2}} \end{pmatrix}$. In particular, they have the same wfr factor $\Phi_b = \sqrt{\mathcal{Z}_\phi}\Phi_R$.

In practice, we can set $\mathcal{Z}_H = \mathcal{Z}_\pi = \mathcal{Z}_\phi$ in the EChL's expressions. However!

$-i\hat{\Sigma}_{\pi\pi}(q^2)|_{SM} = -i\Sigma_{\pi\pi}^{\rm Loop}(q^2)|_{SM} + i\left(\left(q^2 - \xi_R M_{\rm W}^{R\,2}\right)\delta\mathcal{Z}_\phi - \xi_R \delta M_{\rm W}^2 - \xi_R M_{\rm W}^{R\,2}\delta\xi_2^W - \delta T/v_R\right)$

$\Rightarrow$ The unrenormalized ST-identity in the SM differs from the EChL

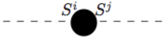
$q^2\Sigma_{WW}^{L\,\rm Loop} + 2q^2\Sigma_{W\pi}^{\rm Loop} - M_{\rm W}^{R\,2}\Sigma_{\pi\pi}^{\rm Loop} - \frac{g_R M_{\rm W}^R}{2} T^{\rm Loop}|_{\rm SM} = 0$ However the renormalized ST-identity is the same in both when $\hat{\Gamma}_{Tadpole} = 0$

$$q^2\hat{\Sigma}_{WW}^L(q^2) + 2q^2\hat{\Sigma}_{W\pi}(q^2) - M_{\rm W}^{R\,2}\hat{\Sigma}_{\pi\pi}(q^2) \xcancel{- \frac{g_R M_{\rm W}^R}{2} T^{\rm Loop}}|_{\rm SM} =$$

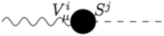
$$= \frac{q^2 - \xi_R M_{\rm W}^{R\,2}}{\xi_R}\left(-\left(\delta\mathcal{Z}_W - \delta\xi_1\right)q^2 + \xi_R\delta M_{\rm W}^2 + \xi_R M_{\rm W}^{R\,2}\left(\delta\mathcal{Z}_\phi + \delta\xi_2\right)\right) \xcancel{- M_{\rm W}^{R\,2}\frac{\delta T}{v_R}}|_{\rm SM}$$

Our diagrammatic conventions for the renormalized 1PI

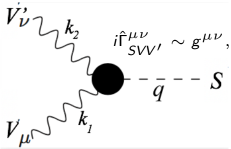
$(T^{\mu\nu} = g^{\mu\nu} - q^\mu q^\nu / q^2$, $L^{\mu\nu} = q^\mu q^\nu / q^2)$ $\quad i\,\Sigma_{ij}^{\mu\nu} = i\left(\Sigma_{ij}^T T^{\mu\nu} + \Sigma_{ij}^L L^{\mu\nu}\right)$



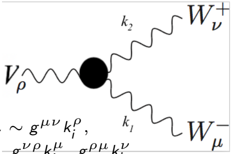
$-i\Sigma_{ij}$



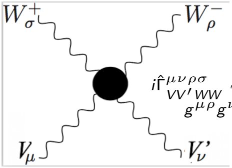
$\Sigma_{ij}^{\mu} = \frac{q^\mu}{m_{Vj}} \Sigma_{ij}$



$i\hat{\Gamma}^{\mu\nu}_{SVV'} \sim g^{\mu\nu}, \, k_i^\mu k_j^\nu$



$i\hat{\Gamma}^{\mu\nu\rho}_{WWV} \sim g^{\mu\nu} k_i^\rho, \; g^{\nu\rho} k_i^\mu, \; g^{\rho\mu} k_i^\nu$



$i\hat{\Gamma}^{\mu\nu\rho\sigma}_{VV'WW} \sim g^{\mu\nu} g^{\rho\sigma}, \; g^{\mu\rho} g^{\nu\sigma}, \; g^{\mu\sigma} g^{\nu\rho}$

$$\Gamma^{\text{(Bosonic) Loop}}_{n-legs} = \Gamma^{\text{GB}}_{n-legs} + \Gamma^{\text{mix}}_{n-legs} + \Gamma^{\text{gauge}}_{n-legs} + \Gamma^{\text{ghost}}_{n-legs}$$

[FeynRules+FeynArts+ FormCalc,FeynCalc,Package-X]

$\mathcal{L}_2$ tree-level relations

$\mu_b^2 \equiv -M_{\rm H}^{b\,2} + 3\lambda_b v_b^2$ and minimum in $-\mu_b^2 + \lambda_b v_b^2 = 0 \to T_b = (-M_{\rm H}^{b\,2}/2 + \lambda_b v_b^2)v_b$
Renormalized vev definition $M_{\rm H}^{R\,2} = 2\lambda_R v_R^2$
and potential $V(H) = \frac{1}{2} M_{\rm H}^2 H^2 + \kappa_3 \lambda v H^3 + \kappa_4 \frac{\lambda}{4} H^4$

Physical gauge bosons definition: $\{B_b, \vec{W}_b\} \to \{B_R, \vec{W}_R\} \to \{W_R^\pm, A_R, Z_R\}$

$$B_{R\,\mu} = c_{\rm w} A_{R\,\mu} - s_{\rm w} Z_{R\,\mu} \quad W_{R\,\mu}^3 = s_{\rm w} A_{R\,\mu} + c_{\rm w} Z_{R\,\mu}$$

$$M_{\rm W}^b = \frac{g_b v_b}{2} \quad M_{\rm Z}^b = \frac{\sqrt{g_b^2 + g_b'^2}\, v_b}{2} \qquad c_{\rm w} = \frac{g_R}{\sqrt{g_R^2 + g_R'^2}} \quad s_{\rm w} = \frac{g_R'}{\sqrt{g_R^2 + g_R'^2}} \quad e = \frac{g_R g_R'}{\sqrt{g_R^2 + g_R'^2}}$$

G_μ-scheme inputs

$M_{\rm W} = 80.38\text{GeV},\ M_{\rm Z} = 91.18\text{GeV},\ M_{\rm H} = 125.1\text{GeV},\ G_\mu = 1.16637 \cdot 10^{-5}\text{GeV}^{-2}$

$\Rightarrow$

$$\frac{e^2}{4\pi} = \alpha_{\rm EM} = \sqrt{2} G_\mu M_{\rm W}^2 (1 - M_{\rm W}^2/M_{\rm Z}^2)/\pi$$

$$c_{\rm w} = \frac{M_{\rm W}}{M_{\rm Z}} \quad s_{\rm w} = \sqrt{1 - \frac{M_{\rm W}^2}{M_{\rm Z}^2}} \quad g = \sqrt{2^{5/2} G_\mu} M_{\rm W} \quad g' = \sqrt{2^{5/2} G_\mu} M_{\rm Z} \sqrt{1 - \frac{M_{\rm W}^2}{M_{\rm Z}^2}}$$

$$v = \frac{1}{\sqrt{\sqrt{2} G_\mu}} \qquad \lambda = \frac{G_\mu M_{\rm H}^2}{\sqrt{2}}$$

$$S \to (g^2 + g'^2)(a_1 + a_{13}) = \frac{\Delta\Sigma_{ZZ}^{T\,\mathrm{Loop}}(M_{\mathrm{Z}}^{R\,2}) - \Delta\Sigma_{ZZ}^{T\,\mathrm{Loop}}(0)}{M_{\mathrm{Z}}^{R\,2}} - \frac{c_{\mathrm{w}}^2 - s_{\mathrm{w}}^2}{s_{\mathrm{w}}\,c_{\mathrm{w}}}\frac{d\Delta\Sigma_{ZA}^{T\,\mathrm{Loop}}}{dq^2}(0) - \frac{d\Delta\Sigma_{AA}^{T\,\mathrm{Loop}}}{dq^2}(0)$$

$$T \to -2g'^2 a_0 = \frac{\Delta\Sigma_{WW}^{T\,\mathrm{Loop}}(0)}{M_{\mathrm{W}}^{R\,2}} - \frac{\Delta\Sigma_{ZZ}^{T\,\mathrm{Loop}}(0)}{M_{\mathrm{Z}}^{R\,2}} + 2\frac{s_{\mathrm{w}}}{c_{\mathrm{w}}}\frac{\Delta\Sigma_{ZA}^{T\,\mathrm{Loop}}(0)}{M_{\mathrm{Z}}^{R\,2}}$$

$$U \to -g^2(a_8 + a_{13}) = \frac{\Delta\Sigma_{WW}^{T\,\mathrm{Loop}}(M_{\mathrm{W}}^{R\,2}) - \Delta\Sigma_{WW}^{T\,\mathrm{Loop}}(0)}{M_{\mathrm{W}}^{R\,2}} - c_{\mathrm{w}}^2\frac{\Delta\Sigma_{ZZ}^{T\,\mathrm{Loop}}(M_{\mathrm{Z}}^{R\,2}) - \Delta\Sigma_{ZZ}^{T\,\mathrm{Loop}}(0)}{M_{\mathrm{Z}}^{R\,2}} - 2s_{\mathrm{w}}\,c_{\mathrm{w}}\frac{d\Delta\Sigma_{ZA}^{T\,\mathrm{Loop}}}{dq^2}(0) - s_{\mathrm{w}}^2\frac{d\Delta\Sigma_{AA}^{T\,\mathrm{Loop}}}{dq^2}(0)$$

with $\Delta\Sigma_{VV'}^{T\,\mathrm{Loop}} = \Sigma_{VV'}^{T\,\mathrm{Loop}}|_{EChL} - \Sigma_{VV'}^{T\,\mathrm{Loop}}|_{SM}$

a_{13} removable by e.o.m redefining $a_1 + a_{13}, a_4 - a_{13}, a_5 + a_{13}, a_6 - a_{13}, a_7 + a_{13}, a_8 + a_{13}$

Finite parts: OS scheme conditions

8 On-Shell conditions for $\mathcal{L}_2$ parameters (${a}_{0,1,8,13}$ from Oblique parameters)

(1) $\hat{\Gamma}_{Tadpole} = 0: \; T^{\rm Loop} - \delta T = 0 \Rightarrow \frac{\delta\lambda}{\lambda_R} = \frac{2T^{\rm Loop}}{v_R M_{\rm H}^{R\,2}} + \frac{\delta M_{\rm H}^2}{M_{\rm H}^{R\,2}} - 2\left(\frac{\delta v}{v_R} + \frac{\delta \mathcal{Z}_\pi}{2}\right) + 2\delta\mathcal{Z}_H$

(2) $M_{\rm H}^F = M_{\rm H}^R$: ${\rm Re}\left[\hat{\Sigma}_{HH}(M_{\rm H}^{R\,2})\right] = 0 \Rightarrow \delta M_{\rm H}^2 = -{\rm Re}\left[\Sigma_{HH}^{\rm Loop}(M_{\rm H}^{R\,2})\right] + \frac{2M_{\rm H}^{R\,4}}{v_R^2}\delta a_{\Box H}$

(3) $\mathcal{R}_H = 1$: ${\rm Re}\left[\frac{d\hat{\Sigma}_{HH}}{dq^2}(M_{\rm H}^{R\,2})\right] = 0 \Rightarrow \delta\mathcal{Z}_H = {\rm Re}\left[\frac{d\Sigma_{HH}^{\rm Loop}}{dq^2}(M_{\rm H}^{R\,2})\right] - \frac{4M_{\rm H}^{R\,2}}{v_R^2}\delta a_{\Box H}$

(4) $M_{\rm W}^F = M_{\rm W}^R$: ${\rm Re}\left[\hat{\Sigma}_{WW}^T(M_{\rm W}^{R\,2})\right] = 0 \Rightarrow \delta M_{\rm W}^2 = -{\rm Re}\left[\Sigma_{WW}^{T\,\rm Loop}(M_{\rm W}^{R\,2})\right]$

(5) $M_{\rm Z}^F = M_{\rm Z}^R$: ${\rm Re}\left[\hat{\Sigma}_{ZZ}^T(M_{\rm Z}^{R\,2})\right] = 0$

$\Rightarrow \delta M_{\rm Z}^2 = -{\rm Re}\left[\Sigma_{ZZ}^{T\,\rm Loop}(M_{\rm Z}^{R\,2})\right] + M_{\rm Z}^2(2g'^2 a_0 + 2g^2 s_{\rm w}^2 a_1 + g^2 c_{\rm w}^2 a_8 + (g^2+g'^2)a_{13})$

(6) $\mathcal{R}_A = 1$: ${\rm Re}\left[\frac{d\hat{\Sigma}_{AA}^T}{dq^2}(0)\right] = 0 \Rightarrow \delta\mathcal{Z}_A = {\rm Re}\left[\frac{d\Sigma_{AA}^{T\,\rm Loop}}{dq^2}(0)\right] + g_R^2 s_{\rm w}^2(2a_1 - a_8)$

(7) NO mix $Z-A$: $\hat{\Sigma}_{ZA}^T(0) = 0 \Rightarrow M_{\rm Z}^{R\,2} s_{\rm w} c_{\rm w}\left(\frac{\delta g}{g_R} - \frac{\delta g'}{g'_R}\right) = -\Sigma_{ZA}^{T\,\rm Loop}(0)$

(8) electric charge as in QED: $\hat{\Gamma}_{Aee}^\mu|_{\rm OS} = ie\gamma^\mu \Rightarrow \delta g' = 0$

OS prescription to the CT in physical sector

$$\begin{pmatrix}\delta\mathcal{Z}_A\\ \delta\mathcal{Z}_Z\end{pmatrix} = \begin{pmatrix}c_{\rm w}^2 & s_{\rm w}^2\\ s_{\rm w}^2 & c_{\rm w}^2\end{pmatrix}\begin{pmatrix}\delta\mathcal{Z}_B\\ \delta\mathcal{Z}_W\end{pmatrix} \quad \delta\mathcal{Z}_{ZA} = s_{\rm w}c_{\rm w}(\delta\mathcal{Z}_W - \delta\mathcal{Z}_B) = \frac{s_{\rm w}c_{\rm w}}{c_{\rm w}^2 - s_{\rm w}^2}(\delta\mathcal{Z}_Z - \delta\mathcal{Z}_A)$$

$$\begin{aligned}
\delta M_{\rm W}^2 =& M_{\rm W}^{R\,2}\left(-\delta\mathcal{Z}_W + \delta\mathcal{Z}_\pi + 2\frac{\delta g}{g_R} + \frac{2\delta v}{v_R}\right)\\
\delta M_{\rm Z}^2 =& M_{\rm Z}^{R\,2}\left(-\delta\mathcal{Z}_Z + \delta\mathcal{Z}_\pi + 2c_{\rm w}^2\frac{\delta g}{g_R} + 2s_{\rm w}^2\frac{\delta g'}{g'_R} + \frac{2\delta v}{v_R}\right)
\end{aligned}$$

$$M_{\rm Z}^{R\,2}s_{\rm w}c_{\rm w}\left(\frac{\delta g}{g_R} - \frac{\delta g'}{g'_R}\right) = \frac{M_{\rm Z}^{R\,2}}{2}\left(\delta\mathcal{Z}_{ZA} + \frac{c_{\rm w}}{s_{\rm w}}\left(\frac{\delta M_{\rm W}^2}{M_{\rm W}^{R\,2}} - \frac{\delta M_{\rm Z}^2}{M_{\rm Z}^{R\,2}}\right)\right)$$

resulting in $\frac{\delta g}{g_R} = -\frac{1}{s_{\rm w}c_{\rm w}}\frac{\Sigma_{ZA}^{T\,{\rm Loop}}(0)}{M_{\rm Z}^{R\,2}}$

$$\begin{aligned}
\delta\mathcal{Z}_Z =& \delta\mathcal{Z}_A - \frac{c_{\rm w}^2 - s_{\rm w}^2}{s_{\rm w}^2}\left(\frac{\delta M_{\rm W}^2}{M_{\rm W}^{R\,2}} - \frac{\delta M_{\rm Z}^2}{M_{\rm Z}^{R\,2}}\right) + 2(c_{\rm w}^2 - s_{\rm w}^2)\left(\frac{\delta g}{g_R} - \frac{\delta g'}{g'_R}\right)\\
=& {\rm Re}\left[\frac{d\Sigma_{AA}^{T\,{\rm Loop}}}{dq^2}(0)\right] + \frac{c_{\rm w}^2 - s_{\rm w}^2}{s_{\rm w}^2}{\rm Re}\left[\frac{\Sigma_{WW}^{T\,{\rm Loop}}(M_{\rm W}^{R\,2})}{M_{\rm W}^{R\,2}} - \frac{\Sigma_{ZZ}^{T\,{\rm Loop}}(M_{\rm Z}^{R\,2})}{M_{\rm Z}^{R\,2}}\right] - \frac{2(c_{\rm w}^2 - s_{\rm w}^2)}{s_{\rm w}c_{\rm w}}\frac{\Sigma_{ZA}^{T\,{\rm Loop}}(0)}{M_{\rm Z}^{R\,2}}\\
&+ 2\frac{c_{\rm w}^2 - s_{\rm w}^2}{s_{\rm w}^2}g'^2 a_0 + 2g_R^2c_{\rm w}^2 a_1 + g^2(-s_{\rm w}^2 + c_{\rm w}^2\frac{c_{\rm w}^2 - s_{\rm w}^2}{s_{\rm w}^2})a_8 + \frac{c_{\rm w}^2 - s_{\rm w}^2}{s_{\rm w}^2}(g^2 + g'^2)a_{13}
\end{aligned}$$

OS prescription to the CT in physical sector

$$
\begin{aligned}
\delta\mathcal{Z}_B =& \frac{1}{c_{\rm w}^2 - s_{\rm w}^2}\left(c_{\rm w}^2\delta\mathcal{Z}_A - s_{\rm w}^2\delta\mathcal{Z}_Z\right)\\
=&\mathrm{Re}\left[\frac{d\Sigma_{AA}^{T\,\mathrm{Loop}}}{dq^2}(0) - \frac{\Sigma_{WW}^{T\,\mathrm{Loop}}(M_{\rm W}^{R\,2})}{M_{\rm W}^{R\,2}} + \frac{\Sigma_{ZZ}^{T\,\mathrm{Loop}}(M_{\rm Z}^{R\,2})}{M_{\rm Z}^{R\,2}}\right] + \frac{2s_{\rm w}}{c_{\rm w}}\frac{\Sigma_{ZA}^{T\,\mathrm{Loop}}(0)}{M_{\rm Z}^{R\,2}}\\
&- 2g'^2 a_0 - g^2(a_8 + a_{13}) - g'^2 a_{13}\\
\delta\mathcal{Z}_W =& \frac{1}{c_{\rm w}^2 - s_{\rm w}^2}\left(-s_{\rm w}^2\delta\mathcal{Z}_A + c_{\rm w}^2\delta\mathcal{Z}_Z\right)\\
=&\mathrm{Re}\left[\frac{d\Sigma_{AA}^{T\,\mathrm{Loop}}}{dq^2}(0) + \frac{c_{\rm w}^2}{s_{\rm w}^2}\frac{\Sigma_{WW}^{T\,\mathrm{Loop}}(M_{\rm W}^{R\,2})}{M_{\rm W}^{R\,2}} - \frac{c_{\rm w}^2}{s_{\rm w}^2}\frac{\Sigma_{ZZ}^{T\,\mathrm{Loop}}(M_{\rm Z}^{R\,2})}{M_{\rm Z}^{R\,2}}\right] - \frac{2c_{\rm w}}{s_{\rm w}}\frac{\Sigma_{ZA}^{T\,\mathrm{Loop}}(0)}{M_{\rm Z}^{R\,2}}\\
&+ 2\frac{c_{\rm w}^2}{s_{\rm w}^2}g'^2 a_0 + 2g^2(a_1 + a_{13}) + \frac{c_{\rm w}^2 - s_{\rm w}^2}{s_{\rm w}^2}g^2(a_8 + a_{13})\\
\frac{\delta v}{v_R} + \frac{\delta\mathcal{Z}_\pi}{2} =& \frac{\delta M_{\rm W}^2}{2M_{\rm W}^{R\,2}} + \frac{\delta\mathcal{Z}_W}{2} - \frac{\delta g}{g_R}\\
=& \frac{1}{2}\mathrm{Re}\left[\frac{d\Sigma_{AA}^{T\,\mathrm{Loop}}}{dq^2}(0) + \frac{c_{\rm w}^2 - s_{\rm w}^2}{s_{\rm w}^2}\frac{\Sigma_{WW}^{T\,\mathrm{Loop}}(M_{\rm W}^{R\,2})}{M_{\rm W}^{R\,2}} - \frac{c_{\rm w}^2}{s_{\rm w}^2}\frac{\Sigma_{ZZ}^{T\,\mathrm{Loop}}(M_{\rm Z}^{R\,2})}{M_{\rm Z}^{R\,2}}\right] + \frac{s_{\rm w}}{c_{\rm w}}\frac{\Sigma_{ZA}^{T\,\mathrm{Loop}}(0)}{M_{\rm Z}^{R\,2}}\\
&+ \frac{c_{\rm w}^2}{s_{\rm w}^2}g'^2 a_0 + g^2(a_1 + a_{13}) + \frac{c_{\rm w}^2 - s_{\rm w}^2}{2s_{\rm w}^2}g^2(a_8 + a_{13})
\end{aligned}
$$

Finite parts of the Unphysical charged sector ($\delta\xi_1$, $\delta\xi_2$, $\delta\mathcal{Z}_\pi$, a_{11})

Unphysical poles of the renormalized propagators in $q^2 = \xi_R M_{\mathrm{W}}^{R\,2}$

The renormalized ST-identity relates the $\hat{\Sigma}^L_{WW}(q^2)$, $\hat{\Sigma}_{W\pi}(q^2)$ and $\hat{\Sigma}_{\pi\pi}(q^2)$

$$q^2\hat{\Sigma}^L_{WW}(q^2) + 2q^2\hat{\Sigma}_{W\pi}(q^2) - M_{\mathrm{W}}^{R\,2}\hat{\Sigma}_{\pi\pi}(q^2) \quad = \\ = \frac{q^2 - \xi_R M_{\mathrm{W}}^{R\,2}}{\xi_R}\Big(\underbrace{-\left(\delta\mathcal{Z}_W - \delta\xi_1\right)q^2 + \xi_R\delta M_{\mathrm{W}}^2 + \xi_R M_{\mathrm{W}}^{R\,2}\left(\delta\mathcal{Z}_\pi + \delta\xi_2\right)}_{f^{\pm}_{ST}(q^2)}\Big)$$

$\Rightarrow \hat{\Sigma}^L_{WW}(\xi_R M_{\mathrm{W}}^{R\,2}) = 0$ and $\hat{\Sigma}_{\pi\pi}(\xi_R M_{\mathrm{W}}^{R\,2}) = 0$ guarantees that $\hat{\Sigma}_{W\pi}(\xi_R M_{\mathrm{W}}^{R\,2}) = 0$

Arbitrariness in the remaining 2 conditions:

- Validity of the renormalized ST-identity $\forall q^2$: $f^{\pm}_{ST}(q^2) = 0 \Rightarrow \delta\xi_1 = \delta\mathcal{Z}_W, \ldots$
- 'Low-Energy Theorems' like: $\mathcal{R}_\pi = _{1+\mathrm{Re}}\left[\frac{d\hat{\Sigma}_{\pi\pi}}{dq^2}(\xi^R_{\pm} M_{\mathrm{W}}^{R\,2})\right] = 1$ and $\hat{\Sigma}_{\pi\pi}(0) = 0$

$$\Rightarrow \hat{f}^{\pm}_{ST}(q^2)|_{LET} = -\left(\delta\mathcal{Z}_W + \frac{\Sigma^{L\,\mathrm{Loop}}_{WW}(\xi_R M_{\mathrm{W}}^{R\,2}) + \delta M_{\mathrm{W}}^2 - \Sigma^{\mathrm{Loop}}_{\pi\pi}(\xi_R M_{\mathrm{W}}^{R\,2})/\xi_R}{M_{\mathrm{W}}^{R\,2}} + \frac{d\Sigma^{\mathrm{Loop}}_{\pi\pi}}{dq^2}(\xi_R M_{\mathrm{W}}^{R\,2})\right)q^2$$

They provide the same divergencies but differ on their finite parts!

Unphysical CT prescriptions

- Validity of the renormalized ST-identity $\forall q^2$: $f_{ST}^{\pm}(q^2) = 0$

$$\delta\xi_1 = \delta\mathcal{Z}_W \qquad \delta\xi_2 = -\frac{1}{M_{\rm W}^{R\,2}}\left(\Sigma_{\pi\pi}^{\rm Loop}(\xi_R M_{\rm W}^{R\,2})/\xi_R + \Sigma_{WW}^{L\,\rm Loop}(\xi_R M_{\rm W}^{R\,2}) + 2\delta M_{\rm W}^2\right) - \delta\mathcal{Z}_W$$

$$\delta\mathcal{Z}_\pi = \frac{1}{M_{\rm W}^{R\,2}}\left(\Sigma_{\pi\pi}^{\rm Loop}(\xi_R M_{\rm W}^{R\,2})/\xi_R + \Sigma_{WW}^{L\,\rm Loop}(\xi_R M_{\rm W}^{R\,2}) + \delta M_{\rm W}^2\right) + \delta\mathcal{Z}_W$$

$$g_R^2 a_{11} = \frac{1}{\xi_R M_{\rm W}^{R\,2}}\left(\Sigma_{WW}^{L\,\rm Loop}(\xi_R M_{\rm W}^{R\,2}) + \delta M_{\rm W}^2 + M_{\rm W}^{R\,2}\mathcal{Z}_W\right)$$

$$\frac{\delta v}{v_R} = \frac{1}{2}{\rm Re}\left[\frac{d\Sigma_{AA}^{T\,\rm Loop}}{dq^2}(0) + \frac{c_{\rm w}^2 - s_{\rm w}^2}{s_{\rm w}^2}\frac{\Sigma_{WW}^{T\,\rm Loop}(M_{\rm W}^{R\,2})}{M_{\rm W}^{R\,2}} - \frac{c_{\rm w}^2}{s_{\rm w}^2}\frac{\Sigma_{ZZ}^{T\,\rm Loop}(M_{\rm Z}^{R\,2})}{M_{\rm Z}^{R\,2}}\right] + \frac{s_{\rm w}}{c_{\rm w}}\frac{\Sigma_{ZA}^{T\,\rm Loop}(0)}{M_{\rm Z}^{R\,2}}$$

- 'Low-Energy Theorems' like: $\mathcal{R}_\pi = 1 + {\rm Re}\left[\frac{d\hat{\Sigma}_{\pi\pi}}{dq^2}(\xi_{\pm}^R M_{\rm W}^{R\,2})\right] = 1$ and $\hat{\Sigma}_{\pi\pi}(0) = 0$

$$\delta\xi_1 = -\frac{1}{M_{\rm W}^{R\,2}}\left(\Sigma_{WW}^{L\,\rm Loop}(\xi_R M_{\rm W}^{R\,2}) + \delta M_{\rm W}^2 - \Sigma_{\pi\pi}^{\rm Loop}(\xi_R M_{\rm W}^{R\,2})/\xi_R\right) - \frac{d\Sigma_{\pi\pi}^{\rm Loop}}{dq^2}(\xi_R M_{\rm W}^{R\,2})$$

$$\delta\xi_2 = -\frac{1}{M_{\rm W}^{R\,2}}\left(2\Sigma_{\pi\pi}^{\rm Loop}(\xi_R M_{\rm W}^{R\,2})/\xi_R + \delta M_{\rm W}^2\right) + \frac{d\Sigma_{\pi\pi}^{\rm Loop}}{dq^2}(\xi_R M_{\rm W}^{R\,2})$$

$$\delta\mathcal{Z}_\pi = \frac{2\Sigma_{\pi\pi}^{\rm Loop}(\xi_R M_{\rm W}^{R\,2})}{\xi_R M_{\rm W}^{R\,2}} - \frac{d\Sigma_{\pi\pi}^{\rm Loop}}{dq^2}(\xi_R M_{\rm W}^{R\,2}) \qquad g_R^2 a_{11} = \frac{1}{\xi_R}\left(\frac{\Sigma_{\pi\pi}^{\rm Loop}(\xi_R M_{\rm W}^{R\,2})}{\xi_R M_{\rm W}^{R\,2}} - \frac{d\Sigma_{\pi\pi}^{\rm Loop}}{dq^2}(\xi_R M_{\rm W}^{R\,2})\right)$$

$$\frac{\delta v}{v_R} = \frac{1}{2}{\rm Re}\left[\frac{d\Sigma_{AA}^{T\,\rm Loop}}{dq^2}(0) + \frac{c_{\rm w}^2 - s_{\rm w}^2}{s_{\rm w}^2}\frac{\Sigma_{WW}^{T\,\rm Loop}(M_{\rm W}^{R\,2})}{M_{\rm W}^{R\,2}} - \frac{c_{\rm w}^2}{s_{\rm w}^2}\frac{\Sigma_{ZZ}^{T\,\rm Loop}(M_{\rm Z}^{R\,2})}{M_{\rm Z}^{R\,2}}\right] + \frac{s_{\rm w}}{c_{\rm w}}\frac{\Sigma_{ZA}^{T\,\rm Loop}(0)}{M_{\rm Z}^{R\,2}}$$
$$- \frac{\Sigma_{\pi\pi}^{\rm Loop}(\xi_{\pm}^R M_{\rm W}^{R\,2})}{\xi_R M_{\rm W}^{R\,2}} + \frac{1}{2}\frac{d\Sigma_{\pi\pi}^{\rm Loop}}{dq^2}(\xi_R M_{\rm W}^{R\,2}) + \frac{c_{\rm w}^2}{s_{\rm w}^2}g'^2 a_0 + g^2(a_1 + a_{13}) + \frac{c_{\rm w}^2 - s_{\rm w}^2}{2s_{\rm w}^2}g^2(a_8 + a_{13})$$

Bosonic Loop's divergencies in 1- and 2-legs 1PI

$$T^{\mathrm{Loop}} = \frac{\Delta_\epsilon}{16\pi^2} \frac{3\left(\kappa_3 M_{\mathrm{H}}^4 + 2a\left(2M_{\mathrm{W}}^4 + M_{\mathrm{Z}}^4\right)\right)}{2v}$$

$$-\Sigma_{HH}^{\mathrm{Loop}}(q^2) = \frac{\Delta_\epsilon}{16\pi^2} \frac{3\left(a^2 q^4 - 2a^2(2M_{\mathrm{W}}^2 + M_{\mathrm{Z}}^2)q^2 + (3\kappa_3^2 + \kappa_4)M_{\mathrm{H}}^4 + (4a^2 + 2b)(2M_{\mathrm{W}}^4 + M_{\mathrm{Z}}^4)\right)}{2v^2}$$

$$\Sigma_{WW}^{L\,\mathrm{Loop}}(q^2) = \frac{\Delta_\epsilon}{16\pi^2} \frac{g^2}{4} \left(a^2 q^2 + (a^2 - b)M_{\mathrm{H}}^2 + (9 - 3a^2 + 4\xi)M_{\mathrm{W}}^2 - 3M_{\mathrm{Z}}^2\right)$$

$$\Sigma_{W\pi}^{\mathrm{Loop}}(q^2) = \frac{\Delta_\epsilon}{16\pi^2} \frac{g^2}{4} \left(-a^2 q^2 - (a^2 - b)M_{\mathrm{H}}^2 - (3 - 3a^2 + 8\xi/3)M_{\mathrm{W}}^2 + (3 - 2\xi/3)M_{\mathrm{Z}}^2\right)$$

$$-\Sigma_{\pi\pi}^{\mathrm{Loop}}(q^2) = \frac{\Delta_\epsilon}{16\pi^2} \left(\frac{a^2}{v^2} q^4 + \frac{q^2}{v^2}((a^2 - b)M_{\mathrm{H}}^2 - (3 + 3a^2 - 4\xi/3)M_{\mathrm{W}}^2 - (3 - 4\xi/3)M_{\mathrm{Z}}^2)\right)$$

$$\Sigma_{WW}^{T\,\mathrm{Loop}}(q^2) = \frac{\Delta_\epsilon}{16\pi^2} \frac{g^2}{12} \left((51 - a^2 - 12\xi)q^2 + 3(a^2 - b)M_{\mathrm{H}}^2 + 3(9 - 3a^2 + 4\xi)M_{\mathrm{W}}^2 - 9M_{\mathrm{Z}}^2\right)$$

$$\Sigma_{AA}^{T\,\mathrm{Loop}}(q^2) = \frac{\Delta_\epsilon}{16\pi^2} e^2(4 - \xi)q^2$$

$$\Sigma_{ZZ}^{T\,\mathrm{Loop}}(q^2) = \frac{\Delta_\epsilon}{16\pi^2} \frac{g^2}{12} \Bigg((4 - \frac{1 + a^2}{c_{\mathrm{w}}^2} + 12c_{\mathrm{w}}^2(4 - \xi))q^2 + 3(a^2 - b)\frac{M_{\mathrm{H}}^2}{c_{\mathrm{w}}^2} + 12M_{\mathrm{W}}^2(3 + \xi) - 18M_{\mathrm{Z}}^2 - 9a^2\frac{M_{\mathrm{Z}}^2}{c_{\mathrm{w}}^2}\Bigg)$$

$$\Sigma_{ZA}^{T\,\mathrm{Loop}}(q^2) = \frac{\Delta_\epsilon}{16\pi^2} \frac{eg}{2c_{\mathrm{w}}} \left((1/3 + 2c_{\mathrm{w}}^2(4 - \xi))q^2 + M_{\mathrm{W}}^2(3 + \xi)\right)$$

$$\Gamma^{\rm Loop}_{HAA} = 0 \qquad \Gamma^{\rm Loop}_{HAZ} = \frac{\Delta_\epsilon}{16\pi^2}\frac{g^2 s_{\rm w}}{v} a M_{\rm W} M_{\rm Z}(\xi+3) g^{\mu\nu}$$

$$\begin{aligned}\Gamma^{\rm Loop}_{HZZ} =& \frac{\Delta_\epsilon}{16\pi^2}\frac{g^2}{12 v c_{\rm w}^2}\Big(\big(3a(2+a^2)q^2 + a(a^2-b)(k_1^2+k_2^2)\\ &-3(a^2-b)(2a-3\kappa_3)M_{\rm H}^2 - 18abM_{\rm Z}^2 + 24aM_{\rm W}^2 c_{\rm w}^2(\xi+3) - 36aM_{\rm W}^2\big) g^{\mu\nu}\\ &+2a(a^2+2b)(k_1^\mu k_1^\nu + k_2^\mu k_2^\nu) + 12a^3 k_1^\mu k_2^\nu\Big)\end{aligned}$$

$$\begin{aligned}\Gamma^{\rm Loop}_{HWW} =& \frac{\Delta_\epsilon}{16\pi^2}\frac{g^2}{12 v}\Big(\big(3a(2+a^2)q^2 + a(a^2-b)(k_1^2+k_2^2)\\ &-3(a^2-b)(2a-3\kappa_3)M_{\rm H}^2 - 18abM_{\rm W}^2 + 6aM_{\rm W}^2(4\xi+9) - 18aM_{\rm Z}^2\big) g^{\mu\nu}\\ &+2a(a^2+2b)(k_1^\mu k_1^\nu + k_2^\mu k_2^\nu) + 12a^3 k_1^\mu k_2^\nu\Big)\end{aligned}$$

$$\begin{aligned}\Gamma^{\rm Loop}_{\pi WA} =& i\frac{\Delta_\epsilon}{16\pi^2}\frac{g^2 s_{\rm w}}{6v}\Big(g^{\mu\nu}\big(3a^2q^2 + (1-a^2)k_2^2\\ &+3(a^2-b)M_{\rm H}^2 + 9(1-a^2)M_{\rm W}^2 - 9M_{\rm Z}^2 + 2(4M_{\rm W}^2+M_{\rm Z}^2)\xi\big)\\ &+6a^2k_1^\mu k_1^\nu + 3a^2 k_1^\mu k_2^\nu - (1-a^2)k_2^\mu k_2^\nu\Big)\end{aligned}$$

$$\begin{aligned}\Gamma^{\rm Loop}_{\pi WZ} =& -i\frac{\Delta_\epsilon}{16\pi^2}\frac{g^2}{12 v c_{\rm w}}\Big(g^{\mu\nu}\big(6s_{\rm w}^2a^2q^2 + (1-a^2)k_1^2 - (1-a^2)(c_{\rm w}^2 - s_{\rm w}^2)k_2^2\\ &+2s_{\rm w}^2\big(3(a^2-b)M_{\rm H}^2 + 9(1-a^2)M_{\rm W}^2 - 9a^2M_{\rm Z}^2 + 2(4M_{\rm W}^2+M_{\rm Z}^2)\xi\big)\big)\\ &+(-1+7a^2-12c_{\rm w}^2a^2)k_1^\mu k_1^\nu + 6s_{\rm w}^2a^2k_1^\mu k_2^\nu + (-1+2c_{\rm w}^2+7a^2-2c_{\rm w}^2a^2)k_2^\mu k_2^\nu\Big)\end{aligned}$$

Bosonic Loop's divergencies in WWV and WWVV' 1PI

$$\Gamma^{\rm Loop}_{WWA} = \frac{\Delta_\epsilon}{16\pi^2}\frac{g^3 s_{\rm w}}{12}\left(g^{\mu\nu}\left(k_1^\rho\left(33-4a^2-18\xi\right)-k_2^\rho\left(33+2a^2-18\xi\right)\right)\right.$$
$$+g^{\nu\rho}(k_1^\mu(33-4a^2-18\xi)-2k_2^\mu(-33+a^2+18\xi))$$
$$\left.+(-33+a^2+18\xi)g^{\rho\mu}(2k_1^\nu+k_2^\nu)\right)$$

$$\Gamma^{\rm Loop}_{WWZ} = \frac{\Delta_\epsilon}{16\pi^2}\frac{g^3}{24c_{\rm w}}\left(g^{\mu\nu}\left(k_1^\rho\left(1+5a^2+c_{\rm w}^2(66-8a^2-36\xi)\right)\right.\right.$$
$$\left.-k_2^\rho\left(1-7a^2+c_{\rm w}^2(66+4a^2-36\xi)\right)\right)$$
$$+g^{\nu\rho}(k_1^\mu(1+5a^2+c_{\rm w}^2(66-8a^2-36\xi))-2k_2^\mu(-1+a^2+c_{\rm w}^2(-66+2a^2+36\xi)))$$
$$\left.+(-1+a^2+c_{\rm w}^2(-66+2a^2+36\xi))g^{\rho\mu}(2k_1^\nu+k_2^\nu)\right)$$

$\mathcal{L}_4$ contributions to 3- and 4-legs 1PI

$$\Gamma_{HAA}|_{\mathcal{L}_4} = \frac{g^2 s_{\rm w}^2}{v}(a_{HBB} + a_{HWW} - a_{H1} + a_{H8}/2)(g^{\mu\nu}(q^2 - k_1^2 - k_2^2) - 2k_2^\mu k_1^\nu)$$

$$\Gamma_{HAZ}|_{\mathcal{L}_4} = \frac{g^2 s_{\rm w}}{v c_{\rm w}}(-s_{\rm w}^2 a_{HBB} + c_{\rm w}^2 a_{HWW} + (1/2 - c_{\rm w}^2)a_{H1} + c_{\rm w}^2 a_{H8}/2)(g^{\mu\nu}(q^2 - k_1^2 - k_2^2) - 2k_2^\mu k_1^\nu$$
$$+ \frac{g^2 s_{\rm w}}{4 v c_{\rm w}}(a_{d1} + a_{d2} + 2a_{d4})(g^{\mu\nu}(q^2 + k_1^2 - k_2^2) - 2(k_1^\mu + k_2^\mu)k_1^\nu)$$

$$\Gamma_{HZZ}|_{\mathcal{L}_4} = -\frac{g^2}{2 v c_{\rm w}^2}\left(g^{\mu\nu}\left(4 s_{\rm w}^2 M_{\rm Z}^2 a_{H0}\right.\right.$$
$$+(-2c_{\rm w}^4 a_{HWW} - 2s_{\rm w}^4 a_{HBB} - 2s_{\rm w}^2 c_{\rm w}^2 a_{H1} - c_{\rm w}^4 a_{H8} - a_{H13} + s_{\rm w}^2 a_{d1} - c_{\rm w}^2 a_{d2} - 2c_{\rm w}^2 a_{d4} - 4s_{\rm w}^2 M_{\rm Z}^2 a_{\square 0} -$$
$$+(2c_{\rm w}^4 a_{HWW} + 2s_{\rm w}^4 a_{HBB} + 2s_{\rm w}^2 c_{\rm w}^2 a_{H1} + c_{\rm w}^4 a_{H8} + a_{H13})(k_1^2 + k_2^2))$$
$$+ \left(-s_{\rm w}^2 a_{d1} + c_{\rm w}^2 a_{d2} + a_{d3} + 2c_{\rm w}^2 a_{d4} + 2a_{d5}\right)\left(k_1^\mu k_1^\nu + k_2^\mu k_2^\nu\right)$$
$$+2\left(a_{d3} + 2a_{d5} - a_{H11}\right) k_1^\mu k_2^\nu$$
$$+2\left(2c_{\rm w}^4 a_{HWW} + 2s_{\rm w}^4 a_{HBB} + 2s_{\rm w}^2 c_{\rm w}^2 a_{H1} + c_{\rm w}^4 a_{H8} - s_{\rm w}^2 a_{d1} + c_{\rm w}^2 a_{d2} + 2c_{\rm w}^2 a_{d4}\right) k$$

$$\Gamma_{HWW}|_{\mathcal{L}_4} = -\frac{g^2}{2v}\left(g^{\mu\nu}\left(-(2a_{HWW} + a_{d2} + 2a_{\square\mathcal{V}\mathcal{V}})q^2 + 2a_{HWW}(k_1^2 + k_2^2)\right)\right.$$
$$+\left(a_{d2} + a_{d3}\right)\left(k_1^\mu k_1^\nu + k_2^\mu k_2^\nu\right) + 2\left(a_{d3} - a_{H11}\right) k_1^\mu k_2^\nu + 2\left(2a_{HWW} + a_{d2}\right) k_2^\mu k_1^\nu$$

$\mathcal{L}_4$ contributions to 3- and 4-legs 1PI

$$\Gamma_{\pi WA}|_{\mathcal{L}_4} = -i\frac{g^2 s_{\rm w}}{v}\left(g^{\mu\nu}\left(-(a_1 - a_2 + a_3 - a_8 + a_9 - 2a_{11})q^2\right.\right.$$
$$+(a_1 - a_2 + a_3 - a_8 + a_9)k_1^2 + (a_1 + a_2 - a_3 - a_8 - a_9)k_2^2))$$
$$+4a_{11}k_1^\mu k_1^\nu + 2a_{11}k_1^\mu k_2^\nu + 2\left(a_1 - a_2 + a_3 - a_8 + a_9\right)k_2^\mu k_1^\nu - 2(a_2 - a_3 - \ldots$$

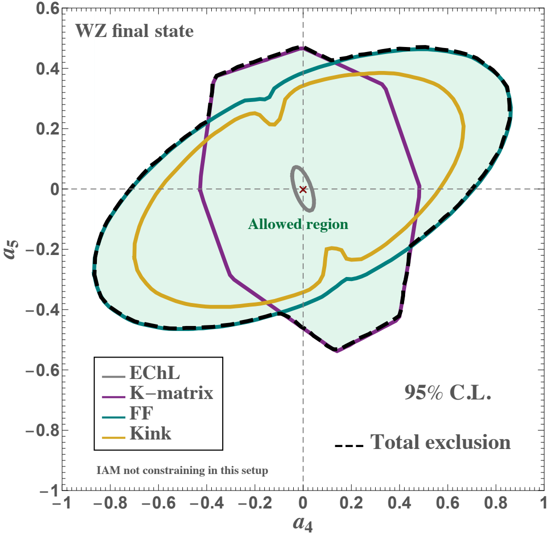
$$\Gamma_{\pi WZ}|_{\mathcal{L}_4} = i\frac{g^2}{v c_{\rm w}}\left(g^{\mu\nu}\left(4s_{\rm w}^2 M_{\rm Z}^2 a_0 - (s_{\rm w}^2 a_1 - s_{\rm w}^2 a_2 + s_{\rm w}^2 a_3 + c_{\rm w}^2 a_8 - c_{\rm w}^2 a_9 - 2s_{\rm w}^2 a_{11} + 2a_{12} + a_{13})q^2\right.\right.$$
$$+(s_{\rm w}^2 a_1 - s_{\rm w}^2 a_2 - (1 + c_{\rm w}^2)a_3 + c_{\rm w}^2 a_8 - c_{\rm w}^2 a_9 + a_{13})k_1^2$$
$$+(s_{\rm w}^2 a_1 + s_{\rm w}^2 a_2 + (1 + c_{\rm w}^2)a_3 + c_{\rm w}^2 a_8 + c_{\rm w}^2 a_9 + a_{13})k_2^2)$$
$$+2\left(a_3 + (s_{\rm w}^2 - c_{\rm w}^2)a_{11} - a_{12}\right)k_1^\mu k_1^\nu + 2(s_{\rm w}^2 a_{11} - a_{12})k_1^\mu k_2^\nu$$
$$+2\left(-s_{\rm w}^2 a_2 - c_{\rm w}^2 a_3 - c_{\rm w}^2 a_9 + a_{11} - 2a_{12} + a_{13})\right)k_2^\mu k_1^\nu$$
$$+2\left(s_{\rm w}^2 a_1 - s_{\rm w}^2 a_2 + s_{\rm w}^2 a_3 + c_{\rm w}^2 a_8 - c_{\rm w}^2 a_9 + a_{13})\right)k_2^\mu k_1^\nu)$$

$$V^{\rm CT}_{WWA}|_{\mathcal{L}_4} = g^3 s_{\rm w}\left(g^{\mu\nu}(a_{11}k_1^\rho - (a_1 - a_2 + a_3 - a_8 + a_9 - a_{11})k_2^\rho)\right.$$
$$\left.+g^{\rho\nu}(a_{11}k_1^\mu + (a_1 - a_2 + a_3 - a_8 + a_9)k_2^\mu)\right)$$

$$V^{\rm CT}_{WWZ}|_{\mathcal{L}_4} = \frac{g^3}{c_{\rm w}}\left(g^{\mu\nu}((a_3 - s_{\rm w}^2 a_{11} + a_{12})k_1^\rho + (s_{\rm w}^2 a_1 - s_{\rm w}^2 a_2 - c_{\rm w}^2 a_3 + c_{\rm w}^2 a_8 - c_{\rm w}^2 a_9 - s_{\rm w}^2 a_{11} + a_{12} + \ldots\right.$$
$$+g^{\nu\rho}((a_3 - s_{\rm w}^2 a_{11} + a_{12})k_1^\mu - (s_{\rm w}^2 a_1 - s_{\rm w}^2 a_2 - (1 + c_{\rm w}^2)a_3 + c_{\rm w}^2 a_8 - c_{\rm w}^2 a_9 + a_{13})\ldots$$
$$\left.+g^{\rho\mu}(-2a_3 k_1^\nu - a_3 k_2^\nu)\right)$$

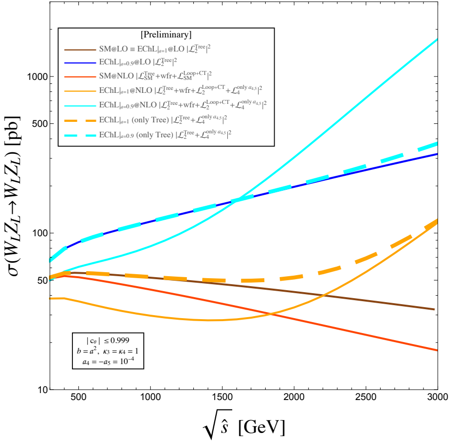
Unitarization effects [Phys. Rev. D 100 no. 9, 096003 (2019)]

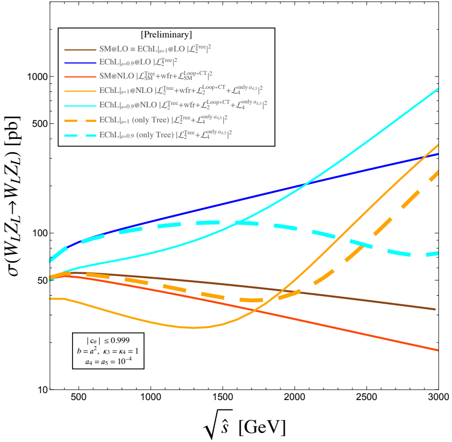
Parameter uncertainty in (a_4, a_5) plane from $WZ \to WZ$



Numerical evaluation of the bosonic contributions in the Feynman gauge ($\xi = 1$)

Only a_4 and a_5 effects of $\mathcal{L}_4$: $a_4 = -a_5 = 10^{-4}$ (left) and $a_4 = a_5 = 10^{-4}$ (right)





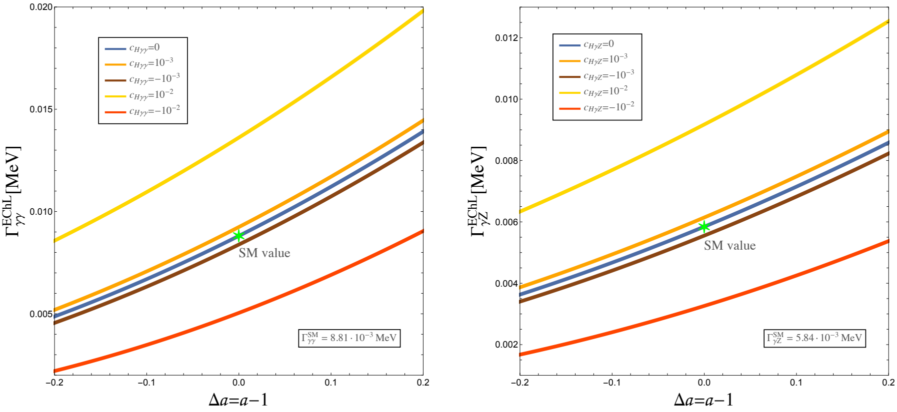
Here, $a_5^{scattering} = a_5 + a_7 - \frac{a}{2} a_{\Box\mathcal{V}\mathcal{V}} + \frac{a^2}{4} a_{\Box\Box}$

G_μ-scheme inputs: $M_{W,Z,H} = 80.38, 91.18, 125.1$ GeV and $G_\mu = 1.16637 \cdot 10^{-5}\text{GeV}^{-2}$

EChL's prediction for $H \to \gamma\gamma, \gamma Z$ decays

[Phys. Rev. D 102 no. 7, 075040 (2020)]

Partial widths for different a and $c_{H\gamma\gamma, H\gamma Z}$ (fermionic contributions included as in the SM)



Next, consider the Higgs boson off-shell (arbitrary q^2)

$H^* \to \gamma V$ 'decays': off- vs on-shell Higgs (Analytic Results)

[Phys. Rev. D 102 no. 7, 075040 (2020)]

Summing all the contributions: $\mathcal{M}^{\mu\nu}_{\gamma V} = V_{H\gamma V}(q, k_1, k_2)\left(g^{\mu\nu} - \frac{k_2^\mu k_1^\nu}{k_1 \cdot k_2}\right)$

$$V_{H\gamma\gamma} = \frac{e^2 g}{2M_{\rm W}}\left(\frac{a}{16\pi^2}\left(2q^2 + 12M_{\rm W}^2\left(1 + \left(2 - \frac{4M_{\rm W}^2}{q^2}\right) f\left(\frac{4M_{\rm W}^2}{q^2}\right)\right)\right) + c_{H\gamma\gamma}q^2\right)$$

$$\begin{aligned} V_{H\gamma Z} =& \frac{eg^2 c_{\rm w}}{2M_{\rm W}}\left(\frac{a}{16\pi^2}\left(2q^2 + 12M_{\rm W}^2 - \frac{M_{\rm Z}^2}{M_{\rm W}^2}q^2 - 2M_{\rm Z}^2\right.\right. \\ &- \frac{4M_{\rm W}^2}{q^2 - M_{\rm Z}^2}\left(-6q^2 + 12M_{\rm W}^2 + \frac{M_{\rm Z}^2}{M_{\rm W}^2}q^2 + 6M_{\rm Z}^2 - \frac{2M_{\rm Z}^4}{M_{\rm W}^2}\right)\left(f\left(\frac{4M_{\rm W}^2}{q^2}\right) - f\left(\frac{4M_{\rm W}^2}{M_{\rm Z}^2}\right)\right) \\ &\left.- \frac{2M_{\rm Z}^2}{q^2 - M_{\rm Z}^2}\left(2q^2 + 12M_{\rm W}^2 - \frac{M_{\rm Z}^2}{M_{\rm W}^2}q^2 - 2M_{\rm Z}^2\right)\left(g\left(\frac{4M_{\rm W}^2}{q^2}\right) - g\left(\frac{4M_{\rm W}^2}{M_{\rm Z}^2}\right)\right)\right) \\ &\left. + c_{H\gamma Z}(q^2 - M_{\rm Z}^2)\right) \end{aligned}$$

Finite and ξ independent for arbitrary q^2! (It is not longer true in the SM)
SM recovered only for $a = 1$, $c_i = 0$ and $q^2 = M_{\rm H}^2$.
Useful in Higgs mediated processes: $WW \to \gamma\gamma\,,\, \gamma Z$ and $\mu^+\mu^- \to \gamma\gamma\,,\, \gamma Z$