Effective field theory approach to low energy probes of physics beyond the standard model

Xiao-Dong Ma

Tsung-Dao Lee Institute, Shanghai

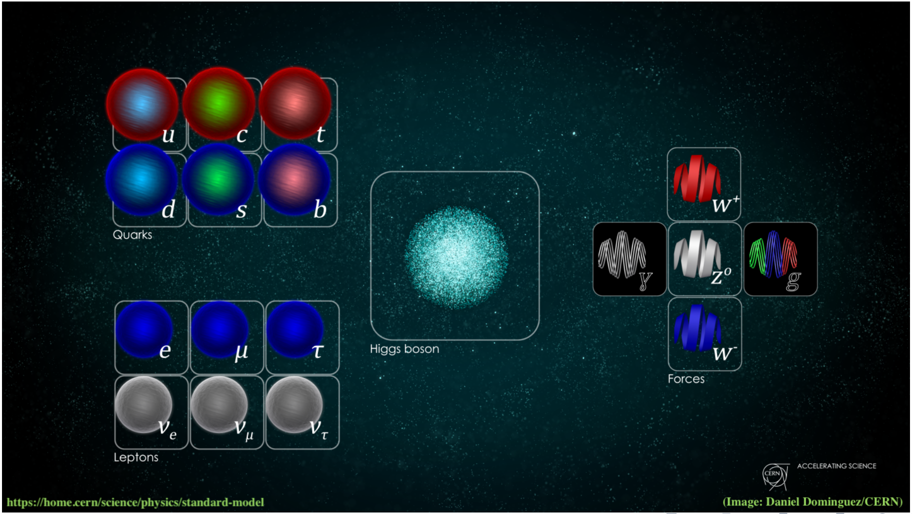
The 2021 EFT School on Collider Phenomenology

https://indico.ihep.ac.cn/event/13633/

April 11, 2021

Outline

Particles of the Standard Model of particle physics



Problems unanswered by the SM

- Experimental side:
  - Neutrino oscillation $\to$ non-vanishing neutrino mass; ◡̈
  - BAU: the asymmetry of matter and anti-matter in the universe; ◡̈
  - The nature of dark matter and dark energy; ◡̈
  - Anomalies from terrestrial experiments: Neutron lifetime, muon $g-2$, R_K, R_K^* in $b \to s\ell^+\ell^-, \cdots$ (??!)
- Theoretical side:
  - Strong CP: $\theta_{\rm CP} < 10^{-10}$;
  - Hierarchy problem: $\delta m_h^2 \propto \Lambda^2$;
  - Vacuum stability: $\lambda(\Lambda) \leq 0$;
  - The nature of electroweak symmetry breaking;
  - Understanding of the SM itself;
  - Quantum gravity;

Remarks on the problems

- Probably the anomalies are due to the uncertainties both from theoretical calculation and experimental measurement;
- Naturalness problems could be the biased taste of beauty;
- Nevertheless, NP must be involved in m_ν and DM;
- Two popular ways towards NP:
  - Top-down/Model building: SUSY, Seesaw mechanism, 2HDM, $\cdots$
  - Bottom-up/EFT: SMEFT (✓), HEFT, DMEFT, LEFT (✓), χPT (✓), χEFT, HQET, NRQCD/NRQED, SCET, $\cdots$

EFT and physical processes

LEFT, LNEFT, DMEFT, HQET, NRQCD, NRQED
χPT, HBχPT, χEFT

$$\Delta L = 2,\ \Delta B = 1,\ \Delta B = 2$$

- **Neutrinoless double beta decay**: $(A, Z) \to (A-2, Z+2) + 2e^-$
- **Rare nucleon decay**: $p \to e^+\pi^0, K^+\nu, \cdots$
- **Neutron-antineutron oscillation**: $n - \bar{n}$
- **dinucleon decay**: $pp \to P_1^+ P_2^+, \ell_1^+\ell_2^+, \cdots$
- **Muon anomalous magnetic moment**: a_μ
- $R_K, R_K^* : b \to s\mu^+\mu^-$
- **Neutrino Non-standard interaction**
- **DM direct/indirect detection**
- $\cdots$

SMEFT
HEFT

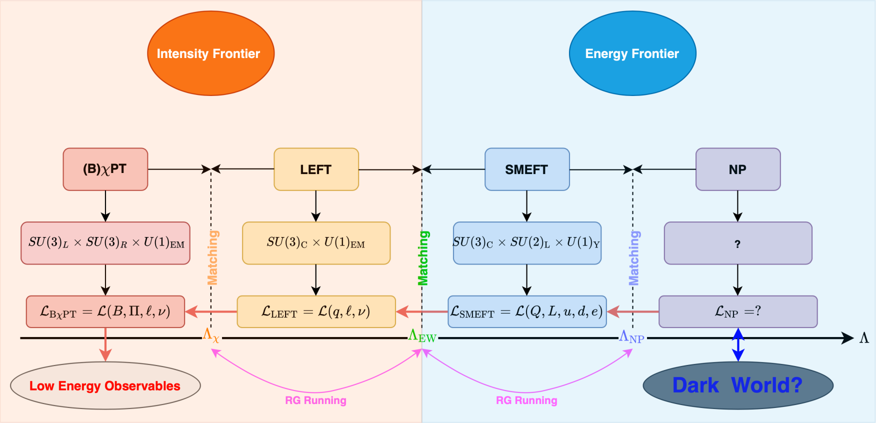
SMEFT at the LHC (Eleni Vryonidou)
Higgs EFT (Xiaochuan Lu)
SMEFT in collider experiments (Hongtao Yang)
SMEFT at future lepton colliders (Jiayin Gu)
EFT operator construction (Jianghao Yu)
On-shell amplitude techniques (Jing Shu)
EFT matching and STrEAM (Zhengkang Zhang)
Tutorials on EFT tools (Jack Y. Araz, Ilaria Brivio, Ken Mimasu)

Λ_χ — Λ_{EW} — Λ_{NP} → Λ

I will focus on the low energy part in this lecture and leave high energy part to the experts in this school!

A general picture for EFT calculation of low energy obs.

Physics is scale-dependent!



Goal in this lecture

1. **Give a brief introduction on the SMEFT, LEFT and χPT;**
2. **Clarify some techniques for EFT calculation;**
3. **Take $K^- \to \pi^+ \ell_\alpha^- \ell_\beta^-$ as an example to show how to connect NP and low energy observables in the EFT framework.**

An incomplete list of references on EFTs used in this talk

1 Lectures and reviews:

- Effective Field Theory [Georgi, 1993]
- Introduction to Effective Field Theories [Manohar, help-ph/9606222,1804.05863]
- Introduction to Effective Field Theory [Burgess, 0701053]
- Lectures on effective field theory [Kaplan, 2016]
- Chiral dynamics in nucleons and nuclei [Bernard-Kaiser-Meissner, hep-ph/9501384]
- Chiral Effective Field Theory and Nuclear Forces [Machleidt-Entem, 2011]

2 Books:

- Dynamics of the Standard Model [Donoghue-Golowich-Holstein]
- A Primer for Chiral Perturbation Theory [Scherer-Schindler]
- Effective Field Theories [Petrov-Blechman]
- An Introduction to Effective Field Theory [Burgess]

3 χPT:

- Chiral Perturbation Theory to One Loop & Chiral Perturbation Theory: Expansions in the Mass of the Strange Quark [Gasser-Leutwyler, 1983, 1984]
- Chiral perturbation theory with tensor sources [Catà-Mateu, 0705.2948]
- Nuclear forces from chiral lagrangians & Effective chiral Lagrangians for nucleon - pion interactions and nuclear forces [Weinberg, 1990, 1991]

4 This talk based on: 1607.07309, 1612.04527, 1901.10302, 1909.06272, 2001.07378, 2005.08013, 2007.08125, 2101.01405, 2102.02562 [Liao-He-Ma-Wang-Wang]

Convention for SM

- Field contents: $H, Q, L, u, d, e, B_\mu, W^I_\mu, G^A_\mu$,
- Symmetry:

$$\text{Poincare} \otimes \text{Gauge} = T_{1,3} \ltimes SO^{\uparrow}_{+}(1,3) \otimes SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$$

- SM Lagrangian:

$$\mathcal{L}_{\text{SM}} = -\frac{1}{4} G^A_{\mu\nu} G^{A\mu\nu} - \frac{1}{4} W^I_{\mu\nu} W^{I\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + (D_\mu H)^\dagger (D^\mu H) + \lambda v^2 (H^\dagger H) - \lambda (H^\dagger H)^2 + \sum_{\Psi = Q,L,u,d,e} \bar{\Psi} i \not{D} \Psi - \left[\bar{Q} Y_u u \tilde{H} + \bar{Q} Y_d d H + \bar{L} Y_e e H + \text{h.c.} \right]$$

- $D_\mu = \partial_\mu - i g_3 T^A G^A_\mu - i g_2 T^I W^I_\mu - i g_1 Y B_\mu, \quad \tilde{H}_i = \epsilon_{ij} H^*_j$
- $Y_{u,d,e}$: the Yukawa couplings;

The ingredients of the SMEFT

- **Assumption**: NP beyond the SM exists, with $\Lambda_{\text{NP}} \gg v$;
- **Ingredients**: SM fields + SM gauge symmetry;
- SMEFT= all possible local, SM gauge invariant operators built from SM fields ordered by the inverse power of Λ_{NP}, i.e.,

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \frac{1}{\Lambda_{\text{NP}}^{D-4}} \sum_{D\geq 5} C_i^D \mathcal{O}_i^D,$$

  Wilson coefficients C_i^D encode the contribution from unknown NP.
- Merit: play physics in a model-independent way;
- **Warning**: not all operators need to be considered $\leftarrow$ *the equivalence theorem* $\Rightarrow$ focus on a minimal basis in each dimension! (>>**JHY**)

Techniques used for operator reduction

- **Integration by parts(IBP)**: the total derivative term has no contribution to the action $D(\mathcal{Q}_1\mathcal{O}_2) = D(\mathcal{Q}_1)\mathcal{O}_2(\checkmark) + \mathcal{Q}_1 D(\mathcal{O}_2)(\times), \cdots$
- **Fierz identity(FI)**: rearrangement of the fermion bilinears

$$(\overline{\Psi_{1L}}\Psi_{2R})(\overline{\Psi^{\mathtt{C}}_{3R}}\Psi_{4R}) = -(\overline{\Psi_{1L}}\Psi_{3R})(\overline{\Psi^{\mathtt{C}}_{4R}}\Psi_{2R}) - (\overline{\Psi_{1L}}\Psi_{4R})(\overline{\Psi^{\mathtt{C}}_{2R}}\Psi_{3R}), \cdots$$

- **Equation of motion(EoM) & Field redefinition**: $i\not{D}Q = Y_u u\tilde{H} + Y_d dH, \cdots$
- **Group identity (GI)**: $(T^A)_{ab}(T^A)_{cd} = \frac{1}{2}\delta_{ad}\delta_{bc} - \frac{1}{2N}\delta_{ab}\delta_{cd}[SU(N)], \cdots$
- **Schouten identity (SI)**: $\epsilon_{ij}\epsilon_{kl} = \epsilon_{il}\epsilon_{kj} - \epsilon_{ik}\epsilon_{lj}, \cdots$
- **Dirac gamma matrix(DG)**:$\sigma_{\mu\nu}P_\pm = \mp\frac{1}{2}i\epsilon_{\mu\nu\rho\sigma}\sigma^{\rho\sigma}P_\pm, \cdots$
- **Bianchi identity (BI)**: $D_\nu\tilde{X}^{\mu\nu} = 0$.
- References: hep-ph/9304230, 1206.5141, 1607.07309, 2007.08125

More on the Fierz identities

The FIs are a very common tool in actual EFT calculations involving four or more fermion fields, here I'd like to share some personal experience on how to deal with them efficiently!

- Fermion bilinear property:

$$\overline{\psi_{1L(R)}}\psi_{2R(L)} = \overline{\psi^{\mathsf{C}}_{2R(L)}}\psi^{\mathsf{C}}_{1L(R)}\,, \overline{\psi_{1L(R)}}\gamma^\mu\psi_{2L(R)} = \overline{\psi^{\mathsf{C}}_{2L(R)}}\gamma^\mu\psi^{\mathsf{C}}_{1L(R)}\,, \overline{\psi_{1L(R)}}\sigma^{\mu\nu}\psi_{2R(L)} = -\overline{\psi^{\mathsf{C}}_{2R(L)}}\sigma^{\mu\nu}\psi^{\mathsf{C}}_{1L(R)}\,.$$

- The two basic ones:

$$(\overline{\psi_{1L}}\gamma^\mu\psi_{2L})(\overline{\psi_{3L}}\gamma_\mu\psi_{4L}) = (\overline{\psi_{1L}}\gamma^\mu\psi_{4L})(\overline{\psi_{3L}}\gamma_\mu\psi_{2L})(\dagger), (\overline{\psi_{1L}}\gamma^\mu\psi_{2L})(\overline{\psi_{3R}}\gamma_\mu\psi_{4R}) = -2(\overline{\psi_{1L}}\psi_{4R})(\overline{\psi_{3R}}\psi_{2L})(\dagger\dagger),$$

- Then $(\dagger) \overset{\dagger\dagger}{\Rightarrow} (\overline{\psi_{1L}}\gamma^\mu\psi_{2L})(\overline{\psi_{3L}}\gamma_\mu\psi_{4L}) = 2(\overline{\psi_{1L}}\psi^{\mathsf{C}}_{3L})(\overline{\psi_{4L}{}^{\mathsf{C}}}\psi_{2L})(*)$, and $\gamma^\mu\gamma^\nu = 2g^{\mu\nu} - \gamma^\nu\gamma^\mu(**)$
- The FI on the previous page is derived as follows:

$$\begin{aligned}(\overline{\Psi_{1L}}\Psi_{2R})(\overline{\Psi^{\mathsf{C}}_{3R}}\Psi_{4R}) &= -\frac{1}{4}(\overline{\Psi_{1L}}\Psi_{2R})(\overline{(\gamma_\mu\Psi_{3R})^{\mathsf{C}}}\gamma^\mu\Psi_{4R}) \overset{*}{=} -\frac{1}{8}(\overline{\psi_{1L}}\gamma^\mu\gamma^\nu\psi_{4R})(\overline{\psi^{\mathsf{C}}_{2R}}\gamma_\mu\gamma_\nu\psi_{3R})\\ &\overset{**}{=} -(\overline{\psi_{1L}}\psi_{4R})(\overline{\psi^{\mathsf{C}}_{2R}}\psi_{3R}) - \frac{1}{8}(\overline{\psi_{1L}}\gamma^\nu\gamma^\mu\psi_{4R})(\overline{(\gamma_\mu\psi_{2R})^{\mathsf{C}}}\gamma_\nu\psi_{3R})\\ &\overset{\dagger\dagger}{=} -(\overline{\psi_{1L}}\psi_{4R})(\overline{\psi^{\mathsf{C}}_{2R}}\psi_{3R}) - (\overline{\psi_{1L}}\psi_{3R})(\overline{\psi^{\mathsf{C}}_{2R}}\psi_{4R})(\spadesuit)\end{aligned}$$

- In principle, all FIs can be manipulated in this way!

The state of the art of operator basis in SMEFT

Lehman 2014

Weinberg, 1979 | Liao, XDMA, 2016 | Murphy, 2020 | Liao, XDMA, 2020

$$\mathscr{L}_{\rm SMEFT} = \mathscr{L}_{\rm SM} + \sum_{{\rm dim}\,5,i} \frac{\hat{C}_{5,i}}{\Lambda}\mathcal{O}^i_{\rm dim-5} + \sum_{{\rm dim}\,6,i} \frac{\hat{C}_{6,i}}{\Lambda^2}\mathcal{O}^i_{\rm dim-6} + \sum_{{\rm dim}\,7,i} \frac{\hat{C}_{7,i}}{\Lambda^3}\mathcal{O}^i_{\rm dim-7} + \sum_{{\rm dim}\,8,i} \frac{C_{8,i}}{\Lambda^4}\mathcal{O}^i_{\rm dim-8} + \sum_{{\rm dim}\,9,i} \frac{C_{9,i}}{\Lambda^5}\mathcal{O}^i_{\rm dim-9} + \cdots$$

Buchmuller, Wyler 1986
Grzadkowski, Iskrzynski, Misiak Rosiek 2010

Li, Ren, Xiao, Yu, Zheng, 2020
Li, Ren, Shu, Xiao, Yu, Zheng, 2020

Hilbert series method: *Henning, Lu, Melia, Murayama 2015, 2017*

- $D \in$ even(odd) if $|B - L|/2$ is even(odd) for SMEFT [1604.05726];
- $D = 6$: $|B - L| = 0$ vs $D = 7$: $|B - L| = 2$;
- $D \in$ odd: L/B is violated;
- Note: The true counting of complete and independent operators must consider fermion flavors.

Basis of dim-7 SMEFT operators [Lehman 14; 1607.07309]

	$\psi^2 H^4$ + h.c.		$\psi^2 H^3 D$ + h.c.
$\mathcal{O}_{LH}$	$\epsilon_{ij}\epsilon_{mn}(L^i C L^m)H^j H^n (H^\dagger H)$	$\mathcal{O}_{LeHD}$	$\epsilon_{ij}\epsilon_{mn}(L^i C\gamma_\mu e)H^j H^m iD^\mu H^n$
	$\psi^2 H^2 D^2$ + h.c.		$\psi^2 H^2 X$ + h.c.
$\mathcal{O}_{LDH1}$ $\mathcal{O}_{LDH2}$	$\epsilon_{ij}\epsilon_{mn}(\overline{L^{C,i}}\overleftrightarrow{D}_\mu L^j)(H^m D^\mu H^n)$ $\epsilon_{im}\epsilon_{jn}(L^i C D^\mu L^j)H^m(D_\mu H^n)$	$\mathcal{O}_{LHB}$ $\mathcal{O}_{LHW}$	$g_1\epsilon_{ij}\epsilon_{mn}(L^i C\sigma_{\mu\nu}L^m)H^j H^n B^{\mu\nu}$ $g_2\epsilon_{ij}(\epsilon\tau^I)_{mn}(L^i C\sigma_{\mu\nu}L^m)H^j H^n W^{I\mu\nu}$
	$\psi^4 D$ + h.c.		$\psi^4 H$ + h.c.
$\mathcal{O}_{\bar{d}uLDL}$	$\epsilon_{ij}(\bar{d}\gamma_\mu u)(\overline{L^{C,i}} i\overleftrightarrow{D}^\mu L^j)$	$\mathcal{O}_{\bar{e}LLLH}$ $\mathcal{O}_{\bar{d}QLLH1}$ $\mathcal{O}_{\bar{d}QLLH2}$ $\mathcal{O}_{\bar{d}uLeH}$ $\mathcal{O}_{\bar{Q}uLLH}$	$\epsilon_{ij}\epsilon_{mn}(\bar{e}L^i)(L^j C L^m)H^n$ $\epsilon_{ij}\epsilon_{mn}(\bar{d}Q^i)(\overline{L^{C,j}}L^m)H^n$ $\epsilon_{ij}\epsilon_{mn}(\bar{d}\sigma_{\mu\nu}Q^i)(\overline{L^{C,j}}\sigma^{\mu\nu}L^m)H^n$ $\epsilon_{ij}(\bar{d}\gamma_\mu u)(\overline{L^{C,i}}\gamma^\mu e)H^j$ $\epsilon_{ij}(\bar{Q}u)(LCL^i)H^j$
$\mathcal{O}_{\bar{L}QddD}$ $\mathcal{O}_{\bar{e}dddD}$	$(\bar{L}\gamma_\mu Q)(dCiD^\mu d)$ $(\bar{e}\gamma_\mu d)(dCiD^\mu d)$	$\mathcal{O}_{\bar{L}dud\tilde{H}}$ $\mathcal{O}_{\bar{L}dddH}$ $\mathcal{O}_{\bar{e}Qdd\tilde{H}}$ $\mathcal{O}_{\bar{L}dQQ\tilde{H}}$	$(\bar{L}d)(uCd)\tilde{H}$ $(\bar{L}d)(dCd)H$ $\epsilon_{ij}(\bar{e}Q^i)(dCd)\tilde{H}^j$ $\epsilon_{ij}(\bar{L}d)(QCQ^i)\tilde{H}^j$
	Redundant operators		
$\mathcal{O}^{(2)}_{\bar{d}uLLD}$	$\epsilon_{ij}(\bar{d}\gamma_\mu u)(L^i C\sigma^{\mu\nu}D_\nu L^j)$	$\mathcal{O}^{(2)}_{\bar{L}dQdD}$	$(\bar{L}iD^\mu d)(QC\gamma_\mu d)$

Operators contributing to $K^- \to \pi^+ \ell^-_\alpha \ell^-_\beta$ and $0\nu\beta\beta$

- **Mass mechanism**: $\mathcal{O}^{5\dagger}_{LH}$, $\mathcal{O}^{\dagger}_{LH}$;
- **Long-range interaction**: $\mathcal{O}^\dagger_{LeHD}$, $\mathcal{O}^\dagger_{\bar{d}QLLH1}$, $\mathcal{O}^\dagger_{\bar{d}QLLH2}$, $\mathcal{O}^\dagger_{\bar{d}uLeH}$, $\mathcal{O}^\dagger_{\bar{Q}uLLH}$, $\mathcal{O}^\dagger_{LDH1}$, $\mathcal{O}^\dagger_{LHW}$, $\mathcal{O}^\dagger_{\bar{d}uLLD}$;
- **Short-range interaction**: $\mathcal{O}^\dagger_{LDH1}$, $\mathcal{O}^\dagger_{LDH2}$, $\mathcal{O}^\dagger_{LHW}$, $\mathcal{O}^\dagger_{\bar{d}uLLD}$

Proof of redundancy

$$
\begin{aligned}
\mathcal{O}^{(2)prst}_{\bar{d}uLLD} &= \epsilon_{ij}(\bar{d}_p\gamma_\mu u_r)(L^i_s C\sigma^{\mu\nu} D_\nu L^j_t) \\
&\overset{\text{DG}}{=\!=} \epsilon_{ij}(\bar{d}_p\gamma_\mu u_r)(L^i_s C\gamma^\mu\gamma^\nu iD_\nu L^j_t) - \epsilon_{ij}(\bar{d}_p\gamma_\mu u_r)(L^i_s CiD^\mu L^j_t) \\
&\overset{\text{EoM}}{=\!=} (Y_e)_{tu}\epsilon_{ij}(\bar{d}_p\gamma_\mu u_r)(L^i_s C\gamma^\mu e_u)H^j - \mathcal{O}^{prst}_{\bar{d}uLLD} \\
&= 2(Y_e)_{tu}\mathcal{O}^{psru}_{\bar{d}LueH} - \mathcal{O}^{prst}_{\bar{d}uLLD}, \\
\mathcal{O}^{prst}_{\bar{L}QddD} &= \epsilon_{\alpha\beta\sigma}\delta_{ij}(\bar{L}_{ip}\gamma_\mu Q_{j\alpha r})(d_{\beta s}CiD^\mu_{\sigma\rho}d_{\rho t}) \\
&\overset{\text{FI}}{=\!=} \epsilon_{\alpha\beta\sigma}\delta_{ij}\Big((\bar{L}_{ip}d_{\beta s})(Q_{j\alpha r}Ci\gamma_\mu D^\mu_{\sigma\rho}d_{\rho t}) + (\bar{L}_{ip}iD^\mu_{\sigma\rho}d_{\rho t})(Q_{j\alpha r}C\gamma_\mu d_{\beta s})\Big) \\
&\overset{\text{EoM}}{=\!=} (Y_d^\dagger)_{tu}\epsilon_{\alpha\beta\sigma}\delta_{ij}\delta_{kl}(\bar{L}_{ip}d_{\beta s})(Q_{j\alpha r}CQ_{k\sigma u})H^*_l + \mathcal{O}^{ptrs}_{\bar{L}dQdD} \\
&= (Y_d^\dagger)_{tu}\mathcal{O}^{psru}_{\bar{L}dQQ\tilde{H}} + \mathcal{O}^{ptrs}_{\bar{L}dQdD},
\end{aligned}
$$

Key low energy observables from dim-7 interactions

- **Neutrino magnetic moment** [1901.10302];
- Nuclear sector:
  - **Neutrinoless double beta decay($0\nu\beta\beta$)** [1708.09390, 1901.10302];
  - Nuclear muon-positron/anti-muon conversion: $\mu^- X \to e^+(\mu^+)X'$;
  - Trimuon production from neutrino-neutron collision: $\nu N \to \mu^-\mu^+\mu^+ X$;
- Meson and baryon sector:
  - **Kaon:** $K^\pm \to \pi^\mp \ell_\alpha^\pm \ell_\beta^\pm$ [1909.06272, 2001.07378];
  - **Tau:** $\tau^- \to M_1^- M_2^- \ell^+$ [2102.03491];
  - Heavy meson LNV 3-body decays: $M_1^\pm \to M_2^\mp \ell_1^\pm \ell_2^\pm$ with $M_1 \in \{D, D_s, B, B_c\}$ [Belle, LHCb,$\cdots$];
  - Heavy baryon LNV 3-body decays: $B_1^\pm \to B_2^\mp \ell_1^\pm \ell_2^\pm$ with $B_1 \in \{\Sigma, \Xi, \Omega, \cdots\}$;
- Collider search: LHC, ...,

Experimental limits on meson LNV 3-body decays

Decay	Exp. UL	Decay	Exp. UL	Decay	Exp. UL
$K^- \to \pi^+\mu^-\mu^-$	4.2×10^{-11}	$K^- \to \pi^+e^-e^-$	2.2×10^{-10}	$K^- \to \pi^+\mu^-e^-$	5.0×10^{-10}
$D^- \to \pi^+\mu^-\mu^-$	2.2×10^{-8}	$D^- \to \pi^+e^-e^-$	1.1×10^{-6}	$D^- \to \pi^+\mu^-e^-$	2.0×10^{-6}
$D^- \to K^+\mu^-\mu^-$	1.0×10^{-5}	$D^- \to K^+e^-e^-$	9×10^{-7}	$D^- \to K^+\mu^-e^-$	1.9×10^{-6}
$D^- \to \rho^+\mu^-\mu^-$	5.6×10^{-4}	$D^- \to \rho^+e^-e^-$	–	$D^- \to \rho^+\mu^-e^-$	–
$D^- \to K^{*+}\mu^-\mu^-$	8.5×10^{-4}	$D^- \to K^{*+}e^-e^-$	–	$D^- \to K^{*+}\mu^-e^-$	–
$D_s^- \to \pi^+\mu^-\mu^-$	1.2×10^{-7}	$D_s^- \to \pi^+e^-e^-$	4.1×10^{-6}	$D_s^- \to \pi^+\mu^-e^-$	8.4×10^{-6}
$D_s^- \to K^+\mu^-\mu^-$	1.3×10^{-5}	$D_s^- \to K^+e^-e^-$	5.2×10^{-6}	$D_s^- \to K^+\mu^-e^-$	6.1×10^{-6}
$D_s^- \to K^{*+}\mu^-\mu^-$	1.4×10^{-3}	$D_s^- \to K^{*+}e^-e^-$	–	$D_s^- \to K^{*+}\mu^-e^-$	–
$B^- \to \pi^+\mu^-\mu^-$	4.0×10^{-9}	$B^- \to \pi^+e^-e^-$	2.3×10^{-8}	$B^- \to \pi^+\mu^-e^-$	1.5×10^{-7}
$B^- \to K^+\mu^-\mu^-$	4.1×10^{-8}	$B^- \to K^+e^-e^-$	3.0×10^{-8}	$B^- \to K^+\mu^-e^-$	1.6×10^{-7}
$B^- \to K^{*+}\mu^-\mu^-$	5.9×10^{-7}	$B^- \to K^{*+}e^-e^-$	4.0×10^{-7}	$B^- \to K^{*+}\mu^-e^-$	3.0×10^{-7}
$B^- \to \rho^+\mu^-\mu^-$	4.2×10^{-7}	$B^- \to \rho^+e^-e^-$	1.7×10^{-7}	$B^- \to \rho^+\mu^-e^-$	4.7×10^{-7}
$B^- \to D^+\mu^-\mu^-$	6.9×10^{-7}	$B^- \to D^+e^-e^-$	2.6×10^{-6}	$B^- \to D^+\mu^-e^-$	1.8×10^{-6}
$B^- \to D_s^+\mu^-\mu^-$	5.8×10^{-7}	$B^- \to D_s^+e^-e^-$	–	$B^- \to D_s^+\mu^-e^-$	–
$B^- \to D^{*+}\mu^-\mu^-$	2.4×10^{-6}	$B^- \to D^{*+}e^-e^-$	–	$B^- \to D^{*+}\mu^-e^-$	–
$\tau^- \to e^+\pi^-\pi^-$	2.0×10^{-8}	$\tau^- \to e^+\pi^-K^-$	3.2×10^{-8}	$\tau^- \to e^+K^-K^-$	3.3×10^{-8}
$\tau^- \to \mu^+\pi^-\pi^-$	3.9×10^{-8}	$\tau^- \to \mu^+\pi^-K^-$	4.8×10^{-8}	$\tau^- \to \mu^+K^-K^-$	4.7×10^{-8}

NA62 [1905.07770]; E865 [0006003]; LHCb [1110.0730, 1201.5600, 1304.6365, 1401.5361]; CLEO [1009.1606]; BaBar [1107.4465, 1202.3650, 1310.8238]; E653 [Phys. Lett. B 345, 85 (1995)]; Belle [1107.0642, 1206.5595].

The SMNEFT

$$\mathcal{L}_{\text{SM}+N} = \mathcal{L}_{\text{SM}} + \bar{N}\not{\partial}N - \left[\frac{1}{2}(NCM_N N) + \bar{L}Y_N N\tilde{H} + \text{h.c.}\right]$$

Besides the basis operators in SMEFT at each dimension, there are additional operators involving sterile neutrino N:

- dim 5: $2(\not{L} \cap B)$ [0904.3244]
- dim 6: $16(L \cap B) + 1(\not{L} \cap B) + 2(\not{L} \cap \not{B})$ [0806.0876; **1612.04527**]
- dim 7: $47(\not{L} \cap B) + 5(\not{L} \cap \not{B})$ [1505.05264; **1612.04527**]
- Important for the study of general neutrino interactions like CEνNS.

LEFT and its development

- **Working scale**: below $\Lambda_{\rm EW}$;
- **Fields**: $u,\ d,\ s,\ c,\ b,\ e,\ \nu_e, \mu,\ \nu_\mu,\ \tau,\ \nu_\tau, \gamma, g$;
- **Symmetry**: $U(1)_{\rm em} \times SU(3)_{\rm color}$;
- LEFT=all possible local, $U(1)_{\rm EM} \times SU(3)_C$ invariant operators constructed from the above fields ordered by the inverse power of $\Lambda_{\rm EW}$, i.e.,

Jenkins, Manohar, Stoffer, 2018 — Li, Ren, Xiao, Yu, Zheng, 2020

$$\mathscr{L}_{\rm LEFT} = \mathscr{L}_{\dim\leq 4} + \sum_{\dim 5,i} \frac{\hat{C}_{5,i}}{\Lambda} \mathscr{Q}^i_{\dim-5} + \sum_{\dim 6,i} \frac{\hat{C}_{6,i}}{\Lambda^2} \mathscr{Q}^i_{\dim-6} + \sum_{\dim 7,i} \frac{\hat{C}_{7,i}}{\Lambda^3} \mathscr{Q}^i_{\dim-7} + \sum_{\dim 8,i} \frac{\hat{C}_{8,i}}{\Lambda^4} \mathscr{Q}^i_{\dim-8} + \sum_{\dim 9,i} \frac{\hat{C}_{9,i}}{\Lambda^5} \mathscr{Q}^i_{\dim-9} + \cdots$$

Liao, XDMA, Wang, 2020 — Murphy, 2020 — Liao, XDMA, Wang, 2019

Dim-6 and dim-7 $|\Delta L| = 2$ operators$(\ell\nu)$ in LEFT

- At dim 6 [Manohar *et al* 17]:

$$\mathcal{O}^{LL,V}_{pr\alpha\beta} = (\overline{u^p_L}\gamma_\mu d^r_L)(\overline{\ell_\alpha}\gamma^\mu P_R \nu^{\mathsf{C}}_\beta), \qquad \mathcal{O}^{RR,V}_{pr\alpha\beta} = (\overline{u^p_R}\gamma_\mu d^r_R)(\overline{\ell_\alpha}\gamma^\mu P_R \nu^{\mathsf{C}}_\beta),$$
$$\mathcal{O}^{RL,S}_{pr\alpha\beta} = (\overline{u^p_R} d^r_L)(\overline{\ell_\alpha} P_R \nu^{\mathsf{C}}_\beta), \qquad \mathcal{O}^{LR,S}_{pr\alpha\beta} = (\overline{u^p_L} d^r_R)(\overline{\ell_\alpha} P_R \nu^{\mathsf{C}}_\beta),$$
$$\mathcal{O}^{LR,T}_{pr\alpha\beta} = (\overline{u^p_L}\sigma_{\mu\nu} d^r_R)(\overline{\ell_\alpha}\sigma^{\mu\nu} P_R \nu^{\mathsf{C}}_\beta),$$

- At dim 7 [2005.08013]:

$$\mathcal{O}^{LL,VD}_{pr\alpha\beta} = (\overline{u^p_L}\gamma_\mu d^r_L)(\overline{\ell_\alpha} i \overleftrightarrow{D}^\mu P_R \nu^{\mathsf{C}}_\beta),$$
$$\mathcal{O}^{RR,VD}_{pr\alpha\beta} = (\overline{u^p_R}\gamma_\mu d^r_R)(\overline{\ell_\alpha} i \overleftrightarrow{D}^\mu P_R \nu^{\mathsf{C}}_\beta),$$
$$\mathcal{O}^{LR,TD}_{pr\alpha\beta} = (\overline{u^p_L}\sigma_{\mu\nu} d^r_R)(\overline{\ell_\alpha}\gamma^{[\mu} \overleftrightarrow{D}^{\nu]} P_R \nu^{\mathsf{C}}_\beta),$$
$$\mathcal{O}^{RL,TD}_{pr\alpha\beta} = (\overline{u^p_R}\sigma_{\mu\nu} d^r_L)(\overline{\ell_\alpha}\gamma^{[\mu} \overleftrightarrow{D}^{\nu]} P_R \nu^{\mathsf{C}}_\beta),$$

Dim-9 $|\Delta L| = 2$ operators($\ell\ell$) in LEFT [1909.06272]

Notation	Operator	Notation	Operator
$\mathcal{O}_{prst}^{LLLL,S/P}$	$(\overline{u_L^p}\gamma^\mu d_L^r)[\overline{u_L^s}\gamma_\mu d_L^t](j^{\alpha\beta}/j_5^{\alpha\beta})$	$\mathcal{O}_{prst}^{RRRR,S/P}$	$(\overline{u_R^p}\gamma^\mu d_R^r)[\overline{u_R^s}\gamma_\mu d_R^t](j^{\alpha\beta}/j_5^{\alpha\beta})$
$\mathcal{O}_{prst}^{LLLL,T}$	$(\overline{u_L^p}\gamma^\mu d_L^r)[\overline{u_L^s}\gamma^\nu d_L^t](j_{\mu\nu}^{\alpha\beta})$	$\mathcal{O}_{prst}^{RRRR,T}$	$(\overline{u_R^p}\gamma^\mu d_R^r)[\overline{u_R^s}\gamma^\nu d_R^t](j_{\mu\nu}^{\alpha\beta})$
$\tilde{\mathcal{O}}_{prst}^{LLLL,T}$	$(\overline{u_L^p}\gamma^\mu d_L^r][\overline{u_L^s}\gamma^\nu d_L^t)(j_{\mu\nu}^{\alpha\beta})$	$\tilde{\mathcal{O}}_{prst}^{RRRR,T}$	$(\overline{u_R^p}\gamma^\mu d_R^r][\overline{u_R^s}\gamma^\nu d_R^t)(j_{\mu\nu}^{\alpha\beta})$
$\mathcal{O}_{prst}^{LRLR,S/P}$	$(\overline{u_L^p} d_R^r)[\overline{u_L^s} d_R^t](j^{\alpha\beta}/j_5^{\alpha\beta})$	$\mathcal{O}_{prst}^{RLRL,S/P}$	$(\overline{u_R^p} d_L^r)[\overline{u_R^s} d_L^t](j^{\alpha\beta}/j_5^{\alpha\beta})$
$\tilde{\mathcal{O}}_{prst}^{LRLR,S/P}$	$(\overline{u_L^p} d_R^r][\overline{u_L^s} d_R^t)(j^{\alpha\beta}/j_5^{\alpha\beta})$	$\tilde{\mathcal{O}}_{prst}^{RLRL,S/P}$	$(\overline{u_R^p} d_L^r][\overline{u_R^s} d_L^t)(j^{\alpha\beta}/j_5^{\alpha\beta})$
$\mathcal{O}_{prst}^{LRLR,T}$	$(\overline{u_L^p} i\sigma^{\mu\nu} d_R^r)[\overline{u_L^s} d_R^t](j_{\mu\nu}^{\alpha\beta})$	$\mathcal{O}_{prst}^{RLRL,T}$	$(\overline{u_R^p} i\sigma^{\mu\nu} d_L^r)[\overline{u_R^s} d_L^t](j_{\mu\nu}^{\alpha\beta})$
$\tilde{\mathcal{O}}_{prst}^{LRLR,T}$	$(\overline{u_L^p}\sigma^{\mu\rho} d_R^r)[\overline{u_L^s}\sigma^{\nu}{}_{\rho} d_R^t](j_{\mu\nu}^{\alpha\beta})$	$\tilde{\mathcal{O}}_{prst}^{RLRL,T}$	$(\overline{u_R^p}\sigma^{\mu\rho} d_L^r)[\overline{u_R^s}\sigma^{\nu}{}_{\rho} d_L^t](j_{\mu\nu}^{\alpha\beta})$
$\mathcal{O}_{prst}^{LRLL,V/A}$	$(\overline{u_L^p} d_R^r)[\overline{u_L^s}\gamma^\mu d_L^t](j_\mu^{\alpha\beta}/j_{5\mu}^{\alpha\beta})$	$\mathcal{O}_{prst}^{RLRR,V/A}$	$(\overline{u_R^p} d_L^r)[\overline{u_R^s}\gamma^\mu d_R^t](j_\mu^{\alpha\beta}/j_{5\mu}^{\alpha\beta})$
$\tilde{\mathcal{O}}_{prst}^{LRLL,V/A}$	$(\overline{u_L^p} d_R^r][\overline{u_L^s}\gamma^\mu d_L^t)(j_\mu^{\alpha\beta}/j_{5\mu}^{\alpha\beta})$	$\tilde{\mathcal{O}}_{prst}^{RLRR,V/A}$	$(\overline{u_R^p} d_L^r][\overline{u_R^s}\gamma^\mu d_R^t)(j_\mu^{\alpha\beta}/j_{5\mu}^{\alpha\beta})$
$\mathcal{O}_{prst}^{LRRR,V/A}$	$(\overline{u_L^p} d_R^r)[\overline{u_R^s}\gamma^\mu d_R^t](j_\mu^{\alpha\beta}/j_{5\mu}^{\alpha\beta})$	$\mathcal{O}_{prst}^{RLLL,V/A}$	$(\overline{u_R^p} d_L^r)[\overline{u_L^s}\gamma^\mu d_L^t](j_\mu^{\alpha\beta}/j_{5\mu}^{\alpha\beta})$
$\tilde{\mathcal{O}}_{prst}^{LRRR,V/A}$	$(\overline{u_L^p} d_R^r][\overline{u_R^s}\gamma^\mu d_R^t)(j_\mu/j_{5\mu}^{\alpha\beta})$	$\tilde{\mathcal{O}}_{prst}^{RLLL,V/A}$	$(\overline{u_R^p} d_L^r][\overline{u_L^s}\gamma^\mu d_L^t)(j_\mu/j_{5\mu}^{\alpha\beta})$
$\mathcal{O}_{prst}^{LRRL,T}$	$(\overline{u_L^p} i\sigma^{\mu\nu} d_R^r)[\overline{u_R^s} d_L^t](j_{\mu\nu}^{\alpha\beta})$	$\mathcal{O}_{prst}^{RLLR,T}$	$(\overline{u_R^p} i\sigma^{\mu\nu} d_L^r)[\overline{u_L^s} d_R^t](j_{\mu\nu}^{\alpha\beta})$
$\tilde{\mathcal{O}}_{prst}^{LRRL,T}$	$(\overline{u_L^p} i\sigma^{\mu\nu} d_R^r][\overline{u_R^s} d_L^t)(j_{\mu\nu}^{\alpha\beta})$	$\tilde{\mathcal{O}}_{prst}^{RLLR,T}$	$(\overline{u_R^p} i\sigma^{\mu\nu} d_L^r][\overline{u_L^s} d_R^t)(j_{\mu\nu}^{\alpha\beta})$
$\mathcal{O}_{prst}^{LRRL,S/P}$	$(\overline{u_L^p} d_R^r)[\overline{u_R^s} d_L^t](j^{\alpha\beta}/j_5^{\alpha\beta})$		
$\tilde{\mathcal{O}}_{prst}^{LRRL,S/P}$	$(\overline{u_L^p} d_R^r][\overline{u_R^s} d_L^t)(j^{\alpha\beta}/j_5^{\alpha\beta})$		

- $j^{\alpha\beta} = (\overline{l_\alpha} l_\beta^C)$, $j_5^{\alpha\beta} = (\overline{l_\alpha}\gamma_5 l_\beta^C)$, $j_{5,\mu}^{\alpha\beta} = (\overline{l_\alpha}\gamma_\mu\gamma_5 l_\beta^C)$ (symmetric)
- $j_\mu^{\alpha\beta} = (\overline{l_\alpha}\gamma_\mu l_\beta^C)$, $j_{\mu\nu}^{\alpha\beta} = (\overline{l_\alpha}\sigma_{\mu\nu} l_\beta^C)$ (anti-symmetric)

Basics for operator renormalization

- Motivation: tame the large logs from perturbative expansion;
- Except the field strength renormalization, there exists operator renormalization: **operator mixing effect**;
- The deminant contributions are from the 1-loop SM correction;
- The renormalization group equation: $16\pi^2 \mu \frac{dC_d}{d\mu} = \hat{\gamma} C_d$, $\hat{\gamma}$ as the anomalous dimension matrix;
- 1-loop calculation: **dimensional regularizaton**$(d = 4 - 2\epsilon)$ + $\overline{\textbf{MS}}$ **scheme** + **background gauge** R_ξ^B
- ξ independent as a check for the calculation.

Basics for operator renormalization

Calculation of β-function($\hat{\gamma}$) $\Leftarrow$ the counterterm $\Leftarrow$ 1-loop correction:

$$\hat{\gamma}_{ij} = -\sum_{g_\alpha} \rho_\alpha \frac{\partial}{\partial g_\alpha} [\Sigma^T + \frac{1}{2} A]_{ij}$$

with $\rho_\alpha = 1$ for $g_\alpha \in \{g_{1,2,3}, Y_{e,d,u}\}$ and $\rho_\alpha = 2$ for $g_\alpha = \lambda$.

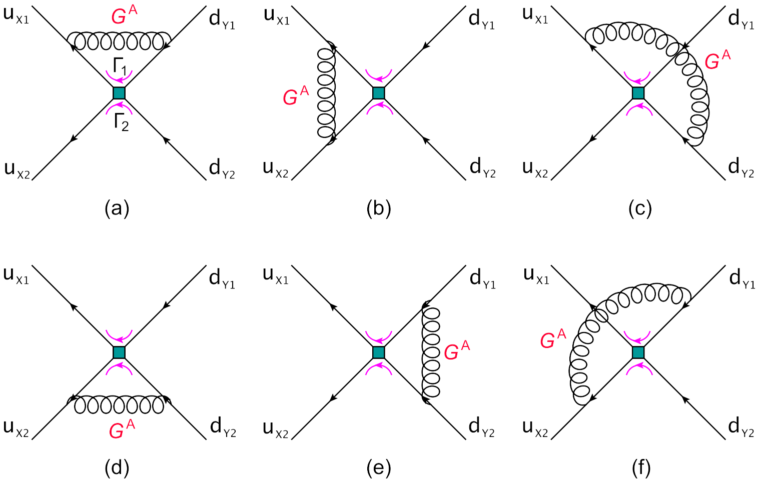
$$C_d^T \mathcal{O}_d \xrightarrow{\text{1-loop irred.}} \frac{1}{16\pi^2\epsilon} C_d^T \Sigma \mathcal{O}_d + \text{finite.}$$

$$C_d^T \mathcal{O}_d \xrightarrow{\text{1-loop red.}} \frac{1}{16\pi^2\epsilon} C_d^T \frac{1}{2} A \mathcal{O}_d + \text{finite.},$$

Total 1-loop correction

$$\langle C_d^T \mathcal{O}_d \rangle^{\text{1-L}} = \frac{1}{16\pi^2\epsilon} C_d^T [\Sigma + \frac{1}{2} A] \mathcal{O}_d$$

Example: QCD running effect for the dim-9 $|\Delta L| = 2$ operators($\ell\ell$) in LEFT



The final renormalization group running equations

$$\mu \frac{d}{d\mu} \begin{pmatrix} C_{prst}^{LLLL,S/P} \\ C_{ptsr}^{LLLL,S/P} \end{pmatrix} = -\frac{\alpha_s}{2\pi} \begin{pmatrix} \frac{3}{N} & -3 \\ -3 & \frac{3}{N} \end{pmatrix} \begin{pmatrix} C_{prst}^{LLLL,S/P} \\ C_{ptsr}^{LLLL,S/P} \end{pmatrix},$$

$$\mu \frac{d}{d\mu} \begin{pmatrix} C_{prst}^{LRLR,S/P} \\ C_{ptsr}^{LRLR,S/P} \\ \tilde{C}_{prst}^{LRLR,S/P} \\ \tilde{C}_{ptsr}^{LRLR,S/P} \end{pmatrix} = -\frac{\alpha_s}{2\pi} \begin{pmatrix} \frac{2}{N} + 6C_F & -4 & 2 & \frac{2}{N} - 4C_F \\ -4 & \frac{2}{N} + 6C_F & \frac{2}{N} - 4C_F & 2 \\ -2 & \frac{4}{N} & -\frac{2}{N} - 2C_F & -2 \\ \frac{4}{N} & -2 & -2 & -\frac{2}{N} - 2C_F \end{pmatrix} \begin{pmatrix} C_{prst}^{LRLR,S/P} \\ C_{ptsr}^{LRLR,S/P} \\ \tilde{C}_{prst}^{LRLR,S/P} \\ \tilde{C}_{ptsr}^{LRLR,S/P} \end{pmatrix},$$

$$\mu \frac{d}{d\mu} \begin{pmatrix} C_{prst}^{LRLL,A} \\ C_{srpt}^{LRLL,A} \\ \tilde{C}_{prst}^{LRLL,A} \\ \tilde{C}_{srpt}^{LRLL,A} \end{pmatrix} = -\frac{\alpha_s}{2\pi} \begin{pmatrix} \frac{1}{N} + 3C_F & -2 & 1 & \frac{1}{N} - 2C_F \\ -2 & \frac{1}{N} + 3C_F & \frac{1}{N} - 2C_F & 1 \\ -1 & \frac{2}{N} & -\frac{1}{N} - C_F & -1 \\ \frac{2}{N} & -1 & -1 & -\frac{1}{N} - C_F \end{pmatrix} \begin{pmatrix} C_{prst}^{LRLL,A} \\ C_{srpt}^{LRLL,A} \\ \tilde{C}_{prst}^{LRLL,A} \\ \tilde{C}_{srpt}^{LRLL,A} \end{pmatrix},$$

$$\mu \frac{d}{d\mu} \begin{pmatrix} C_{prst}^{LRRR,A} \\ C_{ptsr}^{LRRR,A} \\ \tilde{C}_{prst}^{LRRR,A} \\ \tilde{C}_{ptsr}^{LRRR,A} \end{pmatrix} = -\frac{\alpha_s}{2\pi} \begin{pmatrix} \frac{1}{N} + 3C_F & -2 & 1 & \frac{1}{N} - 2C_F \\ -2 & \frac{1}{N} + 3C_F & \frac{1}{N} - 2C_F & 1 \\ -1 & \frac{2}{N} & -\frac{1}{N} - C_F & -1 \\ \frac{2}{N} & -1 & -1 & -\frac{1}{N} - C_F \end{pmatrix} \begin{pmatrix} C_{prst}^{LRRR,A} \\ C_{ptsr}^{LRRR,A} \\ \tilde{C}_{prst}^{LRRR,A} \\ \tilde{C}_{prst}^{LRRR,A} \end{pmatrix},$$

$$\mu \frac{d}{d\mu} \begin{pmatrix} C_{prst}^{LRRL,S/P} \\ \tilde{C}_{prst}^{LRRL,S/P} \end{pmatrix} = -\frac{\alpha_s}{2\pi} \begin{pmatrix} 6C_F & 3 \\ 0 & -\frac{3}{N} \end{pmatrix} \begin{pmatrix} C_{prst}^{LRRL,S/P} \\ \tilde{C}_{ptsr}^{LRRL,S/P} \end{pmatrix}, \ldots,$$

The QCD running effect from $\Lambda_{\rm EW}$ to Λ_χ

- Non-diagonal elements $\Rightarrow$ **operator mixing effect**;
- Restrict to the first generation fermions: the running result for the operators contributing to $0\nu\beta\beta$;
- Running effect from $\Lambda_{\rm EW}$ to Λ_χ:

$$C_{prst}^{LLLL,S/P}(\Lambda_\chi) = 1.22\, C_{prst}^{LLLL,S/P} - 0.44\, C_{ptsr}^{LLLL,S/P},$$
$$C_{prst}^{LRLR,S/P}(\Lambda_\chi) = 3.12\, C_{prst}^{LRLR,S/P} - 1.3\, C_{ptsr}^{LRLR,S/P} + 0.75\, \tilde{C}_{prst}^{LRLR,S/P} - 1.09\, \tilde{C}_{ptsr}^{LRLR,S/P},$$
$$\tilde{C}_{prst}^{LRLR,S/P}(\Lambda_\chi) = 0.54\, \tilde{C}_{prst}^{LRLR,S/P} - 0.02\, \tilde{C}_{ptsr}^{LRLR,S/P} - 0.5\, C_{prst}^{LRLR,S/P} + 0.42\, C_{ptsr}^{LRLR,S/P},$$
$$C_{prst}^{LRRL,S/P}(\Lambda_\chi) = 2.74\, C_{prst}^{LRRL,S/P} + 0.62\, \tilde{C}_{ptsr}^{LRRL,S/P},$$
$$\tilde{C}_{prst}^{LRRL,S/P}(\Lambda_\chi) = 0.88\, \tilde{C}_{prst}^{LRRL,S/P}, \cdots,$$

- Running effect can be as large/small as multiple times.

Basics

- Here we first focus on mesonic part without nucleons;
- **Origin**: QCD has an approximate $G = SU(3)_L \times SU(3)_R$ flavor symmetry for u, d, s quarks, which spontaneously breaks into $SU(3)_V$ by quark condensation;
- **Characteristic scale**: chiral symmetry breaking scale $\Lambda_\chi \approx 1$ GeV ;
- **Fields**: Pseudo Nambu-Goldstone bosons, i.e., light mesons, represented as $\xi = \sqrt{\Sigma} = \exp[i\pi^a \lambda^a / 2F_0]$; and possibly, the external sources;
- Under G : $q_L \to L q_L$, $q_R \to R q_R$, $\xi \to L\xi U^\dagger = U\xi R^\dagger$;
- Mesonic χPT=all possible local, G invariant operators constructed via D_μ and $\xi(\Sigma)$, and ordered by number of derivatives $\mathcal{O}(p^n)$: e.g.,

$$\mathcal{L}_{p^2} = \frac{F_0^2}{4}\mathrm{Tr}(\partial_\mu \Sigma \partial^\mu \Sigma^\dagger) + \frac{F_0^2}{4}(2B_0)\mathrm{Tr}(M^\dagger \Sigma^\dagger + \Sigma M),$$

- **Development**: $\mathcal{L}_{p^4}$[Gasser-Leutwyler], $\mathcal{L}_{p^6}$[9408346, 9902437], $\mathcal{L}_{p^8}$[1810.06834], Hilbert series[2009.01239]

Hadronic realization of 2-quark LEFT operators — focus on dim-6(7) $(\bar{q}\Gamma q)(\ell\Gamma\nu)(D_\mu)$

The method of external sources

- Step 1: Take the QCD-like Lagrangian:

  $\mathcal{L} = \mathcal{L}_{\rm QCD} + \overline{q_L} l_\mu \gamma^\mu q_L + \overline{q_R} r_\mu \gamma^\mu q_R + (\overline{q_R}(s + ip) q_L + \overline{q_L}(t_l^{\mu\nu}\sigma_{\mu\nu}) q_R + \text{h.c.})\,,$

  where $\{l_\mu,\ r_\mu,\ s,\ p, t_l^{\mu\nu}, t_r^{\mu\nu} = t_l^{\mu\nu\dagger}\}$ are external sources;
- Step 2: Identify the corresponding external sources hidden in dim-6(7) LEFT operators;
- Step 3: To linear term in external sources and LO in χPT power counting, they enter through

$$\mathcal{L}_{p^2} = \frac{F_0^2}{4}\text{Tr}(\xi_\mu \xi^\mu) + \frac{F_0^2}{4}\text{Tr}(\chi_+),$$

$$\xi_\mu = i\Big(\xi(\partial_\mu - i r_\mu)\xi^\dagger - \xi^\dagger(\partial_\mu - i l_\mu)\xi^\dagger\Big), \chi_+ = \xi\chi\xi \pm \xi^\dagger\chi^\dagger\xi^\dagger, \chi = 2B(s - ip).$$

- Tensor sources first appear at p^4 [JHEP09(2007)078, Cata-Mateu]

$$\mathcal{L}_{p^4} \supset i\Lambda_2\,\text{Tr}\left(t_l^{\mu\nu}\,(D_\mu U)^\dagger\,U\,(D_\nu U)^\dagger + t_r^{\mu\nu} D_\mu U U^\dagger D_\nu U\right)$$

Hadronic realization of *n*-quark ($n \geq 3$) LEFT operators — focus on dim-9 $(\bar{q}qqq)(\ell\ell)$ operators I

The method of spurion analysis

- Step 1: Take the quark level operator(irrep. under G) as
$$\mathcal{O} = T_{cd}^{\ ab}(\overline{q^{c}_{X_1}}\Gamma_1 q_{Y_1,a})(\overline{q^{d}_{X_2}}\Gamma_2 q_{Y_2,b}),$$
Under chiral group G, the quark fields transform as
$$q_{L,a} \to L_a^{\ p} q_{L,p},\ \overline{q_R}^b \to \overline{q_R}^p (R^\dagger)_p^{\ b},\ q_{R,a} \to R_a^{\ p} q_{R,p},\ \overline{q_L}^b \to \overline{q_L}^p (L^\dagger)_p^{\ b}$$
Require $\mathcal{O}$ to be invariant under G $\Rightarrow$ treat $T_{cd}^{\ ab}$ as a spurion field with a proper transformation law under G.
- Step 2: Construct the corresponding hadronic operators by $T_{cd}^{\ ab}$ together with the Nambu-Goldstone matrix ξ, ..., and require the resulting operators to be invariant under G;
- Step 3: For each independent operator, accompany an unknown low energy constant(LEC).

Hadronic realization of n-quark ($n \geq 3$) LEFT operators — focus on dim-9 $(\bar{q}qqq)(\ell\ell)$ operators II

The LO matching can be finished through the simple replacement:

$$q_{L,a} \to \xi_a{}^{\alpha},\ \overline{q_L}^a \to \xi^{\dagger}_{\alpha}{}^a,\ q_{R,a} \to \xi^{\dagger}_a{}^{\alpha},\ \overline{q_R}^a \to \xi_{\alpha}{}^a,$$

NLO or NNLO matching:

$$q_{L,a} \to ((D_\mu \xi^\dagger)^\dagger)_a{}^\alpha,\ \overline{q_L}^a \to (D_\mu \xi^\dagger)_\alpha^{\ a},\ q_{R,a} \to (D_\mu \xi)_a^{\dagger\alpha},\ \overline{q_R}^a \to (D_\mu \xi)_\alpha^{\ a},$$
$$q_{L,a} \to (M^\dagger \xi^\dagger)_a{}^\alpha,\ \overline{q_L}^a \to (\xi M)_\alpha^{\ a},\ q_{R,a} \to (M\xi)_a{}^\alpha,\ \overline{q_R}^a \to (\xi^\dagger M^\dagger)_\alpha^{\ a},$$

here $D_\mu = \partial_\mu + (\xi^\dagger \partial_\mu \xi + \xi \partial_\mu \xi^\dagger)/2$, with the Greek letters contracted with each other.

- The matching is **irrelevant** with the color contraction of quark level operators, which means the operators with **different** color contractions will match onto the **same** hadronic operators but with **different** LECs;
- The LECs can be determined by chiral symmetry, LQCD, or experimental data, or NDA.

Baryons and more complicated cases

- 2-flavor baryon case: $\Psi = (p, n)^{\rm T} \xrightarrow{G} h\Psi$ and $u \to Ruh^\dagger = huL^\dagger$ where

$$u = \exp\left(\frac{i\Pi}{2F_0}\right), \Pi = \pi^a\tau^a = \begin{pmatrix} \pi^0 & \sqrt{2}\pi^+ \\ \sqrt{2}\pi^- & -\pi^0 \end{pmatrix}.$$

- LO Lagrangian

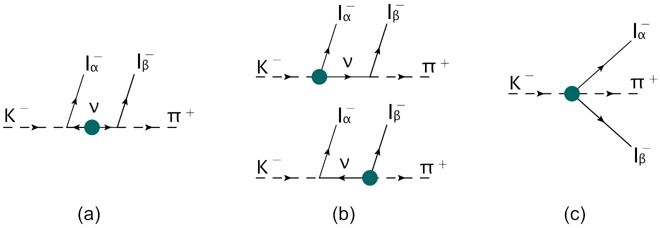
$$\mathcal{L}^{(1)}_{\pi N} = \bar\Psi\left(i\not{D} - m_N + \frac{g_A}{2}\gamma^\mu\gamma_5 u_\mu\right)\Psi\,, D_\mu\Psi = (\partial_\mu + \Gamma_\mu)\Psi\,, \Gamma_\mu = \frac{1}{2}\left(u^\dagger(\partial_\mu - ir_\mu)u + u(\partial_\mu - il_\mu)u^\dagger\right)\,,$$

- Nucleon $\Delta B = 1$ decays — 3-quark operators $qqq\ell$ and $qqq\nu$: Chiral Lagrangian for Deep Mine Physics [Claudson-Wise-Hall] and Nucleon Decay Matrix Elements from Lattice QCD [JLQCD Collaboration]
- $\Delta L = 2$ neutrinoless double beta decay — 4-quark operators $\bar{q}qqq\ell\ell$: [Cirigliano-Dekens-Mereghetti-de Vries-$\cdots$ series of papers]
- $\Delta B = 2$ Neutron-antineutron oscillation and dinucleon decays — 6-quark operators $qqqqqq$ and $qqqqqq\{\ell\ell, \ell\nu, \nu\nu\}$: [Bijnens-Kofoed, 2017; He-Ma, 2020]
- Heavy baryon χPT [Manohar-Georgi, 1984], $\cdots$

Feynman diagrams for $K^- \to \pi^+ \ell_\alpha^- \ell_\beta^-$ in EFTs

EFT: NP $\Leftarrow$ SMEFT $\Leftarrow$**LEFT** $\Leftarrow \chi$PT $\Leftarrow K^- \to \pi^+ \ell_\alpha^- \ell_\beta^-$ exp. data

- The mass mechanism (dim 3)
- The long-distance (LD) interaction (dim 6, dim 7)
- The short-distance (SD) interaction (dim 9)



Matching between LEFT and χPT at Λ_χ for LEFT dim-6 and dim-7 interactions

The relevant LD interactions in LEFT:

- At dim 6 and dim 7

$$\mathcal{O}^{LL,V}_{ui\alpha\beta} = (\overline{u_L}\gamma_\mu d^i_L)(\overline{\ell_\alpha}\gamma^\mu P_R \nu^{\tt C}_\beta) , \qquad \mathcal{O}^{RR,V}_{ui\alpha\beta} = (\overline{u_R}\gamma_\mu d^i_R)(\overline{\ell_\alpha}\gamma^\mu P_R \nu^{\tt C}_\beta) ,$$

$$\mathcal{O}^{LL,VD}_{ui\alpha\beta} = (\overline{u_L}\gamma_\mu d^i_L)(\overline{\ell_\alpha} i \overleftrightarrow{D}^\mu P_R \nu^{\tt C}_\beta) , \qquad \mathcal{O}^{RR,VD}_{ui\alpha\beta} = (\overline{u_R}\gamma_\mu d^i_R)(\overline{\ell_\alpha} i \overleftrightarrow{D}^\mu P_R \nu^{\tt C}_\beta) ,$$

$$\mathcal{O}^{RL,S}_{ui\alpha\beta} = (\overline{u_R} d^i_L)(\overline{\ell_\alpha} P_R \nu^{\tt C}_\beta) , \qquad \mathcal{O}^{LR,S}_{ui\alpha\beta} = (\overline{u_L} d^i_R)(\overline{\ell_\alpha} P_R \nu^{\tt C}_\beta) ,$$

$$\mathcal{O}^{LR,T}_{ui\alpha\beta} = (\overline{u_L}\sigma_{\mu\nu} d^i_R)(\overline{\ell_\alpha}\sigma^{\mu\nu} P_R \nu^{\tt C}_\beta) , \qquad \mathcal{O}^{LR,TD}_{ui\alpha\beta} = (\overline{u_L}\sigma_{\mu\nu} d^i_R)(\overline{\ell_\alpha}\gamma^{[\mu} \overleftrightarrow{D}^{\nu]} P_R \nu^{\tt C}_\beta) ,$$

$$\mathcal{O}^{RL,TD}_{ui\alpha\beta} = (\overline{u_R}\sigma_{\mu\nu} d^i_L)(\overline{\ell_\alpha}\gamma^{[\mu} \overleftrightarrow{D}^{\nu]} P_R \nu^{\tt C}_\beta) ,$$

Recognize the corresponding external sources:

$$(l^\mu)_{ui} = -2\sqrt{2} G_F V_{ui} (\overline{\ell_\alpha}\gamma^\mu P_L \nu_\alpha) + C^{LL,V}_{ui\alpha\beta}(\overline{\ell_\alpha}\gamma^\mu P_R \nu^{\tt C}_\beta) + C^{LL,VD}_{ui\alpha\beta}(\overline{\ell_\alpha} i \overleftrightarrow{D}^\mu P_R \nu^{\tt C}_\beta),$$

$$(l^\mu)_{ui} = C^{RR,V}_{ui\alpha\beta}(\overline{\ell_\alpha}\gamma^\mu P_R \nu^{\tt C}_\beta) + C^{RR,VD}_{ui\alpha\beta}(\overline{\ell_\alpha} i \overleftrightarrow{D}^\mu P_R \nu^{\tt C}_\beta),$$

$$(\chi^\dagger)_{ui} = 2BC^{RL,S}_{ui\alpha\beta}(\overline{\ell_\alpha} P_R \nu^{\tt C}_\beta) , \quad (\chi)_{ui} = 2BC^{LR,S}_{ui\alpha\beta}(\overline{\ell_\alpha} P_R \nu^{\tt C}_\beta),$$

$$(t^{\mu\nu}_l)_{ui} = C^{LR,T}_{ui\alpha\beta}(\overline{\ell_\alpha}\sigma^{\mu\nu} P_R \nu^{\tt C}_\beta) + C^{LR,TD}_{ui\alpha\beta}(\overline{\ell_\alpha}\gamma^{[\mu} \overleftrightarrow{D}^{\nu]} P_R \nu^{\tt C}_\beta) , \quad (t^{\mu\nu}_r)_{ui} = C^{RL,TD}_{ui\alpha\beta}(\overline{\ell_\alpha}\gamma^{[\mu} \overleftrightarrow{D}^{\nu]} P_R \nu^{\tt C}_\beta),$$

To linear term of external sources and LO in χPT power counting $\Rightarrow \mathcal{O}(p^2)$ chiral Lagrangian:

$$\mathcal{L}_{p^2} = \frac{F_0^2}{4}\mathrm{Tr}(\xi_\mu \xi^\mu) + \frac{F_0^2}{4}\mathrm{Tr}(\chi_+),$$

Matching between LEFT and χPT at Λ_χ for dim-9 LEFT interactions

Notation	Quark operator	chiral irrep	Hadronic operator
$\mathcal{O}^{LLLL,S/P}_{udus}$	$(\overline{u_L}\gamma^\mu d_L)[\overline{u_L}\gamma_\mu s_L](j/j_5)$	$27_L \times 1_R$	$\frac{5}{12} g_{27\times 1} F_0^4 (\Sigma i\partial_\mu \Sigma^\dagger)_2^1 (\Sigma i\partial^\mu \Sigma^\dagger)_3^1$
$\mathcal{O}^{LRLR,S/P}_{udus}$	$(\overline{u_L} d_R)[\overline{u_L} s_R](j/j_5)$	$\bar{6}_L \times 6_R$	$-g^a_{6\times 6} \frac{F_0^4}{4} (\Sigma^\dagger)_2^1 (\Sigma^\dagger)_3^1$
$\tilde{\mathcal{O}}^{LRLR,S/P}_{udus}$	$(\overline{u_L} d_R][\overline{u_L} s_R)(j/j_5)$	$\bar{6}_L \times 6_R$	$-g^b_{6\times 6} \frac{F_0^4}{4} (\Sigma^\dagger)_2^1 (\Sigma^\dagger)_3^1$
$\mathcal{O}^{LRLL,A}_{udus}$	$(\overline{u_L} d_R)[\overline{u_L}\gamma^\mu s_L] j_{\mu 5}$	$\overline{15}_L \times 3_R$	$-g^a_{\overline{15}\times 3} \frac{F_0^4}{4} (\Sigma i\partial_\mu \Sigma^\dagger)_3^1 (\Sigma^\dagger)_2^1$
$\tilde{\mathcal{O}}^{LRLL,A}_{udus}$	$(\overline{u_L} d_R][\overline{u_L}\gamma^\mu s_L) j_{\mu 5}$	$\overline{15}_L \times 3_R$	$-g^b_{\overline{15}\times 3} \frac{F_0^4}{4} (\Sigma i\partial_\mu \Sigma^\dagger)_3^1 (\Sigma^\dagger)_2^1$
$\mathcal{O}^{LRLL,A}_{usud}$	$(\overline{u_L} s_R)[\overline{u_L}\gamma^\mu d_L] j_{\mu 5}$	$\overline{15}_L \times 3_R$	$-g^c_{\overline{15}\times 3} \frac{F_0^4}{4} (\Sigma i\partial_\mu \Sigma^\dagger)_2^1 (\Sigma^\dagger)_3^1$
$\tilde{\mathcal{O}}^{LRLL,A}_{usud}$	$(\overline{u_L} s_R][\overline{u_L}\gamma^\mu d_L) j_{\mu 5}$	$\overline{15}_L \times 3_R$	$-g^d_{\overline{15}\times 3} \frac{F_0^4}{4} (\Sigma i\partial_\mu \Sigma^\dagger)_2^1 (\Sigma^\dagger)_3^1$
$\mathcal{O}^{LRRL,S/P}_{udus}$	$(\overline{u_L} d_R)[\overline{u_R} s_L](j/j_5)$	$8_L \times 8_R$	$g^a_{8\times 8} \frac{F_0^4}{4} (\Sigma^\dagger)_2^1 (\Sigma)_3^1$
$\tilde{\mathcal{O}}^{LRRL,S/P}_{udus}$	$(\overline{u_L} d_R][\overline{u_R} s_L)(j/j_5)$	$8_L \times 8_R$	$g^b_{8\times 8} \frac{F_0^4}{4} (\Sigma^\dagger)_2^1 (\Sigma)_3^1$
...	...	...	...

- The lepton current $j = (\overline{l_\alpha} l^C_\beta)$, $j_5 = (\overline{l_\alpha}\gamma_5 l^C_\beta)$, $j_{5,\ \mu} = (\overline{l_\alpha}\gamma_\mu\gamma_5 l^C_\beta)$,
- g_X are LECs,
- $g_{27\times 1}$, $g^a_{8\times 8}$, $g^b_{8\times 8}$ can be extracted from the matrix elements of $\pi^- \to \pi^+$, $K^+ \to \pi^+\pi^0$ and $K^0 \to \bar{K}^0$:

$$g_{27\times 1} = 0.38 \pm 0.08,\ g^a_{8\times 8} = 5.5 \pm 2\ \text{GeV}^2,\ g^b_{8\times 8} = 1.55 \pm 0.65\ \text{GeV}^2,$$ [Cirigliano, *et al*: 1708.09390]

Matching between LEFT and SMEFT at electroweak scale

Only consider the LO dim-7 SMEFT interactions:

- At dim 3: $m_{\alpha\beta} = -v^2 C_{LH5}^{\alpha\beta *} - \frac{1}{2} v^4 C_{LH}^{\alpha\beta *}$,
- At dim 6:

$$C_{pr\alpha\beta}^{LL,V} = \frac{\sqrt{2}}{2} v V_{pr} C_{LeHD}^{\beta\alpha *}, \qquad C_{pr\alpha\beta}^{RR,V} = \frac{\sqrt{2}}{2} v C_{\bar{d}uLeH}^{rp\beta\alpha *},$$

$$C_{pr\alpha\beta}^{RL,S} = \frac{\sqrt{2}}{2} v V_{wr} C_{\bar{Q}uLLH}^{wp\alpha\beta *}, \qquad C_{pr\alpha\beta}^{LR,S} = \frac{\sqrt{2}}{2} v C_{\bar{d}QLLH1}^{rp\alpha\beta *},$$

- At dim 7: $C_{pr\alpha\beta}^{LL,VD} = -V_{pr}\left(4C_{LHW}^{\beta\alpha *} + C_{LDH1}^{\alpha\beta *}\right), \quad C_{pr\alpha\beta}^{RR,VD} = 2C_{\bar{d}uLDL}^{rp\alpha\beta *}$,
- At dim 9:

$$C_{udus}^{LLLL,S/P} = -2\sqrt{2} G_F V_{ud} V_{us}\left(2C_{LDH1}^{\alpha\beta *} + C_{LDH2}^{\alpha\beta *} + 2C_{LHW}^{\alpha\beta *} + 2C_{LHW}^{\beta\alpha *}\right),$$

$$\tilde{C}_{udus}^{LRRL\ S/P} = -4\sqrt{2} G_F V_{us}\left(2C_{\bar{d}uLDL}^{11\alpha\beta *}\right),$$

$$\tilde{C}_{usud}^{LRRL\ S/P} = -4\sqrt{2} G_F V_{ud}\left(2C_{\bar{d}uLDL}^{21\alpha\beta *}\right),$$

Effective Chiral Lagrangian for $K^- \to \pi^+ \ell_\alpha^- \ell_\beta^-$

$$\mathcal{L}_{\rm LD} \supset F_0 \Big[G_F \left(V_{ud} \partial_\mu \pi^- + V_{us} \partial_\mu K^- \right) \left(\overline{\ell_{L\alpha}} \gamma^\mu \nu_\alpha \right) + iB \left(c_{\pi 1}^{\alpha\beta} \pi^- + c_{K1}^{\alpha\beta} K^- \right) \left(\overline{\ell_{L\alpha}} \nu_\beta^{\rm C} \right)$$
$$- \left(c_{\pi 2}^{\alpha\beta} \partial_\mu \pi^- + c_{K2}^{\alpha\beta} \partial_\mu K^- \right) \left(\overline{\ell_{R\alpha}} \gamma^\mu \nu_\beta^{\rm C} \right) - \left(c_{\pi 3}^{\alpha\beta} \partial_\mu \pi^- + c_{K3}^{\alpha\beta} \partial_\mu K^- \right) \left(\overline{\ell_{L\alpha}} i \overleftrightarrow{D}^\mu \nu_\beta^{\rm C} \right) \Big],$$
$$\mathcal{L}_{\rm SD} = F_0^2 G_F \left[c_1^{\alpha\beta} K^- \pi^- \overline{\ell_{L\alpha}} \ell_{L\beta}^{\rm C} + c_5^{\alpha\beta} \partial^\mu K^- \partial_\mu \pi^- \overline{\ell_{L\alpha}} \ell_{L\beta}^{\rm C} \right],$$

At chiral symmetry breaking scale Λ_χ:

$$c_{P_i 1}^{\alpha\beta} = \frac{v}{2}(1.656)\mathcal{Y}_{P_i 1}^{\alpha\beta}, \quad c_{P_i 2}^{\alpha\beta} = \frac{v}{4}\mathcal{Y}_{P_i 2}^{\alpha\beta}, \quad c_{P_i 3}^{\alpha\beta} = \frac{\sqrt{2}}{4}\mathcal{Y}_{P_i 3}^{\alpha\beta},$$
$$c_1^{\alpha\beta} = -2\sqrt{2}\left(0.62 g_{8\times 8}^a + 0.88 g_{8\times 8}^b\right)\mathcal{X}_1^{\alpha\beta}, \quad c_5^{\alpha\beta} = -2\sqrt{2}(1.3 g_{27\times 1}) V_{ud} V_{us} \mathcal{X}_2^{\alpha\beta}.$$

and

$$\mathcal{Y}_{P_i 1}^{\alpha\beta} = \frac{\sqrt{2}}{2}\left(C_{ui\alpha\beta}^{RL,S} - C_{ui\alpha\beta}^{LR,S}\right), \quad \mathcal{Y}_{P_i 2}^{\alpha\beta} = \frac{\sqrt{2}}{4}\left(C_{ui\alpha\beta}^{LL,V} - C_{ui\alpha\beta}^{RR,V}\right),$$
$$\mathcal{Y}_{P_i 3}^{\alpha\beta} = \frac{\sqrt{2}}{4}\left(C_{ui\alpha\beta}^{LL,VD} - C_{ui\alpha\beta}^{RR,VD}\right),$$
$$\mathcal{X}_1^{\alpha\beta} = 2\left(V_{us} C_{\bar{d}uLDL}^{11\alpha\beta *} + V_{ud} C_{\bar{d}uLDL}^{21\alpha\beta *}\right), \quad \mathcal{X}_2^{\alpha\beta} = 2C_{LHW}^{\alpha\beta *} + 2C_{LHW}^{\beta\alpha *} + 2C_{LDH1}^{\alpha\beta *} + C_{LDH2}^{\alpha\beta *},$$

Master formula for the branching ratio in SMEFT

$$\frac{\mathcal{B}(e^-e^-)}{\mathrm{GeV}^6} = \frac{1.7\times10^{-33}}{\mathrm{GeV}^6}\frac{|m_{ee}|^2}{\mathrm{eV}^2} + 80\,|\mathcal{Y}^{ee}_{K1}|^2 + 4.3\,|\mathcal{Y}^{ee}_{\pi1}|^2$$
$$+\,10^{-3}\times\left(48\,|\mathcal{X}^{ee}_1|^2 + 45\,|\mathcal{Y}^{ee}_{K2}|^2 + 2.4\,|\mathcal{Y}^{ee}_{\pi2}|^2\right)$$
$$+\,10^{-8}\times\left(29\,|\mathcal{Y}^{ee}_{K3}|^2 + 23\,|\mathcal{X}^{ee}_2|^2 + 1.6\,|\mathcal{Y}^{ee}_{\pi3}|^2\right) + \mathrm{int.},$$

$$\frac{\mathcal{B}(\mu^-\mu^-)}{\mathrm{GeV}^6} = \frac{4.5\times10^{-34}}{\mathrm{GeV}^6}\frac{|m_{\mu\mu}|^2}{\mathrm{eV}^2} + 16\,\left|\mathcal{Y}^{\mu\mu}_{K1}\right|^2 + 2.2\,|\mathcal{Y}^{\mu\mu}_{\pi1}|^2$$
$$+\,10^{-3}\times\left(17\,|\mathcal{X}^{\mu\mu}_1|^2 + 19\,\left|\mathcal{Y}^{\mu\mu}_{K2}\right|^2 + |\mathcal{Y}^{\mu\mu}_{\pi2}|^2\right)$$
$$+\,10^{-9}\times\left(67\,|\mathcal{X}^{\mu\mu}_2|^2 + 49\,\left|\mathcal{Y}^{\mu\mu}_{K3}\right|^2 + 6.6\,|\mathcal{Y}^{\mu\mu}_{\pi3}|^2\right) + \mathrm{int.},$$

$$\frac{\mathcal{B}(e^-\mu^-)}{\mathrm{GeV}^6} = \frac{2.1\times10^{-33}}{\mathrm{GeV}^6}\frac{|m_{e\mu}|^2}{\mathrm{eV}^2} + 26\,\left|\mathcal{Y}^{\mu e}_{K1}\right|^2 + 17\,\left|\mathcal{Y}^{e\mu}_{K1}\right|^2 + 2\,|\mathcal{Y}^{e\mu}_{\pi1}|^2 + 1.4\,|\mathcal{Y}^{\mu e}_{\pi1}|^2$$
$$+\,10^{-3}\times\left(61\,|\mathcal{X}^{e\mu}_1|^2 + 35\,\left|\mathcal{Y}^{\mu e}_{K2}\right|^2 + 24\,\left|\mathcal{Y}^{e\mu}_{K2}\right|^2 + 1.9\,|\mathcal{Y}^{e\mu}_{\pi2}|^2 + 1.3\,|\mathcal{Y}^{\mu e}_{\pi2}|^2\right)$$
$$+\,10^{-9}\times\left(280\,|\mathcal{X}^{e\mu}_2|^2 + 110\,\left|\mathcal{Y}^{e\mu}_{K3}\right|^2 + 55\,\left|\mathcal{Y}^{\mu e}_{K3}\right|^2 + 6.7\,|\mathcal{Y}^{\mu e}_{\pi3}|^2 + 5.7\,|\mathcal{Y}^{e\mu}_{\pi3}|^2\right) + \mathrm{int.},$$

For $\mathcal{O}(1)$ coupling constant, the dominant contribution from the SD!

Upper limit to Wilson coefficients from current exp. data

$K^- \to \pi^+ e^- e^-$		$K^- \to \pi^+ \mu^- \mu^-$		$K^- \to \pi^+ e^- \mu^-$			
names	bounds	names	bounds	names	bounds	names	bounds
$\lvert\mathcal{Y}^{ee}_{K1}\rvert^{-\frac{1}{3}}$	84.5	$\lvert\mathcal{Y}^{\mu\mu}_{K1}\rvert^{-\frac{1}{3}}$	85.1	$\lvert\mathcal{Y}^{\mu e}_{K1}\rvert^{-\frac{1}{3}}$	61.1	$\lvert\mathcal{Y}^{e\mu}_{K1}\rvert^{-\frac{1}{3}}$	56.9
$\lvert\mathcal{Y}^{ee}_{\pi1}\rvert^{-\frac{1}{3}}$	51.9	$\lvert\mathcal{Y}^{\mu\mu}_{\pi1}\rvert^{-\frac{1}{3}}$	61.2	$\lvert\mathcal{Y}^{e\mu}_{\pi1}\rvert^{-\frac{1}{3}}$	39.8	$\lvert\mathcal{Y}^{\mu e}_{\pi1}\rvert^{-\frac{1}{3}}$	37.5
$\lvert\mathcal{X}^{ee}_{1}\rvert^{-\frac{1}{3}}$	24.5	$\lvert\mathcal{X}^{\mu\mu}_{1}\rvert^{-\frac{1}{3}}$	32.3	$\lvert\mathcal{X}^{e\mu}_{1}\rvert^{-\frac{1}{3}}$	22.3		
$\lvert\mathcal{Y}^{ee}_{K2}\rvert^{-\frac{1}{3}}$	24.3	$\lvert\mathcal{Y}^{\mu\mu}_{K2}\rvert^{-\frac{1}{3}}$	27.7	$\lvert\mathcal{Y}^{\mu e}_{K2}\rvert^{-\frac{1}{3}}$	20.3	$\lvert\mathcal{Y}^{e\mu}_{K2}\rvert^{-\frac{1}{3}}$	19.1
$\lvert\mathcal{Y}^{ee}_{\pi2}\rvert^{-\frac{1}{3}}$	14.9	$\lvert\mathcal{Y}^{\mu\mu}_{\pi2}\rvert^{-\frac{1}{3}}$	17	$\lvert\mathcal{Y}^{e\mu}_{\pi2}\rvert^{-\frac{1}{3}}$	12.5	$\lvert\mathcal{Y}^{\mu e}_{\pi2}\rvert^{-\frac{1}{3}}$	11.7
$\lvert\mathcal{X}^{ee}_{2}\rvert^{-\frac{1}{3}}$	3.2	$\lvert\mathcal{X}^{\mu\mu}_{2}\rvert^{-\frac{1}{3}}$	3.4	$\lvert\mathcal{X}^{e\mu}_{2}\rvert^{-\frac{1}{3}}$	2.9		
$\lvert\mathcal{Y}^{ee}_{K3}\rvert^{-\frac{1}{3}}$	3.3	$\lvert\mathcal{Y}^{\mu\mu}_{K3}\rvert^{-\frac{1}{3}}$	3.2	$\lvert\mathcal{Y}^{e\mu}_{K3}\rvert^{-\frac{1}{3}}$	2.6	$\lvert\mathcal{Y}^{\mu e}_{K3}\rvert^{-\frac{1}{3}}$	2.2
$\lvert\mathcal{Y}^{ee}_{\pi3}\rvert^{-\frac{1}{3}}$	2	$\lvert\mathcal{Y}^{\mu\mu}_{\pi3}\rvert^{-\frac{1}{3}}$	2.3	$\lvert\mathcal{Y}^{\mu e}_{\pi3}\rvert^{-\frac{1}{3}}$	1.5	$\lvert\mathcal{Y}^{e\mu}_{\pi3}\rvert^{-\frac{1}{3}}$	1.5

- $0\nu\beta\beta$: $\Lambda > \mathcal{O}(10 \text{ TeV})$; [Cirigliano *et al*: :1708.09390; Liao and Ma: 1607.07309];
- Second generation of quarks and leptons, constrain NP and parameters in a complementary way.

New bound for the branching ratio with 1TeV NP scale

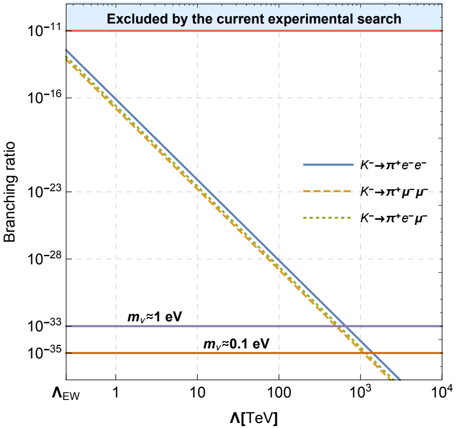
If we take the NP scale
$\Lambda_{\rm NP} \geq 1$TeV and $\mathcal{O}(1)$ coupling constant:

$$\mathcal{B}(e^- e^-) < 8.0 \times 10^{-17},$$
$$\mathcal{B}(\mu^- \mu^-) < 1.6 \times 10^{-17},$$
$$\mathcal{B}(e^- \mu^-) < 2.6 \times 10^{-17},$$

Comparing with the current exps.

$$\mathcal{B}_{\rm exp}(e^- e^-) < 2.2 \times 10^{-10},$$
$$\mathcal{B}_{\rm exp}(\mu^- \mu^-) < 4.2 \times 10^{-11},$$
$$\mathcal{B}_{\rm exp}(e^- \mu^-) < 5 \times 10^{-10},$$

$\Rightarrow$ **Neutrino mass contribution severely suppressed.**
$\Rightarrow$ **NP contribution is far away from the current exp. sensitivity!**



Summary

- Due to the high intensity of low energy experiments, the relevant low energy processes (like the forbidden $\Delta B \neq 0$ and $\Delta L \neq 0$ processes) are pretty sensitive to NP;
- The SMEFT, LEFT and χPT are important tools for model-independent study of low energy observables in search of NP;
- In this talk, the three EFTs have been reviewed and briefly discussed;
- Last, we take the $\Delta L = 2$ process $K^- \to \pi^+ \ell_\alpha^- \ell_\beta^-$ as an example to show how to study high NP using low energy observables.

Thanks for your attention!