第七届XYZ粒子研讨会-山东大学

对全重味五夸克态的研究

LANZHOU UNIVERSITY

安洪涛

兰州大学物理科学与技术学院

H. T. An, K. Chen, Z. W. Liu and X. Liu, "Fully heavy pentaquarks," Phys. Rev. D **103** (2021) no.7, 074006.

青岛 2021. 5. 17

Outline

- Background
- The effective Hamiltonian
- Wave functions of pentaquarks
- Mass spectra and decay behaviors
- Summary

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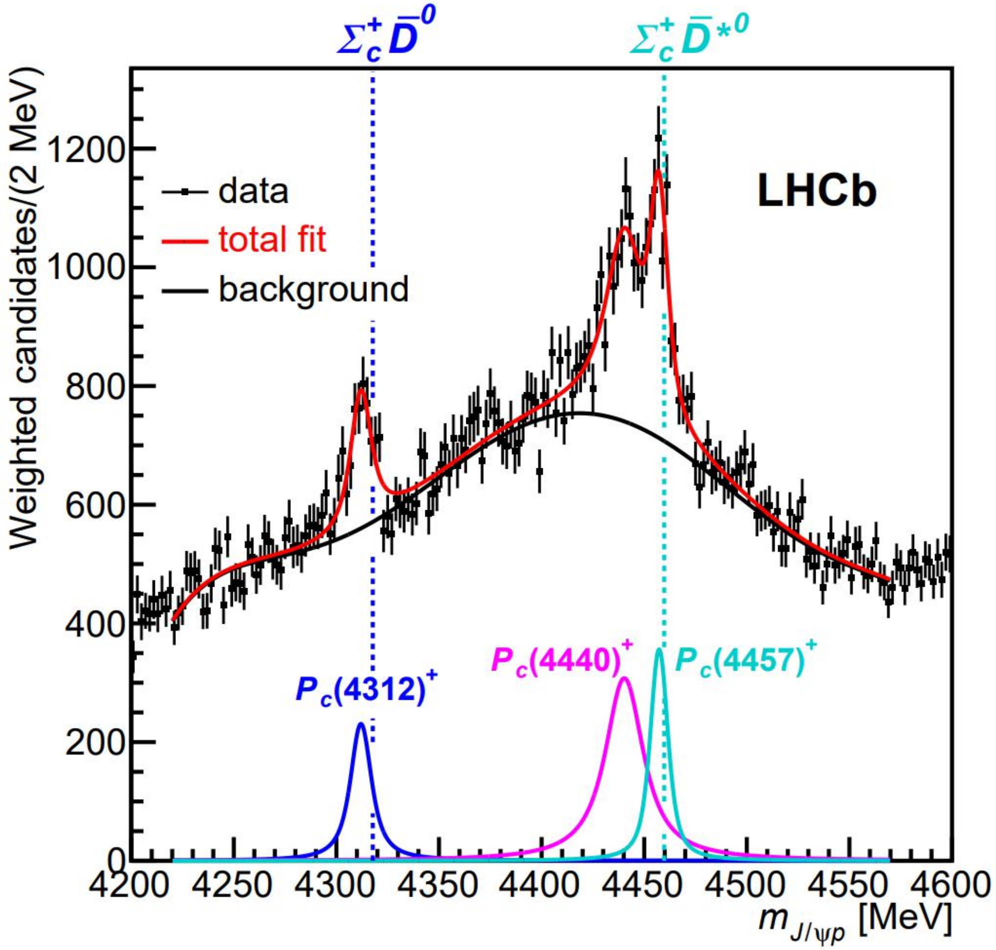
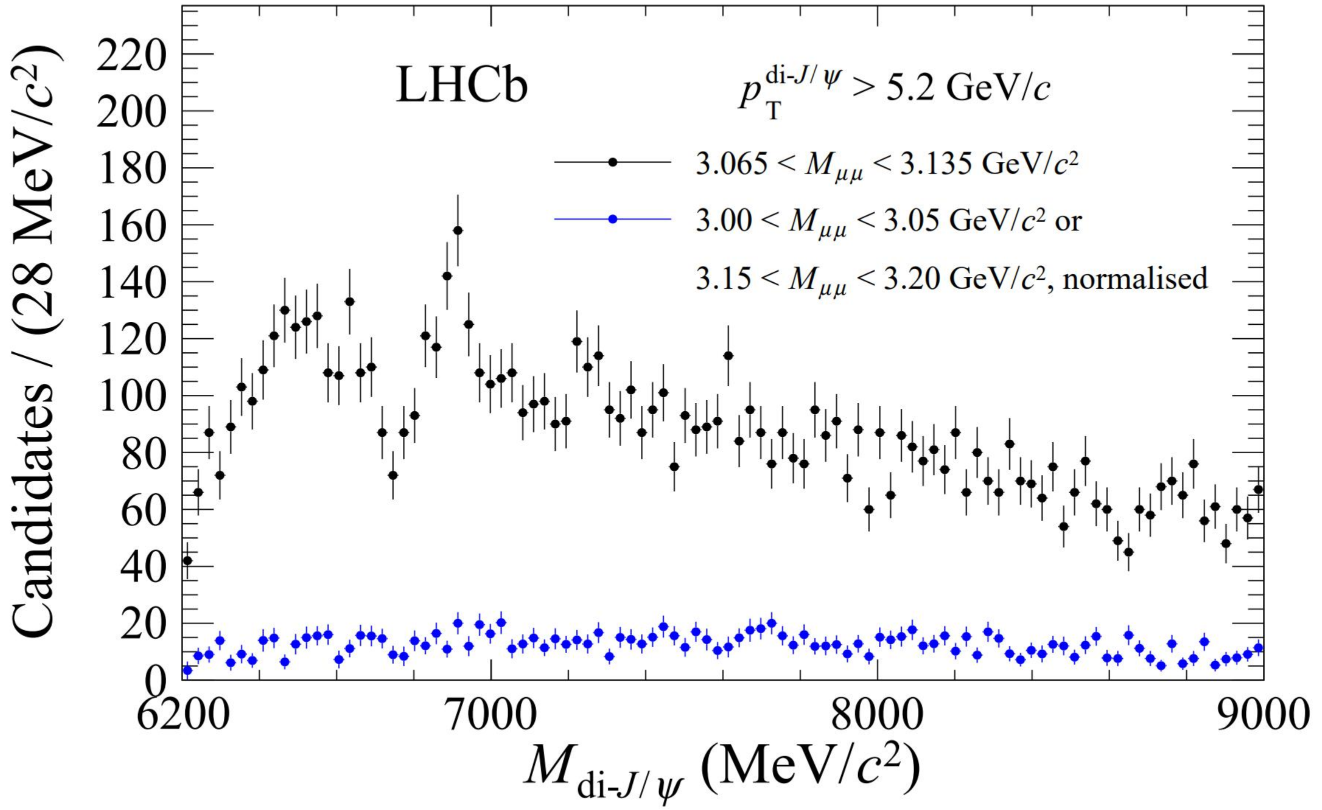
Background



R. Aaij *et al.* [LHCb Collaboration], "Observation of a narrow pentaquark state, $P_c(4312)^+$, and of two-peak structure of the $P_c(4450)^+$," Phys. Rev. Lett. **122**, no. 22, 222001 (2019).



R. Aaij *et al.* [LHCb], "Observation of structure in the J/ψ -pair mass spectrum," Sci. Bull. **65** (2020) no.23, 1983-1993.

The discoveries of fully heavy tetraquark states and P_c states make one speculate that the pentaquark state with fully heavy quarks $QQQQ\bar{Q}$ may also exist.

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- **The effective Hamiltonian**
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The effective Hamiltonian at quark level

$$H = \sum_i m_i + H_{\rm CMI} = \sum_i m_i - \sum_{i<j} C_{ij} \vec{\lambda}_i \cdot \vec{\lambda}_j \vec{\sigma}_i \cdot \vec{\sigma}_j ,$$

The effective quark masses are not enough to absorb all the two-body chromoelectric effects.

Hadron	CMI	Th.	Ex.	(Th.-Ex.)	Hadron	CMI	Th.	Ex.	(Th.-Ex.)
π	$-16C_{n\bar{n}}$	246.6	139.6	107	ρ	$\frac{16}{3}C_{n\bar{n}}$	882.3	775.3	107
K	$-16C_{n\bar{s}}$	602.8	493.7	109	K^*	$\frac{16}{3}C_{n\bar{s}}$	1001.7	891.8	110
					ω	$\frac{16}{3}C_{n\bar{n}}$	882.3	782.7	100
					ϕ	$\frac{16}{3}C_{s\bar{s}}$	1136.7	1019.5	117
D	$-16C_{c\bar{n}}$	1980.7	1869.7	111	D^*	$\frac{16}{3}C_{c\bar{n}}$	2121.5	2010.3	111
D_s	$-16C_{c\bar{s}}$	2157.7	1968.3	189	D_s^*	$\frac{16}{3}C_{c\bar{s}}$	2300.6	2112.2	188
B	$-16C_{b\bar{n}}$	5380.9	5279.5	102	B^*	$\frac{16}{3}C_{b\bar{n}}$	5425.7	5324.7	101
B_s	$-16C_{b\bar{s}}$	5556.3	5366.9	189	B_s^*	$\frac{16}{3}C_{b\bar{s}}$	5605.4	5415.4	190
η_c	$-16C_{c\bar{c}}$	3364.4	2983.9	381	J/ψ	$\frac{16}{3}C_{c\bar{c}}$	3477.5	3096.9	381
η_b	$-16C_{b\bar{b}}$	10059.2	9399.0	660	Υ	$\frac{16}{3}C_{b\bar{b}}$	10121.1	9460.3	661

[1] Y. R. Liu, H. X. Chen, W. Chen, X. Liu and S. L. Zhu, "Pentaquark and Tetraquark states," Prog. Part. Nucl. Phys. **107** (2019), 237-320 [arXiv:1903.11976].

The effective Hamiltonian at quark level

(1) $$M = (M_{ref} - \langle H_{\rm CMI}\rangle_{ref}) + \langle H_{\rm CMI}\rangle.$$

(2) $$H = \sum_i m_i^0 - \sum_{i<j} A_{ij}\vec{\lambda}_i \cdot \vec{\lambda}_j - \sum_{i<j} v_{ij}\vec{\lambda}_i \cdot \vec{\lambda}_j \vec{\sigma}_i \cdot \vec{\sigma}_j$$

$$= -\frac{3}{4}\sum_{i<j} m_{ij} V_{ij}^{\rm C} - \sum_{i<j} v_{ij} V_{ij}^{\rm CMI}.$$

$$V_{ij}^{\rm C} = \vec{\lambda}_i \cdot \vec{\lambda}_j \quad V_{ij}^{\rm CMI} = \vec{\lambda}_i \cdot \vec{\lambda}_j \vec{\sigma}_i \cdot \vec{\sigma}_j$$

(3) $$H = \sum_{i=1}^{5}\left(m_i + \frac{\mathbf{p}_i^2}{2m_i}\right) - \frac{3}{16}\sum_{i<j}^{4} \lambda_i^c \lambda_j^c (V_{ij}^C + V_{ij}^{SS}),$$

$$V_{ij}^C = -\frac{\kappa}{r_{ij}} + \frac{r_{ij}}{a_0^2} - D,$$

$$V_{ij}^{SS} = \frac{\hbar^2 c^2 \kappa}{m_i m_j c^4} \frac{1}{r_0^2 r_{ij}} e^{-r_{ij}/r_0} \sigma_i \cdot \sigma_j.$$

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The flavor ⊗ color ⊗ spin wave functions of pentaquarks

We divide the $QQQQ\bar{Q}$ pentaquark system into the following three groups:

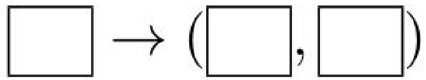
A. The $cccc\bar{Q}$ and $bbbb\bar{Q}$ pentaquark subsystems;

B. The $cccb\bar{Q}$ and $bbbc\bar{Q}$ pentaquark subsystems;

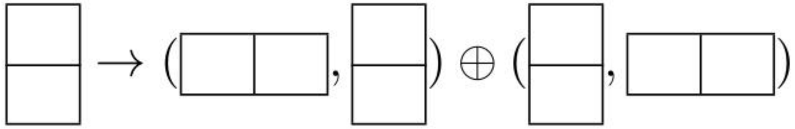
C. The $ccbb\bar{Q}$ pentaquark subsystem.

The flavor ⊗ color ⊗ spin wave functions of pentaquarks

Young tableau, which represents the irreducible bases of the permutation group, enable us to easily identify the multi-quark configuration with certain symmetry property [1].

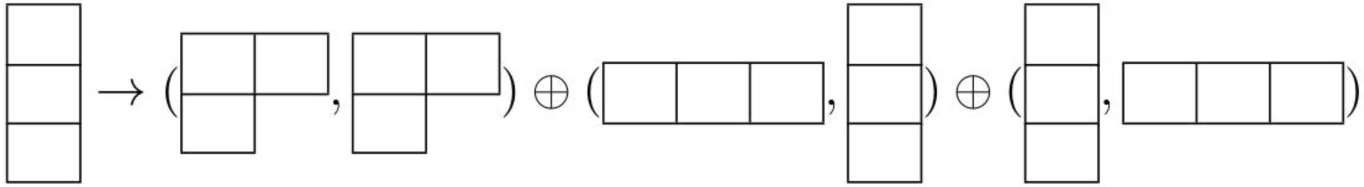


$(mn) \rightarrow (m, n)$



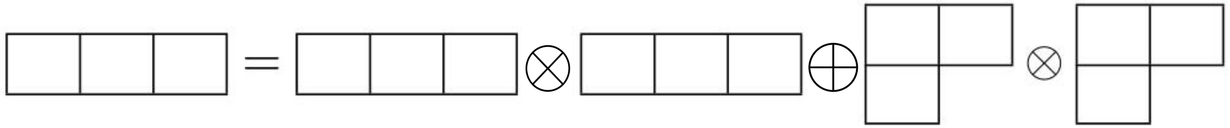
$[\frac{1}{2}(mn)(mn-1)] \rightarrow [\frac{1}{2}m(m+1), \frac{1}{2}n(n-1)] + [\frac{1}{2}m(m-1), \frac{1}{2}n(n+1)]$

For the $SU(12)_{CSI} \supset SU(3)_C \otimes SU(4)_{IS}$:

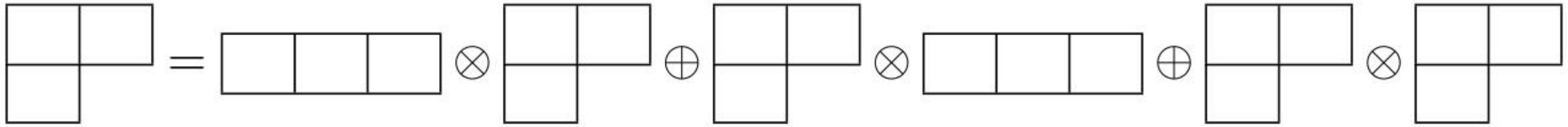


$$220_A = (8, 20) \oplus (10, 4) \oplus (1, 20)$$

For the $SU(4)_{IS} \supset SU(2)_I \otimes SU(2)_S$:



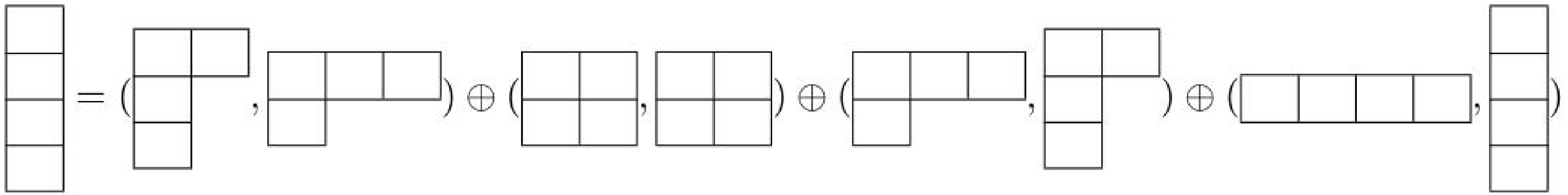
$$20 = (4, 4) \oplus (2, 2)$$



$$20 = (4, 2) \oplus (2, 4) \oplus (2, 2)$$

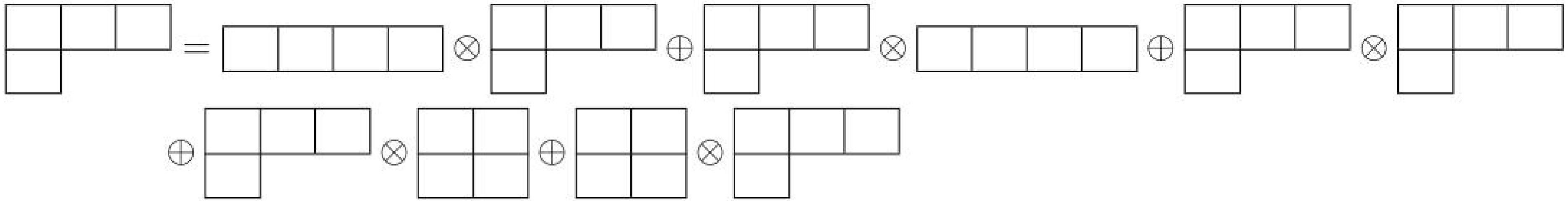
The flavor ⊗ color ⊗ spin wave functions of pentaquarks

For the $SU(12)_{CSI} \supset SU(3)_C \otimes SU(4)_{IS}$:



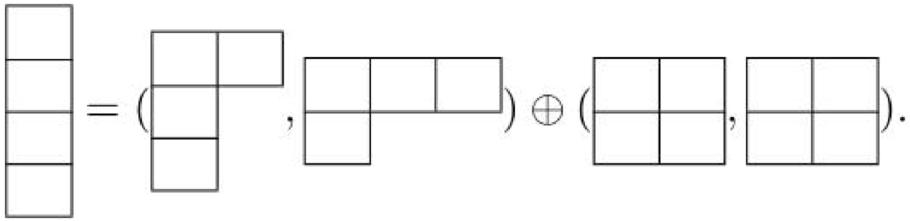
$$495_A = (3, 45) \oplus (6, 20) \oplus (15, 15) \oplus (15, 1)$$

For the $SU(4)_{IS} \supset SU(2)_I \otimes SU(2)_S$:



$$45 = (5, 3) \oplus (3, 5) \oplus (3, 3) \oplus (3, 1) \oplus (1, 3)$$

For the $SU(6)_{CS} \supset SU(3)_C \otimes SU(2)_S$:



$$15_A = (3, 3) \oplus (6, 1)$$

The flavor ⊗ color ⊗ spin wave functions of pentaquarks

Color Wave Functions:

$$[(12)_6\,34]_3 = \begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 & \multicolumn{1}{c}{} \\ \cline{1-1} 4 & \multicolumn{1}{c}{} \\ \cline{1-1}\end{array}_{\,C_1} \qquad [(12)_{\bar{3}}\,34]_3 = \begin{array}{|c|c|}\hline 1 & 3 \\ \hline 2 & \multicolumn{1}{c}{} \\ \cline{1-1} 4 & \multicolumn{1}{c}{} \\ \cline{1-1}\end{array}_{\,C_2} \qquad [(123)_1\,4]_3 = \begin{array}{|c|c|}\hline 1 & 4 \\ \hline 2 & \multicolumn{1}{c}{} \\ \cline{1-1} 3 & \multicolumn{1}{c}{} \\ \cline{1-1}\end{array}_{\,C_3}$$

Spin Wave Functions:

$$S = \frac{5}{2}: \quad \begin{array}{|c|c|c|c|}\hline 1 & 2 & 3 & 4 \\ \hline\end{array}_{\,S_1},$$

$$S = \frac{3}{2}: \quad \begin{array}{|c|c|c|c|}\hline 1 & 2 & 3 & 4 \\ \hline\end{array}_{\,S_1}, \begin{array}{|c|c|c|}\hline 1 & 2 & 3 \\ \hline 4 & \multicolumn{2}{c}{} \\ \cline{1-1}\end{array}_{\,S_2}, \begin{array}{|c|c|c|}\hline 1 & 2 & 4 \\ \hline 3 & \multicolumn{2}{c}{} \\ \cline{1-1}\end{array}_{\,S_3}, \begin{array}{|c|c|c|}\hline 1 & 3 & 4 \\ \hline 2 & \multicolumn{2}{c}{} \\ \cline{1-1}\end{array}_{\,S_4},$$

$$S = \frac{1}{2}: \quad \begin{array}{|c|c|c|}\hline 1 & 2 & 3 \\ \hline 4 & \multicolumn{2}{c}{} \\ \cline{1-1}\end{array}_{\,S_1}, \begin{array}{|c|c|c|}\hline 1 & 2 & 4 \\ \hline 3 & \multicolumn{2}{c}{} \\ \cline{1-1}\end{array}_{\,S_2}, \begin{array}{|c|c|c|}\hline 1 & 3 & 4 \\ \hline 2 & \multicolumn{2}{c}{} \\ \cline{1-1}\end{array}_{\,S_3}, \begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 & 4 \\ \hline\end{array}_{\,S_4}, \begin{array}{|c|c|}\hline 1 & 3 \\ \hline 2 & 4 \\ \hline\end{array}_{\,S_5}.$$

Based on

$$\begin{array}{|c|}\hline \\ \hline \\ \hline \\ \hline \\ \hline\end{array} = \left(\begin{array}{|c|c|}\hline & \\ \hline & \multicolumn{1}{c}{} \\ \cline{1-1} & \multicolumn{1}{c}{} \\ \cline{1-1}\end{array}, \begin{array}{|c|c|c|}\hline & & \\ \hline & \multicolumn{2}{c}{} \\ \cline{1-1}\end{array} \right) \oplus \cdots$$

$$S = \frac{3}{2}: \quad \begin{array}{|c|}\hline 1 \\ \hline 2 \\ \hline 3 \\ \hline 4 \\ \hline\end{array}_{\,CS_1} = a \times \begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 & \multicolumn{1}{c}{} \\ \cline{1-1} 4 & \multicolumn{1}{c}{} \\ \cline{1-1}\end{array}_{\,C_1} \otimes \begin{array}{|c|c|c|}\hline 1 & 3 & 4 \\ \hline 2 & \multicolumn{2}{c}{} \\ \cline{1-1}\end{array}_{\,S_4} + b \times \begin{array}{|c|c|}\hline 1 & 3 \\ \hline 2 & \multicolumn{1}{c}{} \\ \cline{1-1} 4 & \multicolumn{1}{c}{} \\ \cline{1-1}\end{array}_{\,C_2} \otimes \begin{array}{|c|c|c|}\hline 1 & 2 & 4 \\ \hline 3 & \multicolumn{2}{c}{} \\ \cline{1-1}\end{array}_{\,S_3} + c \times \begin{array}{|c|c|}\hline 1 & 4 \\ \hline 2 & \multicolumn{1}{c}{} \\ \cline{1-1} 3 & \multicolumn{1}{c}{} \\ \cline{1-1}\end{array}_{\,C_3} \otimes \begin{array}{|c|c|c|}\hline 1 & 2 & 3 \\ \hline 4 & \multicolumn{2}{c}{} \\ \cline{1-1}\end{array}_{\,S_2};$$

?

The flavor ⊗ color ⊗ spin wave functions of pentaquarks

Any CG coefficient of S_n can be factorized into an isoscalar factor, called K matrix, and a CG coefficient

$$S_n \supset S_{n-1} \supset S_{n-2} \supset \ldots \supset S_2$$

The isoscalar factor denoted by K is defined by

$$S([f']p'q'y'[f'']p''q''y''|[f]pqy) = K([f']p'[f'']p''|[f]p)S([f'_{p'}]q'y'[f''_{p''}]q''y''|[f_p]qy),$$

	[211]1	$[1^4]4$
[31]2 [211]1	$\sqrt{2/3}$	$\sqrt{1/3}$
[31]1 [211]3	$\sqrt{1/3}$	$-\sqrt{2/3}$

	[21]1	$[1^3]3$
[21]2 [21]1	$-\sqrt{1/2}$	$\sqrt{1/2}$
[21]1 [21]2	$-\sqrt{1/2}$	$-\sqrt{1/2}$

Flavor ⊗ Color ⊗ Spin Wave Functions satisfied with fully antisymmetric:

$$S=\frac{3}{2}:\ \ytableaushort{1,2,3,4}_{CS_1} = \frac{1}{\sqrt{3}}\ytableaushort{12,3,4}_{C_1} \otimes \ytableaushort{134,2}_{S_4} - \frac{1}{\sqrt{3}}\ytableaushort{13,2,4}_{C_2} \otimes \ytableaushort{124,3}_{S_3} + \frac{1}{\sqrt{3}}\ytableaushort{14,2,3}_{C_3} \otimes \ytableaushort{123,4}_{S_2}$$

[1] F. Stancu and S. Pepin, "Isoscalar factors of the permutation group," Few Body Syst. **26**, 113 (1999).

The flavor ⊗ color ⊗ spin wave functions of pentaquarks

J	The expressions of CMI Hamiltonian for $cccc\bar{c}$ subsystems
$J=3/2$	$\frac{56}{3}C_{cc}-\frac{16}{3}C_{c\bar{c}}$
$J=1/2$	$\frac{56}{3}C_{cc}+\frac{32}{3}C_{c\bar{c}}$
J	The expressions of CMI Hamiltonian for $cccb\bar{c}$ subsystems
$J=5/2$	$8C_{cc}+\frac{16}{3}C_{b\bar{c}}$
$J=3/2$	$\begin{pmatrix} \frac{28}{3}C_{cc}+\frac{28}{3}C_{cb}-4C_{c\bar{c}}-\frac{4}{3}C_{b\bar{c}} & \frac{2\sqrt{2}}{3}(-C_{cc}+C_{cb}+C_{c\bar{b}}-C_{b\bar{c}}) & -\frac{8\sqrt{5}}{3}(C_{c\bar{c}}-C_{b\bar{c}}) \\ \frac{2\sqrt{2}}{3}(-C_{cc}+C_{cb}+C_{c\bar{b}}-C_{b\bar{c}}) & \frac{26}{3}C_{cc}-6C_{cb}+\frac{2}{3}C_{c\bar{b}}-2C_{b\bar{c}} & \frac{4\sqrt{10}}{3}(C_{c\bar{c}}+2C_{b\bar{c}}) \\ -\frac{8\sqrt{5}}{3}(C_{c\bar{c}}-C_{b\bar{c}}) & \frac{4\sqrt{10}}{3}(C_{c\bar{c}}+2C_{b\bar{c}}) & 8(C_{cc}-C_{b\bar{c}}) \end{pmatrix}$
$J=1/2$	$\begin{pmatrix} \frac{28}{3}C_{cc}+\frac{28}{3}C_{cb}+8C_{c\bar{c}}+\frac{8}{3}C_{b\bar{c}} & \frac{2\sqrt{2}}{3}(-C_{cc}+C_{cb}-2C_{c\bar{c}}+2C_{b\bar{c}}) & \frac{2\sqrt{2}}{3}(C_{c\bar{c}}-C_{b\bar{c}}) \\ \frac{2\sqrt{2}}{3}(-C_{cc}+C_{cb}-2C_{c\bar{c}}+2C_{b\bar{c}}) & \frac{26}{3}C_{cc}-6C_{cb}-\frac{4}{3}C_{c\bar{c}}+4C_{b\bar{c}} & -\frac{2}{3}(13C_{c\bar{c}}-C_{b\bar{c}}) \\ \frac{2\sqrt{2}}{3}(C_{c\bar{c}}-C_{b\bar{c}}) & -\frac{2}{3}(13C_{c\bar{c}}-C_{b\bar{c}}) & 10(C_{cc}-C_{cb}) \end{pmatrix}$

For $I=\frac{3}{2}$ $nnnc\bar{c}$ states (3 baryons in 10_f),

$$\langle H_{CM}\rangle_{J=\frac{3}{2}} = 10C_{nn}+\frac{10}{3}(C_{nc}-C_{n\bar{c}})-\frac{2}{3}C_{c\bar{c}},$$

$$\langle H_{CM}\rangle_{J=\frac{1}{2}} = \begin{pmatrix} 10C_{nn}-\frac{20}{3}(C_{nc}-C_{n\bar{c}})-\frac{2}{3}C_{c\bar{c}} & \frac{10}{\sqrt{3}}(C_{nc}+C_{n\bar{c}}) \\ & 10C_{nn}+2C_{c\bar{c}} \end{pmatrix}.$$

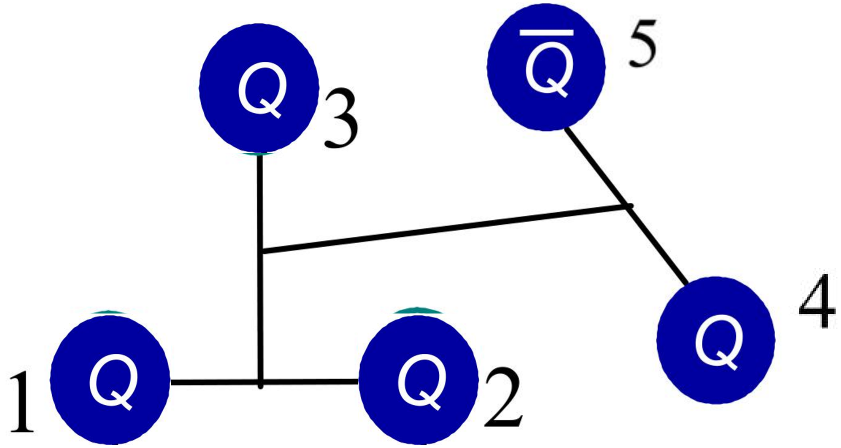
J. Wu, Y. R. Liu, K. Chen, X. Liu and S. L. Zhu, "Hidden-charm pentaquarks and their hidden-bottom and B_c-like partner states," Phys. Rev. D **95**, no. 3, 034002 (2017).

The space wave functions of pentaquarks

We take the spatial function to be a Gaussian which wasextensively used with the variational method to handle calculations in many body problem.

$$\boldsymbol{x}_1 = \frac{1}{2}(\mathbf{r}_1 - \mathbf{r}_2 + \mathbf{r}_3 - \mathbf{r}_4),$$
$$\boldsymbol{x}_2 = \frac{1}{2}(\mathbf{r}_1 - \mathbf{r}_2 - \mathbf{r}_3 + \mathbf{r}_4),$$
$$\boldsymbol{x}_2 = \frac{1}{2}(\mathbf{r}_1 + \mathbf{r}_2 - \mathbf{r}_3 - \mathbf{r}_4),$$
$$\boldsymbol{x}_4 = \frac{1}{2\sqrt{5}}(\mathbf{r}_1 + \mathbf{r}_2 + \mathbf{r}_3 + \mathbf{r}_4 - 4\mathbf{r}_5).$$
$$\boldsymbol{y}_1 = \frac{1}{\sqrt{2}}(\mathbf{r}_1 - \mathbf{r}_2),$$
$$\boldsymbol{y}_2 = \sqrt{\frac{2}{3}}(\mathbf{r}_3 - \frac{1}{2}\mathbf{r}_1 - \frac{1}{2}\mathbf{r}_2),$$
$$\boldsymbol{y}_3 = \frac{1}{\sqrt{2}}(\mathbf{r}_4 - \mathbf{r}_5),$$
$$\boldsymbol{y}_4 = \sqrt{\frac{6}{5}}(\frac{1}{3}(\mathbf{r}_1 + \mathbf{r}_2 + \mathbf{r}_3) - \frac{1}{2}(\mathbf{r}_4 + \mathbf{r}_5)).$$

$$\mathrm{Ux} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & -\frac{1}{2\sqrt{2}} & \frac{\sqrt{\frac{5}{6}}}{2} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{6}} & \frac{1}{2\sqrt{2}} & -\frac{\sqrt{\frac{5}{6}}}{2} \\ 0 & -\sqrt{\frac{2}{3}} & -\frac{1}{2\sqrt{2}} & \frac{\sqrt{\frac{5}{6}}}{2} \\ 0 & 0 & \frac{\sqrt{\frac{5}{2}}}{2} & \frac{\sqrt{\frac{3}{2}}}{2} \end{pmatrix};$$



$$R^s = \exp[-C_{11}(\boldsymbol{x}_1)^2 - C_{11}(\boldsymbol{x}_2)^2 - C_{11}(\boldsymbol{x}_3)^2 - C_{22}(\boldsymbol{x}_4)^2],$$
$$R^s = \exp[-Z^T C^s Z],$$

$$C^s = \begin{pmatrix} C_{11} & 0 & 0 & 0 \\ 0 & C_{11} & 0 & 0 \\ 0 & 0 & C_{11} & 0 \\ 0 & 0 & 0 & C_{22} \end{pmatrix}$$

The space wave functions of pentaquarks

We give some details about the calculation of the potential matrix elements.

Based on

$$A^s = U_x C^s U_x^{-1} \quad A^{st} = A^s + A^t \quad \langle R^s|f|R^t\rangle = \int d^3y_1 d^3y_2 d^3y_3 d^3y_4 f(y_1) exp[-y^T A^{st} y]$$

$$y = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{pmatrix} \quad A = \begin{pmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{23} & A_{24} \\ A_{31} & A_{32} & A_{33} & A_{34} \\ A_{41} & A_{42} & A_{43} & A_{44} \end{pmatrix} y' = \begin{pmatrix} y_1 \\ y_2' \\ y_3' \\ y_4' \end{pmatrix} \quad A' = \begin{pmatrix} A_{11} & X_2 & X_3 & X_4 \\ X_2 & \bar{A}_{22} & 0 & 0 \\ X_3 & 0 & \bar{A}_{33} & 0 \\ X_4 & 0 & 0 & \bar{A}_{44} \end{pmatrix}$$

$$y_1, y_2, y_3, y_4 \rightarrow y_1, y_2', y_3', y_4' \quad y^T A y = y'^T A' y'$$

$$\int d^3y_1 d^3y_2 d^3y_3 d^3y_4 f(y_1) exp[-y^T A^{st} y] = \int d^3y_1 d^3y_2' d^3y_3' d^3y_4' f(y_1) exp[-y'^T A'^{st} y']$$

$$= \int d^3y_1 d^3y_2' d^3y_3' d^3y_4' f(y_1) exp[-(A_{11}X_1^2 + \bar{A}_{22}x_2'^2 + ... + \bar{A}_{44}x_4'^2 + 2X_2x_1x_2' + ... + 2X_4x_1x_4')]$$

$$\int exp[-ar^2 + 2br]d^3r = (\frac{\pi}{a})^{3/2} exp[\frac{b^2}{a}]$$

$$= \frac{(\pi^{3/2})^3}{\bar{A}_{22}^{3/2}\bar{A}_{33}^{3/2}\bar{A}_{44}^{3/2}} \int exp[-(A_{11} - \frac{y_2^2}{\bar{A}_{22}} - \frac{y_3^2}{\bar{A}_{33}} - \frac{y_4^2}{\bar{A}_{44}})y_1^2] f(y_1) d^3y_1$$

$$= \int_0^\infty \frac{(\pi^{3/2})^3}{(det|M^{11}|)^{3/2}} exp[-\frac{det|A|}{det|M^{11}|} y_1^2] 4\pi y_1^2 f(y_1) dy_1$$

$$A_{11} - \frac{y_2^2}{\bar{A}_{22}} - \frac{y_3^2}{\bar{A}_{33}} - \frac{y_4^2}{\bar{A}_{44}} = \frac{det|A'|}{det|M'^{11}|} = \frac{det|A|}{det|M^{11}|}$$

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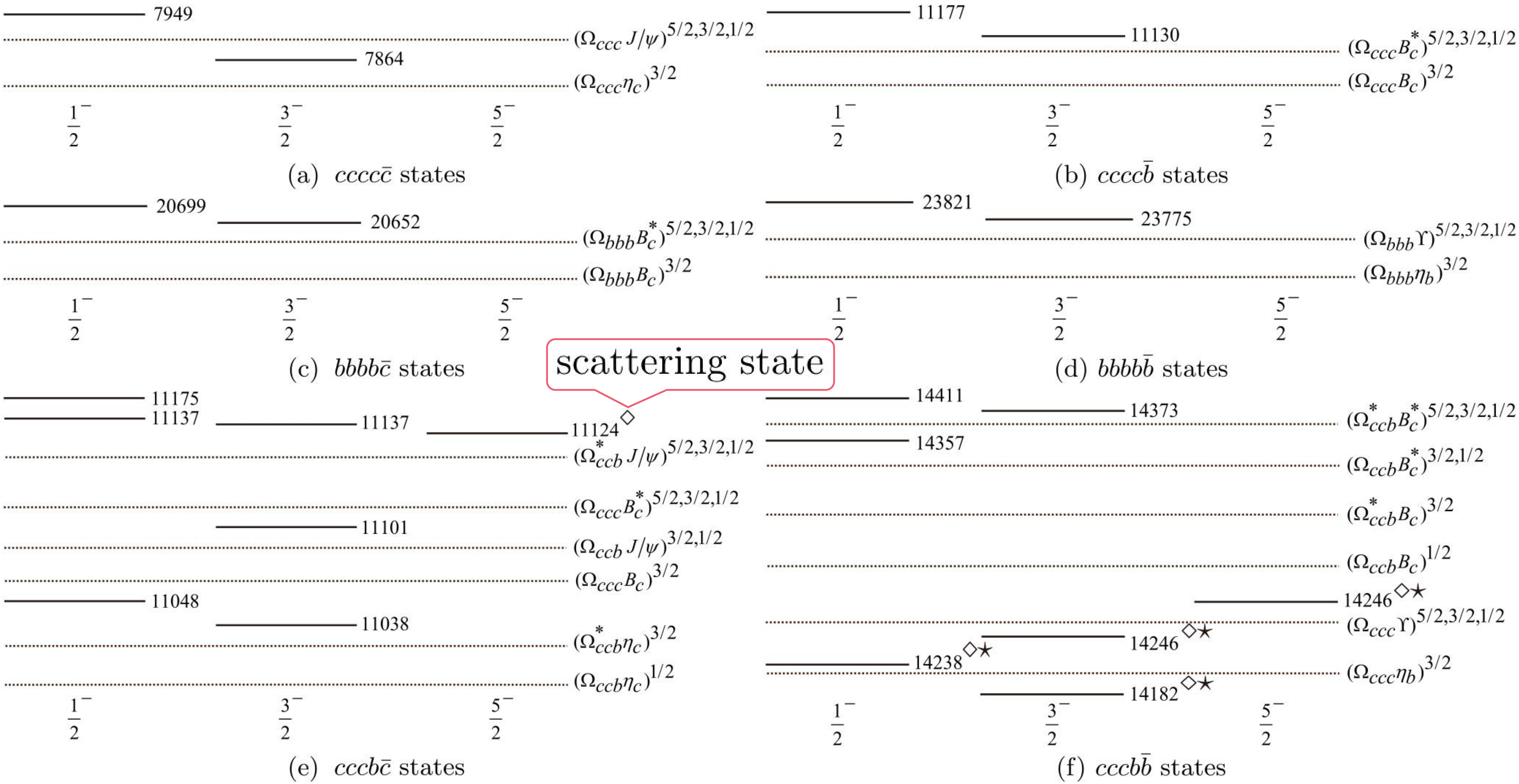
Mass spectra and decay behaviors

According to these scheme, we label the relatively positions of the $QQQQ\bar{Q}$ pentaquark states and the relevant baryon-meson thresholds.

	J^P	traditional CMI model		modified CMI	constituent quark model							QCD sum rule
		Eigenvalue	$(J/\psi\Omega_{ccc})$	model	Mass	Parameter	Parameter	m_i	K.E.	V^C	V^S	approach
$cccc\bar{c}$	$\frac{3}{2}^-$	26.4	7861	7864	8197	$C_{11}=7.1$	$C_{22}=6.4$	9350	868	-2042	21	$7.41^{+0.27}_{-0.31}$
	$\frac{1}{2}^-$	118.1	7946	7949	8222	$C_{11}=7.0$	$C_{22}=6.2$		850	-2023	46	
$cccc\bar{b}$	$\frac{3}{2}^-$	44.0	11131	11130	11520	$C_{11}=7.4$	$C_{22}=8.0$	12739	859	-2106	12	
	$\frac{1}{2}^-$	96.8	11184	11177	11530	$C_{11}=7.3$	$C_{22}=7.9$		852	-2099	37	
$bbbb\bar{c}$	$\frac{3}{2}^-$	16.0	20639	20652	20963	$C_{11}=19.7$	$C_{22}=12.7$	22906	923	-2868	1	
	$\frac{1}{2}^-$	68.8	20692	20699	20981	$C_{11}=19.3$	$C_{22}=12.4$		908	-2853	19	
$bbbb\bar{b}$	$\frac{3}{2}^-$	18.1	23759	23775	24200	$C_{11}=22.1$	$C_{22}=19.6$	26295	954	-3055	6	$21.6^{+0.74}_{-0.22}$
	$\frac{1}{2}^-$	64.5	23805	23821	24207	$C_{11}=21.9$	$C_{22}=19.5$		947	-3049	13	
$cccc\bar{n}$	$\frac{3}{2}^-$	26.4	6761	6761	6933	$C_{11}=6.4$	$C_{22}=3.1$	7817	922	-1792	-14	
	$\frac{1}{2}^-$	132.0	6867	6867	7041	$C_{11}=5.9$	$C_{22}=2.9$		845	-1712	91	
$cccc\bar{s}$	$\frac{3}{2}^-$	25.9	6861	6864	7077	$C_{11}=6.7$	$C_{22}=4.2$	8080	895	-1901	3	
	$\frac{1}{2}^-$	133.1	6968	6972	7145	$C_{11}=6.3$	$C_{22}=3.9$		846	-1850	69	
$bbbb\bar{n}$	$\frac{3}{2}^-$	22.4	19631	19647	19902	$C_{11}=14.4$	$C_{22}=4.4$	21373	890	-2345	-17	
	$\frac{1}{2}^-$	56.0	19664	19681	19966	$C_{11}=13.6$	$C_{22}=4.2$		841	-2295	46	
$bbbb\bar{s}$	$\frac{3}{2}^-$	21.3	19720	19736	19973	$C_{11}=16.2$	$C_{22}=6.5$	21636	893	-2548	-9	
	$\frac{1}{2}^-$	58.1	19757	19773	20016	$C_{11}=15.6$	$C_{22}=6.3$		859	-2513	34	

[1] J. R. Zhang, "Fully-heavy pentaquark states," Phys. Rev. D **103** (2021) no.7, 074016 [arXiv:2011.04594].

Mass spectra and decay behaviors



Relative positions (units: MeV) for partial fully heavy pentaquark system.

Mass spectra and decay behaviors

The eigenvectors of pentaquark states will also provide important information about the two-body strong decay of multiquark states. Thus we calculate the overlaps of wave functions between a fully-heavy pentaquark state and a particular baryon $\otimes$ meson state.

For the two body decay via L-wave process, the expression describing partial decay width can be parameterized as

$$\Gamma_i = \gamma_i \alpha \frac{k^{2L+1}}{m^{2L}} \cdot |c_i|^2,$$

where α is an effective coupling constant, m is the mass of the initial state, k is the momentum of the final states in the rest frame. c_i is the coefficient related to the corresponding baryon-meson component.

In the quark model in the heavy quark limit, the spatial wave functions of the ground S-wave pseudoscalar and vector meson are the same.

As a rough estimation, we introduce the following approximations to calculate the relative partial decay widths of the pentaquark states.

$$\begin{aligned}
&\gamma_{\Omega_{ccc}J/\psi} = \gamma_{\Omega_{ccc}\eta_c}, \quad \gamma_{\Omega_{ccc}B_c^*} = \gamma_{\Omega_{ccc}B_c}, \\
&\gamma_{\Omega_{bbb}B_c^*} = \gamma_{\Omega_{bbb}B_c}, \quad \gamma_{\Omega_{bbb}\Upsilon} = \gamma_{\Omega_{bbb}\eta_b}, \\
&\gamma_{\Omega_{ccc}B_c^*} = \gamma_{\Omega_{ccc}B_c}, \quad \gamma_{\Omega_{ccc}\Upsilon} = \gamma_{\Omega_{ccc}\eta_b}, \\
&\gamma_{\Omega_{bbb}J/\psi} = \gamma_{\Omega_{bbb}\eta_c}, \quad \gamma_{\Omega_{bbb}B_c^*} = \gamma_{\Omega_{bbb}B_c}\cdots
\end{aligned}$$

Mass Spectra and Decay Behaviors

The overlaps of wave functions between a fully-heavy pentaquark state and a particular baryon $\otimes$ meson state.

J^P	$cccc\bar{c}$ Mass	$ccc\otimes c\bar{c}$: $\Omega_{ccc}J/\psi$	$\Omega_{ccc}\eta_c$	$cccc\bar{b}$ Mass	$ccc\otimes c\bar{b}$: $\Omega_{ccc}B_c^*$	$\Omega_{ccc}B_c$	$bbbb\bar{c}$ Mass	$bbb\otimes b\bar{c}$: $\Omega_{bbb}B_c^*$	$\Omega_{bbb}B_c$	$bbbb\bar{b}$ Mass	$bbb\otimes b\bar{b}$: $\Omega_{bbb}\Upsilon$	$\Omega_{bbb}\eta_b$
$\frac{3}{2}^-$	7864	0.456	-0.354	11130	0.456	-0.354	20652	0.456	-0.354	23775	0.456	-0.354
$\frac{1}{2}^-$	7949	-0.577		11177	0.577		20699	-0.577		23821	0.577	

J^P	$cccb\bar{c}$ Mass	$ccc\otimes b\bar{c}$: $\Omega_{ccc}B_c^*$	$\Omega_{ccc}B_c$	$ccb\otimes c\bar{c}$: Ω_{ccb}^*J/ψ	$\Omega_{ccb}^*\eta_c$	$\Omega_{ccb}J/\psi$	$\Omega_{ccb}\eta_c$	$cccb\bar{b}$ Mass	$ccc\otimes b\bar{b}$: $\Omega_{ccc}\Upsilon$	$\Omega_{ccc}\eta_b$	$ccb\otimes c\bar{b}$: $\Omega_{ccb}^*B_c^*$	$\Omega_{ccb}^*B_c$	$\Omega_{ccb}B_c^*$	$\Omega_{ccb}B_c$
$\frac{5}{2}^-$	11124◇	1.000		0.333				14246◇	1.000		0.333			
$\frac{3}{2}^-$	11137	0.812	0.236	-0.361	-0.008	0.275		14373	-0.046	-0.120	0.521	-0.352	-0.209	
	11101	0.569	-0.524	0.396	-0.279	0.102		14246◇	0.999	-0.016	0.030	0.229	-0.242	
	11038	0.130	0.818	0.095	-0.380	-0.250		14182 ◇ ⋆	0.011	0.993	0.154	0.214	0.215	
$\frac{1}{2}^-$	11175	-0.543		-0.587		0.034	-0.001	14411	0.130		-0.626		0.199	0.087
	11137	-0.657		0.172		-0.519	-0.039	14357	-0.206		0.126		0.571	-0.297
	11048	0.523		0.180		0.316	-0.470	14238 ◇ ⋆	0.969		0.004		0.068	0.355

Mass spectra and decay behaviors

The values of $k \cdot |c_i|^2$ for the fully-heavy pentaquark states.

The decay channel is marked with "×" if kinetically forbidden.

	$cccc\bar{c}$	$ccc \otimes c\bar{c}$		$cccc\bar{b}$	$ccc \otimes c\bar{b}$			$bbbb\bar{c}$	$bbb \otimes b\bar{c}$		$bbbb\bar{b}$	$bbb \otimes b\bar{b}$	
J^P	Mass	$\Omega_{ccc}J/\psi$	$\Omega_{ccc}\eta_c$	Mass	$\Omega_{ccc}B_c^*$	$\Omega_{ccc}B_c$		Mass	$\Omega_{bbb}B_c^*$	$\Omega_{bbb}B_c$	Mass	$\Omega_{bbb}\Upsilon$	$\Omega_{bbb}\eta_b$
$\frac{3}{2}^-$	7864	×	74	11130	×	77		20652	39	96	23775	49	108
$\frac{1}{2}^-$	7949	167		11177	181			20699	224		23821	254	

	$cccb\bar{c}$	$ccc \otimes b\bar{c}$		$ccb \otimes c\bar{c}$				$cccb\bar{b}$	$ccc \otimes b\bar{b}$		$ccb \otimes c\bar{b}$			
J^P	Mass	$\Omega_{ccc}B_c^*$	$\Omega_{ccc}B_c$	Ω_{ccb}^*J/ψ	$\Omega_{ccb}^*\eta_c$	$\Omega_{ccb}J/\psi$	$\Omega_{ccb}\eta_c$	Mass	$\Omega_{ccc}\Upsilon$	$\Omega_{ccc}\eta_b$	$\Omega_{ccb}^*B_c^*$	$\Omega_{ccb}^*B_c$	$\Omega_{ccb}B_c^*$	$\Omega_{ccb}B_c$
$\frac{5}{2}^-$	11124◇	14		×				14246◇	15		×			
$\frac{3}{2}^-$	11137	177	36	37	0.05	36		14373	2	16	82	91	25	
	11101	×	130	×	50	3		14246◇	×	0.2	×	×	×	
	11038	×	×	×	54	×		14182 ◇ ⋆	×	×	×	×	×	
$\frac{1}{2}^-$	11175	156		173		0.7	0.002	14411	18		236		30	8
	11137	117		9		127	1	14357	36		×		147	71
	11048	×		×		×	125	14238 ◇ ⋆	×		×		×	×

Outline

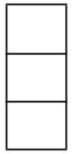
- Background
- The effective Hamiltonian
- Wave functions of pentaquarks
- Mass spectra and decay behaviors
- **Summary**

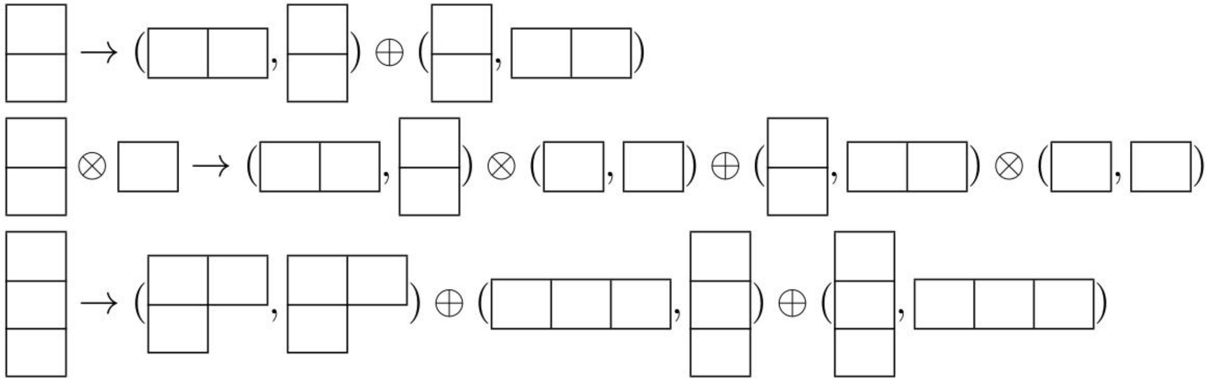
Summary

- We construct the wave functions $\psi_{space} \otimes \psi_{flavor} \otimes \psi_{color} \otimes \psi_{spin}$ based on the Pauli Principle.
- We provide the eigenvectors to extract useful information about the decay properties for the $QQQQ\bar{Q}$ pentaquark systems.
- We only find two $cccc\bar{c}$ states due to the constraint from Pauli principle: a $J^P = 3/2^-$ state $\mathrm{P}_{\mathrm{c}^4\bar{\mathrm{c}}}(7864,\ 0,\ 3/2^-)$ and a $J^P = 1/2^-$ state $\mathrm{P}_{\mathrm{c}^4\bar{\mathrm{c}}}(7949,\ 0,\ 1/2^-)$, and there exists no ground $J^P = 5/2^-$ $cccc\bar{c}$ pentaquark state.
- We think there is also no the ground $J^P = 5/2^-$ $cccb\bar{Q}$ and $bbbc\bar{Q}$ states since such states are identified as scattering states in the quark model.
- We find one good stable candidates: the $\mathrm{P}_{\mathrm{c}^2\mathrm{b}^2\bar{\mathrm{b}}}(17416, 0, 3/2^-)$. It lies only below the allowable decay channel $\Omega^*_{ccb}\eta_b$ 4 MeV, and thus can only decay through electromagnetic or weak interactions.
- To produce the lightest $cccc\bar{c}$ pentaquark state, one needs to simultaneously produce at least four pairs of $c\bar{c}$, this seems to be a difficult task in experiment.

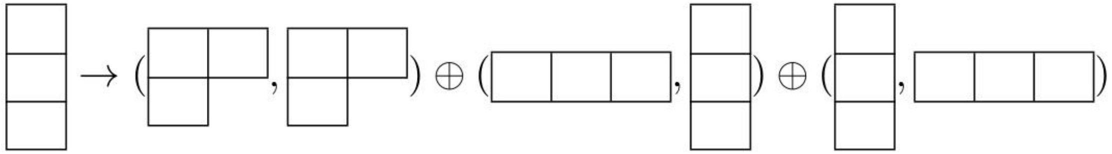
Thanks for your attention.

The flavor ⊗ color ⊗ spin wave functions of pentaquarks

Now let us obtain  of SU(mn). We need

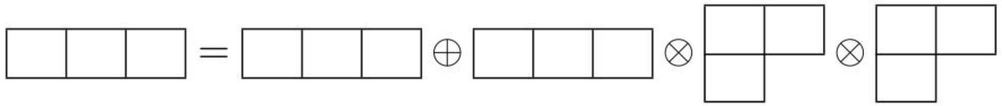


For the $SU(12)_{CSI} \supset SU(3)_C \otimes SU(4)_{IS}$:

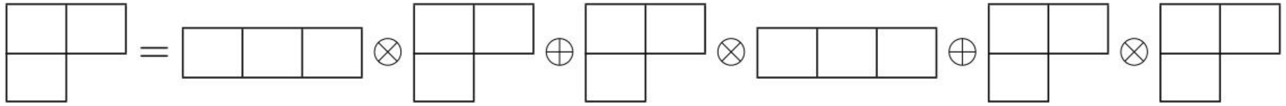


$$220_A = (8, 20) \oplus (10, 4) \oplus (1, 20)$$

For the $SU(4)_{IS} \supset SU(2)_I \otimes SU(2)_S$:



$$20 = (4, 4) \oplus (2, 2)$$



$$20 = (4, 2) \oplus (2, 4) \oplus (2, 2)$$

The flavor ⊗ color ⊗ spin wave functions of pentaquarks

Young tableau, which represents the irreducible bases of the permutation group, enable us to easily identify the multi-quark configuration with certain symmetry property [1].

The Reduction $SU(mn) \to SU(m) \otimes SU(n)$

$$\square \to (\square, \square) \qquad (mn) \to (m, n)$$

$$\square \otimes \square \to (\square, \square) \otimes (\square, \square) \qquad (mn)(mn) \to (m, n)(m, n)$$

$$\square \otimes \square \to (\square \otimes \square, \square \otimes \square) \qquad (mn)(mn) \to (mm, nn)$$

$$(\square\square \oplus \substack{\square \\ \square}) \to (\square \otimes \square, \square \otimes \square) \qquad [\frac{1}{2}(mn)(mn+1) + \frac{1}{2}(mn)(mn-1)] \to (mm, nn)$$

$$=(\square\square \oplus \substack{\square \\ \square}, \square\square \oplus \substack{\square \\ \square}) \qquad = [\frac{1}{2}m(m+1) + \frac{1}{2}m(m-1), \frac{1}{2}n(n+1) + \frac{1}{2}n(n-1)]$$

$$=(\square\square, \square\square) \oplus (\square\square, \substack{\square \\ \square}) \oplus (\square\square, \substack{\square \\ \square}) \oplus (\substack{\square \\ \square}, \substack{\square \\ \square})$$

$$=(\frac{1}{2}m(m+1), \frac{1}{2}n(n+1)) + (\frac{1}{2}m(m-1), \frac{1}{2}n(n+1)) + (\frac{1}{2}m(m+1), \frac{1}{2}n(n-1)) + (\frac{1}{2}m(m-1), \frac{1}{2}n(n-1))$$

$$=(\frac{1}{2}m(m+1), \frac{1}{2}n(n+1)) + (\frac{1}{2}m(m-1), \frac{1}{2}n(n+1)) + (\frac{1}{2}m(m+1), \frac{1}{2}n(n-1)) + (\frac{1}{2}m(m-1), \frac{1}{2}n(n-1))$$

Here we take

$$\square\square \to (\square\square, \square\square) \oplus (\substack{\square \\ \square}, \substack{\square \\ \square}) \qquad [\frac{1}{2}(mn)(mn+1)] \to [\frac{1}{2}m(m+1), \frac{1}{2}n(n+1)] + [\frac{1}{2}m(m-1), \frac{1}{2}n(n-1)]$$

$$\substack{\square \\ \square} \to (\square\square, \substack{\square \\ \square}) \oplus (\substack{\square \\ \square}, \square\square) \qquad [\frac{1}{2}(mn)(mn-1)] \to [\frac{1}{2}m(m+1), \frac{1}{2}n(n-1)] + [\frac{1}{2}m(m-1), \frac{1}{2}n(n+1)]$$

[1] F. Stancu and S. Pepin, "Isoscalar factors of the permutation group," Few Body Syst. **26**, 113 (1999).