# Constructing MASSIVE on-shell operator basis

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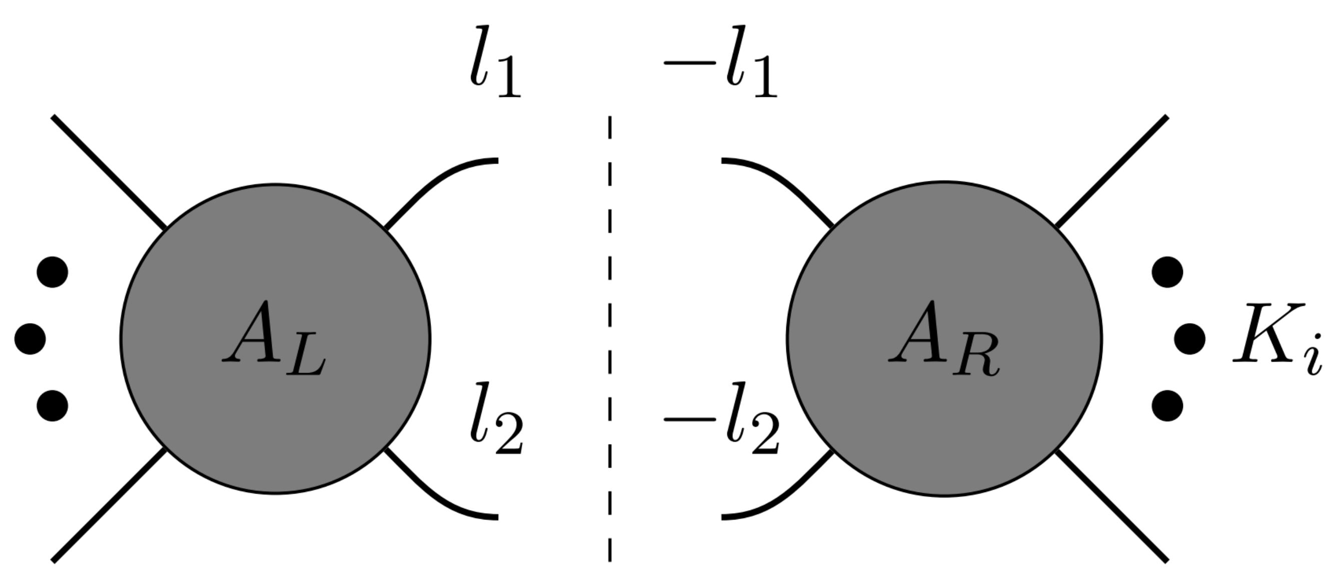
ITP-CAS

Z-Y Dong, T. Ma, J. Shu., arxiv:2103.15837

# On-shell scattering amplitude

Z. Bern, J. Parra-Martinez and E. Sawyer, JHEP **10**, 211 (2020) doi:10.1007/JHEP10(2020)211 [arXiv:2005.12917 [hep-ph]].
M. Jiang, T. Ma and J. Shu, [arXiv:2005.10261 [hep-ph]].

- Efficient in massless EFT calculations




RG-running

$$\frac{\mathrm{d}c_i(\mu)}{\mathrm{d}\log\mu} = \sum_j \frac{1}{16\pi^2}\gamma_{ij}c_j$$

Selection rules

$$\gamma_{ij} = 0$$

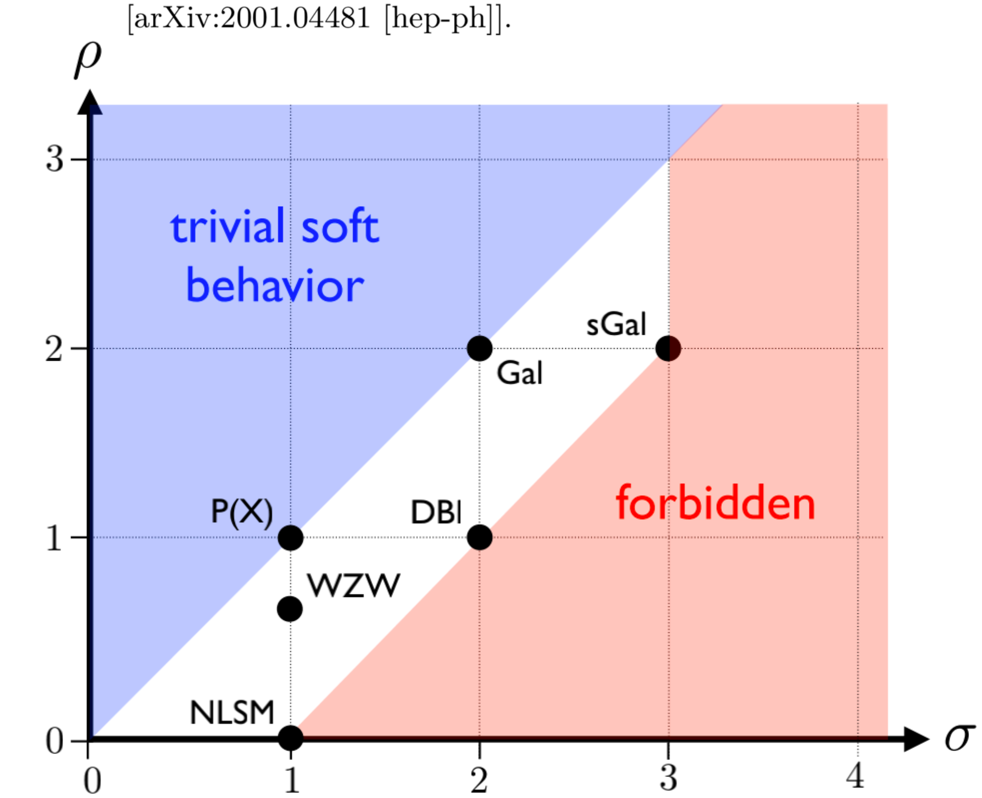
C. Cheung and C. H. Shen, Phys. Rev. Lett. **115**, no. 7, 071601 (2015) doi:10.1103/PhysRevLett.115.071601 [arXiv:1505.01844 [hep-ph]].
M. Jiang, J. Shu, M. L. Xiao and Y. H. Zheng, [arXiv:2001.04481 [hep-ph]].

- Construct scalar EFT through soft limit

$$A_n \sim p^\sigma \quad \text{for} \quad p \to 0$$

C. Cheung, K. Kampf, J. Novotny and J. Trnka, Phys. Rev. Lett. **114**, no.22, 221602 (2015) doi:10.1103/PhysRevLett.114.221602 [arXiv:1412.4095 [hep-th]].
C. Cheung, K. Kampf, J. Novotny, C. H. Shen and J. Trnka, JHEP **02**, 020 (2017) doi:10.1007/JHEP02(2017)020 [arXiv:1611.03137 [hep-th]].

I. Low, Phys. Rev. D **91**, no.10, 105017 (2015) doi:10.1103/PhysRevD.91.105017 [arXiv:1412.2145 [hep-th]].
I. Low, Phys. Rev. D **91**, no.11, 116005 (2015) doi:10.1103/PhysRevD.91.116005 [arXiv:1412.2146 [hep-ph]].

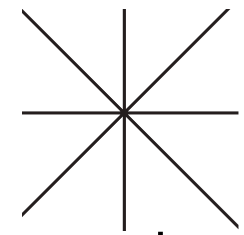


# On-shell amplitude basis

- Efficient in constructing operator basis of massless EFT

H. Elvang, D. Z. Freedman and M. Kiermaier, JHEP **1011**, 016 (2010) doi:10.1007/JHEP11(2010)016 [arXiv:1003.5018 [hep-th]].
Y. Shadmi and Y. Weiss, arXiv:1809.09644 [hep-ph].
T. Ma, J. Shu and M. L. Xiao, [arXiv:1902.06752 [hep-

$\mathcal{O}_{d\geq 4}$



Operator basis

Unfactorizable amplitude basis

- Massless amplitude basis is free of EOMs automatically

Null EOM $\quad p\,|p] = 0, \quad p\,|p\rangle = 0$

- IBP redundancy can be systematically removed by $U(N) \supset \otimes_{i=1}^{N} U(1)_i$

Momentum conservation

B. Henning and T. Melia, Phys. Rev. D **100**, no.1, 016015 (2019) doi:10.1103/PhysRevD.100.016015 [arXiv:1902.06754 [hep-ph]].
B. Henning and T. Melia, [arXiv:1902.06747 [hep-th]].

- The amplitude basis is the basis of some special $U(N)$ representations

H. L. Li, Z. Ren, J. Shu, M. L. Xiao, J. H. Yu and Y. H. Zheng, [arXiv:2005.00008 [hep-ph]].

- It be constructed by the computer programs

(**Field theory can not do it!!!**)

# Introduction of on-shell amplitude

- Massive spinor and its LG $SU(2)_i$

Quantum number $SU(2)_l \otimes SU(2)_r \otimes SU(2)_i$

$$(p_i)_{\dot{\alpha}\alpha} \equiv (p_i)_\mu (\sigma^\mu)_{\dot{\alpha}\alpha} = |i^I]_{\dot{\alpha}} \langle i_I|_\alpha$$

$$|i^I]_{\dot{\alpha}} = (1,2,2) \qquad |i^I\rangle_\alpha = (2,1,2)$$

$SO(3,1)$ Lorentz

- For massless spinor, its little group is $U(1)_j$

$$|j] \to e^{-i\theta_j}|j] \qquad |j\rangle \to e^{i\theta_j}|j\rangle$$

- Spinor product $\quad [ij]^{IJ} \equiv \epsilon^{\dot{\alpha}\dot{\beta}} |i^I]_{\dot{\beta}} |j^J]_{\dot{\alpha}}\,, \quad \langle ij\rangle^{IJ} \equiv \epsilon^{\alpha\beta} |i^I\rangle_\beta |j^J]_\alpha$

- External massless particle-j with helicity $h_j$

E. Witten, Commun. Math. Phys. **252**, 189 (2004) doi:10.1007/s00220-004-1187-3 [hep-th/0312171].

$$\mathcal{M}(e^{-\theta_j}|j], e^{\theta_j}|j\rangle) = e^{-2h_j\theta_j}\mathcal{M}(|j], |j\rangle)$$

- Massive particle-i with spin $s_i$ , the amplitude should be in $2s_i$ indices symmetric representation of $SU(2)_i$

N. Arkani-Hamed, T. C. Huang and Y. t. Huang, arXiv:1709.04891 [hep-th].

Spin 1: $\mathcal{M}^{\{I_1,I_2\}}\left(w^i_{II'}|i^{I'}], w^i_{II'}|i^{I'}\rangle, \ldots\right) = w^i_{I_1I_1'} w^i_{I_2I_2'} \mathcal{M}^{\{I_1',I_2'\}}\left(|i^{I'}], |i^{I'}\rangle, \ldots\right)$

# Massless amplitude basis

T. Ma, J. Shu and M. L. Xiao, [arXiv:1902.06752 [hep-

- Basic structure of massless amplitudes

$$\mathcal{M}(\{p_i, h_i\}) = f(|i], |i\rangle) g(s_{ij}) T^{\{\alpha\}}$$

Gauge structure

Massless LG charged

Massless LG neutral

- An amplitude basis just corresponds to the leading interaction of an operator

$$F_{\mu\nu} \to \partial_\mu A_\nu - \partial_\nu A_\mu \qquad D_\mu \to \partial_\mu$$

- The complete amplitude bases of a scattering process can be obtained by finding all its independent unfactorizable amplitudes allowed by LG, gauge symmetry and spin statistic

Three gluons

$$\mathcal{M}(G^{A+}G^{B+}G^{C+}) = [12][23][31] f^{ABC} \quad \longleftrightarrow \quad f^{ABC} G^A_{\mu\nu} G^B_{\nu\rho} G^C_{\rho\mu}$$

$$\mathcal{M}(G^{A-}G^{B-}G^{C-}) = \langle 12\rangle\langle 23\rangle\langle 31\rangle f^{ABC} \qquad f^{ABC} \tilde{G}^A_{\mu\nu} G^B_{\nu\rho} G^C_{\rho\mu}$$

# Massless amplitude basis

- Systematically construct the complete massless amplitude bases of N external particles without IBP via $U(\bar{N}) \supset \otimes_{i=1}^{N} U(1)_i$

Quantum number under

$SU(2)_l \otimes SU(2)_r \otimes U(N)$

$$\tilde{\lambda}^{k}_{\dot{\alpha}} \equiv |k] = (1, 2, N)$$

$$\lambda_{k\alpha} \equiv |k\rangle = (2, 1, \bar{N})$$

- A polynomial of spinors is a basis of a U(N) representation

$$\begin{array}{|c|}\hline \phantom{i} \\ \hline \phantom{j} \\ \hline \end{array}_{\,r} \otimes \begin{array}{|c|}\hline i \\ \hline j \\ \hline \end{array} = \frac{1}{2!}\epsilon^{\beta\alpha}\left(\tilde{\lambda}^{i}_{\dot{\alpha}}\tilde{\lambda}^{j}_{\dot{\beta}} - \tilde{\lambda}^{j}_{\dot{\alpha}}\tilde{\lambda}^{i}_{\dot{\beta}}\right) = [ij]$$

B. Henning and T. Melia, Phys. Rev. D **100**, no.1, 016015 (2019) doi:10.1103/PhysRevD.100.016015 [arXiv:1902.06754 [hep-ph]].
B. Henning and T. Melia, [arXiv:1902.06747 [hep-th]].

$$\begin{array}{|c|}\hline \phantom{k} \\ \hline \phantom{k} \\ \hline \end{array}_{\,l} \otimes \left.\begin{array}{|c|}\hline k_1 \\ \hline \cdot \\ \hline \cdot \\ \hline k_{N-2} \\ \hline \end{array}\right|_{N-2} = \frac{\left(\epsilon^{ijk_1 \ldots k_{N-2}} + \text{anti-sym in } k_1..k_{N-2}\right)}{(N-2)!}\epsilon^{\beta\alpha}\lambda_{i\alpha}\lambda_{j\beta}$$

$$= \langle ij\rangle \epsilon^{ijk_1 \ldots k_{N-2}}$$

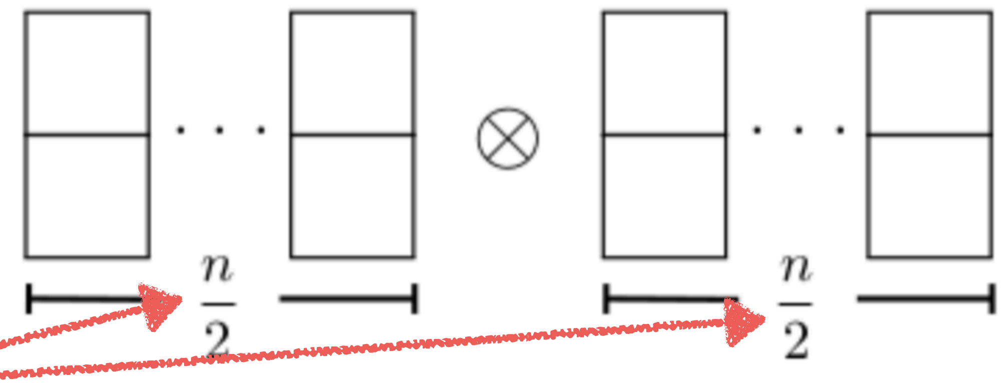
Blue for left-handed spinors

# Massless amplitude basis

- IBP redundancy can be removed by $U(N) \supset \otimes_{i=1}^{N} U(1)_i$ symmetry

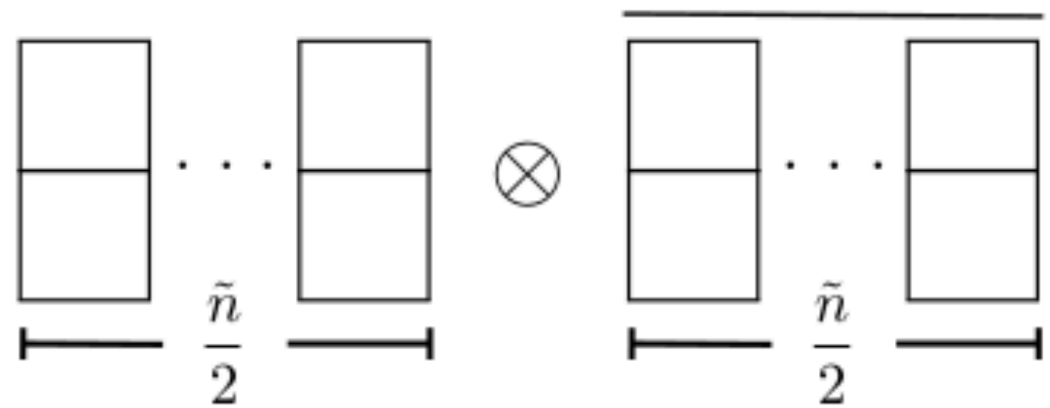
Holomorphic case

$$f_{\text{hol}}^{(n)}(\{\tilde{\lambda}\}) = g_{SU(2)_r} \otimes g_{U(N)} =$$




Same shape

$$f_{\text{anti-hol}}^{(\tilde{n})}(\{\lambda\}) = g_{SU(2)_l} \otimes \bar{g}_{U(N)} =$$

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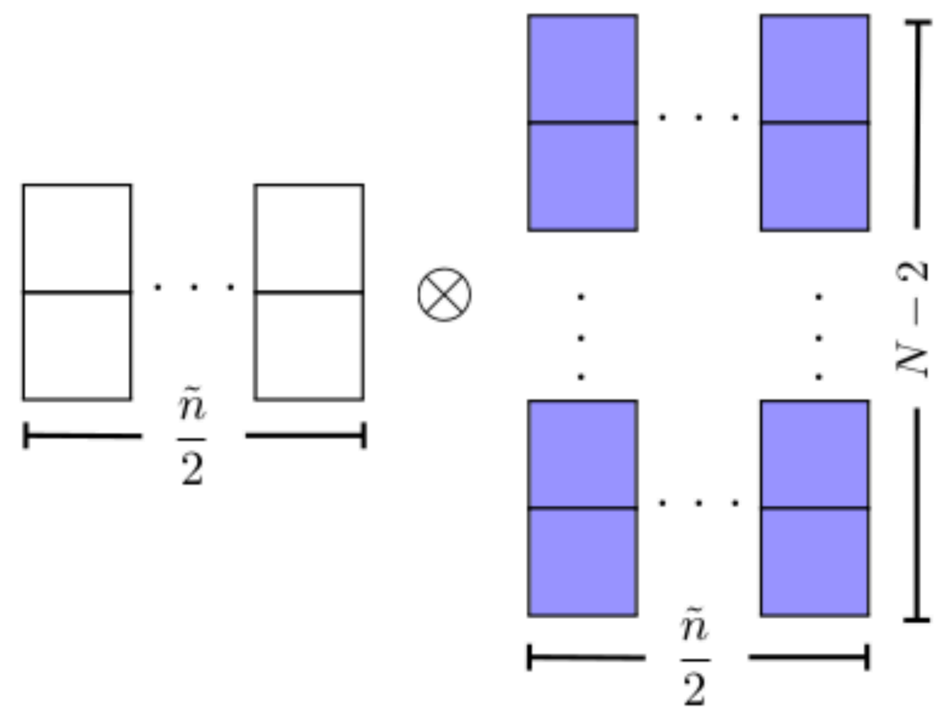


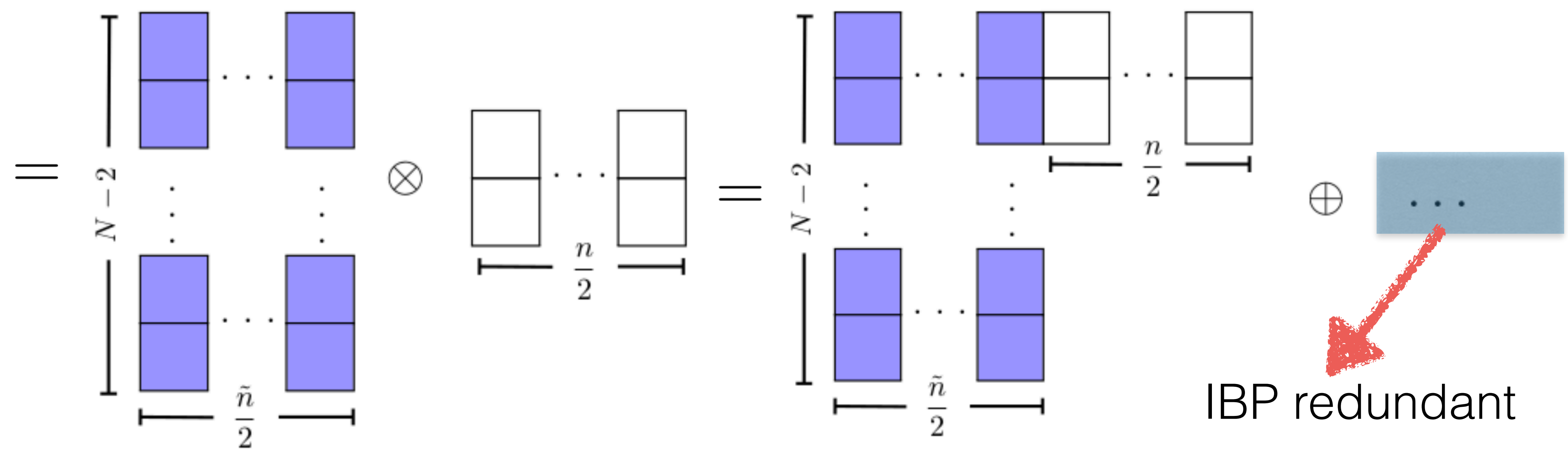

- Holomorphic polynomials are not IBP redundant

# Massless amplitude basis

- IBP redundancy can be removed by $U(N) \supset \otimes_{i=1}^{N} U(1)_i$ symmetry

Non-holomorphic case

$$f^{(n,\tilde{n})}(\{\lambda,\widetilde{\lambda}\}) = (\overline{g}_{U(N)} \otimes g_{U(N)})$$




- The amplitude bases without IBP are the bases of the first U(N) representation

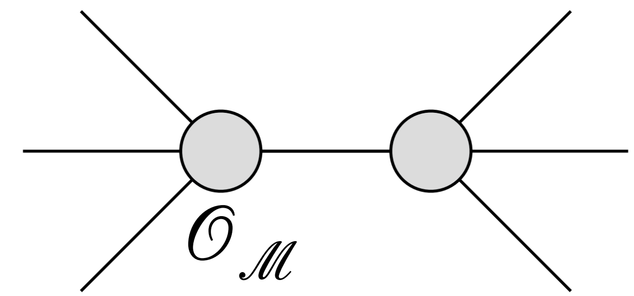
# Problems in massless amplitude basis

- Massless amplitude basis is fail at EWSB phase

G. Durieux, T. Kitahara, Y. Shadmi and Y. Weiss, JHEP **01**, 119 (2020) doi:10.1007/JHEP01(2020)119 [arXiv:1909.10551 [hep-ph]].
G. Durieux, T. Kitahara, C. S. Machado, Y. Shadmi and Y. Weiss, JHEP **12**, 175 (2020) doi:10.1007/JHEP12(2020)175 [arXiv:2008.09652 [hep-ph]].
A. Falkowski, G. Isabella and C. S. Machado,

- Troublesome in calculation at EWSB phase

$$\mathcal{M}_{\mathcal{O}} \to \mathcal{O}_{\mathcal{M}} \xrightarrow{\langle H \rangle \neq 0}$$




- Massless EFT is not concise in describing physics at EWSB

$$W^+ - W^- - Z$$

Three massive gauge bosons

Seven massive amplitudes

$$\left(W^a_{\mu\nu}\right)^3 |H|^{2n}$$

Infinite massless operator bases

- Massive EFT is more useful and convenient at EWSB phase, studying higher spin parties and DM

# Massive amplitude basis

- The scattering amplitude can be factorized in two parts:

$m$ massive
$n$ massless

$$\mathcal{M}^I_{m,n} = \sum_{\{\dot\alpha\}} \mathcal{A}^I_{\{\dot\alpha\}}\left(\{\epsilon_{s_i}\}\right) G^{\{\dot\alpha\}}\left(|j], |j\rangle, p_i\right)$$

Massive LG charged

Massless LG charged

- MLGTS $\mathcal{A}^I$ is required to be the holomorphic function of $|i^I]$ (EOM $|i^I\rangle = p_i|i^I]/m_i$)

Linear in massive polarisation tensor

$$\epsilon_{s_i} \equiv |i]^{\{I_1}_{\dot\alpha_1}, \ldots, |i]^{I_{2s_i}\}}_{\dot\alpha_{2s_i}}$$

$$\in (2s_i + 1, 2s_i + 1) = SU(2)_l \otimes SU(2)_r$$

- MLGNS $G(|j], |j\rangle, p_i)$ is the function of massless spinors $|j]$ or $|j\rangle$ and massive momentum $p_i$

# Massive amplitude basis

- The MLGTS $\mathcal{A}^{\{I\}}_{\{\dot\alpha\}}(\epsilon_i) \sim \otimes_{i=1}^{m}\epsilon_{s_i}$ can be classified by $SU(2)_r$ representation

$$\mathcal{A}^{I}_{\{\dot\alpha\}}(\{\epsilon_{s_i}\}) \subset \bigotimes_{i=1}^{m} \underbrace{\boxed{i}\cdots\boxed{i}}_{(2s_i)}{}_r = \begin{array}{|c|c|c|c|c|}\hline \; & \cdots & \; & \cdots & \; \\ \hline \; & \cdots & \; \\ \cline{1-3}\end{array} \oplus \cdots$$

- The $\mathcal{A}^{I}_{\{\dot\alpha\}}$ can be completely constructed by finding all the $SU(2)_r$ irreducible representations from the outer product of all $\epsilon_{s_i}$

Take $\psi\psi' Z h$ as an example $\psi \sim \boxed{1}$ $\psi' \sim \boxed{2}$ $Z \sim \begin{array}{|c|c|}\hline 3 & 3 \\ \hline\end{array}$ $h \sim \bullet$

$$\mathcal{A}^{I}_{\{\dot\alpha\}}(\{\epsilon_{s_i}\}) \subset \boxed{1} \times \boxed{2} \times \begin{array}{|c|c|}\hline 3 & 3 \\ \hline\end{array} \times \bullet$$

$$= \begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 & 3 \\ \hline\end{array} \oplus \begin{array}{|c|c|c|}\hline 1 & 2 & 3 \\ \hline 3 \\ \cline{1-1}\end{array} \oplus \begin{array}{|c|c|c|}\hline 1 & 3 & 3 \\ \hline 2 \\ \cline{1-1}\end{array} \oplus \begin{array}{|c|c|c|c|}\hline 1 & 2 & 3 & 3 \\ \hline\end{array}$$

# Massive amplitude basis

$$\mathcal{A}^I_{\{\dot\alpha\}}(\{\epsilon_{s_i}\}) \subset \boxed{1} \times \boxed{2} \times \boxed{3\,|\,3} \times \bullet$$

$$= \begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 & 3 \\ \hline \end{array} \oplus \begin{array}{|c|c|c|}\hline 1 & 2 & 3 \\ \hline 3 \\ \cline{1-1} \end{array} \oplus \begin{array}{|c|c|c|}\hline 1 & 3 & 3 \\ \hline 2 \\ \cline{1-1} \end{array} \oplus \begin{array}{|c|c|c|c|}\hline 1 & 2 & 3 & 3 \\ \hline \end{array}$$

- The MLGTS can be read from above YDs. For the first one YD

$$\mathcal{A}^I_{[2,2]} \equiv \begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 & 3 \\ \hline \end{array} = (|1^I]_{\dot\alpha}|2^J]_{\dot\beta}|3^{K_1}]_{\dot\gamma_1}|3^{K_2}]_{\dot\gamma_2} + \text{perms in } SU(2)_r \text{ indices})$$

$$= [1^I 3^{\{K_1}][2^J 3^{K_2\}}].$$

# Massive amplitude basis

- MLGNS $G(|j],|j\rangle,p_i)$ is the function of massless spinors $|j]$ or $|j\rangle$ and massive momentum $p_i$

Number of $|k^K\rangle$ or $|k\rangle$

Number of $|i^I]$

$SU(2)_l$ singlet $\longrightarrow$ $\sum_{k=1}^{N} n_k = \text{even}$

Massive LG neutral $\longrightarrow$ $\tilde{n}_i - n_i = 0 \quad , \text{ with } i = 1,\ldots,m$

Massless LG $\longrightarrow$ $\tilde{n}_j - n_j = 2h_j \; , \text{ with } j = m+1,\ldots,N$

- MLGNS $G(|j],|j\rangle,p_i)$ is bothered by EOM and IBP redundancy

Construct massless limits $\quad p_{i,\dot{\alpha}\alpha} \to |i]_{\dot{\alpha}}\langle i|_{\alpha} : G(|j],|j\rangle,p_i) \to g \;\equiv\; G(|j],|j\rangle,|i]\langle i|)$

Special $U(N)$ representation $\xrightarrow{\cancel{\text{IBP}}}$ massless limits $\{g\}$ $\xrightarrow[|i]_{\dot{\alpha}}\langle i|_{\alpha} \to p_{i,\dot{\alpha}\alpha}]{\cancel{\text{EOM}}\ \text{one to one}}$ Original massive $\{G\}$

# Massive amplitude basis

- One to one map

$$g(|j],|j\rangle,|i]\langle i|) \xrightarrow{|i]'_{\dot\alpha}\langle i|'_{\alpha} \to p_{i,\dot\alpha\alpha}} G'(|j],|j\rangle,p_i)$$

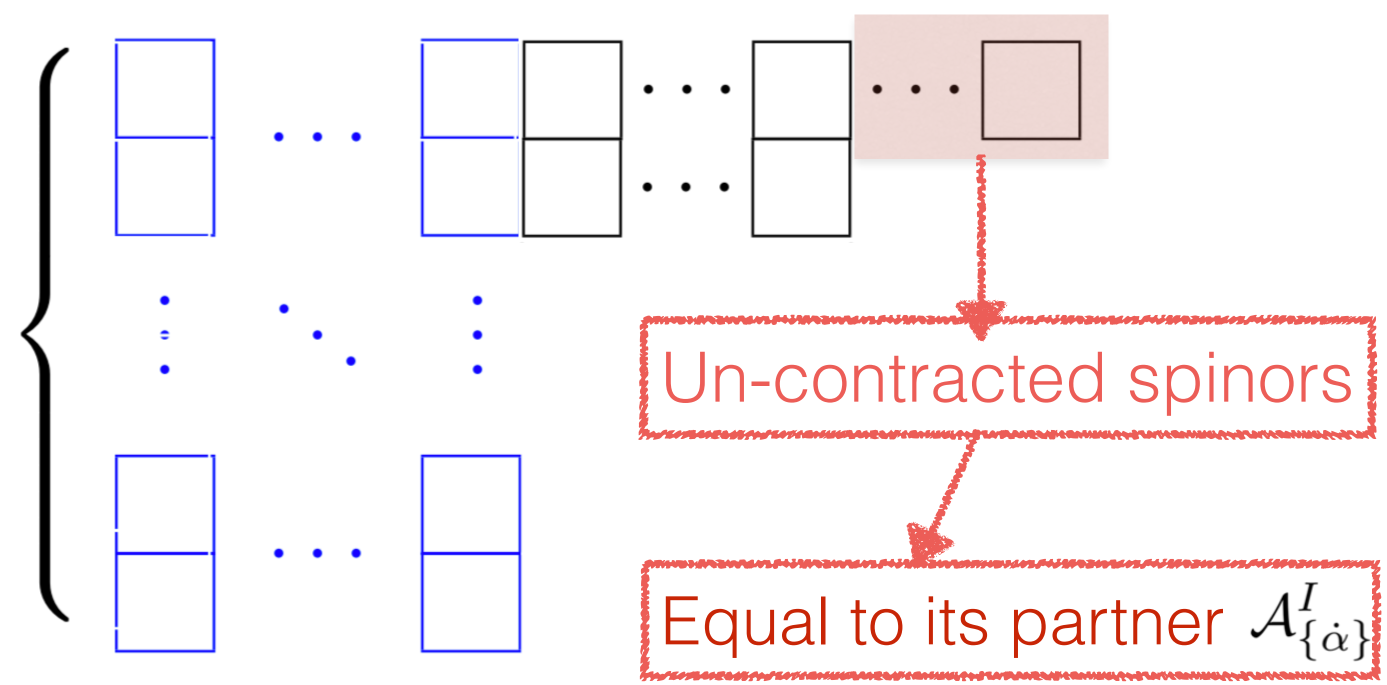
$$g(|j],|j\rangle,|i]\langle i|) \xrightarrow{|i]_{\dot\alpha}\langle i|_{\alpha} \to p_{i,\dot\alpha\alpha}} G(|j],|j\rangle,p_i)$$

$$\text{EOM} \quad p_i|i^I] = m_i|i^I\rangle$$

$$G'(|j],|j\rangle,p_i) - G(|j],|j\rangle,p_i) = \mathcal{O}(m_i^2)$$

- General shape of $g$

$$g(\tilde\lambda,\lambda) = N-2$$



# Massive amplitude basis

- Independence proof:

Case I: $\mathcal{A}^{I}_{[\lambda]} \cdot G(|j], |j\rangle, p_i) \neq \mathcal{A}^{I}_{[\lambda']} \cdot G'(|j], |j\rangle, p_i)\,,\ \lambda \neq \lambda'$

Case II: $\mathcal{A}^{I}_{[\lambda]} \cdot \left(\sum_{\eta} Z_{[\eta]} G_d^{[\eta]} + \sum_{i,\eta'} Z_{i[\eta']} m_i^2 G_{d-2}^{[\eta']} + \cdots\right) = 0$

$$\Rightarrow \sum_{\eta} Z_{[\eta]} G_d^{[\eta]} + \sum_{i,\eta'} Z_{i[\eta']} m_i^2 G_{d-2}^{[\eta']} + \cdots = 0$$

- Example $W^+ - W^- - Z$ **amplitude bases**

MLGTS $\mathcal{A}^{I}_{\{\dot\alpha\}}(\{\epsilon_{s_i}\}) \subset \begin{array}{|c|c|}\hline 1 & 1 \\ \hline \end{array} \times \begin{array}{|c|c|}\hline 2 & 2 \\ \hline \end{array} \times \begin{array}{|c|c|}\hline 3 & 3 \\ \hline \end{array}$

$$= \begin{array}{|c|c|c|}\hline 1 & 1 & 2 \\ \hline 2 & 3 & 3 \\ \hline \end{array} \oplus \begin{array}{|c|c|c|c|}\hline 1 & 1 & 2 & 2 \\ \hline 3 & 3 \\ \cline{1-2} \end{array} \oplus \begin{array}{|c|c|c|c|}\hline 1 & 1 & 2 & 3 \\ \hline 2 & 3 \\ \cline{1-2} \end{array} \oplus \begin{array}{|c|c|c|c|}\hline 1 & 1 & 3 & 3 \\ \hline 2 & 2 \\ \cline{1-2} \end{array}$$

$$\oplus \begin{array}{|c|c|c|c|c|}\hline 1 & 1 & 2 & 2 & 3 \\ \hline 3 \\ \cline{1-1} \end{array} \oplus \begin{array}{|c|c|c|c|c|}\hline 1 & 1 & 2 & 3 & 3 \\ \hline 2 \\ \cline{1-1} \end{array} \oplus \begin{array}{|c|c|c|c|c|c|}\hline 1 & 1 & 2 & 2 & 3 & 3 \\ \hline \end{array}$$

$$\equiv \mathcal{A}^{I}_{[3,3]} \oplus \mathcal{A}^{I}_{[(4,2)^1]} \oplus \mathcal{A}^{I}_{[(4,2)^2]} \oplus \mathcal{A}^{I}_{[(4,2)^3]}$$

$$\oplus \mathcal{A}^{I}_{[(5,1)^1]} \oplus \mathcal{A}^{I}_{[(5,1)^2]} \oplus \mathcal{A}^{I}_{[6]}$$

# Massive amplitude basis

- Examples:

$$\mathcal{A}^{\{I_1,I_2\},\{J_1,J_2\},\{K_1,K_2\}}_{[3,3]} \equiv \begin{array}{|c|c|c|}\hline 1 & 1 & 2 \\ \hline 2 & 3 & 3 \\ \hline \end{array} = 4[1^{I_1}2^{J_1}][1^{I_2}3^{K_1}][2^{J_2}3^{K_2}]$$

$G$ Blue column length: N-2 =1

$$\begin{array}{|c|c|}\hline 1 & 2 \\ \hline & 3 \\ \hline \end{array}$$

Not valid $\longrightarrow$ $G^{\bullet}_{d=0} = 1$

$$2s_{ij} = 2p_i \cdot p_j = (\epsilon_{ijk}m_k)^2 - m_i^2 - m_j^2$$

$$\mathcal{A}^{\{I_1,I_2\},\{J_1,J_2\},\{K_1,K_2\}}_{[(4,2)^1]} \equiv \begin{array}{|c|c|c|c|}\hline 1 & 1 & 2 & 2 \\ \hline 3 & 3 \\ \cline{1-2} \end{array} = 2[1^{I_1}3^{K_1}][1^{I_2}3^{K_2}][2^{J_1}]_{\{\dot\alpha}[2^{J_2}]_{\dot\alpha'\}} + 2[2^{J_1}3^{K_1}][2^{J_2}3^{K_2}][1^{I_1}]_{\{\dot\alpha}[1^{I_2}]_{\dot\alpha'\}} + 8[1^{I_1}3^{K_1}][2^{J_1}3^{K_2}][1^{I_2}]_{\{\dot\alpha}[2^{J_2}]_{\dot\alpha'\}}$$

Only valid

$$G^{[3]}_{d=2} \equiv \begin{array}{|c|c|c|}\hline 1 & 2 & 3 \\ \hline \end{array} = \Big(\langle i_1 i_2\rangle \epsilon^{1 i_1 i_2}|2]^{\{\dot\alpha}|3]^{\dot\alpha'\}} + \langle i_1 i_2\rangle \epsilon^{2 i_1 i_2}|3]^{\{\dot\alpha}|1]^{\dot\alpha'\}} + \langle i_1 i_2\rangle \epsilon^{3 i_1 i_2}|1]^{\{\dot\alpha}|2]^{\dot\alpha'\}}\Big)\Big|_{|i]\langle i| \to p_i}$$

$$= \langle 2_I 3_J\rangle |2^I]^{\{\dot\alpha}|3^J]^{\dot\alpha'\}} + \langle 3_I 1_J\rangle |3^I]^{\{\dot\alpha}|1^J]^{\dot\alpha'\}} \langle 1_I 2_J\rangle |1^I]^{\{\dot\alpha}|2^J]^{\dot\alpha'\}}.$$

# Massive amplitude basis

- Total seven $W^+ - W^- - Z$ **amplitude bases**

$$\mathcal{A}^I_{[3,3]} \ : \ G^{\bullet}_{d=0} = 1$$

$$\mathcal{A}^I_{[(4,2)^{1,2,3}]} \ : \ G^{[3]}_{d=2} = \begin{array}{|c|c|c|}\hline 1 & 2 & 3 \\ \hline\end{array}$$

$$\mathcal{A}^I_{[(5,1)^{1,2}]} \ : \ G^{[6]}_{d=4} = \begin{array}{|c|c|c|c|c|c|}\hline 1 & 1 & 2 & 2 & 3 & 3 \\ \hline\end{array}$$

$$\mathcal{A}^I_{[6]} \ : \ G^{[9]}_{d=6} = \begin{array}{|c|c|c|c|c|c|c|c|c|}\hline 1 & 1 & 1 & 2 & 2 & 2 & 3 & 3 & 3 \\ \hline\end{array}$$

G. Durieux, T. Kitahara, Y. Shadmi and Y. Weiss, JHEP **01**, 119 (2020) doi:10.1007/JHEP01(2020)119 [arXiv:1909.10551 [hep-ph]].

# Summary

☑ Propose a method to completely construct massive amplitude basis

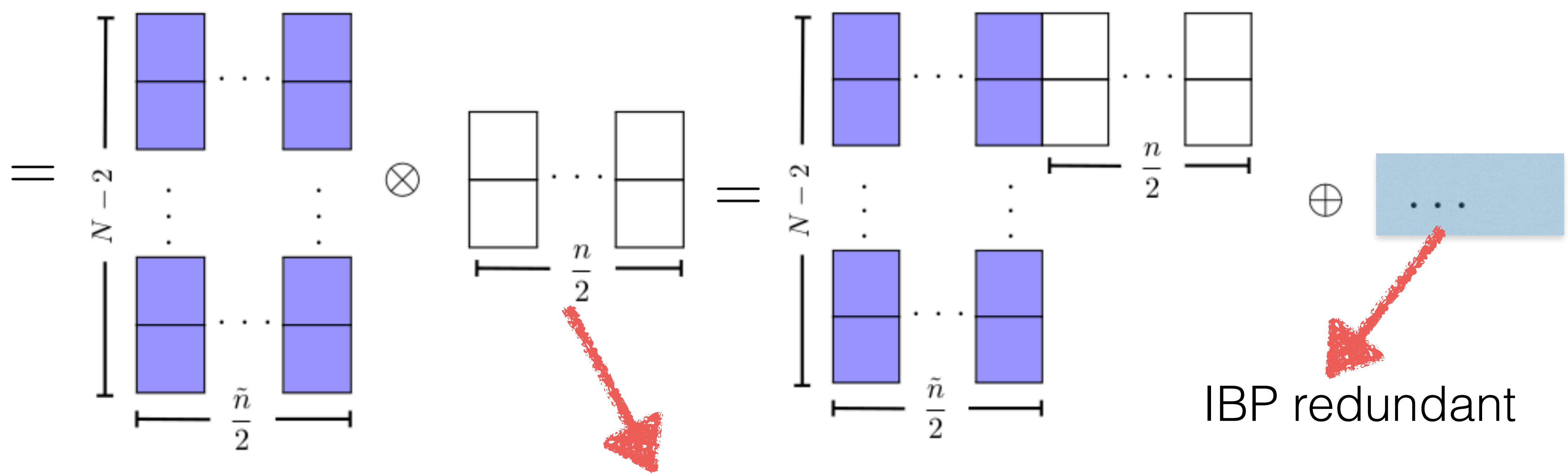
☑ Computer programs can automatically construct it based on it

# BACK UP

# On-shell amplitude basis

- IBP redundancy can be removed by $U(N) \supset \otimes_{i=1}^{N} U(1)_i$ symmetry

$$f^{(n,\tilde{n})}(\{\lambda, \widetilde{\lambda}\}) = (\overline{g}_{U(N)} \otimes g_{U(N)})$$




Same as $SU(2)_r$ YD

- The amplitude basis is the basis of the first U(N) representation

- The computer programs can construct massless basis based on it

# EFT

- Null new physics signals at the detections on ground

$$\Lambda \gg \mathcal{O}(1)\text{TeV} \xrightarrow{\text{precision measurement}} \delta g = g_{exp} - g_{SM} \Leftrightarrow \mathcal{O}_{d\geq 4}$$

- Dark matter detections

$$\mathcal{O}_{DM} \sim ee\phi_{DM}\phi_{DM}$$

- Higher spin particles

$$\mathcal{O} \sim (D_{\mu_1}e)\gamma_{\mu_2}\gamma_{\mu_3}e\rho^{\mu_1\mu_2\mu_3}$$

# Massless amplitude basis

- Basic structure of massless amplitudes

T. Ma, J. Shu and M. L. Xiao, [arXiv:1902.06752 [hep-

$$\mathscr{M}(\{p_i, h_i\}) = f(|i], |i\rangle) g(s_{ij}) T^{\{\alpha\}}$$

Gauge structure

Massless LG charged

Massless LG neutral

- An amplitude basis just corresponds to the leading interaction of a operator

- IBP redundancy can be systematically removed by $U(N) \supset \otimes_{i=1}^{N} U(1)_i$

Quantum number under $SU(2)_r \otimes U(N)$

$$\lambda_\alpha^k \equiv |k] = (2, N)$$

Quantum number under $SU(2)_l \otimes U(N)$

$$\tilde{\lambda}_{k\dot{\alpha}} \equiv |k\rangle = (2, \bar{N})$$

$$\begin{array}{|c|}\hline i \\ \hline j \\ \hline \end{array} = (\tilde{\lambda}^i_{\dot{\alpha}} \tilde{\lambda}^j_{\dot{\beta}} - \tilde{\lambda}^j_{\dot{\alpha}} \tilde{\lambda}^i_{\dot{\beta}}) = [ij]$$

$$\begin{array}{|c|}\hline k_1 \\ \hline \cdot \\ \hline \cdot \\ \hline k_{N-2} \\ \hline \end{array} = \frac{(\epsilon^{ijk_1\ldots k_{N-2}} + \text{anti-sym in } k_1..k_{N-2})}{(N-2)!} \lambda_{i\alpha} \lambda_{j\beta}$$

$$= \langle ij \rangle \epsilon^{ijk_1\ldots k_{N-2}} \,.$$

# UV Completion

- Toy model: a vector-like doublet fermion $\Psi_{\mathbf{2}}$


$$\mathcal{L}_{\rm int} = \lambda_{1L} \bar{\Psi}_{2_R} \Delta_L H + \lambda_{2L} \bar{\Psi}_{2_R} H \eta_L + (L \leftrightarrow R) + h.c.$$

- Effective Lagrange

$$\mathcal{L}_{\rm eff}^{\rm mix} = \frac{-1}{M^2 - p^2} \left( \lambda_{1L} \lambda_{2L} H^\dagger \bar{\Delta}_L H \not{p} \eta_L + M \lambda_{1L} \lambda_{2R} H^\dagger \bar{\Delta}_L H \eta_R \right)$$

Positive Negative

- For pure chiral coupling, Higgs quartic is positive.