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Threshold effects as the origin of several newly observed exotic hadrons

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Y.H. Ge, XHL, H.W. Ke, arXiv:2103.05282

XHL, M.J. Yan, H.W. Ke, G. Li, J.J. Xie, arXiv:2008.07190

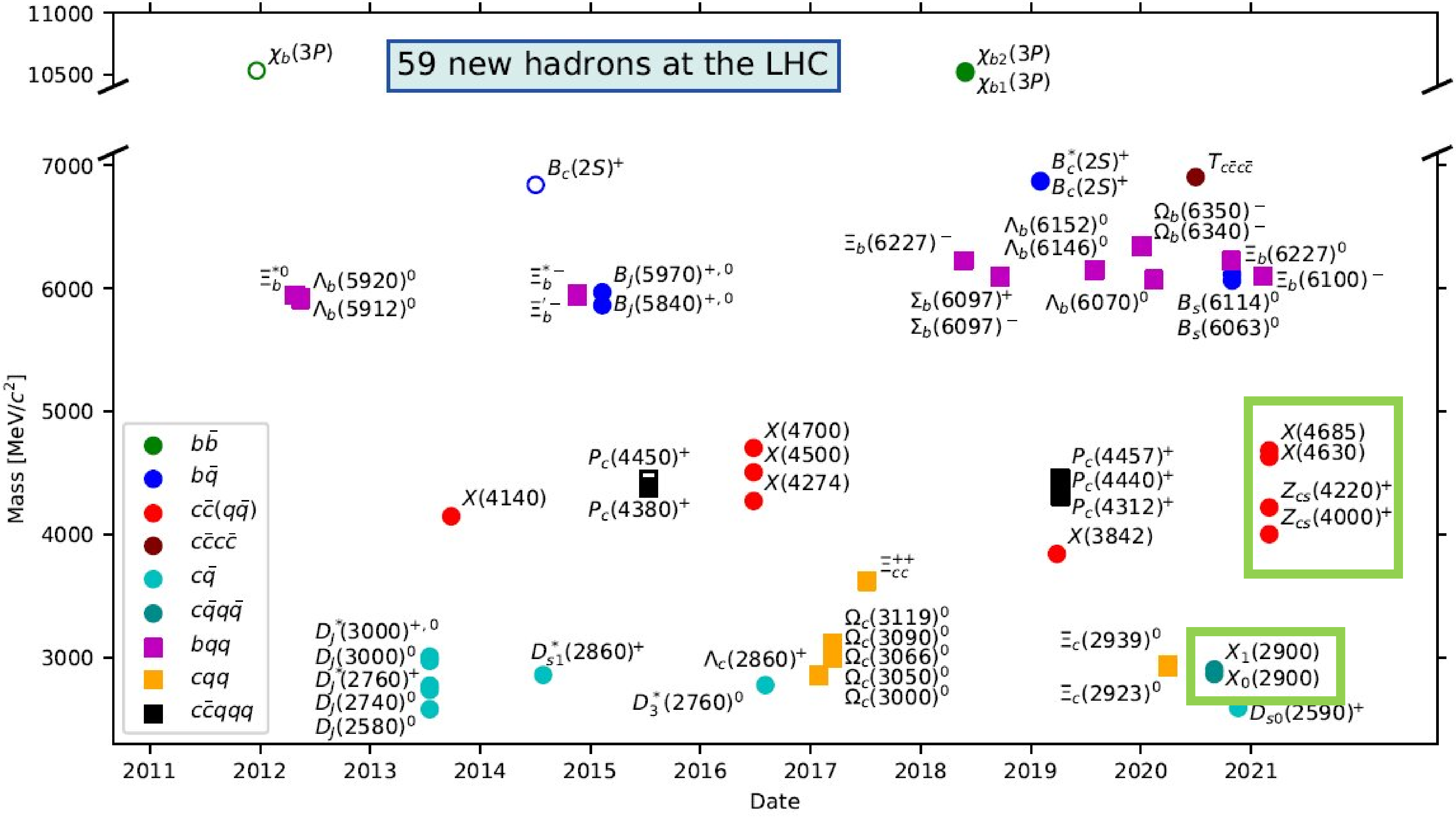
F.K. Guo, XHL, S. Sakai, PPNP 112, 103757 (2020)

第七届XYZ粒子研讨会，山东大学，青岛，2021年5月

Outline

- Brief introduction to exotic hadrons
- Cusp effect
- Triangle singularity (TS) phenomena
- Threshold effects and newly observed XYZ states (X(2900), Zcs(4000/3985), X(4700))
- Summary

The Large Hadron Collider's official tally:
59 new hadrons



Renaissance of Hadron Spectroscopy!

Theoretical Interpretation

- ✓**Hadronic molecule**
- ✓ **Tetraquark, Pentaquark**
- ✓**Hybrid**
- ✓**Hadrocharmonium**

Genuine resonance interpretations

- ✓**Threshold effect (cusp, triangle singularity, …)** (*Non-resonance interpretation*)

"Resonance-like" structure $\overset{?}{=}$ Genuine particle

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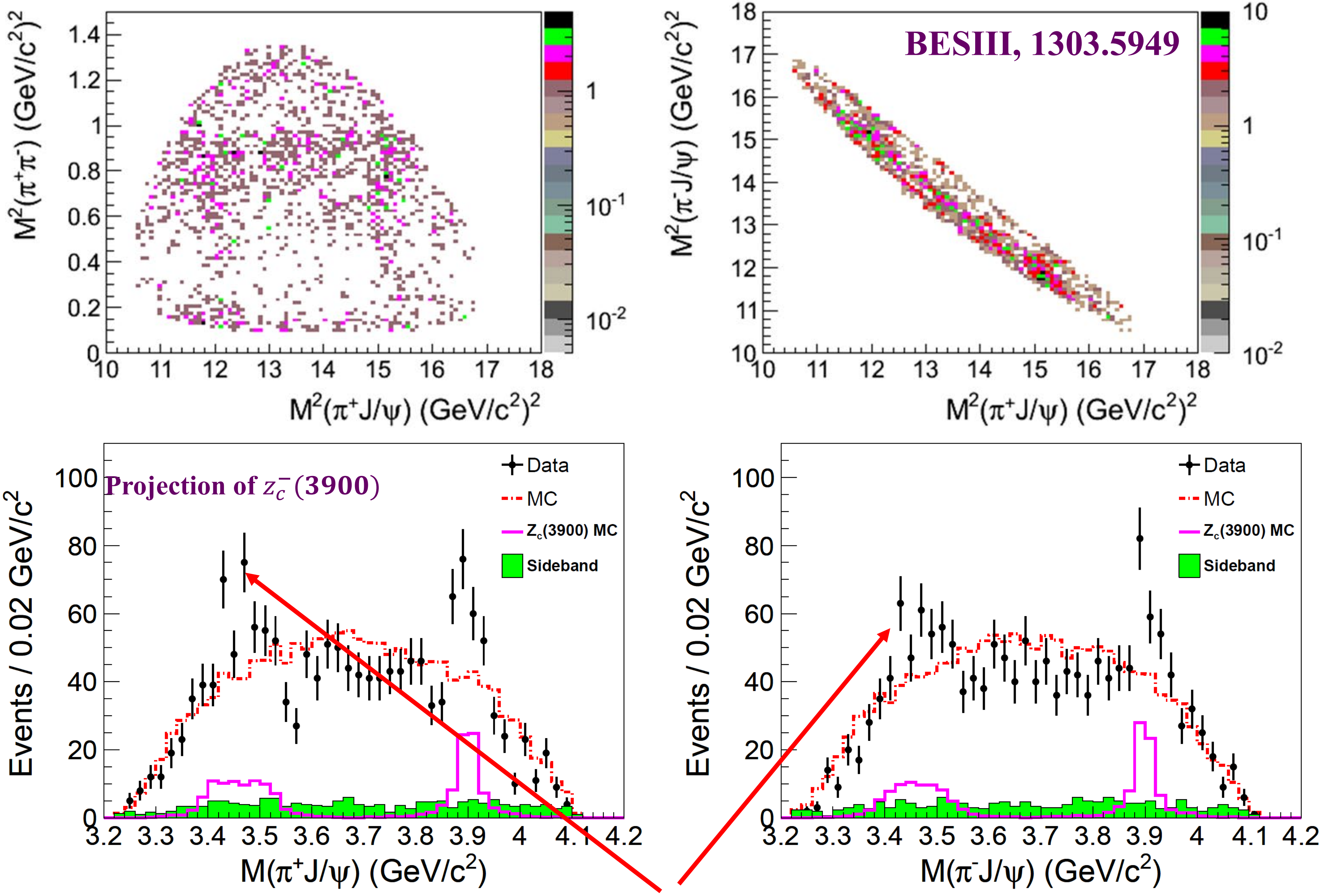


BESIII, 1303.5949

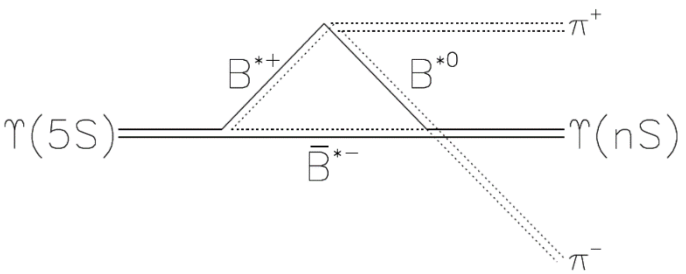
Projection of $Z_c^-(3900)$

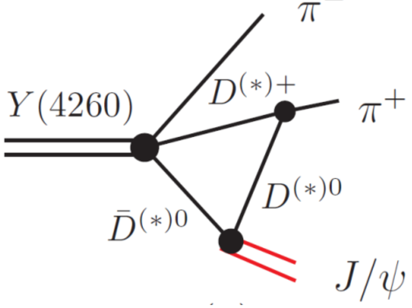
“Reflection” in Dalitz plot

Cusp effect

F.K. Guo, XHL, S. Sakai, PPNP 112, 103757 (2020)

- **Possible correlation with some XYZ states: $Z_b(10610/10650)$, $Z_c(3900)$, $Z_c(4020)$**





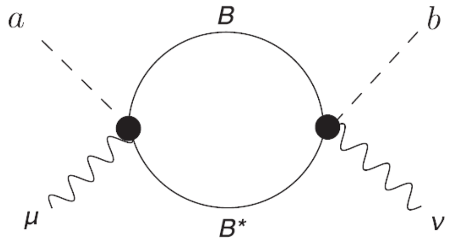


FIG. 1. Coupled channels in $\Upsilon\pi$ scattering.

D.V. Bugg,
EPL96, 11002(2011)

D.Y. Chen, X. Liu,
PRD88, 11002(2013)

E. Swanson,
PRD91, 034009(2015)

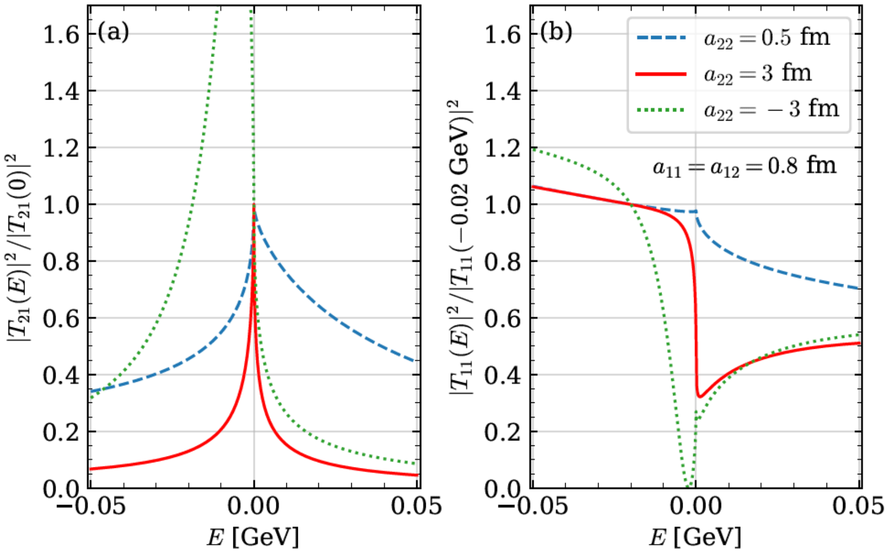
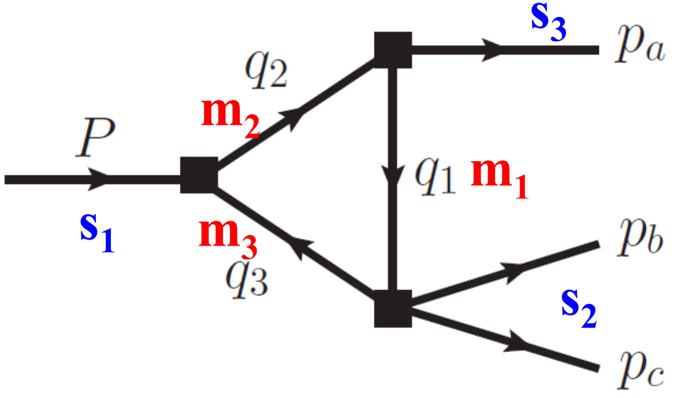


FIG. 2. Illustration of threshold behaviors. Here we use the masses of the π^- and J/ψ for channel-1 and those of the D^0 and D^{*-} for channel-2, and the values of used a_{ij} parameters

X.K. Dong, F.K. Guo, B.S. Zou,
arXiv:2011.14517

J/ψπ-DD* interaction

Triangle Singularity Mechanism



$$P^2 = s_1,\ (p_b + p_c)^2 = s_2$$

$$p_a^2 = s_3$$

$$\Gamma_3(s_1, s_2, s_3) = \frac{-1}{16\pi^2}\int_0^1\int_0^1\int_0^1 da_1\, da_2\, da_3\, \frac{\delta(1-a_1-a_2-a_3)}{D-i\epsilon}$$

$$D = \sum_{i,j=1}^{3} a_i a_j Y_{ij},\quad Y_{ij} = \frac{1}{2}\left[m_i^2 + m_j^2 - (q_i - q_j)^2\right]$$

✓ Singularity in the complex space

Necessary conditions (Landau Equation)

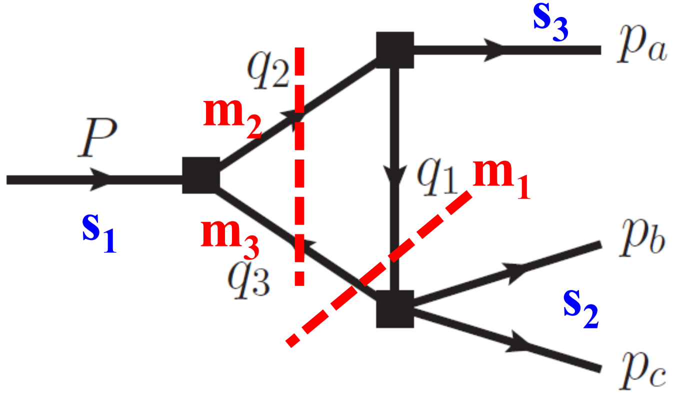
$$D = 0,$$

$$either\ \ a_j = 0\ \ or\ \boxed{\frac{\partial D}{\partial a_j} = 0.}$$

Leading singularity

Landau, Nucl.Phys.13,181(1959)

Triangle Singularity Mechanism



$$P^2 = s_1,\ (p_b + p_c)^2 = s_2$$

$$p_a^2 = s_3$$

✓ **Singularity in the complex space**

The position of the singularity is obtained by solving

$$\det[Y_{ij}] = 0$$

Normal Threshold

s_1, s_3, $m_{1,2,3}$ fixed

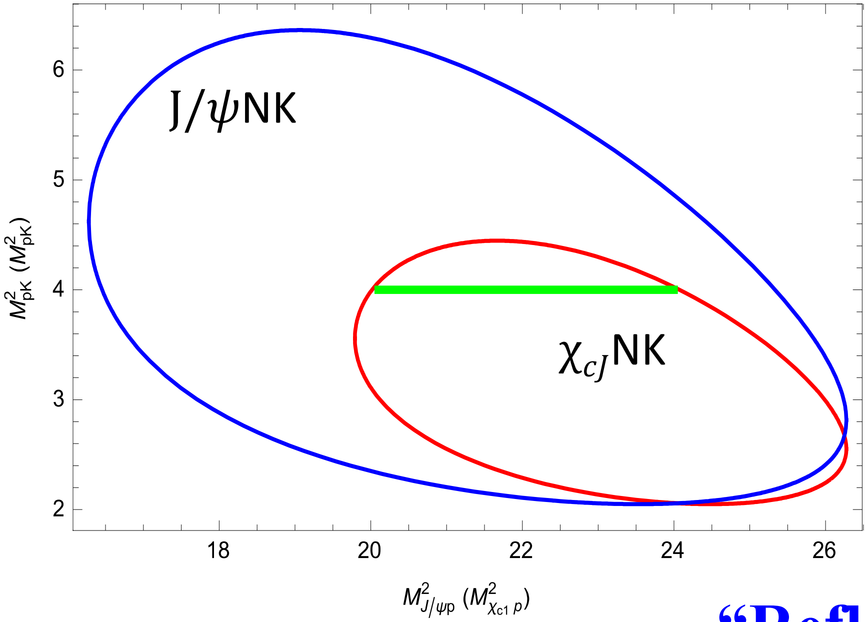
$$\boxed{s_2^{\pm}} = \boxed{(m_1+m_3)^2} + \frac{1}{2m_2^2}\left[(m_1^2+m_2^2-s_3)(s_1-m_2^2-m_3^2) - 4m_2^2 m_1 m_3 \pm \lambda^{1/2}(s_1,m_2^2,m_3^2)\lambda^{1/2}(s_3,m_1^2,m_2^2)\right], \quad \lambda(x,y,z) \equiv (x-y-z)^2 - 4yz$$

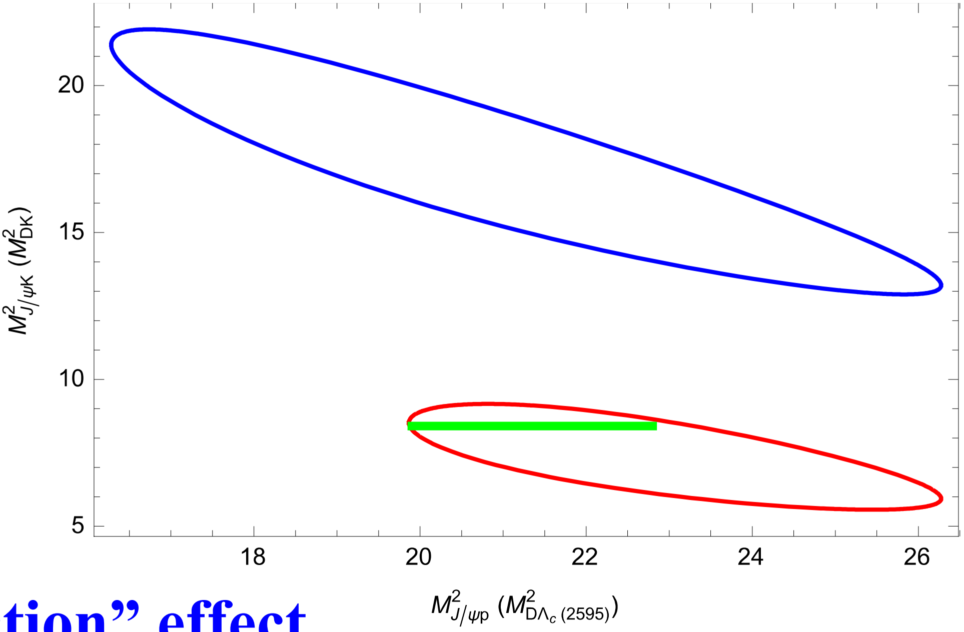
Anomalous Threshold

$$\boxed{s_1^{\pm}} = (m_2+m_3)^2 + \frac{1}{2m_1^2}\left[(m_1^2+m_2^2-s_3)(s_2-m_1^2-m_3^2) - 4m_1^2 m_2 m_3 \pm \lambda^{1/2}(s_2,m_1^2,m_3^2)\lambda^{1/2}(s_3,m_1^2,m_2^2)\right].$$

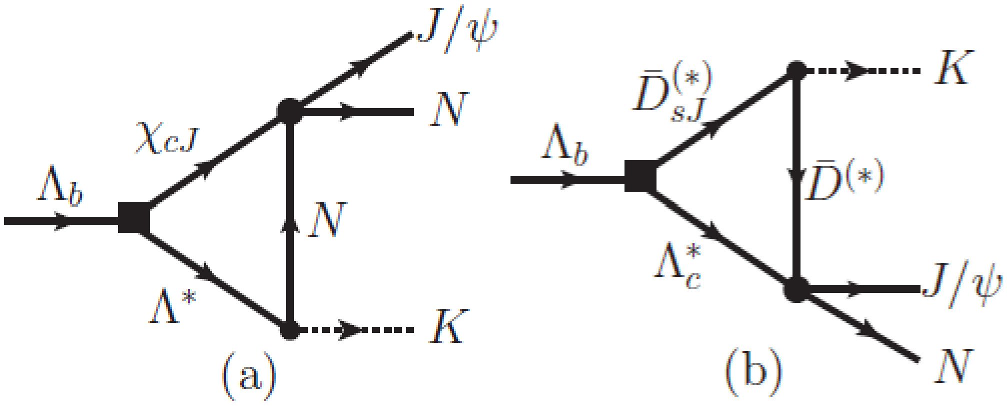
s_2, s_3, $m_{1,2,3}$ fixed

TS mechanism: Dalitz plot



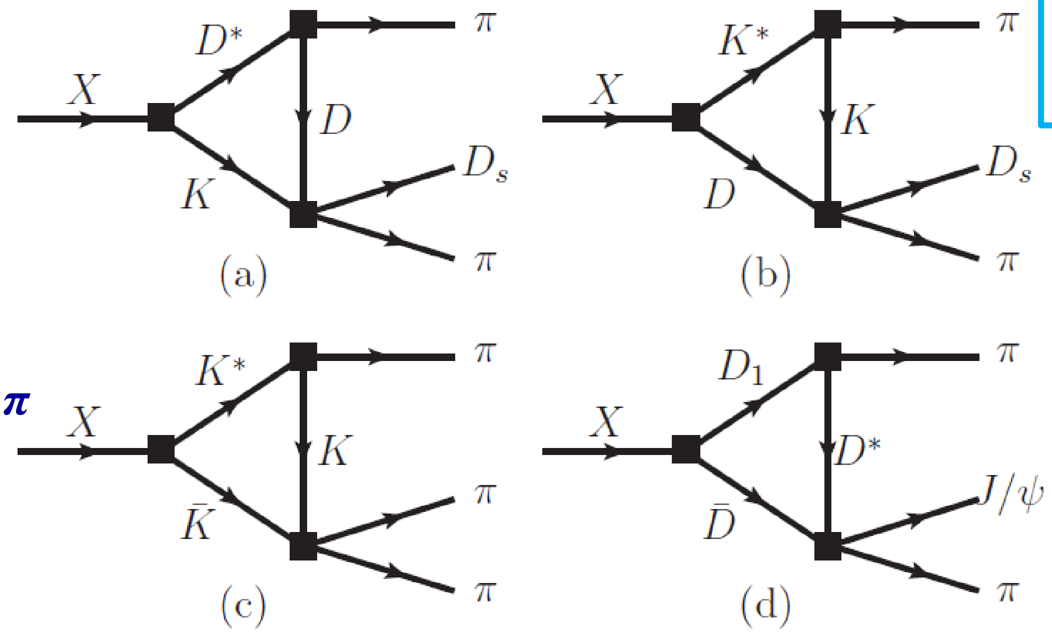


"Reflection" effect



Triangle Singularity Phenomena

also see talks by Feng-Kun Guo, Jun Jiang, Hao-Jie Jing



$\eta(1405) \rightarrow f_0(980)\pi$

$Y(4260) \rightarrow Z_c(3900)\pi$

Wu, Liu, Zhao & Zou, PRL108,081803(2012)

Wang,Hanhart,Zhao,PRL111,132003(2013)

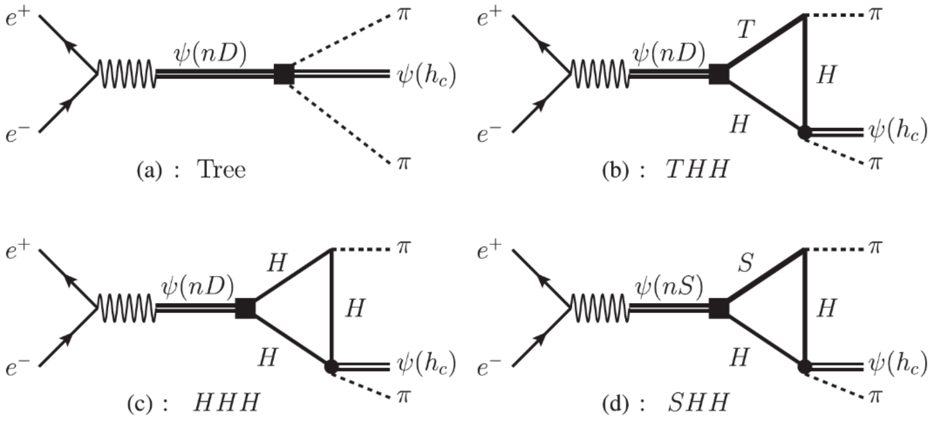
Kinematic region of ATS

[MeV]	Fig. 3(a)	Fig. 3(b)	Fig. 3(c)	Fig. 3(d)
$\Delta_{s1}^{\max}$	0.089	96	49	16
$\Delta_{s2}^{\max}$	0.087	62	38	15

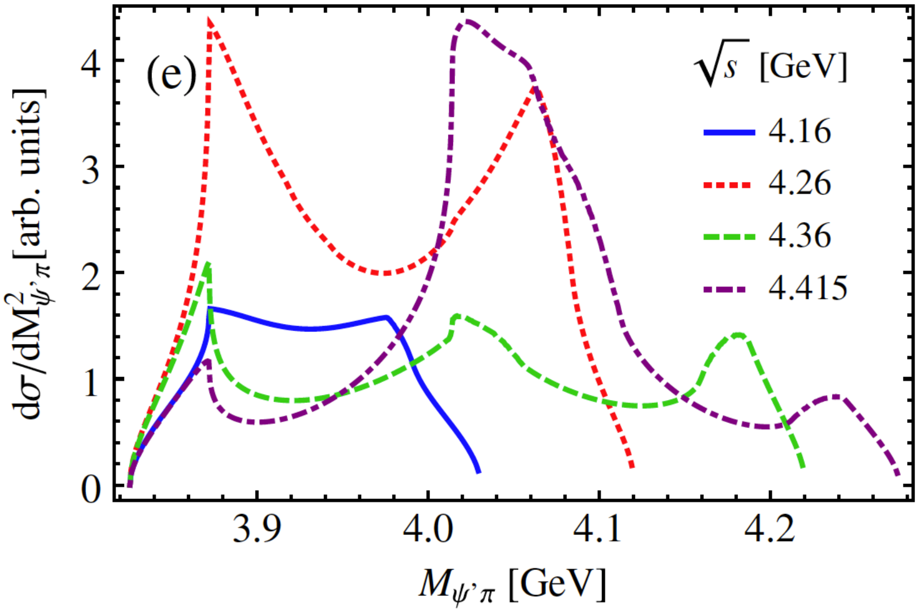
$$\Delta_{s_1} = \sqrt{s_1^-} - \sqrt{s_{1N}},$$
$$\Delta_{s_2} = \sqrt{s_2^-} - \sqrt{s_{2N}}.$$

Liu, Oka, Zhao, PLB753, 297(2016)

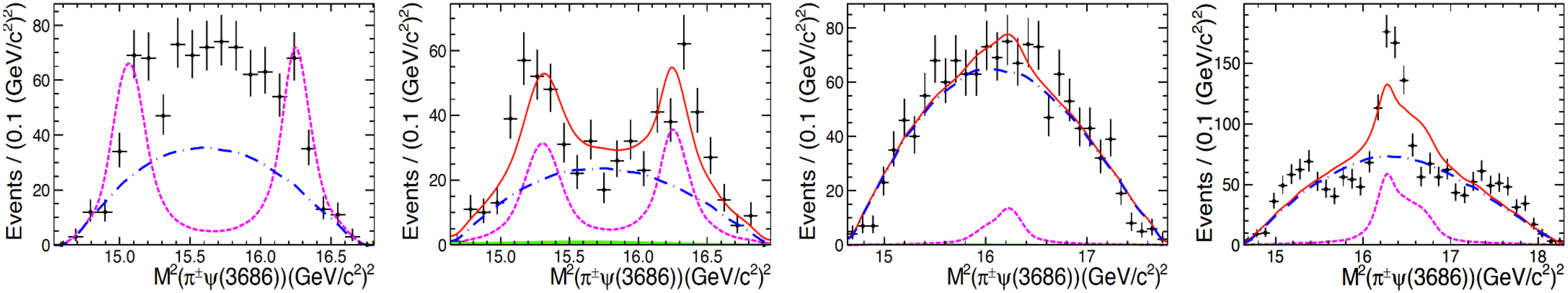
TS mechanism and structures in $e^+e^-\rightarrow\psi(3686)\pi\pi$



X.H. Liu, PRD90,074004(2014)



BESIII, arXiv:1703.08787



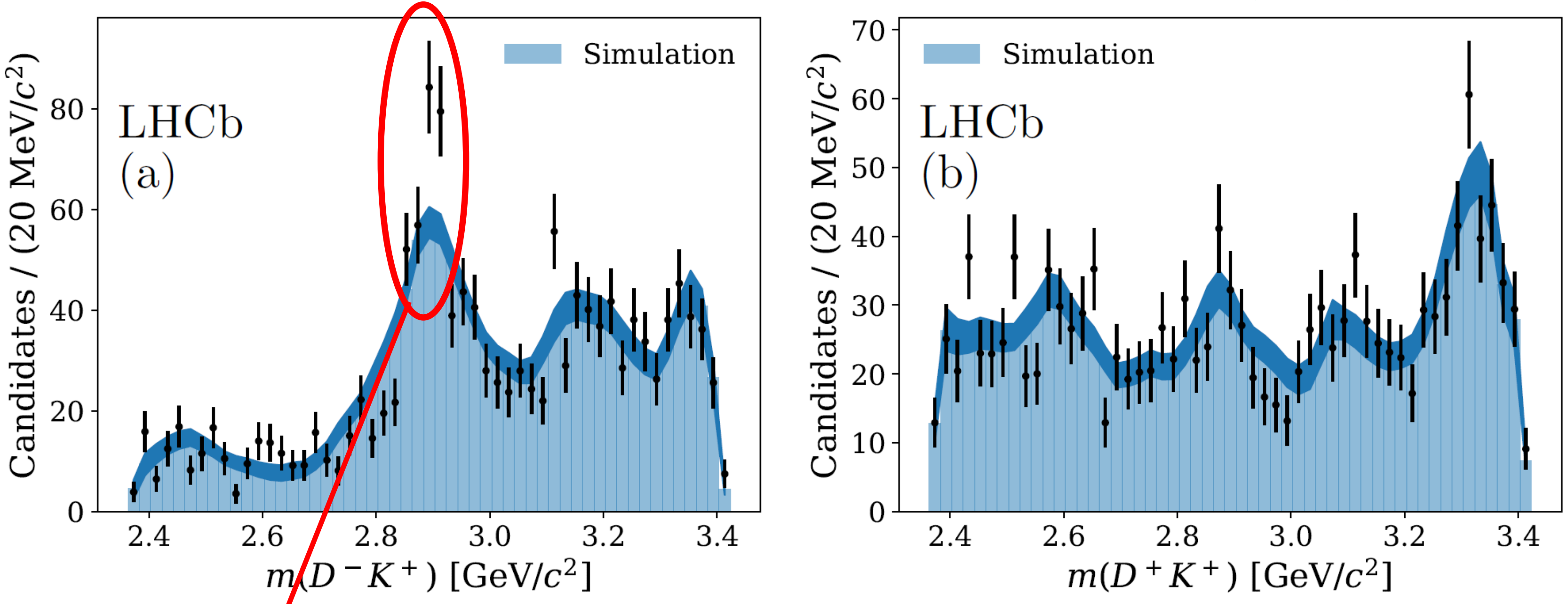
Theoretical predictions are consistent with the observed ψ(3686)pi invariant mass distributions at various CM energies

Threshold effects and newly observed X(2900), Zcs(4000/3985), X(4700)

Observation of D^-K^+ ($\bar{c}\bar{s}ud$) structure

$$B^+ \to D^+ D^- K^+$$

LHCb, PRL125, 242001(2020); PRD102, 112003(2020)



States	Mass/MeV	Width/MeV	Fraction/%	J^P
$X_0(2900)$	$2866 \pm 7 \pm 2$	$57 \pm 12 \pm 4$	$5.6 \pm 1.4 \pm 0.5$	0^+
$X_1(2900)$	$2904 \pm 5 \pm 1$	$110 \pm 11 \pm 4$	$30.6 \pm 2.4 \pm 2.1$	1^-

Observation of D^-K^+ ($\bar{c}\bar{s}ud$) structure

States	Mass/MeV	Width/MeV	Fraction/%
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J^P

0^+

1^-

Two close thresholds :

D^*K^* ~2902 MeV

D_1K ~2914 MeV

Interpretations :

J. He, D.Y. Chen, 2020

- **$\bar{D}^*K^*$, $\bar{D}_1K$ molecular state**
- **Tightly bound tetraquark state**

Predictions: an excited 0^+ tetraquark with mass 2850 MeV, and a 1^+ state with mass 2902 MeV are predicted. Many other states are also predicted

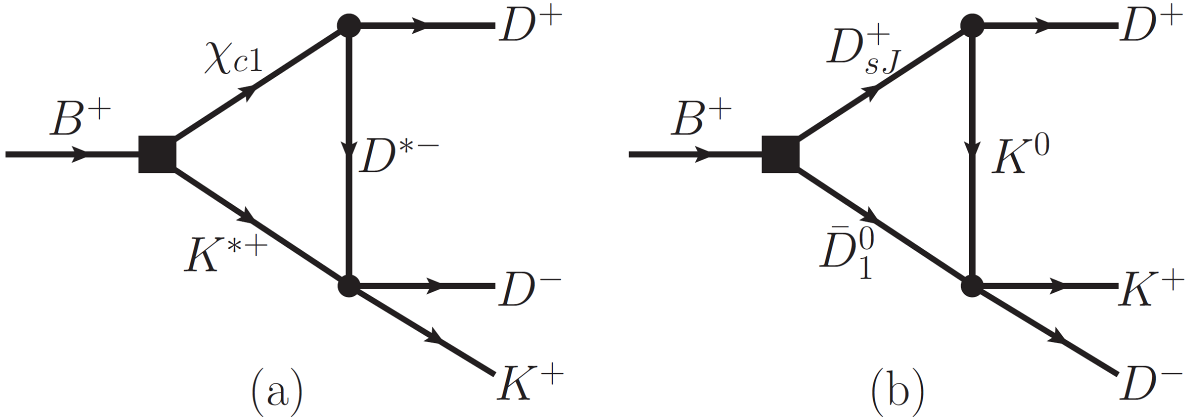
Y.R. Liu et al, PRD101, 114017(2020)

Puzzle

Chiral symmetry implies:

$\bar{D}_1K$ molecule ⟷ $\bar{D}K$ molecule

Threshold effects and $X_{0,1}(2900)$



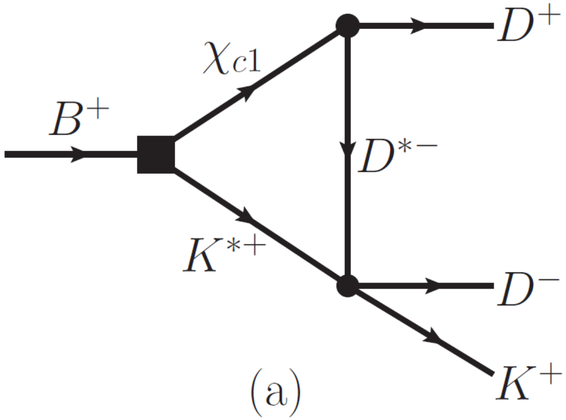
(a) (b)

$$\mathcal{A}^{[\chi_{c1}K^{*+}D^{*-}]}_{B^+\to D^+D^-K^+} = -i\int\frac{d^4q_1}{(2\pi)^4}\frac{\mathcal{A}(B^+\to\chi_{c1}K^{*+})}{(q_1^2-m_{K^*}^2+im_{K^*}\Gamma_{K^*})} \times\frac{\mathcal{A}(\chi_{c1}\to D^+D^{*-})\mathcal{A}(D^{*-}K^{*+}\to D^-K^+)}{(q_2^2-m_{\chi_{c1}}^2+im_{\chi_{c1}}\Gamma_{\chi_{c1}})(q_3^2-m_{D^*}^2)}, \tag{7}$$

$$\mathcal{A}^{[D^+_{sJ}\bar{D}^0_1K^0]}_{B^+\to D^+D^-K^+} = -i\int\frac{d^4q_1}{(2\pi)^4}\frac{\mathcal{A}(B^+\to D^+_{sJ}\bar{D}^0_1)}{(q_1^2-m_{\bar{D}_1}^2+im_{\bar{D}_1}\Gamma_{\bar{D}_1})} \times\frac{\mathcal{A}(D^+_{sJ}\to D^+K^0)\mathcal{A}(\bar{D}^0_1K^0\to D^-K^+)}{(q_2^2-m_{D_{sJ}}^2+im_{D_{sJ}}\Gamma_{D_{sJ}})(q_3^2-m_K^2)}. \tag{8}$$

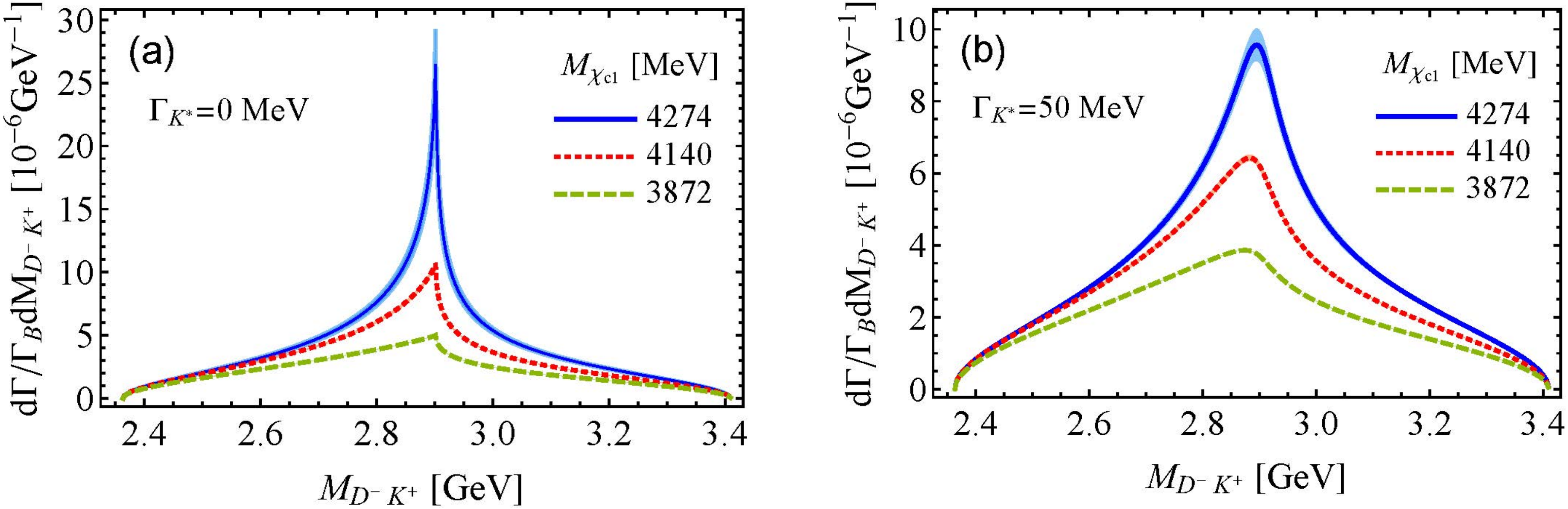
XHL, M.J. Yan, H.W. Ke, G. Li, J.J. Xie, arXiv:2008.07190

Threshold effects and $X_0(2900)$



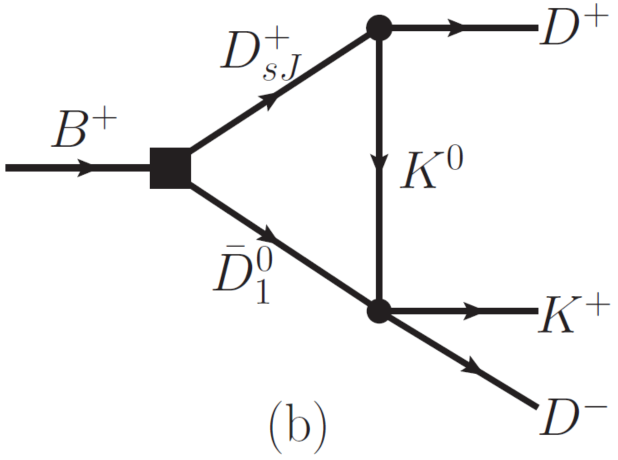
$$\mathcal{A}(D^{*-}K^{*+} \to D^-K^+) = C_a \epsilon_{D^{*-}} \cdot \epsilon_{K^{*+}}$$

$J^P=0^+$



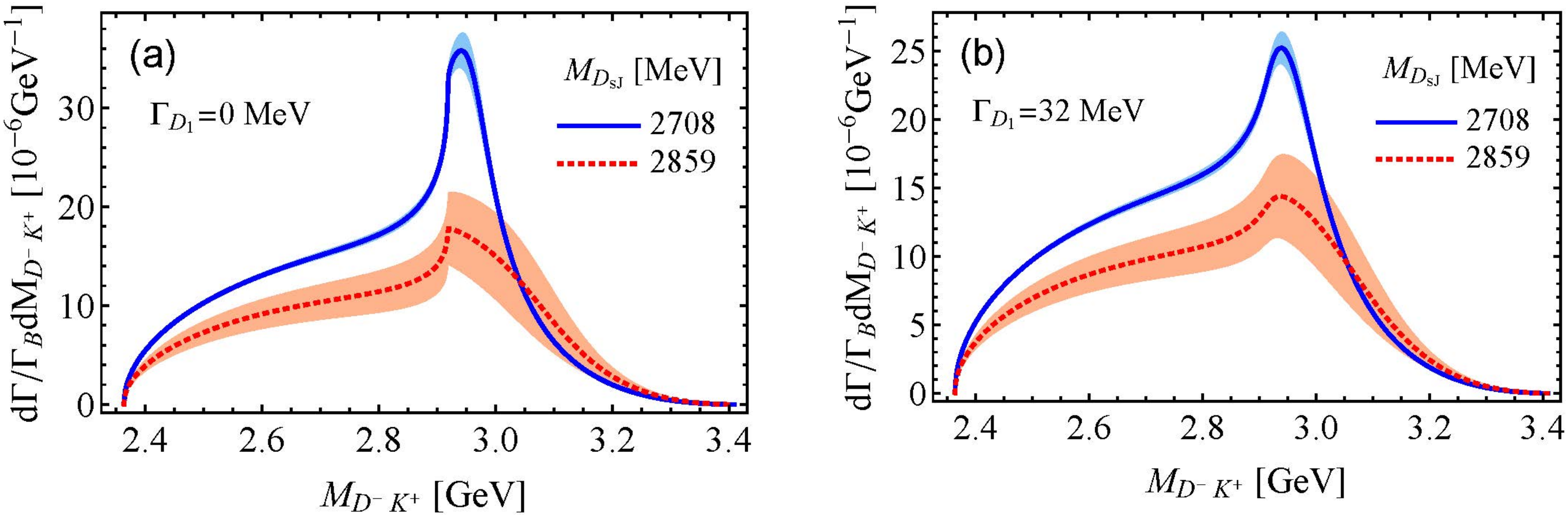
XHL, M.J. Yan, H.W. Ke, G. Li, J.J. Xie, arXiv:2008.07190

Threshold effects and $X_1(2900)$



$$\mathcal{A}(\bar{D}_1^0 K^0 \to D^- K^+) \;=\; C_b(p_{K^0} + p_{K^+}) \cdot \epsilon_{\bar{D}_1^0}$$

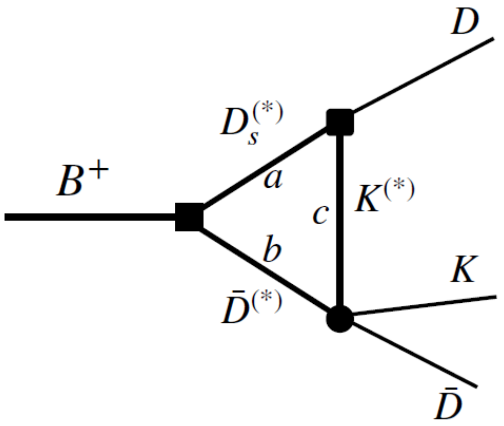
$J^P=1^-$

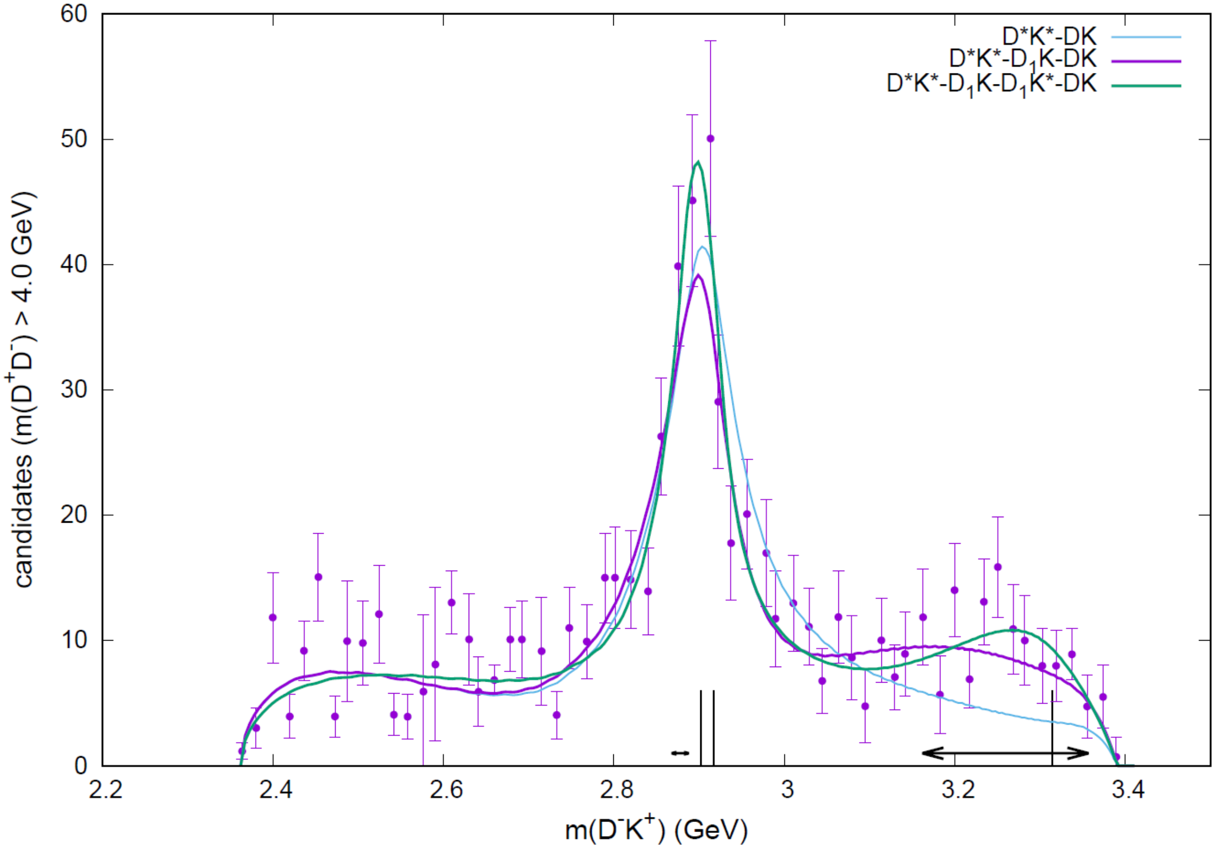


XHL, M.J. Yan, H.W. Ke, G. Li, J.J. Xie, arXiv:2008.07190

Threshold effects and $X_1(2900)$

T.J. Burns, E.S. Swanson, PLB813, 106057(2021)





$$\triangle_{a\alpha}(s_{\bar{D}K}) = \int \frac{d^3q}{(2\pi)^3} F_{\rm ew}(\boldsymbol{q}+\boldsymbol{k}/2)\, F_{\rm 3P0}(3\boldsymbol{k}/4-\boldsymbol{q}/2)\, F_{L_\alpha}(q) Y_{L_\alpha M_\alpha}(\hat{q})\cdot$$
$$[m_B - m_a^\alpha - m_b^\alpha - (\boldsymbol{q}+\boldsymbol{k}/2)^2/(2\mu_{ab}) + i\Gamma_a^\alpha/2 + i\Gamma_b^\alpha/2]^{-1}\cdot$$
$$[m_B - E_D - m_b^\alpha - m_c^\alpha - (\boldsymbol{q}+\boldsymbol{k}/2)^2/(2m_b^\alpha) - (\boldsymbol{q}-\boldsymbol{k}/2)^2/(2m_c^\alpha) + i\Gamma_b^\alpha/2 + i\Gamma_c^\alpha/2]^{-1}.$$

$$F_L(x) = \frac{x^L}{1+x^2}, \quad x = \frac{p}{\beta},$$

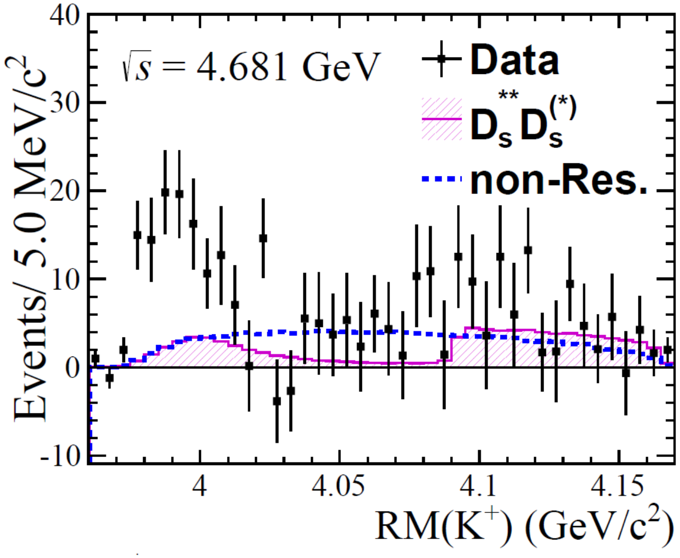
$Z_{cs}(3985)$ and $Z_{cs}(4000)$

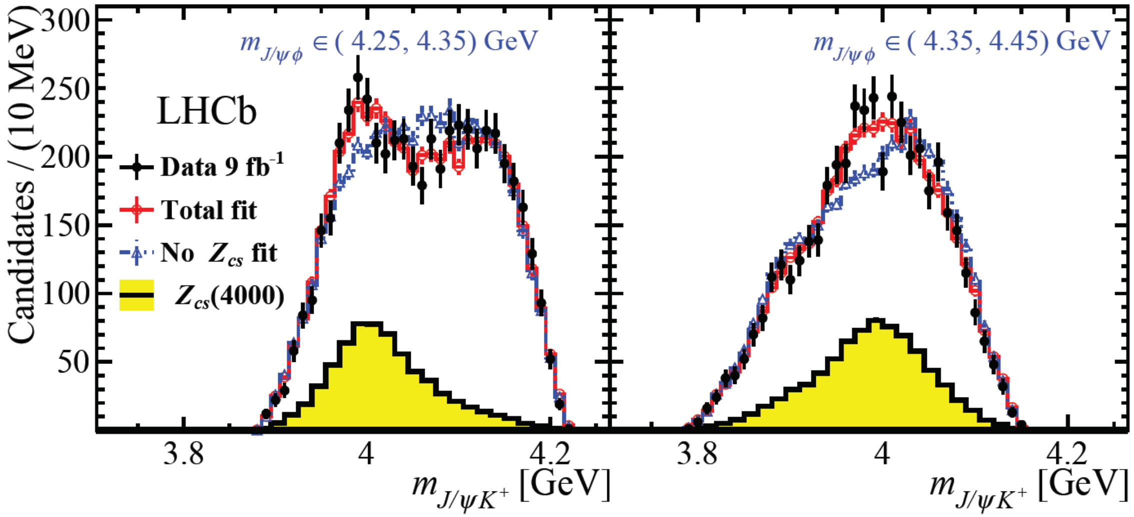
BESIII, 2011.07855

LHCb, 2103.01803

$$e^+e^- \to K^+(D_s^- D^{*0} + D_s^{*-} D^0)$$

$$B^+ \to J/\psi \phi K^+$$





$$m_{\rm pole}(Z_{cs}(3985)^-) = (3982.5^{+1.8}_{-2.6} \pm 2.1)\,{\rm MeV}/c^2,$$

$$\boxed{\Gamma_{\rm pole}(Z_{cs}(3985)^-) = (12.8^{+5.3}_{-4.4} \pm 3.0)\,{\rm MeV}.}$$

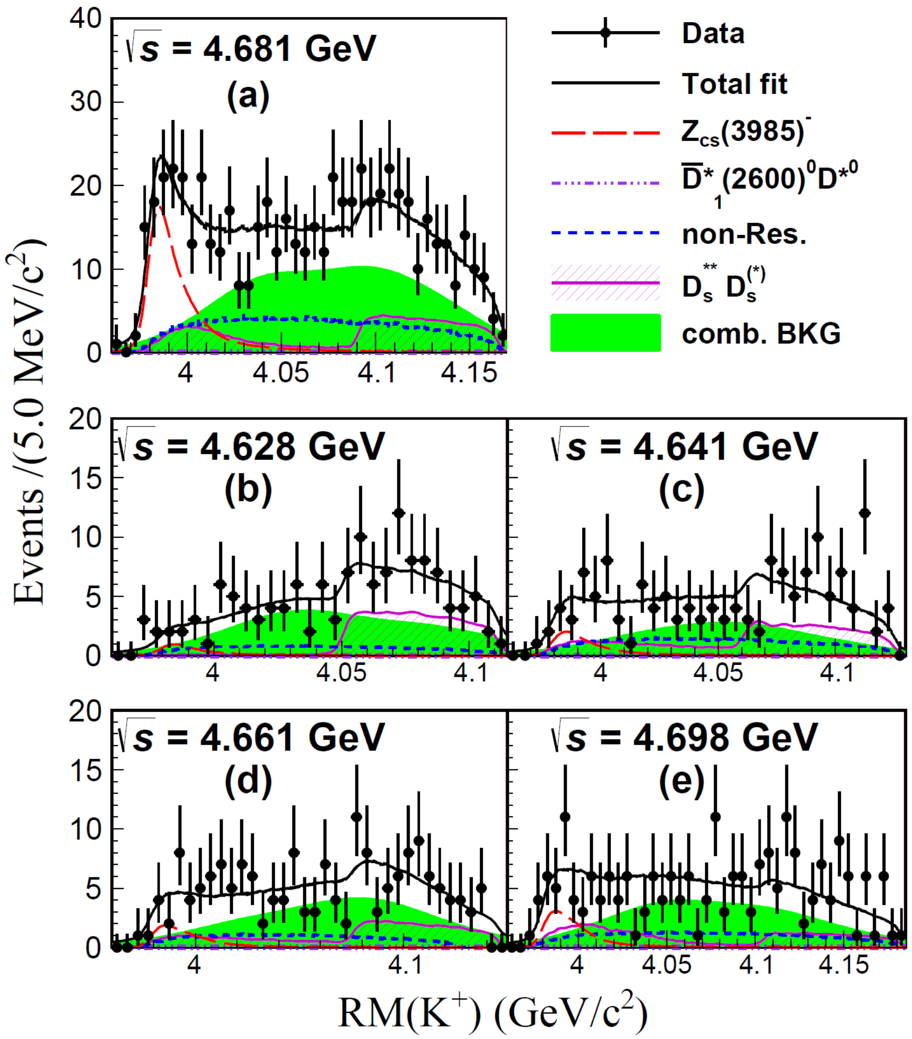
$$Z_{cs}(4000)$$

$$M = 4003 \pm 6^{+4}_{-14}\ {\rm MeV},\ \boxed{\Gamma = 131 \pm 15 \pm 26\ {\rm MeV}}$$

Widths are quite different

Different origin?

$Z_{cs}(3985)$ and $Z_{cs}(4000)$



Theoretical interpretations:

- Molecular partner of Zc(3900), DsD*+Ds*D interactions via exchanging σ, f_0, η, J/ψ …
- Virtual state

Ortega, Entem, Fernandez, 2103.07781

- Compact tetraquark

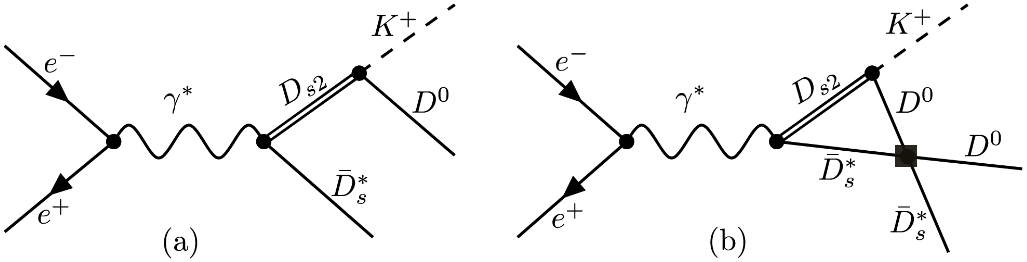
Yang, Cao, Guo, Nieves, Valderrama 2011.08725

- Threshold effects
- Reflection effects

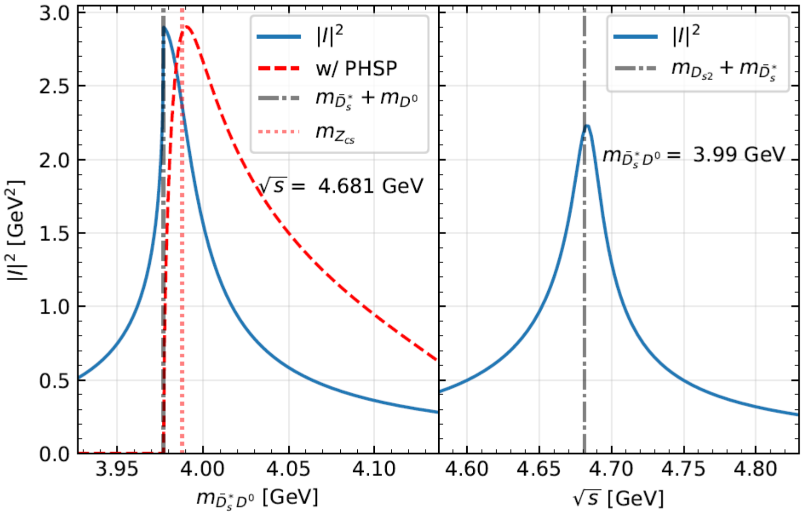
Wang, Zhou, Liu, Matsuki, 2011.08628

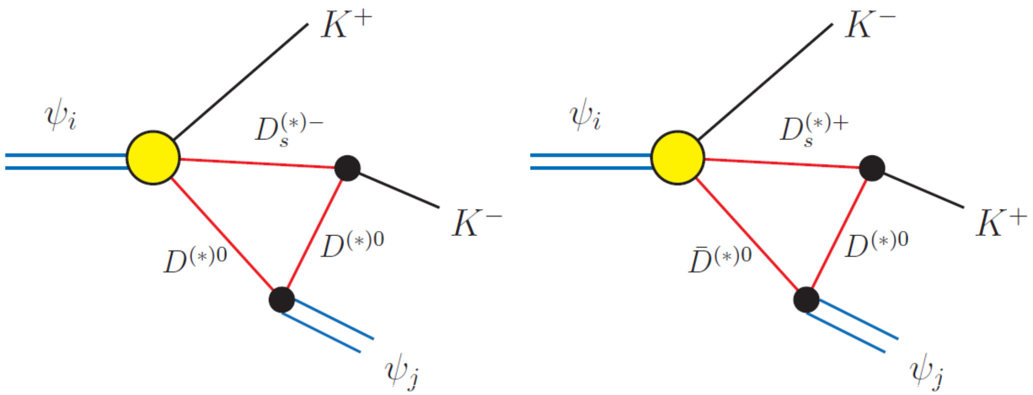
$D_s^{*-} D_{s2}^*(2573)^+$ threshold~ 4681 MeV

$Z_{cs}(3985)$ and $Z_{cs}(4000)$

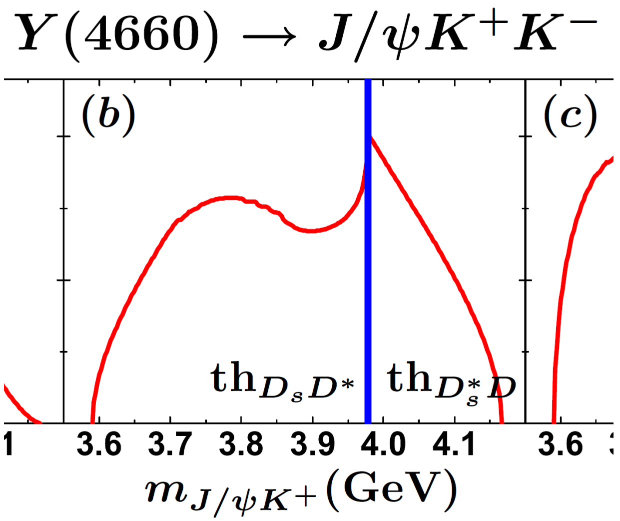


Yang, Cao, Guo, Nieves, Valderrama, 2011.08725

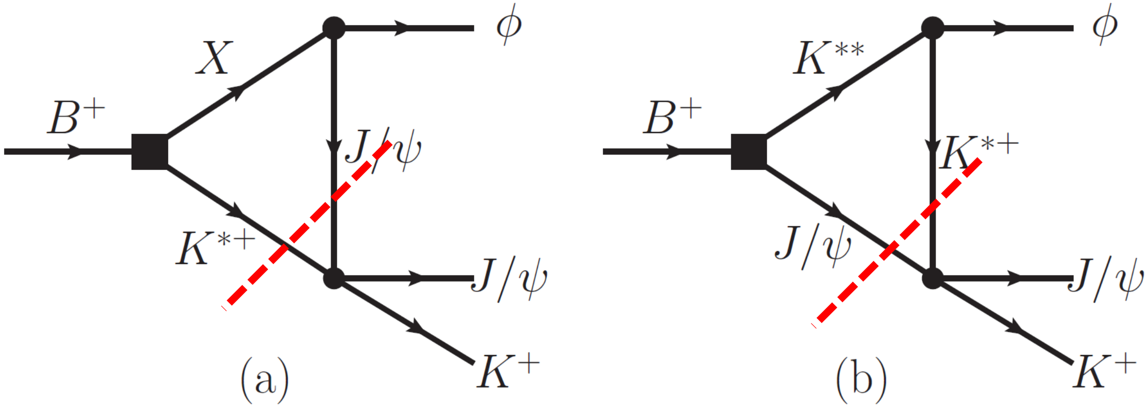




D.Y. Chen, X. Liu, T. Matsuki, PRL110, 232001(2013)



Threshold effects and $Z_{cs}(4000)$



$J/\psi K^*$ threshold~ 3989 MeV

TS kinematic region

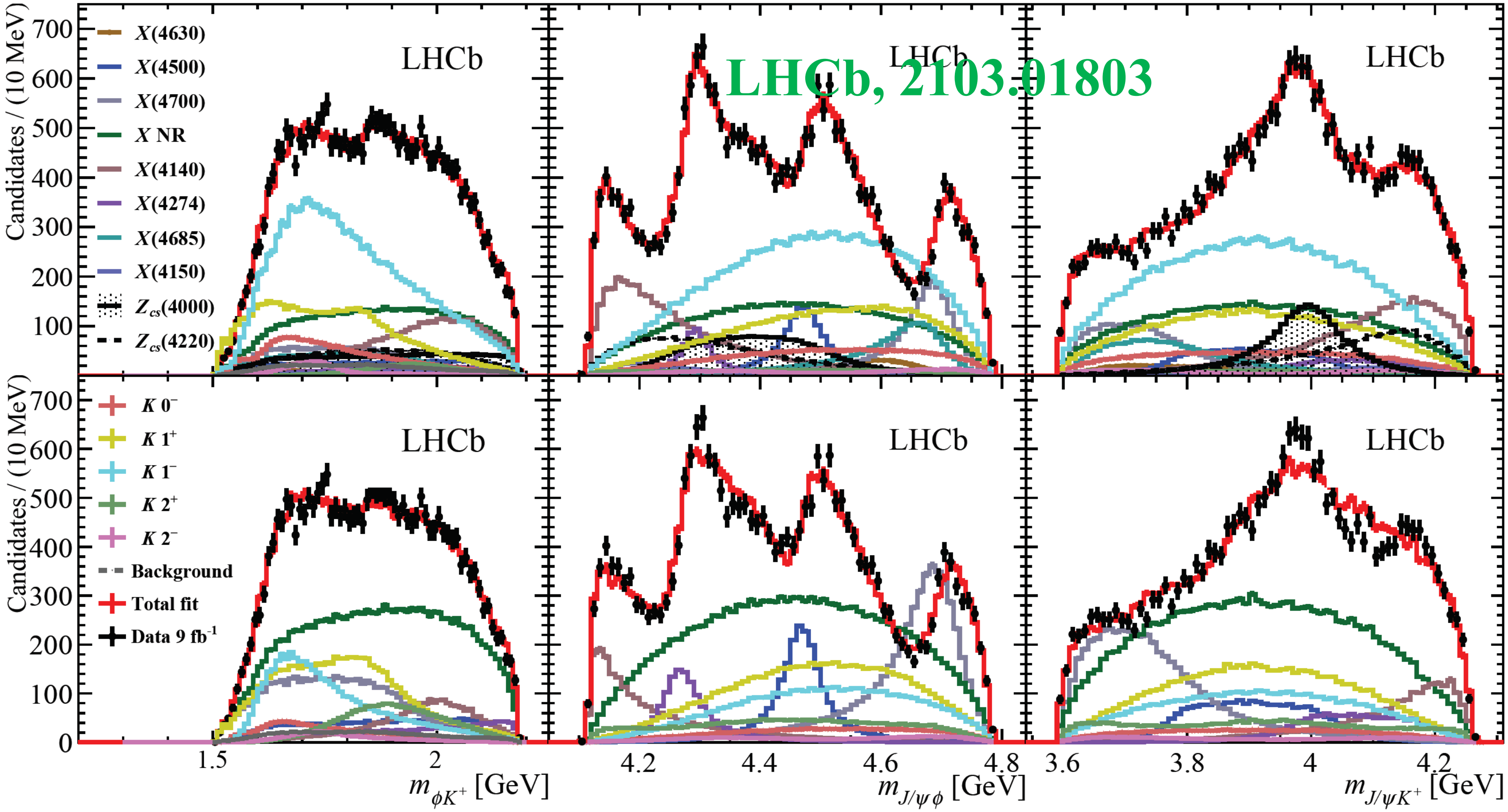
Diagram	$M_X/M_{K^{**}}$	$M_{J/\psi K^+}$
Fig. 1(a)	M_X: 4372~4388	3989~4005
Fig. 1(b)	$M_{K^{**}}$: 2068~2182	3989~4099

Y.H. Ge, XHL, H.W. Ke, arXiv:2103.05282

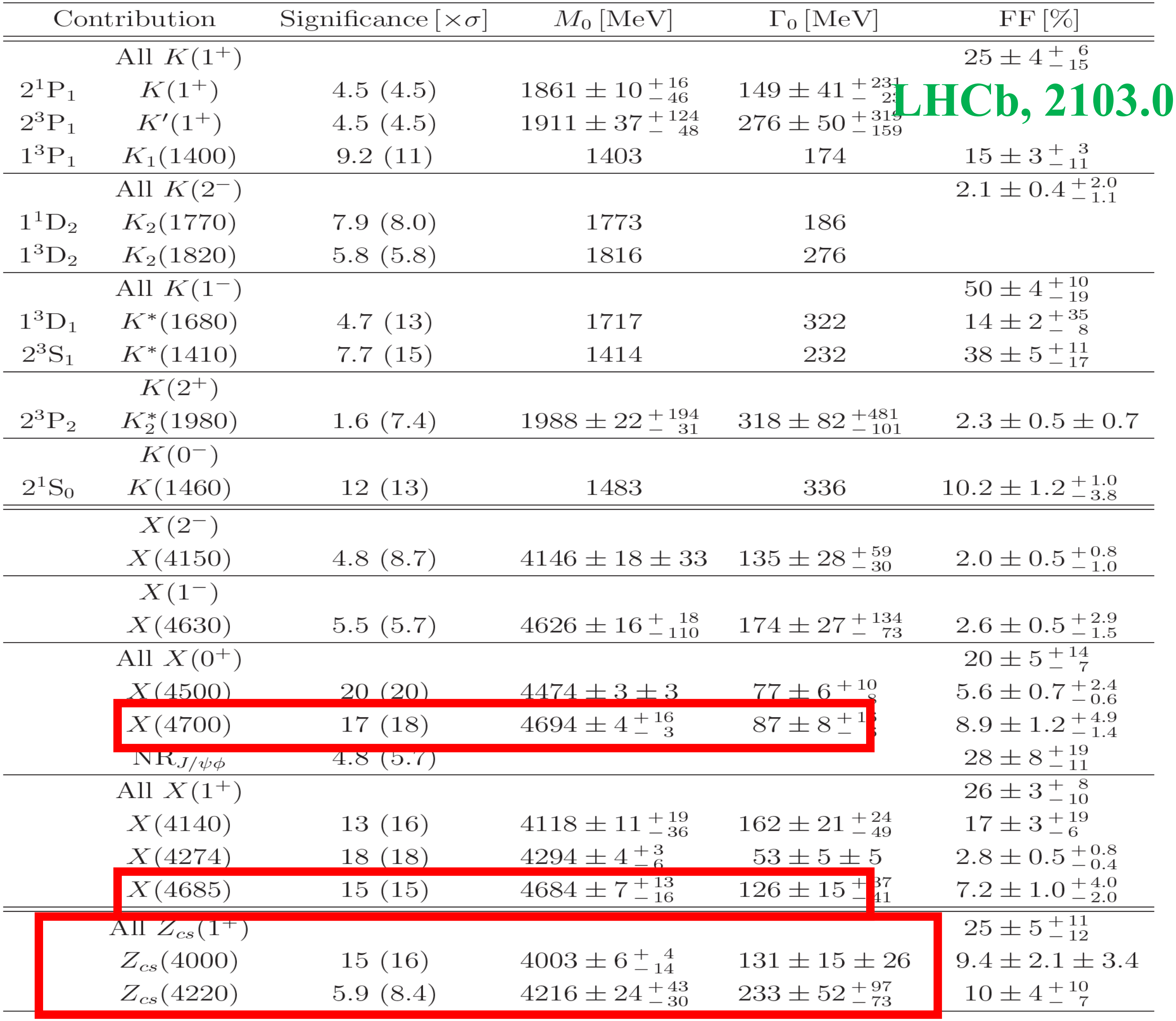
LHCb, 2103.01803

$B^+ \to J/\psi\phi K^+$

K states** **X states** **Zcs states**



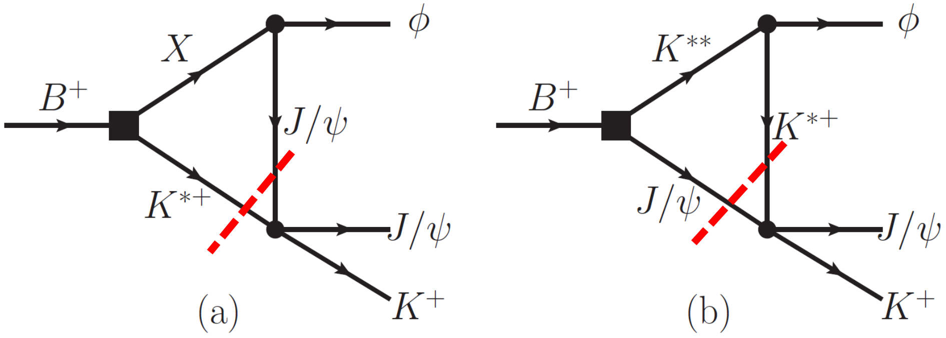
LHCb, 2103.01803



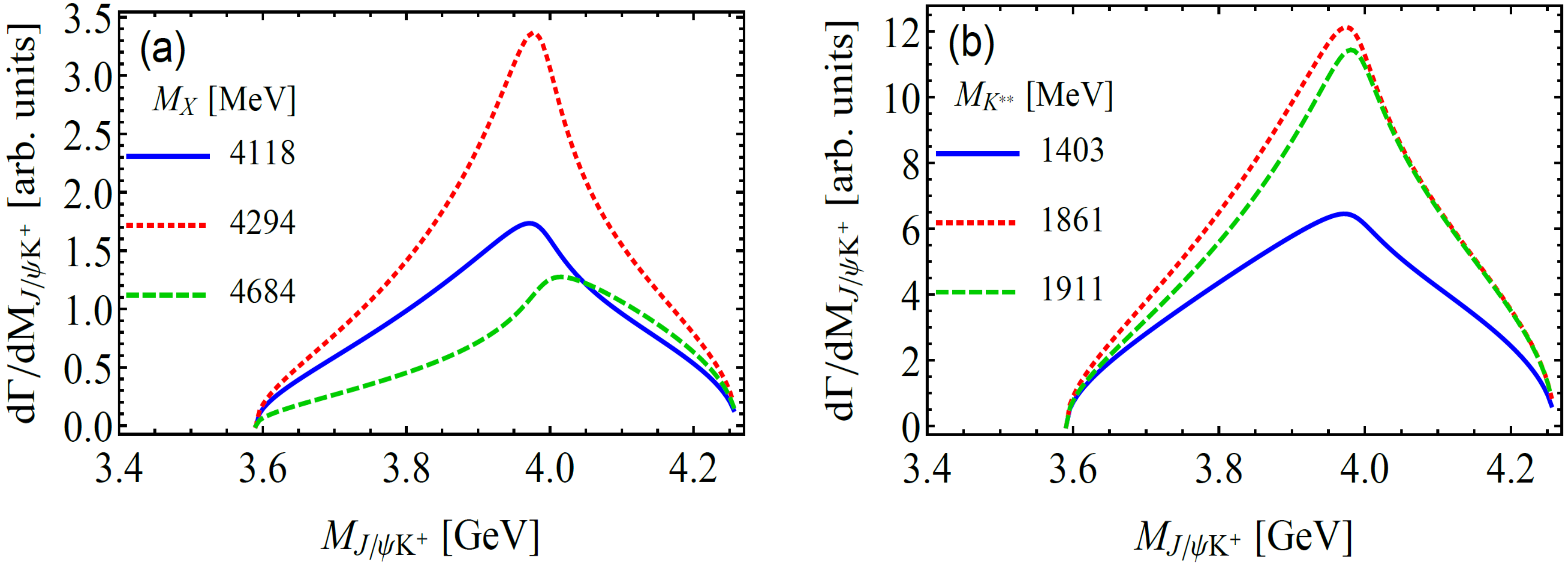
LHCb, 2103.01803

Contribution		Significance [$\times\sigma$]	M_0 [MeV]	Γ_0 [MeV]	FF [%]
	All $K(1^+)$				$25 \pm 4 \,^{+\ 6}_{-\,15}$
2^1P_1	$K(1^+)$	4.5 (4.5)	$1861 \pm 10 \,^{+16}_{-46}$	$149 \pm 41 \,^{+231}_{-\ 23}$	
2^3P_1	$K'(1^+)$	4.5 (4.5)	$1911 \pm 37 \,^{+124}_{-\ 48}$	$276 \pm 50 \,^{+319}_{-159}$	
1^3P_1	$K_1(1400)$	9.2 (11)	1403	174	$15 \pm 3 \,^{+\ 3}_{-\,11}$
	All $K(2^-)$				$2.1 \pm 0.4 \,^{+2.0}_{-1.1}$
1^1D_2	$K_2(1770)$	7.9 (8.0)	1773	186	
1^3D_2	$K_2(1820)$	5.8 (5.8)	1816	276	
	All $K(1^-)$				$50 \pm 4 \,^{+10}_{-19}$
1^3D_1	$K^*(1680)$	4.7 (13)	1717	322	$14 \pm 2 \,^{+35}_{-\ 8}$
2^3S_1	$K^*(1410)$	7.7 (15)	1414	232	$38 \pm 5 \,^{+11}_{-17}$
	$K(2^+)$				
2^3P_2	$K_2^*(1980)$	1.6 (7.4)	$1988 \pm 22 \,^{+194}_{-\ 31}$	$318 \pm 82 \,^{+481}_{-101}$	$2.3 \pm 0.5 \pm 0.7$
	$K(0^-)$				
2^1S_0	$K(1460)$	12 (13)	1483	336	$10.2 \pm 1.2 \,^{+1.0}_{-3.8}$
	$X(2^-)$				
	$X(4150)$	4.8 (8.7)	$4146 \pm 18 \pm 33$	$135 \pm 28 \,^{+59}_{-30}$	$2.0 \pm 0.5 \,^{+0.8}_{-1.0}$
	$X(1^-)$				
	$X(4630)$	5.5 (5.7)	$4626 \pm 16 \,^{+\ 18}_{-110}$	$174 \pm 27 \,^{+134}_{-\ 73}$	$2.6 \pm 0.5 \,^{+2.9}_{-1.5}$
	All $X(0^+)$				$20 \pm 5 \,^{+14}_{-\ 7}$
	$X(4500)$	20 (20)	$4474 \pm 3 \pm 3$	$77 \pm 6 \,^{+10}_{-\ 8}$	$5.6 \pm 0.7 \,^{+2.4}_{-0.6}$
	$X(4700)$	17 (18)	$4694 \pm 4 \,^{+16}_{-\ 3}$	$87 \pm 8 \,^{+1}_{-}$	$8.9 \pm 1.2 \,^{+4.9}_{-1.4}$
	$NR_{J/\psi\phi}$	4.8 (5.7)			$28 \pm 8 \,^{+19}_{-11}$
	All $X(1^+)$				$26 \pm 3 \,^{+\ 8}_{-\,10}$
	$X(4140)$	13 (16)	$4118 \pm 11 \,^{+19}_{-36}$	$162 \pm 21 \,^{+24}_{-49}$	$17 \pm 3 \,^{+19}_{-\ 6}$
	$X(4274)$	18 (18)	$4294 \pm 4 \,^{+3}_{-6}$	$53 \pm 5 \pm 5$	$2.8 \pm 0.5 \,^{+0.8}_{-0.4}$
	$X(4685)$	15 (15)	$4684 \pm 7 \,^{+13}_{-16}$	$126 \pm 15 \,^{+37}_{-41}$	$7.2 \pm 1.0 \,^{+4.0}_{-2.0}$
	All $Z_{cs}(1^+)$				$25 \pm 5 \,^{+11}_{-12}$
	$Z_{cs}(4000)$	15 (16)	$4003 \pm 6 \,^{+\ 4}_{-\,14}$	$131 \pm 15 \pm 26$	$9.4 \pm 2.1 \pm 3.4$
	$Z_{cs}(4220)$	5.9 (8.4)	$4216 \pm 24 \,^{+43}_{-30}$	$233 \pm 52 \,^{+97}_{-73}$	$10 \pm 4 \,^{+10}_{-\ 7}$

Threshold effects and $Z_{cs}(4000)$

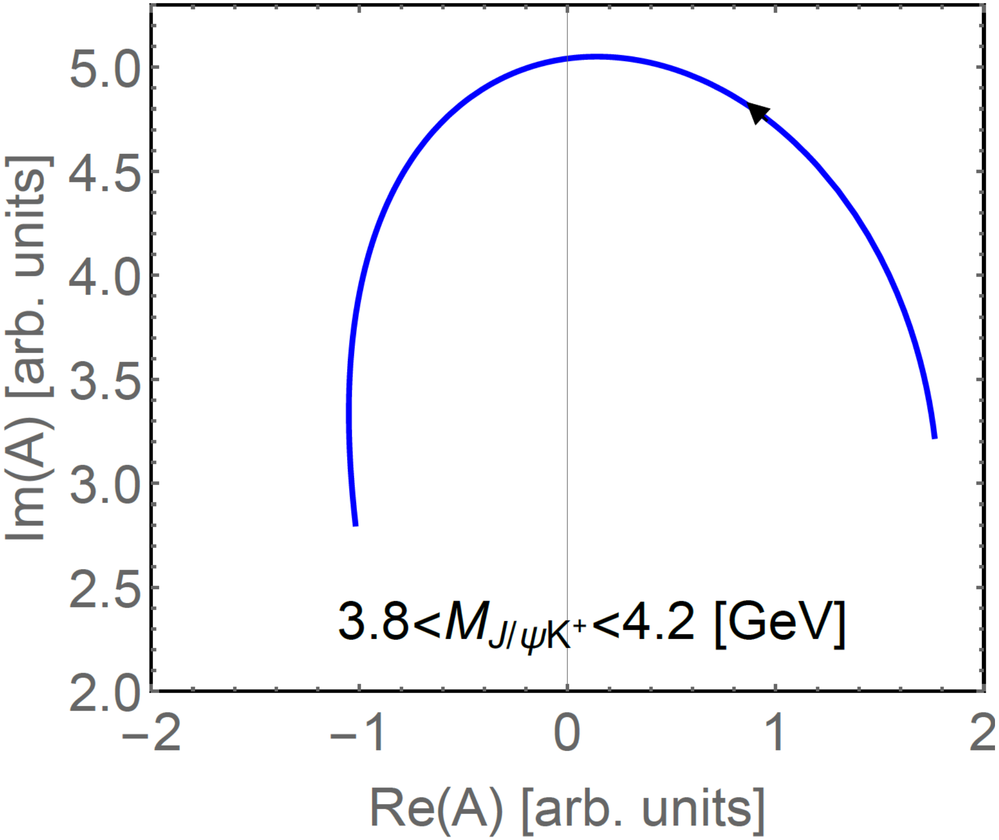


$$\mathcal{A}^{[K^{**}\psi K^*]}_{B^+\to J/\psi\phi K^+} = -i\int \frac{d^4q_1}{(2\pi)^4}\frac{\mathcal{A}(B^+\to J/\psi K^{**})}{(q_1^2-M_{J/\psi}^2)} \times \frac{\mathcal{A}(K^{**}\to \phi K^{*+})\mathcal{A}(J/\psi K^{*+}\to J/\psi K^+)}{(q_2^2-M_{K^{**}}^2+iM_{K^{**}}\Gamma_{K^{**}})(q_3^2-M_{K^*}^2+iM_{K^*}\Gamma_{K^*})}$$



Y.H. Ge, XHL, H.W. Ke, arXiv:2103.05282

Threshold effects and $Z_{cs}(4000)$

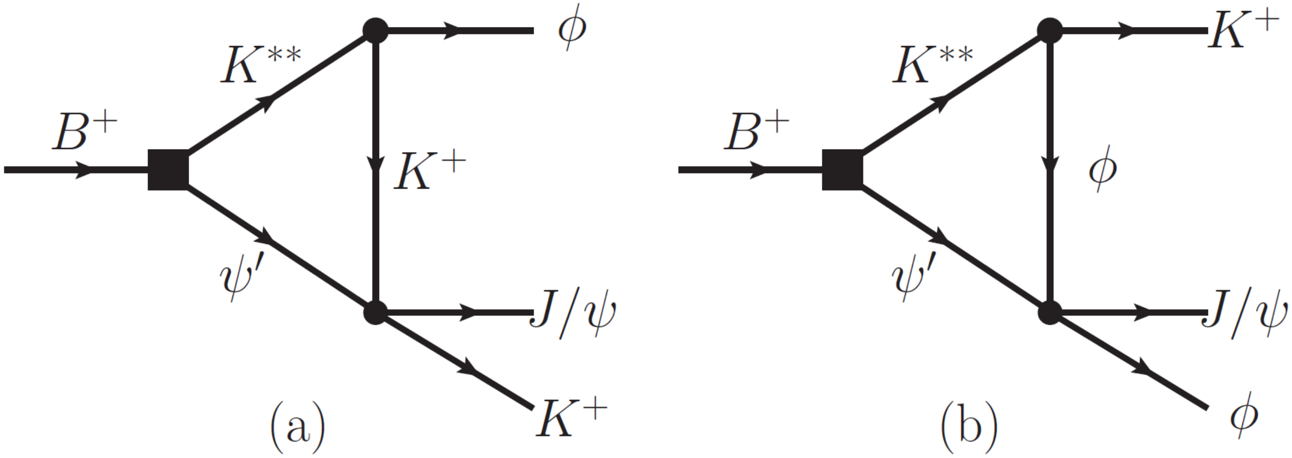


Argand plot

$$\mathcal{A}^{[K^{**}\psi K^{*}]}_{B^+\to J/\psi\phi K^+} = -i\int \frac{d^4q_1}{(2\pi)^4}\frac{\mathcal{A}(B^+\to J/\psi K^{**})}{(q_1^2-M_{J/\psi}^2)}$$

$$\times\frac{\mathcal{A}(K^{**}\to\phi K^{*+})\mathcal{A}(J/\psi K^{*+}\to J/\psi K^+)}{(q_2^2-M_{K^{**}}^2+iM_{K^{**}}\Gamma_{K^{**}})(q_3^2-M_{K^*}^2+iM_{K^*}\Gamma_{K^*})}$$

Threshold effects and Z_{cs}(4220), X(4700)



X.H. Liu, PLB766, 117 (2017)

ψ(2S)K threshold~ 4180 MeV

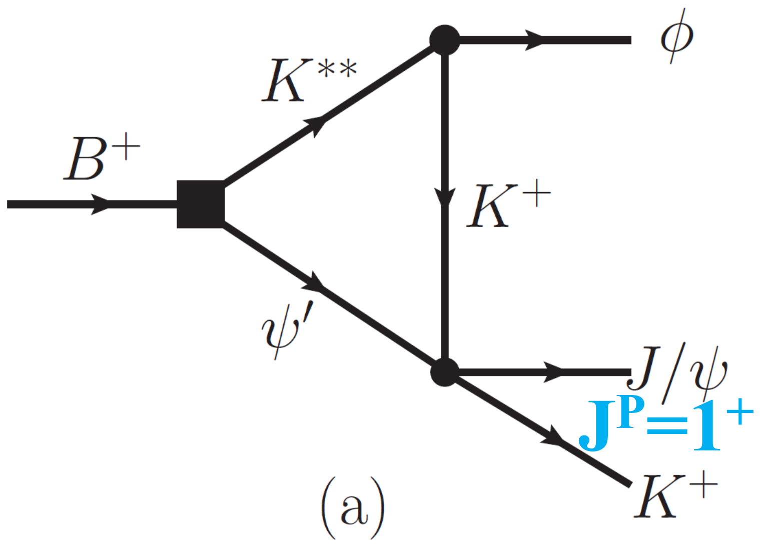
ψ(2S)ϕ threshold~ 4706 MeV

TS kinematic region

Diagram	$M_{K^{**}}$	$M_{J/\psi K^+}/M_{J/\psi\phi}$
Fig. 4(a)	1546∼1593	$M_{J/\psi K^+}$: 4180∼4226
Fig. 4(b)	1572∼1593	$M_{J/\psi\phi}$: 4706∼4727

Y.H. Ge, XHL, H.W. Ke, arXiv:2103.05282

Threshold effects and Z_{cs}(4220), X(4700)



$J^P=1^+$

$$\mathcal{A}(B^+ \to \psi' K^{**}) \;=\; a'\, \epsilon^*(\psi') \cdot \epsilon^*(K^{**})$$

$$K^{**}(1^+) \to \phi K^+$$

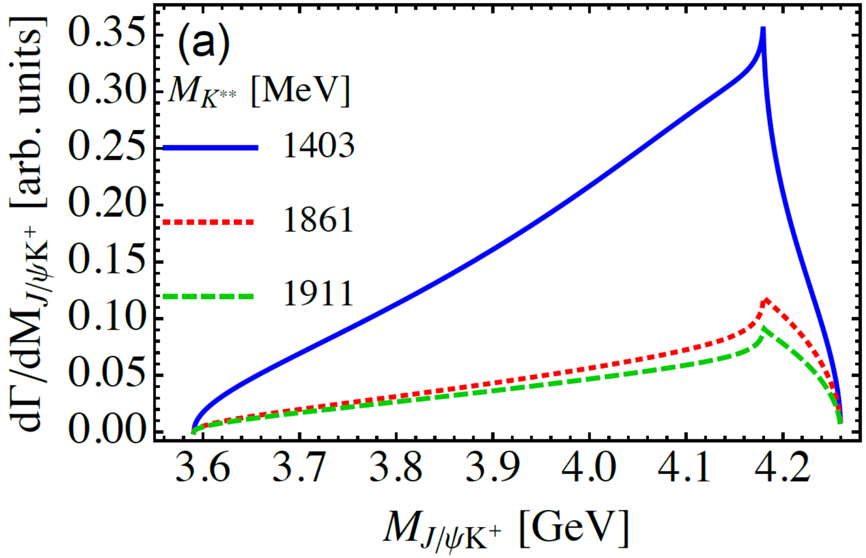
$$\mathcal{A}(K^{**} \to \phi K^+) = g_A \epsilon(K^{**}) \cdot \epsilon^*(\phi)$$

$$K^{**}(1^-) \to \phi K^+$$

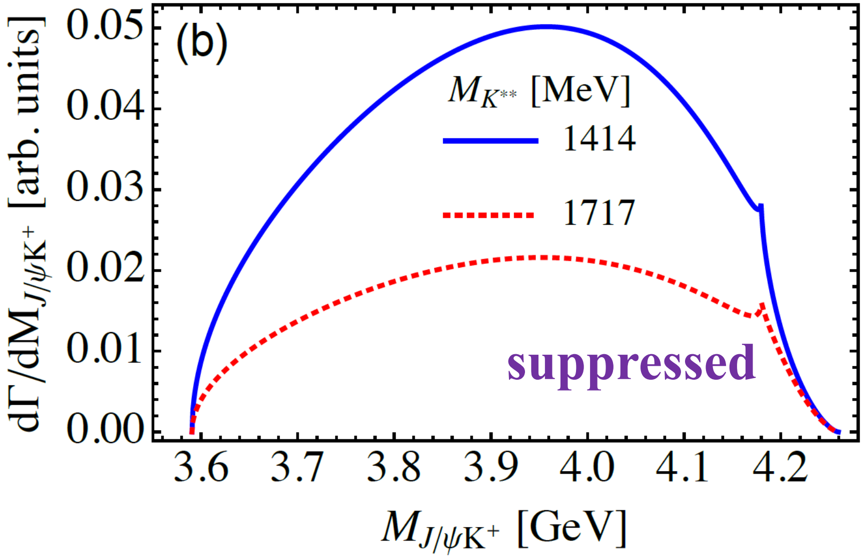
$$\mathcal{A}(K^{**} \to \phi K^+) = g_V \varepsilon_{\mu\nu\alpha\beta} p^{\mu}_{K^{**}} p^{\nu}_{\phi} \epsilon^{\alpha}(K^{**}) \epsilon^{*\beta}(\phi)$$

$$\mathcal{A}(\psi' K^+ \to J/\psi K^+) = g_{\psi' K}\; \epsilon(\psi') \cdot \epsilon(J/\psi)$$

$K^{**}(J^P=1^+)$



$K^{**}(J^P=1^-)$



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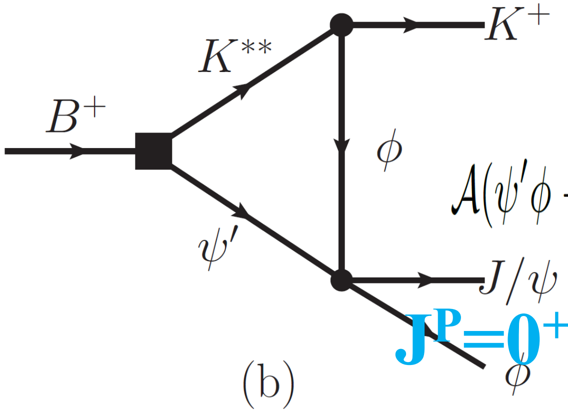
Y.H. Ge, XHL, H.W. Ke, arXiv:2103.05282

Threshold effects and $Z_{cs}(4220)$, X(4700)



$J^P=0^+$, 1^+

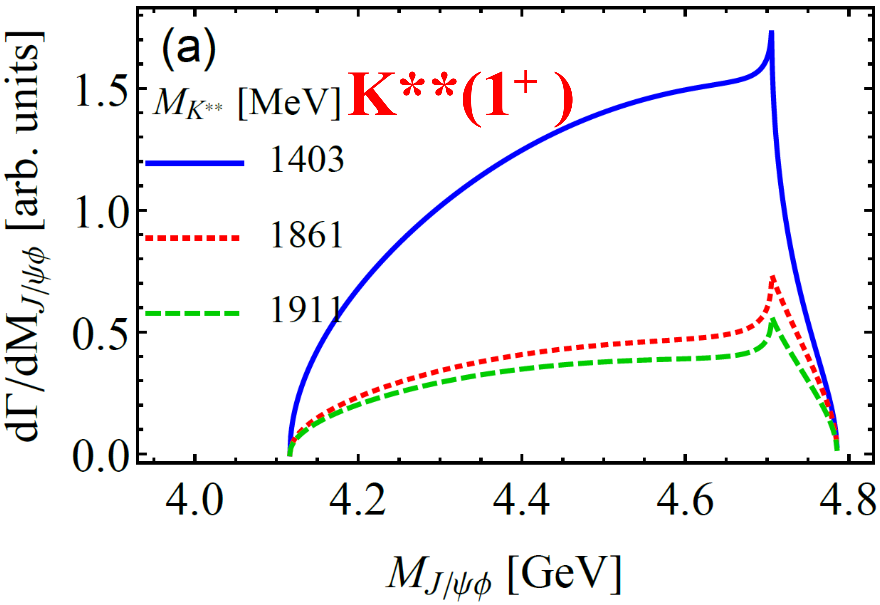
$J/\psi\phi$ ($J^P=0^+$)

$$\mathcal{A}(\psi'\phi \to J/\psi\phi) = g_{\psi\phi}\,\epsilon(\psi')\cdot\epsilon(\phi)\,\epsilon^*(J/\psi)\cdot\epsilon^*(\phi)$$

$$\mathcal{A}(\psi'\phi \to J/\psi\phi) = \tilde{g}_{\psi\phi}\,\varepsilon_{\mu\nu\alpha\beta}\varepsilon_{\gamma\delta\lambda\rho}(p^{\mu}_{J/\psi}+p^{\mu}_{\phi})(p^{\gamma}_{J/\psi}+p^{\gamma}_{\phi})g^{\nu\delta}\epsilon^{\alpha}(\psi')\epsilon^{\beta}(\phi)\epsilon^{*\lambda}(J/\psi)\epsilon^{*\rho}(\phi)$$

$J/\psi\phi$ ($J^P=1^+$)

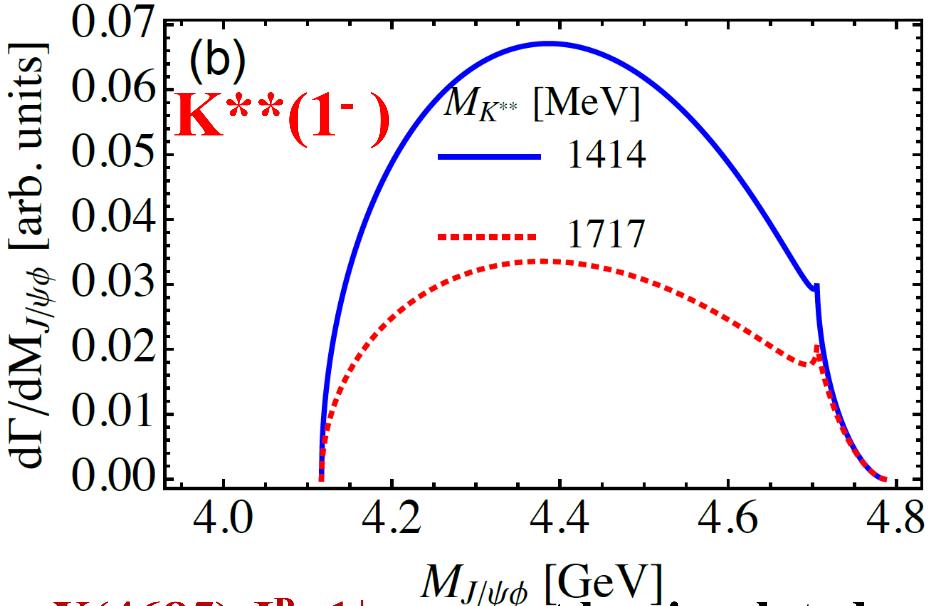
$J/\psi\phi$ ($J^P=0^+$)



$K^{**}(1^+)$

X(4700) $J^P=0^+$

$J/\psi\phi$ ($J^P=1^+$)



$K^{**}(1^-)$

X(4685) $J^P=1^+$, cannot be simulated by the threshold effect

Summary

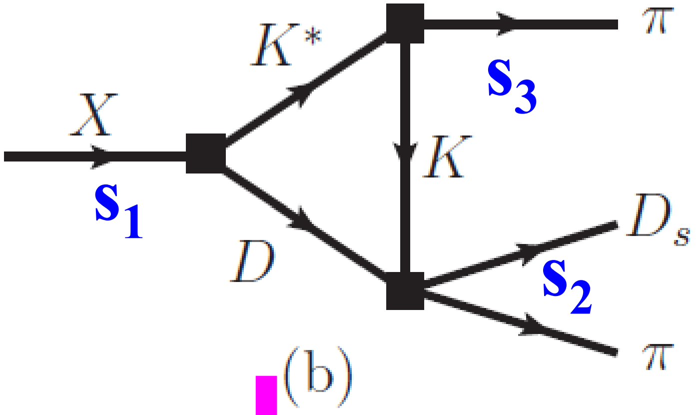
- Kinematic singularities can simulate resonance-like peaks in the invariant mass distribution, which implies that non-resonance interpretation for some exotic hadron candidates ($X_{0,1}(2900)$, $Z_{cs}(3985/4000)$, $Z_{cs}(4220)$, X(4700) in this talk) is possible.
- X(4685) ($J^P=1^+$) could be a genuine resonance, since the threshold effect cannot simulate it well.
- Being different from the genuine resonances, the TS mechanism is a process-dependent mechanism, and sensitive to the kinematic configurations.

  Model independent but Process dependent.
- Study on threshold effects is necessary before claiming that a resonance-like structure is a genuine particle.

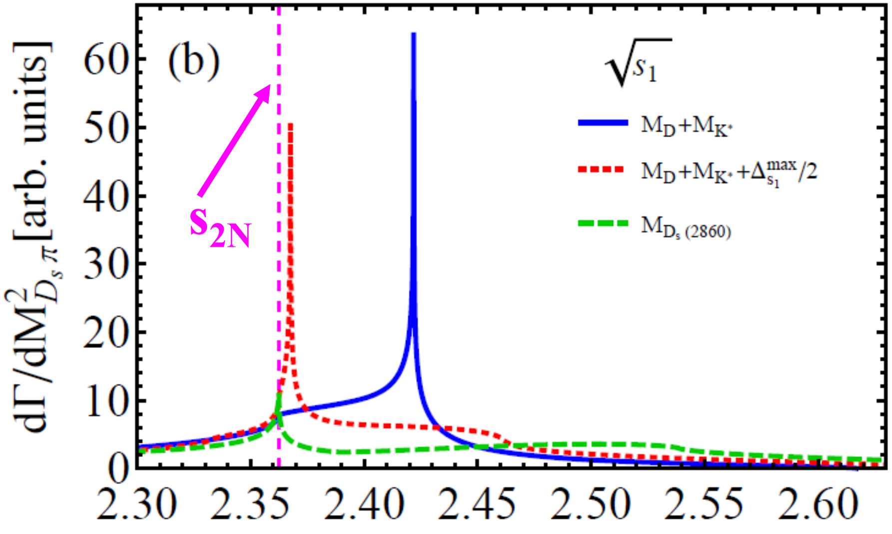
Thanks!

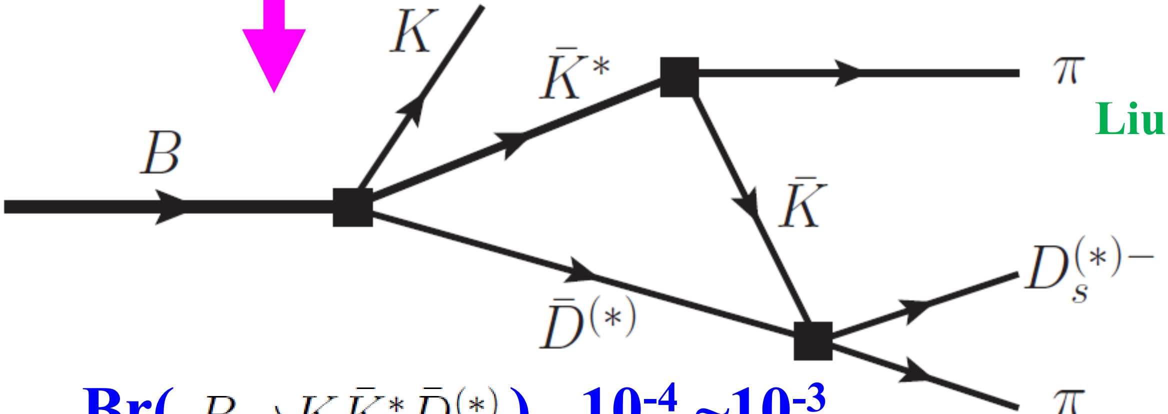
Distinguish Kinematic Singularities **from** Dynamic Poles

Criterion: Movement of TS peak



Movement of TS peak





Liu, Oka, Zhao, PLB753, 297(2016)

Br($B\to K\bar{K}^*\bar{D}^{(*)}$ **) $10^{-4}\sim10^{-3}$**

Br($B\to KD_s^{(*)-}\pi\pi$ **) $\sim10^{-4}$**

[MeV]	Fig. 3(a	Fig. 3(b)	Fig. 3(c)	Fig. 3(d)
$\Delta_{s_1}^{\max}$	0.089	96	49	16
$\Delta_{s_2}^{\max}$	0.087	62	38	15