Production of Fully-Heavy Tetraquarks Using NRQCD Factorization

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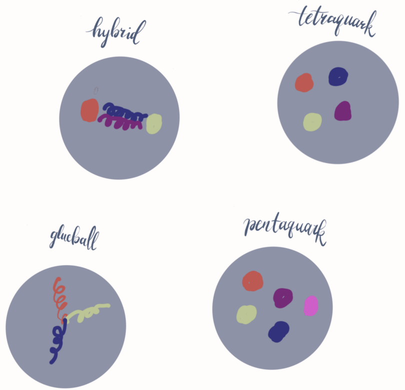
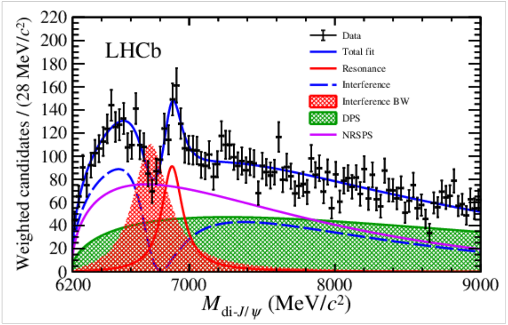
Motivation

Exotic Hadrons



Discovery of $X(6900)$



Invariant mass spectrum of J/ψ-pair candidates $(\mathrm{LHCb},\ 2020)$

- First fullly-charm tetraquark candidate
- Strong decay to two J/ψ, $C = +$ **See Zehua Xu**

Fully-heavy Tetraquark

- Theoretical investigations on the fully heavy tetraquarks date back to late 1970s (Iwasaki, 1976; Chao, 1981).
- Phenomenological studies of spectra and decay properties: Badalian *et al.*, 1987; *et al.*, 2006; Wang, 2017,2020; W. Chen *et al.*, 2017,2018; Wu *et al.*, 2018; Liu *et al.*, 2019; Wang, Di, 2019; H.-X. Chen *et al.*, 2020; Jin *et al.*, 2020; Guo, Oller, 2020....
- Search for the fully-bottom tetraquark on Lattice NRQCD: found no indication of any states below $2\eta_b$ threshold in the $0^{++}, 1^{+-}$ and 2^{++} channels (Hughes *et al.*, 2018).

Workshop on the XYZ Particles

Production Mechanism

- Duality relations: Berezhnoy *et al.*, 2011, 2012; Kaliner *et al.*, 2017
- Color evaporation model: Carvalho *et al.*, 2016; Maciuła *et al.*, 2020

Production Mechanism

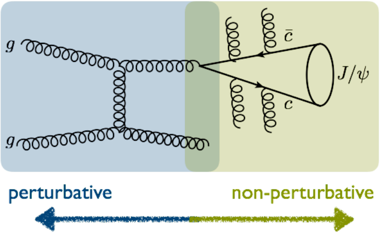
- Duality relations: Berezhnoy *et al.*, 2011, 2012; Kaliner *et al.*, 2017
- Color evaporation model: Carvalho *et al.*, 2016; Maciuła *et al.*, 2020
- NRQCD-inspired factorization: Y.-Q. Ma, Zhang, 2020; Feng et al., 2020,2021; R.-L. Zhu, 2020

NRQCD Factorization

QCD Factorization Theorem

The inclusive production of high-$P_\perp$ hadrons in the high-energy hadron collision experiments is dominated by the fragmentation mechanism (Collins *et al.*, 1989).

$$\mathrm{d}\sigma\left[A+B\to H\left(P_\perp\right)+X\right] = \sum_i \mathrm{d}\hat{\sigma}\left[A+B\to i\left(\frac{P_\perp}{z}\right)+X\right]\otimes \boldsymbol{D_{i\to H}}\left(z,\mu\right)+\mathcal{O}\left(\frac{1}{P_\perp^2}\right)$$



PARTICLEBITES, 2016

$\hat{\sigma}$: partonic cross section,
$D_{i\to H}\left(z,\mu\right)$: fragmentation function

Fragmentation Function

Gauge-invariant operator definition for the gluon fragmentation function (Collins, Soper, 1982)

$$D_{g\to H}(z,\mu_\Lambda) = \frac{-g_{\mu\nu} z^{d-3}}{2\pi k^+ (N_c^2-1)(d-2)} \int_{-\infty}^{+\infty} dx^- e^{-ik^+x^-} \sum_X \left\langle 0 \middle| G_c^{+\mu}(0) \mathcal{E}^\dagger(0,0,\mathbf{0}_\perp)_{cb} \middle| H(P)+X \right\rangle \left\langle H(P)+X \middle| \mathcal{E}\left(0,x^-,\mathbf{0}_\perp\right)_{ba} G_a^{+\nu}\left(0,x^-,\mathbf{0}_\perp\right) \middle| 0 \right\rangle$$

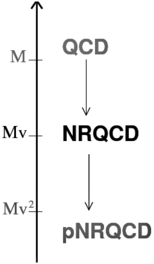
G: field-strength tensor of gluons,
k: momentum of G,
$\mathcal{E}$: gauge link,
$d := 4-2\epsilon$: spacetime dimension,
$z := P^+/k^+$

DGLAP Equation

Fragmentation function follows Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution equation

$$\mu \frac{\partial}{\partial \mu} D_{g \to T_{4c}}(z, \mu) = \frac{\alpha_s(\mu)}{\pi} \sum_{j \in \{g,c\}} \int_z^1 \frac{\mathrm{d}y}{y} P_{g \leftarrow j} \left(\frac{z}{y}, \mu \right) D_{g \to T_{4c}}(y, \mu)$$

$P_{g \leftarrow j}$ is the splitting kernel.

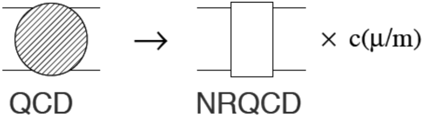


Vairo,
Hadron 2011

- Quarkonium energy scale (Braaten, 1997)

	$c\bar{c}$	$b\bar{b}$	$t\bar{t}$
M	1.5 GeV	4.7 GeV	180 GeV
Mv	0.9 GeV	1.5 GeV	16 GeV
Mv^2	0.5 GeV	0.5 GeV	1.5 GeV

- Integrate out the heavy($\sim M$) degrees of freedom



Qiu, 2011

- For inclusive processes, we use the vacuum-saturation approximation.
- Factorization formula for the fragmentation function $D_{g\to T_{4c}}$

$$
\begin{aligned}
D_{g\to T_{4c}}(z,\mu_\Lambda) =& \frac{d_{3\times 3}\left[g\to cc\bar{c}\bar{c}^{(J)}\right]}{m^9}\left|\left\langle T_{4c}^{(J)}\left|\mathcal{O}_{\bar{\mathbf{3}}\otimes\mathbf{3}}^{(J)}\right|0\right\rangle\right|^2 \\
&+\frac{d_{6\times 6}\left[g\to cc\bar{c}\bar{c}^{(J)}\right]}{m^9}\left|\left\langle T_{4c}^{(J)}\left|\mathcal{O}_{\mathbf{6}\otimes\bar{\mathbf{6}}}^{(J)}\right|0\right\rangle\right|^2 \\
&+\frac{d_{3\times 6}\left[g\to cc\bar{c}\bar{c}^{(J)}\right]}{m^9}2\mathrm{Re}\left[\left\langle T_{4c}^{(J)}\left|\mathcal{O}_{\bar{\mathbf{3}}\otimes\mathbf{3}}^{(J)}\right|0\right\rangle\left\langle 0\left|\mathcal{O}_{\mathbf{6}\otimes\bar{\mathbf{6}}}^{(J)\dagger}\right|T_{4c}^{(J)}\right\rangle\right]
\end{aligned}
$$

NRQCD Factorization

- For the exclusive production at B factory

$$\sigma(e^+e^- \to T_{4c}^J + \gamma) = \frac{F_{3,3}^{[J]}}{m_c^8}(2M_{T_{4c}}) \left| \left\langle T_{4c}^{(J)} \middle| \mathcal{O}_{\bar{\mathbf{3}}\otimes\mathbf{3}}^{(J)} \middle| 0 \right\rangle \right|^2 + \frac{F_{6,6}^{[J]}}{m_c^8}(2M_{T_{4c}}) \left| \left\langle T_{4c}^{(J)} \middle| \mathcal{O}_{\mathbf{6}\otimes\bar{\mathbf{6}}}^{(J)} \middle| 0 \right\rangle \right|^2 + \frac{F_{3,6}^{[J]}}{m_c^8}(2M_{T_{4c}}) 2\mathrm{Re}\left[\left\langle T_{4c}^{(J)} \middle| \mathcal{O}_{\bar{\mathbf{3}}\otimes\mathbf{3}}^{(J)} \middle| 0 \right\rangle \left\langle 0 \middle| \mathcal{O}_{\mathbf{6}\otimes\bar{\mathbf{6}}}^{(J)\dagger} \middle| T_{4c}^{(J)} \right\rangle \right]$$

- For the inclusive production at B factory

$$\sigma(e^+e^- \to T_{4c}^{1+-} + X) = \frac{F_{\bar{\mathbf{3}}\otimes\mathbf{3}}}{m_c^8}(2M_{T_{4c}}) \left| \left\langle T_{4c}^{(1)} \middle| \mathcal{O}_{\bar{\mathbf{3}}\otimes\mathbf{3}}^{(1)} \middle| 0 \right\rangle \right|^2$$

NRQCD Operators

We construct the NRQCD local operators for the S-wave tetraquark with $J^{PC} = 0^{++}, 1^{+-}, 2^{++}$

$$\mathcal{O}^{(0)}_{\bar{\mathbf{3}}\otimes\mathbf{3}} = -\frac{1}{\sqrt{3}}[\psi_a^\dagger \sigma^i (i\sigma^2)\psi_b^*][\chi_c^T (i\sigma^2)\sigma^i \chi_d]\, \mathcal{C}^{ab;cd}_{\bar{\mathbf{3}}\otimes\mathbf{3}},$$

$$\mathcal{O}^{(0)}_{\mathbf{6}\otimes\bar{\mathbf{6}}} = \frac{1}{\sqrt{6}}[\psi_a^\dagger (i\sigma^2)\psi_b^*][\chi_c^T (i\sigma^2)\chi_d]\, \mathcal{C}^{ab;cd}_{\mathbf{6}\otimes\bar{\mathbf{6}}},$$

$$\mathcal{O}^{(1)i}_{\bar{\mathbf{3}}\otimes\mathbf{3}} = \frac{\mathrm{i}}{\sqrt{2}}\epsilon^{ijk}\left(\psi_a^\dagger \sigma^j \mathrm{i}\sigma^2 \psi_b^*\right)\left(\chi_c^T \mathrm{i}\sigma^2 \sigma^k \chi_d\right)\mathcal{C}^{ab;cd}_{\mathbf{3}\otimes\bar{\mathbf{3}}}$$

$$\mathcal{O}^{(2)kl}_{\bar{\mathbf{3}}\otimes\mathbf{3}} = [\psi_a^\dagger \sigma^m (i\sigma^2)\psi_b^*][\chi_c^T (i\sigma^2)\sigma^n \chi_d]\, \Gamma^{kl;mn}\, \mathcal{C}^{ab;cd}_{\bar{\mathbf{3}}\otimes\mathbf{3}}$$

$$\mathcal{C}^{ab;cd}_{\bar{\mathbf{3}}\otimes\mathbf{3}} := \frac{1}{2\sqrt{3}}(\delta^{ac}\delta^{bd} - \delta^{ad}\delta^{bc}), \quad \mathcal{C}^{ab;cd}_{\mathbf{6}\otimes\bar{\mathbf{6}}} := \frac{1}{2\sqrt{6}}(\delta^{ac}\delta^{bd} + \delta^{ad}\delta^{bc})$$

$$\Gamma^{kl;mn} := \frac{1}{2}\left(\delta^{km}\delta^{ln} + \delta^{kn}\delta^{lm} - \frac{2}{3}\delta^{kl}\delta^{mn}\right)$$

NRQCD Operators

- The operators manifest the correct C-parity under the charge conjugation transformations

$$\psi \to \mathrm{i}\left(\chi^\dagger \sigma^2\right)^t, \quad \chi \to -\mathrm{i}\left(\psi^\dagger \sigma^2\right)^t$$

- We use the basis in which the quark and anti-quark pairs in the color-triplet and color-sexet, respectively. The operators can also be constructed from quark-antiquark pairs in the color-singlet and color-octet.
- These NRQCD operators in can also be inferred by performing the Foldy-Wouthuysen-Tani transformation from the QCD interpolating currents(H.-X. Chen *et al.*,2020).

Perturbative Matching

We use the perturbative matching procedure to determine the short-distance coefficients(SDCs).

- Replace the physical tetraquark state T_{4c}^{J} with a free 4-quark state
- Calculate both sides of factorization formula in perturbative QCD and perturbative NRQCD
- Solving the factorization formula to determine the SDCs.

The SDCs are insensitive to the long-distance physics.

For convenience, we use the eigenstates of the angular momentum, manifesting the same quantum numbers as the physical tetraquark states.

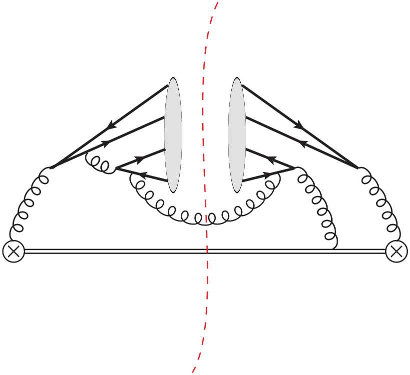
$$\left|\mathcal{T}^{J,m_j}_{\bar{\mathbf{3}}\otimes\mathbf{3}}(Q)\right\rangle = \frac{1}{2}\sum_{s_*,\lambda_*}\left\langle\frac{1}{2}\lambda_1\frac{1}{2}\lambda_2\middle|1s_1\right\rangle\left\langle\frac{1}{2}\lambda_3\frac{1}{2}\lambda_4\middle|1s_2\right\rangle\langle 1s_1 1s_2|Jm_j\rangle \mathcal{C}^{ab;cd}_{\bar{\mathbf{3}}\otimes\mathbf{3}}\left|c_a^{\lambda_1}(q_1)c_b^{\lambda_2}(P-q_1)\bar{c}_c^{\lambda_3}(q_2)\bar{c}_d^{\lambda_4}(Q-P-q_2)\right\rangle$$

$$\left|\mathcal{T}^{0,0}_{\mathbf{6}\otimes\bar{\mathbf{6}}}(Q)\right\rangle = \frac{1}{2}\sum_{\lambda_*}\left\langle\frac{1}{2}\lambda_1\frac{1}{2}\lambda_2\middle|00\right\rangle\left\langle\frac{1}{2}\lambda_3\frac{1}{2}\lambda_4\middle|00\right\rangle \mathcal{C}^{ab;cd}_{\mathbf{6}\otimes\bar{\mathbf{6}}}\left|c_a^{\lambda_1}(q_1)c_b^{\lambda_2}(P-q_1)\bar{c}_c^{\lambda_3}(q_2)\bar{c}_d^{\lambda_4}(Q-P-q_2)\right\rangle,$$

Fragmentation Production of T_{4c} at LHC

Feynman Diagrams

- We employ the self-written program `HepLib`, which employ `Qgraf` and `GiNaC` internally to generate the Feynman diagrams (Feng *et al.*, 2021).
- There are about 100 diagrams for the amplitude.



Perturbative QCD Calculation

To project the $QQ\bar{Q}\bar{Q}$ into correct spin/color quantum number of tetraquark, we use the following projector

$$\bar{u}_i^a \bar{u}_j^b v_k^c v_l^d \to (C\Pi_\mu)^{ij}(\Pi_\nu C)^{lk}\mathcal{C}_{\bar{\mathbf{3}}\otimes\mathbf{3}}^{abcd} J_{0,1,2}^{\mu\nu}$$
$$\bar{u}_i^a \bar{u}_j^b v_k^c v_l^d \to (C\Pi_0)^{ij}(\Pi_0 C)^{lk}\mathcal{C}_{\mathbf{6}\otimes\bar{\mathbf{6}}}^{abcd}$$

C is the charge conjugate operator, $\Pi_\mu(\Pi_0)$ is the spin-triplet(singlet) projector of quarks (Petrelli *et al.*, 1997), $J_{0,1,2}^{\mu\nu}$ are the spin projectors of quark and anti-quark pairs (Braaten, Lee, 2003).

$$J_0^{\mu\nu} = \frac{1}{\sqrt{3}}\eta^{\mu\nu}(P), \quad J_1^{\mu\nu}(\epsilon) = -\frac{i}{\sqrt{2P^2}}\varepsilon^{\mu\nu\rho\sigma}\epsilon_\rho P_\sigma,$$
$$J_2^{\mu\nu}(\epsilon) = \epsilon_{\rho\sigma}\left\{\frac{1}{2}\left[\eta^{\mu\rho}(P)\eta^{\nu\sigma}(P) + \eta^{\mu\sigma}(P)\eta^{\nu\rho}(P)\right] - \frac{1}{3}\eta^{\mu\nu}(P)\eta^{\rho\sigma}(P)\right\}$$

SDCs

$$d_{3\times 3}\left(g\to 0^{++}\right) = \frac{\pi^2\alpha_s^4}{497664z(2-z)^2(3-z)}\Big[186624-430272z+511072z^2-425814z^3 + 217337z^4-61915z^5+7466z^6+42(1-z)(2-z)(3-z)(-144+634z -385z^2+70z^3)\log(1-z)+36(2-z)(3-z)\left(144-634z+749z^2-364z^3 +74z^4\right)\log\left(1-\frac{z}{2}\right)+12(2-z)(3-z)\left(72-362z+361z^2-136z^3+23z^4\right) \times\log\left(1-\frac{z}{3}\right)\Big].$$

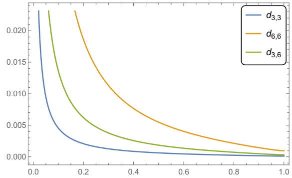
$$d_{6\times 6}\left(g\to 0^{++}\right) = \frac{\pi^2\alpha_s^4}{55296z(2-z)^2(3-z)}\Big[186624-430272z+617824z^2-634902z^3 + 374489z^4-115387z^5+14378z^6-6(1-z)(2-z)(3-z)(-144-2166z +1015z^2+70z^3)\log(1-z)-156(2-z)(3-z)\left(144-1242z+1693z^2-876z^3 +170z^4\right)\log\left(1-\frac{z}{2}\right)+300(2-z)(3-z)\left(72-714z+953z^2-472z^3+87z^4\right) \times\log\left(1-\frac{z}{3}\right)\Big].$$

$$d_{3\times 6}\left(g\to 0^{++}\right) = -\frac{\pi^2\alpha_s^4}{165888z(2-z)^2(3-z)}\Big[186624-430272z+490720z^2-394422z^3 + 199529z^4-57547z^5+7082z^6+6(1-z)(2-z)(3-z)(-432+3302z -1855z^2+210z^3)\log(1-z)-12(2-z)(3-z)\left(720-2258z+2329z^2-1052z^3 +226z^4\right)\log\left(1-\frac{z}{2}\right)+12(2-z)(3-z)\left(936-4882z+4989z^2-1936z^3+331z^4\right) \times\log\left(1-\frac{z}{3}\right)\Big].$$

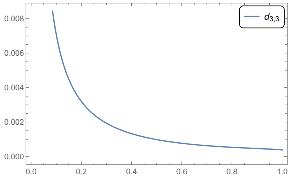
SDCs

$$
\begin{aligned}
d_{3\times 3}\left(g\to 2^{++}\right) =& \frac{\pi^2\alpha_s^4}{622080z^2(2-z)^2(3-z)}\Big[2\left(46656-490536z+1162552z^2\right.\\
&-1156308z^3+595421z^4-170578z^5+21212z^6\left.\right)z\\
&+3(1-z)(2-z)(3-z)(-20304-31788z)(1296+1044z\\
&+73036z^2-36574z^3+7975z^4\left.\right)\log(1-z)\Big]+33(2-z)(3-z)(1296+25)\\
&-9224z^2+9598z^3-3943z^4+725z^5\left.\right)\log\left(1-\frac{z}{3}\right)\Big],\\
d_{6\times 6}\left(g\to 2^{++}\right) =& d_{3\times 6}\left(g\to 2^{++}\right)=0.
\end{aligned}
$$

- There is NO IR divergence.



SDCs for T_{4c}^{0++}



SDC for T_{4c}^{2++}

Long-Distance Matrix Elements(LDMEs)

- The NRQCD LDMEs should be calculated in lattice QCD in principle since they are non-perturbative.
- We use the diquark model to calculate the LDMEs, resulting in the product of wave functions at the origin.
- The fock component of $\mathbf{6}\otimes\bar{\mathbf{6}}$ is neglected for simplicity while there are some results in literature(Lü, *et al.*, 2020; Zhao, *et al.*, 2020).

$$\left|\left\langle T_{4c}^{0}\left|\mathcal{O}_{\bar{\mathbf{3}}\otimes\mathbf{3}}^{(0)}\right|0\right\rangle\right|^2=\frac{1}{4\pi^3}\left|R_{\mathcal{D}}(0)\right|^4\left|R_{\mathcal{T}}(0)\right|^2,$$

$$\sum_{m_j}\left|\left\langle T_{4c}^{2,m_j}\left|\mathcal{O}_{\bar{\mathbf{3}}\otimes\mathbf{3}}^{(2)kl}\right|0\right\rangle\right|^2=\frac{5}{4\pi^3}\left|R_{\mathcal{D}}(0)\right|^4\left|R_{\mathcal{T}}(0)\right|^2.$$

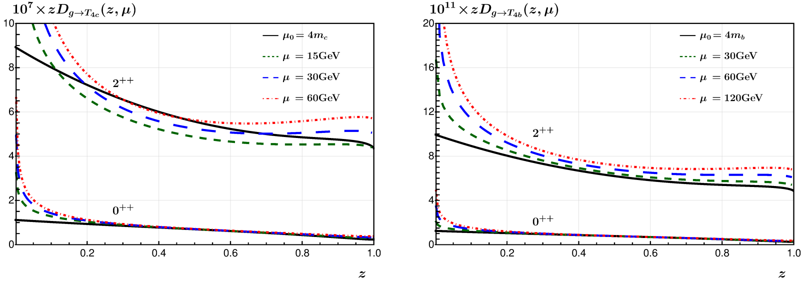
- The phenomenological results we use $(\mathrm{GeV}^{3/2})$(Kiselev *et al.*, 2002; Debastiani, Navarra, 2019):

	$R_{\mathcal{D}}(0)$	$R_{\mathcal{T}^0}(0)$	$R_{\mathcal{T}^2}(0)$
T_{4c}	0.523	2.902	2.583
T_{4b}(Coulomb)	0.703	5.57909	5.57909

Evolution of Fragmentation Function

Since the process is gluon dominance, the leading order splitting kernels read (n_f: number of active light quark flavors):

$$P_{g\leftarrow g}(z) = 6\left[\frac{(1-z)}{z} + \frac{z}{(1-z)_+} + z(1-z) + \left(\frac{11}{12} - \frac{n_f}{18}\right)\delta(1-z)\right]$$

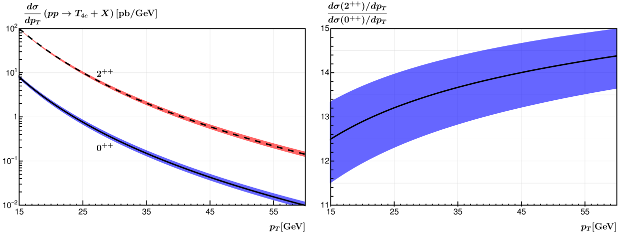


Evolution of $g \to T_{4c/4b}$ fragmentation functions.

Phenomenology at LHC

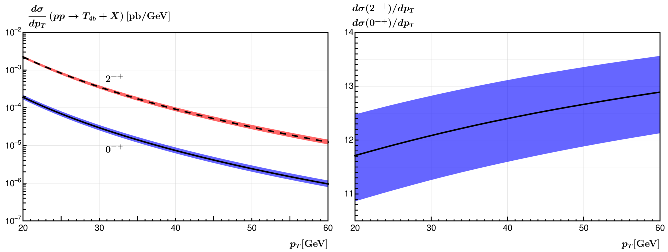
- Parameters: $\sqrt{s} = 13\,\mathrm{TeV}$; `CTEQ14` PDF sets; factorization scale $\mu \in [p_T/2, 2p_T]$
  $m_c = 1.5\,\mathrm{GeV}$

	0^{++}		2^{++}	
p_T/GeV	σ/pb	N_{events}	σ/pb	N_{events}
$[15, 60]$	33^{+4}_{-4}	$9.9^{+1.2}_{-1.2} \times 10^7$	424^{+13}_{-21}	$1.27^{+0.04}_{-0.06} \times 10^9$



- Parameters: $\sqrt{s} = 13\,\text{TeV}$; `CTEQ14` PDF sets; factorization scale $\mu \in [p_T/2, 2p_T]$
  $m_b = 4.8\,\text{GeV}$

	0^{++}		2^{++}	
p_T/GeV	$\sigma/10^{-3}\text{pb}$	$N_{\text{events}}/10^3$	$\sigma/10^{-2}\text{pb}$	$N_{\text{events}}/10^4$
$[20, 60]$	$1.04^{+0.17}_{-0.15}$	$3.12^{+0.51}_{-0.45}$	$1.24^{+0.11}_{-0.11}$	$3.72^{+0.33}_{-0.33}$



Production of T_{4c} at B Factory

Factorization Formula

The differential cross section can be expressed in terms of the differential decay rate of a virtual photon.

$$\frac{\mathrm{d}\sigma\left[e^+e^- \to \gamma(\lambda_1) + T^J_{4c}(\lambda_2)\right]}{\mathrm{d}\cos\theta} = \frac{2\pi\alpha}{s^{3/2}} \sum_{S_z=\pm 1} \frac{\mathrm{d}\Gamma\left[\gamma^*(S_z) \to \gamma(\lambda_1) + T^J_{4c}(\lambda_2)\right]}{\mathrm{d}\cos\theta}$$

$$\frac{\mathrm{d}\sigma\left[e^+e^- \to T^{1+-}_{4c}(P) + X\right]}{\mathrm{d}\cos\theta} = \frac{4\pi\alpha}{s^{3/2}} \frac{\mathrm{d}\Gamma\left[\gamma^* \to T^{1+-}_{4c}(P) + X\right]}{\mathrm{d}\cos\theta}$$

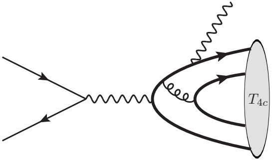
The factorization holds true at the helicity amplitude level for the exclusive process.

$$\frac{\mathrm{d}\Gamma\left[\gamma^*(S_z) \to \gamma(\lambda_1) + T^J_{4c}(\lambda_2)\right]}{\mathrm{d}\cos\theta} = \frac{|\mathbf{p}_f|}{16\pi s} \frac{3}{4\pi} \left|\mathcal{M}^J_{\lambda_1,\lambda_2}\right|^2 \left|d^1_{S_z,\lambda}(\theta)\right|^2$$

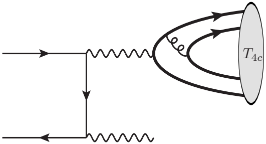
$$\mathcal{M}^J_{\lambda_1,\lambda_2} = \frac{\mathcal{A}^{3[J]}_{\lambda_1,\lambda_2}}{m_c^4} \sqrt{2M_{T_{4c}}} \langle T^J_{4c}|\mathcal{O}^{(J)}_{\bar{\mathbf{3}}\otimes\mathbf{3}}|0\rangle + \frac{\mathcal{A}^{6[J]}_{\lambda_1,\lambda_2}}{m_c^4} \sqrt{2M_{T_{4c}}} \langle T^J_{4c}|\mathcal{O}^{(J)}_{\mathbf{6}\otimes\bar{\mathbf{6}}}|0\rangle$$

Feynman Diagrams(Exclusive)

- There are roughly 40 s-channel diagrams in total.
- Due to C-parity conservation, the t-channel process in $b)$ does not contribute.



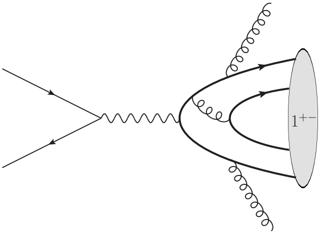
$a)$



$b)$

Feynman Diagrams(Inclusive)

- There are roughly 400 Feynman diagrams in total.
- C-parity conservation requires that two additional gluons emitted accompanied with C-odd T_{4c}



Kinematics(Inclusive)

To deduce the SDC affiliated with the differential inclusive production rate of T_{4c}^{1+-}, we need further integrate over the phase space integration of the gluons recoiling against T_{4c}^{1+-}

$$\int d\Phi_3 = \frac{s}{2(4\pi)^3} \int_{2\sqrt{r}}^{1+r} \mathrm{d}z \int_{x_1^-}^{x_1^+} \mathrm{d}x_1,$$

where $z = 2P^0/\sqrt{s}$ and $x_1 = 2k_1^0/\sqrt{s}$ are the energy fractions of the tetraquark and one gluon, respectively, $r = 16m_c^2/s$.
The integration boundaries of x_1 are

$$x_1^{\pm} = \frac{1}{2}(2-z) \pm \frac{1}{2}\sqrt{z^2-4r}$$

SDCs(Exclusive)

At large $\sqrt{s}$ limit, the polarized cross section scales as $\sigma \propto s^{-2-|\lambda|}$, which is compatible with the helicity selection rule.

$$\mathcal{A}^{3[0]}_{1,0} = \mathcal{A}^{3[0]}_{-1,0} = -\frac{16\pi^{5/2}\alpha\alpha_s\left(10-17r+9r^2\right)}{27\sqrt{3}(3-r)(2-r)},$$

$$\mathcal{A}^{6[0]}_{1,0} = \mathcal{A}^{6[0]}_{-1,0} = -\frac{16\pi^{5/2}\alpha\alpha_s\left(10-9r+r^2\right)}{9\sqrt{3}(3-r)(2-r)},$$

$$\mathcal{A}^{3[2]}_{1,0} = \mathcal{A}^{3[2]}_{-1,0} = \frac{128\pi^{5/2}\alpha\alpha_s}{27\sqrt{6}(3-r)},$$

$$\mathcal{A}^{3[2]}_{1,1} = \mathcal{A}^{3[2]}_{-1,-1} = \frac{512\pi^{5/2}\alpha\alpha_s}{27\sqrt{2}(3-r)}\left(\frac{m_c}{s^{1/2}}\right),$$

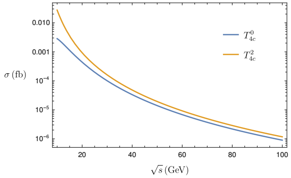
$$\mathcal{A}^{3[2]}_{1,2} = \mathcal{A}^{3[2]}_{-1,-2} = \frac{2048\pi^{5/2}\alpha\alpha_s}{27(3-r)}\left(\frac{m_c}{s^{1/2}}\right)^2.$$

SDCs(Inclusive)

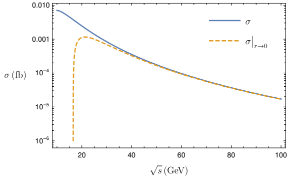
The full analytical expression is too lengthy to be presented here. We choose to present its limiting value near the upper endpoint:

$$\left.\frac{\mathrm{d}F_{\bar{\mathbf{3}}\otimes\mathbf{3}}}{\mathrm{d}z}\right|_{z\to 1+r} = \frac{2^2\pi^3\alpha^2\alpha_s^4}{3^8 s^2(3-r)^2(2-r)^2(3+r)(6+r)}$$
$$\times \Big(550800 + 482112\ln 2 - 803628r - 183168r\ln 2 + 275616r\ln r$$
$$+ 27\left(17856 - 16992r - 844r^2 + 4764r^3 - 779r^4 - 336r^5 + 70r^6 + r^7\right)\ln(2-r)$$
$$+ 16\left(-30132 + 11448r - 3897r^2 + 8403r^3 - 2489r^4 - 475r^5 + 166r^6\right)\ln(3-r)$$
$$+ 235854r^2 + 62352r^2\ln 2 + 85140r^2\ln r + 62742r^3 - 134448r^3\ln 2 - 263076r^3\ln r$$
$$- 50316r^4 + 39824r^4\ln 2 + 60857r^4\ln r + 2706r^5 + 7600r^5\ln 2 + 16672r^5\ln r$$
$$+ 1842r^6 - 2656r^6\ln 2 - 4546r^6\ln r - 27r^7\ln r\Big).$$

Phenomenology



Exlusive production



Inclusive production

The total cross sections for both exclusive and inclusive processes decline quite fast with increasing $\sqrt{s}$. If we adopt the diquark model and neglect the $\mathbf{6} \otimes \bar{\mathbf{6}}$ component as before, at the B factory energy $\sqrt{s} = 10.58$ GeV the cross sections are

$$\sigma\left[e^+e^- \to T^0_{4c} + \gamma\right] \approx 0.0026\ \mathrm{fb},$$
$$\sigma\left[e^+e^- \to T^2_{4c} + \gamma\right] \approx 0.020\ \mathrm{fb},$$
$$\sigma\left[e^+e^- \to T^1_{4c} + X\right] \approx 0.0069\ \mathrm{fb}.$$

If we adopt a four-body potential model based on hyperspherical expansion(Zhao *et al.*, 2020) to give an approximate estimation, the nonperturbative vacuum-to-tetraquark NRQCD matrix element turns to be

$$\left\langle T^{(J)}_{\text{color}}(m_j)\left|\mathcal{O}^{(J)}_{\text{color}}\right|0\right\rangle \approx \sqrt{\frac{105}{32}}\frac{1}{\pi^2}R^{(J),\text{color}}_{[4]}(0)\left\langle \mathcal{T}^{J,m_j}_{\text{color}}(m_j)\left|\mathcal{O}^{(J)}_{\text{color}}\right|0\right\rangle$$

The cross sections and the event numbers then become

	0^{++}	2^{++}	1^{+-}
σ/fb	17	14	7.3
$N_{\text{events}}/10^5$	8.6	7.0	3.7

The designed luminosity is about $50\,\text{ab}^{-1}$.

Summary

- We propose a model-independent approach to study the production of fully heavy tetraquark, based on NRQCD factorization.
- The production rates of T_{4c} appears to be significant on the LHC due to the huge luminosity.
- Different models provide contradicting predictions at `Belle 2` experiment.
- More reliable phenomenological estimate beyond the naive diquark model will be useful.


Thanks