Institute of Theoretical Physics
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Operator Basis for Effective Field Theories

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Outline

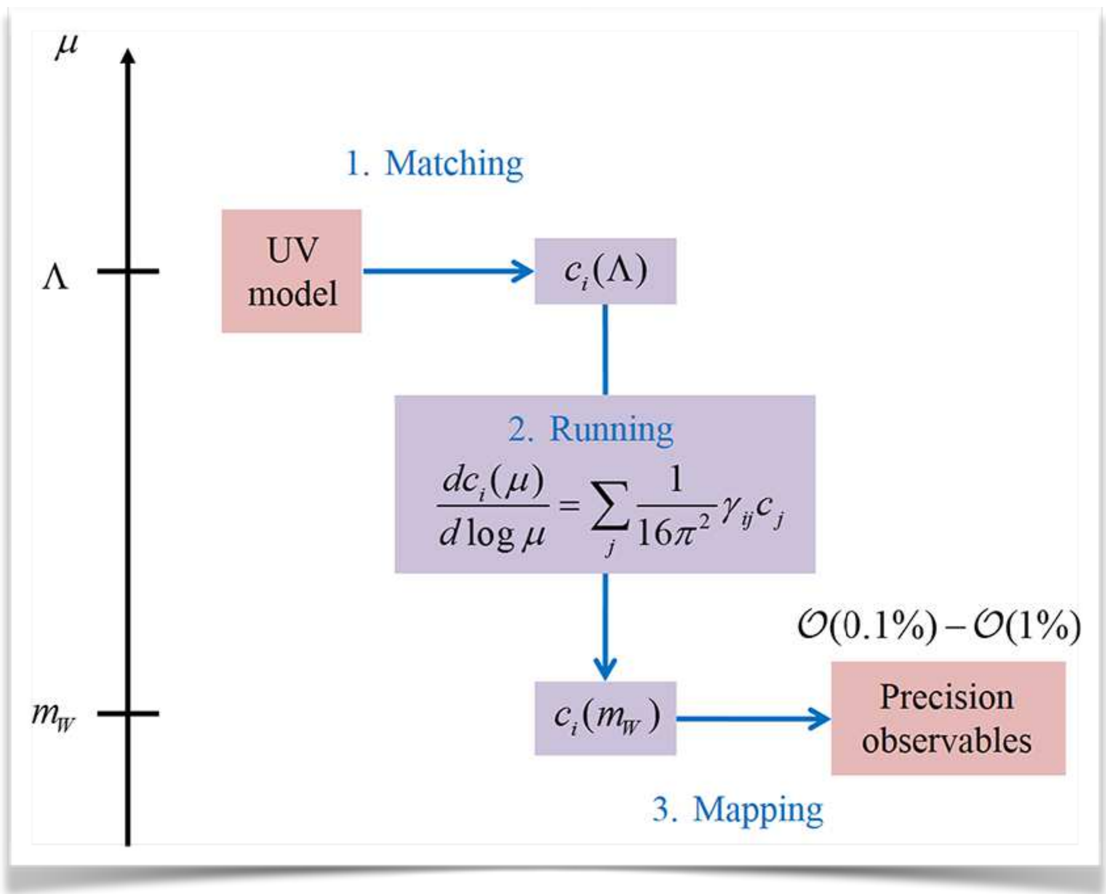
- Introduction: core of EFT
- Group theory for Lagrangian invariant
- Construct higher dimensional operators
- Systematical procedure: Young tensor basis
- Linear vs nonlinear Lagrangian
- Summary and outlook

Introduction

Core of EFTs

Scales

Decoupling Theorem



Field d.o.f

Symmetries

Wilsonian

Wilsonian Lagrangian

$$Z[J_\Phi, J_\varphi] = \int [D\Phi]\,[D\varphi]\; e^{i\int d^dx(\mathscr{L}[\Phi,\varphi]+J_\Phi\Phi+J_\varphi\varphi)}$$

Equation of motion (EOM): $\frac{\delta\mathscr{L}_{\rm UV}}{\delta\Phi} = 0\,.$

Set $\Phi = \Phi_c[\phi]$ (EOM solution)

$$Z[J_\varphi] = \int\,[D\varphi]\; e^{i\int d^dx(\mathscr{L}_{\rm eft}[\varphi,\Phi_c]+J_\varphi\varphi)}$$

top-down approach

Wilsonian

Symmetry determines Lagrangian

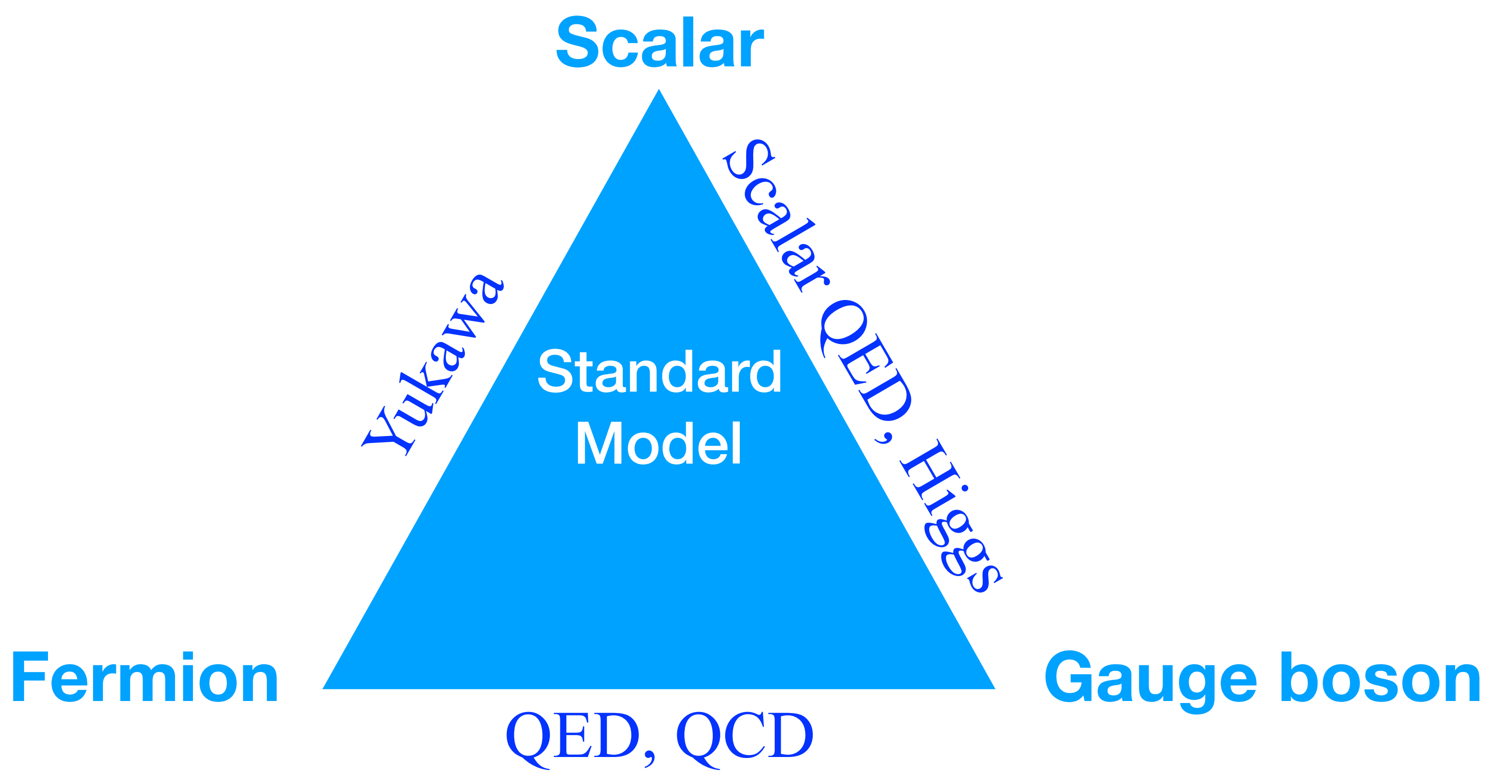
$$W = \overbrace{\int_{k<\Lambda} [Dg][DA][D\psi][D\Phi]\, \exp}^{\text{quantum mechanics}} \left\{ i \overbrace{\int d^4x \sqrt{-g}}^{\text{spacetime}} \left[\overbrace{\frac{m_p^2}{2} R}^{\text{gravity}} \right.\right.$$

$$\left.\left. \underbrace{-\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu}}_{\text{other forces}} + \underbrace{i\bar{\psi}^i \gamma^\mu D_\mu \psi^i + \left(\bar{\psi}^i_L V_{ij} \Phi \psi^j_R + \text{h.c.} \right)}_{\text{matter}} \underbrace{- |D_\mu \Phi|^2 - V(\Phi)}_{\text{Higgs}} \right] \right\}$$

bottom-up approach

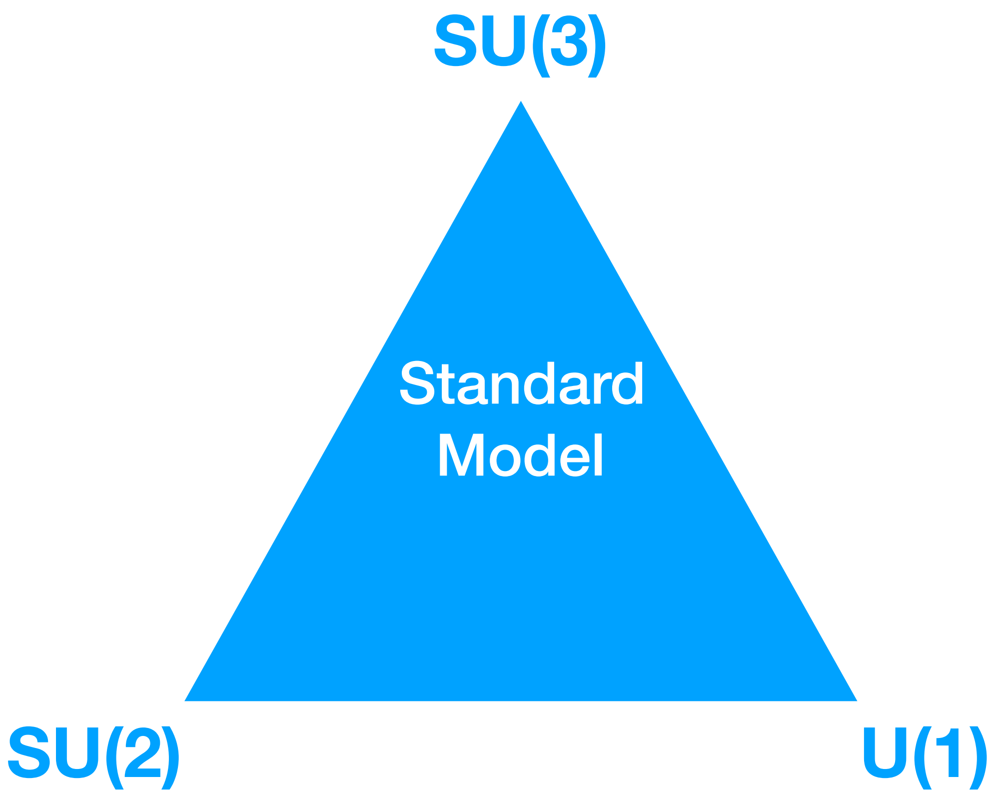
Fields and Interactions

Quantum field theory course:



Symmetry

History of particle physics: find more symmetries



Custodial symmetry, flavor symmetry, …

Standard Model Extensions

$$SU(3)_C \times SU(2)_L \times U(1)_Y$$

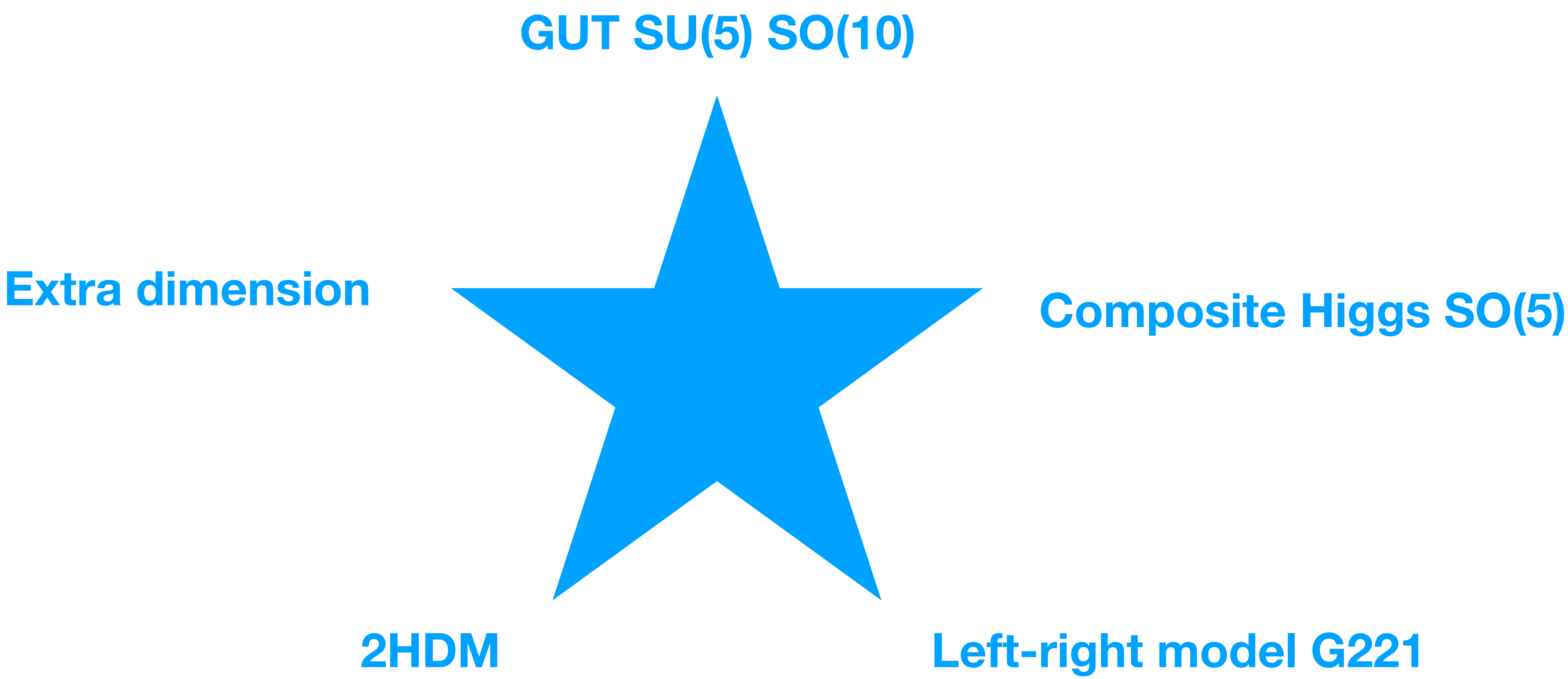
Adding more fields and higher representations

Considering bigger gauge group

SUSY not classified here

More Symmetry!!!

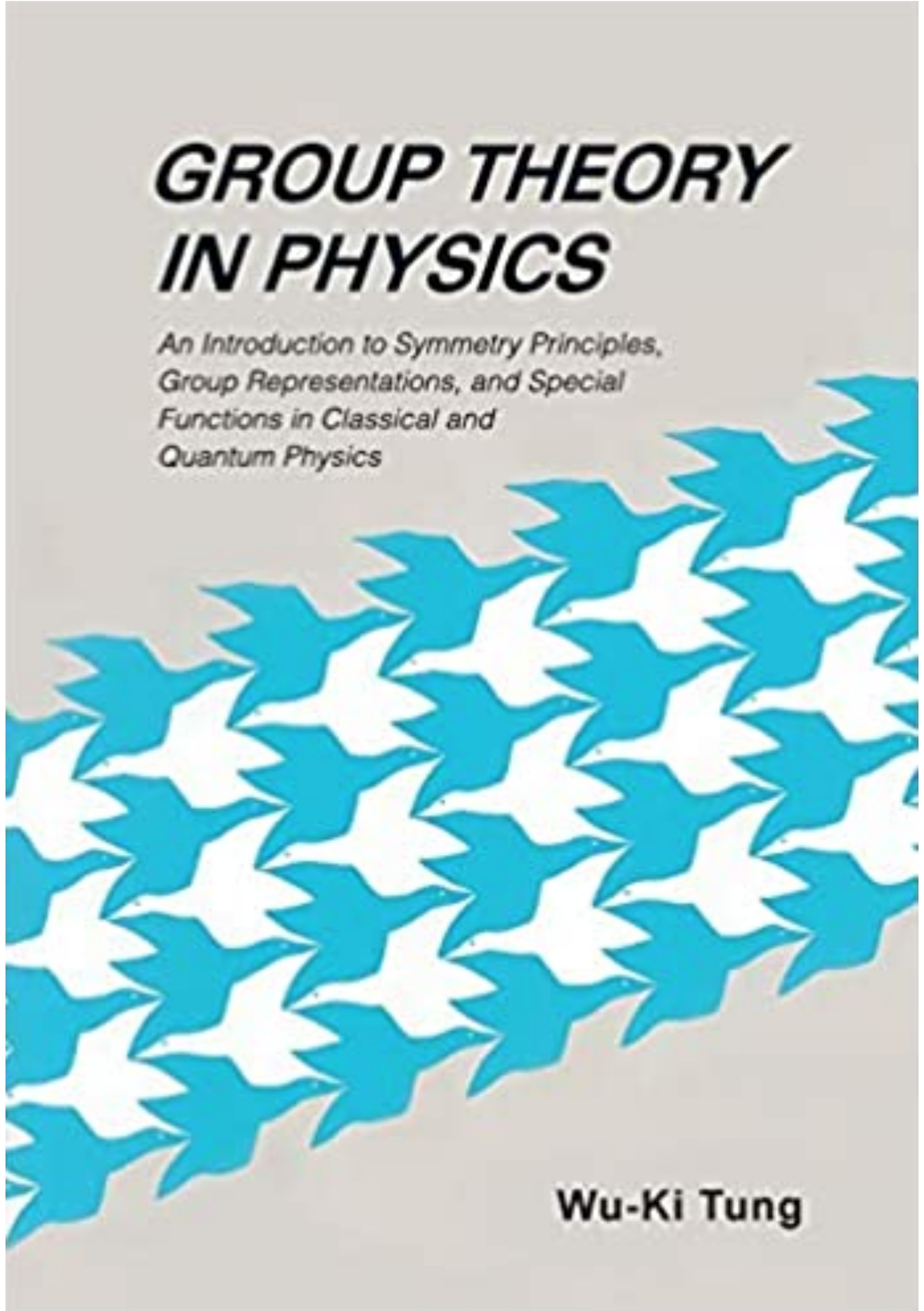
Finding new symmetries in searching new physics

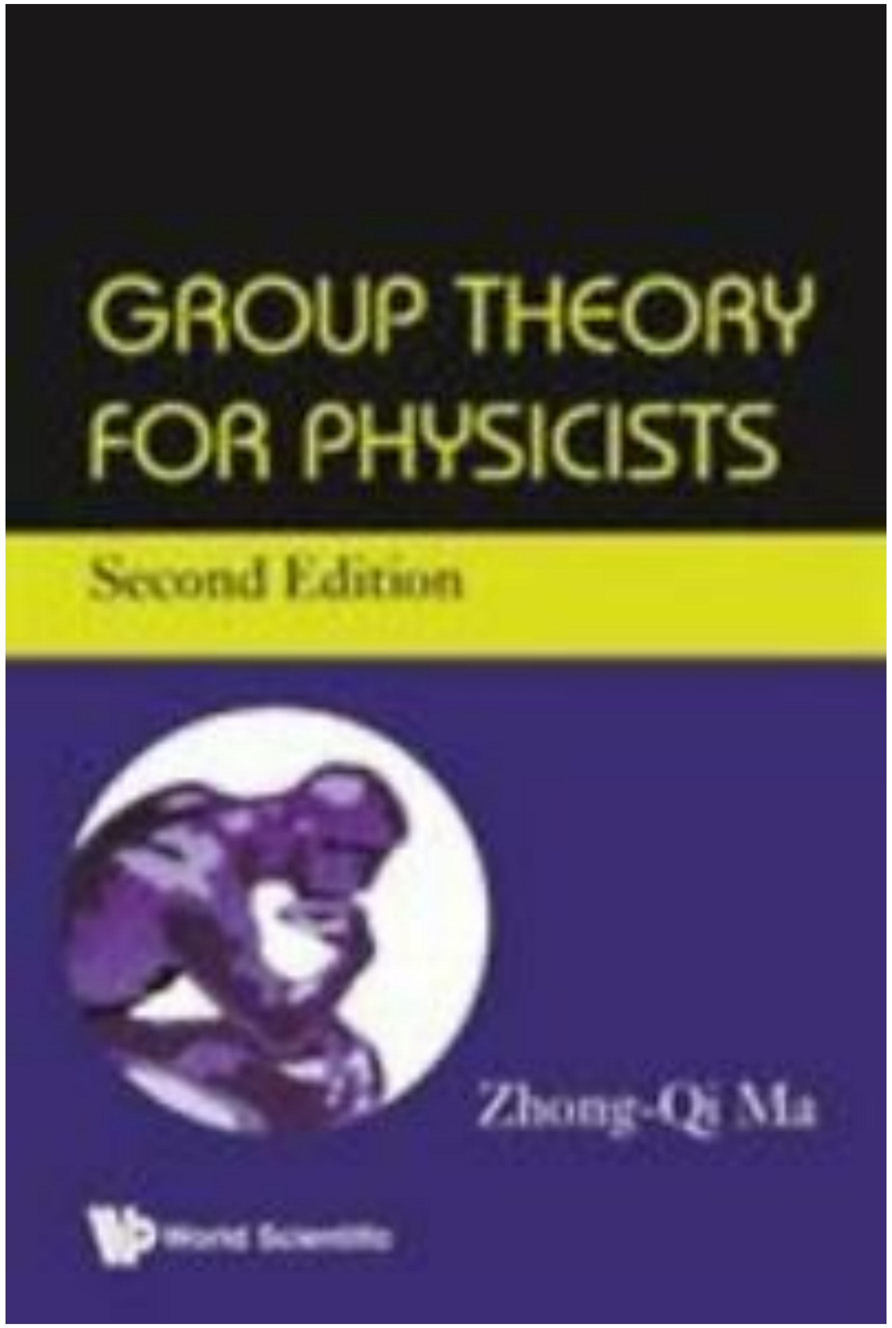


SUSY not classified here

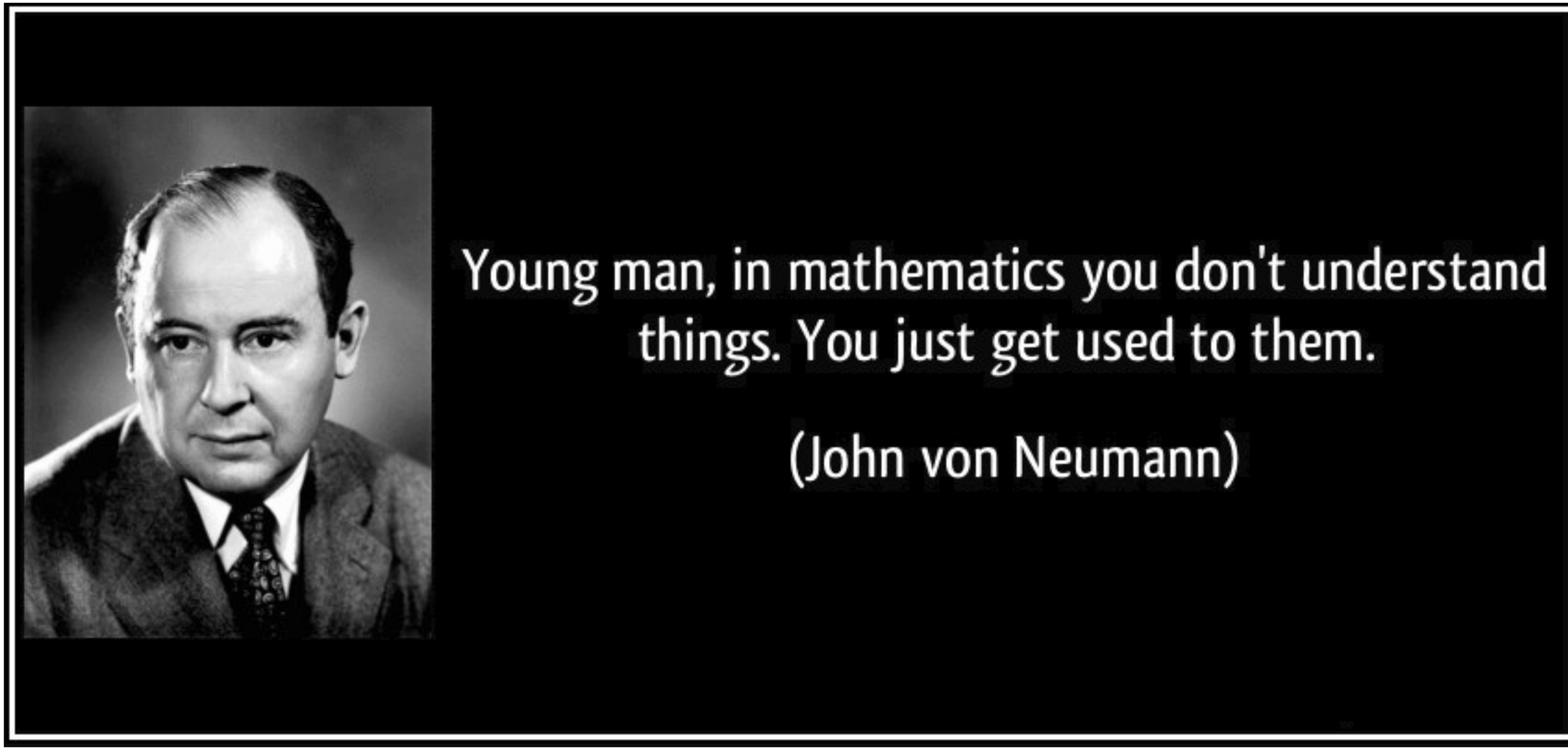
Internal Symmetry

Group Theory





Group Theory

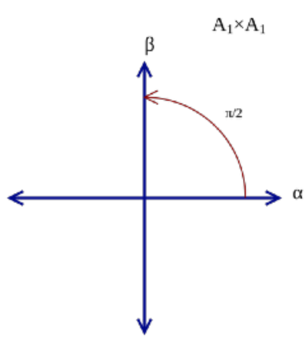


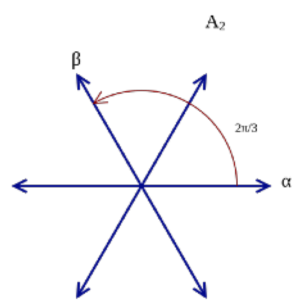
SU(N) Group

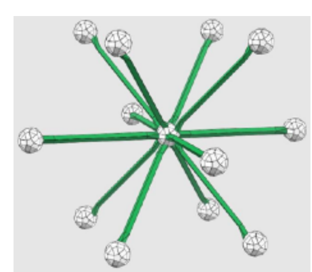
$$[A^{\mu}{}_{\nu}, A^{\alpha}{}_{\beta}] = \delta^{\alpha}_{\nu} A^{\mu}{}_{\beta} - \delta^{\mu}_{\beta} A^{\alpha}{}_{\nu}, \quad A^{\mu}{}_{\mu} = 0. \quad \mu, \nu = 1, 2, \cdots, N.$$

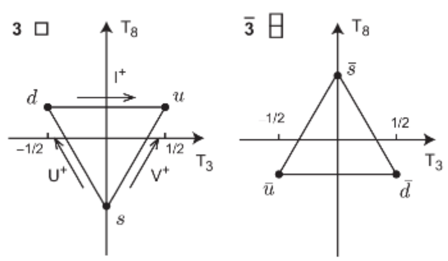
Root-weight: obtain transformation matrices for any irrep

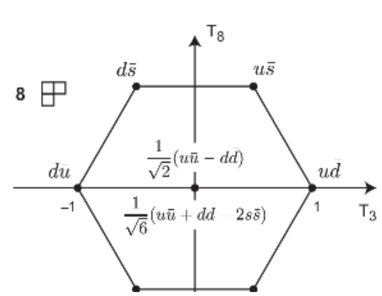
$$[H_i, H_j] = 0 \quad [H_i, E_\alpha] = a_i E_\alpha \quad [E_\alpha, E_{-\alpha}] = a^i H_i$$











Young tensor: obtain tensor basis for any irrep

$$\xi'^{\alpha} = U^{\alpha}{}_{\beta} \xi^{\beta} \quad \eta'_{\alpha} = \eta_{\beta} (U^{\dagger})^{\beta}{}_{\alpha} \quad (U^{\dagger})^{\alpha}{}_{\beta} U^{\beta}{}_{\gamma} = \delta^{\alpha}{}_{\gamma}\,.$$

$$\epsilon_{\beta_1 \ldots \beta_n} U^{\beta_1}{}_{\alpha_1} \ldots U^{\beta_n}{}_{\alpha_n} = \epsilon_{\alpha_1 \ldots \alpha_n} \qquad \det U = 1$$

$\xi^a = 2$ in SU(2); 3 in SU(3); ... $\quad \xi_a = \bar{2}$ in SU(2); $\bar{3}$ in SU(3); ...

$$\zeta^{ab} \equiv \xi^a \xi^b = \frac{1}{2}(\zeta^{ab} + \zeta^{ba}) + \frac{1}{2}(\zeta^{ab} - \zeta^{ba}) = \zeta^{\{ab\}} + \zeta^{[ab]}$$

$$\boxed{a} \times \boxed{b} = \boxed{a\,|\,b} + \boxed{\begin{matrix} a \\ b \end{matrix}}\,.$$

$$2 \otimes 2 = 3 \oplus 1$$

$$3 \otimes 3 = \bar{3} \oplus 6$$

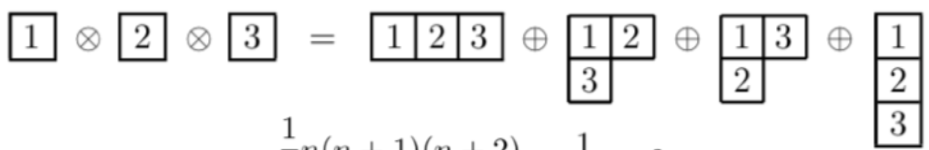
$$\zeta^{\alpha}{}_{\beta} = \xi^{\alpha} \xi_{\beta} = \hat{\zeta}^{\alpha}{}_{\beta} + \frac{1}{n} \delta^{\alpha}{}_{\beta} \xi^{\gamma} \xi_{\gamma} \qquad \hat{\zeta}^{\alpha}{}_{\beta} = \zeta^{\alpha}{}_{\beta} - \frac{1}{n} \delta^{\alpha}{}_{\beta} \xi^{\gamma} \xi_{\gamma}$$

$$3 \otimes \bar{3} = 1 \oplus 8$$
$$n \otimes \bar{n} = 1 \oplus (n^2 - 1).$$

SU(N) Tensor Product

Tensor decomposition

$$\zeta^{\alpha_1\alpha_2\alpha_3} = \xi^{\alpha_1}\xi^{\alpha_2}\xi^{\alpha_3} = \zeta^{\{\alpha_1\alpha_2\alpha_3\}} \oplus \zeta^{\{\alpha_1\alpha_2\}\alpha_3} \oplus \zeta^{\{\alpha_1\alpha_3\}\alpha_2} \oplus \zeta^{[\alpha_1\alpha_2\alpha_3]}$$

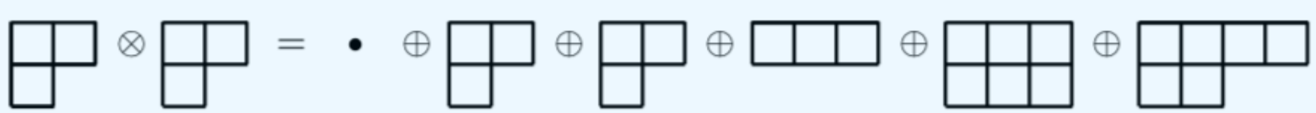


$$\frac{1}{6}n(n+1)(n+2) \qquad \frac{1}{3}n(n^2-1) \qquad \frac{1}{6}n(n-1)(n-2)$$

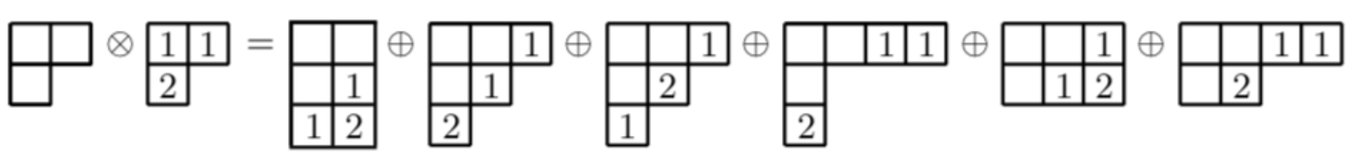
$$2 \otimes 2 \otimes 2 = 4 \oplus 2 \oplus 2$$

$$3 \otimes 3 \otimes 3 = 10 \oplus 8 \oplus 8 \oplus 1$$

Tensor Product



$$8 \otimes 8 = 1 \oplus 8 \oplus 8 \oplus 10 \oplus \overline{10} \oplus 27$$



LieART 2.0 – A Mathematica Application for Lie Algebras and Representation Theory

Robert Feger[a,*], Thomas W. Kephart[b], Robert J. Saskowski [1b]

http://lieart.hepforge.org

```
In[16]:= DecomposeProduct[Irrep[SU3][8],Irrep[SU3][8]]
```

Out[16]:= $\mathbf{1} + 2(\mathbf{8}) + \mathbf{10} + \overline{\mathbf{10}} + \mathbf{27}$

```
:= DecomposeProduct[Irrep[SU4][4],Irrep[SU4][4],Irrep[SU4][6],Irrep[SU4][15]]
```

Out[20]:= $2(\mathbf{1}) + 7(\mathbf{15}) + 4(\mathbf{20'}) + \mathbf{35} + 5(\mathbf{45}) + 3(\overline{\mathbf{45}}) + 3(\mathbf{84}) + 2(\mathbf{175}) + \mathbf{256}$

GroupMath: A Mathematica package for group theory calculations

Renato M. Fonseca

renatofonseca.net/groupmath

```
In[]:= ReduceRepProduct[SU3, {3, 3, 8, 8, 8, -3, 8}]
Out[]= {{{6, 5}, 1}, {{7, 3}, 4}, {{5, 4}, 26}, {{4, 6}, 5}, {{3, 5}, 64}, {{6, 2}, 48},
  {{4, 3}, 166}, {{2, 7}, 9}, {{1, 6}, 66}, {{2, 4}, 260}, {{5, 1}, 176},
  {{3, 2}, 434}, {{0, 8}, 5}, {{0, 5}, 137}, {{1, 3}, 448}, {{8, 1}, 5},
  {{7, 0}, 27}, {{4, 0}, 235}, {{2, 1}, 510}, {{0, 2}, 297}, {{1, 0}, 217}}
```

SM Lagrangian

Particle(s)	Field(s)	Content	Charge	Spin	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
Quarks	Q_i	$(u,d)_L$	(⅔, −⅓)	½	3	2	⅓
(Three generations)	u_{Ri}	u_R	⅔	½	$\overline{3}$	1	4/3
	d_{Ri}	d_R	−⅓	½	$\overline{3}$	1	−⅔
Leptons	L_i	$(\nu_e,e)_L$	(0, −1)	½	1	2	−1
(Three generations)	l_{Ri}	e_R	−1	½	1	1	−2
Gluons	G_μ^a	g	0	1	8	1	0
W bosons	$W_\mu^{1,2}$	$W^\pm$	±1	1	1	3	0
Photon, Z boson	W_μ^3, B_μ	γ, Z^0	0	1	1	3, 1	0
Higgs boson	ϕ	H	0	0	1	2	1

SU(2)

SU(2)	$\mathbf{2}\times\mathbf{2}=\mathbf{1}+\mathbf{3}$
	$\mathbf{3}\times\mathbf{2}=\mathbf{2}+\mathbf{4}$
	$\mathbf{3}\times\mathbf{3}=\mathbf{1}+\mathbf{3}+\mathbf{5}$
	$\mathbf{4}\times\mathbf{2}=\mathbf{3}+\mathbf{5}$
	$\mathbf{4}\times\mathbf{3}=\mathbf{2}+\mathbf{4}+\mathbf{6}$
	$\mathbf{4}\times\mathbf{4}=\mathbf{1}+\mathbf{3}+\mathbf{5}+\mathbf{7}$

SU(3)	$\overline{\mathbf{3}}\times\mathbf{3}=\mathbf{1}+\mathbf{8}$
	$\mathbf{3}\times\mathbf{3}=\overline{\mathbf{3}}+\mathbf{6}$
	$\overline{\mathbf{6}}\times\mathbf{3}=\overline{\mathbf{3}}+\overline{\mathbf{15}}$
	$\mathbf{6}\times\mathbf{3}=\mathbf{8}+\mathbf{10}$
	$\overline{\mathbf{6}}\times\overline{\mathbf{6}}=\mathbf{6}+\overline{\mathbf{15}}+\overline{\mathbf{15}}'$
	$\mathbf{6}\times\overline{\mathbf{6}}=\mathbf{1}+\mathbf{8}+\mathbf{27}$
	$\mathbf{8}\times\mathbf{3}=\mathbf{3}+\overline{\mathbf{6}}+\mathbf{15}$
	$\mathbf{8}\times\mathbf{6}=\mathbf{3}+\overline{\mathbf{6}}+\mathbf{15}+\mathbf{24}$
	$\mathbf{8}\times\mathbf{8}=\mathbf{1}+2(\mathbf{8})+\mathbf{10}+\overline{\mathbf{10}}+\mathbf{27}$
	$\overline{\mathbf{10}}\times\mathbf{3}=\overline{\mathbf{6}}+\mathbf{24}$
	$\mathbf{10}\times\mathbf{3}=\mathbf{15}+\mathbf{15}'$
	$\overline{\mathbf{10}}\times\overline{\mathbf{6}}=\mathbf{15}+\mathbf{21}+\mathbf{24}$
	$\mathbf{10}\times\overline{\mathbf{6}}=\mathbf{3}+\mathbf{15}+\mathbf{42}$
	$\mathbf{10}\times\mathbf{8}=\mathbf{8}+\mathbf{10}+\mathbf{27}+\mathbf{35}$
	$\overline{\mathbf{10}}\times\mathbf{10}=\mathbf{1}+\mathbf{8}+\mathbf{27}+\mathbf{64}$
	$\mathbf{10}\times\mathbf{10}=\overline{\mathbf{10}}+\mathbf{27}+\mathbf{28}+\mathbf{35}$

Decompose tensor products of product irreps $(\mathbf{3},\ \overline{\mathbf{3}},\ \mathbf{1})\otimes(\overline{\mathbf{3}},\ \mathbf{3},\ \mathbf{1})$ of SU(3)⊗SU(3)⊗SU(3):

```
In[23]:= DecomposeProduct[
          ProductIrrep[Irrep[SU3][3],Irrep[SU3][Bar[3]],Irrep[SU3][1]],
          ProductIrrep[Irrep[SU3][Bar[3]],Irrep[SU3][3],Irrep[SU3][1]]]
Out[23]:= (1,1,1) + (8,1,1) + (1,8,1) + (8,8,1)
```

$$\mathcal{L} = \underbrace{-\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{8}tr(\mathbf{W}_{\mu\nu}\mathbf{W}^{\mu\nu}) - \frac{1}{2}tr(\mathbf{G}_{\mu\nu}\mathbf{G}^{\mu\nu})}_{\text{U(1), SU(2) and SU(3) gauge terms}}$$

$$\underbrace{+(\bar{\nu}_L,\bar{e}_L)\,\tilde{\sigma}^\mu iD_\mu \begin{pmatrix}\nu_L\\ e_L\end{pmatrix} + \bar{e}_R\sigma^\mu iD_\mu e_R + \bar{\nu}_R\sigma^\mu iD_\mu\nu_R + \text{Hermitian conjugate}}_{\text{lepton dynamical term}}$$

$$\underbrace{-\frac{\sqrt{2}}{v}\left[(\bar{\nu}_L,\bar{e}_L)\,\phi M^e e_R + \bar{e}_R\bar{M}^e\bar{\phi}\begin{pmatrix}\nu_L\\ e_L\end{pmatrix}\right]}_{\text{electron, muon, tauon mass term}}$$

$$\underbrace{-\frac{\sqrt{2}}{v}\left[(-\bar{e}_L,\bar{\nu}_L)\,\phi^* M^\nu \nu_R + \bar{\nu}_R\bar{M}^\nu\phi^T\begin{pmatrix}-e_L\\ \nu_L\end{pmatrix}\right]}_{\text{neutrino mass term}}$$

$$\underbrace{+(\bar{u}_L,\bar{d}_L)\,\tilde{\sigma}^\mu iD_\mu \begin{pmatrix}u_L\\ d_L\end{pmatrix} + \bar{u}_R\sigma^\mu iD_\mu u_R + \bar{d}_R\sigma^\mu iD_\mu d_R + \text{Hermitian conjugate}}_{\text{quark dynamical term}}$$

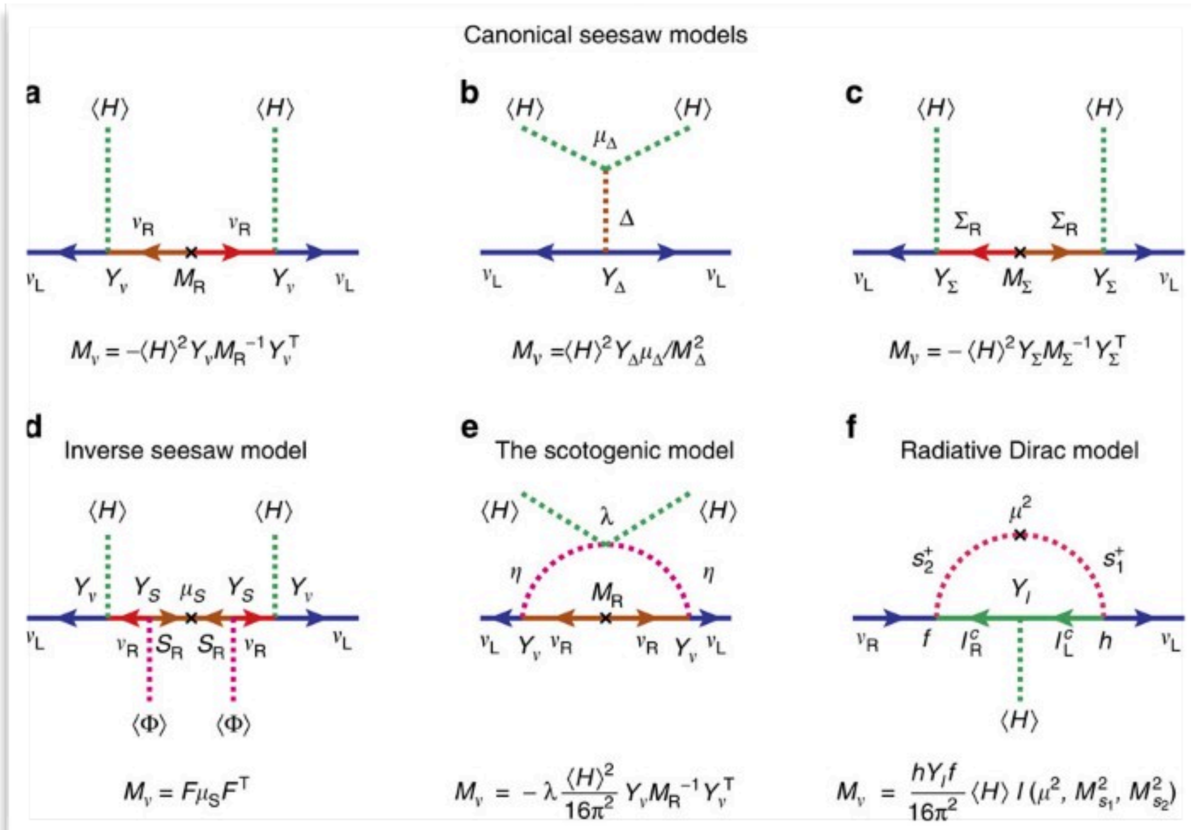
$$\underbrace{-\frac{\sqrt{2}}{v}\left[(\bar{u}_L,\bar{d}_L)\,\phi M^d d_R + \bar{d}_R\bar{M}^d\bar{\phi}\begin{pmatrix}u_L\\ d_L\end{pmatrix}\right]}_{\text{down, strange, bottom mass term}}$$

$$\underbrace{-\frac{\sqrt{2}}{v}\left[(-\bar{d}_L,\bar{u}_L)\,\phi^* M^u u_R + \bar{u}_R\bar{M}^u\phi^T\begin{pmatrix}-d_L\\ u_L\end{pmatrix}\right]}_{\text{up, charmed, top mass term}}$$

$$\underbrace{+\overline{(D_\mu\phi)}D^\mu\phi - m_h^2[\bar{\phi}\phi - v^2/2]^2/2v^2.}_{\text{Higgs dynamical and mass term}}$$

Beyond SM Lagrangian

First evidence of beyond SM: neutrino masses



Integrate out heavy particles

$$\underbrace{\underbrace{\bar{L}^c \otimes \Phi}_{1} \otimes \underbrace{\Phi \otimes L}_{1}}_{\text{Type I}}\,, \quad \underbrace{\underbrace{\bar{L}^c \otimes L}_{3} \otimes \underbrace{\Phi \otimes \Phi}_{3}}_{\text{Type II}}\,, \quad \underbrace{\underbrace{\bar{L}^c \otimes \Phi}_{3} \otimes \underbrace{\Phi \otimes L}_{3}}_{\text{Type III}}$$

$$\mathbf{2} \times \mathbf{2} = \mathbf{1} + \mathbf{3}$$

$$\mathbf{3} \times \mathbf{3} = \mathbf{1} + \mathbf{3} + \mathbf{5}$$

$$Q_{\nu\nu} = \varepsilon_{jk}\varepsilon_{mn}\varphi^j\varphi^m (l_p^k)^T C l_r^n \equiv (\widetilde{\varphi}^\dagger l_p)^T C (\widetilde{\varphi}^\dagger l_r),$$

$$\kappa_{ij} L_i L_j H H$$

3 generation = 9 parameters!?

Dim-5 Weinberg Operator

Repeated fields leads to symmetries in the coupling parameters

$$2 \times 2 = 1_A + 3_S \qquad SU(2)$$ in QM

Only one Higgs doublet in SM

$$\underbrace{L \times L}_{\cancel{1}_A + 3_S} \times \underbrace{H \times H}_{\cancel{1}_A + 3_S}$$

$$\kappa_{ij} L_i L_j H H$$

κ_{ij} is symmetric: $\kappa_{ij} = \kappa_{ji}$

3 generation = 6 parameters!

Higgs self couplings

$$g^{(\alpha)}_{ijkl} H^*_i H^*_j H_k H_l$$

$\{\{S, S\}, \frac{1}{4} n^2 (1+n)^2, 1\}$ | $\{\{A, A\}, \frac{1}{4} (-1+n)^2 n^2, 1\}$

2HDM = 10 parameters

If more than one Higgs doublet

$$(1_A + 3_S) \times (1_A + 3_S) = 1_{SS} + 1_{AA} + \cdots$$

$$\kappa^{SS}_{ijkl} (L_i L_j H_k H_l)_{SS} + \kappa^{AA}_{ijkl} (L_i L_j H_k H_l)_{AA}$$

4-fermion operators

$$L^* L^* L L$$

$$2^* \times 2^* \times 2 \times 2 = (1_A + 3_S) \times (1_A + 3_S)$$

$$\underbrace{g^{(SS)}_{ijkl}}_{\text{36 par.}} \left(L^*_i L^*_j L_k L_l\right)_{(SS)} + \underbrace{g^{(AA)}_{ijkl}}_{\text{9 par.}} \left(L^*_i L^*_j L_k L_l\right)_{(AA)}$$

45 parameters in 4-fermion operator

[Fonseca, GroupMath]

More Repeated Fields

So we need to keep track of S and A symmetries for repeated field

$\phi\phi\phi\phi$

doublet $SU(2)$ [and no hypercharge]

$2 \times 2 \times 2 \times 2 = (2 \times 2) \times (2 \times 2) =$

$(1_A + 3_S) \times (1_A + 3_S) = 1_{SS} + 1_{AA} + \cdots$

Keep SS

1 coupling parameters?

$Q_i Q_j Q_k L_l$

3^4 = 81 coupling parameters?

Keep AA only?

$\zeta^{\alpha_1\alpha_2\alpha_3} = \xi^{\alpha_1}\xi^{\alpha_2}\xi^{\alpha_3} = \zeta^{\{\alpha_1\alpha_2\alpha_3\}} \oplus \zeta^{\{\alpha_1\alpha_2\}\alpha_3} \oplus \zeta^{\{\alpha_1\alpha_3\}\alpha_2} \oplus \zeta^{[\alpha_1\alpha_2\alpha_3]}$

1 x 3 = 3 coupling parameters?

[Fonseca, GroupMath]

More Repeated Fields

So we need to keep track of S and A symmetries for repeated field

$\phi\phi\phi\phi$

doublet $SU(2)$ [and no hypercharge]

$$2 \times 2 \times 2 \times 2 = (2 \times 2) \times (2 \times 2) =$$
$$(1_A + 3_S) \times (1_A + 3_S) = 1_{SS} + 1_{AA} + \cdots$$

Keep SS

1 coupling parameters?

Answer: 0!!!

$Q_i Q_j Q_k L_l$

3^4 = 81 coupling parameters?

Keep AA only?

$$\zeta^{\alpha_1\alpha_2\alpha_3} = \xi^{\alpha_1}\xi^{\alpha_2}\xi^{\alpha_3} = \zeta^{\{\alpha_1\alpha_2\alpha_3\}} \oplus \zeta^{\{\alpha_1\alpha_2\}\alpha_3} \oplus \zeta^{\{\alpha_1\alpha_3\}\alpha_2} \oplus \zeta^{[\alpha_1\alpha_2\alpha_3]}$$

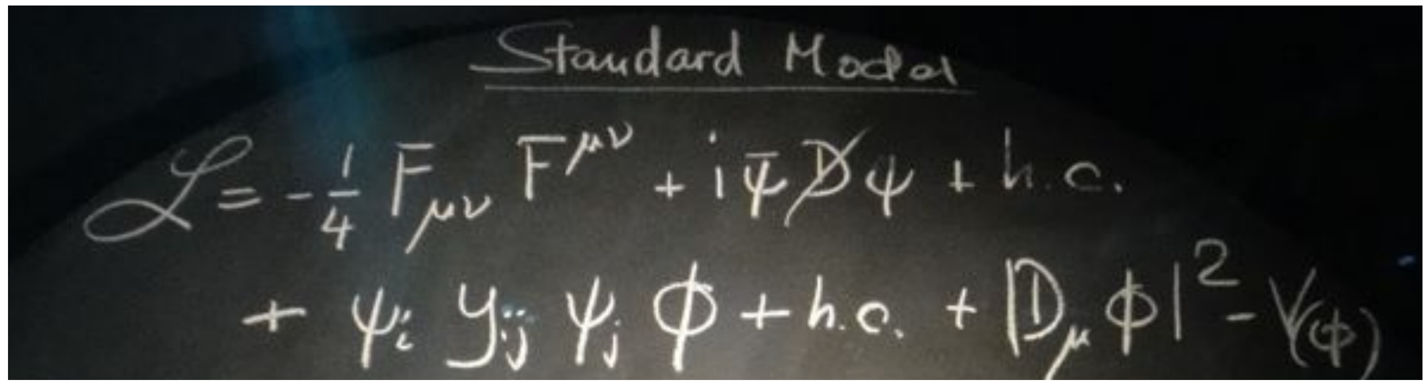
1 x 3 = 3 coupling parameters?

Answer: 19 x 3 = 57 parameters!!!

It is not enough by keeping only S and A symmetries for repeated field!

Operator with Derivatives

New types of operators beyond SM Lagrangian: operator with more derivatives or more fields than just kinetic term in SM



Dim-5 & dim-6 operators contain D

	$\psi^2 D^2,\ F\phi D^2$
$\bar\psi^2 D^2,\ \bar F\phi D^2$	$\phi^3 D^2,\ \bar\psi\psi\phi D$

		$F^2 D^2$
	$\bar\psi\psi D^3,\ \bar F F D^2,\ \phi^2 D^4$	$F\bar\psi\psi D,\ \psi^2\phi D^2,\ F\phi^2 D^2$
$\bar F^2 D^2$	$\bar F\bar\psi\psi D,\ \bar\psi^2\phi D^2,\ \bar F\phi^2 D^2$	$\bar\psi\psi\phi^2 D,\ \bar\psi^2\psi^2,\ \phi^4 D^2$

Which fields Ds should act to?

$[\bar\psi\psi\, D_\mu D^\mu\varphi,\ \ \varphi\bar\psi D_\mu D^\mu\psi,\ \ (D_\mu\varphi)\bar\psi\sigma^{\mu\nu}D_\nu\psi\ \text{ and }\ (D^\mu\varphi)\bar\psi D_\mu\psi]\qquad \bar\psi\sigma^{\mu\nu}\psi\, D_\mu D_\nu\varphi\qquad \varphi\bar\psi\sigma^{\mu\nu}D_\mu D_\nu\psi$

$X^{\mu\nu}\bar\psi\gamma_\mu D_\nu\psi\qquad \bar\psi\gamma_\rho\gamma_\mu\gamma_\nu\psi\, D^\rho X^{\mu\nu}\qquad \bar\psi\gamma^\mu\psi\, D^\rho X_{\rho\mu}$

$(\varphi^\dagger D^\mu\varphi)^\star\,(\varphi^\dagger D_\mu\varphi)\qquad (\varphi^\dagger\varphi)\,[(D_\mu\varphi)^\dagger(D^\mu\varphi)]\qquad (\varphi^\dagger\varphi)\Box(\varphi^\dagger\varphi)$

$(\varphi^\dagger\tau^I\varphi)\,[(D_\mu\varphi)^\dagger\tau^I(D^\mu\varphi)]$

Covariant Derivative

SMEFT should be gauge invariant: all derivatives should be covariant derivatives

$$D_\mu = \partial_\mu - i g_s \frac{1}{2}\lambda^A G^A_\mu - i g \frac{1}{2} T^I W^I - i g' Y B$$

$$\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi \equiv i\varphi^\dagger \left(D_\mu - \overleftarrow{D}_\mu\right)\varphi \quad \text{and} \quad \varphi^\dagger i \overleftrightarrow{D}^I_\mu \varphi \equiv i\varphi^\dagger \left(\tau^I D_\mu - \overleftarrow{D}_\mu \tau^I\right)\varphi. \qquad \varphi^\dagger \overleftarrow{D}_\mu \varphi \equiv (D_\mu\varphi)^\dagger \varphi$$

Gauge field should be gauge field tensor:

$$X_{\mu\nu} \in \{G^A_{\mu\nu}, W^I_{\mu\nu}, B_{\mu\nu}\} \qquad \widetilde{X}_{\mu\nu} = \tfrac{1}{2}\varepsilon_{\mu\nu\rho\sigma} X^{\rho\sigma}$$

$$\begin{aligned} G^A_{\mu\nu} &= \partial_\mu G^A_\nu - \partial_\nu G^A_\mu - g_s f^{ABC} G^B_\mu G^C_\nu, & (D_\rho G_{\mu\nu})^A &= \partial_\rho G^A_{\mu\nu} - g_s f^{ABC} G^B_\rho G^C_{\mu\nu}, \\ W^I_{\mu\nu} &= \partial_\mu W^I_\nu - \partial_\nu W^I_\mu - g\varepsilon^{IJK} W^J_\mu W^K_\nu, & (D_\rho W_{\mu\nu})^I &= \partial_\rho W^I_{\mu\nu} - g\varepsilon^{IJK} W^J_\rho W^K_{\mu\nu}, \\ B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu, & D_\rho B_{\mu\nu} &= \partial_\rho B_{\mu\nu}. \end{aligned}$$

Bianchi identity (BI):

$$D_{[\rho} X_{\mu\nu]} = 0 \qquad D^\rho D_\rho X_{\mu\nu} = -\left(D^\rho D_\mu X_{\nu\rho} + D^\rho D_\nu X_{\rho\mu}\right)$$

Covariant Derivative Commutator (CDC):

$$[D_\rho, D_\alpha] \sim X_{\rho\alpha} \qquad \bar\psi\sigma^{\mu\nu}\psi\, D_\mu D_\nu \varphi \quad \varphi\bar\psi\sigma^{\mu\nu} D_\mu D_\nu \psi \longrightarrow \psi^2 X \varphi$$

Field Redefinition

$$Z[j_i] = \int \prod_i \mathcal{D}\varphi_i \exp\left(i \int d^4x \left[\mathcal{L}_0 + \eta\mathcal{L}_1 + \sum_i j_i\varphi_i + \mathcal{O}(\eta^2)\right]\right)$$

$$\phi^\dagger = (\phi')^\dagger + \eta T[\varphi]$$

$T[\varphi]$ is any local function of any of the fields φ

$$Z[j_i] = \int \prod_i \mathcal{D}\varphi'_i \left|\frac{\delta\phi^\dagger}{\delta(\phi')^\dagger}\right| \exp\left(i \int d^4x \left[\mathcal{L}'_0 + \delta\mathcal{L}'_0 + \eta\mathcal{L}'_1 + \eta\delta\mathcal{L}'_1 + \sum_i j_i\varphi_i + j_{\phi^\dagger}\eta T + \mathcal{O}(\eta^2)\right]\right)$$

$$\mathcal{L}'_i \equiv \mathcal{L}_i\left((\phi')^\dagger, \partial_\mu(\phi')^\dagger\right) \qquad \delta\phi^\dagger \equiv \phi^\dagger - (\phi')^\dagger = \eta T[\varphi]$$

$$\delta\mathcal{L}'_i \equiv \frac{\delta\mathcal{L}'_i}{\delta(\phi')^\dagger}\delta\phi^\dagger - \frac{\delta\mathcal{L}'_i}{\delta\partial_\mu(\phi')^\dagger}\delta\partial_\mu\phi^\dagger = \left(\frac{\delta\mathcal{L}'_i}{\delta(\phi')^\dagger} - \partial_\mu\frac{\delta\mathcal{L}'_i}{\delta\partial_\mu(\phi')^\dagger}\right)\delta\phi^\dagger = \left(\frac{\delta\mathcal{L}'_i}{\delta(\phi')^\dagger} - \partial_\mu\frac{\delta\mathcal{L}'_i}{\delta\partial_\mu(\phi')^\dagger}\right)\eta T[\varphi]$$

$$Z[j_i] = \int \prod_i \mathcal{D}\varphi'_i \left|\frac{\delta\phi^\dagger}{\delta(\phi')^\dagger}\right| \exp\left(i \int d^4x \left[\mathcal{L}'_0 + \left(\frac{\delta\mathcal{L}'_0}{\delta(\phi')^\dagger} - \partial_\mu\frac{\delta\mathcal{L}'_0}{\delta\partial_\mu(\phi')^\dagger}\right)\eta T[\varphi] + \eta\mathcal{L}'_1 + \sum_i j_i\varphi_i + j_{\phi^\dagger}\eta T + \mathcal{O}(\eta^2)\right]\right).$$

the source term and the Jacobian can be neglected [hep-ph/9304230].

Gaussian theorem on action

Equation of Motion (EOM)

Two equivalent operators related by EOM

Integration by part (IBP) $\partial_\mu \mathcal{O}^\mu$

Total derivatives are removed

EOM and IBP

SM Equations of motion:

$$(D^\mu D_\mu \varphi)^j = m^2 \varphi^j - \lambda \left(\varphi^\dagger \varphi\right) \varphi^j - \bar{e}\,\Gamma_e^\dagger l^j + \varepsilon_{jk} \bar{q}^k \,\Gamma_u u - \bar{d}\,\Gamma_d^\dagger q^j$$

$$i\not{D} l = \Gamma_e e \varphi, \qquad i\not{D} e = \Gamma_e^\dagger \varphi^\dagger l, \qquad i\not{D} q = \Gamma_u u \widetilde{\varphi} + \Gamma_d d \varphi, \qquad i\not{D} u = \Gamma_u^\dagger \widetilde{\varphi}^\dagger q, \qquad i\not{D} d = \Gamma_d^\dagger \varphi^\dagger q.$$

$$\begin{aligned} (D^\rho G_{\rho\mu})^A &= g_s \left(\bar{q}\gamma_\mu T^A q + \bar{u}\gamma_\mu T^A u + \bar{d}\gamma_\mu T^A d\right), \\ (D^\rho W_{\rho\mu})^I &= \frac{g}{2}\left(\varphi^\dagger i \overleftrightarrow{D}^I_\mu \varphi + \bar{l}\gamma_\mu \tau^I l + \bar{q}\gamma_\mu \tau^I q\right), \end{aligned}$$

$$\partial^\rho B_{\rho\mu} = g' Y_\varphi \, \varphi^\dagger i \overleftrightarrow{D}_\mu \varphi + g' \sum_{\psi \in \{l,e,q,u,d\}} Y_\psi \, \bar{\psi}\gamma_\mu \psi.$$

Total derivative examples:

$$\begin{aligned} (D_\mu \varphi)^\dagger \varphi + \varphi^\dagger D_\mu \varphi &= (\partial_\mu \varphi^\dagger)\varphi + i(A_\mu \varphi)^\dagger \varphi + \varphi^\dagger \partial_\mu \varphi - i\varphi^\dagger A_\mu \varphi \\ &= (\partial_\mu \varphi^\dagger)\varphi + \varphi^\dagger \partial_\mu \varphi = \partial_\mu (\varphi^\dagger \varphi) \end{aligned}$$

$$(D^n \varphi)^\dagger (D^m \varphi) = -(D^{n+1}\varphi)^\dagger (D^{m-1}\varphi) + \partial \left[(D^n \varphi)^\dagger (D^{m-1}\varphi)\right]$$

EOM and CDC

Dim-5 Operators:

$N-h$ \ $N+h$	1	3	5
1		$\psi^2 D^2,\ F\phi D^2$	$F^2\phi,\ F\psi^2$
3	$\bar\psi^2 D^2,\ \bar F\phi D^2$	$\phi^3 D^2,\ \bar\psi\psi\phi D$	$\psi^2\phi^2$
5	$\bar F^2\phi,\ \bar F\bar\psi^2$	$\bar\psi^2\phi^2$	ϕ^5

Odd power of scalar, and SU(2)L transformation $\bar\psi_L\sigma^{\mu\nu}\psi_R$

Dim-6 Operators:

$N-h$ \ $N+h$	0	2	4	6
0			$F^2 D^2$	F^3
2		$\bar\psi\psi D^3,\ \bar F F D^2,\ \phi^2 D^4$	$F\bar\psi\psi D,\ \psi^2\phi D^2,\ F\phi^2 D^2$	$F^2\phi^2,\ F\psi^2\phi,\ \psi^4$
4	$\bar F^2 D^2$	$\bar F\bar\psi\psi D,\ \bar\psi^2\phi D^2,\ \bar F\phi^2 D^2$	$\bar\psi\psi\phi^2 D,\ \bar\psi^2\psi^2,\ \phi^4 D^2$	$\psi^2\phi^3$
6	$\bar F^3$	$\bar F^2\phi^2,\ \bar F\bar\psi^2\phi,\ \bar\psi^4$	$\bar\psi^2\phi^3$	ϕ^6

SMEFT Warsaw Basis

Grzadkowski, Iskrzynski, Misiak, Rosiek 2010

X^3		φ^6 and $\varphi^4 D^2$		$\psi^2\varphi^3$	
Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	Q_φ	$(\varphi^\dagger\varphi)^3$	$Q_{e\varphi}$	$(\varphi^\dagger\varphi)(\bar l_p e_r \varphi)$
$Q_{\widetilde G}$	$f^{ABC} \widetilde G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$Q_{\varphi\Box}$	$(\varphi^\dagger\varphi)\Box(\varphi^\dagger\varphi)$	$Q_{u\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p u_r \widetilde\varphi)$
Q_W	$\varepsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$Q_{\varphi D}$	$\left(\varphi^\dagger D^\mu\varphi\right)^\star \left(\varphi^\dagger D_\mu\varphi\right)$	$Q_{d\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p d_r \varphi)$
$Q_{\widetilde W}$	$\varepsilon^{IJK} \widetilde W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$				
$X^2\varphi^2$		$\psi^2 X\varphi$		$\psi^2\varphi^2 D$	
$Q_{\varphi G}$	$\varphi^\dagger\varphi\, G_{\mu\nu}^A G^{A\mu\nu}$	Q_{eW}	$(\bar l_p \sigma^{\mu\nu} e_r)\tau^I \varphi W_{\mu\nu}^I$	$Q_{\varphi l}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar l_p \gamma^\mu l_r)$
$Q_{\varphi\widetilde G}$	$\varphi^\dagger\varphi\, \widetilde G_{\mu\nu}^A G^{A\mu\nu}$	Q_{eB}	$(\bar l_p \sigma^{\mu\nu} e_r)\varphi B_{\mu\nu}$	$Q_{\varphi l}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu^I \varphi)(\bar l_p \tau^I \gamma^\mu l_r)$
$Q_{\varphi W}$	$\varphi^\dagger\varphi\, W_{\mu\nu}^I W^{I\mu\nu}$	Q_{uG}	$(\bar q_p \sigma^{\mu\nu} T^A u_r)\widetilde\varphi\, G_{\mu\nu}^A$	$Q_{\varphi e}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar e_p \gamma^\mu e_r)$
$Q_{\varphi\widetilde W}$	$\varphi^\dagger\varphi\, \widetilde W_{\mu\nu}^I W^{I\mu\nu}$	Q_{uW}	$(\bar q_p \sigma^{\mu\nu} u_r)\tau^I \widetilde\varphi\, W_{\mu\nu}^I$	$Q_{\varphi q}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar q_p \gamma^\mu q_r)$
$Q_{\varphi B}$	$\varphi^\dagger\varphi\, B_{\mu\nu} B^{\mu\nu}$	Q_{uB}	$(\bar q_p \sigma^{\mu\nu} u_r)\widetilde\varphi\, B_{\mu\nu}$	$Q_{\varphi q}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu^I \varphi)(\bar q_p \tau^I \gamma^\mu q_r)$
$Q_{\varphi\widetilde B}$	$\varphi^\dagger\varphi\, \widetilde B_{\mu\nu} B^{\mu\nu}$	Q_{dG}	$(\bar q_p \sigma^{\mu\nu} T^A d_r)\varphi\, G_{\mu\nu}^A$	$Q_{\varphi u}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar u_p \gamma^\mu u_r)$
$Q_{\varphi WB}$	$\varphi^\dagger\tau^I\varphi\, W_{\mu\nu}^I B^{\mu\nu}$	Q_{dW}	$(\bar q_p \sigma^{\mu\nu} d_r)\tau^I \varphi\, W_{\mu\nu}^I$	$Q_{\varphi d}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar d_p \gamma^\mu d_r)$
$Q_{\varphi\widetilde WB}$	$\varphi^\dagger\tau^I\varphi\, \widetilde W_{\mu\nu}^I B^{\mu\nu}$	Q_{dB}	$(\bar q_p \sigma^{\mu\nu} d_r)\varphi\, B_{\mu\nu}$	$Q_{\varphi ud}$	$i(\widetilde\varphi^\dagger D_\mu\varphi)(\bar u_p \gamma^\mu d_r)$

SMEFT Warsaw Basis

Grzadkowski, Iskrzynski, Misiak, Rosiek 2010

$(\bar{L}L)(\bar{L}L)$		$(\bar{R}R)(\bar{R}R)$		$(\bar{L}L)(\bar{R}R)$	
Q_{ll}	$(\bar{l}_p\gamma_\mu l_r)(\bar{l}_s\gamma^\mu l_t)$	Q_{ee}	$(\bar{e}_p\gamma_\mu e_r)(\bar{e}_s\gamma^\mu e_t)$	Q_{le}	$(\bar{l}_p\gamma_\mu l_r)(\bar{e}_s\gamma^\mu e_t)$
$Q_{qq}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{q}_s\gamma^\mu q_t)$	Q_{uu}	$(\bar{u}_p\gamma_\mu u_r)(\bar{u}_s\gamma^\mu u_t)$	Q_{lu}	$(\bar{l}_p\gamma_\mu l_r)(\bar{u}_s\gamma^\mu u_t)$
$Q_{qq}^{(3)}$	$(\bar{q}_p\gamma_\mu\tau^I q_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$	Q_{dd}	$(\bar{d}_p\gamma_\mu d_r)(\bar{d}_s\gamma^\mu d_t)$	Q_{ld}	$(\bar{l}_p\gamma_\mu l_r)(\bar{d}_s\gamma^\mu d_t)$
$Q_{lq}^{(1)}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{q}_s\gamma^\mu q_t)$	Q_{eu}	$(\bar{e}_p\gamma_\mu e_r)(\bar{u}_s\gamma^\mu u_t)$	Q_{qe}	$(\bar{q}_p\gamma_\mu q_r)(\bar{e}_s\gamma^\mu e_t)$
$Q_{lq}^{(3)}$	$(\bar{l}_p\gamma_\mu\tau^I l_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$	Q_{ed}	$(\bar{e}_p\gamma_\mu e_r)(\bar{d}_s\gamma^\mu d_t)$	$Q_{qu}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{u}_s\gamma^\mu u_t)$
		$Q_{ud}^{(1)}$	$(\bar{u}_p\gamma_\mu u_r)(\bar{d}_s\gamma^\mu d_t)$	$Q_{qu}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{u}_s\gamma^\mu T^A u_t)$
		$Q_{ud}^{(8)}$	$(\bar{u}_p\gamma_\mu T^A u_r)(\bar{d}_s\gamma^\mu T^A d_t)$	$Q_{qd}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{d}_s\gamma^\mu d_t)$
				$Q_{qd}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{d}_s\gamma^\mu T^A d_t)$

$(\bar{L}R)(\bar{R}L)$ and $(\bar{L}R)(\bar{L}R)$		B-violating	
Q_{ledq}	$(\bar{l}_p^j e_r)(\bar{d}_s q_t^j)$	Q_{duq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(q_s^{\gamma j})^T C l_t^k\right]$
$Q_{quqd}^{(1)}$	$(\bar{q}_p^j u_r)\varepsilon_{jk}(\bar{q}_s^k d_t)$	Q_{qqu}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{quqd}^{(8)}$	$(\bar{q}_p^j T^A u_r)\varepsilon_{jk}(\bar{q}_s^k T^A d_t)$	Q_{qqq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jn}\varepsilon_{km}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$
$Q_{lequ}^{(1)}$	$(\bar{l}_p^j e_r)\varepsilon_{jk}(\bar{q}_s^k u_t)$	Q_{duu}	$\varepsilon^{\alpha\beta\gamma}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{lequ}^{(3)}$	$(\bar{l}_p^j \sigma_{\mu\nu} e_r)\varepsilon_{jk}(\bar{q}_s^k \sigma^{\mu\nu} u_t)$		

Dim-6 SMEFT

Using an effective lagrangian technique we have studied systematically deviations from the standard model which could be the low-energy manifestations of $SU(3)\times SU(2)\times U(1)$ invariant new interactions with scales Λ in the TeV range. The effective lagrangian contains 1 dimension-five operator, which violates lepton number, and 80 baryon- and lepton-number conserving dimension-six operators.

Buchmuller, Wyler 1986

16 EOM $X^{\mu\nu}\bar{\psi}\gamma_\mu D_\nu\psi$ $(D^\mu\varphi)\bar{\psi}D_\mu\psi$

5 Fierz

When the Standard Model is considered as an effective low-energy theory, higher dimensional interaction terms appear in the Lagrangian. Dimension-six terms have been enumerated in the classical article by Buchmueller and Wyler [3]. Although redundance of some of those operators has been already noted in the literature, no updated complete list has been published to date. Here we perform their classification once again from the outset. Assuming baryon number conservation, we find 15 + 19 + 25 = 59 independent operators (barring flavour structure and Hermitian conjugations), as compared to 16 + 35 + 29 = 80 in Ref.[3]. The three summed numbers refer to operators containing 0, 2 and 4 fermion fields. If the assumption of baryon number conservation is relaxed, 5 new operators arise in the four-fermion sector.

Grzadkowski, Iskrzynski, Misiak, Rosiek 2010

QQQL

When the Standard Model is considered as an effective low-energy theory, higher dimensional interaction terms appear in the Lagrangian. Dimension-six terms have been enumerated in the classical article by Buchmueller and Wyler [3]. Although redundance of some of those operators has been already noted in the literature, no updated complete list has been published to date. Here we perform their classification once again from the outset. Assuming baryon number conservation, we find 15 + 19 + 25 = 59 independent operators (barring flavour structure and Hermitian conjugations), as compared to 16 + 35 + 29 = 80 in Ref.[3]. The three summed numbers refer to operators containing 0, 2 and 4 fermion fields. If the assumption of baryon number conservation is relaxed, 4 new operators arise in the four-fermion sector.

V3 2017

The leading order BNV operators arise at dimension $d = 6$. The most general dimension-six Lagrangian can be cast in 63 independent operators [7–11]. Out of these 63 operators, 59 operators preserve baryon number, and

Alonso, Chang, Jenkins, Manohar, Shotwell 2014

Higher Dim Operators?

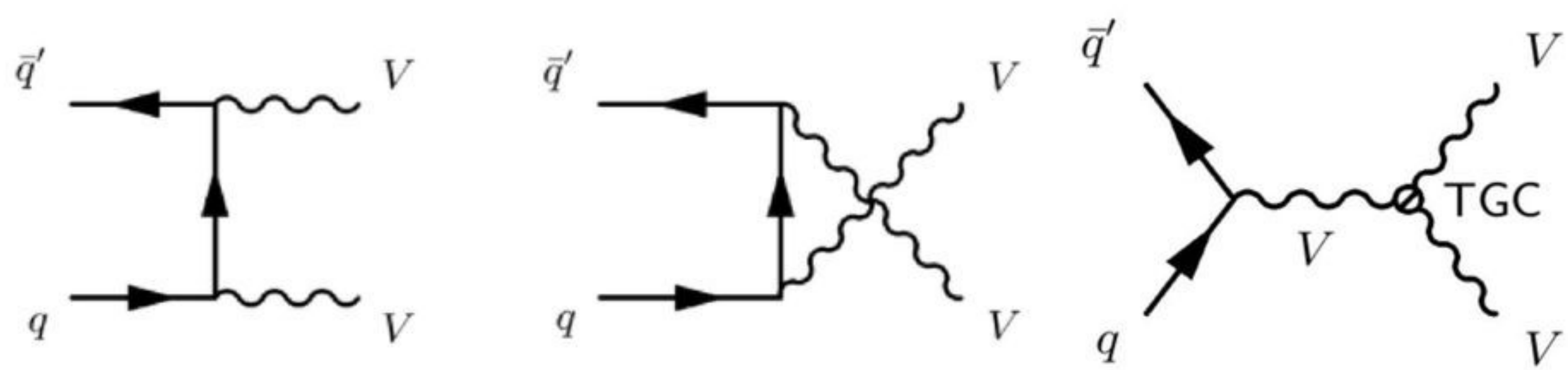
Expect much smaller effects for dim > 7

$$|\mathcal{A}|^2 \sim \left| A_{\rm SM} + \frac{A_{\rm dim-6}}{\Lambda^2} + \frac{A_{\rm dim-8}}{\Lambda^4} + \ldots \right|^2$$

$$\sim \boxed{|A_{\rm SM}|^2 + \frac{2}{\Lambda^2} A_{\rm dim-6} A^*_{\rm SM}} + \frac{1}{\Lambda^4} |A_{\rm dim-6}|^2 + \frac{2}{\Lambda^4} A_{\rm dim-8} A^*_{\rm SM}$$

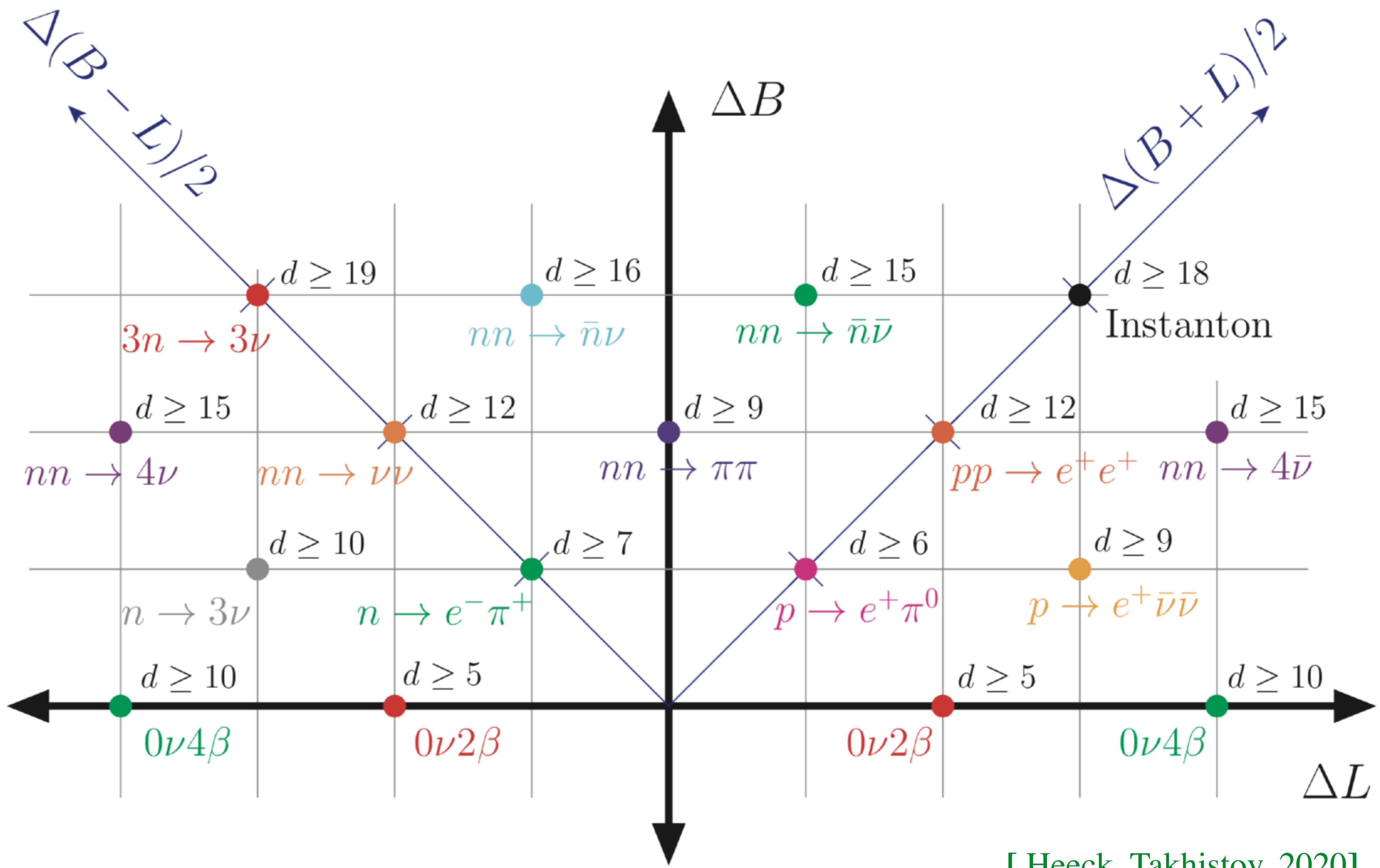
HL-LHC starts to probe dim-8

Leading operators at dim-8



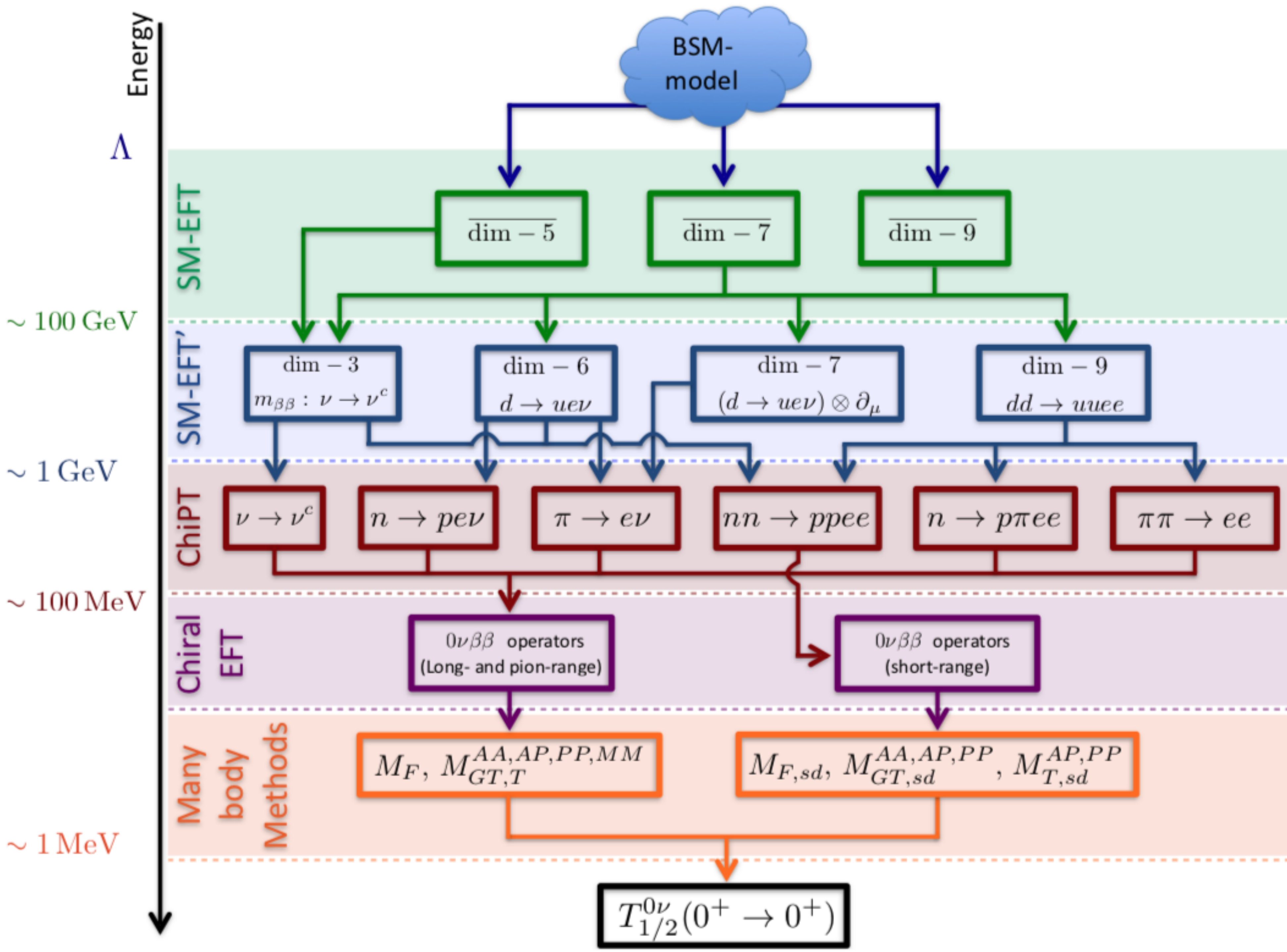
Neutral triple gauge boson ZZZ, ZZA, ZAA couplings

Higher Dim Operators



[Heeck, Takhistov, 2020]

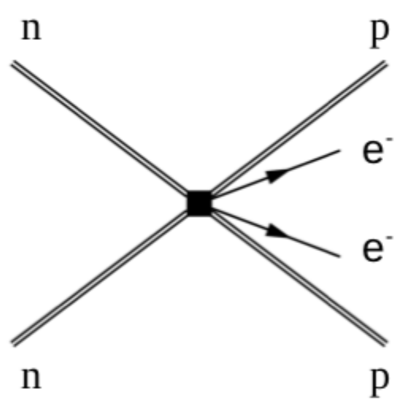
Higher Dim Operators

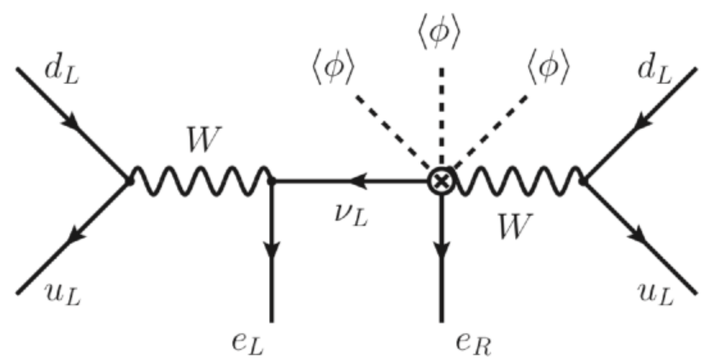


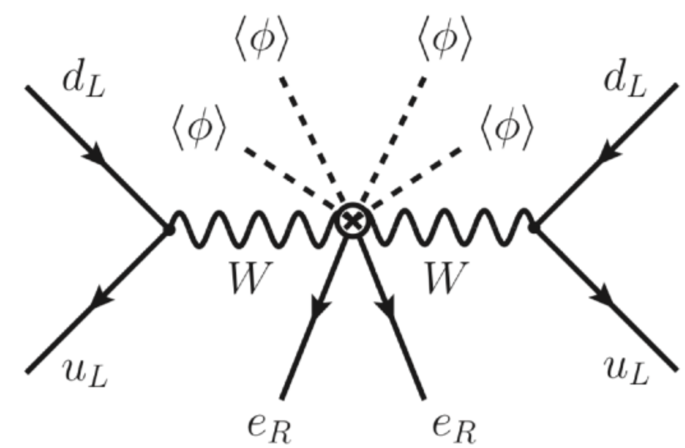
[Cirigliano, Dekens, de Vries, Graesser, **2018**]

Partial Set of EFT Operator

Can we only write down partial set of operators?







Operators convert to each other, and to other types!

Hard to pick up unless complete set is known.

Two Difficulties

Difficult to write down!

Repeated fields

Repeatedly use EOM, IBP, CDC

Derivatives

Symmetric group and SU(nf)

More symmetries?

Real Scalar EFT

Real scalar Lagrangian: ϕ^n

$$\partial^{2n}\phi^2,\ \partial^{2n}\phi^3,\ \partial^{2n}\phi^4,\ \cdots$$

How many independent operators of the form $\partial^{2n}\phi^4$?

$2n$	0	2	4	6	8	10	12	14	16
# independent $\partial^{2n}\phi^4$ operators	1	0	1	1	1	1	2	1	2

1

$s+t+u=0$

$s^2+t^2+u^2$

$s^3+t^3+u^3 \sim stu$

$(s^2+t^2+u^2)^2$

$stu\,(s^2+t^2+u^2)$

$(s^2+t^2+u^2)^3$ & $(stu)^2$

$stu\,(s^2+t^2+u^2)^2$

$(s^2+t^2+u^2)^4$ & $(stu)^2(s^2+t^2+u^2)$

Magic things happen?!

[Elvang 2020]

Spacetime Symmetry

SO(N) Group

$$
\begin{aligned}
[X_{\mu\nu}, X_{\alpha\beta}] &= X_{\mu\nu}X_{\alpha\beta} - X_{\alpha\beta}X_{\mu\nu} \qquad X_{\mu\nu} = -X_{\nu\mu}, \quad \mu,\nu = 1,2,\cdots,N \\
&= i\left(\delta_{\mu\alpha}X_{\nu\beta} + \delta_{\nu\beta}X_{\mu\alpha} - \delta_{\mu\beta}X_{\nu\alpha} - \delta_{\nu\alpha}X_{\mu\beta}\right).
\end{aligned}
$$

SO(n)

$$J_{j,k} = \frac{\hbar}{i}\left(x_j\frac{\partial}{\partial x_k} - x_k\frac{\partial}{\partial x_j}\right)$$

$$[J_{ij}, J_{k\ell}] = i\hbar\left(\delta_{ik}J_{j\ell} + \delta_{j\ell}J_{ik} - \delta_{i\ell}J_{jk} - \delta_{jk}J_{i\ell}\right)$$

$$[J_x, J_y] = iJ_z, [J_y, J_z] = iJ_x, [J_z, J_x] = iJ_y.$$

tensor rep

$$R(\alpha,\beta,\gamma) = \begin{pmatrix} \cos\alpha\cos\beta\cos\gamma - \sin\alpha\sin\gamma \\ -\cos\beta\cos\gamma\sin\alpha - \cos\alpha\sin\gamma \\ -\cos\gamma\sin\beta \end{pmatrix}$$

$$\mathrm{SU}(2)/Z_2,\ Z_2 = \{-1, 1\}$$

covering group

$$U(\alpha,\beta,\gamma) = \begin{pmatrix} e^{i(\alpha+\gamma)/2}\cos\beta/2 & e^{i(\alpha-\gamma)/2}\sin\beta/2 \\ -e^{-i(\alpha-\gamma)/2}\sin\beta/2 & e^{-i(\alpha+\gamma)/2}\cos\beta/2 \end{pmatrix} = \begin{pmatrix} a & b \\ -b^* & a^* \end{pmatrix}$$

SO(n-1,1)

$$L_{j,k} = \frac{\hbar}{i}\left(x_j\frac{\partial}{\partial x_k} - x_k\frac{\partial}{\partial x_j}\right), \quad L_{0,j} = \frac{\hbar}{i}\left(x_0\frac{\partial}{\partial x_j} + x_j\frac{\partial}{\partial x_0}\right)$$

$$
\begin{aligned}
&[L_{0,j}, L_{0,k}] = -i\hbar L_{j,k}, \quad [L_{0,j}, L_{k,n}] = i\hbar\left(\delta_{jn}L_{0,k} - \delta_{jk}L_{0,n}\right), \\
&[L_{k,j}, L_{m,n}] = i\hbar\left(\delta_{km}L_{j,n} + \delta_{nj}L_{k,m} - \delta_{kn}L_{j,m} - \delta_{jm}L_{k,n}\right).
\end{aligned}
$$

SO(3,1)

$$
\begin{aligned}
(M_{12}, M_{23}, M_{31}) &= (J_3, J_1, J_2)\ , \qquad \tfrac{1}{2}M^{\mu\nu}M_{\mu\nu} = \mathbf{J}^2 - \mathbf{K}^2\ , \\
(M_{01}, M_{02}, M_{03}) &= (K_1, K_2, K_3)\ , \qquad \tfrac{1}{2}\epsilon^{\mu\nu\sigma\tau}M_{\mu\nu}M_{\sigma\tau} = -\mathbf{J}\cdot\mathbf{K}\ ,
\end{aligned}
$$

$$[M_{\lambda\rho}, M_{\mu\nu}] = -i(g_{\lambda\mu}M_{\rho\nu} + g_{\rho\nu}M_{\lambda\mu} - g_{\lambda\nu}M_{\rho\mu} - g_{\rho\mu}M_{\lambda\nu})$$

$$
\begin{aligned}
M_i &= \tfrac{1}{2}(J_i + iK_i)\ , \\
N_i &= \tfrac{1}{2}(J_i - iK_i)\ ,
\end{aligned}
$$

$$
\begin{aligned}
[M_i, M_j] &= i\epsilon_{ijk}M_k\ , \\
[N_i, N_j] &= i\epsilon_{ijk}N_k\ , \\
[M_i, N_j] &= 0\ .
\end{aligned}
$$

SO(3,1) Irreps

SO(3,1) not simply connected, find irrep by its covering group SL(2,C)

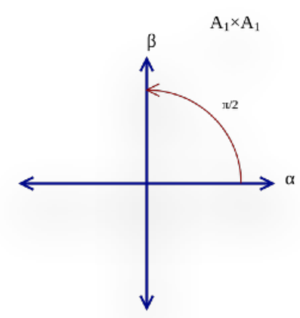
$$x^\mu = (x^0, x^1, x^2, x^3)$$

complexification

$$X = \sigma_\mu x^\mu = \begin{pmatrix} x^0 + x^3 & x^1 - ix^2 \\ x^1 + ix^2 & x^0 - x^3 \end{pmatrix}$$

$$x^\mu = \frac{1}{2}\mathrm{Tr}(\underline{\sigma}^\mu X)$$

Spinor rep under SU(2) x SU(2)



The $(\frac{1}{2}, 0)$ Representation

$$N_i^+ = \frac{1}{2}(J_i + iK_i) = iK_i = \frac{1}{2}\sigma_i,$$

$$N_i^- = \frac{1}{2}(J_i - iK_i) = 0$$

$$K_i = -i\frac{1}{2}\sigma_i, \qquad J_i = iK_i = \frac{1}{2}\sigma_i.$$

Complex conjugate

The $(0, \frac{1}{2})$ Representation

$$N_i^+ = \frac{1}{2}(J_i + iK_i) = 0$$

$$N_i^- = \frac{1}{2}(J_i - iK_i) = -iK_i = \frac{1}{2}\sigma_i,$$

$$J_i = \frac{1}{2}\sigma_i, \qquad K_i = i\frac{1}{2}\sigma_i.$$

The $(\frac{1}{2}, \frac{1}{2})$ Representation

$$v^{\dot{a}b} \longrightarrow v'^{\dot{c}d} = \left(e^{\frac{i}{2}\boldsymbol{\theta}\cdot\boldsymbol{\sigma} - \frac{1}{2}\boldsymbol{\phi}\cdot\boldsymbol{\sigma}}\right)^{\dot{c}}_{\dot{a}} \left(e^{\frac{i}{2}\boldsymbol{\theta}\cdot\boldsymbol{\sigma} + \frac{1}{2}\boldsymbol{\phi}\cdot\boldsymbol{\sigma}}\right)^{d}_{b} v^{\dot{a}b}$$

$$v^{\dot{a}b} = v^\mu \sigma_\mu^{\dot{a}b} = \begin{pmatrix} v^0 & v^1 & v^2 & v^3 \end{pmatrix} \begin{pmatrix} \sigma_0 \\ \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{pmatrix}$$

tensor rep

$$\begin{pmatrix} v^0 & v^1 & v^2 & v^3 \end{pmatrix} = v^\mu$$

Jiang-Hao Yu

SM Fields Under SL(2,C)

$\psi_\alpha \in (1/2, 0), \quad \psi^\dagger_{\dot\alpha} \in (0, 1/2), \qquad H_i \in (0,0)\,, \quad H^{\dagger i} \in (0,0)\,,$

$F_{\mathrm{L}\alpha\beta} = \frac{i}{2} F_{\mu\nu}\sigma^{\mu\nu}_{\alpha\beta} \in (1,0), \quad F_{\mathrm{R}\dot\alpha\dot\beta} = -\frac{i}{2} F_{\mu\nu}\bar\sigma^{\mu\nu}_{\dot\alpha\dot\beta} \in (0,1). \qquad X^{\mu\nu}_{L,R} = \frac{1}{2}(X^{\mu\nu} \mp i\widetilde{X}^{\mu\nu})$

$h = j_r - j_l$

Fields	$SU(2)_l \times SU(2)_r$	h	$SU(3)_C$	$SU(2)_W$	$U(1)_Y$	Flavor
$G^A_{\mathrm{L}\alpha\beta}$	$(1,0)$	-1	$\mathbf{8}$	$\mathbf{1}$	0	1
$W^I_{\mathrm{L}\alpha\beta}$	$(1,0)$	-1	$\mathbf{1}$	$\mathbf{3}$	0	1
$B_{\mathrm{L}\alpha\beta}$	$(1,0)$	-1	$\mathbf{1}$	$\mathbf{1}$	0	1
$L_{\alpha i}$	$(\frac{1}{2},0)$	$-1/2$	$\mathbf{1}$	$\mathbf{2}$	$-1/2$	n_f
$e_{\mathrm{c}\alpha}$	$(\frac{1}{2},0)$	$-1/2$	$\mathbf{1}$	$\mathbf{1}$	1	n_f
$Q_{\alpha a i}$	$(\frac{1}{2},0)$	$-1/2$	$\mathbf{3}$	$\mathbf{2}$	$1/6$	n_f
$u^a_{\mathrm{c}\alpha}$	$(\frac{1}{2},0)$	$-1/2$	$\overline{\mathbf{3}}$	$\mathbf{1}$	$-2/3$	n_f
$d^a_{\mathrm{c}\alpha}$	$(\frac{1}{2},0)$	$-1/2$	$\overline{\mathbf{3}}$	$\mathbf{1}$	$1/3$	n_f
H_i	$(0,0)$	0	$\mathbf{1}$	$\mathbf{2}$	$1/2$	1

Hermitian conjugate $\qquad (F_{\mathrm{L}\alpha\beta})^\dagger = F_{\mathrm{R}\dot\alpha\dot\beta}$

$H^\dagger$ (and $L^\dagger, Q^\dagger$) as a 2 of $SU(2)$ $\qquad \epsilon^{ij} H^\dagger_i H_j. \quad H^\dagger_{\bar 2} = \epsilon H^\dagger_2 \qquad e_R = e^\dagger_C, u_R = u^\dagger_C, d_R = d^\dagger_C$

$\tilde{H} = \epsilon H^\dagger$

SO(3,1) Spinor Rep

Clifford algebra $\{\Gamma_i, \Gamma_j\} = 2\delta_{ij}, \quad i = 1, \cdots, N$

$\Gamma^{(f)}(i_1, \cdots, i_f) \equiv (-)^{\frac{f(f-1)}{2}} \Gamma_{i_1} \cdots \Gamma_{i_f}$

$$\gamma_\mu \to S^{-1}(a)\gamma_\mu S(a) = \sum_{\nu=1}^{N} a_{\mu\nu}\gamma_\nu$$

S(a) rep of SO(N)

SO(2n)

SO(2n+1)

$$\gamma_\mu = \begin{pmatrix} 0 & \sigma_\mu \\ \sigma_\mu^\dagger & 0 \end{pmatrix} \quad S(a) = \begin{pmatrix} S_1(a) & 0 \\ 0 & S_2(a) \end{pmatrix}$$

Reducible rep of SO(N)

$$S_1^{-1}(a)\sigma_\mu S_1(a) = \sum_{\nu=1}^{N} a_{\mu\nu}\sigma_\nu, \quad S_2^{-1}(a)\sigma_\mu^\dagger S_2(a) = \sum_{\nu=1}^{N} a_{\mu\nu}\sigma_\nu^\dagger,$$

$$J_{\mu\nu}^{(+)} = \frac{1}{4}\left(\sigma_\mu\sigma_\nu^\dagger - \sigma_\nu\sigma_\mu^\dagger\right)$$

$$J_{\mu\nu}^{(-)} = \frac{1}{4}\left(\sigma_\mu^\dagger\sigma_\nu - \sigma_\nu^\dagger\sigma_\mu\right)$$

$$[J_{\mu\nu}^{(\pm)}, J_{\alpha\beta}^{(\pm)}] = -\delta_{\mu\alpha}J_{\nu\beta}^{(\pm)} - \delta_{\nu\beta}J_{\mu\alpha}^{(\pm)} + \delta_{\mu\beta}J_{\nu\alpha}^{(\pm)} + \delta_{\nu\alpha}J_{\mu\beta}^{(\pm)}$$

irreducible rep of SO(N)

$$\Gamma_A$$

$$\mathbb{1},\ \gamma_\mu,\ \gamma_\mu\gamma_\nu\ (\mu<\nu),\ \gamma_\mu\gamma_\nu\gamma_\lambda\ (\mu<\nu<\lambda),$$

$$\cdots,\ \gamma_1\gamma_2\gamma_3\cdots\gamma_N.$$

$$\mathrm{Tr}\left[\Gamma_a^{\mathcal{A}}\Gamma_b^{\mathcal{B}}\right] = \delta^{\mathcal{A}\mathcal{B}}g_{ab}, \quad \mathcal{A}, \mathcal{B} = S, V, T, A, P,$$

$$\Lambda = \gamma_1\gamma_2\gamma_3\cdots\gamma_N.$$

$$\gamma_\mu\Lambda = \begin{cases} \Lambda\gamma_\mu, & \text{if} \quad N = 2M+1(\text{odd}), \\ -\Lambda\gamma_\mu, & \text{if} \quad N = 2M(\text{even}) \end{cases}$$

$$[\Lambda, \Gamma_A] = 0$$

For odd N only

Spinor Notation

$$\Psi = \begin{pmatrix} \xi_\alpha \\ \chi^{\dagger\dot{\alpha}} \end{pmatrix}, \quad \bar{\Psi} = \Psi^\dagger \gamma^0 = \left(\chi^\alpha,\ \xi^\dagger_{\dot{\alpha}}\right)$$

$$\begin{aligned}
\bar{\Psi}_1 \Psi_2 =& \chi_1^\alpha \xi_{2\alpha} + \xi^\dagger_{1\dot{\alpha}} \chi_2^{\dagger\dot{\alpha}} , \\
\bar{\Psi}_1 \gamma^\mu \Psi_2 =& \chi_1^\alpha \sigma^\mu_{\alpha\dot{\alpha}} \chi_2^{\dagger\dot{\alpha}} + \xi^\dagger_{1\dot{\alpha}} \bar{\sigma}^{\mu\dot{\alpha}\alpha} \xi_{2\alpha} , \\
\bar{\Psi}_1 \sigma^{\mu\nu} \Psi_2 =& \chi_1^\alpha \left(\sigma^{\mu\nu}\right)_\alpha{}^\beta \xi_{2\beta} + \xi^\dagger_{1\dot{\alpha}} \left(\bar{\sigma}^{\mu\nu}\right)^{\dot{\alpha}}{}_{\dot{\beta}} \chi_2^{\dot{\beta}} , \\
\Psi_1^{\mathrm{T}} C \Psi_2 =& \xi_1^\alpha \xi_{2\alpha} + \chi^\dagger_{1\dot{\alpha}} \chi_2^{\dagger\dot{\alpha}} , \\
\Psi_1^{\mathrm{T}} C \gamma^\mu \Psi_2 =& \xi_1^\alpha \sigma^\mu_{\alpha\dot{\alpha}} \chi_2^{\dagger\dot{\alpha}} + \chi^\dagger_{1\dot{\alpha}} \bar{\sigma}^{\mu\dot{\alpha}\alpha} \xi_{2\alpha} , \\
\Psi_1^{\mathrm{T}} C \sigma^{\mu\nu} \Psi_2 =& \xi_1^\alpha \left(\sigma^{\mu\nu}\right)_\alpha{}^\beta \xi_{2\beta} + \chi^\dagger_{1\dot{\alpha}} \left(\bar{\sigma}^{\mu\nu}\right)^{\dot{\alpha}}{}_{\dot{\beta}} \chi_2^{\dagger\dot{\beta}} , \\
\bar{\Psi}_1 C \bar{\Psi}_2^{\mathrm{T}} =& \xi^\dagger_{1\dot{\alpha}} \xi_2^{\dagger\dot{\alpha}} + \chi_1^\alpha \chi_{2\alpha} , \\
\bar{\Psi}_1 \gamma^\mu C \bar{\Psi}_2^{\mathrm{T}} =& \chi_1^\alpha \sigma^\mu_{\alpha\dot{\alpha}} \xi_2^{\dagger\dot{\alpha}} + \xi^\dagger_{1\dot{\alpha}} \bar{\sigma}^{\mu\dot{\alpha}\alpha} \chi_{2\alpha} , \\
\bar{\Psi}_1 \sigma^{\mu\nu} C \bar{\Psi}_2^{\mathrm{T}} =& \xi^\dagger_{1\dot{\alpha}} \left(\bar{\sigma}^{\mu\nu}\right)^{\dot{\alpha}}{}_{\dot{\beta}} \xi_2^{\dagger\dot{\beta}} + \chi_1^\alpha \left(\sigma^{\mu\nu}\right)_\alpha{}^\beta \chi_{2\beta} .
\end{aligned}$$

$$q_{\mathrm{L}} = \begin{pmatrix} Q \\ 0 \end{pmatrix}, \quad u_{\mathrm{R}} = \begin{pmatrix} 0 \\ u_{\mathbb{C}}^\dagger \end{pmatrix}, \quad d_{\mathrm{R}} = \begin{pmatrix} 0 \\ d_{\mathbb{C}}^\dagger \end{pmatrix}, \quad l_{\mathrm{L}} = \begin{pmatrix} L \\ 0 \end{pmatrix}, \quad e_{\mathrm{R}} = \begin{pmatrix} 0 \\ e_{\mathbb{C}}^\dagger \end{pmatrix}.$$

$$\bar{q}_{\mathrm{L}} = \left(0\,,\ Q^\dagger\right), \quad \bar{u}_{\mathrm{R}} = \left(u_{\mathbb{C}}\,,\ 0\right), \quad \bar{d}_{\mathrm{R}} = \left(d_{\mathbb{C}}\,,\ 0\right), \quad \bar{l}_{\mathrm{L}} = \left(0\,,\ L^\dagger\right), \quad \bar{e}_{\mathrm{R}} = \left(e_{\mathbb{C}}\,,\ 0\right).$$

$$u_{\mathbb{C}} \sigma^\mu u_{\mathbb{C}}^\dagger = \bar{u}\gamma^\mu u, \quad e_{\mathbb{C}} L = \bar{e}\, l, \quad u_{\mathbb{C}}^\dagger d_{\mathbb{C}}^\dagger = u^T C d\,.$$

Fierz and Schouten Identities

$$
\begin{pmatrix} \delta_{ij}\delta_{kl} \\ (\gamma^\mu)_{ij}(\gamma_\mu)_{kl} \\ \frac{1}{2}(\sigma^{\mu\nu})_{ij}(\sigma_{\mu\nu})_{kl} \\ (\gamma^\mu\gamma_5)_{ij}(\gamma_\mu\gamma_5)_{kl} \\ (\gamma_5)_{ij}(\gamma_5)_{kl} \end{pmatrix} = \begin{pmatrix} 1/4 & 1/4 & 1/4 & -1/4 & 1/4 \\ 1 & -1/2 & 0 & -1/2 & -1 \\ 3/2 & 0 & -1/2 & 0 & 3/2 \\ -1 & -1/2 & 0 & -1/2 & 1 \\ 1/4 & -1/4 & 1/4 & 1/4 & 1/4 \end{pmatrix} \begin{pmatrix} \delta_{il}\delta_{kj} \\ (\gamma^\mu)_{il}(\gamma_\mu)_{kj} \\ \frac{1}{2}(\sigma^{\mu\nu})_{il}(\sigma_{\mu\nu})_{kj} \\ (\gamma^\mu\gamma_5)_{il}(\gamma_\mu\gamma_5)_{kj} \\ (\gamma_5)_{il}(\gamma_5)_{kj} \end{pmatrix}
$$

$$
g_{\mu\nu}\sigma^\mu_{\alpha\dot\alpha}\sigma^\nu_{\beta\dot\beta} = 2\epsilon_{\alpha\beta}\epsilon_{\dot\alpha\dot\beta},
$$

$$
\epsilon^{\alpha\beta}\delta^\gamma_\kappa + \epsilon^{\beta\gamma}\delta^\alpha_\kappa + \epsilon^{\gamma\alpha}\delta^\beta_\kappa = 0.
$$

$$
\tilde\epsilon_{\dot\alpha\dot\beta}\delta^{\dot\kappa}_{\dot\gamma} + \tilde\epsilon_{\dot\beta\dot\gamma}\delta^{\dot\kappa}_{\dot\alpha} + \tilde\epsilon_{\dot\gamma\dot\alpha}\delta^{\dot\kappa}_{\dot\beta} = 0
$$

$$
\begin{aligned}
(\bar d l)(\bar l d) &= -\frac{1}{4}(\bar d d)(\bar l l) - \frac{1}{4}(\bar d\gamma^\mu d)(\bar l\gamma_\mu l) - \frac{1}{8}(\bar d\sigma^{\mu\nu} d)(\bar l\sigma_{\mu\nu} l) + \frac{1}{4}(\bar d\gamma^\mu\gamma_5 d)(\bar l\gamma_\mu\gamma_5 l) - \frac{1}{4}(\bar d\gamma_5 d)(\bar l\gamma_5 l) \\
&= -\frac{1}{2}(\bar d\gamma^\mu d)(\bar l\gamma_\mu l) \,, \\
(\bar l C\bar q)(l C q) &= -\frac{1}{4}(\bar l l)(\bar q q) + \frac{1}{4}(\bar l\gamma^\mu l)(\bar q\gamma_\mu q) + \frac{1}{8}(\bar l\sigma^{\mu\nu} l)(\bar q\sigma_{\mu\nu} q) + \frac{1}{4}(\bar l\gamma^\mu\gamma_5 l)(\bar q\gamma_\mu\gamma_5 q) - \frac{1}{4}(\bar l\gamma_5 l)(\bar q\gamma_5 q) \\
&= \frac{1}{2}(\bar l\gamma^\mu l)(\bar q\gamma_\mu q) \,.
\end{aligned}
$$

LLHH

Symmetry of the coupling parameters:

	LL	HH
$SU(3)_C$	\	\
$SU(2)_W$	$\ydiagram{2}$	$\ydiagram{2}$
$SU(2)_l$	$\ydiagram{1,1}$	\
$SU(2)_r$	\	\
Grassmann	$\ydiagram{1,1}$	\
Flavor	$\ydiagram{2} \times \ydiagram{1,1} \times \ydiagram{1,1} = \ydiagram{2}$	$\ydiagram{2}$

$$3 + 3 \quad = 6$$

LLHH

	LL	HH
$SU(3)_C$	$\backslash$	$\backslash$
$SU(2)_W$	$\left\{ \ydiagram{2} , \ydiagram{1,1} \right\}$	$\left\{ \ydiagram{2} , \ydiagram{1,1} \right\}$
$SU(2)_l$	$\ydiagram{1,1}$	$\backslash$
$SU(2)_r$	$\backslash$	$\backslash$
Grassmann	$\ydiagram{1,1}$	$\backslash$
Flavor	$\left\{ \ydiagram{2} , \ydiagram{1,1} \right\}$	$\left\{ \ydiagram{2} , \ydiagram{1,1} \right\}$

QQQL Operator

	Q^3	L
$SU(3)_C$	$\ydiagram{1,1,1}$	\
$SU(2)_W$	$\ydiagram{2,1}$	$\ydiagram{1}$
$SU(2)_l$	$\ydiagram{2,1}$	$\ydiagram{1}$
$SU(2)_r$	\	\
Grassmann	$\ydiagram{1,1,1}$	\
Flavor	$\ydiagram{1,1,1} \times \ydiagram{2,1} \times \ydiagram{2,1} \times \ydiagram{1,1,1} = \ydiagram{3} + \ydiagram{2,1} + \ydiagram{1,1,1}$	$\ydiagram{1} \times \ydiagram{1} = \ydiagram{1}$

$$10 + 8 + 1 \qquad \times 3 = 57$$

$$\ytableaushort{r,s,t} : \quad \ytableaushort{1,2,3},$$

$$\ytableaushort{rs,t} : \quad \ytableaushort{11,2}, \ytableaushort{11,3}, \ytableaushort{22,3}, \ytableaushort{12,2}, \ytableaushort{13,3}, \ytableaushort{23,3}, \ytableaushort{12,3}, \ytableaushort{13,2},$$

$$\ytableaushort{rst} : \quad \ytableaushort{111}, \ytableaushort{112}, \ytableaushort{113}, \ytableaushort{122}, \ytableaushort{123}, \ytableaushort{133}, \ytableaushort{222}, \ytableaushort{223}, \ytableaushort{233}, \ytableaushort{333},$$

Sn x SU(m) Group

Three spin-1/2 particle wave function:

$$2 \otimes 2 \otimes 2 = 4 \oplus 2 \oplus 2$$

$$\begin{array}{c}
\psi_+(1)\psi_+(2)\psi_+(3) \\
\psi_+(1)\psi_+(2)\psi_-(3) + \psi_+(1)\psi_-(2)\psi_+(3) + \psi_-(1)\psi_+(2)\psi_+(3) \\
\psi_+(1)\psi_-(2)\psi_-(3) + \psi_-(1)\psi_+(2)\psi_-(3) + \psi_-(1)\psi_-(2)\psi_+(3) \\
\psi_-(1)\psi_-(2)\psi_-(3)
\end{array}$$

SU(2) 4-dim irrrep 4

S3 1-dim irrrep [3]

$$\begin{aligned}
|1\rangle &= 2\psi_+(1)\psi_+(2)\psi_-(3) - \psi_+(1)\psi_-(2)\psi_+(3) - \psi_-(1)\psi_+(2)\psi_+(3) \\
|2\rangle &= 2\psi_-(1)\psi_-(2)\psi_+(3) - \psi_+(1)\psi_-(2)\psi_-(3) - \psi_-(1)\psi_+(2)\psi_-(3) \\
|3\rangle &= 2\psi_+(1)\psi_-(2)\psi_+(3) - \psi_+(1)\psi_+(2)\psi_-(3) - \psi_-(1)\psi_+(2)\psi_+(3) \\
|4\rangle &= 2\psi_-(1)\psi_+(2)\psi_-(3) - \psi_+(1)\psi_-(2)\psi_-(3) - \psi_-(1)\psi_-(2)\psi_+(3)
\end{aligned}$$

$$\begin{array}{cc}
|1\rangle & |2\rangle \\
|3\rangle & |4\rangle
\end{array}$$

SU(2) 2-dim irrrep

S3 2-dim irrrep [2 1]

Flavor Tensor

Three 1/2-particle with 3 flavors:

$$3 \otimes 3 \otimes 3 = 10 \oplus 8 \oplus 8 \oplus 1$$

SU(3) 1-d irrrep	$\begin{array}{\|c\|}\hline r\\ \hline s\\ \hline t\\ \hline\end{array}:$	$\begin{array}{\|c\|}\hline 1\\ \hline 2\\ \hline 3\\ \hline\end{array},$	S3 1-dim irrrep [111]
SU(3) 8-d irrrep	$\begin{array}{\|c\|c\|}\hline r & s\\ \hline t\\ \cline{1-1}\end{array}:$	$\begin{array}{\|c\|c\|}\hline 1&1\\ \hline 2\\ \cline{1-1}\end{array}, \begin{array}{\|c\|c\|}\hline 1&1\\ \hline 3\\ \cline{1-1}\end{array}, \begin{array}{\|c\|c\|}\hline 2&2\\ \hline 3\\ \cline{1-1}\end{array}, \begin{array}{\|c\|c\|}\hline 1&2\\ \hline 2\\ \cline{1-1}\end{array}, \begin{array}{\|c\|c\|}\hline 1&3\\ \hline 3\\ \cline{1-1}\end{array}, \begin{array}{\|c\|c\|}\hline 2&3\\ \hline 3\\ \cline{1-1}\end{array}, \begin{array}{\|c\|c\|}\hline 1&2\\ \hline 3\\ \cline{1-1}\end{array}, \begin{array}{\|c\|c\|}\hline 1&3\\ \hline 2\\ \cline{1-1}\end{array},$	S3 2-dim irrrep [21]
SU(3) 10-d irrrep	$\boxed{r}\boxed{s}\boxed{t}:$	$\boxed{1}\boxed{1}\boxed{1}, \boxed{1}\boxed{1}\boxed{2}, \boxed{1}\boxed{1}\boxed{3}, \boxed{1}\boxed{2}\boxed{2}, \boxed{1}\boxed{2}\boxed{3}, \boxed{1}\boxed{3}\boxed{3}, \boxed{2}\boxed{2}\boxed{2}, \boxed{2}\boxed{2}\boxed{3}, \boxed{2}\boxed{3}\boxed{3}, \boxed{3}\boxed{3}\boxed{3},$	S3 1-dim irrrep [3]

$$\begin{array}{cccccccc} |1\rangle & |2\rangle & |3\rangle & |4\rangle & |5\rangle & |6\rangle & |7\rangle & |8\rangle \\ |1'\rangle & |2'\rangle & |3'\rangle & |4'\rangle & |5'\rangle & |6'\rangle & |7'\rangle & |8'\rangle \end{array}$$

SU(3) 8-dim irrrep

S3 2-dim irrrep [2 1]

Schur-Weyl theorem:

$$\underbrace{(\{1\} \otimes (1)) \otimes (\{1\} \otimes (1)) \otimes \cdots \otimes (\{1\} \otimes (1))}_{N \text{ factors}} = \sum_{\lambda \vdash N} \{\lambda\} \otimes (\lambda)$$

Plethysm

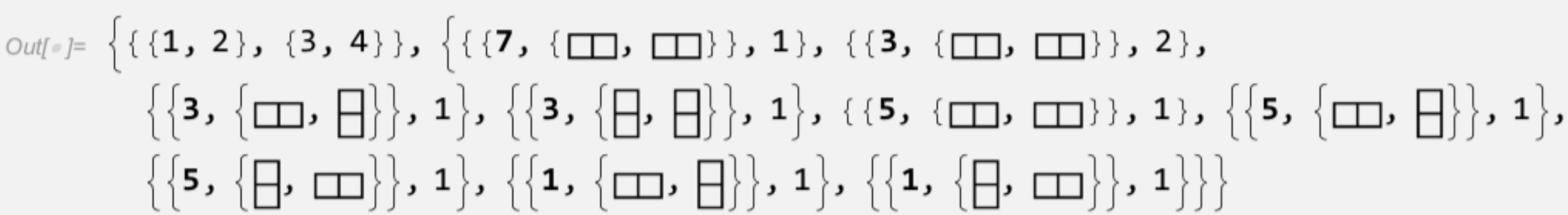
N Identical non-fundamental particles with nf flavors

$$6\times 6\times 6 = \mathbf{1}_{\{3\}} + \mathbf{27}_{\{3\}} + \overline{\mathbf{28}}_{\{3\}} + \mathbf{8}_{\{2,1\}} + \mathbf{27}_{\{2,1\}} + \overline{\mathbf{35}}_{\{2,1\}} + \mathbf{10}_{\{1,1,1\}} + \overline{\mathbf{10}}_{\{1,1,1\}}\,.$$

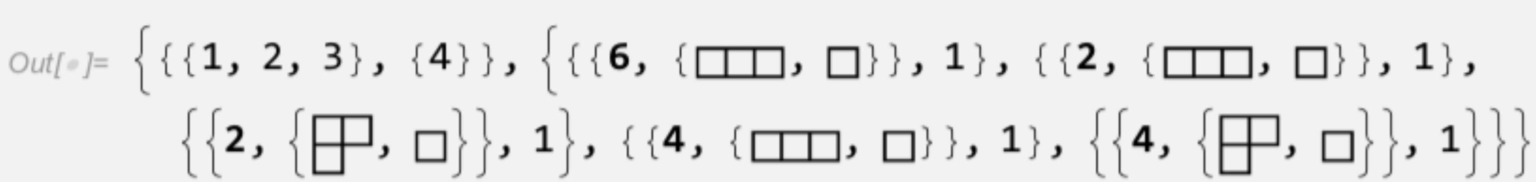
```
In[∘]:= Plethysms[SU3, 6, {3}, UseName → True]
Out[∘]= {{28̄, 1}, {1, 1}, {27, 1}}

In[∘]:= Plethysms[SU3, 6, {2, 1}, UseName → True]
Out[∘]= {{35̄, 1}, {8, 1}, {27, 1}}

In[∘]:= Plethysms[SU3, 6, {1, 1, 1}, UseName → True]
Out[∘]= {{10̄, 1}, {10, 1}}
```

[Fonseca, GroupMath]

```
In[∘]:= PermutationSymmetry[SU2, {2, 2, 3, 3}, UseName → True]
```



```
In[∘]:= PermutationSymmetry[SU2, {2, 2, 2, 3}, UseName → True]
```



Characters of SU(N)

Character of discrete Sn group:

Class	(1^3)	$(2,1)$	(3)
n_ρ	1	3	2
$\chi^{(3)}$	1	1	1
$\chi^{(21)}$	2	0	-1
$\chi^{(1^3)}$	1	-1	1

Character of semi-simple group SU(N): n-1 complex variables

$$\chi(g) = \chi(x_k) = \mathrm{tr}\left[e^{i\sum_k \theta_k H_k}\right] \qquad x_k = e^{i\theta_k}$$

SU(2) rep:

$$e^{i\theta T_3} = \mathrm{diag}\left(e^{ij\theta}, e^{i(j-1)\theta}, \cdots, e^{i(-j)\theta}\right) = \left(y^{2j}, y^{2j-2}, \cdots y^{-2j}\right)$$

$$\chi = y^{2j} + y^{2j-2} + \cdots + y^{-2j} = y^{2j}\frac{1-y^{-4j-2}}{1-y^{-2}} = \frac{y^{2j+1} - y^{-2j-1}}{y - y^{-1}}$$

SU(3) rep:

$$\chi_{(SU(3))_8}(z_1, z_2) = z_1 z_2 + \frac{1}{z_1 z_2} + 2 + \frac{z_1}{z_2^2} + \frac{z_1^2}{z_2} + \frac{z_2}{z_1^2} + \frac{z_2^2}{z_1}.$$

Weyl character formula in general

Plethysm of Characters

S_N and $U(n)$ characters

Schur-Weyl duality

U(n) fund. rep

Plethysm

U(n) higher rep

U(n) characters

$$s_\lambda(z) = \frac{1}{N!}\sum_{\rho\vdash N} n_\rho \chi^\lambda_\rho p_\rho(z)\,,$$

$$p_k(z) = \sum_{i=1}^{n} z_i^k\,.$$

S_3 has three irreps, labeled by $\lambda = (3)$, (21),

three classes, $\rho = (1^3)$, $(2,1)$, and (3),

Schur symmetric function

$$s_\kappa \circledR s_\lambda = \chi^\lambda \cdot \Xi^N_\kappa\,.$$

$$s_\lambda = s_1 \circledR s_\lambda = \chi^\lambda \cdot \Xi^N_1 = \frac{1}{N!}\sum_\rho n_\rho \chi^\lambda_\rho p_\rho.$$

U(3) x S2

$$[s_2 \circledR s_2](x) = s_2(z^{(2)}(x)) = \sum_{i\le j}^{6} z_i^{(2)}(x) z_j^{(2)}(x)$$

$$= x_1^4 + x_1^2x_2^2 + x_1^2x_3^2 + x_1^3x_2 + x_1^2x_2x_3 + x_1^3x_3 + \ldots.$$

$$\chi_{\mathrm{Sym}^n(\mathbf{R})}(z)$$

Plethystic Exponential (PE)

Sum over all possible Plethysm, add spurion (not field) for repeated field

$$\mathrm{PE}[\phi_{\mathbf{R}}\,\chi_{\mathbf{R}}(z)] = \sum_{n=0}^{\infty} \phi_{\mathbf{R}}^{n}\,\chi_{\mathrm{Sym}^{n}(\mathbf{R})}(z) = \exp\left[\sum_{r=1}^{\infty} \frac{1}{r}\phi_{\mathbf{R}}^{r}\,\chi_{\mathbf{R}}(z^{r})\right]$$

$$z = \{z_1, \ldots, z_{\mathrm{rank(G)}}\}$$

$$\mathrm{PEF}[\phi_{\mathbf{R}}\,\chi_{\mathbf{R}}(z)] = \sum_{n=0}^{\infty} \phi_{\mathbf{R}}^{n}\,\chi_{\wedge^{n}(\mathbf{R})}(z) = \exp\left[\sum_{r=1}^{\infty} \frac{(-1)^{r+1}}{r}\phi_{\mathbf{R}}^{r}\,\chi_{\mathbf{R}}(z^{r})\right]$$

$$\mathrm{PE}[\phi_{\mathbf{R}}, \ldots, \varphi_{\mathbf{R}'}] = \mathrm{PE}[\phi_{\mathbf{R}}]\cdots\mathrm{PE}[\varphi_{\mathbf{R}'}]$$

Tensor product of spurions

$$\begin{aligned}\mathrm{PE}[\phi_{\mathbf{2}}] = \exp\left[\sum_{r=1}^{\infty}\frac{1}{r}\phi_{\mathbf{2}}^{r}\,\chi_{\mathbf{2}}(y^{r})\right] &= 1 + \chi_{\mathbf{2}}(y)\,\phi + \frac{1}{2}(\chi_{\mathbf{2}}(y^{2}) + \chi_{\mathbf{2}}(y)^{2})\,\phi^{2} + \mathcal{O}(\phi^{3})\\ &= 1 + \chi_{\mathbf{2}}(y)\,\phi + \chi_{\mathbf{3}}(y)\phi^{2} + \mathcal{O}(\phi^{3})\,,\end{aligned}$$

[Feng, Hanany, He, 2007]

[Hanany, Jenkins, Manohar, Torri, 2010]

[Lehmann, Martin, 2015]

Hilbert Series

$$\mathrm{PE}[\phi_{\mathbf{R}}, \ldots, \varphi_{\mathbf{R}'}] = \mathrm{PE}[\phi_{\mathbf{R}}] \cdots \mathrm{PE}[\varphi_{\mathbf{R}'}]$$

Schur theorem: orthonormal with Haar measure integral

$$\int d\mu_G(g)\, \chi_{\mathbf{R}}(g)\, \chi^*_{\mathbf{R}'}(g) \;=\; \delta_{\mathbf{R}\,\mathbf{R}'}.$$

Project onto trivial representation

$$\mathcal{H}(\phi_{\mathbf{R}}, \ldots, \varphi_{\mathbf{R}'}) = \int d\mu_G\, \mathrm{PE}[\phi_{\mathbf{R}}, \ldots, \varphi_{\mathbf{R}'}]\,.$$

Molien-Weyl formula

Hilbert series: series for group invariant (q)

$$\mathcal{H}(q) = \sum_{r=0}^{\infty} c_r\, q^r$$

generating function that counts the number (c) of independent group invariants (q)

$$\begin{aligned}\mathcal{H}(\phi_{\mathbf{2}}, \phi^\dagger_{\mathbf{2}}) &= \int d\mu_{SU(2)}(y)\,\big(1 + (\phi_{\mathbf{2}} + \phi^\dagger_{\mathbf{2}})\chi_{\mathbf{2}}(y) + (\phi^2_{\mathbf{2}} + \phi^{\dagger\,2}_{\mathbf{2}})\chi_{\mathbf{3}}(y) + (\phi_{\mathbf{2}}\phi^\dagger_{\mathbf{2}})\chi_{\mathbf{2}}(y)\chi_{\mathbf{2}}(y) + \ldots\big) \\ &= 1 + \phi_{\mathbf{2}}\phi^\dagger_{\mathbf{2}} + \mathcal{O}(\phi_{\mathbf{2}}, \phi^\dagger_{\mathbf{2}})^3\,,\end{aligned}$$

$$\int d\mu_{SU(2)}(y) = \frac{1}{2\pi i}\oint_{|y|=1} \frac{dy}{y}(1 - y^2)$$

Character and Haar Measure

Group	Representation	Character	Haar measure
Lorentz	$(0,0)$	1	$\frac{1}{(2\pi \mathrm{i})^2}\oint_{\lvert y_1\rvert=1}\frac{\mathrm{d}y_1}{y_1}(1-y_1^2) \times \oint_{\lvert y_2\rvert=1}\frac{\mathrm{d}y_2}{y_2}(1-y_2^2)$
	$(\frac{1}{2},0)$	$y_1+\frac{1}{y_1}$	
	$(0,\frac{1}{2})$	$y_2+\frac{1}{y_2}$	
	$(\frac{1}{2},0)\oplus(0,\frac{1}{2})$	$y_1+\frac{1}{y_1}+y_2+\frac{1}{y_2}$	
	$(\frac{1}{2},\frac{1}{2})$	$(y_1+\frac{1}{y_1})(y_2+\frac{1}{y_2})$	
	$(1,0)\oplus(0,1)$	$y_1^2+1+\frac{1}{y_1^2}+(y_1\leftrightarrow y_2)$	
	$(2,0)\oplus(0,2)$	$y_1^4+y_1^2+1+\frac{1}{y_1^2}+\frac{1}{y_1^4}+(y_1\leftrightarrow y_2)$	
	$(\frac{3}{2},\frac{1}{2})$	$(y_1^3+y_1+\frac{1}{y_1}+\frac{1}{y_1^3})(y_2+\frac{1}{y_2})$	
	$(\frac{1}{2},\frac{3}{2})$	$(y_1+\frac{1}{y_1})(y_2^3+y_1+\frac{1}{y_2}+\frac{1}{y_2^3})$	
$U(1)$	charge Q	x^Q	$\frac{1}{2\pi \mathrm{i}}\oint_{\lvert x\rvert=1}\frac{\mathrm{d}x}{x}$
$SU(2)$	singlet	1	$\frac{1}{2\pi \mathrm{i}}\oint_{\lvert y\rvert=1}\frac{\mathrm{d}y}{y}(1-y^2)$
	fundamental/doublet	$y+\frac{1}{y}$	
	triplet/adjoint	$y^2+1+\frac{1}{y^2}$	
$SU(3)$	singlet	1	$\frac{1}{(2\pi \mathrm{i})^2}\oint_{\lvert z_1\rvert=1}\frac{\mathrm{d}z_2}{z_2}\oint_{\lvert z_2\rvert=1}\frac{\mathrm{d}z_2}{z_2} \times(1-z_1z_2)(1-\frac{z_1^2}{z_2})(1-\frac{z_2^2}{z_1})$
	fundamental/3	$z_1+\frac{z_2}{z_1}+\frac{1}{z_2}$	
	antifundamental/$\bar{3}$	$z_2+\frac{z_1}{z_2}+\frac{1}{z_1}$	
	adjoint	$z_1z_2+\frac{z_2^2}{z_1}+\frac{z_1^2}{z_2}+2+\frac{z_2}{z_1^2}+\frac{z_1}{z_2^2}+\frac{1}{z_1z_2}$	

Jiang-Hao Yu

SMEFT Counting

$$\chi_a(x_1, x_2, x, y, z_1, z_2) = \tilde{\chi}_{[SU(2)_l \times SU(2)_r]}(x_1, x_2)\chi_a^{U(1)}(x)\chi_a^{SU(2)}(y)\chi_a^{SU(3)}(z_1, z_2)$$

$$\chi_Q = \chi_{[\frac{3}{2},(\frac{1}{2},0)]}(\mathcal{D}, \alpha, \beta)\ \chi_{1/6}^{U(1)}(x)\ \chi_{\mathbf{2}}^{SU(2)}(y)\ \chi_{\mathbf{3}}^{SU(3)}(z_1, z_2)$$

$$\mathrm{PE}\Big[\frac{Q}{\mathcal{D}^{\frac{3}{2}}}\chi_Q\Big] = \exp\Bigg(\sum_{r=1}^{\infty} \frac{1}{r}(-1)^{r+1}\frac{Q^r}{\mathcal{D}^{\frac{3}{2}r}}\chi_{[\frac{3}{2},(\frac{1}{2},0)]}(\mathcal{D}^r; \alpha^r, \beta^r) \times \chi_{\frac{1}{6}}^{U(1)}(x^r)\ \chi_{\mathbf{2}}^{SU(2)}(y^r)\ \chi_{\mathbf{3}}^{SU(3)}(z_1^r, z_2^r)\Bigg)$$

$$\mathcal{H}(q) = \sum_{r=0}^{\infty} c_r\, q^r$$

[Lehmann, Martin, 2015]

$$G^3 + 57\, L\, Q^3 + 45\, d^2\, d^{*2} + 81\, d\, e\, d^*\, e^* + 36\, e^2\, e^{*2} + G^{*3} + B^2\, H\, H^* + G^2\, H\, H^* + 9\, B\, e\, L\, H^* + 9\, B\, d\, Q\, H^* + 9\, d\, G\, Q\, H^* +$$
$$H\, B^{*2}\, H^* + H\, G^{*2}\, H^* + 9\, e\, H\, L\, H^{*2} + 9\, d\, H\, Q\, H^{*2} + H^3\, H^{*3} + 81\, d\, L\, d^*\, L^* + 81\, e\, L\, e^*\, L^* + 81\, d\, Q\, e^*\, L^* + 9\, H\, B^*\, e^*\, L^* +$$
$$9\, H^2\, e^*\, H^*\, L^* + 45\, L^2\, L^{*2} + 81\, e\, L\, d^*\, Q^* + 162\, d\, Q\, d^*\, Q^* + 9\, H\, B^*\, d^*\, Q^* + 81\, e\, Q\, e^*\, Q^* + 9\, H\, d^*\, G^*\, Q^* + 9\, H^2\, d^*\, H^*\, Q^* +$$
$$162\, L\, Q\, L^*\, Q^* + 90\, Q^2\, Q^{*2} + 57\, L^*\, Q^{*3} + 81\, L\, Q\, d^*\, u^* + 54\, Q^2\, e^*\, u^* + 9\, B^*\, H^*\, Q^*\, u^* + 9\, G^*\, H^*\, Q^*\, u^* + 9\, H\, H^{*2}\, Q^*\, u^* +$$
$$162\, e^*\, L^*\, Q^*\, u^* + 162\, d^*\, Q^{*2}\, u^* + 81\, d^*\, e^*\, u^{*2} + H\, B^*\, H^*\, W^* + 9\, H\, e^*\, L^*\, W^* + 9\, H\, d^*\, Q^*\, W^* + 9\, H^*\, Q^*\, u^*\, W^* + H\, H^*\, W^{*2} + W$$
$$9\, B\, H\, Q\, u + 9\, G\, H\, Q\, u + 162\, e\, L\, Q\, u + 162\, d\, Q^2\, u + 9\, H^2\, Q\, H^*\, u + 81\, d\, L^*\, Q^*\, u + 54\, e\, Q^{*2}\, u + 162\, d\, d^*\, u^*\, u + 81\, e\, e^*\, u^*\, u$$
$$81\, L\, L^*\, u^*\, u + 162\, Q\, Q^*\, u^*\, u + 81\, d\, e\, u^2 + 45\, u^{*2}\, u^2 + B\, H\, H^*\, W + 9\, e\, L\, H^*\, W + 9\, d\, Q\, H^*\, W + 9\, H\, Q\, u\, W + H\, H^*\, W^2 + W^3 +$$

Operators with Derivative

Integration by part (IBP):

$$\left.\begin{array}{r} \mathcal{O} \\ \partial_\mu \mathcal{O} \\ \partial_{\mu_1}\partial_{\mu_2}\mathcal{O} \\ \vdots \end{array}\right\} \quad \textbf{Operators}$$

Symmetrize operator Lorentz indices

Descendants (total derivatives)

$$\begin{array}{r} \mathcal{O} \\ \partial_\mu \mathcal{O} \\ \partial_{\{\mu_1}\partial_{\mu_2\}}\mathcal{O} \\ \vdots \end{array}$$

Form free field irrep of conformal group

Conformal Group SO(4,2)

From SO(3,1) to SO(4,2)

$$\eta = \mathrm{diag}(\underbrace{+1}_{1},\underbrace{-1,-1,-1}_{3}) \to \mathrm{diag}(\underbrace{+1,+1}_{2},\underbrace{-1,-1,-1,-1}_{4})$$

$$[X_{\mu\nu}, X_{\alpha\beta}] = i\left(\delta_{\mu\alpha}X_{\nu\beta} + \delta_{\nu\beta}X_{\mu\alpha} - \delta_{\mu\beta}X_{\nu\alpha} - \delta_{\nu\alpha}X_{\mu\beta}\right).$$

$$X_{\mu\nu} \equiv M_{\mu\nu}, \quad X_{-1\mu} \equiv \frac{1}{2}(P_\mu - K_\mu), \quad X_{0\mu} \equiv \frac{1}{2}(P_\mu + K_\mu), \quad \text{and} \quad X_{-1,0} \equiv D$$

$$\begin{aligned} P_\mu &= -i\partial_\mu \\ M_{\mu\nu} &= i(x_\mu\partial_\nu - x_\nu\partial_\mu) \\ D &= -ix^\mu\partial_\mu \\ K_\mu &= i(x^2\partial_\mu - 2x_\mu x^\nu\partial_\nu) \end{aligned}$$

$$\begin{aligned} [D, P_\mu] &= -iP_\mu, \\ [D, K_\mu] &= iK_\mu, \\ [K_\mu, P_\nu] &= 2i(\eta_{\mu\nu}D + M_{\mu\nu}), \\ [M_{\mu\nu}, K_\rho] &= i(\eta_{\mu\rho}K_\nu - \eta_{\nu\rho}K_\mu), \end{aligned}$$

$$\begin{aligned} [M_i, M_j] &= i\epsilon_{ijk}M_k\ , \\ [N_i, N_j] &= i\epsilon_{ijk}N_k\ , \\ [M_i, N_j] &= 0\ . \end{aligned}$$

Cartan subalgebra:

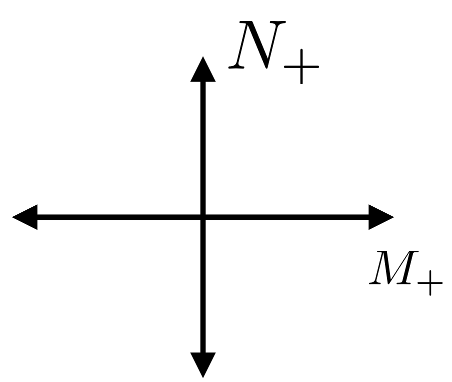
$$H_0 = \{D, M_3, N_3\}, E_+ = \{P_\mu, M_+, N_+\}, \text{and } E_- = \{K_\mu, M_-, N_-\}$$

Conformal Group Rep

SO(3,1)

$$\tfrac{1}{2}M^{\mu\nu}M_{\mu\nu} = \mathbf{J}^2 - \mathbf{K}^2 \,,$$
$$\tfrac{1}{2}\epsilon^{\mu\nu\sigma\tau}M_{\mu\nu}M_{\sigma\tau} = -\mathbf{J}\cdot\mathbf{K} \,,$$

$$M_i = \tfrac{1}{2}(J_i + iK_i) \,,$$
$$N_i = \tfrac{1}{2}(J_i - iK_i) \,,$$



Fields

$(0,0), (1/2,0), (0,1/2),$
$(1/2,1/2), (1,0), (0,1)$

Poincare

Semidirect product of a semisimple (Lorentz) and an Abelian group (translations)

$$P^2 = P_\mu P^\mu$$
$$W^2 = W_\mu W^\mu$$

$$W_\mu = \epsilon_{\mu\nu\sigma\tau}M^{\nu\sigma}P^\tau$$

$$W_0 = \mathbf{P}\cdot\mathbf{J} \,,$$
$$\mathbf{W} = P_0\,\mathbf{J} - \mathbf{P}\times\mathbf{K} \,.$$

P invariant subgroup:

$$[w_0, w_\pm] = \pm m w_\pm$$
$$[w_+, w_-] = 2m w_0$$

States

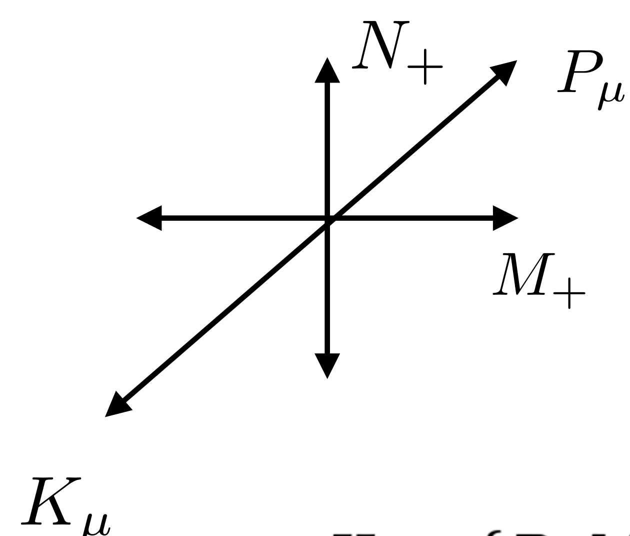
$|m, \mathbf{p}, j, \sigma\rangle$

SO(4,2)

3 casimirs

$$\tfrac{1}{2}M^{\mu\nu}M_{\mu\nu} \;+ D(D-d) \;-P^\mu K_\mu$$

$$E_+ = \{P_\mu, M_+, N_+\} \qquad E_- = \{K_\mu, M_-, N_-\}$$



$$H_0 = \{D, M_3, N_3\}$$

(Δ, ℓ_1, ℓ_2)

$$P_\mu : (\Delta, \ell) \to \left(\Delta + 1, \ell \otimes \left(\frac{1}{2}, \frac{1}{2}\right)\right)$$

$$\ell = (\ell_1, \ell_2)$$

Operators and Character

Primary operator and descendants:

$$R_{\Delta,\ell} \sim \begin{pmatrix} \phi \\ \partial_{\mu_1}\phi \\ \partial_{\mu_1}\partial_{\mu_2}\phi \\ \partial_{\mu_1}\partial_{\mu_2}\partial_{\mu_3}\phi \\ \vdots \end{pmatrix}$$

	Scaling dim.	Spin
ϕ	Δ	ℓ
$\partial_{\mu_1}\phi$	$\Delta+1$	$(\frac{1}{2},\frac{1}{2})\otimes\ell$
$\partial_{\mu_1}\partial_{\mu_2}\phi$	$\Delta+2$	$\mathrm{sym}^2(\frac{1}{2},\frac{1}{2})\otimes\ell$
$\partial_{\mu_1}\partial_{\mu_2}\partial_{\mu_3}\phi$	$\Delta+2$	$\mathrm{sym}^3(\frac{1}{2},\frac{1}{2})\otimes\ell$
$\vdots$	$\vdots$	$\vdots$

Characters

$$\chi_{\Delta,\ell} = \mathrm{tr}_{|\Delta,\ell\rangle^*}\left[e^{i\theta D + i\theta_L J_L^3 + i\theta_R J_R^3}\right] \qquad q = e^{i\theta D},\ x_1 = e^{i\theta_L J_L^3} \text{ and } x_2 = e^{i\theta_R J_R^3}$$

$$= \sum_{n=0}^{\infty} q^{\Delta+n}\chi_{\mathrm{sym}^n(\frac{1}{2},\frac{1}{2})\otimes\ell}(x) = q^{\Delta}\chi_\ell(x)\sum_{n=0}^{\infty} q^n \chi_{\mathrm{sym}^n(\frac{1}{2},\frac{1}{2})}(x) \equiv q^{\Delta}\chi_\ell(x_1,x_2)P(q,x)$$

$$x = (x_1, x_2)$$

Haar measure

$$P(q,x) \equiv \sum_{n=0}^{\infty} q^n \chi_{\mathrm{sym}^n(\frac{1}{2},\frac{1}{2})}(x) = \mathrm{PE}\left[q\chi_{(\frac{1}{2},\frac{1}{2})}(x)\right]$$

$$\int d\mu_{SO(4,2)} = \int d\mu_L \oint \frac{dq}{2\pi i q}\frac{1}{P^*(q,x)P(q,x)}$$

[Henning, Lu, Melia, Murayama, 2017]

IBP and EOM

Keep only the primary operator contribution = solve IBP

$$\mathrm{PE}\left[\phi\chi_{\Delta,\ell}(q,x)\right] = 1+\sum_{n=1}^{\infty}\phi^n\chi_{\mathrm{sym}^n(\Delta,\ell)}(q,x)$$

tensor decomposition

$$\chi_{\mathrm{sym}^n(\Delta,\ell)}(q,x) = \sum_{\Delta',\ell'} b^{(n)}_{\Delta',\ell'}\chi_{\Delta',\ell'}(q,x)$$

$$= 1+\sum_{\Delta',\ell'}\sum_{n=1}^{\infty}\phi^n b^{(n)}_{\Delta',\ell'}\chi_{\Delta',\ell'}(q,x) = 1+\sum_{\Delta',\ell'} c_{\Delta',\ell'}(\phi)\chi_{\Delta',\ell'}(q,x)$$

project onto primary states

$$\int \mathrm{d}\mu_{SO(4,2)}\,\chi^*_{\Delta',(0,0)}\ \ \mathrm{PE}\left[\phi\chi_{\Delta,\ell}(q,x)\right]$$

$$H(\mathcal{D},\phi) = 1+\sum_{\Delta'} c_{\Delta',(0,0)}\left(\frac{\phi}{\mathcal{D}^{\Delta}}\right)\mathcal{D}^{\Delta'} = 1+\sum_{n=0}^{\infty} c_{(\Delta+n),(0,0)}\left(\frac{\phi}{\mathcal{D}^{\Delta}}\right)\mathcal{D}^{\Delta+n}$$

$$H(\mathcal{D},\phi) = 1+\int \mathrm{d}\mu_L \oint \frac{\mathrm{d}q}{2\pi i}\left(\frac{\mathcal{D}}{q}\right)^{\Delta}\frac{1}{q-\mathcal{D}}\frac{1}{P(q,x)}\left(\mathrm{PE}\left[\frac{\phi}{\mathcal{D}^{\Delta}}\chi_{\Delta,\ell}(q,x)\right]-1\right)$$

[Henning, Lu, Melia, Murayama, 2017]

IBP and EOM

EOM: identify EOM symmetry and remove it

$$D_{\mu_1} D_{\mu_2}\phi = \underbrace{(D_{\mu_1} D_{\mu_2} - \tfrac{1}{4}\eta_{\mu_1\mu_2} D^2)\phi}_{\text{Traceless}} + \tfrac{1}{4}\underbrace{\eta_{\mu_1\mu_2} D^2\phi}_{\text{Trace}},$$

$$\boxed{\chi_{\text{sym}^n(\frac{1}{2},\frac{1}{2})}(x) = \widetilde{\chi}_n(x) + \chi_{\text{sym}^{n-1}(\frac{1}{2},\frac{1}{2})}(x)\cdot 1}$$

$$\widetilde{\chi}_{[1,(0,0)]}(q,x) = \sum_{n=0}^{\infty} q^{1+n}\widetilde{\chi}_n(x) = q\left(\sum_{n=0}^{\infty} q^n \chi_{\text{sym}^n(\frac{1}{2},\frac{1}{2})}(x) - \sum_{n=2}^{\infty} q^n \chi_{\text{sym}^{n-2}(\frac{1}{2},\frac{1}{2})}(x)\right)$$

$$= q\left(1-q^2\right)\sum_{n=0}^{\infty} q^n \chi_{\text{sym}^n(\frac{1}{2},\frac{1}{2})}(x) = q\left(1-q^2\right)P(q,x).$$

$$\boxed{\chi_{\text{sym}^n(\frac{1}{2},\frac{1}{2})}(x)\chi_{(\frac{1}{2},0)}(x) = \widetilde{\chi}(x) + \chi_{\text{sym}^{n-1}(\frac{1}{2},\frac{1}{2})}(x)\chi_{(0,\frac{1}{2})}(x).}$$

$$\boxed{\chi_{\text{sym}^n(\frac{1}{2},\frac{1}{2})}(x)\chi_{(1,0)}(x) = \widetilde{\chi}(x) + \left(\chi_{\text{sym}^{n-1}(\frac{1}{2},\frac{1}{2})}(x)\chi_{(\frac{1}{2},\frac{1}{2})}(x) - \chi_{\text{sym}^{n-2}(\frac{1}{2},\frac{1}{2})}(x)\cdot 1\right)}$$

$$D_\mu J^\mu = 0.$$

Fields Under Conformal Group

Character of product Irrep

Representation	Character
$[1,(0,0)]$	$\mathcal{D}\left(1-\mathcal{D}^2\right)P(\mathcal{D},x_1,x_2)$
$[\frac{3}{2},(\frac{1}{2},0)]$	$\mathcal{D}^{\frac{3}{2}}\left(\chi_{(\frac{1}{2},0)}(x_1,x_2)-\mathcal{D}\chi_{(0,\frac{1}{2})}(x_1,x_2)\right)P(\mathcal{D},x_1,x_2)$
$[\frac{3}{2},(0,\frac{1}{2})]$	$\mathcal{D}^{\frac{3}{2}}\left(\chi_{(0,\frac{1}{2})}(x_1,x_2)-\mathcal{D}\chi_{(\frac{1}{2},0)}(x_1,x_2)\right)P(\mathcal{D},x_1,x_2)$
$[\frac{3}{2},(\frac{1}{2},0)\oplus(0,\frac{1}{2})]$	$\mathcal{D}^{\frac{3}{2}}\left(\chi_{(\frac{1}{2},0)\oplus(0,\frac{1}{2})}(x_1,x_2)-\mathcal{D}\chi_{(\frac{1}{2},0)\oplus(0,\frac{1}{2})}(x_1,x_2)\right)P(\mathcal{D},x_1,x_2)$
$[2,(1,0)\oplus(0,1)]$	$\mathcal{D}^2\left(\chi_{(1,0)\oplus(0,1)}(x_1,x_2)-2\mathcal{D}\chi_{(\frac{1}{2},\frac{1}{2})}(x_1,x_2)+2\mathcal{D}^2\right)P(\mathcal{D},x_1,x_2)$
$[3,(2,0)\oplus(0,2)]$	$\mathcal{D}^3\Big(\chi_{(2,0)\oplus(0,2)}(x_1,x_2)-\mathcal{D}\chi_{(\frac{3}{2},\frac{1}{2})\oplus(\frac{1}{2},\frac{3}{2})}(x_1,x_2)$ $+\mathcal{D}^2\chi_{(1,0)\oplus(0,1)}(x_1,x_2)\Big)P(\mathcal{D},x_1,x_2)$

$$P(\mathcal{D},\alpha,\beta)=\frac{1}{(1-\mathcal{D}\alpha\beta)(1-\mathcal{D}/\alpha\beta)(1-\mathcal{D}\alpha/\beta)(1-\mathcal{D}\beta/\alpha)}.$$

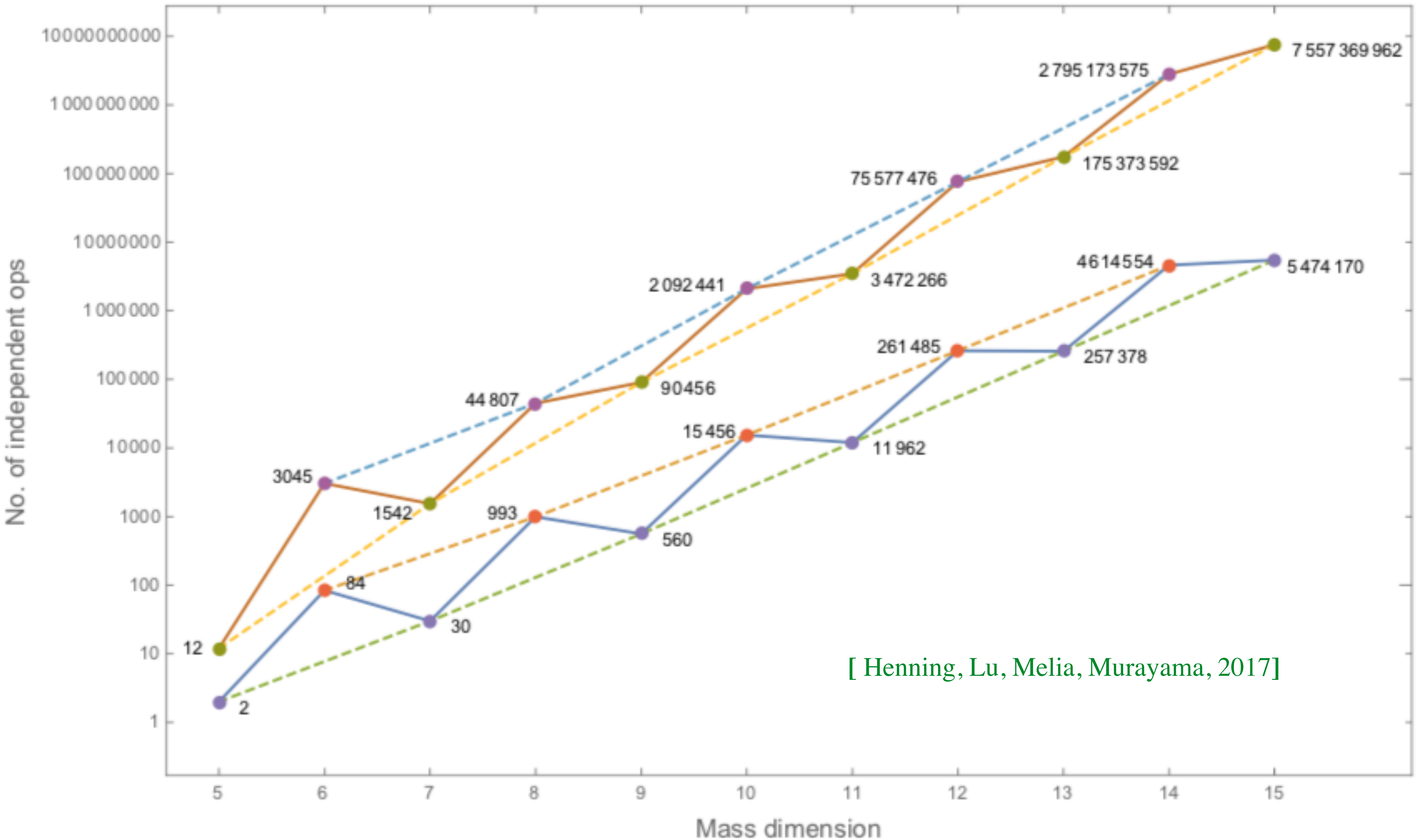
SMEFT Results

$$
\begin{aligned}
&H_{SM}\Big|_{\mathcal{O}(\epsilon^5)} \\
&= \frac{1}{(2\pi \mathrm{i})^6} \oint \frac{\mathrm{d}x_1}{x_1}(1-x_1^2) \oint \frac{\mathrm{d}x_2}{x_2}(1-x_2^2) \oint \frac{\mathrm{d}x}{x} \oint \frac{\mathrm{d}y}{y}(1-y^2) \oint \frac{\mathrm{d}z_2}{z_2} \oint \frac{\mathrm{d}z_2}{z_2}(1-z_1 z_2)\Big(1-\frac{z_1^2}{z_2}\Big)\Big(1-\frac{z_2^2}{z_1}\Big) \\
&\quad \epsilon^5\left[\varphi^2 l^2 \frac{(1+y^2+y^4)(x_1^2+y^4x_1^2+y^2(1+x_1^2)^2)}{y^2x_1^2} + (\varphi^\dagger)^2(l^\dagger)^2 \frac{(1+y^2+y^4)(x_2^2+y^4x_2^2+y^2(1+x_2^2)^2)}{y^2x_2^2} + \ldots\right] \\
&= \varphi^2 l^2 + (\varphi^\dagger)^2(l^\dagger)^2, \qquad (5.7)
\end{aligned}
$$

[Henning, Lu, Melia, Murayama, 2017]

$$
\begin{aligned}
\widehat{H}_6 = {} & H^3H^{\dagger 3} + u^\dagger Q^\dagger H H^{\dagger 2} + 2Q^2Q^{\dagger 2} + Q^{\dagger 3}L^\dagger + Q^3L + 2QQ^\dagger LL^\dagger + L^2L^{\dagger 2} + uQH^2H^\dagger \\
& + 2uu^\dagger QQ^\dagger + uu^\dagger LL^\dagger + u^2u^{\dagger 2} + e^\dagger u^\dagger Q^2 + e^\dagger L^\dagger H^2 H^\dagger + 2e^\dagger u^\dagger Q^\dagger L^\dagger + eLHH^{\dagger 2} + euQ^{\dagger 2} \\
& + 2euQL + ee^\dagger QQ^\dagger + ee^\dagger LL^\dagger + ee^\dagger uu^\dagger + e^2e^{\dagger 2} + d^\dagger Q^\dagger H^2H^\dagger + 2d^\dagger u^\dagger Q^{\dagger 2} + d^\dagger u^\dagger QL \\
& + d^\dagger e^\dagger u^{\dagger 2} + d^\dagger eQ^\dagger L + dQHH^{\dagger 2} + 2duQ^2 + duQ^\dagger L^\dagger + de^\dagger QL^\dagger + deu^2 + 2dd^\dagger QQ^\dagger + dd^\dagger LL^\dagger \\
& + 2dd^\dagger uu^\dagger + dd^\dagger ee^\dagger + d^2d^{\dagger 2} + u^\dagger Q^\dagger H^\dagger G_R + d^\dagger Q^\dagger HG_R + HH^\dagger G_R^2 + G_R^3 + uQHG_L \\
& + dQH^\dagger G_L + HH^\dagger G_L^2 + G_L^3 + u^\dagger Q^\dagger H^\dagger W_R + e^\dagger L^\dagger HW_R + d^\dagger Q^\dagger HW_R + HH^\dagger W_R^2 + W_R^3 \\
& + uQHW_L + eLH^\dagger W_L + dQH^\dagger W_L + HH^\dagger W_L^2 + W_L^3 + u^\dagger Q^\dagger H^\dagger B_R + e^\dagger L^\dagger HB_R \\
& + d^\dagger Q^\dagger HB_R + HH^\dagger B_R W_R + HH^\dagger B_R^2 + uQHB_L + eLH^\dagger B_L + dQH^\dagger B_L + HH^\dagger B_L W_L \\
& + HH^\dagger B_L^2 + 2QQ^\dagger HH^\dagger \mathcal{D} + 2LL^\dagger HH^\dagger \mathcal{D} + uu^\dagger HH^\dagger \mathcal{D} + ee^\dagger HH^\dagger \mathcal{D} + d^\dagger uH^2\mathcal{D} + du^\dagger H^{\dagger 2}\mathcal{D} \\
& + dd^\dagger HH^\dagger \mathcal{D} + 2H^2H^{\dagger 2}\mathcal{D}^2 \,. \qquad (3.20)
\end{aligned}
$$

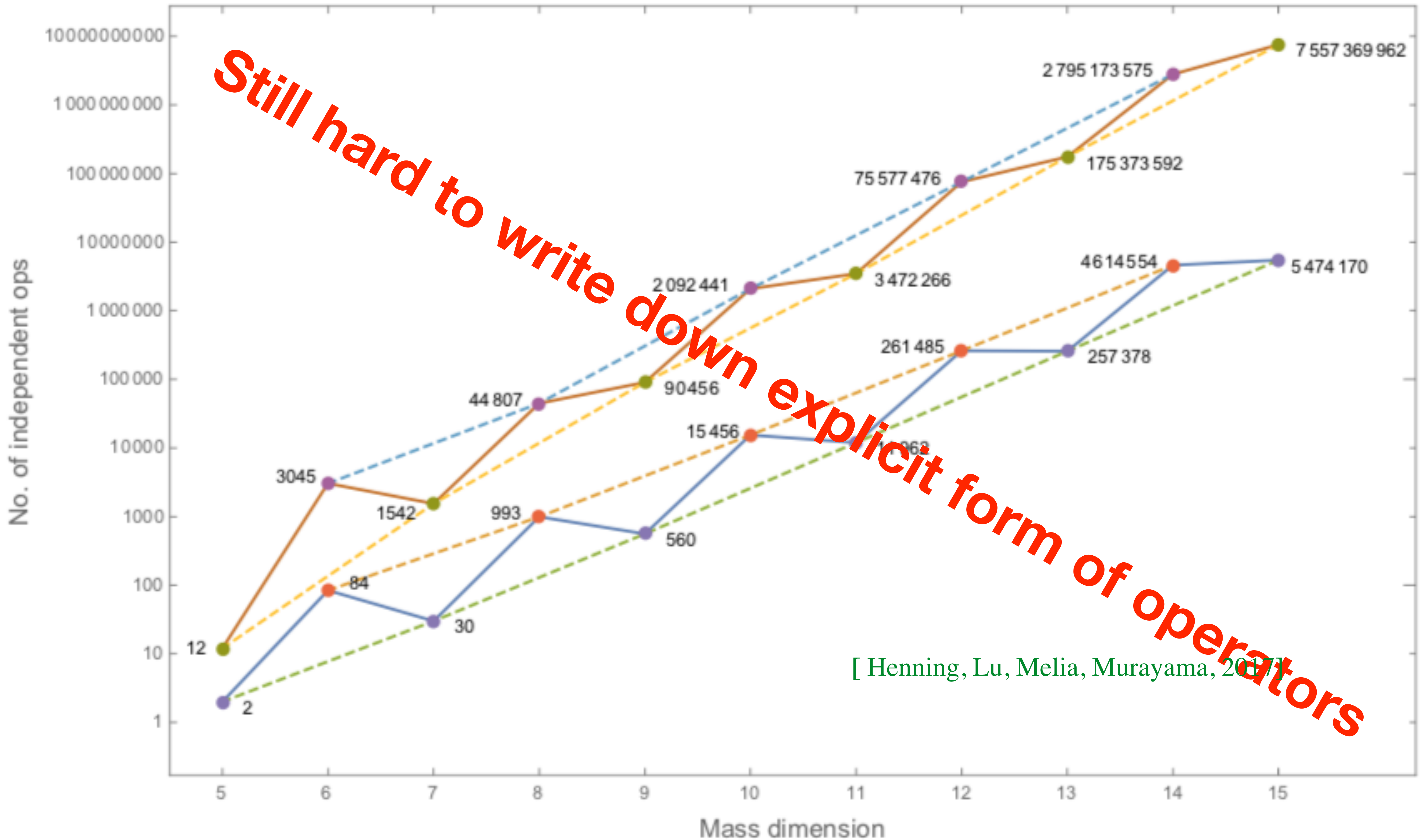
SMEFT Results



[Henning, Lu, Melia, Murayama, 2017]

Jiang-Hao Yu

SMEFT Results



Still hard to write down explicit form of operators

[Henning, Lu, Melia, Murayama, 2017]

Young Tensor Basis

ITP-CAS group

Hao-Lin Li, Zhe Ren, Jing Shu, Ming-Lei Xiao, **JHY**, Yu-Hui Zheng, 2005.00008

Hao-Lin Li, Zhe Ren, Ming-Lei Xiao, **JHY**, Yu-Hui Zheng, 2007.07899

Hao-Lin Li, Zhe Ren, Ming-Lei Xiao, **JHY**, Yu-Hui Zheng, 2012.09188

Hao-Lin Li, Jing Shu, Ming-Lei Xiao, **JHY**, 2012.11615

Higher Dim Operators!?

Given numbers of independent operators

does not mean we know explicit form of operators!

Derivatives

$BWHH^\dagger D^2$

2

$$
\begin{aligned}
&(D^2H^\dagger)HB_{L\mu\nu}W_L^{\mu\nu},\ (D^\mu D_\nu H^\dagger)HB_{L\mu\rho}W_L^{\nu\rho},\ (D_\nu D^\mu H^\dagger)HB_{L\mu\rho}W_L^{\nu\rho},\ (D_\mu H^\dagger)(D^\mu H)B_{L\nu\rho}W_L^{\nu\rho},\\
&(D_\mu H^\dagger)(D^\nu H)B_{L\nu\rho}W_L^{\mu\rho},\ (D^\nu H^\dagger)(D_\mu H)B_{L\nu\rho}W_L^{\mu\rho},\ (D_\mu H^\dagger)H(D^\mu B_{L\nu\rho})W_L^{\nu\rho},\ (D_\mu H^\dagger)H(D^\nu B_{L\nu\rho})W_L^{\mu\rho},\\
&(D^\nu H^\dagger)H(D_\mu B_{L\nu\rho})W_L^{\mu\rho},\ (D_\mu H^\dagger)HB_{L\nu\rho}(D^\mu W_L^{\nu\rho}),\ (D_\mu H^\dagger)HB_{L\nu\rho}(D^\nu W_L^{\mu\rho}),\ (D^\nu H^\dagger)HB_{L\nu\rho}(D_\mu W_L^{\mu\rho}),\\
&H^\dagger(D^2H)B_{L\mu\nu}W_L^{\mu\nu},\ H^\dagger(D^\mu D_\nu H)B_{L\mu\rho}W_L^{\nu\rho},\ H^\dagger(D_\nu D^\mu H)B_{L\mu\rho}W_L^{\nu\rho},\ H^\dagger(D^\mu H)(D_\mu B_{L\nu\rho})W_L^{\nu\rho},\\
&H^\dagger(D^\nu H)(D_\mu B_{L\nu\rho})W_L^{\mu\rho},\ H^\dagger(D_\mu H)(D^\nu B_{L\nu\rho})W_L^{\mu\rho},\ H^\dagger(D^\mu H)B_{L\nu\rho}(D_\mu W_L^{\nu\rho}),\ H^\dagger(D^\nu H)B_{L\nu\rho}(D_\mu W_L^{\mu\rho}),\\
&H^\dagger(D_\mu H)B_{L\nu\rho}(D^\nu W_L^{\mu\rho}),\ H^\dagger H(D^2B_{L\mu\nu})W_L^{\mu\nu},\ H^\dagger H(D^\mu D_\nu B_{L\mu\rho})W_L^{\nu\rho},\ H^\dagger H(D_\nu D^\mu B_{L\mu\rho})W_L^{\nu\rho},\\
&H^\dagger H(D^\mu B_{L\nu\rho})(D_\mu W_L^{\nu\rho}),\ H^\dagger H(D^\nu B_{L\nu\rho})(D_\mu W_L^{\mu\rho}),\ H^\dagger H(D_\mu B_{L\nu\rho})(D^\nu W_L^{\mu\rho}),\ H^\dagger HB_{L\mu\nu}(D^2W_L^{\mu\nu}),\\
&H^\dagger HB_{L\mu\rho}(D^\mu D_\nu W_L^{\nu\rho}),\ H^\dagger HB_{L\mu\rho}(D_\nu D^\mu W_L^{\nu\rho}). \qquad (14)
\end{aligned}
$$

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Which 2 should be picked up?

Repeated fields

$QQQL$

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$$
Q^{qqq\ell}_{prst} = C^{prst}
\begin{matrix}
\epsilon^{abc}\epsilon^{ik}\epsilon^{jl}(L_{pi}Q_{raj})(Q_{sbk}Q_{tcl})\\
\epsilon^{abc}\epsilon^{ik}\epsilon^{jl}(L_{pi}Q_{sbk})(Q_{raj}Q_{tcl})\\
\epsilon^{abc}\epsilon^{ij}\epsilon^{kl}(L_{pi}Q_{sbk})(Q_{raj}Q_{tcl})\\
\epsilon^{abc}\epsilon^{ij}\epsilon^{kl}(L_{pi}Q_{raj})(Q_{sbk}Q_{tcl})
\end{matrix}
\qquad p,r,s,t=1,2,3
$$

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What flavor relations should be imposed?

Jiang-Hao Yu

Operator as Group Invariant

Start from the effective operator contributing to muon g-2:

Gauge invariance: gauge factor

$$2 \otimes 1 \otimes 2 \otimes 3 = 1 \oplus 3 \oplus 3 \oplus 5$$

$$\boxed{(\bar{l}_p \sigma^{\mu\nu} e_r)\tau^I \varphi W^I_{\mu\nu}} \qquad \boxed{(\bar{l}_p \sigma^{\mu\nu} e_r)\varphi B_{\mu\nu}}$$

Lorentz invariance: Lorentz indices contracted in pair

$$\left(\frac{1}{2},0\right) \otimes \left(\frac{1}{2},0\right) \otimes (0,0) \otimes [(1,0) \oplus (0,1)] = (0,0) \oplus (1,0) \oplus (2,0) \oplus (1,1) \oplus (1,0) \oplus (0,1)$$

Redundancies: equation of motion, integration by part, covariant derivative commutator

$$X^{\mu\nu}\bar{\psi}\gamma_\mu D_\nu \psi \qquad \varphi\bar{\psi}\sigma^{\mu\nu} D_\mu D_\nu \psi \qquad (D^\mu \varphi)\bar{\psi} D_\mu \psi$$

$$\varphi \bar{l}_p \sigma^{\mu\nu} D_\mu D_\nu e_r \qquad (\bar{l}_p D_\mu e_r) D^\mu \varphi$$

Operator as On-shell Amplitude

EFT operator = Contact amplitude = Group invariant + little group scaling

ψ_α	λ_α
$\psi^\dagger_{\dot\alpha}$	$\tilde\lambda_{\dot\alpha}$
$F^-_{\alpha\beta} = F_{\mu\nu}\sigma^{\mu\nu}_{\alpha\beta}$	$\lambda_\alpha\lambda_\beta$
$F^+_{\dot\alpha\dot\beta} = F_{\mu\nu}\bar\sigma^{\mu\nu}_{\dot\alpha\dot\beta}$	$\tilde\lambda_\alpha\tilde\lambda_\beta$

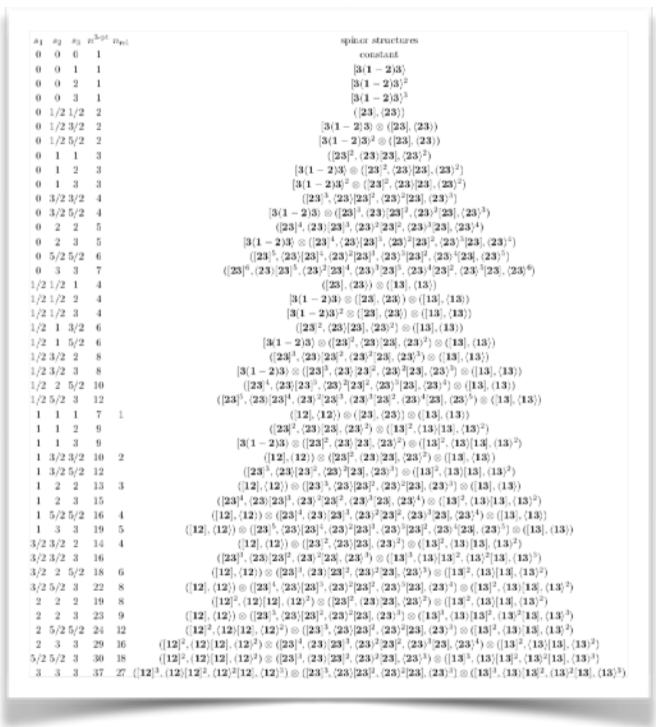
$$\langle ij\rangle = \tilde\lambda_i\epsilon\tilde\lambda_j$$

$$[ij] = \lambda_i\epsilon\lambda_j$$

$$\boxed{(\bar l_p\sigma^{\mu\nu}e_r)\tau^I\varphi W^I_{\mu\nu}} \longrightarrow \lambda_{1\alpha}\lambda_{2\beta}\lambda_4^\alpha\lambda_4^\beta\tau^I \longrightarrow [14][24]\tau^I$$

[Shadmi, Weiss, 2018]
[Ma, Shu, Xiao, 2019]
[Durieux, Kitahara, Shadmi, Weiss, 2019]
[Falkowski, Machado, 2019]

[Durieux, Kitahara, Machado, Shadmi, Weiss, 2020]



Stripped contact term bases for all 4-point amplitudes

[Refer to Machado's talk for more details]

All 3-particle massless amplitudes except F^3 vanish at on-shell

$\cancel{X^{\mu\nu}\bar\psi\gamma_\mu D_\nu\psi}$ $\quad\cancel{\varphi\bar\psi\sigma^{\mu\nu}D_\mu D_\nu\psi}$ $\quad\cancel{(D^\mu\varphi)\bar\psi D_\mu\psi}$

Operator as On-shell Amplitude

Currently hard to systematically construct more than 4-particle amplitude

[Durieux, Kitahara, Machado, Shadmi, Weiss, 2020]

Dim-8 SMEFT

$N+h$ \ $N-h$	0	2	4	6	8
0					F^4
2				$F^2\bar\psi\psi D$, $\psi^4 D^2$, $F\psi^2\phi D^2$, $F^2\phi^2 D^2$	$F\psi^4$, $F^2\psi^2\phi$, $F^3\phi^2$
4			$\bar F^2 F^2$, $\bar F F\bar\psi\psi D$, $\bar\psi^2\psi^2 D^2$, $F\psi^2\phi D^2$, $\bar F F\phi^2 D^2$, $\phi^4 D^4$, $\bar\psi\psi\phi^2 D^3$	$F\bar\psi^2\psi^2$, $F^2\bar\psi^2\phi$, $\bar\psi\psi^3\phi D$, $F\bar\psi\psi\phi^2 D$, $\psi^2\phi^3 D^2$, $F\phi^4 D^2$	$\psi^4\phi^2$, $F\psi^2\phi^3$, $F^2\phi^4$
6		$\bar F^2\bar\psi\psi D$, $\bar\psi^4 D^2$, $\bar F\bar\psi^2\phi D^2$, $\bar F^2\phi^2 D^2$	$\bar F\bar\psi^2\psi^2$, $\bar F^2\psi^2\phi$, $\bar\psi^3\psi\phi D$, $\bar F\bar\psi\psi\phi^2 D$, $\bar\psi^2\phi^3 D^2$, $\bar F\phi^4 D^2$	$\bar\psi^2\psi^2\phi^2$, $\bar\psi\psi\phi^4 D$, $\phi^6 D^2$	$\psi^2\phi^5$
8	$\bar F^4$	$\bar F\bar\psi^4$, $\bar F^2\bar\psi^2\phi$, $\bar F^3\phi^2$	$\bar\psi^4\phi^2$, $\bar F\bar\psi^2\phi^3$, $\bar F^2\phi^4$	$\bar\psi^2\phi^5$	ϕ^8

N = 4

N = 5

N = 6

N = 7

N = 8

g-2 dim-8:

$$\boxed{(\bar l_p\sigma^{\mu\nu}e_r)\tau^I\varphi W^I_{\mu\nu}} \xrightarrow{\text{Add } H^\dagger H} \boxed{\left(\tau^I\right)^k_j W^I_{\mu\nu}\left(e_{\mathbb{C}p}\sigma^{\mu\nu}L_{rk}\right)H^{\dagger j}\left(H^\dagger H\right)}$$

$$\boxed{(\bar l_p\sigma^{\mu\nu}e_r)\tau^I\varphi W^I_{\mu\nu}} \xrightarrow{\text{Add } D^2} \boxed{\left(\tau^I\right)^i_j W^I_{\mu\lambda}\left(e_{\mathbb{C}p}\sigma^{\nu\lambda}L_{ri}\right)D^\mu D_\nu H^{\dagger j}}$$

Operator as Spinor Tensor

Consider dim-8 g-2 operator with derivatives

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$$\boxed{(\bar{l}_p\sigma^{\mu\nu}e_r)\tau^I\varphi W^I_{\mu\nu}} \xrightarrow{\text{Ways to add } D^2}$$

$$\begin{gathered}
D^\mu D^\nu W_{\mu\nu}(e_{cp}L_r)H^\dagger,\ D^\mu W_{\mu\nu}(D^\nu e_{cp}L_r)H^\dagger,\ D^\mu W_{\mu\nu}(e_{cp}D^\nu L_r)H^\dagger,\ D^\mu W_{\mu\nu}(e_{cp}L_r)D^\nu H^\dagger,\\
W_{\mu\nu}(D^\mu D^\nu e_{cp}L_r)H^\dagger,\ W_{\mu\nu}(D^\mu e_{cp}D^\nu L_r)H^\dagger,\ W_{\mu\nu}(D^\mu e_{cp}L_r)D^\nu H^\dagger,\ W_{\mu\nu}(e_{cp}D^\mu D^\nu L_r)H^\dagger,\\
W_{\mu\nu}(e_{cp}D^\mu L_r)D^\nu H^\dagger,\ W_{\mu\nu}(e_{cp}L_r)D^\mu D^\nu H^\dagger,\ D^\mu D_\nu W_{\mu\lambda}(e_{cp}\sigma^{\nu\lambda}L_r)H^\dagger,\ D_\nu D^\mu W_{\mu\lambda}(e_{cp}\sigma^{\nu\lambda}L_r)H^\dagger,\\
D^\mu W_{\mu\lambda}(D_\nu e_{cp}\sigma^{\nu\lambda}L_r)H^\dagger,\ D_\nu W_{\mu\lambda}(D^\mu e_{cp}\sigma^{\nu\lambda}L_r)H^\dagger,\ D^\mu W_{\mu\lambda}(e_{cp}\sigma^{\nu\lambda}D_\nu L_r)H^\dagger,\ D_\nu W_{\mu\lambda}(e_{cp}\sigma^{\nu\lambda}D^\mu L_r)H^\dagger,\\
D^\mu W_{\mu\lambda}(e_{cp}\sigma^{\nu\lambda}L_r)D_\nu H^\dagger,\ D_\nu W_{\mu\lambda}(e_{cp}\sigma^{\nu\lambda}L_r)D^\mu H^\dagger,\ W_{\mu\lambda}(D^\mu D_\nu e_{cp}\sigma^{\nu\lambda}L_r)H^\dagger,\ W_{\mu\lambda}(D_\nu D^\mu e_{cp}\sigma^{\nu\lambda}L_r)H^\dagger,\\
W_{\mu\lambda}(D^\mu e_{cp}\sigma^{\nu\lambda}D_\nu L_r)H^\dagger,\ W_{\mu\lambda}(D_\nu e_{cp}\sigma^{\nu\lambda}D^\mu L_r)H^\dagger,\ W_{\mu\lambda}(D^\mu e_{cp}\sigma^{\nu\lambda}L_r)D_\nu H^\dagger,\ W_{\mu\lambda}(D_\nu e_{cp}\sigma^{\nu\lambda}L_r)D^\mu H^\dagger,\\
W_{\mu\lambda}(e_{cp}\sigma^{\nu\lambda}D^\mu D_\nu L_r)H^\dagger,\ W_{\mu\lambda}(e_{cp}\sigma^{\nu\lambda}D_\nu D^\mu L_r)H^\dagger,\ W_{\mu\lambda}(e_{cp}\sigma^{\nu\lambda}D^\mu L_r)D_\nu H^\dagger,\ W_{\mu\lambda}(e_{cp}\sigma^{\nu\lambda}D_\nu L_r)D^\mu H^\dagger,\\
W_{\mu\lambda}(e_{cp}\sigma^{\nu\lambda}L_r)D^\mu D_\nu H^\dagger,\ W_{\mu\lambda}(e_{cp}\sigma^{\nu\lambda}L_r)D_\nu D^\mu H^\dagger,\ \{W\to\tilde{W}\}
\end{gathered}$$

Each field belongs to a SL(2,C) irrep

$$\boxed{H_i \in (0,0) \quad \psi_\alpha \in (1/2,0) \quad F_{\mathrm{L}\alpha\beta} = \frac{i}{2}F_{\mu\nu}\sigma^{\mu\nu}_{\alpha\beta} \in (1,0) \quad D_{\alpha\dot\alpha} = D_\mu\sigma^\mu_{\alpha\dot\alpha} \in (1/2,1/2),}$$

Operator with explicit spinor indices

$$\epsilon_{\alpha_1\alpha_3}\epsilon_{\alpha_1\alpha_3}\epsilon_{\alpha_2\alpha_4}\epsilon^{\dot\alpha_3\dot\alpha_4}F_1^{\alpha_1^2}\psi_2^{\alpha_2}(D\psi_3)^{\alpha_3^2}_{\dot\alpha_3}(D\phi_4)^{\alpha_4}_{\dot\alpha_4}$$

$$F_1^{\alpha\beta}\psi_2^\gamma(D\psi_3)_{\alpha\beta\dot\alpha}(D\phi_4)_\gamma{}^{\dot\alpha}$$

How to obtain independent operator with derivatives systematically?

Equation of Motion (EOM)

For fields with derivatives, symmetric and antisymmetric indices:

$$(D\psi)_{\alpha\beta\dot{\alpha}} = -\frac{1}{2}\epsilon_{\alpha\beta}(\not{D}\psi)_{\dot{\alpha}} + \frac{1}{2}(D\psi)_{(\alpha\beta)\dot{\alpha}}$$

$$\left(\frac{1}{2},\frac{1}{2}\right)\otimes\left(\frac{1}{2},0\right)=\left(0,\frac{1}{2}\right)\oplus\left(1,\frac{1}{2}\right)$$

$\left(\frac{1}{2},\frac{1}{2}\right)\times\left(\frac{1}{2},0\right)$ **(0,1/2)** **(1,1/2)**

$$(D^2\phi)_{\alpha\beta\dot{\alpha}\dot{\beta}} = \frac{1}{2}\epsilon_{\alpha\beta}\epsilon_{\dot{\alpha}\dot{\beta}}D^\mu D_\mu\phi - \frac{i}{4}\epsilon_{\dot{\alpha}\dot{\beta}}\sigma^{\mu\nu}_{\alpha\beta}[D_\mu, D_\nu]\phi - \frac{i}{4}\epsilon_{\alpha\beta}\bar{\sigma}^{\mu\nu}_{\dot{\alpha}\dot{\beta}}[D_\mu, D_\nu]\phi + \frac{1}{4}(D^2\phi)_{(\alpha\beta)(\dot{\alpha}\dot{\beta})}$$

$\left(\frac{1}{2},\frac{1}{2}\right)\times\left(\frac{1}{2},\frac{1}{2}\right)$ **(0,0)** **(1,0)** **(0,1)** **(1,1)**

Only take the highest weight for field with derivatives

$$\boxed{D^w\Psi \in \left(j_l + \frac{w}{2}, j_r + \frac{w}{2}\right) \oplus \cancel{\text{lower weights}}}$$

with totally symmetric spinor indices

EOM removed by taking highest weight!

Covariant derivative commutator, Bianchi identity also removed

Operator as Spinor Tensor

Any operator can be written with totally symmetric spinor indices:

$$\boxed{\mathcal{O} = (\epsilon^{\alpha_i\alpha_j})^n (\tilde{\epsilon}_{\dot{\alpha}_i\dot{\alpha}_j})^{\tilde{n}} \prod_{i=1}^{N} (D^{r_i-|h_i|}\Phi_i)^{\dot{\alpha}_i^{r_i+h_i}}_{\alpha_i^{r_i-h_i}}}$$

$$\epsilon_{\alpha_1\alpha_3}\epsilon_{\alpha_1\alpha_3}\epsilon_{\alpha_2\alpha_4}\epsilon^{\dot{\alpha}_3\dot{\alpha}_4} F_1^{\alpha_1^2}\psi_2^{\alpha_2}(D\psi_3)^{\alpha_3^2}_{\dot{\alpha}_3}(D\phi_4)^{\alpha_4}_{\dot{\alpha}_4}$$

Transformation under SL(2,C) x SU(N)

$$\boxed{\epsilon^{\alpha_i\alpha_j} \to \sum_{k,l} \mathcal{U}^i_k \mathcal{U}^j_l \epsilon^{\alpha_k\alpha_l}, \quad \tilde{\epsilon}_{\dot{\alpha}_i\dot{\alpha}_j} \to \sum_{k,l} \mathcal{U}^{\dagger k}_{\ i} \mathcal{U}^{\dagger l}_{\ j} \tilde{\epsilon}_{\dot{\alpha}_k\dot{\alpha}_l}.}$$

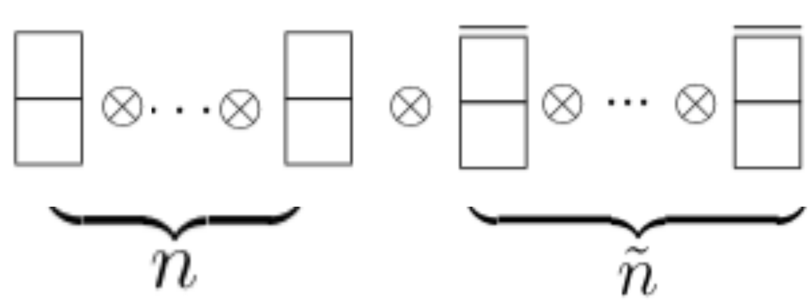
$i, j, k, l = 1 \text{ to } N$

 $= [1^2]$

 $= [1^{N-2}]$

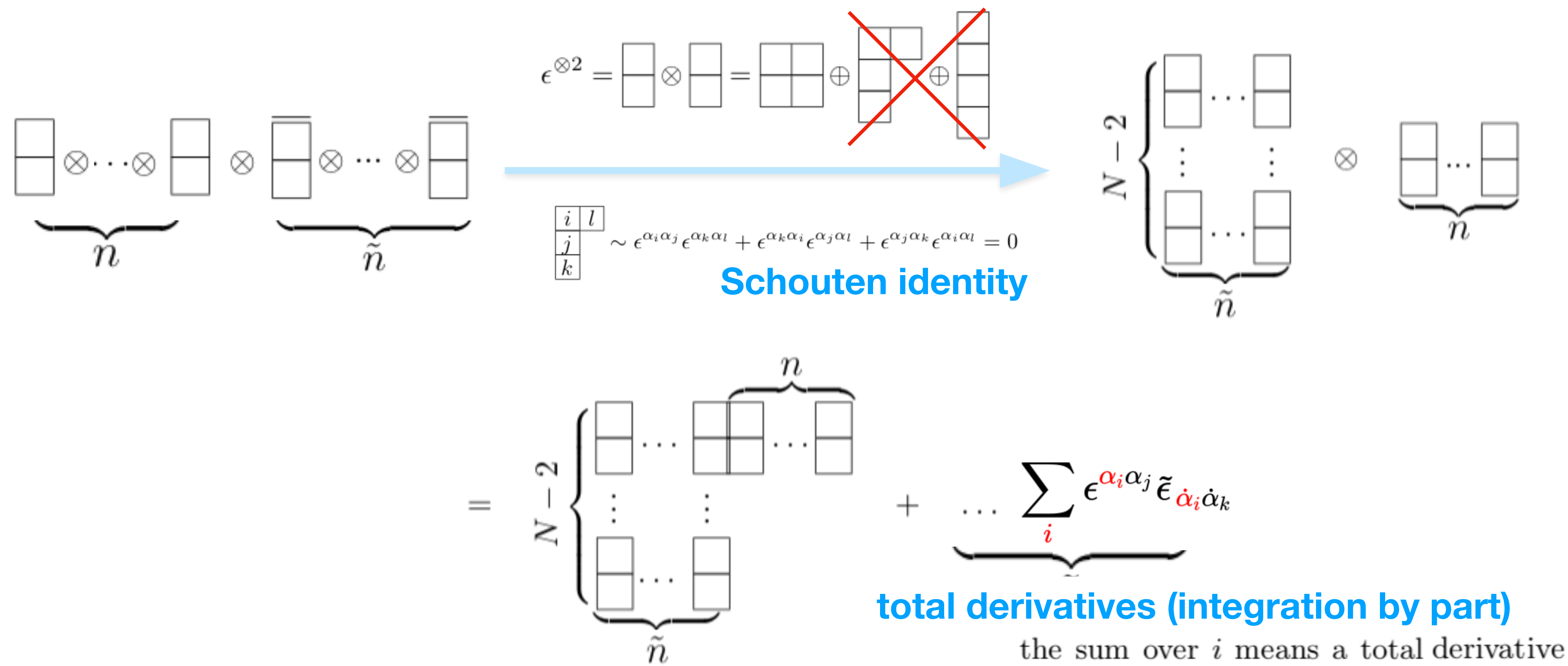
$\epsilon^{\alpha_i\alpha_j}$

$\mathcal{E}^{ijk_1,\ldots,k_{N-2}}\, \tilde{\epsilon}_{\dot{\alpha}_i\dot{\alpha}_j}$



= **Irrep** $\oplus$ … $\oplus$ **Irrep**

Total Derivatives



[Such Young diagram also obtained from conformal K harmonics]

[Henning, Melia, 2019]

Differently we obtain Young diagram using epsilon tensor transformation

No need conformal symmetry!

Independent Lorentz Structure

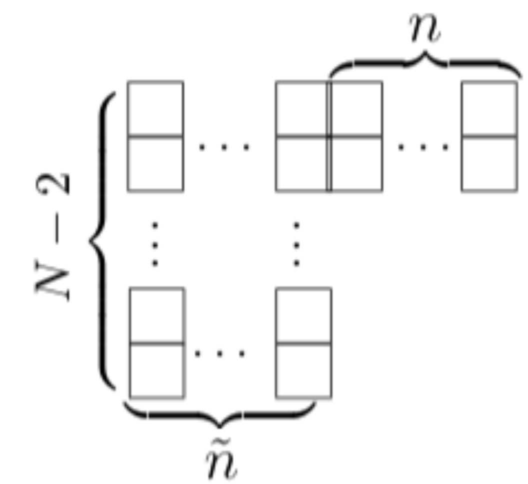
To obtain independent operator, we invent a new Young diagram filling procedure!

Filling rules on semi-standard Young tableau (SSYT)

with given class

$$\{\underbrace{1,\dots,1}_{\#1},\underbrace{2,\dots,2}_{\#2},\dots\} \qquad \#i = \tilde{n} - 2h_i$$

$$Y_{N,n,\tilde{n}} =$$



Fock's condition removes redundancy

Basis
- **YT method guarantees independence!**
- **Filling all SSYT guarantees completeness!**

$$\boxed{\left(\tau^I\right)^i_j W^I_{\mu\lambda}\left(e_{\mathbb{C}p}\sigma^{\nu\lambda}L_{ri}\right)D^\mu D_\nu H^{\dagger j}}$$

$$(\tilde{n}=1, n=3)$$

$$\#1 = 3, \#2 = \#3 = 2, \#4 = 1.$$

New filling: any operator could be converted to this basis

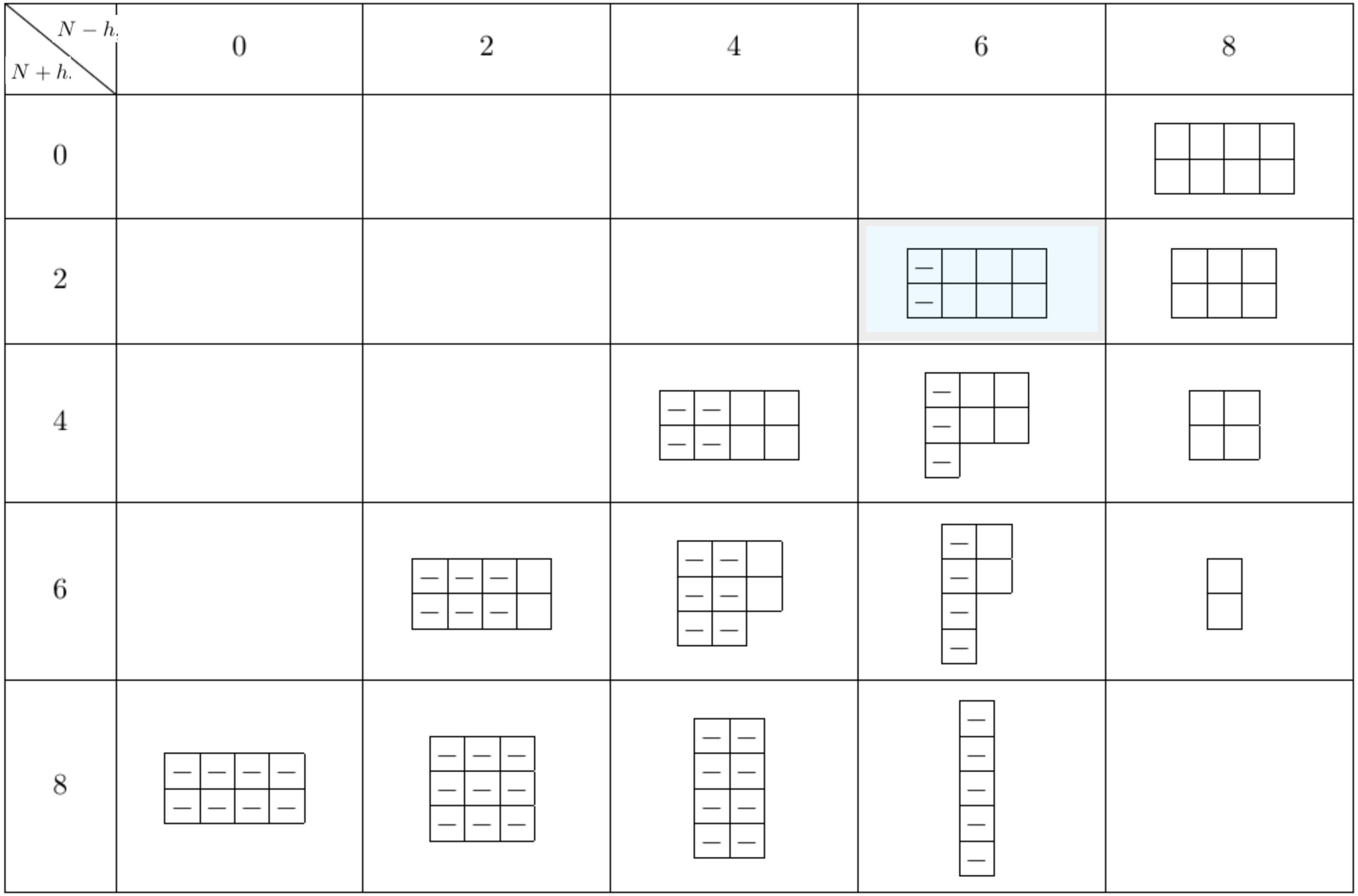
1	1	1	2
2	3	3	4

$\epsilon_{\alpha_1\alpha_3}\epsilon_{\alpha_1\alpha_3}\epsilon_{\alpha_2\alpha_4}\epsilon^{\dot\alpha_3\dot\alpha_4}$ $\quad F_{\rm L1}^{\alpha\beta}\psi_2^{\gamma}\left(D\psi_3\right)_{\alpha\beta\dot\alpha}\left(D\phi_4\right)_{\gamma}{}^{\dot\alpha}$ $\quad \langle 13\rangle\langle 13\rangle\langle 24\rangle[34]$

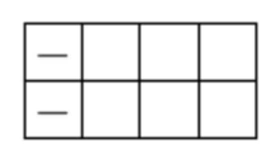
1	1	1	3
2	2	3	4

$\epsilon_{\alpha_1\alpha_2}\epsilon_{\alpha_1\alpha_3}\epsilon_{\alpha_3\alpha_4}\epsilon^{\dot\alpha_3\dot\alpha_4}$ $\quad F_{\rm L1}^{\alpha\beta}\psi_{2\alpha}\left(D\psi_3\right)_{\beta}{}^{\gamma}{}_{\dot\alpha}\left(D\phi_4\right)_{\gamma}{}^{\dot\alpha}$ $\quad \langle 12\rangle\langle 13\rangle\langle 34\rangle[34]$

Dim-8 Young Diagrams



Different filling corresponds different operator!



$F^2\bar{\psi}\psi D$, $\psi^4 D^2$,
$F\psi^2\phi D^2$, $F^2\phi^2 D^2$

Different Operator with Same YD

$(\tilde{n}=1, n=3)$

$\boxed{W e_{\mathbb{C}} L H^\dagger D^2}$ $\quad \#1=3, \#2=\#3=2, \#4=1.$

1	1	1	2
2	3	3	4

1	1	1	3
2	2	3	4

$\epsilon_{\alpha_1\alpha_3}\epsilon_{\alpha_1\alpha_3}\epsilon_{\alpha_2\alpha_4}\epsilon^{\dot{\alpha}_3\dot{\alpha}_4}$ $\quad \epsilon_{\alpha_1\alpha_2}\epsilon_{\alpha_1\alpha_3}\epsilon_{\alpha_3\alpha_4}\epsilon^{\dot{\alpha}_3\dot{\alpha}_4}$

$F_{\rm L1}^{\alpha\beta}\psi_2^{\gamma}\left(D\psi_3\right)_{\alpha\beta\dot{\alpha}}\left(D\phi_4\right)_{\gamma}{}^{\dot{\alpha}}$ $\quad F_{\rm L1}^{\alpha\beta}\psi_{2\alpha}\left(D\psi_3\right)_{\beta}{}^{\gamma}{}_{\dot{\alpha}}\left(D\phi_4\right)_{\gamma}{}^{\dot{\alpha}}$

$\boxed{B W H H^\dagger D^2}$ $\quad \#1=3, \#2=3, \#3=1, \#4=1$

1	1	1	3
2	2	2	4

1	1	1	2
2	2	3	4

$\epsilon_{\dot{\alpha}_3\dot{\alpha}_4}\epsilon^{\alpha_1\alpha_2}\epsilon^{\alpha_1\alpha_2}\epsilon^{\alpha_3\alpha_4}$ $\quad \epsilon_{\dot{\alpha}_3\dot{\alpha}_4}\epsilon^{\alpha_1\alpha_2}\epsilon^{\alpha_1\alpha_3}\epsilon^{\alpha_2\alpha_4}$

$B_L{}^{\alpha\beta}W_{L\alpha\beta}\left(DH^\dagger\right)^{\gamma}{}_{\dot{\alpha}}\left(DH\right)_{\gamma}{}^{\dot{\alpha}},$ $\quad B_L{}^{\alpha\beta}W_{L\alpha}{}^{\gamma}\left(DH^\dagger\right)_{\beta\dot{\alpha}}\left(DH\right)_{\gamma}{}^{\dot{\alpha}}$

Traditional vs Young Tensor

$(\tilde{n}=1, n=3)$

$\boxed{W e_{\mathbb{C}} L H^\dagger D^2}$ $\#1=3, \#2=\#3=2, \#4=1.$

$$\begin{array}{|c|c|c|c|}\hline 1&1&1&2\\\hline 2&3&3&4\\\hline\end{array} \qquad \begin{array}{|c|c|c|c|}\hline 1&1&1&3\\\hline 2&2&3&4\\\hline\end{array}$$

$\epsilon_{\alpha_1\alpha_3}\epsilon_{\alpha_1\alpha_3}\epsilon_{\alpha_2\alpha_4}\epsilon^{\dot\alpha_3\dot\alpha_4}$ $\epsilon_{\alpha_1\alpha_2}\epsilon_{\alpha_1\alpha_3}\epsilon_{\alpha_3\alpha_4}\epsilon^{\dot\alpha_3\dot\alpha_4}$

$F_{\rm L1}^{\alpha\beta}\psi_2^{\gamma}\,(D\psi_3)_{\alpha\beta\dot\alpha}\,(D\phi_4)_\gamma{}^{\dot\alpha}$ $F_{\rm L1}^{\alpha\beta}\psi_{2\alpha}\,(D\psi_3)_\beta{}^\gamma{}_{\dot\alpha}\,(D\phi_4)_\gamma{}^{\dot\alpha}$

$\boxed{B W H H^\dagger D^2}$ $\#1=3, \#2=3, \#3=1, \#4=1$

$$\begin{array}{|c|c|c|c|}\hline 1&1&1&3\\\hline 2&2&2&4\\\hline\end{array} \qquad \begin{array}{|c|c|c|c|}\hline 1&1&1&2\\\hline 2&2&3&4\\\hline\end{array}$$

$\epsilon_{\dot\alpha_3\dot\alpha_4}\epsilon^{\alpha_1\alpha_2}\epsilon^{\alpha_1\alpha_2}\epsilon^{\alpha_3\alpha_4}$ $\epsilon_{\dot\alpha_3\dot\alpha_4}\epsilon^{\alpha_1\alpha_2}\epsilon^{\alpha_1\alpha_3}\epsilon^{\alpha_2\alpha_4}$

$B_L{}^{\alpha\beta}W_{L\alpha\beta}\,(DH^\dagger)^\gamma{}_{\dot\alpha}\,(DH)_\gamma{}^{\dot\alpha}$ 2 $B_L{}^{\alpha\beta}W_{L\alpha}{}^\gamma\,(DH^\dagger)_{\beta\dot\alpha}\,(DH)_\gamma{}^{\dot\alpha}$

Young tensor method (No need EoM&IBP)

Traditional method

$\boxed{B W H H^\dagger D^2}$

[Hays, Martin, Sanz, Setford, 2018]

$$(D^2H^\dagger)HB_{L\mu\nu}W_L^{\mu\nu},\ (D^\mu D_\nu H^\dagger)HB_{L\mu\rho}W_L^{\nu\rho},\ (D_\nu D^\mu H^\dagger)HB_{L\mu\rho}W_L^{\nu\rho},\ (D_\mu H^\dagger)(D^\mu H)B_{L\nu\rho}W_L^{\nu\rho},$$
$$(D_\mu H^\dagger)(D^\nu H)B_{L\nu\rho}W_L^{\mu\rho},\ (D^\nu H^\dagger)(D_\mu H)B_{L\nu\rho}W_L^{\mu\rho},\ (D_\mu H^\dagger)H(D^\mu B_{L\nu\rho})W_L^{\nu\rho},\ (D_\mu H^\dagger)H(D^\nu B_{L\nu\rho})W_L^{\mu\rho},$$
$$(D^\nu H^\dagger)H(D_\mu B_{L\nu\rho})W_L^{\mu\rho},\ (D_\mu H^\dagger)HB_{L\nu\rho}(D^\mu W_L^{\nu\rho}),\ (D_\mu H^\dagger)HB_{L\nu\rho}(D^\nu W_L^{\mu\rho}),\ (D^\nu H^\dagger)HB_{L\nu\rho}(D_\mu W_L^{\mu\rho}),$$
$$H^\dagger(D^2H)B_{L\mu\nu}W_L^{\mu\nu},\ H^\dagger(D^\mu D_\nu H)B_{L\mu\rho}W_L^{\nu\rho},\ H^\dagger(D_\nu D^\mu H)B_{L\mu\rho}W_L^{\nu\rho},\ H^\dagger(D^\mu H)(D_\mu B_{L\nu\rho})W_L^{\nu\rho},$$
$$H^\dagger(D^\nu H)(D_\mu B_{L\nu\rho})W_L^{\mu\rho},\ H^\dagger(D_\mu H)(D^\nu B_{L\nu\rho})W_L^{\mu\rho},\ H^\dagger(D^\mu H)B_{L\nu\rho}(D_\mu W_L^{\nu\rho}),\ H^\dagger(D^\nu H)B_{L\nu\rho}(D_\mu W_L^{\mu\rho}),$$
$$H^\dagger(D_\mu H)B_{L\nu\rho}(D^\nu W_L^{\mu\rho}),\ H^\dagger H(D^2B_{L\mu\nu})W_L^{\mu\nu},\ H^\dagger H(D^\mu D_\nu B_{L\mu\rho})W_L^{\nu\rho},\ H^\dagger H(D_\nu D^\mu B_{L\mu\rho})W_L^{\nu\rho},$$
$$H^\dagger H(D^\mu B_{L\nu\rho})(D_\mu W_L^{\nu\rho}),\ H^\dagger H(D^\nu B_{L\nu\rho})(D_\mu W_L^{\mu\rho}),\ H^\dagger H(D_\mu B_{L\nu\rho})(D^\nu W_L^{\mu\rho}),\ H^\dagger HB_{L\mu\nu}(D^2W_L^{\mu\nu}),$$
$$H^\dagger HB_{L\mu\rho}(D^\mu D_\nu W_L^{\nu\rho}),\ H^\dagger HB_{L\mu\rho}(D_\nu D^\mu W_L^{\nu\rho}). \tag{14}$$

30

EOM

$$(DH^\dagger)_{\alpha\dot\alpha}(DH)_{\beta\dot\beta}B_{L\{\gamma\delta\}}W_{L\{\xi\eta\}}\,\epsilon^{\dot\alpha\dot\beta}\epsilon^{\alpha\beta}\epsilon^{\gamma\xi}\epsilon^{\delta\eta}$$
$$(DH^\dagger)_{\alpha\dot\alpha}(DH)_{\beta\dot\beta}B_{L\{\gamma\delta\}}W_{L\{\xi\eta\}}\,\tfrac{1}{2}\epsilon^{\dot\alpha\dot\beta}\epsilon^{\delta\xi}(\epsilon^{\alpha\gamma}\epsilon^{\beta\eta}+\epsilon^{\beta\gamma}\epsilon^{\alpha\eta})$$
$$(DH^\dagger)_{\alpha\dot\alpha}\,H\,(DB_L)_{\{\beta\gamma\delta\},\dot\beta}W_{L\{\xi\eta\}}\,\epsilon^{\dot\alpha\dot\beta}\epsilon^{\alpha\beta}\epsilon^{\gamma\xi}\epsilon^{\delta\eta}$$
$$(DH^\dagger)_{\alpha\dot\alpha}\,H\,B_{L\{\xi\eta\}}(DW_L)_{\{\beta\gamma\delta\},\dot\beta}\,\epsilon^{\dot\alpha\dot\beta}\epsilon^{\alpha\beta}\epsilon^{\gamma\xi}\epsilon^{\delta\eta}$$
$$H^\dagger\,(DH)_{\alpha\dot\alpha}\,(DB_L)_{\{\beta\gamma\delta\},\dot\beta}W_{L\{\xi\eta\}}\,\epsilon^{\dot\alpha\dot\beta}\epsilon^{\alpha\beta}\epsilon^{\gamma\xi}\epsilon^{\delta\eta}$$
$$H^\dagger\,(DH)_{\alpha\dot\alpha}\,B_{L\{\xi\eta\}}(DW_L)_{\{\beta\gamma\delta\},\dot\beta}\,\epsilon^{\dot\alpha\dot\beta}\epsilon^{\alpha\beta}\epsilon^{\gamma\xi}\epsilon^{\delta\eta}$$
$$H^\dagger H\,(DB_L)_{\{\alpha\beta\gamma\},\dot\alpha}(DW_L)_{\{\xi\eta\delta\},\dot\beta}\,\epsilon^{\dot\alpha\dot\beta}\epsilon^{\alpha\xi}\epsilon^{\beta\eta}\epsilon^{\gamma\delta}$$

7

IBP

$$B_L{}^{\alpha\beta}W_{L\alpha\beta}\,(DH^\dagger)^\gamma{}_{\dot\alpha}\,(DH)_\gamma{}^{\dot\alpha}$$
$$B_L{}^{\alpha\beta}W_{L\alpha}{}^\gamma\,(DH^\dagger)_{\beta\dot\alpha}\,(DH)_\gamma{}^{\dot\alpha}$$

2

Gauge Structure

Gauge structure (internal sym) is easier than Lorentz structure (spacetime sym)

Dim-6 four fermion B-conserving operators: 25

[Grzadkowski, Iskrzynski, Misiak, Rosiek, 2010]

$(\bar{L}L)(\bar{L}L)$		$(\bar{R}R)(\bar{R}R)$		$(\bar{L}L)(\bar{R}R)$	
Q_{ll}	$(\bar{l}_p\gamma_\mu l_r)(\bar{l}_s\gamma^\mu l_t)$	Q_{ee}	$(\bar{e}_p\gamma_\mu e_r)(\bar{e}_s\gamma^\mu e_t)$	Q_{le}	$(\bar{l}_p\gamma_\mu l_r)(\bar{e}_s\gamma^\mu e_t)$
$Q_{qq}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{q}_s\gamma^\mu q_t)$	Q_{uu}	$(\bar{u}_p\gamma_\mu u_r)(\bar{u}_s\gamma^\mu u_t)$	Q_{lu}	$(\bar{l}_p\gamma_\mu l_r)(\bar{u}_s\gamma^\mu u_t)$
$Q_{qq}^{(3)}$	$(\bar{q}_p\gamma_\mu\tau^I q_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$	Q_{dd}	$(\bar{d}_p\gamma_\mu d_r)(\bar{d}_s\gamma^\mu d_t)$	Q_{ld}	$(\bar{l}_p\gamma_\mu l_r)(\bar{d}_s\gamma^\mu d_t)$
$Q_{lq}^{(1)}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{q}_s\gamma^\mu q_t)$	Q_{eu}	$(\bar{e}_p\gamma_\mu e_r)(\bar{u}_s\gamma^\mu u_t)$	Q_{qe}	$(\bar{q}_p\gamma_\mu q_r)(\bar{e}_s\gamma^\mu e_t)$
$Q_{lq}^{(3)}$	$(\bar{l}_p\gamma_\mu\tau^I l_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$	Q_{ed}	$(\bar{e}_p\gamma_\mu e_r)(\bar{d}_s\gamma^\mu d_t)$	$Q_{qu}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{u}_s\gamma^\mu u_t)$
		$Q_{ud}^{(1)}$	$(\bar{u}_p\gamma_\mu u_r)(\bar{d}_s\gamma^\mu d_t)$	$Q_{qu}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{u}_s\gamma^\mu T^A u_t)$
		$Q_{ud}^{(8)}$	$(\bar{u}_p\gamma_\mu T^A u_r)(\bar{d}_s\gamma^\mu T^A d_t)$	$Q_{qd}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{d}_s\gamma^\mu d_t)$
				$Q_{qd}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{d}_s\gamma^\mu T^A d_t)$

$(\bar{L}R)(\bar{R}L)$ and $(\bar{L}R)(\bar{L}R)$		B-violating	
Q_{ledq}	$(\bar{l}_p^j e_r)(\bar{d}_s q_t^j)$	Q_{duq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(q_s^{\gamma j})^T C l_t^k\right]$
$Q_{quqd}^{(1)}$	$(\bar{q}_p^j u_r)\varepsilon_{jk}(\bar{q}_s^k d_t)$	Q_{qqu}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{quqd}^{(8)}$	$(\bar{q}_p^j T^A u_r)\varepsilon_{jk}(\bar{q}_s^k T^A d_t)$	Q_{qqq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jn}\varepsilon_{km}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$
$Q_{lequ}^{(1)}$	$(\bar{l}_p^j e_r)\varepsilon_{jk}(\bar{q}_s^k u_t)$	Q_{duu}	$\varepsilon^{\alpha\beta\gamma}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{lequ}^{(3)}$	$(\bar{l}_p^j\sigma_{\mu\nu}e_r)\varepsilon_{jk}(\bar{q}_s^k\sigma^{\mu\nu}u_t)$		

Buchmuller&Wyler wrote 29: 5 redundant operators (Fierz) + 1 missing

$$(\bar{l}_p\gamma_\mu\tau^I l_r)(\bar{l}_s\tau^I\gamma^\mu l_t) = 2Q_{ll}^{ptsr} - Q_{ll}^{prst}$$

Fierz identity for SU(N): $\boxed{\sum_a (T_a)_{ij}(T_a)_{kl} = \delta_{il}\delta_{kj} - \frac{1}{N}\delta_{ij}\delta_{kl}}$

Gauge Structure

How to obtain independent and complete gauge structure systematically?

g-2 dim 8 operator

$W e_{\mathbb{C}} L H^\dagger D^2$ $\quad \boxed{\left(\tau^I\right)^i_j W^I_{\mu\nu} \left(e_{\mathbb{C}p} D^\mu L_{ri}\right) D^\nu H^{\dagger j}}$ $\quad \boxed{\left(\tau^I\right)^i_j W^I_{\mu\lambda} \left(e_{\mathbb{C}p} \sigma^{\nu\lambda} L_{ri}\right) D^\mu D_\nu H^{\dagger j}}$

$W e_{\mathbb{C}} L H H^{\dagger 2}$ $\quad \boxed{\left(\tau^I\right)^k_j W^I_{\mu\nu} \left(e_{\mathbb{C}p} \sigma^{\mu\nu} L_{rk}\right) H^{\dagger j} \left(H^\dagger H\right)}$

?

We invent Littlewood-Richardson method at Young tableau level

$${\tau^I}_{ij} W^I\!: \begin{array}{|c|c|}\hline i & j \\ \hline\end{array}\,,\ L_k\!: \begin{array}{|c|}\hline k \\ \hline\end{array}\,,\ H_l\!: \begin{array}{|c|}\hline l \\ \hline\end{array}\,,\ H^\dagger_m H^\dagger_n\!: \begin{array}{|c|c|}\hline m & n \\ \hline\end{array}$$

$$\begin{array}{|c|c|}\hline i & j \\ \hline\end{array} \xrightarrow{\begin{array}{|c|}\hline k \\ \hline\end{array}} \begin{array}{|c|c|c|}\hline i & j & k \\ \hline\end{array} \xrightarrow{\begin{array}{|c|}\hline l \\ \hline\end{array}} \begin{array}{|c|c|c|}\hline i & j & k \\ \hline l \\ \cline{1-1}\end{array} \xrightarrow{\begin{array}{|c|c|}\hline m & n \\ \hline\end{array}} \begin{array}{|c|c|c|}\hline i & j & k \\ \hline l & m & n \\ \hline\end{array} \qquad \epsilon^{il}\epsilon^{jm}\epsilon^{kn} \qquad W^I L_k H^{\dagger k}\left(H^\dagger \tau H\right)$$

$$\begin{array}{|c|c|}\hline i & j \\ \hline\end{array} \xrightarrow{\begin{array}{|c|}\hline k \\ \hline\end{array}} \begin{array}{|c|c|}\hline i & j \\ \hline k \\ \cline{1-1}\end{array} \xrightarrow{\begin{array}{|c|}\hline l \\ \hline\end{array}} \begin{array}{|c|c|c|}\hline i & j & l \\ \hline k \\ \cline{1-1}\end{array} \xrightarrow{\begin{array}{|c|c|}\hline m & n \\ \hline\end{array}} \begin{array}{|c|c|c|}\hline i & j & l \\ \hline k & m & n \\ \hline\end{array} \qquad \epsilon^{ik}\epsilon^{jm}\epsilon^{ln} \qquad \left(\tau^I\right)^k_j W^I L_k H^{\dagger j}\left(H^\dagger H\right)$$

Find the 4-th g-2 dim 8 operator: $\quad \boxed{W^I_{\mu\nu} \left(e_{\mathbb{C}p} \sigma^{\mu\nu} L_{ri}\right) H^{\dagger i} \left(H^\dagger \tau^I H\right)}$

Operator Y-Basis

Direct product of Lorentz and gauge structures gives operator Y-basis

$W e_{\mathbb{C}} L H^\dagger D^2$

$$\left(\begin{array}{|c|c|c|c|}\hline 1&1&1&2\\\hline 2&3&3&4\\\hline\end{array} + \begin{array}{|c|c|c|c|}\hline 1&1&1&3\\\hline 2&2&3&4\\\hline\end{array} \right) \times \begin{array}{|c|c|}\hline i&j\\\hline k&l\\\hline\end{array} = \boxed{(\tau^I)^i_j W^I_{\mu\nu} (e_{\mathbb{C}p} D^\mu L_{ri}) D^\nu H^{\dagger j}} + \boxed{(\tau^I)^i_j W^I_{\mu\lambda} (e_{\mathbb{C}p} \sigma^{\nu\lambda} L_{ri}) D^\mu D_\nu H^{\dagger j}}$$

$W e_{\mathbb{C}} L H H^{\dagger 2}$

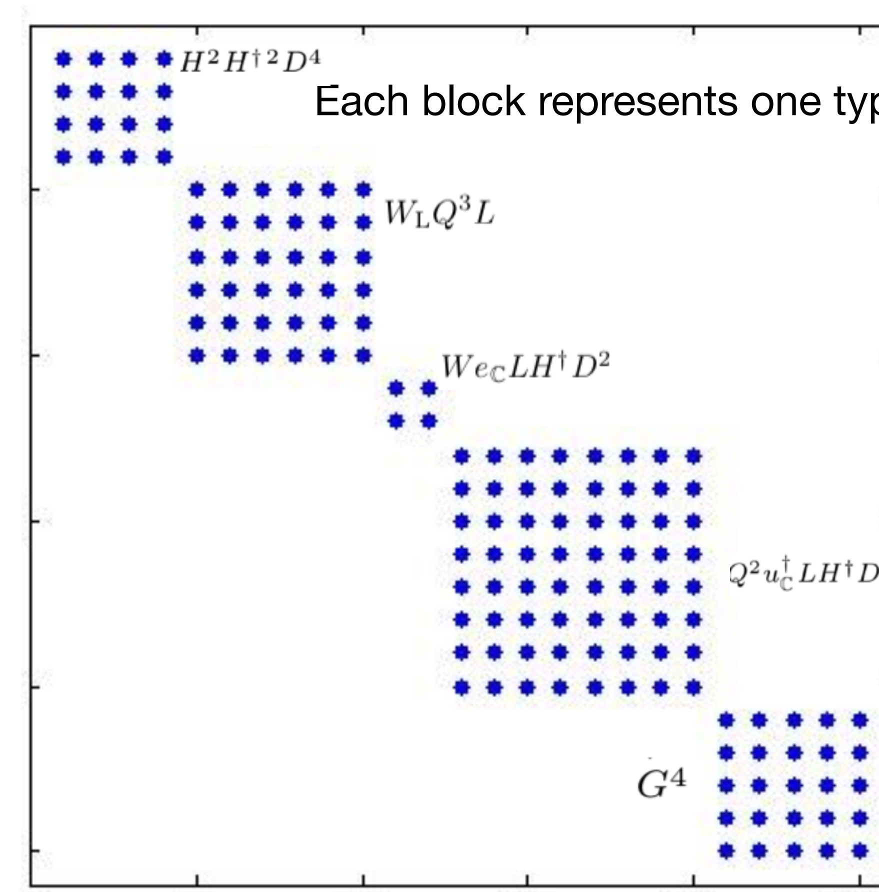
$$\begin{array}{|c|c|}\hline 1&1\\\hline 2&3\\\hline\end{array} \times \left(\begin{array}{|c|c|c|}\hline i&j&l\\\hline k&m&n\\\hline\end{array} + \begin{array}{|c|c|c|}\hline i&j&k\\\hline l&m&n\\\hline\end{array} \right) = \boxed{(\tau^I)^k_j W^I_{\mu\nu} (e_{\mathbb{C}p} \sigma^{\mu\nu} L_{rk}) H^{\dagger j} (H^\dagger H)} + \boxed{W^I_{\mu\nu} (e_{\mathbb{C}p} \sigma^{\mu\nu} L_{ri}) H^{\dagger i} (H^\dagger \tau^I H)}$$

Each type of operator forms

a linear operator space

Complete sets of dim-8 operators forms

a block-diagonal linear space



Each block represents one type of operator

no meaning for x and y axis

Operators with Repeated Field

The Y-basis needs to be reorganized if repeated field in type

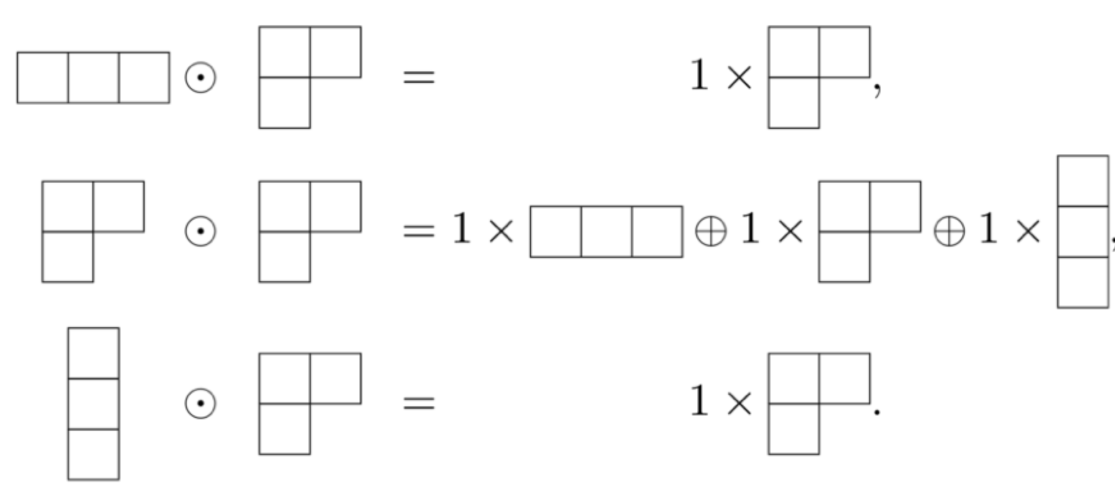
$G_L d_c^3 e_c^\dagger D$

$$\left(\begin{array}{|c|c|c|c|}\hline 1&1&1&2\\\hline 2&3&3&4\\\hline 4\\\cline{1-1}\end{array}, \begin{array}{|c|c|c|c|}\hline 1&1&1&2\\\hline 2&3&4&4\\\hline 3\\\cline{1-1}\end{array}, \begin{array}{|c|c|c|c|}\hline 1&1&1&3\\\hline 2&2&3&4\\\hline 4\\\cline{1-1}\end{array}, \begin{array}{|c|c|c|c|}\hline 1&1&1&3\\\hline 2&2&2&4\\\hline 3\\\cline{1-1}\end{array}, \right) \times \left(\begin{array}{|c|c|c|}\hline e_1&e_2&a_1\\\hline e_3&b_1&c_1\\\hline a_2&b_2&c_2\\\hline\end{array} + \begin{array}{|c|c|c|}\hline e_1&e_2&b_1\\\hline e_3&a_1&c_1\\\hline a_2&b_2&c_2\\\hline\end{array} \right) =$$

$$= (\mathcal{M}_1^{\rm m}+\mathcal{M}_2^{\rm m}+\mathcal{M}_3^{\rm m}+\mathcal{M}_4^{\rm m}) \times (T^{\rm m}_{\rm SU3,1} + T^{\rm m}_{\rm SU3,2}) =$$ **8 operators**

Re-organize by symmetric group on repeated field dc

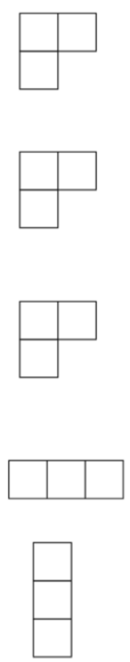
Lorentz $SU(3)_C$ Flavor



Flavor Bland P-Basis

Linear transformation between Y-basis and P-basis:

$$\mathcal{O}^{\rm p} = \mathcal{K}^{\rm py}.\mathcal{O}^{\rm y}$$

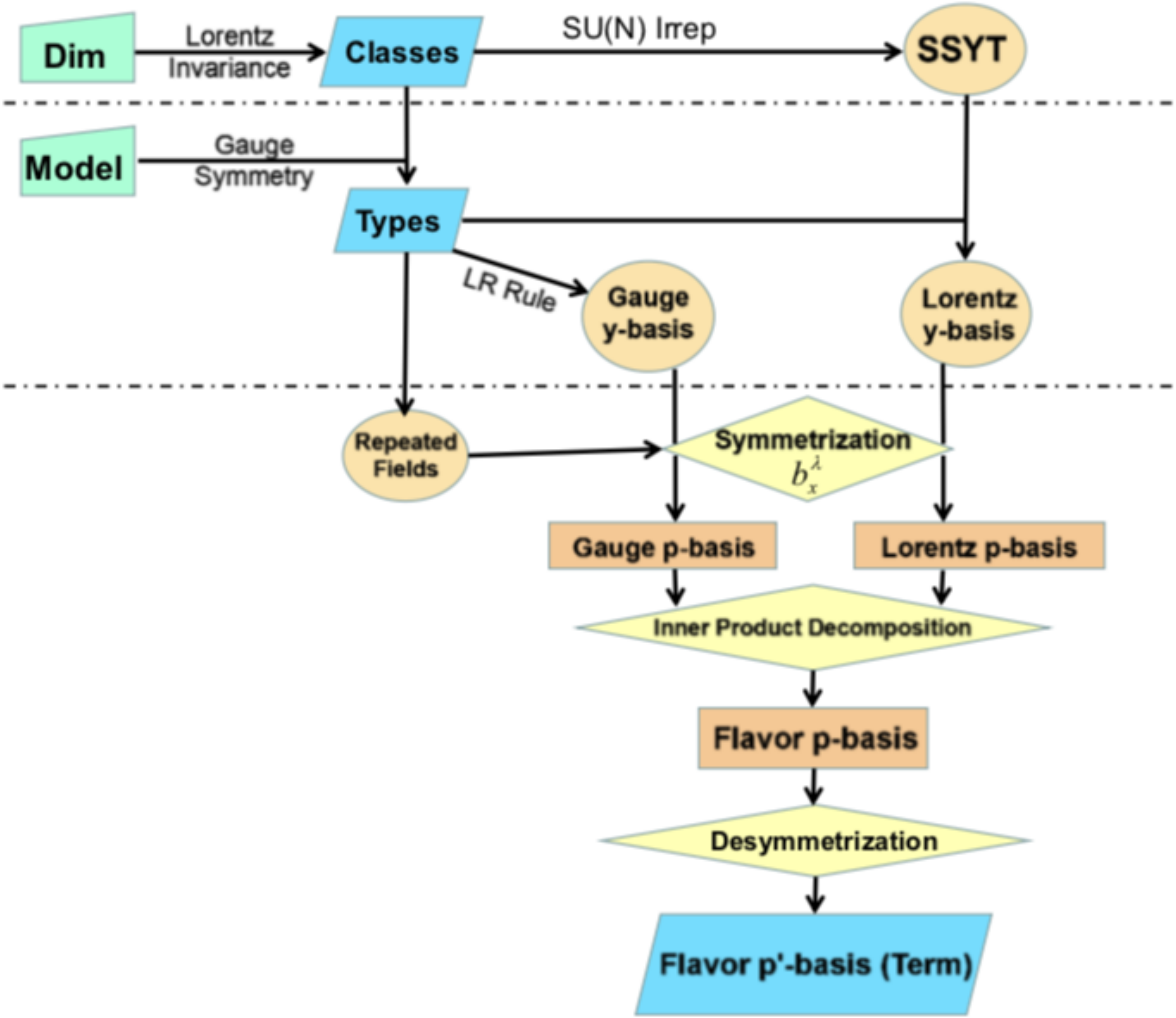


$$\begin{pmatrix} \mathcal{O}^{(\rm p)}_{([2,1],1),1} \\ \mathcal{O}^{(\rm p)}_{([2,1],2),1} \\ \mathcal{O}^{(\rm p)}_{([2,1],1),2} \\ \mathcal{O}^{(\rm p)}_{([2,1],2),2} \\ \mathcal{O}^{(\rm p)}_{([2,1],1),3} \\ \mathcal{O}^{(\rm p)}_{([2,1],2),3} \\ \mathcal{O}^{(\rm p)}_{([3],1),1} \\ \mathcal{O}^{(\rm p)}_{([1^3],1),1} \end{pmatrix} = \begin{pmatrix} 0 & \frac{4}{3} & 0 & \frac{4}{3} & 0 & \frac{4}{3} & 0 & 0 \\ -\frac{4}{3} & 0 & -\frac{4}{3} & 0 & -\frac{4}{3} & 0 & 0 & 0 \\ -\frac{4}{9} & -\frac{4}{9} & -\frac{8}{9} & \frac{4}{9} & \frac{4}{9} & -\frac{8}{9} & -\frac{4}{9} & -\frac{4}{9} \\ -\frac{4}{9} & \frac{8}{9} & \frac{4}{9} & \frac{4}{9} & -\frac{8}{9} & \frac{4}{9} & -\frac{4}{9} & \frac{8}{9} \\ -\frac{8}{3} & \frac{4}{3} & \frac{8}{3} & -\frac{4}{3} & \frac{8}{3} & -\frac{4}{3} & \frac{16}{3} & -\frac{8}{3} \\ \frac{4}{3} & -\frac{8}{3} & -\frac{4}{3} & \frac{8}{3} & -\frac{4}{3} & \frac{8}{3} & -\frac{8}{3} & \frac{16}{3} \\ \frac{2}{3} & -\frac{4}{3} & \frac{4}{3} & -\frac{2}{3} & -\frac{2}{3} & -\frac{2}{3} & \frac{2}{3} & -\frac{4}{3} \\ -\frac{4}{3} & 0 & 0 & -\frac{4}{3} & -\frac{4}{3} & \frac{4}{3} & -\frac{4}{3} & 0 \end{pmatrix} \begin{pmatrix} \mathcal{M}^{\rm m}_1 T^{\rm m}_{\rm SU3,1} \\ \mathcal{M}^{\rm m}_1 T^{\rm m}_{\rm SU3,2} \\ \mathcal{M}^{\rm m}_2 T^{\rm m}_{\rm SU3,1} \\ \mathcal{M}^{\rm m}_2 T^{\rm m}_{\rm SU3,2} \\ \mathcal{M}^{\rm m}_3 T^{\rm m}_{\rm SU3,1} \\ \mathcal{M}^{\rm m}_3 T^{\rm m}_{\rm SU3,2} \\ \mathcal{M}^{\rm m}_4 T^{\rm m}_{\rm SU3,1} \\ \mathcal{M}^{\rm m}_4 T^{\rm m}_{\rm SU3,2} \end{pmatrix}$$

Compared to dim-8 paper, dim-9 paper tackled flavor structure of operators

[Li, Ren, Xiao, **Yu**, Zheng, **2020**]

Automized Procedure



SMEFT

Dimension-5

$$\epsilon_{ij}\epsilon_{mn}(L^i C L^m)H^j H^n$$

[Weinberg, 1979] **2**

Dimension-6

Φ^6 and $\Phi^4 D^2$	$\psi^2\Phi^3$	X^3
$O_\Phi = (\Phi^\dagger\Phi)^3$	$O_{l\Phi} = (\Phi^\dagger\Phi)(\bar{L}_i l_j \Phi)$	$O_G = -f^{ABC}G_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$
$O_{\Phi\Box} = (\Phi^\dagger\Phi)\Box(\Phi^\dagger\Phi)$	$O_{u\Phi} = (\Phi^\dagger\Phi)(\bar{Q}_i u_j \Phi^c)$	$O_{\widetilde{G}} = -f^{ABC}\widetilde{G}_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$
$O_{\Phi D} = (\Phi^\dagger D^\mu\Phi)^*(\Phi^\dagger D_\mu\Phi)$	$O_{d\Phi} = (\Phi^\dagger\Phi)(\bar{Q}_i d_j \Phi)$	$O_W = -\epsilon^{abc}W_\mu^{a\nu}W_\nu^{b\rho}W_\rho^{c\mu}$
		$O_{\widetilde{W}} = -\epsilon^{abc}\widetilde{W}_\mu^{a\nu}W_\nu^{b\rho}W_\rho^{c\mu}$

$X^2\Phi^2$	$\psi^2 X$
$O_{\Phi G} = (\Phi^\dagger\Phi)G^A_{\mu\nu}G^{A\mu\nu}$	O_{uG}
$O_{\Phi\widetilde{G}} = (\Phi^\dagger\Phi)\widetilde{G}^A_{\mu\nu}G^{A\mu\nu}$	O_{dG}
$O_{\Phi W} = (\Phi^\dagger\Phi)W^a_{\mu\nu}W^{a\mu\nu}$	O_{lW}
$O_{\Phi\widetilde{W}} = (\Phi^\dagger\Phi)\widetilde{W}^a_{\mu\nu}W^{a\mu\nu}$	O_{uW}
$O_{\Phi B} = (\Phi^\dagger\Phi)B_{\mu\nu}B^{\mu\nu}$	O_{dW}
$O_{\Phi\widetilde{B}} = (\Phi^\dagger\Phi)\widetilde{B}_{\mu\nu}B^{\mu\nu}$	O_{lB}
$O_{\Phi WB} = -(\Phi^\dagger\tau^a\Phi)W^a_{\mu\nu}B^{\mu\nu}$	O_{uB}
$O_{\Phi\widetilde{W}B} = -(\Phi^\dagger\tau^a\Phi)\widetilde{W}^a_{\mu\nu}B^{\mu\nu}$	O_{dB}

$(\bar{L}L)(\bar{L}L)$	$(\bar{R}R)(\bar{R}R)$	$(\bar{L}L)(\bar{R}R)$
$O_{LL} = (\bar{L}_i\gamma_\mu L_j)(\bar{L}_k\gamma^\mu L_l)$	$O_{ll} = (\bar{l}_i\gamma_\mu l_j)(\bar{l}_k\gamma^\mu l_l)$	$O_{Ll} = (\bar{L}_i\gamma_\mu L_j)(\bar{l}_k\gamma^\mu l_l)$
$O^{(1)}_{QQ} = (\bar{Q}_i\gamma_\mu Q_j)(\bar{Q}_k\gamma^\mu Q_l)$	$O_{uu} = (\bar{u}_i\gamma_\mu u_j)(\bar{u}_k\gamma^\mu u_l)$	$O_{Lu} = (\bar{L}_i\gamma_\mu L_j)(\bar{u}_k\gamma^\mu u_l)$
$O^{(3)}_{QQ} = (\bar{Q}_i\gamma_\mu\tau^a Q_j)(\bar{Q}_k\gamma^\mu\tau^a Q_l)$	$O_{dd} = (\bar{d}_i\gamma_\mu d_j)(\bar{d}_k\gamma^\mu d_l)$	$O_{Ld} = (\bar{L}_i\gamma_\mu L_j)(\bar{d}_k\gamma^\mu d_l)$
$O^{(1)}_{LQ} = (\bar{L}_i\gamma_\mu L_j)(\bar{Q}_k\gamma^\mu Q_l)$	$O_{lu} = (\bar{l}_i\gamma_\mu l_j)(\bar{u}_k\gamma^\mu u_l)$	$O_{Ql} = (\bar{Q}_i\gamma_\mu Q_j)(\bar{l}_k\gamma^\mu l_l)$
$O^{(3)}_{LQ} = (\bar{L}_i\gamma_\mu\tau^a L_j)(\bar{Q}_k\gamma^\mu\tau^a Q_l)$	$O_{ld} = (\bar{l}_i\gamma_\mu l_j)(\bar{d}_k\gamma^\mu d_l)$	$O^{(1)}_{Qu} = (\bar{Q}_i\gamma_\mu Q_j)(\bar{u}_k\gamma^\mu u_l)$
	$O^{(1)}_{ud} = (\bar{u}_i\gamma_\mu u_j)(\bar{d}_k\gamma^\mu d_l)$	$O^{(8)}_{Qu} = \left(\bar{Q}_i\gamma_\mu \frac{\lambda^A}{2}Q_j\right)\left(\bar{u}_k\gamma^\mu\frac{\lambda^A}{2}u_l\right)$
	$O^{(8)}_{ud} = \left(\bar{u}_i\gamma_\mu \frac{\lambda^A}{2}u_j\right)\left(\bar{d}_k\gamma^\mu\frac{\lambda^A}{2}d_l\right)$	$O^{(1)}_{Qd} = (\bar{Q}_i\gamma_\mu Q_j)(\bar{d}_k\gamma^\mu d_l)$
		$O^{(8)}_{Qd} = \left(\bar{Q}_i\gamma_\mu \frac{\lambda^A}{2}Q_j\right)\left(\bar{d}_k\gamma^\mu\frac{\lambda^A}{2}d_l\right)$

$(\bar{L}R)(\bar{R}L)$ and $(\bar{L}R)(\bar{L}R)$	B-violating
$O_{LdQ} = (\bar{L}_i^\sigma l_j)(\bar{d}_k Q_l^\sigma)$	$O_{duQ} = \epsilon^{\alpha\beta\gamma}\epsilon_{\sigma\tau}\left[(d_i^\alpha)^T C u_j^\beta\right]\left[(Q_k^{\gamma\sigma})^T C L_l^\tau\right]$
$O^{(1)}_{QuQd} = (\bar{Q}_i^\sigma u_j)\epsilon_{\sigma\tau}(\bar{Q}_k^\tau d_l)$	$O_{QQu} = \epsilon^{\alpha\beta\gamma}\epsilon_{\sigma\tau}\left[(Q_i^{\alpha\sigma})^T C Q_j^{\beta\tau}\right]\left[(u_k^\gamma)^T C l_l\right]$
$O^{(8)}_{QuQd} = \left(\bar{Q}_i^\sigma \frac{\lambda^A}{2}u_j\right)\epsilon_{\sigma\tau}\left(\bar{Q}_k^\tau\frac{\lambda^A}{2}d_l\right)$	$O_{QQQ} = \epsilon^{\alpha\beta\gamma}\epsilon_{\sigma\tau}\epsilon_{\sigma'\tau'}\left[(Q_i^{\alpha\sigma})^T C Q_j^{\beta\tau}\right]\left[(Q_k^{\gamma\sigma'})^T C L_l^{\tau'}\right]$
$O^{(1)}_{LlQu} = (\bar{L}_i^\sigma l_j)\epsilon_{\sigma\tau}(\bar{Q}_k^\tau u_l)$	$O_{duu} = \epsilon^{\alpha\beta\gamma}\left[(d_i^\alpha)^T C u_j^\beta\right]\left[(u_k^\gamma)^T C l_l\right]$
$O^{(3)}_{LlQu} = (\bar{L}_i^\sigma \sigma_{\mu\nu} l_j)\epsilon_{\sigma\tau}(\bar{Q}_k^\tau\sigma^{\mu\nu} u_l)$	

[Buchmuller, Wyler, 1986] [Grzadkowski, Iskrzynski, Misiak, Rosiek, 2010] **84**

Dimension-7

1 : $\psi^2 X H^2$ + h.c.

$Q_{l^2WH^2}$	$\epsilon_{mn}(\tau^I\epsilon)_{jk}(l_p^m C i\sigma^{\mu\nu}l_r^j)H^n H^k W^I_{\mu\nu}$
$Q_{l^2BH^2}$	$\epsilon_{mn}\epsilon_{jk}(l_p^m C i\sigma^{\mu\nu}l_r^j)H^n H^k B_{\mu\nu}$

2 : $\psi^2 H^4$ + h.c.

$Q_{l^2H^4}$	$\epsilon_{mn}\epsilon_{jk}(l_p^m C l_r^j)H^n H^k(H^\dagger H)$

3(B) : $\psi^4 H$ + h.c.

Q_{l^3eH}	$\epsilon_{jk}\epsilon_{mn}(\bar{e}_p l_r^j)(l_s^k C l_t^m)H^n$
Q_{leudH}	$\epsilon_{jk}(\bar{d}_p l_r^j)(u_s C e_t)H^k$
$Q^{(1)}_{l^2qdH}$	$\epsilon_{jk}\epsilon_{mn}(\bar{d}_p l_r^j)(q_s^k C l_t^m)H^n$
$Q^{(2)}_{l^2qdH}$	$\epsilon_{jm}\epsilon_{kn}(\bar{d}_p l_r^j)(q_s^k C l_t^m)H^n$
Q_{l^2quH}	$\epsilon_{jk}(\bar{q}_p^m u_r)(l_{sm} C l_t^j)H^k$

3($\not{B}$) : $\psi^4 H$ + h.c.

Q_{lud^2H}	$\epsilon_{\alpha\beta\gamma}(\bar{l}_p d_r^\alpha)(u_s^\beta C d_t^\gamma)\tilde{H}$
Q_{lq^2dH}	$\epsilon_{\alpha\beta\gamma}\epsilon_{jk}(\bar{l}_p^m d_r^\alpha)(q_{sm}^\beta C q_t^{j\gamma})\tilde{H}^k$
Q_{ld^3H}	$\epsilon_{\alpha\beta\gamma}(\bar{l}_p d_r^\alpha)(d_s^\beta C d_t^\gamma)H$
Q_{eqd^2H}	$\epsilon_{\alpha\beta\gamma}\epsilon_{jk}(\bar{e}_p q_r^{j\alpha})(d_s^\beta C d_t^\gamma)\tilde{H}^k$

4 : $\psi^2 H^3 D$ + h.c.

Q_{leH^3D}	$\epsilon_{mn}\epsilon_{jk}(l_p^m C\gamma^\mu e_r)H^n H^j iD_\mu H^k$

5(B) : $\psi^4 D$ + h.c.

Q_{l^2udD}	$\epsilon_{jk}(\bar{d}_p\gamma^\mu u_r)(l_s^j C iD_\mu l_t^k)$

6 : $\psi^2 H^2 D^2$ + h.c.

$Q^{(1)}_{l^2H^2D^2}$	$\epsilon_{jk}\epsilon_{mn}(l_p^j C D^\mu l_r^k)H^m(D_\mu H^n)$
$Q^{(2)}_{l^2H^2D^2}$	$\epsilon_{jm}\epsilon_{kn}(l_p^j C D^\mu l_r^k)H^m(D_\mu H^n)$

5($\not{B}$) : $\psi^4 D$ + h.c.

Q_{lqd^2D}	$\epsilon_{\alpha\beta\gamma}(\bar{l}_p\gamma^\mu q_r^\alpha)(d_s^\beta C iD_\mu d_t^\gamma)$
Q_{ed^3D}	$\epsilon_{\alpha\beta\gamma}(\bar{e}_p\gamma^\mu d_r^\alpha)(d_s^\beta C iD_\mu d_t^\gamma)$

[Lehman, 2014] **30**

Dimension-8

[Li, Ren, Shu, Xiao, **Yu**, Zheng, **2020**]

N	$(n,\tilde{n})$	Subclasses	$\mathcal{N}_{\text{type}}$	$\mathcal{N}_{\text{term}}$	$\mathcal{N}_{\text{operator}}$	Equations
4	(4,0)	$F_L^4 + h.c.$	14	26	26	(4.19)
	(3,1)	$F_L^2\psi\psi^\dagger D + h.c.$	22	22	$22n_f^2$	(4.51)
		$\psi^4 D^2 + h.c.$	$4\underline{+4}$	$18\underline{+14}$	$12n_f^4\underline{+n_f^3(5n_f-1)}$	(4.75, 4.78, 4.80)
		$F_L\psi^2\phi D^2 + h.c.$	16	32	$32n_f^2$	(4.44)
		$F_L^2\phi^2 D^2 + h.c.$	8	12	12	(4.14)
	(2,2)	$F_L^2 F_R^2$	14	17	17	(4.19)
		$F_L F_R\psi\psi^\dagger D$	27	35	$35n_f^2$	(4.50, 4.51)
		$\psi^2\psi^{\dagger 2}D^2$	$17\underline{+4}$	$54\underline{+8}$	$\frac{1}{2}n_f^2(75n_f^2+11)\underline{+6n_f^4}$	(4.74, 4.79-4.81)
		$F_R\psi^2\phi D^2 + h.c.$	16	16	$16n_f^2$	(4.44)
		$F_L F_R\phi^2 D^2$	5	6	6	(4.14)
		$\psi\psi^\dagger\phi^2 D^3$	7	16	$16n_f^2$	(4.31, 4.32)
		$\phi^4 D^4$	1	3	3	(4.8)
5	(3,0)	$F_L\psi^4 + h.c.$	$12\underline{+10}$	$66\underline{+54}$	$42n_f^4\underline{+2n_f^3(9n_f+1)}$	(4.86, 4.88, 4.89, 4.91)
		$F_L^2\psi^2\phi + h.c.$	32	60	$60n_f^2$	(4.47, 4.48)
		$F_L^3\phi^2 + h.c.$	6	6	6	(4.16)
	(2,1)	$F_L\psi^2\psi^{\dagger 2} + h.c.$	$84\underline{+24}$	$172\underline{+32}$	$2n_f^2(59n_f^2-2)\underline{+24n_f^4}$	(4.84-4.85), (4.88-4.92)
		$F_R^2\psi^2\phi + h.c.$	32	36	$36n_f^2$	(4.47, 4.48)
		$\psi^3\psi^\dagger\phi D + h.c.$	$32\underline{+14}$	$180\underline{+56}$	$n_f^3(135n_f-1)\underline{+n_f^3(29n_f+3)}$	(4.66, 4.69-4.72)
		$F_L\psi\psi^\dagger\phi^2 D + h.c.$	38	92	$92n_f^2$	(4.39, 4.40)
		$\psi^2\phi^3 D^2 + h.c.$	6	36	$36n_f^2$	(4.28)
		$F_L\phi^4 D^2 + h.c.$	4	6	6	(4.10)
6	(2,0)	$\psi^4\phi^2 + h.c.$	$12\underline{+4}$	$48\underline{+18}$	$5(5n_f^4+n_f^2)\underline{+\frac{2}{3}(8n_f^4+n_f^2)}$	(4.55, 4.59, 4.62, 4.64)
		$F_L\psi^2\phi^3 + h.c.$	16	22	$22n_f^2$	(4.36)
		$F_L^2\phi^4 + h.c.$	8	10	10	(4.12)
	(1,1)	$\psi^2\psi^{\dagger 2}\phi^2$	$23\underline{+10}$	$57\underline{+14}$	$n_f^2(42n_f^2+n_f+2)\underline{+3n_f^3(3n_f-1)}$	(4.54, 4.55, 4.59-4.63)
		$\psi\psi^\dagger\phi^4 D$	7	13	$13n_f^2$	(4.24, 4.25)
		$\phi^6 D^2$	1	2	2	(4.8)
7	(1,0)	$\psi^2\phi^5 + h.c.$	6	6	$6n_f^2$	(4.21)
8	(0,0)	ϕ^8	1	1	1	(4.8)
Total		48	$471\underline{+70}$	$1070\underline{+196}$	$993(n_f=1)$, $44807(n_f=3)$	

[Murphy, 2020] **993**

Dimension-9

[Li, Ren, Xiao, **Yu**, Zheng, **2020**]

N	$(n,\tilde{n})$	Classes	$\mathcal{N}_{\text{type}}$	$\mathcal{N}_{\text{term}}$	$\mathcal{N}_{\text{operator}}$	Equations
4	(3,2)	$\psi^3\psi^\dagger D^3 + h.c.$	$0+4+2+0$	10	$\frac{2}{3}n_f^2(7n_f^2-1)$	(5.50)(5.51)
		$\psi^2\phi^2 D^4 + h.c.$	$0+0+2+0$	6	$3n_f(n_f+1)$	(5.21)
5	(3,1)	$F_L\psi^3\psi^\dagger D + h.c.$	$0+10+6+0$	72	$32n_f^4$	(5.59)(5.60)
		$\psi^4\phi D^2 + h.c.$	$0+4+4+0$	100	$40n_f^4$	(5.45-5.48)
		$F_L\psi^2\phi^2 D^2 + h.c.$	$0+0+4+0$	34	$17n_f^2-n_f$	(5.28)(5.29)
	(2,2)	$F_R\psi^3\psi^\dagger D + h.c.$	$0+10+6+0$	54	$4n_f^3(6n_f+1)$	(5.59)(5.60)
		$\psi^2\psi^{\dagger 2}\phi D^2$	$0+4+4+0$	84	$n_f^3(49n_f+1)$	(5.45-5.48)
		$F_R\psi^2\phi^2 D^2 + h.c.$	$0+0+4+0$	20	$2n_f(5n_f-1)$	(5.28)(5.29)
		$\psi\psi^\dagger\phi^3 D^3$	$0+0+2+0$	6	$6n_f^2$	(5.19)
6	(3,0)	$\psi^6 + h.c.$	$2+4+6+0$	116	$\frac{1}{24}n_f^2(415n_f^4+53n_f^3+59n_f^2+139n_f+6)$	(5.63-5.70)
		$F_L\psi^4\phi + h.c.$	$0+12+10+0$	102	$2n_f^3(21n_f+1)$	(5.54-5.56)
		$F_L^2\psi^2\phi^2 + h.c.$	$0+0+8+0$	20	$2n_f(5n_f+2)$	(5.32)
	(2,1)	$\psi^4\psi^{\dagger 2} + h.c.$	$4+26+20+4$	244	$\frac{1}{6}n_f^3(382n_f^3-9n_f^2+2n_f+21)$	(5.63-5.69)
		$F_L\psi^2\psi^{\dagger 2}\phi + h.c.$	$0+24+24+0$	92	$52n_f^4$	(5.54-5.56)
		$F_L^2\psi^{\dagger 2}\phi^2 + h.c.$	$0+0+8+0$	12	$2n_f(3n_f+2)$	(5.32)
		$\psi^3\psi^\dagger\phi^2 D + h.c.$	$0+12+18+0$	186	$\frac{2}{3}n_f^2(146n_f^2+1)$	(5.39-5.42)
		$F_L\psi\psi^\dagger\phi^3 D + h.c.$	$0+0+8+0$	12	$12n_f^2$	(5.25)
		$\psi^2\phi^4 D^2 + h.c.$	$0+0+4+0$	24	$2n_f(6n_f+1)$	(5.17)
7	(2,0)	$\psi^4\phi^3 + h.c.$	$0+6+6+0$	32	$\frac{4}{3}n_f^2(10n_f^2-1)$	(5.35-5.37)
		$F_L\psi^2\phi^4 + h.c.$	$0+0+4+0$	8	$2n_f(2n_f-1)$	(5.23)
	(1,1)	$\psi^2\psi^{\dagger 2}\phi^3$	$0+6+10+0$	24	$14n_f^4$	(5.35-5.37)
		$\psi\psi^\dagger\phi^5 D$	$0+0+2+0$	2	$2n_f^2$	(5.12)
8	(1,0)	$\psi^2\phi^6 + h.c.$	$0+0+2+0$	2	$n_f^2+n_f$	(5.9)
Total		42	6+122+164+4	1262	$8+204+348+0\ (n_f=1)$ $2862+42234+44874+486\ (n_f=3)$	

[Liao, Ma, 2020] **560**

Low Energy EFT

Dimension-5

Dim-5 operators

N	$(n,\tilde{n})$	Classes	$\mathcal{N}_{\rm type}$
3	$(2,0)$	$F_{\rm L}\psi_{\rm L}{}^2 + h.c.$	$10+0+2+0$

10

Dimension-6

Dim-6 operators

N	$(n,\tilde{n})$	Classes	$\mathcal{N}_{\rm type}$	$\mathcal{N}_{\rm term}$
3	$(3,0)$	$F_{\rm L}^3 + h.c.$	$2+0+0+0$	2
4	$(2,0)$	$\psi_{\rm L}^4 + h.c.$	$14+12+8+2$	78
	$(1,1)$	$\psi_{\rm L}^2\psi_{\rm R}^2$	$40+20+12+0$	84
Total		5	$56+32+20+2$	164

120

[Jenkins, Manohar, Stoffer, 2017]

Dimension-7

Dim-7 operators

N	$(n,\tilde{n})$	Classes	$\mathcal{N}_{\rm type}$	$\mathcal{N}_{\rm term}$
4	$(3,0)$	$F_{\rm L}^2\psi_{\rm L}^2 + h.c.$	$16+0+4+0$	32
	$(2,1)$	$F_{\rm L}^2\psi_{\rm R}^2 + h.c.$	$16+0+4+0$	24
		$\psi_{\rm L}^3\psi_{\rm R}D + h.c.$	$50+32+22+$	
Total		6	$82+32+30+$	

166

[Liao, Ma, Wang, 2020]

Dimension-8

[Li, Ren, Xiao, **Yu**, Zheng, 2020]

[Murphy, 2020]

N	$(n,\tilde{n})$	Subclasses	$\mathcal{N}_{\rm type}$	$\mathcal{N}_{\rm term}$	$\mathcal{N}_{\rm operator}$	Equations
4	$(4,0)$	$F_{\rm L}^4 + h.c.$	14	26	26	(4.19)
	$(3,1)$	$F_{\rm L}^2\psi\psi^\dagger D + h.c.$	22	22	$22n_f^2$	(4.51)
		$\psi^4 D^2 + h.c.$	$4\underline{+4}$	$18\underline{+14}$	$12n_f^4\underline{+n_f^3(5n_f-1)}$	(4.75, 4.78, 4.80)
		$F_{\rm L}\psi^2\phi D^2 + h.c.$	16	32	$32n_f^2$	(4.44)
		$F_{\rm L}^2\phi^2 D^2 + h.c.$	8	12	12	(4.14)
	$(2,2)$	$F_{\rm L}^2F_{\rm R}^2$	14	17	17	(4.19)
		$F_{\rm L}F_{\rm R}\psi\psi^\dagger D$	27	35	$35n_f^2$	(4.50, 4.51)
		$\psi^2\psi^{\dagger 2}D^2$	$17\underline{+4}$	$54\underline{+8}$	$\frac{1}{2}n_f^2(75n_f^2+11)\underline{+6n_f^4}$	(4.74, 4.79-4.81)
		$F_{\rm R}\psi^2\phi D^2 + h.c.$	16	16	$16n_f^2$	(4.44)
		$F_{\rm L}F_{\rm R}\phi^2 D^2$	5	6	6	(4.14)
		$\psi\psi^\dagger\phi^2 D^3$	7	16	$16n_f^2$	(4.31, 4.32)
		$\phi^4 D^4$	1	3	3	(4.8)
5	$(3,0)$	$F_{\rm L}\psi^4 + h.c.$	$12\underline{+10}$	$66\underline{+54}$	$42n_f^4\underline{+2n_f^3(9n_f+1)}$	(4.86, 4.88, 4.89, 4.91)
		$F_{\rm L}^2\psi^2\phi + h.c.$	32	60	$60n_f^2$	(4.47, 4.48)
		$F_{\rm L}^3\phi^2 + h.c.$	6	6	6	(4.16)
	$(2,1)$	$F_{\rm L}\psi^2\psi^{\dagger 2} + h.c.$	$84\underline{+24}$	$172\underline{+32}$	$2n_f^2(59n_f^2-2)\underline{+24n_f^4}$	(4.84-4.85), (4.88-4.92)
		$F_{\rm R}^2\psi^2\phi + h.c.$	32	36	$36n_f^2$	(4.47, 4.48)
		$\psi^3\psi^\dagger\phi D + h.c.$	$32\underline{+14}$	$180\underline{+56}$	$n_f^3(135n_f-1)\underline{+n_f^3(29n_f+3)}$	(4.66, 4.69-4.72)
		$F_{\rm L}\psi\psi^\dagger\phi^2 D + h.c.$	38	92	$92n_f^2$	(4.39, 4.40)
		$\psi^2\phi^3 D^2 + h.c.$	6	36	$36n_f^2$	(4.28)
		$F_{\rm L}\phi^4 D^2 + h.c.$	4	6	6	(4.10)
6	$(2,0)$	$\psi^4\phi^2 + h.c.$	$12\underline{+4}$	$48\underline{+18}$	$5(5n_f^4+n_f^2)\underline{+\frac{2}{3}(8n_f^4+n_f^2)}$	(4.55, 4.59, 4.62, 4.64)
		$F_{\rm L}\psi^2\phi^3 + h.c.$	16	22	$22n_f^2$	(4.36)
		$F_{\rm L}^2\phi^4 + h.c.$	8	10	10	(4.12)
	$(1,1)$	$\psi^2\psi^{\dagger 2}\phi^2$	$23\underline{+10}$	$57\underline{+14}$	$n_f^2(42n_f^2+n_f+2)\underline{+3n_f^3(3n_f-1)}$	(4.54, 4.55, 4.59-4.63)
		$\psi\psi^\dagger\phi^4 D$	7	13	$13n_f^2$	(4.24, 4.25)
		$\phi^6 D^2$	1	2	2	(4.8)
7	$(1,0)$	$\psi^2\phi^5 + h.c.$	6	6	$6n_f^2$	(4.21)
8	$(0,0)$	ϕ^8	1	1	1	(4.8)
Total		48	$471\underline{+70}$	$1070\underline{+196}$	$993(n_f=1)$, $44807(n_f=3)$	

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Dimension-9

[Li, Ren, Xiao, **Yu**, Zheng, 2020]

N	$(n,\tilde{n})$	Classes	$\mathcal{N}_{\rm type}$	$\mathcal{N}_{\rm term}$	$\mathcal{N}_{\rm operator}$	Equations
4	$(3,2)$	$\psi^3\psi^\dagger D^3 + h.c.$	$0+4+2+0$	10	$\frac{2}{3}n_f^2(7n_f^2-1)$	(5.50)(5.51)
		$\psi^2\phi^2 D^4 + h.c.$	$0+0+2+0$	6	$3n_f(n_f+1)$	(5.21)
5	$(3,1)$	$F_{\rm L}\psi^3\psi^\dagger D + h.c.$	$0+10+6+0$	72	$32n_f^4$	(5.59)(5.60)
		$\psi^4\phi D^2 + h.c.$	$0+4+4+0$	100	$40n_f^4$	(5.45-5.48)
		$F_{\rm L}\psi^2\phi^2 D^2 + h.c.$	$0+0+4+0$	34	$17n_f^2-n_f$	(5.28)(5.29)
	$(2,2)$	$F_{\rm R}\psi^3\psi^\dagger D + h.c.$	$0+10+6+0$	54	$4n_f^3(6n_f+1)$	(5.59)(5.60)
		$\psi^2\psi^{\dagger 2}\phi D^2$	$0+4+4+0$	84	$n_f^3(49n_f+1)$	(5.45-5.48)
		$F_{\rm R}\psi^2\phi^2 D^2 + h.c.$	$0+0+4+0$	20	$2n_f(5n_f-1)$	(5.28)(5.29)
		$\psi\psi^\dagger\phi^3 D^3$	$0+0+2+0$	6	$6n_f^2$	(5.19)
6	$(3,0)$	$\psi^6 + h.c.$	$2+4+6+0$	116	$\frac{1}{24}n_f^2(415n_f^4+53n_f^3+59n_f^2+139n_f+6)$	(5.63-5.70)
		$F_{\rm L}\psi^4\phi + h.c.$	$0+12+10+0$	102	$2n_f^3(21n_f+1)$	(5.54-5.56)
		$F_{\rm L}^2\psi^2\phi^2 + h.c.$	$0+0+8+0$	20	$2n_f(5n_f+2)$	(5.32)
	$(2,1)$	$\psi^4\psi^{\dagger 2} + h.c.$	$4+26+20+4$	244	$\frac{1}{6}n_f^3(382n_f^3-9n_f^2+2n_f+21)$	(5.63-5.69)
		$F_{\rm L}\psi^2\psi^{\dagger 2}\phi + h.c.$	$0+24+24+0$	92	$52n_f^4$	(5.54-5.56)
		$F_{\rm L}^2\psi^{\dagger 2}\phi^2 + h.c.$	$0+0+8+0$	12	$2n_f(3n_f+2)$	(5.32)
		$\psi^3\psi^\dagger\phi^2 D + h.c.$	$0+12+18+0$	186	$\frac{2}{3}n_f^2(146n_f^2+1)$	(5.39-5.42)
		$F_{\rm L}\psi\psi^\dagger\phi^3 D + h.c.$	$0+0+8+0$	12	$12n_f^2$	(5.25)
		$\psi^2\phi^4 D^2 + h.c.$	$0+0+4+0$	24	$2n_f(6n_f+1)$	(5.17)
7	$(2,0)$	$\psi^4\phi^3 + h.c.$	$0+6+6+0$	32	$\frac{4}{3}n_f^2(10n_f^2-1)$	(5.35-5.37)
		$F_{\rm L}\psi^2\phi^4 + h.c.$	$0+0+4+0$	8	$2n_f(2n_f-1)$	(5.23)
	$(1,1)$	$\psi^2\psi^{\dagger 2}\phi^3$	$0+6+10+0$	24	$14n_f^4$	(5.35-5.37)
		$\psi\psi^\dagger\phi^5 D$	$0+0+2+0$	2	$2n_f^2$	(5.12)
8	$(1,0)$	$\psi^2\phi^6 + h.c.$	$0+0+2+0$	2	$n_f^2+n_f$	(5.9)
Total		42	6+122+164+4	1262	$8+204+348+0\ (n_f=1)$ $2862+42234+44874+486\ (n_f=3)$	

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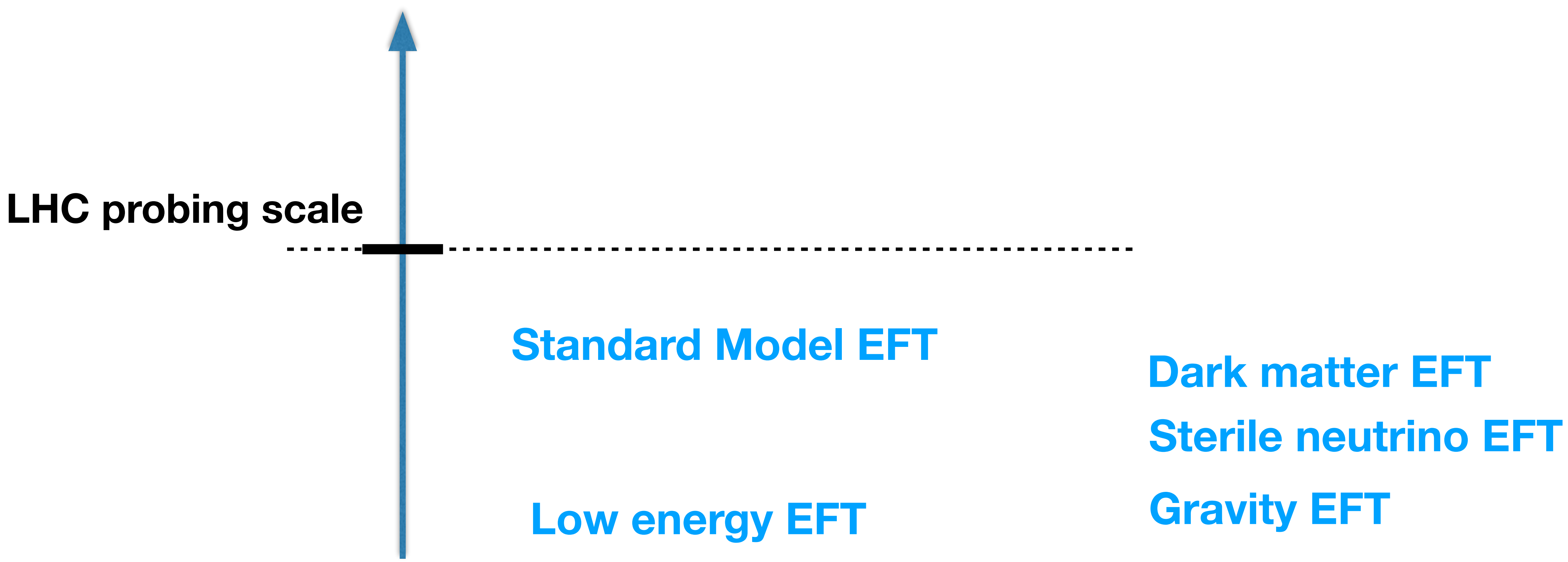
Jiang-Hao Yu

Landscape of Generic EFTs

Any EFT with Lorentz inv. and any gauge symmetries, SU(5), LRSM, etc



Li, Shu, Xiao, **JHYu**, arXiv: 2012.11615

Li, Ren, Xiao, **JHYu**, Zheng, in preparation

Symmetry Nonlinearly!

EFT Framework

Energy

UV models

many models …..

$$\mathcal{L} = \mathcal{L}^{\rm eff}_{\rm Gravity} + \underbrace{\mathcal{L}_{\rm QCD} + \mathcal{L}_{\rm QED} + \mathcal{L}_{\rm EW}}_{\mathcal{L}_{SM}} + \mathcal{L}^{NP}_{\rm heavy}$$

TeV scale

Standard Model EFT

$$\mathcal{L}_{SMEFT} = \mathcal{L}_{SM} + \frac{C_i^{(5)}}{\Lambda_{NP}} Q_i^{(5)} + \frac{C_i^{(6)}}{\Lambda_{NP}^2} Q_i^{(6)} + \cdots$$

EW scale

Low energy EFT

$$\mathcal{L}_{\rm LEFT} = \mathcal{L}_{\rm QED+QCD} + \frac{C_i^{(5)}}{M_W} O_i^{(5)} + \frac{C_i^{(6)}}{M_W^2} O_i^{(6)} + \cdots$$

$\Lambda_{\rm QCD}$

Chiral Lagrangian + Baryon EFT

$$\mathcal{L}_\chi = \mathcal{L}_\pi(s, p, l_\mu, r_\mu) + \mathcal{L}_{\pi N}(s, p, l_\mu, r_\mu) + \cdots$$

MeV scale

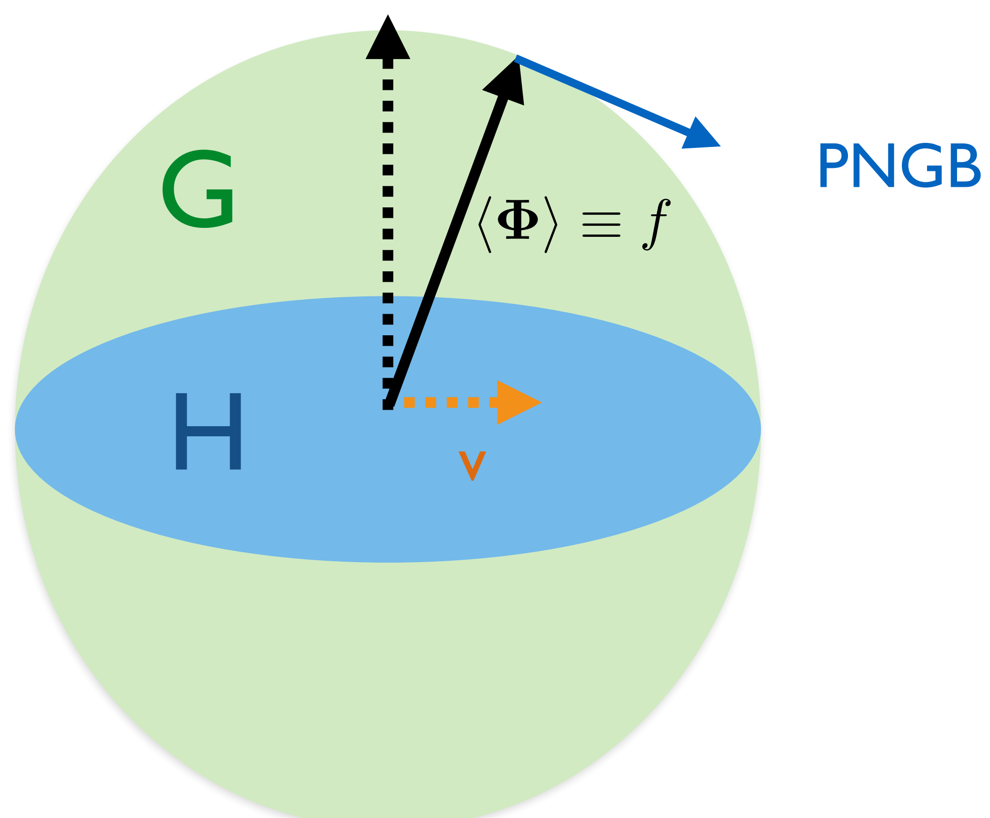
Non-relativistic EFT

Pseudo-Goldstone Boson

Chiral Lagrangian description

G → H

$$\mathbf{\Phi}(x) \equiv \exp\left(\frac{i}{f}\pi^{\hat{a}}(x)T^{\hat{a}}\right)\langle\mathbf{\Phi}\rangle$$



Chiral Lagrangian

Define Goldstone boson matrix, which transform nonlinearly under G

$$U(\Pi) \equiv e^{i\Pi(x)} \to g\, U(\Pi)\, h^\dagger(\Pi(x), g))$$

$[X^a, X^b] = i\cancel{f^{abc}X^c} + i f^{abi} T^i$

Symmetric coset

CCWZ construction

[Callan, Coleman, Wess, Zumino, 1969]

$$-i\, U^\dagger D_\mu U = d^{\hat{a}}_\mu T^{\hat{a}} + E^a_\mu T^a \equiv d_\mu + E_\mu \,.$$

$$\bar{\Sigma} \equiv U(\Pi)^2 = \exp(2i\Pi(x))$$

Transform linearly under G

$$\bar{\Sigma} \to g\, \bar{\Sigma}\, R(g)^\dagger$$

Building block

$$\boxed{\begin{array}{c} d_\mu(\Pi), \quad E_{\mu\nu}(\Pi) \\ f_{\mu\nu} \equiv U^\dagger F_{\mu\nu} U = f^+ T^a + f^- T^{\hat{a}} \end{array}}$$

Building block

$$\boxed{\begin{array}{c} \overline{\mathrm{V}}_\mu = (\mathbf{D}_\mu \mathbf{\Sigma})\, \mathbf{\Sigma}^{-1} \qquad \overline{\mathbf{T}} \equiv \mathbf{\Sigma} Q_Y \mathbf{\Sigma}^{-1} \\ \overline{\mathrm{F}}_{\mu\nu}, \quad \mathbf{\Sigma}\overline{\mathrm{F}}^R_{\mu\nu}\mathbf{\Sigma}^{-1} \end{array}}$$

Chiral Lagrangian

QCD chiral Lagrangian SU(2) x SU(2)/ SU(2)

Electroweak chiral Lagrangian SU(2) x SU(2)/ SU(2)

Higgs EFT: singlet Higgs + SO(4)/SO(3)

Pseudo-Goldstone Higgs chiral Lagrangian G/H

Composite Higgs Models

$\mathcal{G}$	$\mathcal{H}$	C	N_G	$\mathbf{r}_{\mathcal{H}} = \mathbf{r}_{\mathrm{SU}(2)\times\mathrm{SU}(2)}\ (\mathbf{r}_{\mathrm{SU}(2)\times\mathrm{U}(1)})$
SO(5)	SO(4)	✓	4	$\mathbf{4} = (\mathbf{2},\mathbf{2})$
SO(6)	SO(5)	✓	5	$\mathbf{5} = (\mathbf{1},\mathbf{1}) + (\mathbf{2},\mathbf{2})$
SU(3) × U(1)	SU(2) × U(1)		5	$\mathbf{2}_{\pm 1/2} + \mathbf{1}_{\mathbf{0}}$
SU(4)	Sp(4)	✓	5	$\mathbf{5} = (\mathbf{1},\mathbf{1}) + (\mathbf{2},\mathbf{2})$
SO(7)	SO(6)	✓	6	$\mathbf{6} = 2\cdot(\mathbf{1},\mathbf{1}) + (\mathbf{2},\mathbf{2})$
SO(8)	SO(7)	✓	7	$\mathbf{7} = 3\cdot(\mathbf{1},\mathbf{1}) + (\mathbf{2},\mathbf{2})$
SU(4) × U(1)	SU(3) × U(1)		7	$\mathbf{3}_{-1/3} + \bar{\mathbf{3}}_{+1/3} + \mathbf{1}_{\mathbf{0}} = 3\cdot\mathbf{1}_{\mathbf{0}} + \mathbf{2}_{\pm 1/2}$
SO(7)	G_2	✓*	7	$\mathbf{7} = (\mathbf{1},\mathbf{3}) + (\mathbf{2},\mathbf{2})$
SO(9)	SO(8)	✓	8	$\mathbf{8} = 2\cdot(\mathbf{2},\mathbf{2})$
SU(4)	$[\mathrm{SU}(2)]^2 \times \mathrm{U}(1)$	✓*	8	$(\mathbf{2},\mathbf{2})_{\pm\mathbf{2}} = 2\cdot(\mathbf{2},\mathbf{2})$
$[\mathrm{SU}(3)]^2$	SU(3)		8	$\mathbf{8} = \mathbf{1}_{\mathbf{0}} + \mathbf{2}_{\pm 1/2} + \mathbf{3}_{\mathbf{0}}$
Sp(6)	Sp(4) × SU(2)	✓	8	$(\mathbf{4},\mathbf{2}) = 2\cdot(\mathbf{2},\mathbf{2})$
SU(5)	SU(4) × U(1)	✓*	8	$\mathbf{4}_{-5} + \bar{\mathbf{4}}_{+\mathbf{5}} = 2\cdot(\mathbf{2},\mathbf{2})$
$[\mathrm{SO}(5)]^2$	SO(5)	✓*	10	$\mathbf{10} = (\mathbf{1},\mathbf{3}) + (\mathbf{3},\mathbf{1}) + (\mathbf{2},\mathbf{2})$
SO(7)	SO(5) × U(1)	✓*	10	$\mathbf{10}_{\mathbf{0}} = (\mathbf{3},\mathbf{1}) + (\mathbf{1},\mathbf{3}) + (\mathbf{2},\mathbf{2})$
SO(7)	$[\mathrm{SU}(2)]^3$	✓*	12	$(\mathbf{2},\mathbf{2},\mathbf{3}) = 3\cdot(\mathbf{2},\mathbf{2})$
SU(5)	SO(5)	✓*	14	$\mathbf{14} = (\mathbf{3},\mathbf{3}) + (\mathbf{2},\mathbf{2}) + (\mathbf{1},\mathbf{1})$
SU(6)	Sp(6)	✓*	14	$\mathbf{14} = 2\cdot(\mathbf{2},\mathbf{2}) + (\mathbf{1},\mathbf{3}) + 3\cdot(\mathbf{1},\mathbf{1})$
$[\mathrm{SO}(6)]^2$	SO(6)	✓*	15	$\mathbf{15} = (\mathbf{1},\mathbf{1}) + 2\cdot(\mathbf{2},\mathbf{2}) + (\mathbf{3},\mathbf{1}) + (\mathbf{1},\mathbf{3})$
SO(9)	SO(5) × SO(4)	✓*	20	$(\mathbf{5},\mathbf{4}) = (\mathbf{2},\mathbf{2}) + (\mathbf{1}+\mathbf{3},\mathbf{1}+\mathbf{3})$

[Csaki, et. al, 2015]

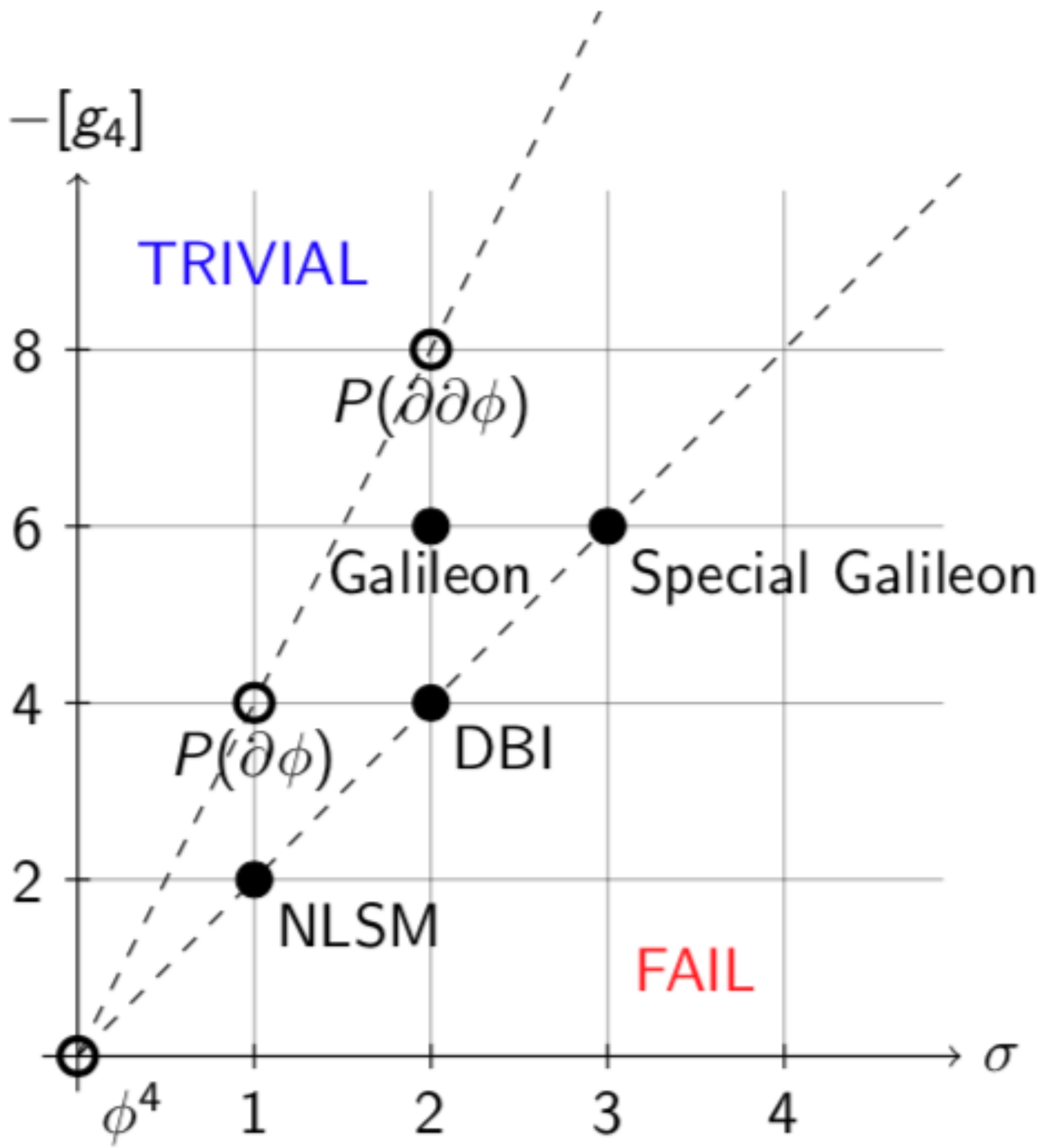
Composite Higgs
[Agashe, Contino, Pomarol 2004]

Minimal Neutral Naturalness
[Xu, **Yu**, Zhu, **2018**]

Twin Higgs
[Chacko, Goh, Harnik 2006]

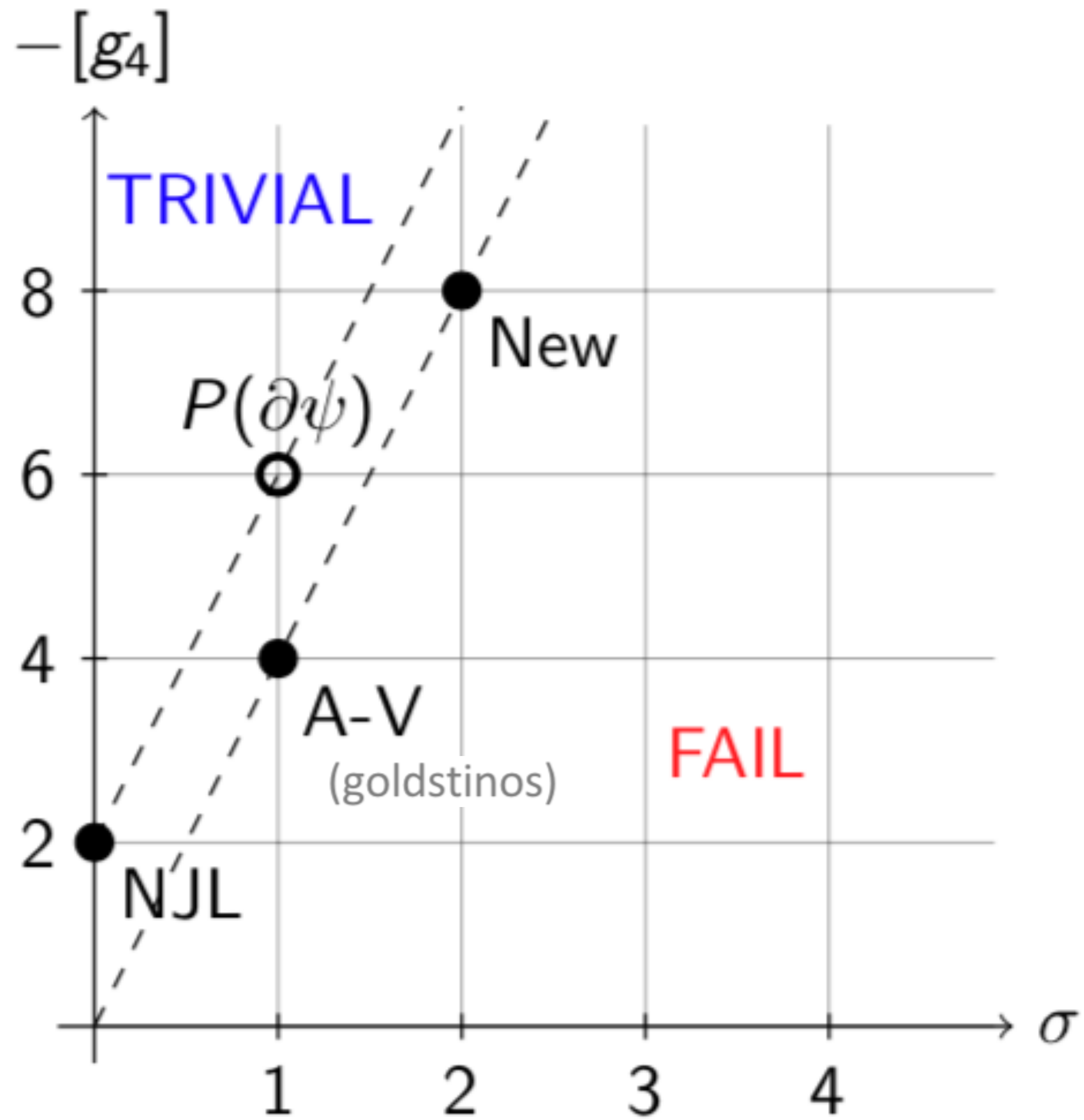
Little Higgs
[Arkani-hamed, et.al. 2000]

More Special EFTs



[Cheung, Kampf, Novotny, Shen, and Trnka 2014 & 2016]

More Special EFTs



Elvang, Hadjiantonis, Jones, Paranjape (2018)

Summary & Outlook

Summary

目录

Internal symmetry SU(n)

Symmetric group S(n)

Lorentz group SO(3,1)

Poincare group

Conformal group SO(4,2)

Coset G/H

Summary

Take home message 1: From operator counting to operator writing systematically

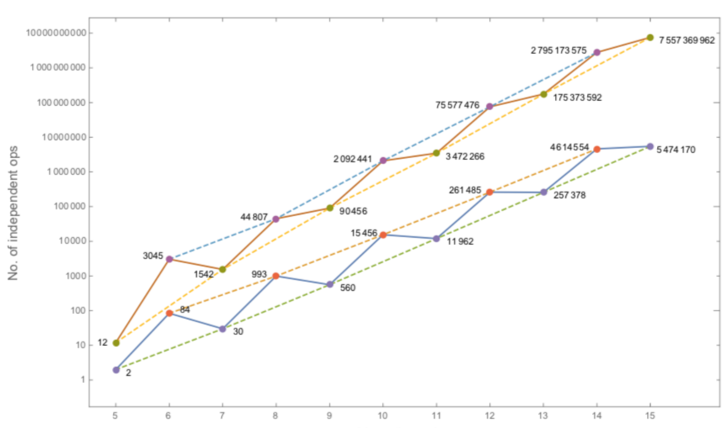
Hilbert series → On-shell EFT → Young tensor

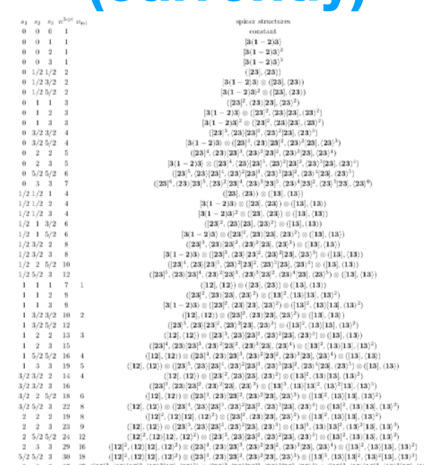
Only counting

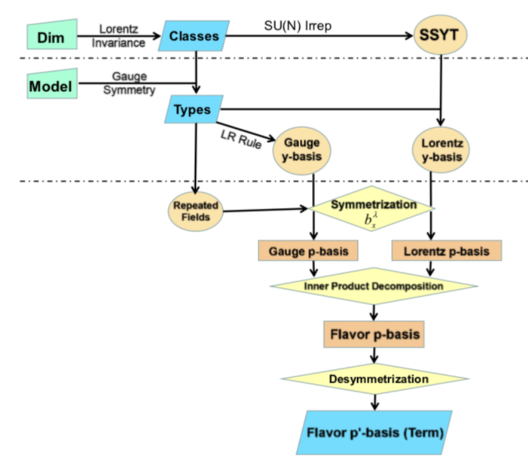
up to 4 fields in operator (currently)

[ITP-CAS group, **2020**]

Any operator to any mass dimension







Take home message 2: Unified construction of Lorentz&gauge by Young tableau

For any generic EFT with Lorentz and any gauge symmetry

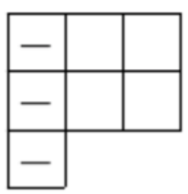
Thanks!

Appreciate all materials from web, prepared for this school

Backup Slides

Different Filling for Young Diagram

[Henning, Melia, 2019]



SSYT for class I

SSYT for class II

....

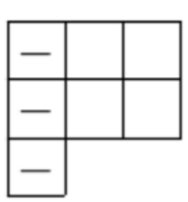
SSYT for helicity = 3/2

SSYT for helicity = 2

Further selection

Operator Lorentz structure
(cannot deal repeated field)

[Li, Ren, Xiao, Yu, Zheng, 2020]



SSYT for unique class
and helicity

One-to-one correspondence
No need selection

Operator Lorentz structure
(can deal repeated field)

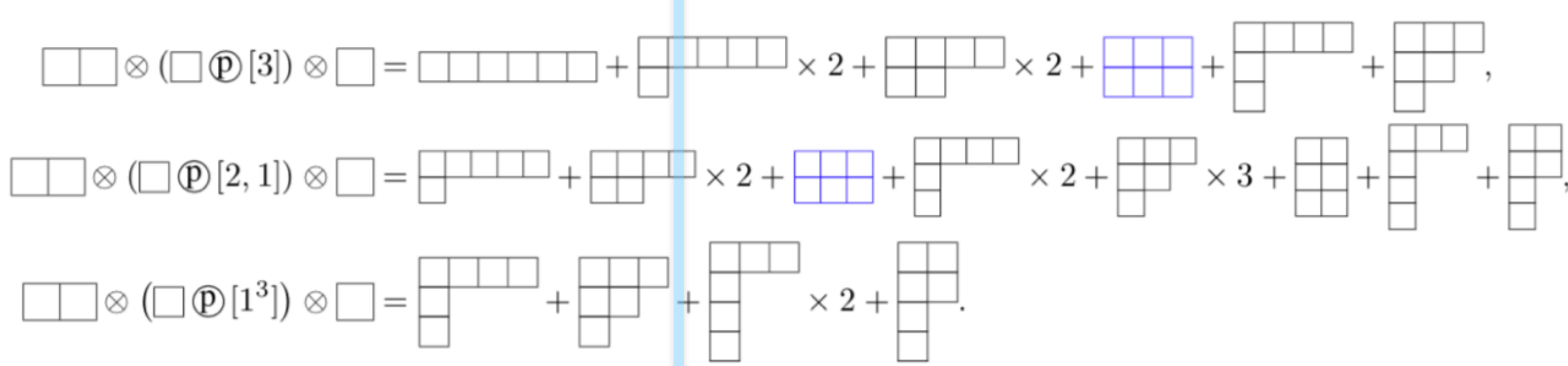
Young Tensor with Repeated Field

$W_{\mathrm{L}}Q^3L$

$n=3, \tilde{n}=0. \quad \#1=2, \#2=\#3=\#4=\#5=1$

$$\begin{array}{|c|c|c|}\hline 1 & 1 & 2 \\ \hline 3 & 4 & 5 \\ \hline \end{array} \qquad \begin{array}{|c|c|c|}\hline 1 & 1 & 3 \\ \hline 2 & 4 & 5 \\ \hline \end{array} \qquad \begin{array}{|c|c|c|}\hline 1 & 1 & 4 \\ \hline 2 & 3 & 5 \\ \hline \end{array}$$

$$\epsilon^{\alpha_1\alpha_3}\epsilon^{\alpha_1\alpha_4}\epsilon^{\alpha_2\alpha_5} \qquad \epsilon^{\alpha_1\alpha_2}\epsilon^{\alpha_1\alpha_4}\epsilon^{\alpha_3\alpha_5} \qquad \epsilon^{\alpha_1\alpha_2}\epsilon^{\alpha_1\alpha_3}\epsilon^{\alpha_4\alpha_5}$$



$$\mathcal{M}^{[1^3]}_{3,1} = \frac{1}{3}(\mathcal{M}_1+\mathcal{M}_2+\mathcal{M}_3)$$

$$\mathcal{M}^{[2,1]}_{3,x} = \left\{ \frac{1}{3}(\mathcal{M}_1+\mathcal{M}_2-2\mathcal{M}_3)\ ,\ \frac{1}{3}(\mathcal{M}_1-2\mathcal{M}_2+\mathcal{M}_3) \right\}_x$$

Jiang-Hao Yu