Higgs and Effective Field Theory - HEFT 2021

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ArXiv: 21.... (to appear)

Higher order renormalisation in scalar effective field theory

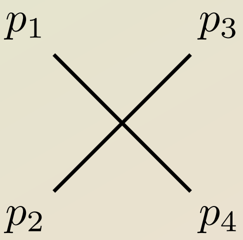
16 April 2021

Outline

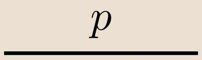
- Introduction:
  - Basis of conformal primaries
  - Anomalous dimensions
- R*-operation
- General form of results
- Example: mass dimension 8

Massless scalar effective field theory

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{g}{4!}\phi^4 + \sum_{d>4}\frac{\sum_i c_i^d \mathcal{O}_i^d}{\Lambda^{d-4}}$$



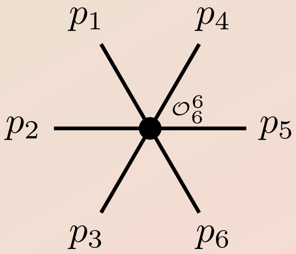
$$= -ig$$



$$= \frac{i}{p^2}$$

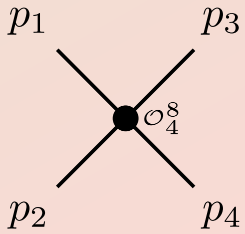
For example :

$$\frac{1}{\Lambda^2}\,\frac{c_6^6}{6!}\phi^6 \implies$$



$$= \frac{ic_6^6}{\Lambda^2}$$

$$\frac{c_4^8}{4\,\Lambda^4}\,\phi^2\,\partial_\mu\partial_\nu\phi\,\partial^\mu\partial^\nu\phi \implies$$



$$= \frac{ic_4^8}{\Lambda^4}\sum_{i<j}(p_i \cdot p_j)^2$$

Operator basis of conformal primaries

"Special" choice of operator basis given by the set of conformal primaries* :

$$K_\mu \, \mathcal{O}_{prim} = 0$$

$$K_\mu = \sum_i^{\mathrm{N}} \left(2 \frac{d}{dp_i^\mu} + 2 p_i^\nu \frac{d}{dp_i^\nu} \frac{d}{dp_i^\mu} - p_{i\mu} \frac{d}{dp_i^\nu} \frac{d}{dp_{i\nu}} \right)$$

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*: see Henning, Lu, Melia & Murayama [1706.08520]
And **Tom Melia**'s talk @HEFT2021 (Thursday)

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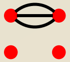
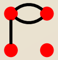
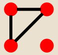
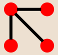
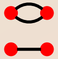
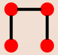
For example :

$$K_\mu \left(-\frac{2}{4} \phi^2 \partial_\mu \partial_\nu \partial_\rho \phi \partial^\mu \partial^\nu \partial^\rho \phi + 9 \phi \partial_\mu \phi \partial_\nu \partial_\rho \phi \partial^\mu \partial^\nu \partial^\rho \phi - \frac{72}{6} \phi \partial_\mu \partial_\nu \phi \partial_\rho \partial^\mu \phi \partial^\nu \partial^\rho \phi - \frac{72}{6} \partial_\mu \phi \partial_\nu \phi \partial_\rho \phi \partial^\mu \partial^\nu \partial^\rho \phi - \frac{126}{4} \partial_\mu \phi \partial^\mu \phi \partial_\nu \partial_\rho \phi \partial^\nu \partial^\rho \phi + \frac{108}{2} \partial_\mu \phi \partial_\nu \phi \partial_\rho \partial^\mu \phi \partial^\rho \partial^\nu \phi \right)$$

$= K_\mu \Big(-\frac{2}{4}$  $+ 9$  $- \frac{72}{6}$  $- \frac{72}{6}$  $- \frac{126}{4}$  $+ \frac{108}{2}$  $\Big) = 0$

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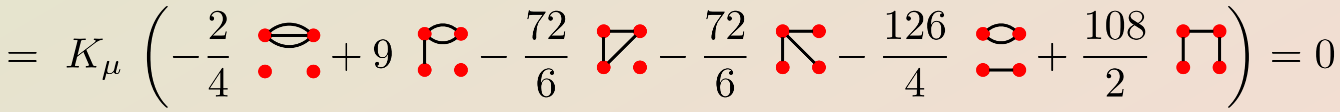
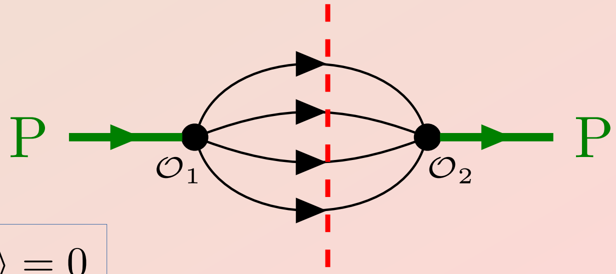


Consider the phase space integral :

$$\langle \mathcal{O}_1 ,\ \mathcal{O}_2 \rangle = \left[\prod_{i=1}^{\text{N}} \int d^4 p_i \ \delta^+(p_i^2) \right] \delta(P - \Sigma_i p_i) \ \mathcal{O}_1(p_i) \cdot \mathcal{O}_2(\text{-}\, p_i) \ \overset{N=4}{=\!=}$$



$$\Longrightarrow \quad \begin{aligned} \langle \partial^2 \phi \, \mathcal{O} ,\ \mathcal{O}_{prim} \rangle &= 0 \\ \langle P \, \mathcal{O} \, ,\ \mathcal{O}_{prim} \rangle &= 0 \end{aligned}$$

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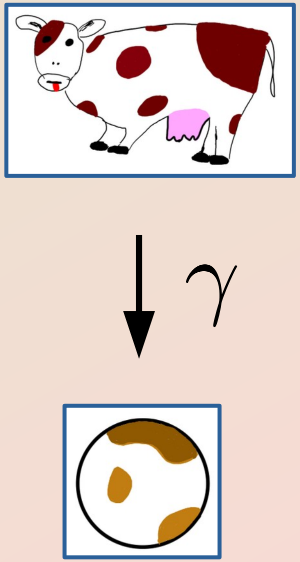
Anomalous dimensions



Definition :

$$\frac{dc_i^d(\mu)}{d\log(\mu)} \equiv \gamma_{ij}^d c_j^d + \sum_{\{d_a\}} \gamma_{ij_1\ldots j_r}^{(d_1\ldots d_r)} \left(c_{j_1}^{d_1} \cdot \ldots \cdot c_{j_r}^{d_r}\right)$$

For example :

$$\begin{aligned}\frac{dc_i^8(\mu)}{d\log(\mu)} = \gamma_{ij}^8 c_j^8 + \gamma_{ijk}^{(6,6)}\; c_j^6 c_k^6 \;&= -\beta\left(Z^8\right)_{iy}^{-1}\frac{dZ_{yj}^8}{dg}c_j^8 - \beta\left(Z^8\right)_{iy}^{-1}\frac{dZ_{yjk}^{(6,6)}}{dg}c_j^6 c_k^6 \\ &\quad - 2\left(Z^8\right)_{iy}^{-1} Z_{yjk}^{(6,6)}\;\gamma_{jj'}^6\; c_{j'}^6 c_k^6\end{aligned}$$

Using :

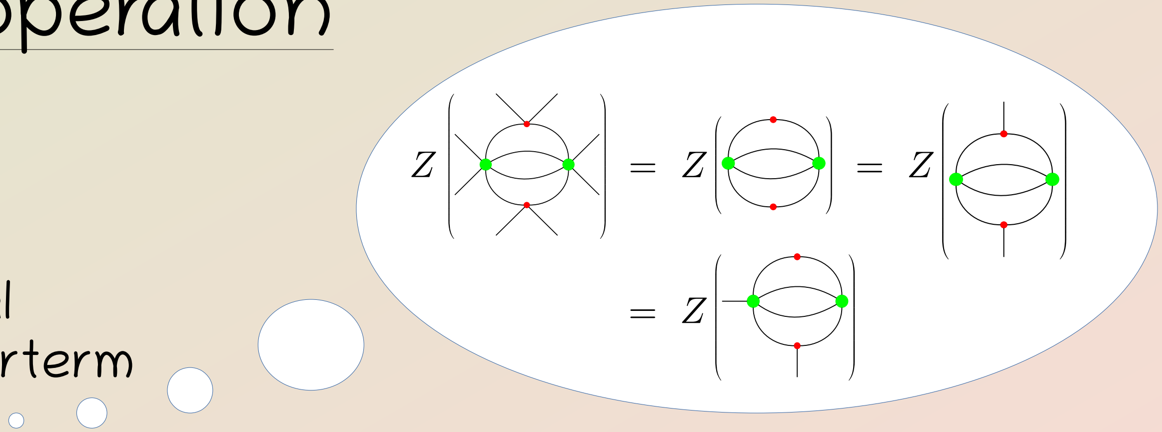
$$(c_b)_i^d = Z_{ij}^d c_j^d + \sum_{\{d_a\}} Z_{ij_1\ldots j_r}^{(d_1\ldots d_r)} \left(c_{j_1}^{d_1} \cdot \ldots \cdot c_{j_r}^{d_r}\right)$$

In MS-scheme of dimensional regularisation

Renormalisation: R*-operation[1]

- Recursively subtracts both UV and IR subdivergences

- Overall UV divergence is local
  → Taylor expand the counterterm
  → Infrared Rearrangement



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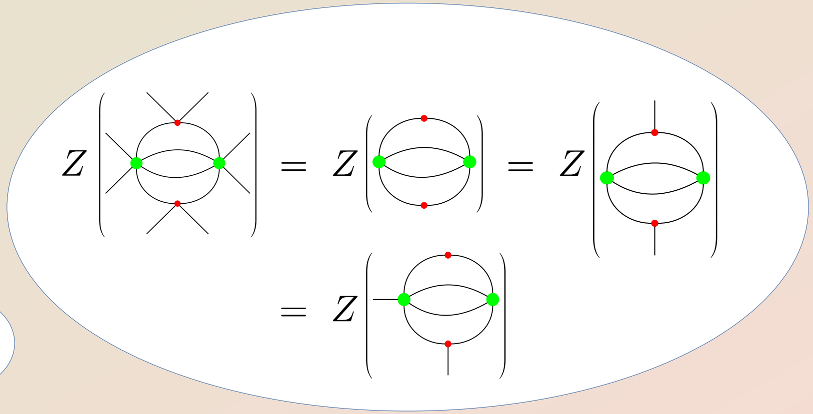
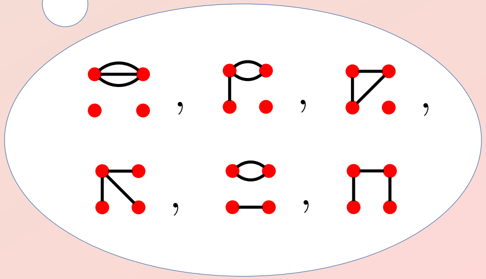
[1] -For original developments, see Chetyrkin & Tkachov (1982), Chetyrkin & Smirnov (1984) and Smirnov & Chetyrkin (1985).
-For recent developments, see Herzog & Ruijl [1703.03776], Beekveldt, Borinsky & Herzog [2003.04301]

Renormalisation: R*-operation[1]



- Recursively subtracts both UV and IR subdivergences
- Overall UV divergence is local
  → Taylor expand the counterterm
  → Infrared Rearrangement
- We renormalise off-shell Greens functions
  - → Counterterms outside the physical operator basis required
  - → Need the full set of operators that are independent under IBP
- No explicit counterterms for subdivergences necessary
  - → Calculations with only V_{PHYS}
- Implemented in `Maple`, `Form`[2], `Forcer`[3]



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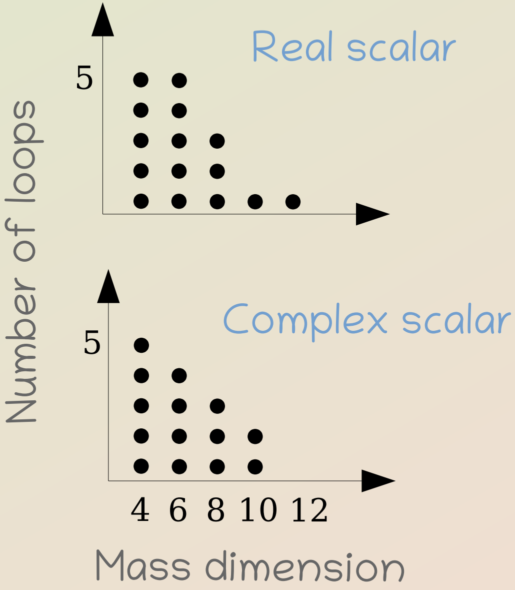
[2] Vermaseren [math-ph/0010025]

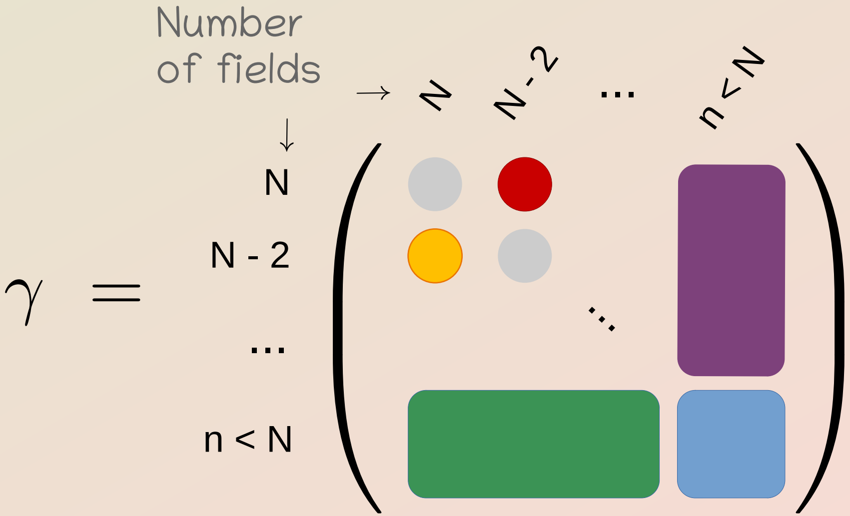
[3] Ueda, Ruijl and Vermaseren [1607.07318]

Results

Considered loop order and mass dimension :



General anomalous dimension matrix γ at mass dimension N



- 🔴 1-loop zero only in a basis of primaries
- 🟪 Generally non-zero in a basis of primaries
- 🟩 Expected zeros up to N - n loops *
- 🟡 Unexpected 3-loop zero in many bases
- 🟦 Symmetric at 1-loop for an orthonormal basis of primaries †

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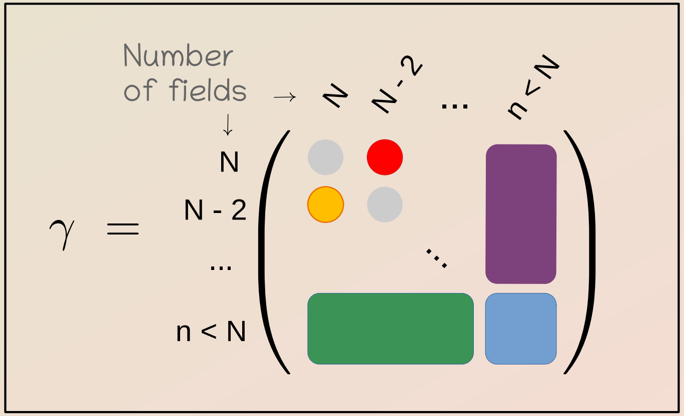
* : See Bern, Parra-Matrinez, Sawyer [1910.05831]

† : See Craigie, Dobrev, Todorov. Annals Phys. (1985)
And Hogervorst, Rychkov & Van Rees [1512.00013.pdf]

Example: Complex scalar
Mass dimension 8



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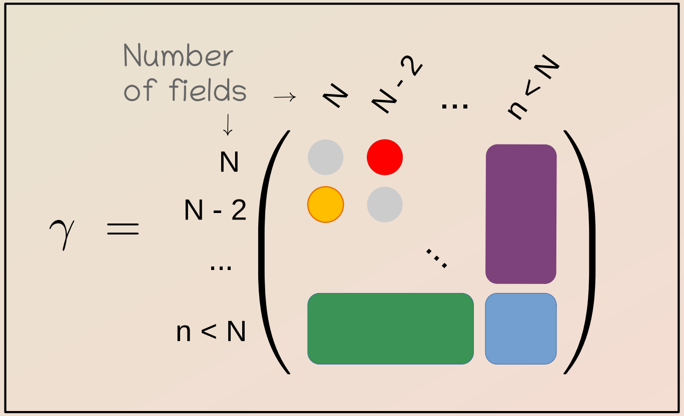
Example: Complex scalar
Mass dimension 8



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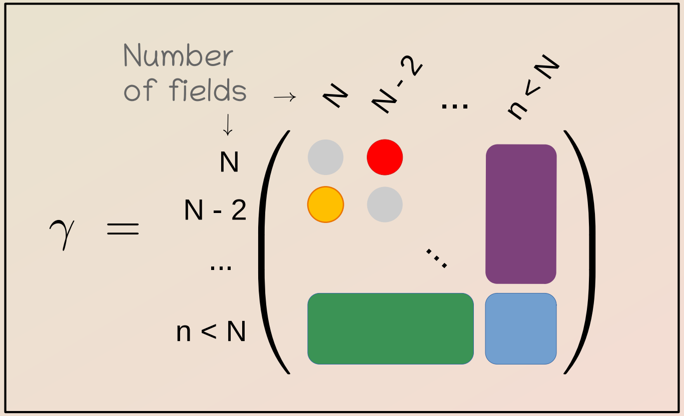

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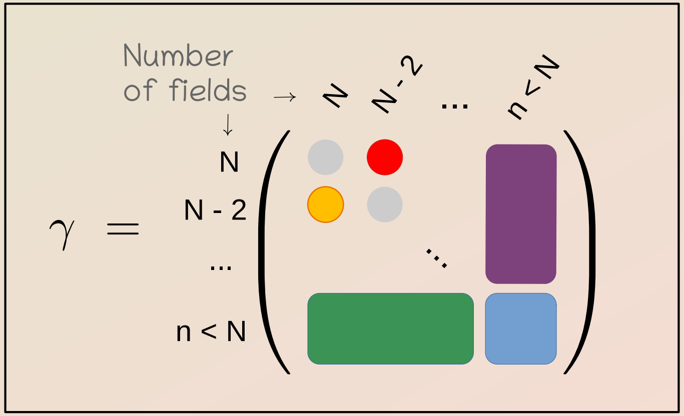

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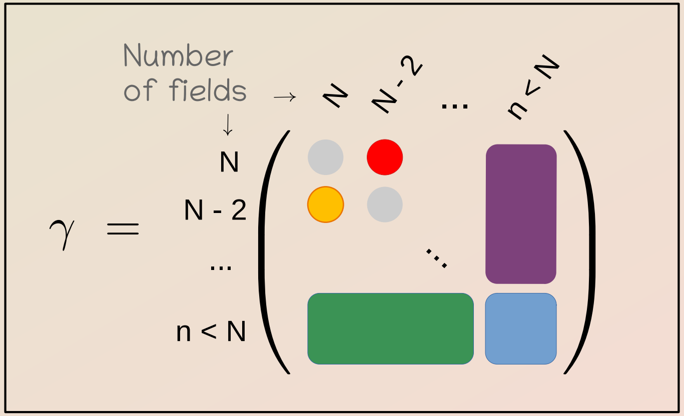
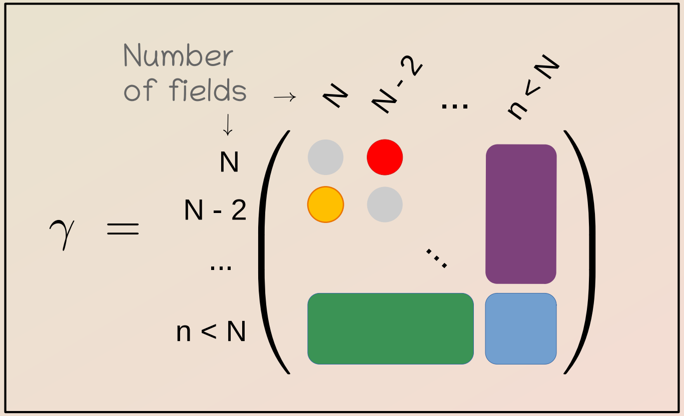
\boxed{0g+0g^2+0g^3} &
\boxed{0g+0g^2} + \frac{115g^3}{324} &
\begin{gathered} \frac{46g}{3} - \frac{14557g^2}{540} \\ +\left(96\zeta_3 + \frac{198001}{2430}\right)g^3 \end{gathered} &
\begin{gathered} -\frac{19g}{3} + \frac{2809g^2}{180} \\ -\left(\frac{176\zeta_3}{3} + \frac{718739}{9720}\right)g^3 \end{gathered}
\end{array}\right)
\end{array}$$

Example: Complex scalar
Mass dimension 8



$$\gamma^{8}_{prim} = \begin{array}{c} \begin{array}{cccc} \mathcal{O}^{(8)}_{8,prim} & \mathcal{O}^{(8)}_{6,prim} & \mathcal{O}^{(8)}_{4,prim\,(1,0)} & \mathcal{O}^{(8)}_{4,prim\,(0,1)} \end{array} \\ \begin{pmatrix} \begin{array}{c} 29g - 409g^2 \\ +\left(2352\zeta_3 + \frac{57765}{8}\right)g^3 \end{array} & \begin{array}{c} 0g - 240g^2 \\ +\left(2304\zeta_3 + 5882\right)g^3 \end{array} & \begin{array}{c} \frac{54216g}{5} - \frac{958314g^2}{5} \\ +\left(\frac{4083264\zeta_3}{5} + \frac{55731313}{15}\right)g^3 \end{array} & \begin{array}{c} -\frac{8856g}{5} + \frac{159894g^2}{5} \\ -\left(\frac{713664\zeta_3}{5} + \frac{9281093}{15}\right)g^3 \end{array} \\ \boxed{0g+0g^2}\boxed{+0g^3} & \begin{array}{c} 4g - \frac{122g^2}{3} \\ +\left(216\zeta_3 + \frac{4559}{12}\right)g^3 \end{array} & \begin{array}{c} -\frac{679g}{5} + \frac{14209g^2}{12} \\ -\left(\frac{29216\zeta_3}{5} + \frac{3248605}{324}\right)g^3 \end{array} & \begin{array}{c} \frac{164g}{5} - \frac{9233g^2}{36} \\ +\left(\frac{6056\zeta_3}{5} + \frac{2740291}{1296}\right)g^3 \end{array} \\ \boxed{0g + 0g^2 + 0g^3} & \boxed{0g+0g^2} + \frac{5g^3}{108} & \begin{array}{c} \frac{11g}{3} - \frac{29g^2}{180} \\ -\left(\frac{32\zeta_3}{3} + \frac{97091}{9720}\right)g^3 \end{array} & \begin{array}{c} -\frac{4g}{3} + \frac{1057g^2}{540} \\ -\left(\frac{16\zeta_3}{3} + \frac{10868}{1215}\right)g^3 \end{array} \\ \boxed{0g + 0g^2 + 0g^3} & \boxed{0g+0g^2} + \frac{115g^3}{324} & \begin{array}{c} \frac{46g}{3} - \frac{14557g^2}{540} \\ +\left(96\zeta_3 + \frac{198001}{2430}\right)g^3 \end{array} & \begin{array}{c} -\frac{19g}{3} + \frac{2809g^2}{180} \\ -\left(\frac{176\zeta_3}{3} + \frac{718739}{9720}\right)g^3 \end{array} \end{pmatrix} \end{array}$$

Example: Complex scalar
Mass dimension 8



$$\gamma^{8}_{prim} = \begin{array}{cccc} \mathcal{O}^{(8)}_{8,prim} & \mathcal{O}^{(8)}_{6,prim} & \mathcal{O}^{(8)}_{4,prim\,(1,0)} & \mathcal{O}^{(8)}_{4,prim\,(0,1)} \\ \left(\begin{array}{c} 29g - 409g^2 \\ +\left(2352\zeta_3 + \frac{57765}{8}\right)g^3 \end{array} \right. & \begin{array}{c} 0g - 240g^2 \\ +\left(2304\zeta_3 + 5882\right)g^3 \end{array} & \begin{array}{c} \frac{54216g}{5} - \frac{958314g^2}{5} \\ +\left(\frac{4083264\zeta_3}{5} + \frac{55731313}{15}\right)g^3 \end{array} & \left. \begin{array}{c} -\frac{8856g}{5} + \frac{159894g^2}{5} \\ -\left(\frac{713664\zeta_3}{5} + \frac{9281093}{15}\right)g^3 \end{array} \right) \\ 0g+0g^2+0g^3 & \begin{array}{c} 4g - \frac{122g^2}{3} \\ +\left(216\zeta_3 + \frac{4559}{12}\right)g^3 \end{array} & \begin{array}{c} -\frac{679g}{5} + \frac{14209g^2}{12} \\ -\left(\frac{29216\zeta_3}{5} + \frac{3248605}{324}\right)g^3 \end{array} & \begin{array}{c} \frac{164g}{5} - \frac{9233g^2}{36} \\ +\left(\frac{6056\zeta_3}{5} + \frac{2740291}{1296}\right)g^3 \end{array} \\ 0g + 0g^2 + 0g^3 & 0g+0g^2 + \frac{5g^3}{108} & \begin{array}{c} \frac{11g}{3} - \frac{29g^2}{180} \\ -\left(\frac{32\zeta_3}{3} + \frac{97091}{9720}\right)g^3 \end{array} & \begin{array}{c} -\frac{4g}{3} + \frac{1057g^2}{540} \\ -\left(\frac{16\zeta_3}{3} + \frac{10868}{1215}\right)g^3 \end{array} \\ 0g + 0g^2 + 0g^3 & 0g+0g^2 + \frac{115g^3}{324} & \begin{array}{c} \frac{46g}{3} - \frac{14557g^2}{540} \\ +\left(96\zeta_3 + \frac{198001}{2430}\right)g^3 \end{array} & \begin{array}{c} -\frac{19g}{3} + \frac{2809g^2}{180} \\ -\left(\frac{176\zeta_3}{3} + \frac{718739}{9720}\right)g^3 \end{array} \end{array}$$

Example: 1-loop submatrix of length-4 operators

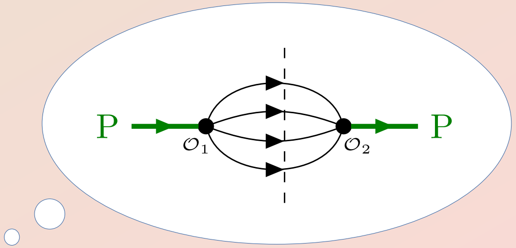
$$\gamma_{\text{sub}} = \boxed{\begin{pmatrix} \frac{11}{3} & -\frac{4}{3} \\ \frac{46}{3} & -\frac{19}{3} \end{pmatrix}}$$

for

$$\mathcal{O}^{(8)}_{4,prim}(1,0)$$
$$\mathcal{O}^{8c}_{4,prim}(0,1)$$

Instead choose

$$\mathcal{O}^{8}_{4,prim}\,(a,\,b) = \mathcal{O}^{8}_{4,prim}\left(3\sin\theta,\;13\sin\theta + 2\sqrt{5}\cos\theta\right)$$
$$\mathcal{O}^{8c}_{4,prim}\,(a',\,b') = \mathcal{O}^{8c}_{4,prim}\left(3\cos\theta,\;13\cos\theta - 2\sqrt{5}\sin\theta\right)$$



with

$$\langle \mathcal{O}^8_4(a,b),\; \mathcal{O}^8_4(a',b')\rangle = \left[\prod_{i=1}^{\text{N}} \int d^4p_i\;\delta^+(p_i^2)\right]\delta(P-\Sigma_i p_i)\;\;\mathcal{O}(p_i)\cdot\mathcal{O}'(\text{-}\,p_i) = 0$$

$$\langle \mathcal{O}^8_4(a,b),\; \mathcal{O}^8_4(a,b)\rangle = \langle \mathcal{O}^8_4(a',b'),\; \mathcal{O}^8_4(a',b')\rangle$$

for any angle θ

Note the effect of a change of basis on the AD γ_{sub} :

$$\vec{c} = R\,\vec{c}\,' \implies \gamma' = R^{\text{-}1}\gamma R$$

Example: 1-loop submatrix of length-4 operators

Orthonormal basis:

$$\mathcal{O}^{8}_{4,prim}\,(a,\,b) = \mathcal{O}^{8}_{4,prim}\left(3\sin\theta,\;\;13\sin\theta + 2\sqrt{5}\cos\theta\right)$$

$$\mathcal{O}^{8c}_{4,prim}\,(a',\,b') = \mathcal{O}^{8c}_{4,prim}\left(3\cos\theta,\;\;13\cos\theta - 2\sqrt{5}\sin\theta\right)$$

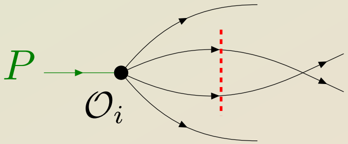
$$\Longrightarrow \quad \gamma'_{\text{sub}} = \begin{pmatrix} a & a' \\ b & b' \end{pmatrix}^{\text{-1}} \begin{pmatrix} \frac{11}{3} & -\frac{4}{3} \\ \frac{46}{3} & -\frac{19}{3} \end{pmatrix} \begin{pmatrix} a & a' \\ b & b' \end{pmatrix}$$

$$= \begin{pmatrix} -\frac{1}{9}(12 - 7\cos 2\theta + 8\sqrt{5}\sin 2\theta) & -\frac{1}{9}(8\sqrt{5}\cos 2\theta + 7\sin 2\theta) \\ -\frac{1}{9}(8\sqrt{5}\cos 2\theta + 7\sin 2\theta) & -\frac{1}{9}(12 + 7\cos 2\theta - 8\sqrt{5}\sin 2\theta) \end{pmatrix}$$

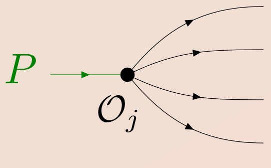
→ Symmetric submatrix !

Submatrix of same length operator mixing

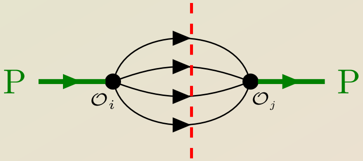
Form Factor method for anomalous dimensions*:



$$+perms = -\pi \sum_j \mathcal{O}_j \, \gamma_{ji} = -\pi \sum_j \gamma_{ji} \Bigg[\quad \Bigg]$$

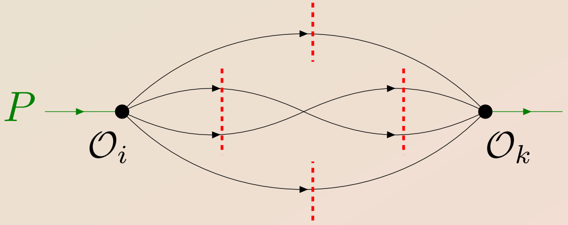


Assume orthonormality w.r.t. the phase space integral :



$$= (P^2)^x \, \delta_{ij}$$

$$\Longrightarrow$$



$$= -\pi \, (P^2)^x \, \gamma_{ki}$$

$$\Longrightarrow \boxed{(\gamma_{\text{sub}})_{ki} = (\gamma_{\text{sub}})_{ik}}^{\,\dagger}$$

Note : $\langle P \, \mathcal{O} \, , \, \mathcal{O}_{prim} \rangle = 0 \quad \Longrightarrow$

Submatrix of physical AD, γ_{sub} , is symmetric if the conformal primary part is orthonormal

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$*$: See Caron-Huot & Wilhelm [1607.06448]

† : See Craigie, Dobrev, Todorov. Annals Phys. (1985)
And Hogervorst, Rychkov & Van Rees [1512.00013.pdf]

Conclusion

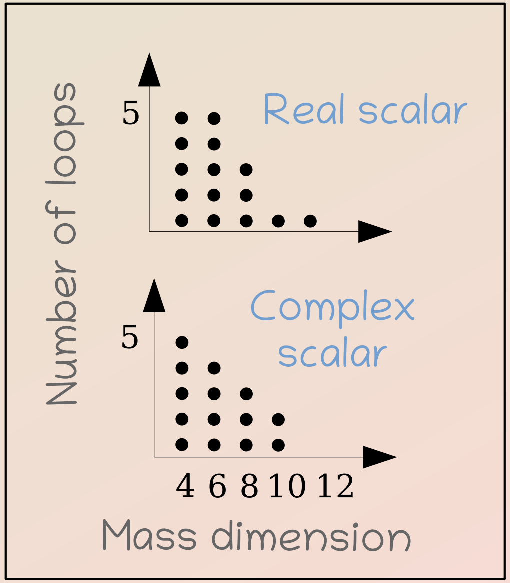
First application of R*-operation in EFT
→ useful for renormalisation at high loop order and high mass dimension

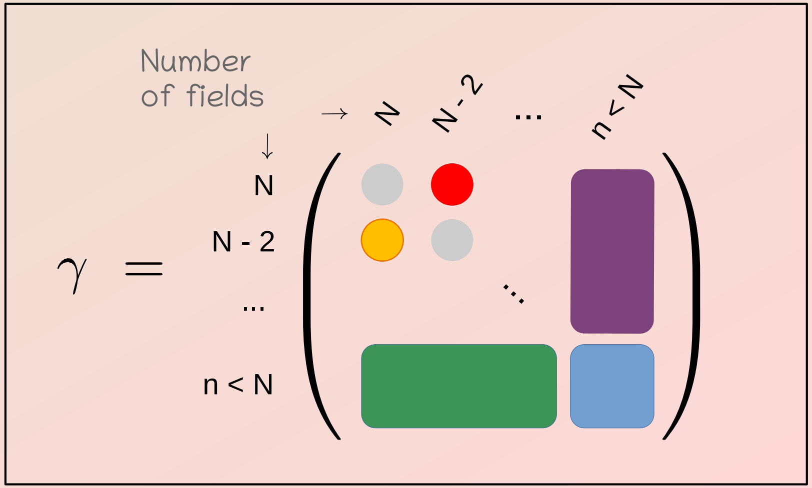
Analysed the general structure of the anomalous dimensions and its basis-dependence

Confirmed existing predictions

Open questions:

- Red and orange zeros





Thank you !