S-matrix Bootstrap for Effective Field Theories

EPFL

João Penedones

phonons with Elias-Miró, Guerrieri, Hebbar, Vieira

pions with Guerrieri, Vieira

TODAY → super-gravitons with Guerrieri, Vieira

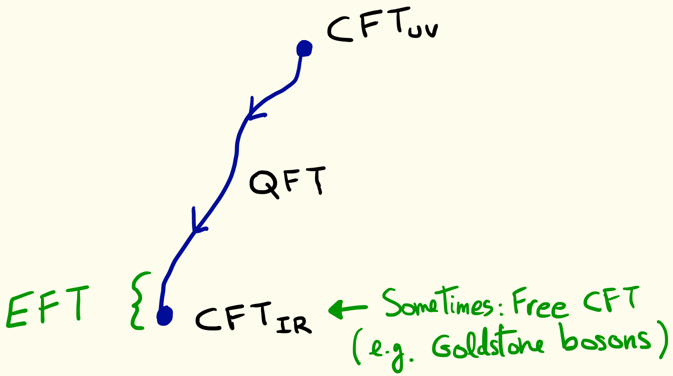
photons with Meineri, Haring, Hebbar, Karateev (to appear)

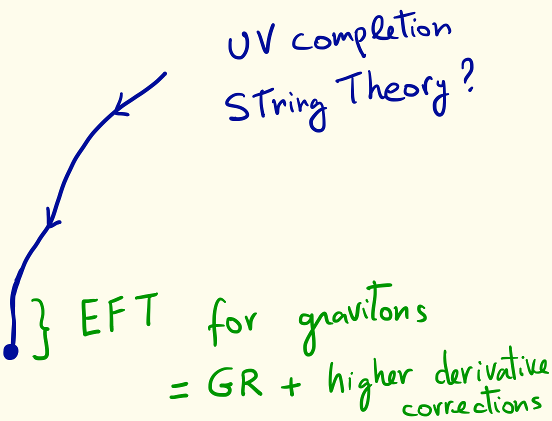
HEFT 2021, Hefei, China, 16/04/2021

Introduction

QFT Bootstrap

Goal: map out the space of QFTs and develop nonperturbative methods to compute their observables.





S-matrix Bootstrap approach: bound the space of theories by imposing the principles of Lorentz invariance, causality and unitarity on scattering amplitudes. [old idea – new (numerical) approach]

Outline

- Introduction

- Supergravitons

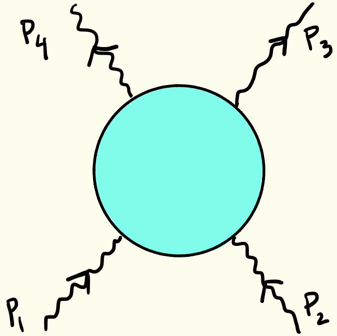
  - S-matrix Bootstrap review

[- Massless pions]

- Discussion

Maximal Supergravity $D=10$

$2 \to 2$ Supergraviton Scattering



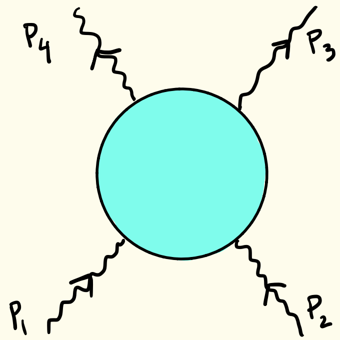
$$\mathbb{A}_{2\to 2} = \mathbf{R}^4 A(s,t,u)\,.$$

pre-factor fixed by SUSY

crossing symmetric function of $\begin{cases} s = (p_1+p_2)^2 \\ t = (p_1-p_3)^2 \\ u = (p_1-p_4)^2 \end{cases}$

$s+t+u = 0$

2 → 2 Supergraviton Scattering



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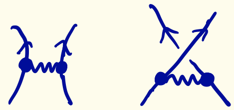
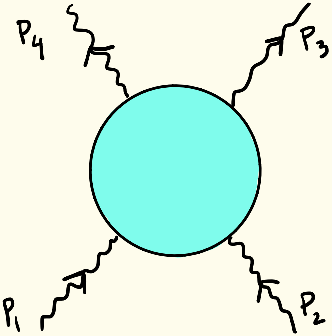


Focus on charged scalar :
[axi-dilaton in IIB]

$$T(s,t,u) \equiv s^4 A(s,t,u) = -8\pi G_N \left(\frac{s^2}{t} + \frac{s^2}{u}\right) + \ldots$$

$$\frac{T(s,t,u)}{8\pi G_N = 64\pi^7 \ell_P^8} = s^4\left(\frac{1}{stu} + \alpha\,\ell_P^6 + O(s)\right)$$

$2 \to 2$ Supergraviton Scattering



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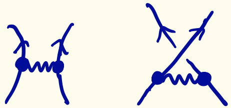

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Wilson coefficient

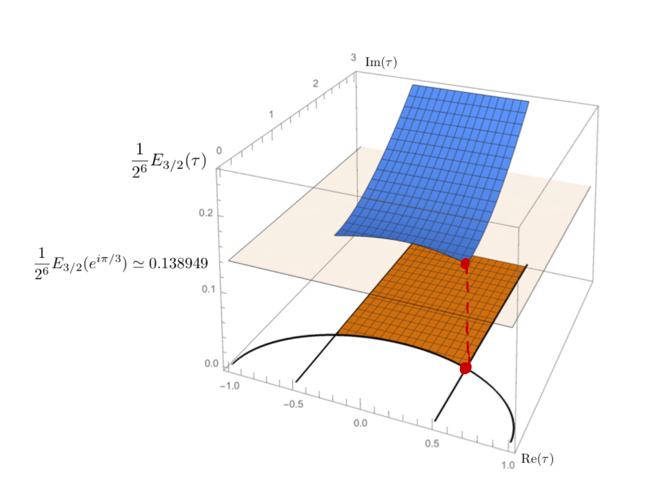
EFT: $$S = \frac{1}{(2\pi)^7 \ell_p^8}\int d^{10}x \sqrt{-g}\left[R + \#\alpha \ell_p^6 R^4 + \ldots\right]$$

Superstrings

IIB

$$\alpha^{\text{IIB}} = \frac{1}{2^6} E_{\frac{3}{2}}(\tau, \bar{\tau}) \geq \frac{1}{2^6} E_{\frac{3}{2}}(e^{i\pi/3}, e^{-i\pi/3}) \approx 0.1389$$

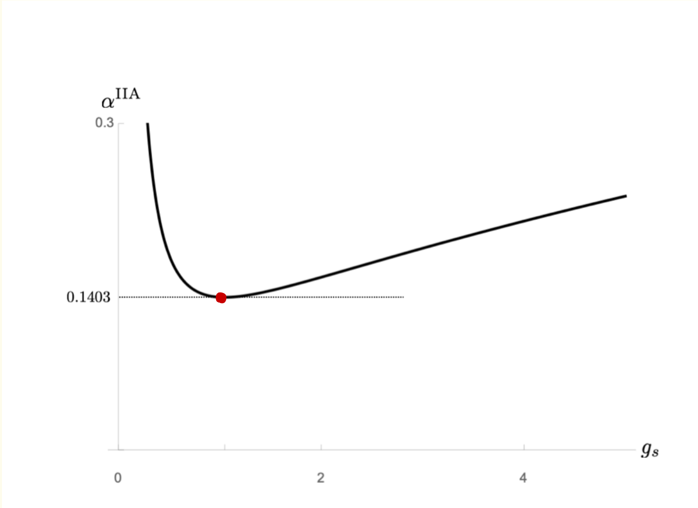
$\tau = x_s + i/g_s$



['97 Green, Gutperle]

IIA

$$\alpha^{\text{IIA}} = \frac{\zeta(3)}{32 g_s^{3/2}} + g_s^{1/2}\frac{\pi^2}{96} \geq \frac{\pi^{3/2}(\zeta(3))^{1/4}}{24\sqrt{3}} \approx 0.1403$$



$$\alpha^{\text{IIB}} = \alpha^{\text{IIA}} + O\left(e^{-\frac{2\pi}{g_s}}\right).$$

Dispersive formula

$$\alpha = \frac{1}{32\pi^8 \ell_P^{14}} \int_0^\infty \frac{ds}{s^5} \underbrace{\mathrm{Im}\, T(s+i\epsilon, t=0)}_{\frac{\pi^{10}}{18} \ell_P^{16} s^5 + O(s^6)} . > 0$$

$\leftarrow$ 1-loop in EFT

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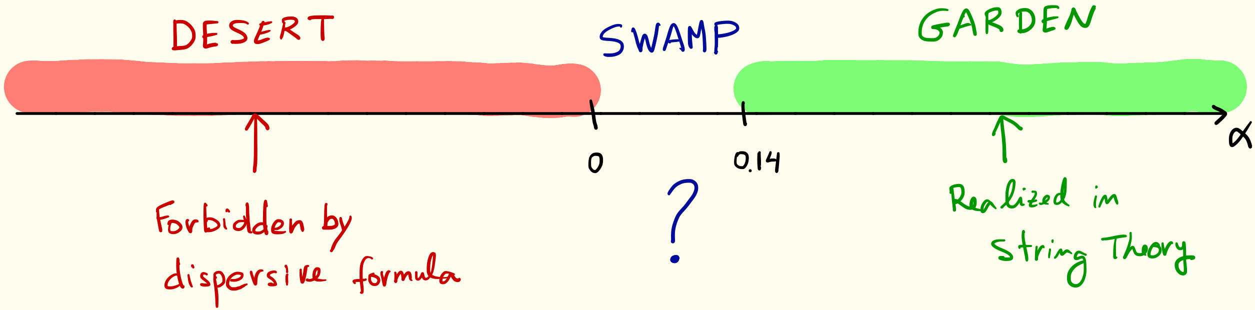

Numerical S-matrix Bootstrap

ALGORITHM

['17 Paulos, JP, Toledo, Van Rees, Vieira]

1. Write amplitude ansatz obeying: $\begin{cases} \text{Lorentz invariance} \\ \text{Crossing symmetry} \\ \text{Analyticity} \\ \text{Low energy from EFT} \end{cases}$

2. Impose unitarity of each partial wave

3. Minimize a linear observable of the amplitude

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   - Lorentz invariance
   - Crossing symmetry
   - Analyticity
   - Low energy from EFT

   ansatz $\rightarrow$ # of parameters $\sim N^2$

2. Impose unitarity of each partial wave $\rightarrow$ spin $\ell \leq L$

3. Minimize a linear observable of the amplitude

4. Extrapolate $L \rightarrow \infty$ and $N \rightarrow \infty$

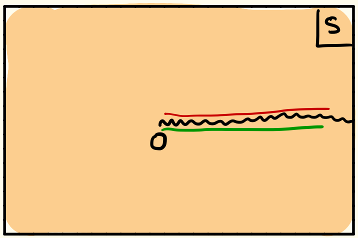
1. Analytic and crossing symmetric ansatz:

free parameters

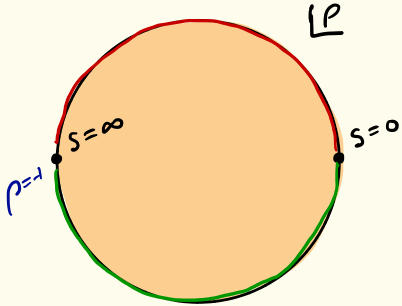
$$\frac{T}{8\pi G_N} = s^4\left(\underbrace{\frac{1}{stu}}_{\texttt{SUGRA}} + \underbrace{\prod_{A=s,t,u}(\rho_A+1)^2 \sum_{a+b+c\leq N}{}' \alpha_{(abc)}\rho_s^a\rho_t^b\rho_u^c}_{\texttt{UV completion}}\right) \qquad (9)$$

$s+t+u=0$

$[\ell_p = 1]$

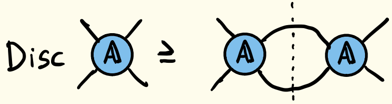


$$\rho(s) = \frac{1-\sqrt{-s}}{1+\sqrt{-s}}$$



2. Unitarity constraints

['98 Bern et al.]



$$\sum_{\text{two pt}} \mathbf{R}^4_{12\to\text{two pt}} \mathbf{R}^4_{\text{two pt}\to 34} = \mathbf{R}^4_{12\to 34} \times s^4$$

$\Rightarrow$

$$\mathrm{Disc}A \geq s^4 \int d\mathrm{LIPS}\, A \times A\,.$$

Unitarity of $\mathbb{A}_{2\to 2} = \mathbf{R}^4 A(s,t,u)\,.$ $\Leftrightarrow$ Unitarity of $T = s^4 A(s,t,u)$

$\Rightarrow$ $\boxed{|S_\ell(s)|^2 \leq 1}$ for $\begin{cases} \ell = 0, 2, 4, \ldots, L \\ s > 0 \ \text{[grid]} \end{cases}$

Gegenbauer polynomial

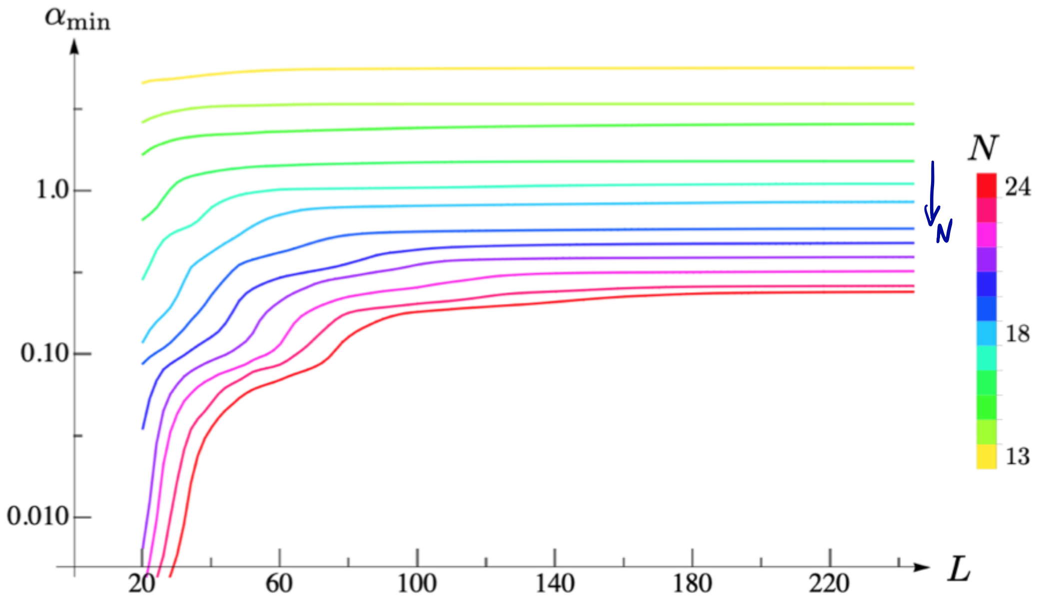
$$S_\ell(s) = 1 + \frac{i}{3 \cdot 2^{18} \pi^4} s^3 \int_{-1}^{1} dx (1-x^2)^3 \frac{C_\ell^{7/2}(x)}{C_\ell^{7/2}(1)} T(s,x)\,,$$

$\begin{cases} t = -\frac{s}{2}(1-x) \\ u = -\frac{s}{2}(1+x) \end{cases}$

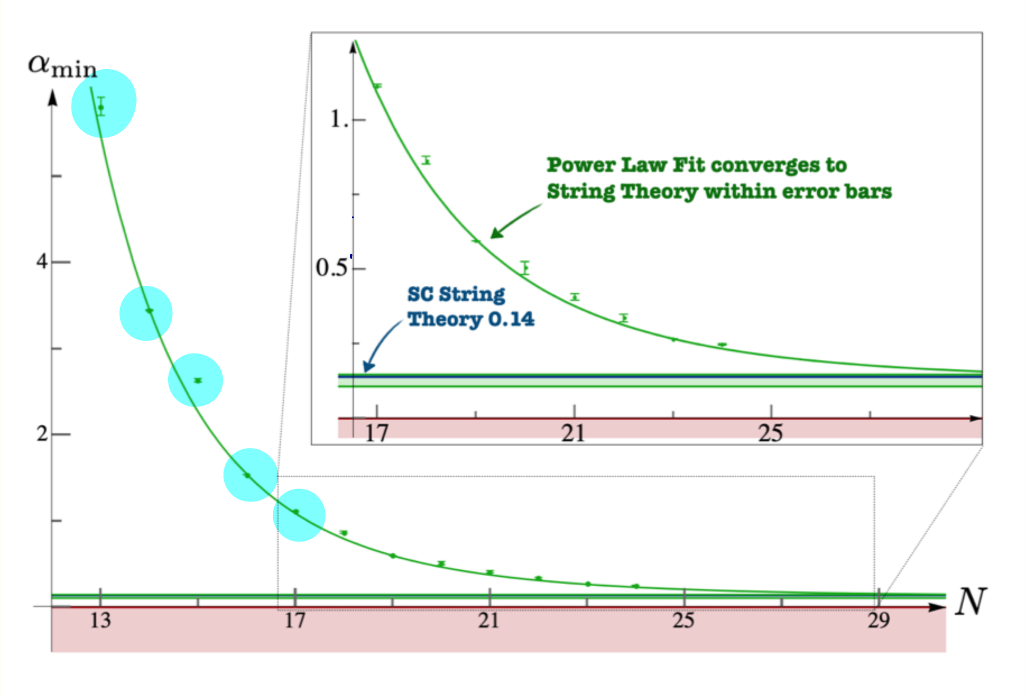
$\cos\theta$ ← scattering angle

3. Minimize α for each N and L.

$\rightarrow$ Semi-definite Programming SDPB ['15 Simmons-Duffin]

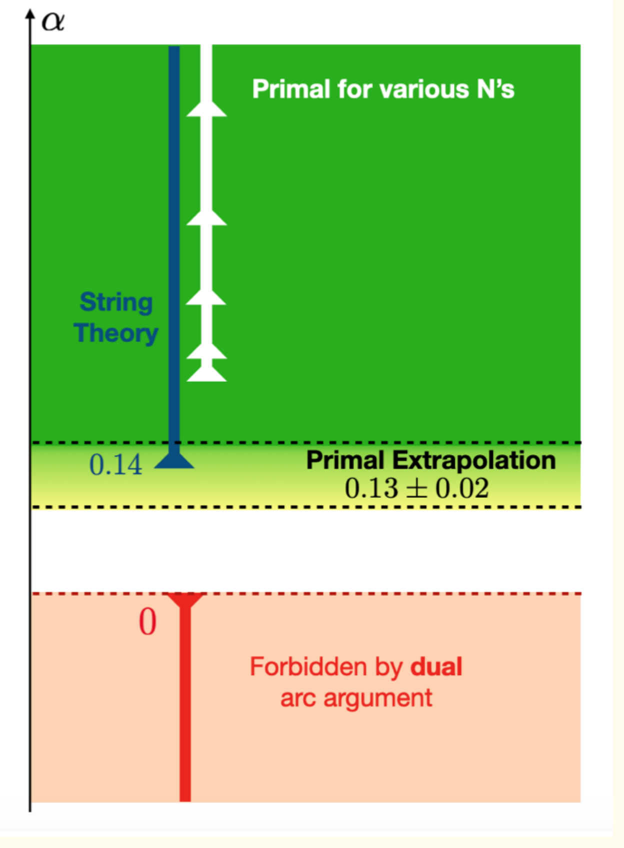


4. Extrapolate $L \to \infty$ and $N \to \infty$



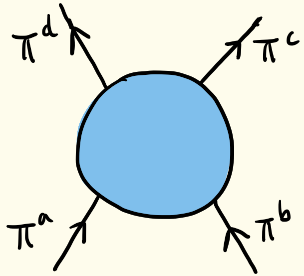
$$\alpha_{\rm min}^{\rm Boot} \equiv \lim_{\substack{N\to\infty \\ L\to\infty}} \alpha_{\rm min}(N, L) \approx 0.13 \pm 0.02$$

Final Result



Massless Pions

$\pi\pi$ scattering



$$= \quad \mathcal{T}_{ab}^{cd} = A(s|t,u)\delta_{ab}\delta^{cd} + A(t|s,u)\delta_a^c\delta_b^d + A(u|s,t)\delta_a^d\delta_b^c\,,$$

crossing : $A(s|t,u) = A(s|u,t)$

$$A(s|t,u) = \frac{s}{f_\pi^2} + \frac{1}{f_\pi^4}\left[\alpha s^2 + \beta(t^2+u^2) \underbrace{- \frac{s^2}{32\pi^2}\log\frac{-s}{f_\pi^2} - \frac{t-u}{96\pi^2}\left(t\log\frac{-t}{f_\pi^2} - u\log\frac{-u}{f_\pi^2}\right)}_{\text{Fixed by unitarity or 1-loop}}\right] + \ldots \tag{3}$$

Physical Wilson coefficients

From the SU(2) Chiral Lagrangian :

[Weinberg '79]
[Gasser, Leutwyler '84]

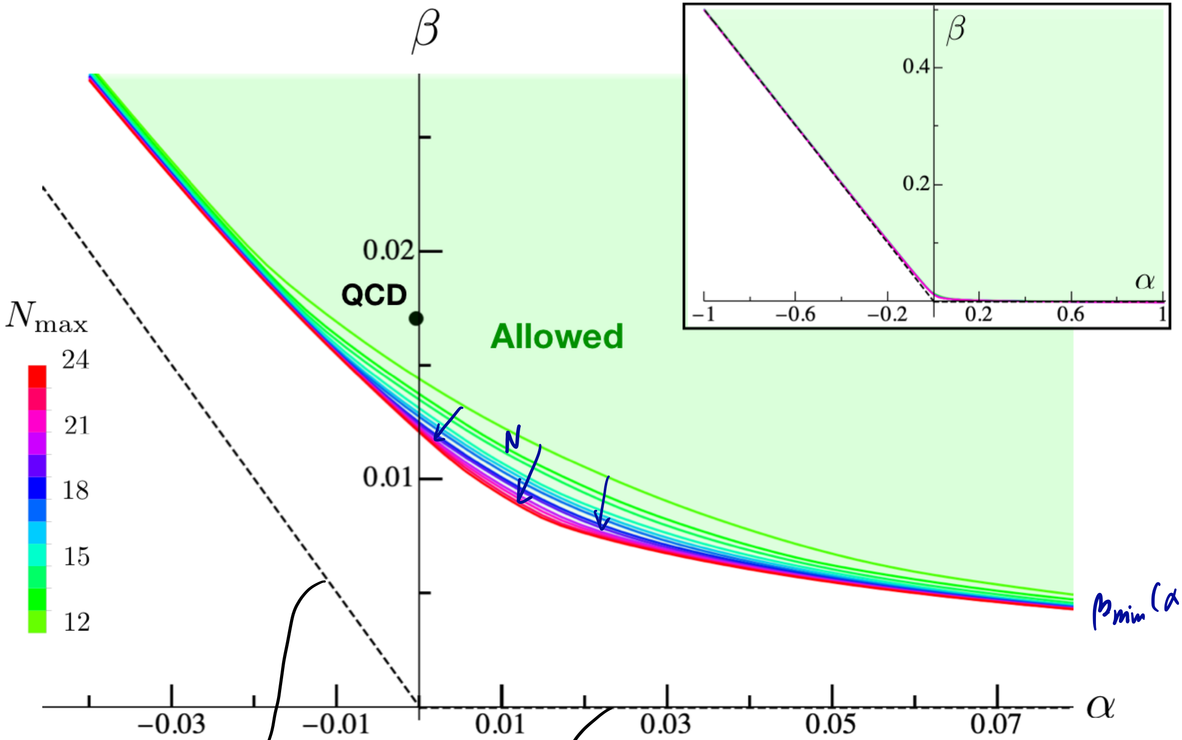
$$\mathcal{L} = \frac{1}{4}f_\pi^2\,\mathrm{tr}\left(\partial_\mu U^\dagger \partial^\mu U\right) + \ell_1\left[\mathrm{tr}\left(\partial_\mu U^\dagger \partial^\mu U\right)\right]^2 + \ell_2\,\mathrm{tr}\left(\partial_\mu U^\dagger \partial_\nu U\right)\mathrm{tr}\left(\partial^\mu U^\dagger \partial^\nu U\right) + \ldots$$

pion decay constant

low energy constants or Wilson coefficients

$U(x) = \exp\left(\frac{i}{f_\pi}\sum_{a=1}^{3}\sigma^a\pi^a(x)\right) \in SU(2)$

QCD from ['95 Bijnens, Colangelo, Ecker, Gasser, Sainio]



$\beta_{min}(\alpha)$

$\boxed{\alpha + 2\beta > 0}$

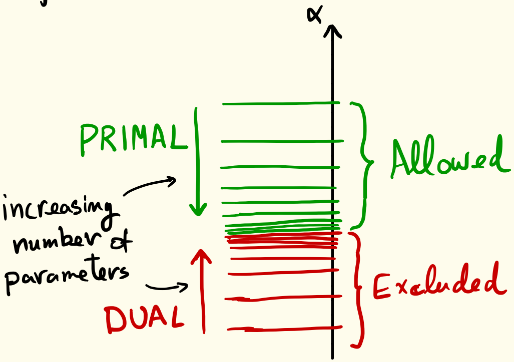
$\boxed{\beta > 0}$

[Pham, Truong '85]

[Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi '06]

Discussion

– Dual formulation would be very useful.



[Guerrieri, Homrich, Vieira '20]
[He, Kruczenski '21]

– Existing dual methods only use $\operatorname{Im} f_\ell(s) \geq 0$ instead of:

$$|S_\ell(s)| \leq 1 \iff 2 \operatorname{Im} f_\ell(s) \geq |f_\ell(s)|^2$$

$[S_\ell(s) = 1 + i f_\ell(s)]$

['20 Bellazzini, Elias-Miró, Rattazzi, Riembau, Riva]
['20 Tolley, Wang, Zhou] ← Next Talk
['20 Caron-Huot, Van Duong]
['20 Arkani-Hamed, Huang, Huang]
['21 Caron-Huot, Mazac, Rastelli, Simmons-Duffin]

Future work for gravitational scattering

- Other spacetime dimensions

$D = 11$ M-theory $\Rightarrow$ $\alpha = \frac{1}{2^7}$

$4 < D < 10$ String compactifications $\Rightarrow$ $\alpha_{min} = ?$

$D = 4$ IR divergences

- Inelasticity from Black Hole production

$$|S_\ell(s)|^2 \leq \begin{cases} 1 & \text{if} \quad b \equiv \frac{\ell}{\sqrt{s}} > R_{Hor}(E=\sqrt{s}) \\ e^{-S_{BH}(E=\sqrt{s},\,\ell)} & \text{otherwise} \end{cases}$$

['08 Giddings, Srednicki]

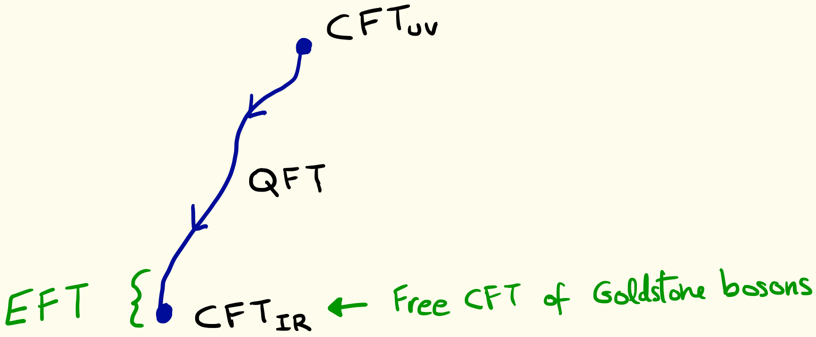
- Other Wilson coefficients

- No SUSY.

['20 Hebbar, Karateev, JP]

['21 Bern, Kosmopoulos, Zhiboedov]

Can we insert UV information into the S-matrix Bootstrap?



Yes, using form factors and spectral densities of T^μ_μ.

['19 Karateev, Kuhn, JP]
['20 Karateev]

Yes, using dilaton scattering amplitudes in 4D.

['11 Komargodski, Schwimmer]

[in progress with Karateev, Marucha, Sahoo]

Thank You!