A Preferred Basis for Effective Field Theories
from Amplitude Basis

Ming-Lei Xiao

Institute of Theoretical Physics, Chinese Academy of Science, Beijing

arXiv:2001.04481 with Minyuan Jiang, Jing Shu and Yu-Hui Zheng
arXiv:2012.11615 with Hao-Lin Li, Jing Shu and Jiang-Hao Yu

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Outline

- A brief review of partial waves.
- Construct the generalized partial wave basis.
- The j-basis amplitudes/operators.
- Applications: selection rules and UV origins.
- Summary & Outlook

A Review of Partial Wave Analysis I

Partial Wave Analysis: solving scattering problems by decomposing the multi-particle wave functions $|\Psi\rangle = |\psi_1, \psi_2, \ldots\rangle$ into its constituent angular momentum components $|P, J, M, \ldots\rangle$.

Group	C-G coefficients
$SO(3)$	$\langle j_1, m_1; j_2, m_2 \| J, M\rangle$
Poincaré	$\langle p_1, s_1, \sigma_1; p_2, s_2, \sigma_2; \ldots \| P, J, M, \ldots\rangle$

S-matrix is block diagonal $\Rightarrow$ $a_J(s = P^2) = \langle P, J, M|S|P, J, M\rangle$

- Perturbative Unitarity Bound Re$a_J \leq \frac{1}{2}$.
  - coupled channel analysis
- Phase Space Integration $\sim \sum_J (2J+1)|a_J|^2$.

A Review of Partial Wave Analysis II

Well known partial waves for 2-particle states ($h_i = \vec{s}_i \cdot \hat{p}_i$):

$$\langle h_1', h_2'; \theta, \phi | h_1, h_2; J, M \rangle = \sqrt{\frac{2J+1}{4\pi}} \delta_{h_1,h_1'} \delta_{h_2,h_2'} \overbrace{\mathcal{D}^J_{M,\alpha}(\phi, \theta, -\phi)}^{\text{Wigner-D matrix}} .$$

Jacob-Wick expansion:

$$\langle h_3, h_4; \theta, \phi | S | h_1, h_2; 0, 0 \rangle = \frac{1}{4\pi} \sum_J (2J+1) a_J(s) d^J_{\alpha,\alpha'}(\theta) e^{i(\alpha-\alpha')\phi} .$$

Inspirations:

- Defined in CoM frame (requires boosts for non-CoM frame).
  Lorentz invariant form?
- Generalization to more particles? (3-particle phase space)
- Implication by amplitude-operator correspondence?

Helicity Amplitudes as Partial Wave Basis

Lorentz invariant form of amplitudes: $\mathcal{A}(|i\rangle^{(I)}, |i]^{(J)})$.

- Helicity spinor variables: $p_{i\mu}\sigma^{\mu}_{\alpha\dot{\alpha}} = |i\rangle_\alpha [i|_{\dot{\alpha}}$.
- Massive version: $p_{i\mu}\sigma^{\mu}_{\alpha\dot{\alpha}} = |i\rangle^I_\alpha [i|_{\dot{\alpha}\, I}$ (arXiv:1709.04891)

Correspondence with d-matrices, e.g.

$$d^1_{1,0}(\theta) \sim \mathcal{A}(h, h \to 1, 0) = \begin{cases} [13][23] & h = -1/2 \\ \langle 12\rangle[13][23] & h = 0 \\ \ldots \end{cases}$$

$$d^0_{0,0}(\theta) \sim \mathcal{A}\left(\frac{1}{2}, \frac{1}{2} \to -\frac{1}{2}, -\frac{1}{2}\right)^{J=0} = \langle 12\rangle\langle 34\rangle.$$

$$d^1_{0,0}(\theta) \sim \mathcal{A}\left(\frac{1}{2}, \frac{1}{2} \to -\frac{1}{2}, -\frac{1}{2}\right)^{J=1} = \langle 13\rangle\langle 24\rangle + \langle 14\rangle\langle 23\rangle.$$

Q: How to construct amplitude basis with given J ?

Intuitive Construction

A1: Auxiliary spinors for total momentum $P_\mu \sigma^\mu_{\alpha\dot{\alpha}} = |\chi\rangle^I_\alpha [\chi|_{\dot{\alpha}\, I}$

- Generalize: $\langle \Psi | P, J, M \rangle = f^J_\Psi(|i\rangle, |i]; |\chi\rangle, |\chi])^{(I_1, \ldots, I_{2J})}$.
  1. Lorentz invariance
  2. Little group

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$$f^J_{h_1,h_2} = [12]^{J+h_1+h_2} \left(\langle 1\chi\rangle^{J-h_1+h_2} \langle 2\chi\rangle^{J+h_1-h_2} \right)^{(I_1,\ldots,I_{2J})}$$

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- Generalized partial wave amplitude basis $\mathcal{B}^J = f^J_{\text{final}} \cdot f^{J*}_{\text{initial}}$.

$$d^J_{h_3-h_4,h_2-h_1} = f^J_{h_3,h_4} \cdot f^{J*}_{h_1,h_2} \qquad \checkmark$$

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- A quick criterion: $2J$ "bridges" among same type spinors.

$$|\chi\rangle^{I_1} \cdots |\chi\rangle^{I_{2J}} \cdot \langle\chi|_{I_1} \cdots \langle\chi|_{I_{2J}} = (-\sqrt{s})^{2J} \epsilon^{\cdot 2J}$$

$$\mathcal{B}^{J=0} = \langle 12\rangle\langle 34\rangle, \quad \mathcal{B}^{J=1} = \langle 13\rangle\langle 24\rangle + \langle 14\rangle\langle 23\rangle \quad \checkmark$$

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- f^J_Ψ not available (yet) for general $N > 2$ states; tricky to get complete basis at given dimension $|\chi] = \frac{P|\chi\rangle}{\sqrt{s}}$.

Poincaré Algebra

A2: Analogy with $SO(3)$ algebra:

- $SO(3)$ — $\mathbf{J}^2|J,M\rangle = J(J+1)|J,M\rangle$:

$$\begin{aligned}\mathbf{L}^2\langle\theta,\phi|l,m\rangle &= -\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial}{\partial\theta}+\frac{1}{\sin^2\theta}\frac{\partial}{\partial\phi}\right]Y_l^m(\theta,\phi)\\ &= l(l+1)Y_l^m(\theta,\phi)\end{aligned}$$

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A2: Analogy with $SO(3)$ algebra: $(\mathbf{W}^{\mu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\mathbf{P}_{\nu}\mathbf{J}_{\rho\sigma})$

- Poincaré — $\mathbf{W}^2|P,J,M\rangle = -P^2 J(J+1)|P,J,M\rangle$:

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Spinor representation for N-particle wave functions

$$\mathbf{P}_\mu = \frac{1}{2}\sum_{i=1}^{N}\langle i|\sigma_\mu|i], \quad \mathbf{J}_{\mu\nu} = \sum_{i=1}^{N}(\langle i|\sigma_{\mu\nu}|\partial_i\rangle + [i|\bar{\sigma}_{\mu\nu}|\partial_i])$$

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Spinor representation for N-particle wave functions

$$\mathbf{W}^2 = \frac{s}{8}\sum_{i,j=1}^{N}\Big(\langle i,\partial_j\rangle\langle j,\partial_i\rangle + [i,\partial_j][j,\partial_i]\Big) - \frac{1}{4}\sum_{i,j,k,l}[i,j]\langle j,\partial_k\rangle\langle k,l\rangle[l,\partial_i]$$

Eigenvalue equation: $\mathbf{W}^2_{\text{initial/final}}\mathcal{B}^J = -sJ(J+1)\mathcal{B}^J$ ✓

$$\mathbf{W}^2\mathcal{B}^{\text{y}} = -s\mathcal{W}.\mathcal{B}^{\text{y}}, \qquad \mathcal{K}.\mathcal{W}.\mathcal{K}^{-1} = \text{diag}\{J(J+1)\}$$

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Complete Partial Wave Amplitude Basis: $\mathcal{B}^{\text{j}} = \mathcal{K}.\mathcal{B}^{\text{y}}$

Examples

- 4-fermion (ψ^4):

$$\mathcal{B}^{\mathrm{y}} = \left\{ \begin{array}{l} \langle 12\rangle\langle 34\rangle \\ \langle 13\rangle\langle 24\rangle \end{array} \right. , \qquad \mathbf{W}_{(1,2)}{}^{2}\mathcal{B}^{\mathrm{y}} = -s_{12} \begin{pmatrix} 0 & 0 \\ -1 & 2 \end{pmatrix} . \mathcal{B}^{\mathrm{y}}$$

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$$\Rightarrow \mathcal{B}^{\mathrm{j}} = \begin{cases} \langle 12\rangle\langle 34\rangle & J=0 \\ -\langle 12\rangle\langle 34\rangle + 2\langle 13\rangle\langle 24\rangle & J=1 \end{cases}$$

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- 4-scalar ($\phi^4 D^2$):

$$\mathcal{B}^{\mathrm{y}} = \left\{ \begin{array}{l} s_{12} \\ s_{13} \end{array} \right. , \qquad \mathbf{W}_{(1,3)}{}^2 \mathcal{B}^{\mathrm{y}} = -s_{13} \begin{pmatrix} 2 & 1 \\ 0 & 0 \end{pmatrix} . \mathcal{B}^{\mathrm{y}}$$

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Amplitude/Operator j-Basis

Construct the j-basis by adding gauge tensor factors, e.g.

$$\mathcal{A}^{(6)}(H_{1,i}, H_3^{\dagger,k} \to H_{2,j}, H_4^{\dagger,l}) = \sum_{J,\mathbf{r}} C^{J,\mathbf{r}} \mathcal{B}^J T^{\mathbf{r}},$$

Lorentz j-basis (partial waves)	Gauge j-basis
$\mathcal{B}^{J=0} = s_{13}/\Lambda^2$	$T^{\mathbf{r=1}} = \delta_i^k \delta_j^l$
$\mathcal{B}^{J=1} = (s_{12} - s_{14})/\Lambda^2$	$T^{\mathbf{r=3}} = (\tau^I)_i^k (\tau^I)_j^l$

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with coefficients $C^{J,\mathbf{r}} \to C^{\mathrm{j}}_{1,2,3,4}$.

$$\underbrace{\mathcal{O}^{\mathrm{j}}.C^{\mathrm{j}} = \mathcal{O}^{\mathrm{p}}.C^{\mathrm{p}}}_{\text{alternative basis}}$$

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$$\mathcal{O}^{\mathrm{j}}.C^{\mathrm{j}} = \underbrace{\mathcal{O}^{\mathrm{p}}.C^{\mathrm{p}} = \mathcal{O}^{\mathrm{j}}.(\mathcal{K}^{\mathrm{pj}})^{\intercal}.C^{\mathrm{p}}}_{\text{basis conversion}}$$

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with coefficients $C^{J,\mathbf{r}} \to C^{\mathrm{j}} = (\mathcal{K}^{\mathrm{pj}})^{\intercal}.C^{\mathrm{p}}$.

$$\underbrace{\begin{pmatrix} C^{0,\mathbf{1}} \\ C^{0,\mathbf{3}} \\ C^{1,\mathbf{1}} \\ C^{1,\mathbf{3}} \end{pmatrix}}_{C^{\mathrm{j}}} = \underbrace{\begin{pmatrix} 3 & 0 \\ -1 & 1 \\ -1 & -1 \\ -1 & 0 \end{pmatrix}}_{(\mathcal{K}^{\mathrm{pj}})^{\intercal}} \cdot \underbrace{\begin{pmatrix} C_{H\Box} \\ C_{HD} \end{pmatrix}}_{C^{\mathrm{p}} \text{ in the Warsaw basis}}$$

j-basis is an alternative flavor-blind operator/amplitude basis, **NOT** independent after considering repeated field issue!

Relation with other basis

Remember we can expand any operators onto the y-basis

$$\mathcal{O}^{\mathrm{p}} = \mathcal{K}^{\mathrm{pj}}.\mathcal{O}^{\mathrm{j}}$$

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$$\mathcal{K}^{\mathrm{py}}.\mathcal{O}^{\mathrm{y}} = \mathcal{O}^{\mathrm{p}} = \mathcal{K}^{\mathrm{pj}}.\mathcal{O}^{\mathrm{j}} = \mathcal{K}^{\mathrm{pj}}.\mathcal{K}^{\mathrm{jy}}.\mathcal{O}^{\mathrm{y}}$$

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Remember we can expand any operators onto the y-basis

$$\mathcal{K}^{\mathrm{pj}} = \mathcal{K}^{\mathrm{py}}.(\mathcal{K}^{\mathrm{jy}})^{-1}$$

Conversion matrix $\mathcal{K}^{\mathrm{jy}} = \mathcal{K}^{\mathrm{jy}}_{\mathcal{B}} \otimes \mathcal{K}^{\mathrm{jy}}_{T}$.

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Using package *AmplitudeBasisCalculator.m*

```
GetJBasisForType[Model, "H"^2 "H†"^2 "D"^2, {{1, 3}, {2, 4}}] // PrintJBasis
```

$\langle| \{1,3\} \to \{1,1,3\} |\rangle \to \begin{pmatrix} 1 & -2 & -\frac{1}{2} & 1 \end{pmatrix}$

$\langle| \{1,3\} \to \{1,1,1\} |\rangle \to \begin{pmatrix} 2 & 0 & -1 & 0 \end{pmatrix}$

$\langle| \{1,3\} \to \{0,1,3\} |\rangle \to \begin{pmatrix} 0 & 0 & -\frac{1}{2} & 1 \end{pmatrix}$

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$$\langle | H \to \square\square, H^\dagger \to \square\square | \rangle \to \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \end{pmatrix}$$

$$\langle | H \to \begin{smallmatrix}\square\\\square\end{smallmatrix}, H^\dagger \to \begin{smallmatrix}\square\\\square\end{smallmatrix} | \rangle \to \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

Selection Rules for Loop Integrals I

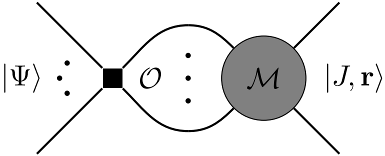
- Corresponding operators annihilate states with mismatch quantum numbers (when Wick contraction is unique)

$$\mathcal{O}^{J',\mathbf{r}'}|J,\mathbf{r}\rangle = 0, \quad \text{if } J \neq J' \text{ or } \mathbf{r} \neq \mathbf{r}'$$

- Non-trivial when the quantum number is inferred by the other side (conservation laws)

$$\langle J,\mathbf{r}|\mathcal{M} \otimes \mathcal{O}^{J',\mathbf{r}'}|\Psi\rangle = 0, \quad \text{if } J \neq J' \text{ or } \mathbf{r} \neq \mathbf{r}'$$

Valid to all orders of $\mathcal{M}$ and rational terms also vanish!



Selection Rules for Loop Integrals II

Two cases when $(J, \mathbf{r})$ are constrained:

- Rule A: $J \geq |h_1 - h_2|$ for two massless particle state $|h_1, h_2\rangle$.
  Ex. type $H^4 D^2$ contribute to $\mathcal{A}(H, H^\dagger, e_R, e_R^\dagger)$ through the combination
  $$C^{1,\mathbf{1}} \sim C_{H\Box} + C_{HD}$$
  .
- Rule B: Permutation symmetry of gauge sector $(-1)^J$ for two identical particles.
  Ex. type $H^2\psi\bar{\psi}D$ do not contribute to $\mathcal{A}(H, H^\dagger, \gamma/g, \gamma/g)$, but may contribute to
  $$\mathcal{A}(H_i, H^{\dagger,j}, W^I, W^J) \sim C_{H\psi}\epsilon^{IJK}(\tau^K)_i^j$$

UV Origins of Effective Operators

UV ansatz with particular heavy resonances directly produces j-basis effective operators

$$\text{Resonance with } J, \mathbf{r} \quad \Rightarrow \quad \mathcal{O}^{J,\mathbf{r}} = (\mathcal{K}^{\rm pj})^{-1}.\mathcal{O}^{\rm p}.$$

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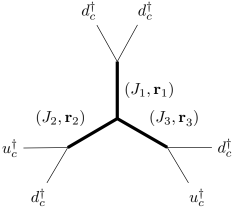
Ex. n-$\bar{n}$ oscillation from effective operators $d_R^4 u_R^2$:



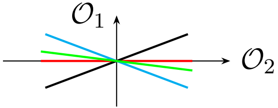
$$\mathcal{O}_1^{\rm p} = \epsilon^{ace}\epsilon^{bdf}(d_{Ra}d_{Rb})(d_{Rc}d_{Rd})(u_{Re}u_{Rf}),$$
$$\mathcal{O}_2^{\rm p} = \epsilon^{acd}\epsilon^{bef}(d_{Ra}d_{Rb})(d_{Rc}u_{Re})(d_{Rd}u_{Rf}).$$

n-$\bar{n}$ Operator Landscape

The $(\mathcal{K}^{\mathrm{pj}})^{-1}$ information (part):

$(\mathbf{r}_i, J_i)$	$(1,1,1)$	$(0,1,1)$	$(1,0,1)$	$(1,1,0)$	$(0,0,0)$
$(\mathbf{6},\mathbf{6},\mathbf{6})$	0	$3\mathcal{O}_1 + 8\mathcal{O}_2$	0	0	$\mathcal{O}_1 - 8\mathcal{O}_2$
$(\mathbf{6},\bar{\mathbf{3}},\bar{\mathbf{3}})$	0	$\mathcal{O}_2$	0	0	$\mathcal{O}_2$
$(\bar{\mathbf{3}},\mathbf{6},\bar{\mathbf{3}})$	$3\mathcal{O}_1 + 8\mathcal{O}_2$	0	$\mathcal{O}_1 - 8\mathcal{O}_2$	$3\mathcal{O}_1 + 8\mathcal{O}_2$	0
$(\bar{\mathbf{3}},\bar{\mathbf{3}},\mathbf{6})$	$3\mathcal{O}_1 + 8\mathcal{O}_2$	0	$3\mathcal{O}_1 + 8\mathcal{O}_2$	$\mathcal{O}_1 - 8\mathcal{O}_2$	0
$(\bar{\mathbf{3}},\bar{\mathbf{3}},\bar{\mathbf{3}})$	$-3\mathcal{O}_1 + 8\mathcal{O}_2$	0	$3\mathcal{O}_1 + 8\mathcal{O}_2$	$3\mathcal{O}_1 + 8\mathcal{O}_2$	0

Target subspaces on the landscape:



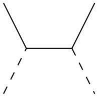
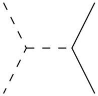
Seesaw Model from Landscape Analysis

- p-basis:

$$\mathcal{O}^S = (HL)(HL), \quad \mathcal{O}^A = (HH)(LL)$$

$\mathcal{O}^A$ requires anti-symmetric flavors for Higgs, ruled out in SM!

- j-basis:

Topology	j-basis	Model
	$\mathcal{O}^{(1/2,\mathbf{1})}_{HL\to HL} = \mathcal{O}^S + \mathcal{O}^A$	type I
	$\mathcal{O}^{(1/2,\mathbf{3})}_{HL\to HL} = \mathcal{O}^S - 3\mathcal{O}^A$	type III
	$\mathcal{O}^{(0,\mathbf{1})}_{HH\to LL} = \mathcal{O}^A$	N/A
	$\mathcal{O}^{(0,\mathbf{3})}_{HH\to LL} = \mathcal{O}^S$	type II

Summary & Outlook

- Partial wave basis is generalized to arbitrary number of external particles, massive or massless alike.
- Systematic j-basis analysis, realized by code.
- Selection rules in loop integrals, partial wave phase space integration.
- Landscape analysis, UV resonance, positivity bound, perturbative unitarity bound.

Thank You

Fine

Backup – spinor values

With momentum $p_{i\mu} = (E, p\sin\theta_i\cos\phi_i, p\sin\theta_i\sin\phi_i, p\cos\theta_i)$

$$|i\rangle_\alpha = |\bar{i}]^{\dot{\alpha}} = \sqrt{E}\begin{pmatrix}\cos\frac{\theta_i}{2} \\ \sin\frac{\theta_i}{2}e^{i\phi_i}\end{pmatrix}, \quad |i]^{\dot{\alpha}} = |\bar{i}\rangle_\alpha = \sqrt{E}\begin{pmatrix}-\sin\frac{\theta_i}{2}e^{-i\phi_i} \\ \cos\frac{\theta_i}{2}\end{pmatrix}$$

When $m^2 = E^2 - p^2 \neq 0$, expand the massive spinors in a basis $\zeta^\pm$,

$$|i\rangle^I = |i\rangle\zeta^{-I} + \frac{m}{E}|\bar{i}\rangle\zeta^{+I}, \quad |i]^I = |i]\zeta^{+I} + \frac{m}{E}|\bar{i}]\zeta^{-I}.$$

with orthonormal condition $\zeta^{+I}\zeta^-_I = 1$. The choice

$$\zeta^+ = \begin{pmatrix}1 \\ 0\end{pmatrix}, \quad \zeta^- = \begin{pmatrix}0 \\ 1\end{pmatrix}$$

represents spin components along the momentum direction $\hat{\mathbf{p}}$.