Study of Ds(2317) in the quark model

谭悦　　南京师范大学

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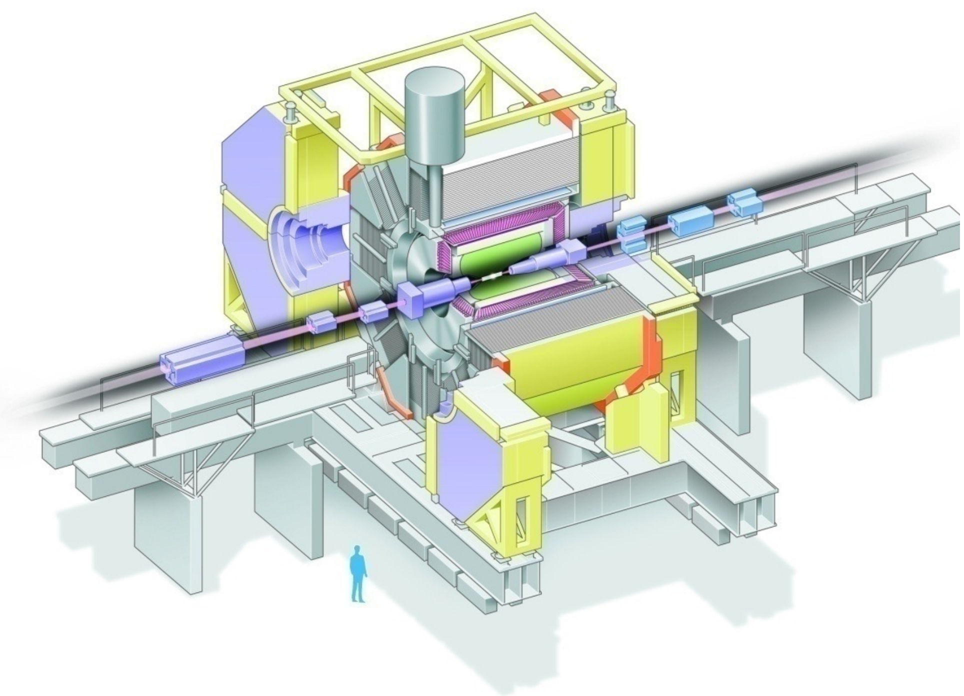
Content

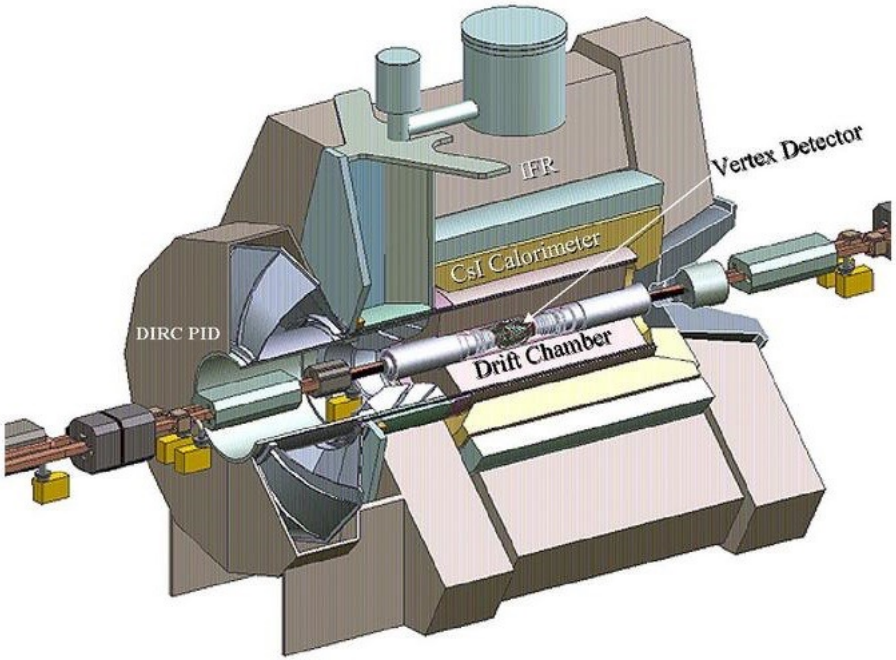
Motivation

Most of the XYZ from B-factories

BELLE

BABAR
Collaboration Home Page





+ BES, CDF, D0，CLEO,CMS

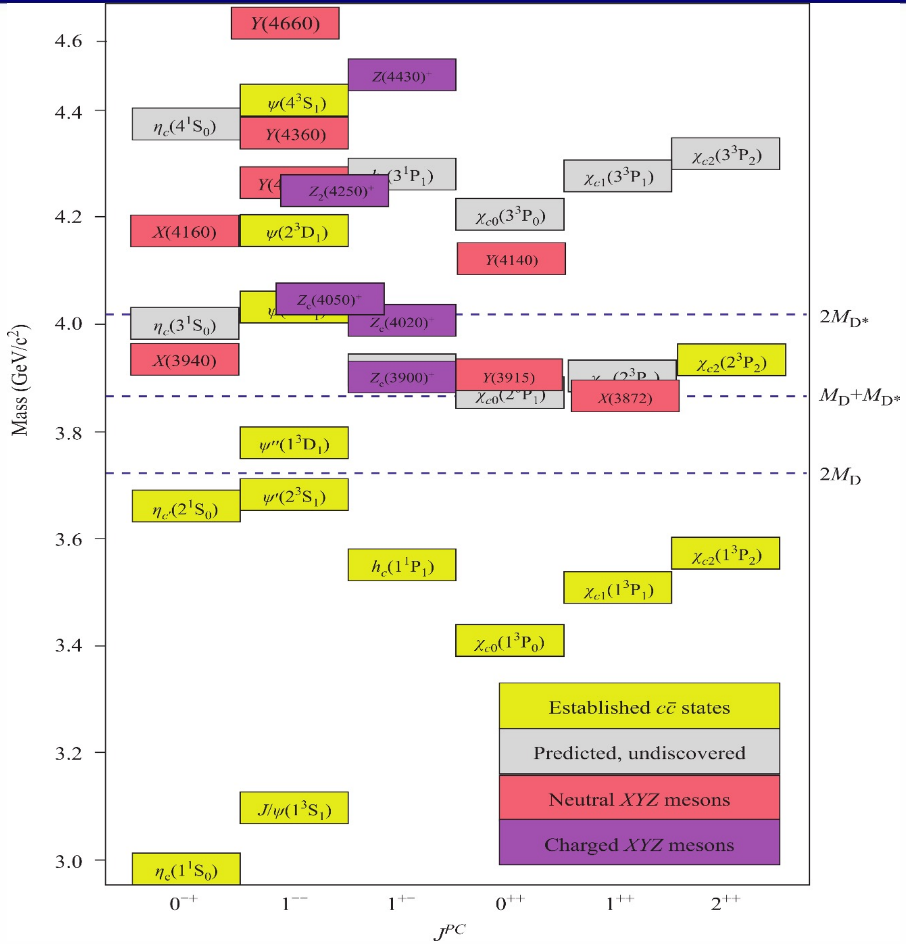
Motivation

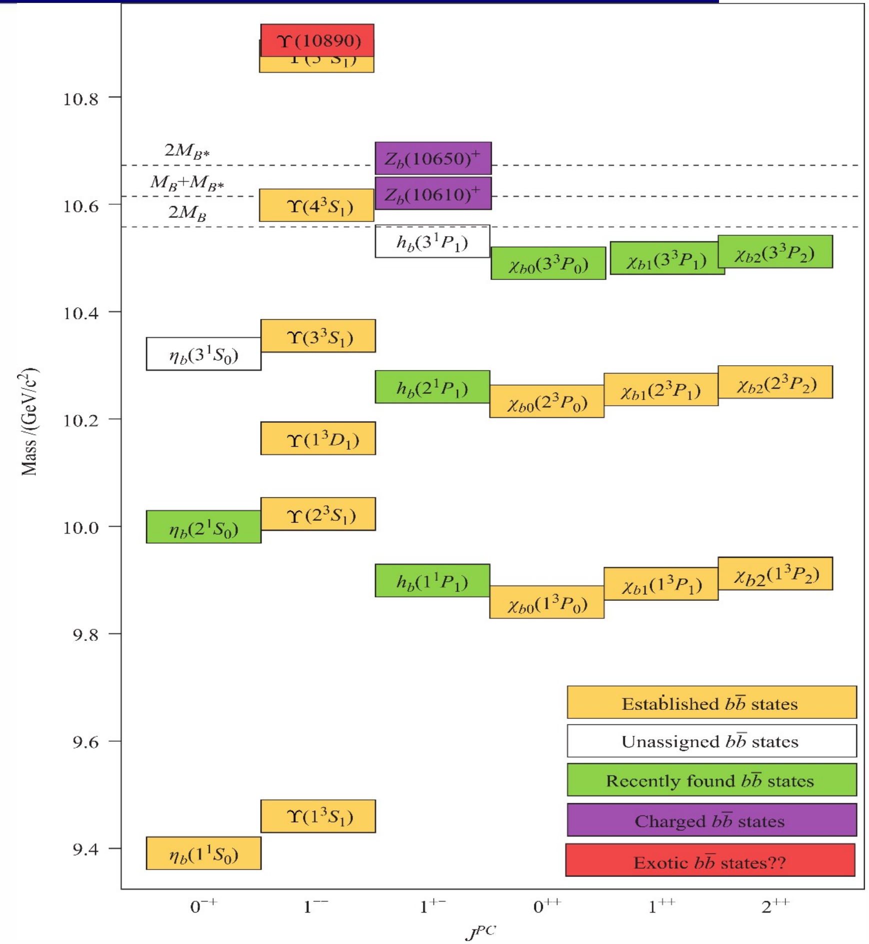
1. Since 2003, a lot of new hadron states reported by Belle, BARBAR, BESIII, D0, CDF, LHCb, ……

2. They are difficult to be accommodated by quark models

   "exotic states"

Motivation





S. L. Olsen, A new hadron spectroscopy, Front. Phys. 10 (2015) 101401

BABAR

Collaboration Home Page

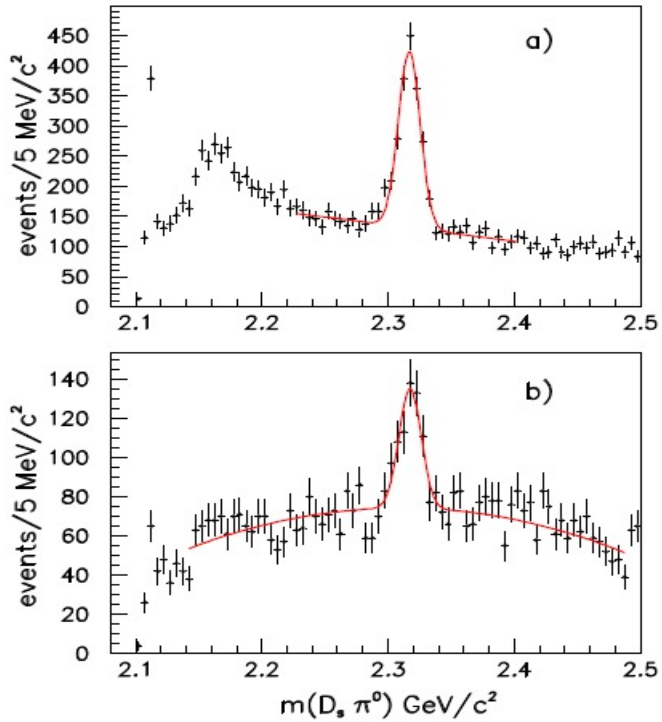
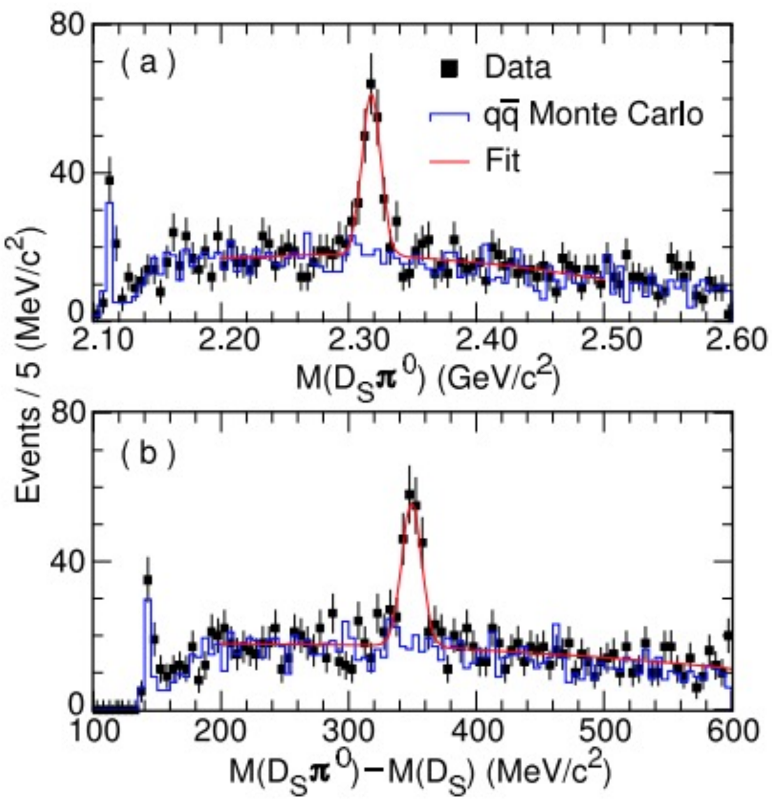


FIG. 2: The $D_s^+\pi^0$ mass distribution for (a) the decay $D_s^+ \to K^+K^-\pi^+$ and (b) the decay $D_s^+ \to K^+K^-\pi^+\pi^0$. The fits to the mass distributions as described in the text are indicated by the curves.

B.Aubert, Observation of a narrow meson decaying to Ds \pi at a mass of 2.32-GeV

--------Phys.Rev.Lett. 90, 242001.

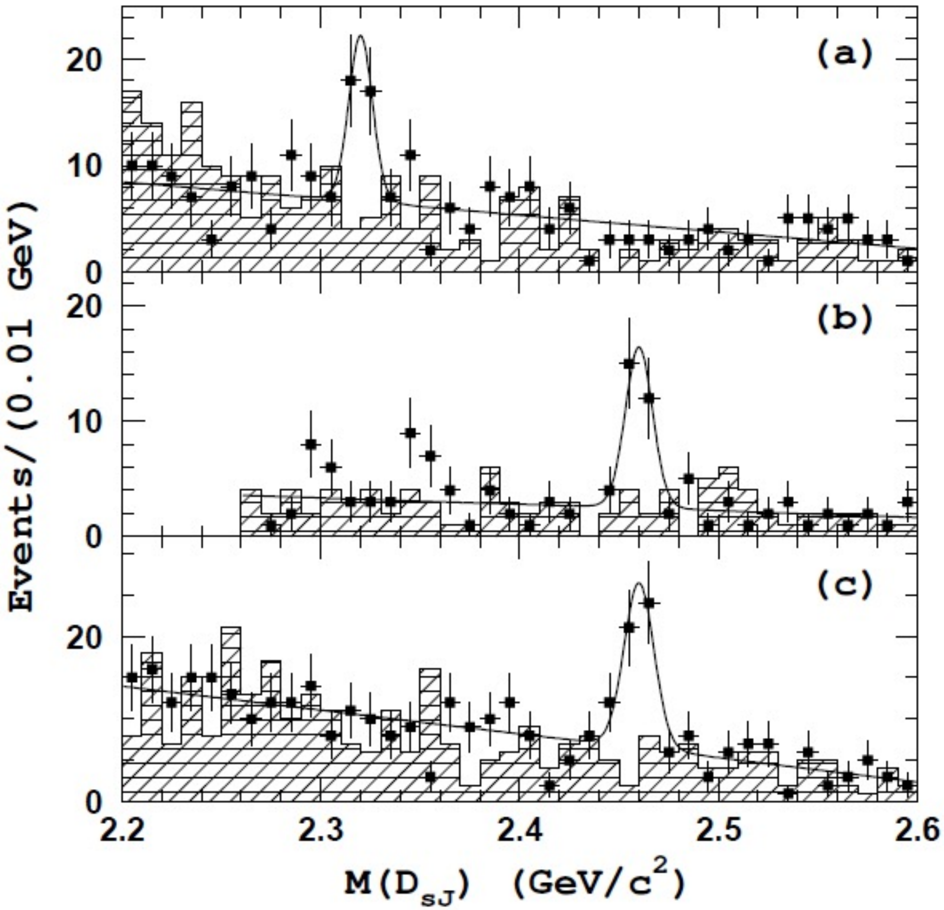


$$\frac{\sigma \cdot \mathcal{B}\left(D_{sJ}^{*}(2317) \rightarrow D_s^{+}\pi^{o}\right)}{\sigma\left(D_s^{+}\right)} = (7.9 \pm 1.2 \pm 0.4) \times 10^{-2},$$

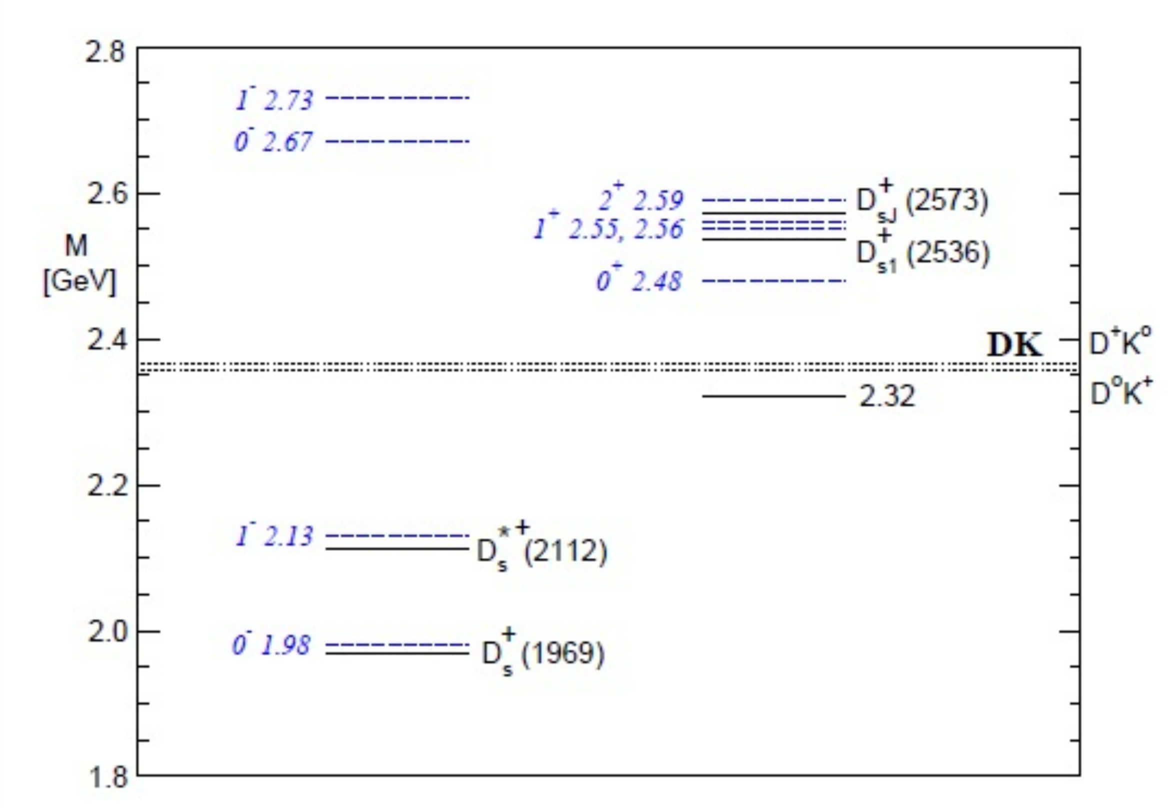
$$\frac{\sigma \cdot \mathcal{B}\left(D_{sJ}(2463) \rightarrow D_s^{*+}\pi^{o}\right)}{\sigma\left(D_s^{+}\right)} = (3.5 \pm 0.9 \pm 0.2) \times 10^{-2},$$

D.Besson,Observation of a narrow resonance of mass 2.46 GeV decaying to D*+(s) pi0 and confirmation of the D*(sJ)(2317) state,

------Phys.Rev.D 68, 032002 (2003)



P.Krokovny, Observation of the D(sJ)(2317) and D(sJ)(2457) in B decays,
------Phys. Rev. Lett.91, 262002 (2003)


2.8
2.6
2.4
2.2
2.0
1.8
M
[GeV]
1⁻ 2.73
0⁻ 2.67
2⁺ 2.59
1⁺ 2.55, 2.56
0⁺ 2.48
D_{sJ}^{+} (2573)
D_{s1}^{+} (2536)
DK
D^+K^0
D^0K^+
2.32
1⁻ 2.13
D_s^{*+}(2112)
0⁻ 1.98
D_s^{+}(1969)

Motivation

Ds(2317):Conventional cs state

Dsj(2317) and Dsj(2463) my be cs Ds0 and Ds1 states by radiative transitions.

$$\Gamma(i \to f+\gamma) \qquad (1)$$
$$= \frac{4}{27}\alpha \langle e_Q\rangle^2 \omega^3 (2J_f+1) |\langle ^{2s+1}S_{J'}|r|^{2s+1}P_J\rangle|^2 S_{if}$$

$$\langle e_Q\rangle = \frac{m_s e_c - m_c e_s}{m_c + m_s}$$

Sif = 1 for the transitions between spin-triplet states

S.Godfrey,Testing the nature of the D(sJ)*(2317)+ and D(sJ)(2463)+ states using radiative transitions, ------ Phys.Lett.B 568, 254 (2003)

Motivation

Ds(2317):Conventional cs state

Phys. Rev. D 68 (2003) 054024
Phys. Lett. B 570 (2003) 180
Phys. Rev. D 68 (2003) 114011
Phys. Rev. D 69 (2004) 114008
Phys. Rev. D 72 (2005) 074004

Motivation

Ds(2317):Molecular DK state

By looking for the pole of scattering matrix on an appropriate Riemann sheet,Ds(2317) and Bs can be a bound state at the same time.

HEAVY CHIRAL UNITARY APPROACH

$$\mathcal{L} = \frac{1}{4f_\pi^2}(\partial^\mu P[\Phi, \partial_\mu \Phi]P^\dagger - P[\Phi, \partial_\mu \Phi]\partial^\mu P^\dagger)$$

octet Goldstone bosons $\Phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}$

F.K.Guo,Dynamically generated 0+ heavy mesons in a heavy chiral unitary approach,
------Phys. Lett. B 641, 278 (2006)

Motivation

Ds(2317):Molecular DK state

a conventional quark model cs assignment is implausible, and they favor a I=0 csnn baryonium assignment and a DK meson molecule assignment.

traditional quark model state appears implausible for two reasons:

1. the mass predicted by Godfrey and Isgur for this cs state is 2.48 GeV, 160 MeV higher than the BaBar state
2. as the scalar 3P0 cs belongs to the j = 1/2 heavy quark symmetry doublet, both the 3P0 cs and its Ds1 partner are expected to be much broader than the states in the j = 3/2 doublet.

T.Barnes, Implications of a DK molecule at 2.32-GeV, Phys.Rev.D 68, 054006 (2003)

Motivation

Ds(2317):Molecular DK state

In lattice QCD, the scattering of light pseudoscalar mesons off charmed mesons favors that Ds(2317) may be a DK molecule.

$$p\cot\delta(p) = \frac{1}{\pi L}\mathbf{S}\left(\left(\frac{pL}{2\pi}\right)^2\right)$$

L.Liu,Interactions of charmed mesons with light pseudoscalar mesons from lattice QCD and implications on the nature of the $D_{s0}^*(2317),----Phys.Rev.D87(2013),014508

Motivation

Ds(2317):Molecular DK state

The DK interaction is strong enough to form a bound state, the Ds0(2317). In addition. both DDk and DDDk are bound states.

$$V_{DK}(\vec{r}; R_c) = C_S \frac{e^{-(r/R_S)^2}}{\pi^{3/2} R_S^3} + C(R_C) \frac{e^{-(r/R_c)^2}}{\pi^{3/2} R_c^3} = C'_S e^{-(r/R_S)^2} + C'_L e^{-(r/R_c)^2}$$

$$V_{DD}(r; \Lambda) = V_\rho(r; \Lambda) + V_\omega(r; \Lambda) + V_\sigma(r; \Lambda)$$

$$V_\sigma(r; \Lambda) = -g_\sigma^2 \, m_\sigma \, W_C(m_\sigma r, \frac{\Lambda}{m_\sigma}),$$

$$V_\rho(r; \Lambda) = +\vec{\tau}_1 \cdot \vec{\tau}_2 \, g_\rho^2 \, m_\rho \, W_C(m_\rho r, \frac{\Lambda}{m_\rho}),$$

$$V_\omega(r; \Lambda) = +g_\omega^2 \, m_\omega \, W_C(m_\omega r, \frac{\Lambda}{m_\omega}),$$

$$W_C(x, \lambda) = \frac{e^{-x}}{4\pi x} - \lambda \frac{e^{-\lambda x}}{4\pi \lambda x} - \frac{(\lambda^2 - 1)}{2\lambda} \frac{e^{-\lambda x}}{4\pi}$$

T.W.Wu,DK,DDK,and DDDK molecules-understanding the nature of the Ds(2317), -------Phys.Rev.D 100,034029 (2019)

Motivation

Ds(2317): unquenched state

Ds0(2317)-----DK
Ds1(2460)-----D*K
X(3872)-------D*D

Mesons with masses below their lowest OZI-allowed strong decay thresholds have very small widths

G.Rupp,Unquenching and unitarising mesons in quark models and on the lattice,
-------Acta Phys.Polon.Supp.10, 1061 (2017)

Motivation

Our work

- Unify the description of ordinary and exotic hadrons

  Unquenched quark model

$$|M\rangle = C_1|q\bar{q}\rangle + C_2|q\bar{q}q\bar{q}\rangle + C_3|q\bar{q}q\bar{q}q\bar{q}\rangle + C_4|q\bar{q}g\rangle + \ldots\ldots$$

- Calculating method

  Gaussion expansion method (GEM)

- Key problem

  The transition operators: $3P_0$ operator

Quark model

- 1964, Gell-Mann / Zweig: quark model / Ace model
- 1964, discovery of Ω (1961, predicted)
- So far, the most successful phenomenological method describing the experimental data
- Ω, predicted, then discovered d* (dibaryon)

The chiral quark model (ChQM)

- The chiral quark model: The most used QM

describes properties of hadrons, hadron-hadron interactions well

- In ChQM:

Confinement: confining potential (phenomenology)

Asmptotic freedom: one-gluon-exchange

Chiral symmetry spontaneous breaking: Goldstone exchange

$$H=\sum_{i=1}^{n}(m_i+\frac{p_i^2}{2m_i})-T_{cm}+\sum_{i=1<j}^{n}\left(V_{ij}^{CON}+V_{ij}^{OGE}+\sum_{\chi=\pi,K,\eta,\sigma}V_{ij}^{\chi}\right)$$

The chiral quark model (ChQM)

$$V_{ij}^{CON} = (-a_c r_{ij}^2 - \Delta)\boldsymbol{\lambda}_i^c \cdot \boldsymbol{\lambda}_j^c$$

$$V_{ij}^{OGE} = \frac{\alpha_s}{4}\boldsymbol{\lambda}_i^c \cdot \boldsymbol{\lambda}_j^c \left[\frac{1}{r_{ij}} - \frac{2\pi}{3m_i m_j}\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \delta(\boldsymbol{r}_{ij})\right],$$

其中：$\delta(\boldsymbol{r}_{ij}) = \frac{e^{-r_{ij}/r_0(\mu_{ij})}}{4\pi r_{ij} r_0^2(\mu_{ij})},$

$$V_{ij}^{\pi} = \frac{g_{ch}^2}{4\pi}\frac{m_\pi^2}{12m_i m_j}\frac{\Lambda_\pi^2 m_\pi}{\Lambda_\pi^2 - m_\pi^2}\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \left[Y(m_\pi r_{ij}) - \frac{\Lambda_\pi^3}{m_\pi^3}Y(\Lambda_\pi r_{ij})\right]\sum_{a=1}^{3}\lambda_i^a \cdot \lambda_j^a,$$

$$V_{ij}^{K} = \frac{g_{ch}^2}{4\pi}\frac{m_K^2}{12m_i m_j}\frac{\Lambda_K^2 m_K}{\Lambda_K^2 - m_K^2}\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \left[Y(m_K r_{ij}) - \frac{\Lambda_K^3}{m_K^3}Y(\Lambda_K r_{ij})\right]\sum_{a=4}^{7}\lambda_i^a \cdot \lambda_j^a,$$

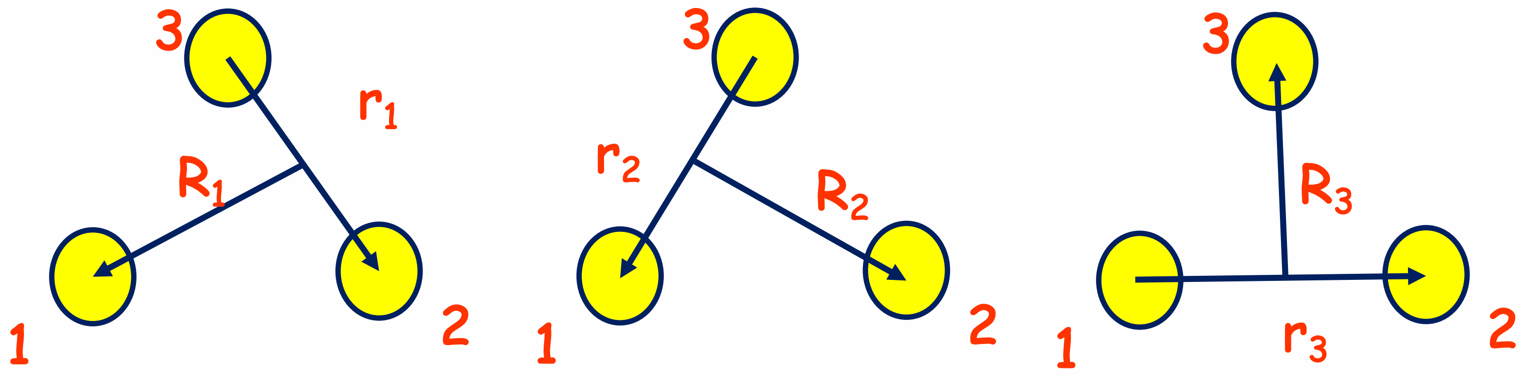
$$V_{ij}^{\eta} = \frac{g_{ch}^2}{4\pi}\frac{m_\eta^2}{12m_i m_j}\frac{\Lambda_\eta^2}{\Lambda_\eta^2 - m_\eta^2}m_\eta \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \left[Y(m_\eta r_{ij}) - \frac{\Lambda_\eta^3}{m_\eta^3}Y(\Lambda_\eta r_{ij})\right]$$
$$\times \left[cos\theta_P(\lambda_i^8 \cdot \lambda_j^8) - sin\theta_P(\lambda_i^0 \cdot \lambda_j^0)\right],$$

$$V_{ij}^{\sigma} = -\frac{g_{ch}^2}{4\pi}\frac{\Lambda_\sigma^2 m_\sigma}{\Lambda_\sigma^2 - m_\sigma^2}\left[Y(m_\sigma r_{ij}) - \frac{\Lambda_\sigma}{m_\sigma}Y(\Lambda_\sigma r_{ij})\right]$$

其中：Yukawa函数定义为$Y(x) = e^{-x}/x$。

J. Phys. G 31 (2005) 481-506

Gaussian Expansion Method



$$\psi_{JM} = \mathcal{A}\phi_{JM}^{(1)}(\boldsymbol{r}_1, \boldsymbol{R}_1) = \phi_{JM}^{(1)}(\boldsymbol{r}_1, \boldsymbol{R}_1) + \phi_{JM}^{(2)}(\boldsymbol{r}_2, \boldsymbol{R}_2) + \phi_{JM}^{(3)}(\boldsymbol{r}_3, \boldsymbol{R}_3)$$

$$\phi_{JM}^{(c)}(r_c, R_c) = \sum_{nl,NL} \boldsymbol{C_{NL,lm}}\, \phi_{nl}^{(c)}(r_c)\psi_{NL}^{(c)}(R_c)[Y_{lm}(\widehat{\boldsymbol{r}_c}) \otimes Y_{LM}(\widehat{\boldsymbol{R}_c})\,]_{JM}$$

Determined by diagonalizing H

Gaussian Expansion Method

Radial part Gaussian function:

$$\phi_{nl}(r) = \sum_{n=1}^{n_{max}} c_n N_{nl}\, r^l\, e^{-\nu_n r^2} \quad , \qquad N_{nl} = \left[\frac{2^{l+2}(2\nu_n)^{l+\frac{3}{2}}}{\sqrt{\pi}\,(2l+1)!!}\right]^{\frac{1}{2}}$$

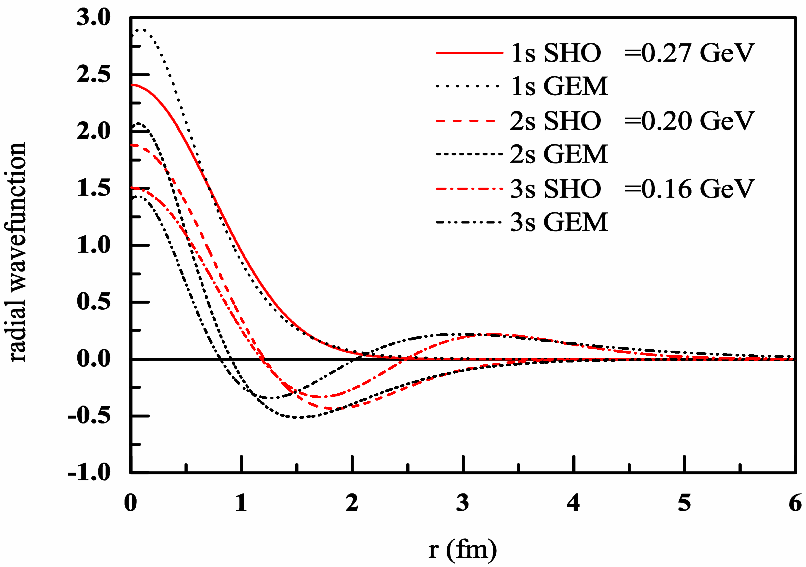
Infinitesimally-shifted Gaussian basis functions(ISG):

$$\phi^G_{nlm}(\boldsymbol{r}) = N_{nl}\, r^l\, e^{-\nu_n r^2} Y_{lm}(\hat{\boldsymbol{r}}) \quad = N_{nl} \lim_{\epsilon \to 0} \frac{1}{\epsilon^l} \sum_{k}^{k_{max}} C_{lm,k}\, e^{-\nu_n(\boldsymbol{r}-\epsilon \boldsymbol{D}_{lm,k})^2}$$

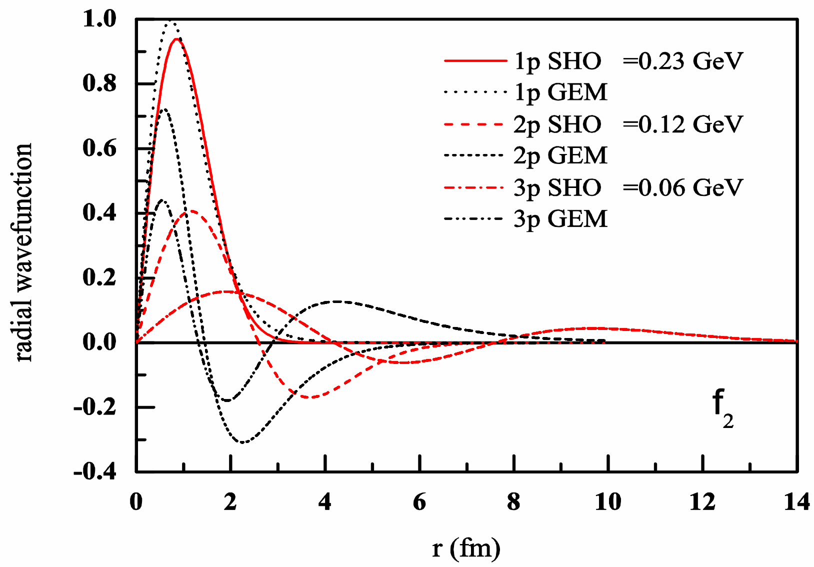
E. HIYAMA, Progress in Particle and Nuclear Physics 51 (2003) 223 -307

Wavefunction of meson

E. Hiyama, Y. Kino and M. Kamimura, *Progress in Particle and Nuclear Physics 51 (2003) 223 -307.*



The radial function of ρ meson(1S, 2S, 3S state).



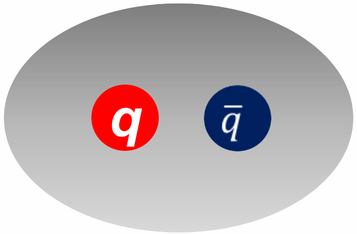
The radial function of f_2 meson(1P, 2P, 3P state).

QQM for meson:

Ds（2317） may be a traditional cs state?

QQM for meson:

$$H\Psi^{JM}(1,2) = E^{J}\Psi^{JM}(1,2)\,.$$



$$\Psi^{IJM}(1,2) = \sum_{\alpha}[\psi_l(\mathbf{r})\chi_s(1,2)]_{JM}\omega^{I}(1,2)\phi(1,2)\,,$$

$$\sum_{n',\alpha'}\left(H^{J}_{n\alpha,n'\alpha'} - E^{J}N^{J}_{n\alpha,n'\alpha'}\right)C^{J}_{n'\alpha'} = 0\,,$$

$$H^{J}_{n\alpha,n'\alpha'} = \langle\Phi^{JM}_{n\alpha}|H|\Phi^{JM}_{n'\alpha'}\rangle\,,\quad N^{J}_{n\alpha,n'\alpha'} = \langle\Phi^{JM}_{n\alpha}|1|\Phi^{JM}_{n'\alpha'}\rangle\,.$$

QQM for meson:

	D_s	D_s^*	D_{s0}	D_{s1}
QQM	1953	2080	2433	2336
Exp	1968	2112	2460	2317

QQM for meson:

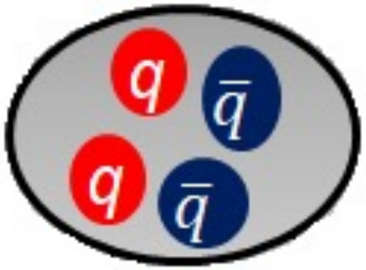
Ds（2317） may be a tetraquark state?

meson-meson system:

wavefunctions for two sub-clusters:

$$\Psi^{I_1J_1}_{M_{I_1}M_{J_1}}(12) = [\psi_{l_1}(\boldsymbol{r}_{12})\chi_{s_1}(12)]^{J_1}_{M_{J_1}}\,\omega^{c_1}(12)\phi^{I_1}_{M_{I_1}}(12),$$

$$\Psi^{I_2J_2}_{M_{I_2}M_{J_2}}(34) = [\psi_{l_2}(\boldsymbol{r}_{34})\chi_{s_2}(34)]^{J_2}_{M_{J_2}}\,\omega^{c_2}(34)\phi^{I_2}_{M_{I_2}}(34),$$



The total function of four-quark:

$$\begin{aligned}\Psi^{IJ}_{M_IM_J} &= \mathcal{A}\left[\Psi^{I_1J_1}(1,2)\Psi^{I_2J_2}(3,4)\psi_{L_r}(\boldsymbol{r}_{1234})\right]^{IJ}_{M_IM_J} \\ &= \Big[[\psi_{l_1}(\boldsymbol{r}_{12})\chi_s(12)]^{J_1}[\psi_{l_2}(\boldsymbol{r}_{34})\chi_s(34)]^{J_2} \\ &\quad \psi_{L_r}(\boldsymbol{r}_{1234})\Big]^J_{M_J}[\omega^{c_1}(12)\omega^{c_2}(34)]^{[222]} \\ &\quad \left[\phi^{I_1}_{M_{I_1}}(12)\phi^{I_2}_{M_{I_2}}(34)\right]^I_{M_I},\end{aligned}$$

Antisymmetric operator:

$$\mathcal{A} = \frac{1}{2}(1 - P_{13} - P_{24} + P_{13}P_{24}).$$

$$\begin{cases} H^J_{4q} = \langle \psi^{IJ}_{M_IM_J}|H|\psi^{IJ}_{M_IM_J}\rangle \\ N^J_{4q} = \langle \psi^{IJ}_{M_IM_J}|1|\psi^{IJ}_{M_IM_J}\rangle \end{cases}$$

orbit

$$\varphi(r) = \sum C_n \phi(r_{12}) \phi(r_{34}) \phi(r_{12,34})$$

spin

$$\chi_{11}^{\sigma} = \alpha\alpha, \quad \chi_{10}^{\sigma} = \frac{1}{\sqrt{2}}(\alpha\beta + \beta\alpha), \quad \chi_{1-1}^{\sigma} = \beta\beta, \quad \chi_{00}^{\sigma} = \frac{1}{\sqrt{2}}(\alpha\beta - \beta\alpha),$$

$$\begin{cases} 0 \times 0 & |S_1\rangle = \chi_0^{\sigma 1} = \chi_{00}^{\sigma}\chi_{00}^{\sigma}; \\ 1 \times 1 & |S_2\rangle = \chi_0^{\sigma 2} = \sqrt{\frac{1}{3}}(\chi_{11}^{\sigma}\chi_{1-1}^{\sigma} - \chi_{10}^{\sigma}\chi_{10}^{\sigma} + \chi_{1-1}^{\sigma}\chi_{11}^{\sigma}) \end{cases}$$

flavor

$$|\frac{1}{2},-\frac{1}{2}\rangle = -c\bar{u} \quad |\frac{1}{2},\frac{1}{2}\rangle = c\bar{d} \quad |\frac{1}{2},\frac{1}{2}\rangle = u\bar{s} \quad |\frac{1}{2},-\frac{1}{2}\rangle = d\bar{s}$$

$$|0,0\rangle = \frac{1}{\sqrt{2}}[c\bar{d}d\bar{s} + c\bar{u}u\bar{s}]$$

color

$$\chi^{m1}_{1\otimes 1} = \frac{1}{\sqrt{9}}(\bar{r}r\bar{r}r + \bar{r}r\bar{g}g + \bar{r}r\bar{b}b + \bar{g}g\bar{r}r + \bar{g}g\bar{g}g + \bar{g}g\bar{b}b + \bar{b}b\bar{r}r + \bar{b}b\bar{g}g + \bar{b}b\bar{b}b),$$

$$\chi^{m2}_{8\otimes 8} = \frac{\sqrt{2}}{12}(3\bar{b}r\bar{r}b + 3\bar{g}r\bar{r}g + 3\bar{b}g\bar{g}b + 3\bar{g}b\bar{b}g + 3\bar{r}g\bar{g}r + 3\bar{r}b\bar{b}r + 2\bar{r}r\bar{r}r + 2\bar{g}g\bar{g}g + 2\bar{b}b\bar{b}b - \bar{r}r\bar{g}g - \bar{g}g\bar{r}r - \bar{b}b\bar{g}g - \bar{b}b\bar{r}r - \bar{g}g\bar{b}b - \bar{r}r\bar{b}b).$$

$$\chi^{d1}_{3\otimes\bar{3}} = \frac{\sqrt{3}}{6}(rg\bar{r}\bar{g} - rg\bar{g}\bar{r} + gr\bar{g}\bar{r} - gr\bar{r}\bar{g} + rb\bar{r}\bar{b} - rb\bar{b}\bar{r} + br\bar{b}\bar{r} - br\bar{r}\bar{b} + gb\bar{g}\bar{b} - gb\bar{b}\bar{g} + bg\bar{b}\bar{g} - bg\bar{g}\bar{b}).$$

$$\chi^{d2}_{6\otimes\bar{6}} = \frac{\sqrt{6}}{12}(2rr\bar{r}\bar{r} + 2gg\bar{g}\bar{g} + 2bb\bar{b}\bar{b} + rg\bar{r}\bar{g} + rg\bar{g}\bar{r} + gr\bar{g}\bar{r} + gr\bar{r}\bar{g} + rb\bar{r}\bar{b} + rb\bar{b}\bar{r} + br\bar{b}\bar{r} + br\bar{r}\bar{b} + gb\bar{g}\bar{b} + gb\bar{b}\bar{g} + bg\bar{b}\bar{g} + bg\bar{g}\bar{b}).$$

physical channel	channel	Energy
DK	0×0 1×1	2359.0221
	0×0 8×8	3155.2939
D*K*	1×1 1×1	2896.4998
	1×1 8×8	3078.9527
diquark-antidiquark	0×0 3×3	3003.4892
	0×0 6×6	3163.2813
	1×1 3×3	3181.0801
	1×1 6×6	3060.6638
couple channel		2358.9008
Threshold		2356.7786

physical channel	channel	Energy	Energy
$D\bar{K}$	0×0 1×1	2357.9321	2359.0221
	0×0 8×8	3117.9508	3155.2939
$D^*\bar{K}^*$	1×1 1×1	2896.1181	2896.4998
	1×1 8×8	2871.5064	3078.9527
diquark-antidiquark	0×0 3×3	2660.2882	3003.4892
	0×0 6×6		3163.2813
	1×1 3×3		3181.0801
	1×1 6×6	2983.3319	3060.6638
couple channel		2342.0181	2358.9008
Threshold		2356.7786	

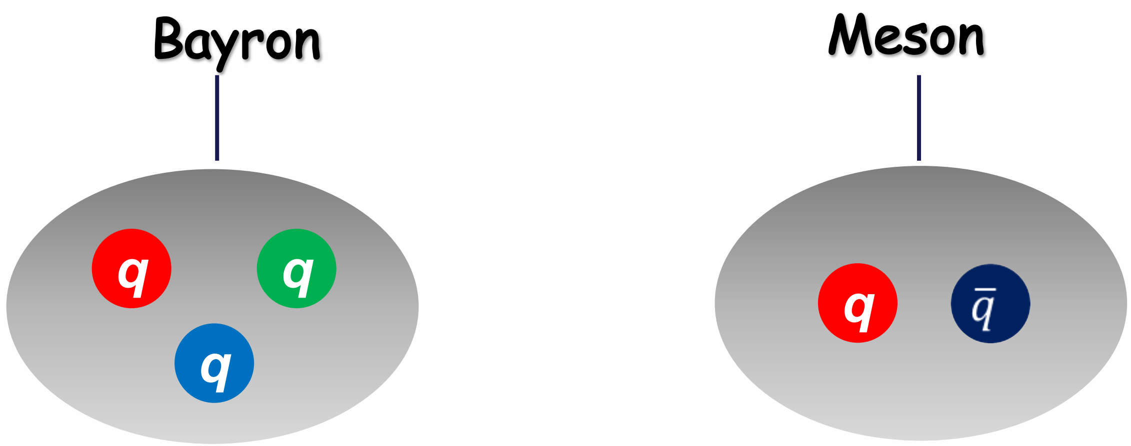
Y.Tan, J.Ping,Systematics of QQqq in a chiral constituent quark model
-------Eur.Phys.J.Plus 135,716 (2020)

UQM for meson:

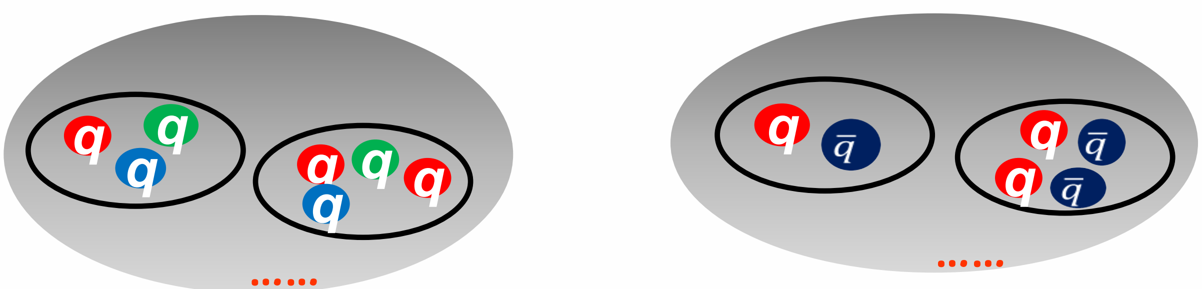
Ds（2317） may be an unquenched state?

The Unquenched quark model (UQM)

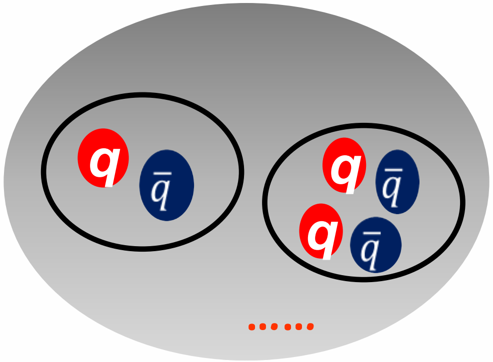
In the conventional quenched quark model:



In the unquenched quark model:



Unquenched quark model



- In unquenched quark model,

$$H\psi = E\psi$$

$$\psi = c_1\psi_{2q} + c_2\psi_{4q}$$

$$H = H_{2q} + H_{4q} + T_{24}$$

- The matrix elements of Hamiltonian:

$$< \psi|H|\psi > = < c_1\Psi_{2q} + c_2\Psi_{4q} \,|\, H_{2q} + H_{4q} + T_{24} \,|\, c_1\Psi_{2q} + c_2\Psi_{4q} >$$

$$= {c_1}^2 < \Psi_{2q} \,|\, H_{2q} \,|\, \Psi_{2q} > + {c_2}^2 < \Psi_{4q} \,|\, H_{4q} \,|\, \Psi_{4q} >$$

$$+ c_1\, {c_2}^* < \Psi_{4q}|T_{24}|\Psi_{2q} > + {c_1}^* c_2 < \Psi_{2q}|\; T_{24}^{\dagger} \;|\Psi_{4q} >$$

3P0 Operator

$$H_{24} = < \psi_{4q} | T_{24} | \psi_{2q} >$$

momentum space:

$$T_{24} = -3\gamma \sum_{m} < 1m-1m|00 > \int d\boldsymbol{p}_3 d\boldsymbol{p}_4 \, \delta^3(\boldsymbol{p}_3 + \boldsymbol{p}_4) y_{1m}\left(\frac{\boldsymbol{p}_3 - \boldsymbol{p}_4}{2}\right) e^{-f^2 p^2} \chi^{34}_{1-m} \phi^{34}_0 \omega^{34}_0 b^{\dagger}_3(\boldsymbol{p}_3) d^{\dagger}_4(\boldsymbol{p}_4)$$

coordinate space:

$$T_{24} = -3\gamma \sum_{m} < 1m1\text{-}m|00 > \int d\boldsymbol{r}_3 d\boldsymbol{r}_4 \left(\frac{1}{2\pi}\right)^{\frac{3}{2}} \mathrm{i}\, \mathrm{r}\, 2^{-\frac{5}{2}} f^{-5} Y_{1m}(\theta_r, \phi_r) e^{-\frac{r^2}{4f^2}} \chi^{34}_{1-m} \phi^{34}_0 \omega^{34}_0 b^{\dagger}_3(\boldsymbol{r}_3) d^{\dagger}_4(\boldsymbol{r}_4)$$

Unquenched quark model

$\gamma = 6.95, f \to 0$

TABLE II. Mass shifts computed for non-strange mesons with quantum numbers IJ^- ($I = 0, 1$; $J = 0, 1$) using the transition matrix constructed from $T_{f\to 0}$ in Eq. (28). (η is an isospin 0 partner to the pion; and all dimensioned quantities are listed in MeV.)

states(IJ^P)	$\pi(10^-)$	$\rho(11^-)$	$\omega(01^-)$	$\eta(00^-)$
bare mass (Theo.)	139.0	772.7	701.9	669.5
$\pi\pi$	-	−130.1	-	-
$\pi\rho$	−847.9	-	−596.4	-
$\pi\omega$	-	−182.5	-	-
$\eta\rho$	-	−159.3	-	-
$\rho\rho$	-	−561.7	-	−834.4
$\rho\omega$	−804.4	-	-	-
$\eta\omega$	-	-	−175.1	-
$\omega\omega$	-	-	-	−271.1
$K\bar{K}$	-	−65.0	−70.7	-
$K\bar{K}^*(\bar{K}K^*)$	−340.2	−122.4	−125.3	−214.0
$K^*\bar{K}^*$	−680.0	−429.2	−481.7	−421.8
Total mass shift	−2672.5	−1650.2	−1449.2	−1741.3

Larger mass shifts are obtained with lighter quarks.

- For $b\bar{b}$ system, ~100MeV;
- For $c\bar{c}$ system, ~800MeV;
- For $s\bar{s}$ system,~1500MeV;
- For $u\bar{u}/d\bar{d}$ system,~2500MeV.

The negative mass shifts are alarmingly large when considering the hadron-loop effects.

Improvement one

3P0 Operator

$$H_{24} = \langle \psi_{4q} | T_{24} | \psi_{2q} \rangle$$

$\gamma = 6.95, f \nrightarrow 0$

momentum space:

$$T_{24} = -3\gamma \sum_m \langle 1m - 1m|00 \rangle \int d\boldsymbol{p}_3 d\boldsymbol{p}_4 \, \delta^3(\boldsymbol{p}_3 + \boldsymbol{p}_4) y_{1m}\left(\frac{\boldsymbol{p}_3 - \boldsymbol{p}_4}{2}\right) e^{-f^2 \boldsymbol{p}^2} \chi^{34}_{1-m} \phi^{34}_0 \omega^{34}_0 b_3^\dagger(\boldsymbol{p}_3) d_4^\dagger(\boldsymbol{p}_4)$$

coordinate space:

$$T_{24} = -3\gamma \sum_m \langle 1m1\text{-}m|00 \rangle \int d\boldsymbol{r}_3 d\boldsymbol{r}_4 \left(\frac{1}{2\pi}\right)^{\frac{3}{2}} \mathrm{i}\, r\, 2^{-\frac{5}{2}} f^{-5} Y_{1m}(\theta_r, \phi_r) e^{-\frac{r^2}{4f^2}} \chi^{34}_{1-m} \phi^{34}_0 \omega^{34}_0 b_3^\dagger(\boldsymbol{r}_3) d_4^\dagger(\boldsymbol{r}_4)$$

X.Chen, Light-meson masses in an unquenched quark model,
------Phys.Rev.D 97,094016 (2018)

Unquenched quark model

$$T_{24} = -3\gamma \sum_{m} < 1m - 1m|00 > \int d\boldsymbol{r}_3 d\boldsymbol{r}_4 \left(\frac{1}{2\pi}\right)^{\frac{3}{2}} \mathrm{i}\, \mathrm{r}\, 2^{-\frac{5}{2}} f^{-5} Y_{1m}(\theta_r, \phi_r) e^{-\frac{r^2}{4f^2}} \chi_{1-m}^{34} \phi_0^{34} \omega_0^{34} b_3^{+}(\boldsymbol{r}_3) d_4^{+}(\boldsymbol{r}_4)$$

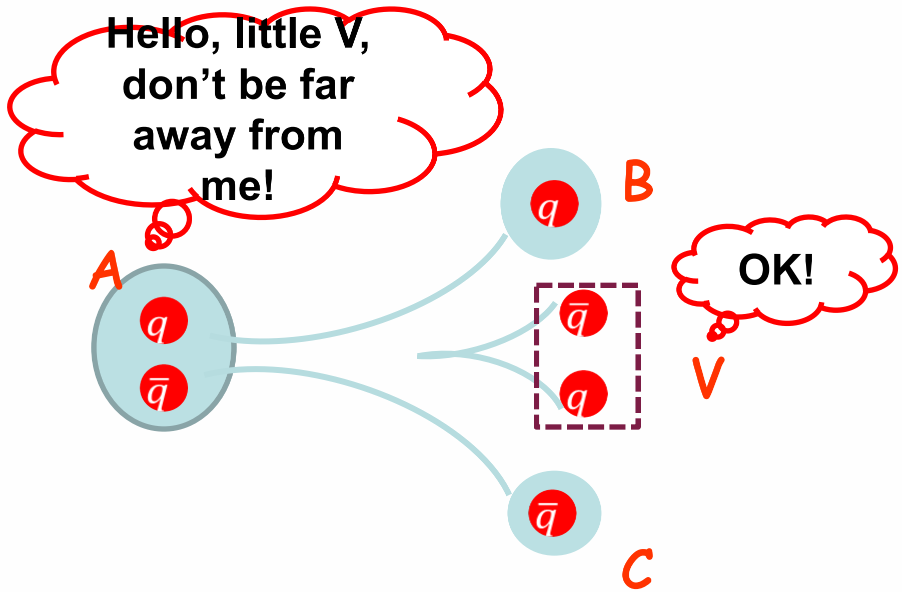
TABLE III: $\pi\rho$ contribution to π with the modified transition operator Eq. (28). (unit: MeV)

f (fm)	0.001	0.01	0.1	0.3	0.5	0.7	0.9	1.0
E_0 (MeV)	−709	−687	−189	100	133	138	139	139
ΔM (MeV)	−848	−826	−328	−39	−6	−1	0	0

X.Chen,Light-meson masses in an unquenched quark model,
------Phys.Rev.D 97,094016 (2018)

Improvement Two

Unquenched quark model



$$T_{24} = -3\gamma \sum_{m} < 1m - 1m|00 > \int d\boldsymbol{r}_3 d\boldsymbol{r}_4 \left(\frac{1}{2\pi}\right)^{\frac{3}{2}} \mathrm{i\,r}\, 2^{-\frac{5}{2}} f^{-5} Y_{1m}(\theta_r, \phi_r) e^{-\frac{r^2}{4f^2}}\; e^{-\frac{R_{AV}^2}{R_0^2}} \chi_{1-m}^{34} \phi_0^{34} \omega_0^{34} b_3^{+}(\boldsymbol{r}_3) d_4^{+}(\boldsymbol{r}_4)$$

X.Chen,Light-meson masses in an unquenched quark model,
------Phys.Rev.D 97,094016 (2018)

Improvement Three:Combined Effect

Unquenched quark model

$$T_2 = -3\gamma \sum_m \langle 1m1(-m)|00\rangle \int d\mathbf{r}_3 d\mathbf{r}_4 \left(\frac{1}{2\pi}\right)^{\frac{5}{2}} ir2^{-\frac{5}{2}} f^{-5}$$

$$\times Y_{1m}(\hat{\mathbf{r}}) e^{-\frac{\mathbf{r}^2}{4f^2}} e^{-\frac{R_{AV}^2}{R_0^2}} \chi_{1-m}^{34} \phi_0^{34} \omega_0^{34} b_3^{\dagger}(\mathbf{r}_3) d_4^{\dagger}(\mathbf{r}_4)$$

$$\gamma = 6.95$$
$$f \to 0$$
$$R_0 = 1\ \mathbf{fm}$$

X.Chen,Light-meson masses in an unquenched quark model,
------Phys.Rev.D 97,094016 (2018)

Unquenched quark model

$$T_{24} = -3\gamma \sum_{m} < 1m - 1m|00 > \int d\boldsymbol{r}_3 d\boldsymbol{r}_4 \left(\frac{1}{2\pi}\right)^{\frac{3}{2}} \mathrm{i}\, \mathrm{r}\, 2^{-\frac{5}{2}} f^{-5} Y_{1m}(\theta_r, \phi_r)$$

$$e^{-\frac{r^2}{4f^2}} e^{-\frac{R_{AV}^2}{f_0^2}} \chi_{1-m}^{34} \phi_0^{34} \omega_0^{34} b_3^{\dagger}(\boldsymbol{r}_3) d_4^{\dagger}(\boldsymbol{r}_4)$$

$\Gamma(\rho \rightarrow \pi\pi) = 150\ MeV$

Mass shifts should neither be too large nor too small

$\longrightarrow$

$\gamma = 32.17$
$f = 0.5$ fm
$R_0 = 1$ fm

X.Chen,Light-meson masses in an unquenched quark model,
------Phys.Rev.D 97,094016 (2018)

UQM for Ds mass spectrum

- check the validity of vaccumulating approach
- understand Ds(2317) in uqm ---our goal
- investigate four Ds states, Ds,Ds*,Ds(2317),Ds(2460)

UQM for Ds mass spectrum

$$P = 1 - \sum Pi$$

$$\Delta m = \sum mi$$

$$(H) = \begin{bmatrix} \langle H_{2q} \rangle & \langle H_{24} \rangle_1 & \langle H_{24} \rangle_2 & \cdots & \langle H_{24} \rangle_n \\ \langle H_{42} \rangle_1 & \langle H_{4q} \rangle_{11} & \langle H_{4q} \rangle_{12} & \cdots & \langle H_{4q} \rangle_{1n} \\ \langle H_{42} \rangle_2 & \langle H_{4q} \rangle_{21} & \langle H_{4q} \rangle_{22} & \cdots & \langle H_{4q} \rangle_{2n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \langle H_{42} \rangle_n & \langle H_{4q} \rangle_{n1} & \langle H_{4q} \rangle_{n2} & \cdots & \langle H_{4q} \rangle_{nn} \end{bmatrix}$$

$$(N) = \begin{bmatrix} \langle N_{2q} \rangle & 0 & 0 & \cdots & 0 \\ 0 & \langle N_{4q} \rangle_{11} & \langle N_{4q} \rangle_{12} & \cdots & \langle N_{4q} \rangle_{1n} \\ 0 & \langle N_{4q} \rangle_{21} & \langle N_{4q} \rangle_{22} & \cdots & \langle N_{4q} \rangle_{2n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & \langle N_{4q} \rangle_{n1} & \langle N_{4q} \rangle_{n2} & \cdots & \langle N_{4q} \rangle_{nn} \end{bmatrix}$$

Y.Tan and J.Ping,X(3872) in an unquenched quark model,
------Phys.Rev.D 100,034022 (2019)

UQM for Ds mass spectrum

	η_c		$\chi_{c1}(2P)$	
bare mass	2986.28		3889.62	
meson-meson	Δm (MeV)	P_i	Δm (MeV)	P_i
$D\bar{D}^*(S)$	-189.75	4.02%	-	-
$D_s^*\bar{D}_s(S)$	-76.50	1.75%	-	-
Total	-266.35	5.77%	-	-
$D\bar{D}^*+D_s^*\bar{D}_s$ (S)	-266.71	5.20%	-	-
$D\bar{D}^*(S)$	-189.75	4.02%	-73.65	43.09%
$D^*\bar{D}(S)$	-189.75	4.02%	-73.65	43.09%
Total	-379.50	8.04%	-147.30	86.18%
$D\bar{D}^*+D^*\bar{D}$ (S)	-369.43	6.4%	-117.11	33.38%

Y.Tan and J.Ping,X(3872) in an unquenched quark model,
------Phys.Rev.D 100,034022 (2019)

UQM for Ds mass spectrum

$$P_{total} = 1 - \sum_{i=1}^{N1}[P_{i,E_i \simeq E_{2q}}] - \sum_{i=N1}^{N} P_i$$

$$\Delta E_{total} = \sum_{i=1}^{N1}[\Delta E_{i,E_i \simeq E_{2q}}] + \sum_{i=N1}^{N} \Delta E_i$$

Y.Tan and J.Ping,X(3872) in an unquenched quark model,
------Phys.Rev.D 100,034022 (2019)

UQM for Ds mass spectrum

bare mass	D_s 2009.3		D_s^* 2131.7		$D_{s0}(2317)$ 2397.9		$D_{s1}(2460)$ 2494.7	
	Δ_M	Per	Δ_M	Per	Δ_M	Per	Δ_M	Per
$DK^*(P)+D^*K(P)+D^*K^*(P)$	-23.4	1.7%	-6.5	0.7%	-	-	-	-
$D_s\phi(P)+D_s^*\eta(P)+D_s^*\phi(P)$	-10.1	0.7%	-2.6	0.2%	-	-	-	-
$DK(P)+D^*K^*(P)$	-	-	-4.2	0.6%	-	-	-	-
$D_s\eta(P)+D_s^*\phi(P)$	-	-	-1.4	0.1%	-	-	-	-
$D^*K^*(P)$	-	-	-15.7	0.5%	-	-	-	-
$D_s^*\phi(P)$	-	-	-7.1	0.6%	-	-	-	-
$DK(S)+D^*K^*(S)$	-	-	-	-	-37.5	53.5%	-	-
$D_s\eta(S)+D_s^*\phi(S)$	-	-	-	-	-3.4	0.7%	-	-
$D^*K^*(D)$	-	-	-	-	-24.3	2.4%	-0.4	0.1%
$D_s^*\phi(D)$	-	-	-	-	-8.4	0.7%	-0.1	0.1%
$DK^*(S)+D^*K(S)+D^*K^*(S)$	-	-	-	-	-		-29.7	53.2%
$D_s\phi(S)+D_s^*\eta(S)+D_s^*\phi(S)$	-	-	-	-	-		-9.5	1.1%
$DK^*(D)+D^*K(D)+D^*K^*(D)$	-	-	-	-	-		-22.6	8.4%
$D_s\phi(D)+D_s^*\eta(D)+D_s^*\phi(D)$	-	-	-	-	-		-2.4	0.4%
Total	-33.5	2.4%	-37.5	2.7%	-76.3	57.3%	-63.2	64.1%
Unquenching	1975.7	97.6%	2094.1	97.3%	2324.2	42.7%	2429.8	35.9%

UQM for Ds mass spectrum

$$\Delta M = M(D_s(2317)) - \frac{1}{4}(M(D_s) + 3M(D_s^*))$$

	this work	Re【1】	Re【2】	Exp
bare mass	287	309	274	241
unquenched mass	258	249	254	

Re[1]:Phys. Rev. D 94, no. 7, 074037 (2016)
Re[2]:PoS Hadron 2017, 024 (2018)

Summary

1. traditional 3P0 operator needs to be reasonably modified.
2. Ds(2317) may be a mixture state with 43% cs meson and 57% molecule states.
3. channel coupling plays an ignore effect on uqm.

Thanks for attention