Comprehensive study of light axial vector mesons with the presence of triangle singularity

arXiv:2103.16861

Meng-chuan Du（杜蒙川）

In collaboration with Prof. Qiang Zhao（赵强）

2021年5月18日，青岛

Content

- Introduction
- Decay with the presence of TS
- Numerical results
- Conclusion

Introduction

$q\bar{q}$ mesons

In quark model, the axial vector mesons (AVM) are states with $^{2s+1}P_J = {}^3P_1$ and $^1P_1$

$$J^{PC} = 1^{+-} \quad \begin{pmatrix} \frac{\cos\alpha_h h_1' + \sin\alpha_h h_1}{\sqrt{2}} + \frac{b_1^0}{\sqrt{2}} & b_1^+ & K_{1B}^+ \\ b_1^- & \frac{\cos\alpha_h h_1' + \sin\alpha_h h_1}{\sqrt{2}} - \frac{b_1^0}{\sqrt{2}} & K_{1B}^0 \\ K_{1B}^- & \bar{K}_{1B}^0 & -\sin\alpha_h h_1' + \cos\alpha_h h_1 \end{pmatrix}$$

$$J^{PC} = 1^{++} \quad \begin{pmatrix} \frac{\cos\alpha_f f_1' + \sin\alpha_f f_1}{\sqrt{2}} + \frac{a_1^0}{\sqrt{2}} & a_1^+ & K_{1A}^+ \\ a_1^- & \frac{\cos\alpha_f f_1' + \sin\alpha_f f_1}{\sqrt{2}} - \frac{a_1^0}{\sqrt{2}} & K_{1A}^0 \\ K_{1A}^- & \bar{K}_{1A}^0 & -\sin\alpha_f f_1' + \cos\alpha_f f_1 \end{pmatrix}$$

In PDG, the assignments and masses are

States	Mass [MeV]
$a_1(1260)$	1230 ± 40
$f_1(1285)$	1281.9 ± 0.5
$f_1(1420)$	1426.3 ± 0.9
$b_1(1235)$	1229.5 ± 3.2

States	Mass [MeV]
$h_1(1170)$	1166 ± 8
$h_1(1415)$	1416 ± 8
$K_1(1270)$	1253 ± 7
$K_1(1400)$	1403 ± 7

Open $K^*\bar{K}$ threshold and TS

- Abnormal isospin breaking in $J/\psi \to \gamma\eta(1405/1475) \to \gamma + 3\pi$

M. Ablikim et al. [BESIII], PRL 108,182001(2012)

- TS mechanism is the key

J. J. Wu, X. H. Liu, Q. Zhao and B. S. Zou, PRL 108,081803(2012)

F. Aceti, W. H. Liang, E. Oset, J. J. Wu and B. S. Zou, PRD 86,114007(2012)

N. N. Achasov, A. A. Kozhevnikov and G. N. Shestakov, PRD 92, 036003(2015)

M. C. Du and Q. Zhao, PRD 100,036005(2019)

- TS mechanism for $J/\psi \to \gamma f_1(1420) \to \gamma + 3\pi$

X. G. Wu, J. J. Wu, Q. Zhao and B. S. Zou, Phys. Rev. D 87, 014023 (2013)

- $a_1(1420)$ observed in $\pi^- p \to p + f_0\pi^- \to p + \pi^+\pi^-\pi^-$

C. Adolph et al. [COMPASS], PRL 115,082001(2015)

H. X. Cheng, E. L. Cui, W. Chen, T. G. Steele, X. Liu and S. L. Zhu, PRD 91,094022 (2015)

Qiang Zhao, Hadron-2013, Nara, Japan

M. Mikhasenko, B. Ketzer and A. Sarantsev, PRD 91,094015 (2015)

F. Aceti, L.R.Dai and E. Oset, PRD 94,096015(2016)

M. G. Alexeev et al. [COMPASS], arXiv:2006.05342(2020)

Hadronic molecules

- $f_1(1420)$ could be a $K^*\overline{K}$ molecule.

R.S.Longacre, PRD 42,874(1990)

- S-wave $K^*\overline{K}$ interaction can dynamically generate $f_1(1285), a_1(1260)$, $h_1(1170), h_1(1415)$ and $b_1(1235)$ states. It is found that $f_1(1420)$ could be accommodated by the molecule framework.

L.Roca, E.Oset and J.Singh PRD 72,014002(2005)

- $f_1(1420)$ is a kinematic effect induced by the TS of $f_1(1285)$

V. R. Debastiani, F. Aceti, W. H. Liang and E. Oset, PRD 95,034015(2017)

- $J/\psi \to \pi b_1, h_1(1170)\eta^{(\prime)}, h_1(1415)\eta^{(\prime)}$ are studied treating these mesons as dynamically generated states by the pseudoscalar and vector S-wave couplings

W. H. Liang, S. Sakai and E. Oset, PRD 99,094020(2019)

Our scheme

- The calculation based on the molecular-state picture is not fully consistent with experiment results.
- In our study we simply take these states as quark model states. The mixing angles are introduced and the production and decay channels are constrained by SU(3) flavor symmetry.
- We will then give a brief introduction to the mixing angle and the production mechanism. After that we will discuss the decay mechanism with the presence of TS.

State mixing and production

Mixing angles

$h_1, h_1', f_1, f_1', K_{1A}, K_{1B}$ are mixing objects of physical states.

$$\begin{pmatrix} K_1(1270) \\ K_1(1400) \end{pmatrix} = \begin{pmatrix} \cos\theta_{K_1} & \sin\theta_{K_1} \\ -\sin\theta_{K_1} & \cos\theta_{K_1} \end{pmatrix} \begin{pmatrix} K_{1B} \\ K_{1A} \end{pmatrix}$$

$$\begin{pmatrix} f_1' \\ f_1 \end{pmatrix} = \begin{pmatrix} \cos\theta_f & -\sin\theta_f \\ \sin\theta_f & \cos\theta_f \end{pmatrix} \begin{pmatrix} \tilde{f}_8 \\ \tilde{f}_1 \end{pmatrix} = \begin{pmatrix} \cos\alpha_f & -\sin\alpha_f \\ \sin\alpha_f & \cos\alpha_f \end{pmatrix} \begin{pmatrix} f_n \\ f_s \end{pmatrix}$$

$$\begin{pmatrix} h_1' \\ h_1 \end{pmatrix} = \begin{pmatrix} \cos\theta_h & -\sin\theta_h \\ \sin\theta_h & \cos\theta_h \end{pmatrix} \begin{pmatrix} \tilde{h}_8 \\ \tilde{h}_1 \end{pmatrix} = \begin{pmatrix} \cos\alpha_h & -\sin\alpha_h \\ \sin\alpha_h & \cos\alpha_h \end{pmatrix} \begin{pmatrix} h_n \\ h_s \end{pmatrix}$$

$\alpha = \theta + \arctan\sqrt{2}$

$$\tilde{f}_8, \tilde{h}_8 = \frac{u\bar{u} + d\bar{d} - 2s\bar{s}}{\sqrt{6}};\ \tilde{f}_1, \tilde{h}_1 = \frac{u\bar{u} + d\bar{d} + s\bar{s}}{\sqrt{3}};\ f_n, h_n = \frac{u\bar{u} + d\bar{d}}{\sqrt{2}};\ f_s, h_s = s\bar{s}$$

$|\theta_{K_1}|$ —Gell-Mann-Okubo relation→ θ_f, α_f ; θ_h, α_h

$|\theta_{K_1}| \sim 34^\circ$ *or* 57°,
$|\theta_{K_1}| < 45^\circ$ is favored ,
$\alpha_f \simeq 84.5^\circ, \alpha_h \simeq 91.8^\circ$

H. Y. Cheng, PLB 707,116

Production in e^+e^- annihilation

- $f_1(f_1')$ can be produced in $J/\psi \to \gamma f_1(f_1')$

$$\frac{g_{f'}}{g_f} = \frac{\langle f_1'\gamma|\hat{H}|J/\psi\rangle}{\langle f_1\gamma|\hat{H}|J/\psi\rangle} = \frac{\sqrt{2}\cos\alpha_f - \sin\alpha_f}{\sqrt{2}\sin\alpha_f + \cos\alpha_f}$$

$$\langle (s\bar{s})_{1^{++}}\gamma|\hat{H}|J/\psi\rangle \simeq \langle (u\bar{u})_{1^{++}}\gamma|\hat{H}|J/\psi\rangle = \langle (d\bar{d})_{1^{++}}\gamma|\hat{H}|J/\psi\rangle$$

- $J/\psi \to \gamma a_1$ is suppressed. a_1 can be searched in $\psi(3686) \to \gamma\chi_{c1} \to \gamma\pi a_1 \to \gamma + 4\pi$

$$B.R(\psi(2S) \to \gamma\chi_{c1}) = (9.79 \pm 0.2)\%$$

- b_1 and $h_1(h_1')$ can be searched in $J/\psi \to \pi b_1$, and $J/\psi \to \eta^{(\prime)} h_1^{(\prime)}$. They are constrained by SU(3) symmetry, i.e.

$$L_{\psi BP} = g_{\psi BP}\psi^{\mu}\langle B_{\mu}P\rangle$$

$$B.R(J/\psi \to \pi^{\pm} b_1^{\mp}) = (3 \pm 0.5) \times 10^{-3},$$
$$B.R.(J/\psi \to \eta' h_1' \to \eta' K^*\bar{K} + c.c.) = (2.16 \pm 0.31) \times 10^{-4}.$$

- Double-OZI suppressed process is possible for $J/\psi \to \eta^{(\prime)} h_1^{(\prime)}$

Decay with the presence of TS

TS enhanced decay channels

There could be strong impact of TS in the following channels

$f_1(f_1') \to a_0\pi \to \eta\pi\pi,$

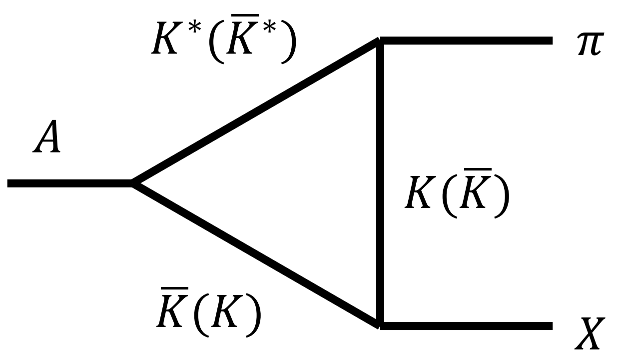
$f_1(f_1') \to a_0\pi \to K\bar{K}\pi,$

$f_1(f_1') \to f_0\pi \to 3\pi,$

$a_1 \to f_0\pi \to 3\pi,$

$b_1 \to \phi\pi \to K\bar{K}\pi,$

$h_1(h_1') \to \phi\pi \to K\bar{K}\pi,$



- $I = 0$ $K\bar{K}$ can couple to f_0 in S-wave, ϕ and ω in P-wave,

  $I = 1$ $K\bar{K}$ can couple to a_0 in S-wave, ρ in P-wave.
- There are other resonant contributions to some of these channels.
- Effective couplings of axial vector mesons are constrained by experiments and symmetry.

$f_1(f_1') \to a_0\pi \to \eta\pi\pi$

- $a_0\pi$ is one of the dominant decay channels of f_1

$$B.R.(f_1 \to a_0\pi \to \eta\pi\pi) = (38 \pm 4)\%$$

- In PDG, $f_1' \to a_0\pi$ is "possibly seen". The branching ratio of $f_1' \to a_0\pi$ is small.
- There is weak signal of f_1' in $\eta\pi^+\pi^-$ final state in pp scattering by WA102 collaboration [1].

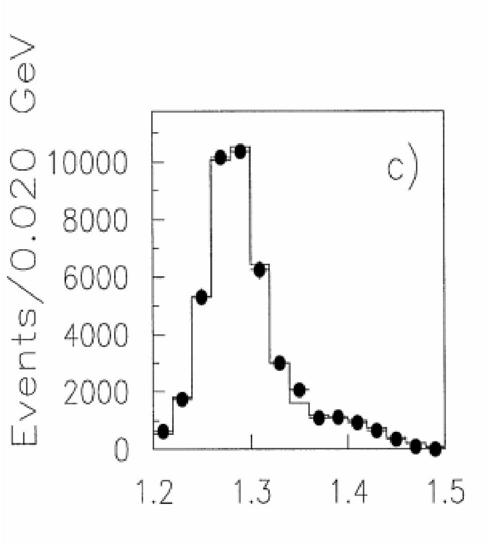
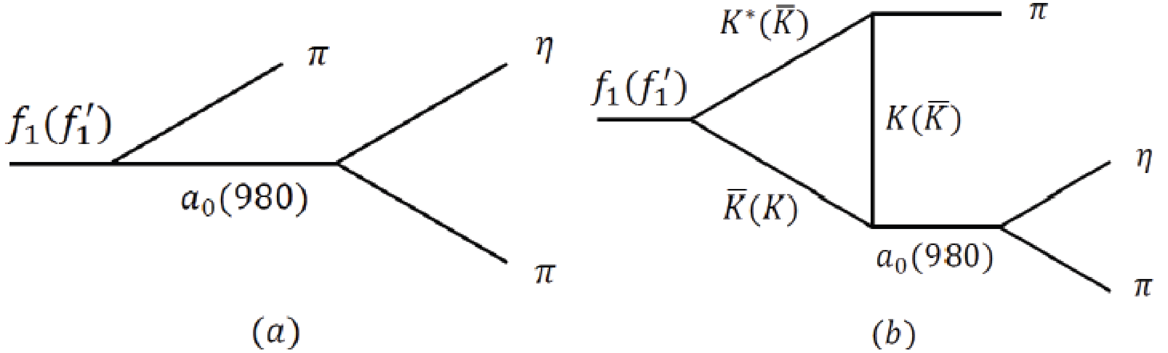


Fig.1 $J^{PC} = 1^{++}$ $a_0\pi$ wave of the $\eta\pi^+\pi^-$ spectrum in $pp \to p_f(\eta\pi^+\pi^-)p_s$. The 0^{-+} wave is suppressed in central production.[1]

[1] D. Baberis et al. [WA102], PLB 440,252

$f_1(f_1') \to a_0\pi \to \eta\pi\pi$



The interactions are given by

$$L_{AVP} = ig_{AVP}\langle A^{\mu}[V_{\mu}, P]\rangle \qquad L_{VPP} = ig_{VPP}\langle V^{\mu}[P, \partial_{\mu}P]\rangle$$

Matric elements of A, V, P are fields of axial vector mesons ($J^{PC} = 1^{++}$), vector mesons and psedoscalar mesons multiplets.

$$L_{f_1} = ig_{AVP}\frac{-2\cos\alpha_f + \sqrt{2}\sin\alpha_f}{2} f_1^{\mu}(K_{\mu}^{*0}\bar{K}^0 - \bar{K}_{\mu}^{*0}K^0 + K_{\mu}^{*+}K^- - K_{\mu}^{*-}K^+)$$

$$L_{f_1'} = ig_{AVP}\frac{\sqrt{2}\cos\alpha_f + 2\sin\alpha_f}{2} f_1'^{\mu}(K_{\mu}^{*0}\bar{K}^0 - \bar{K}_{\mu}^{*0}K^0 + K_{\mu}^{*+}K^- - K_{\mu}^{*-}K^+)$$

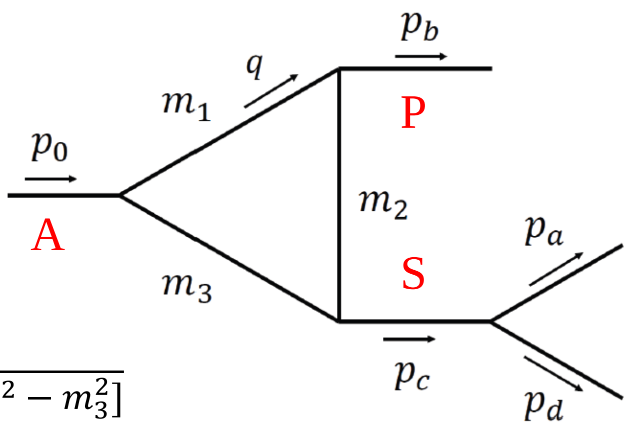
P-wave bare vertices of $f_1(f_1') \to a_0\pi$ are given by the $|n\bar{n}\rangle$ component of $f_1(f_1')$

$$L_{f_1 a_0 \pi} = g_{ASP}\sin\alpha_f \; f_1^{\mu}(\pi\partial_{\mu}a_0 - a_0\partial_{\mu}\pi)$$

$$L_{f_1' a_0 \pi} = g_{ASP}\cos\alpha_f \; f_1^{\mu}(\pi\partial_{\mu}a_0 - a_0\partial_{\mu}\pi)$$

g_{ASP} and g_{AVP} are pending determined

Loop integral for $A \to SP$



For $A \to SP$, the typical loop integral takes the following form

$$I^\mu = \int \frac{d^4q}{(2\pi)^4} \frac{\left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right)(q-2p_b)_\nu F(q^2)}{(q^2 - m_1^2 + i m_1 \Gamma_1)[(q-p_b)^2 - m_2^2][(p_0 - q)^2 - m_3^2]}$$

Monopole form factor is applied to cut the UV divergence of triangle diagrams,

$$\mathrm{F(q^2)} = \prod \frac{m_i^2 - \Lambda_i^2}{p_i^2 - \Lambda_i^2}, \Lambda_i = m_i + \beta \Lambda_{QCD}$$

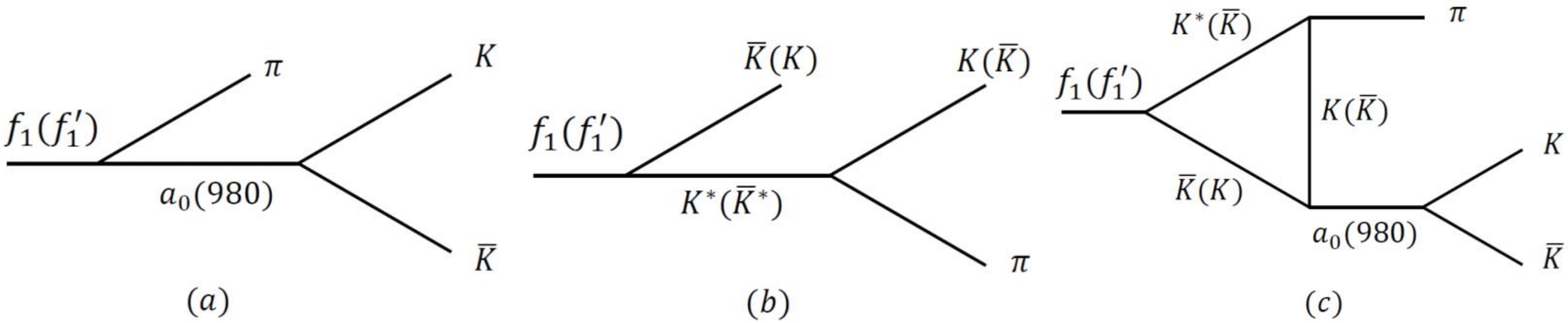
Unitarized propagator for a_0 and f_0 [1,2]

$$\frac{i}{D_{a_0}(k^2)} = \frac{i}{k^2 - m_{a_0}^2 - i \sum_{ab} g_{a_0 ab}^2 \Pi_{ab}(k^2)} \qquad ab = \{\eta\pi, K^0\bar{K}^0, K^+K^-\}$$

$$\frac{i}{D_{f_0}(k^2)} = \frac{i}{k^2 - m_{f_0}^2 - i \sum_{ab} g_{f_0 ab}^2 \Pi_{ab}(k^2)} \qquad ab = \{\pi^+\pi^-, \pi^0\pi^0, K^0\bar{K}^0, K^+K^-\}$$

[1] N.N. Achasov, A.V. Kiselev, PRD 70, 111901(2004)
[2] N.N. Achasov, A.A. Kozhevnikov, G.N. Shestakov , PRD 92,036003

$f_1(f_1') \to K\bar{K}\pi$



We know from experiments that

- $f_1 \to K\bar{K}\pi$ is dominated by P-wave $f_1 \to a_0\pi$ channel
- $f_1' \to K\bar{K}\pi$ is dominated by S-wave $f_1' \to K^*\bar{K}$ channel

$$L_{f_1 a_0 \pi} = g_{ASP} \sin\alpha_f \; f_1^\mu(\pi\partial_\mu a_0 - a_0\partial_\mu\pi)$$

$$L_{f_1' a_0 \pi} = g_{ASP} \cos\alpha_f \; f_1'^\mu(\pi\partial_\mu a_0 - a_0\partial_\mu\pi)$$

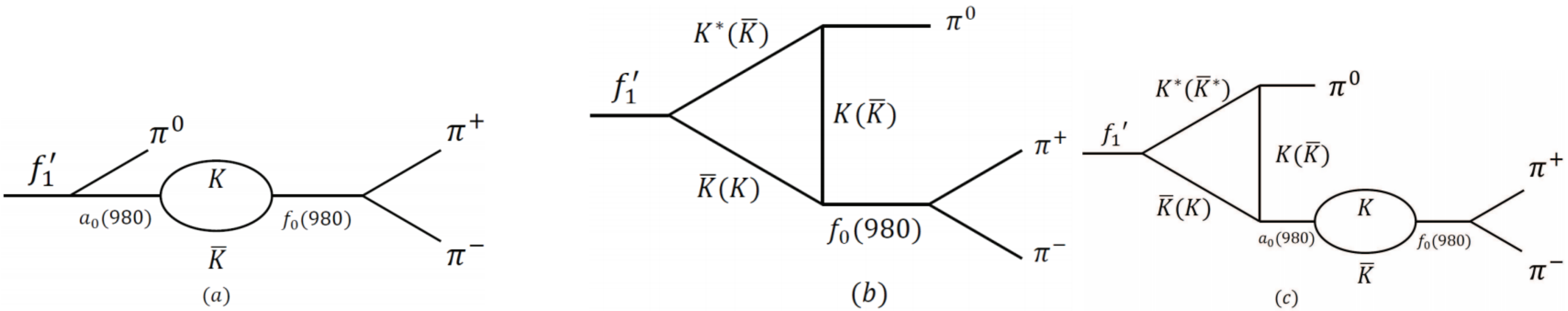
In our scheme

- $f_1 \to K^*\bar{K}$ is suppressed by the phase space
- The tree-level $f_1' \to a_0\pi$ is suppressed by mixing angle
- For f_1' decay, the interference may lead to non-trivial structure in the low-mass region of $K\bar{K}$ spectrum

To determine the couplings

- g_{AVP} can be estimated from $f_1' \to K\bar{K}\pi$
- Given g_{AVP}, g_{ASP} can be determined from $f_1 \to a_0\pi \to \eta\pi\pi$
- The phase between g_{AVP} and g_{ASP} can be fixed by $f_1 \to K\bar{K}\pi$

$f_1(f_1') \to f_0\pi \to 3\pi$



$\sim B_n - B_c$ $\sim \epsilon_\mu(I_n^\mu - I_c^\mu)$ $\sim \epsilon_\mu(I_n^\mu + I_c^\mu)(B_n - B_c)$

- Similar mechanism has been applied to $\eta(1405/1475)$ decays [1]
- The isospin violation is produced by

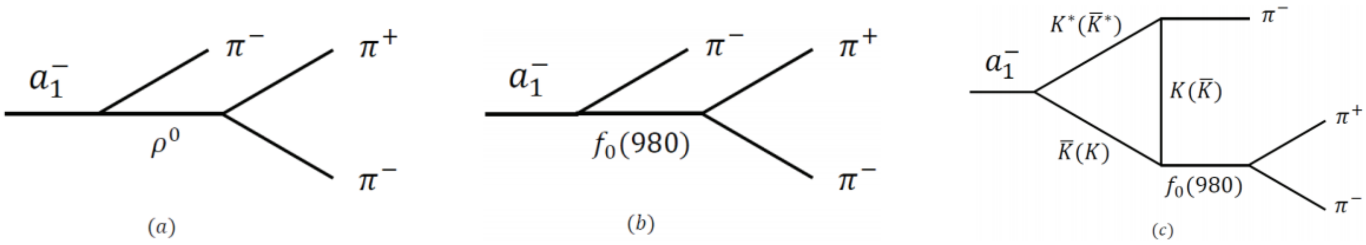
1) Direct f_0 production through triangle diagrams
2) $a_0 - f_0$ mixing, which is enhanced by triangle mechanism

- Model dependence is suppressed for

1) The dispersive parts cancel in the on-shell kinematic region.
2) The masses of a_0 and f_0 are very close to $K\bar{K}$ threshold.

[1]M. C. Du and Q. Zhao, PRD 100,036005(2019)

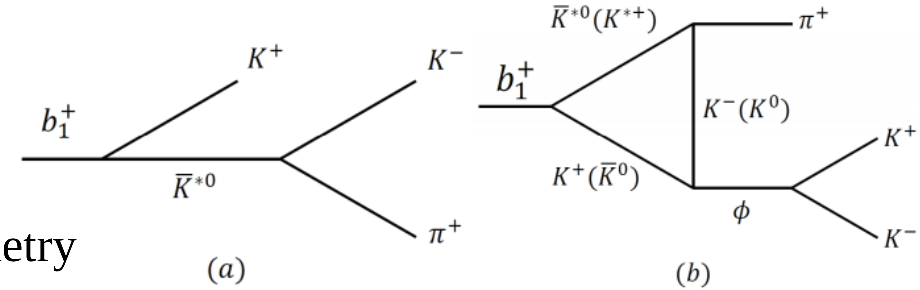
$a_1 \to 3\pi$



- The TS mechanism in $a_1(1260)$ decay has been studied in Ref[1,2]
- The interaction of $a_1 \to \rho\pi, K^*\bar{K}$ is related with $f_1' \to K^*\bar{K}$ by SU(3) symmetry.
- The tree level $a_1(1260) \to f_0\pi$ is allowed, which serves as a probe to detect the structure of $f_0(980)$. If it is treated as a scalar quark model state, the coupling of $a_1 \to f_0\pi$ can be correlated with $f_1 \to a_0\pi$.

[1] M. Mikhasenko, B. Ketzer and A. Sarantsev, PRD 91, 094015 (2015)
[2] F. Aceti, L. R. Dai and E. Oset, PRD 94, 096015 (2016)

$b_1, h_1' \to \phi\pi$



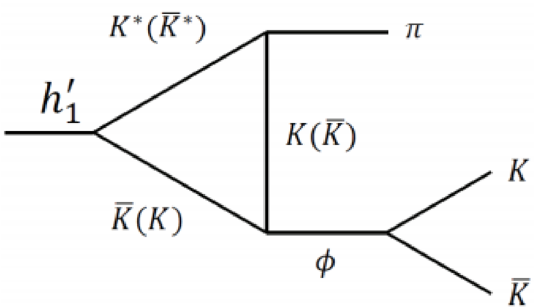
- OZI suppressed for $b_1 \to \phi\pi$.
- $B(1^{+-}) \to VP$ is constrained by symmetry

$$L_{BVP} = g_{BVP}\langle B^\mu\{V_\mu, P\}\rangle$$

g_{BVP} can be determined by $h_1 \to \rho\pi \to 3\pi$ (α_h dep.) and $b_1 \to \omega\pi$ (S-wave, α_h indep.).

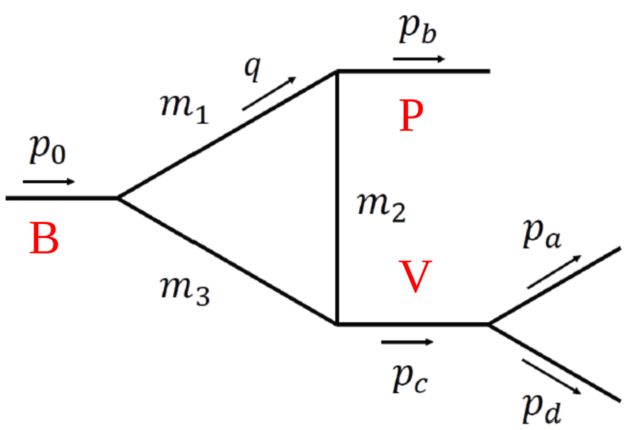
- $J/\psi \to \pi K^* \bar{K} + c.c. \to \phi\pi\pi$ and $J/\psi \to \eta K^* \bar{K} \to \eta\phi\pi$ are studied in Ref. [1].

[1] H. J. Jing, F. K. Guo, B. S. Zou, PRD 100,114010(2019)



- G parity is violated for $h_1(h_1') \to \phi\pi$
- The branching ratios of $J/\psi \to \pi^- b_1^+ + c.c. \to \phi\pi^+\pi^-$ and $J/\psi \to \eta h_1' \to \eta\phi\pi$ can be calculated.

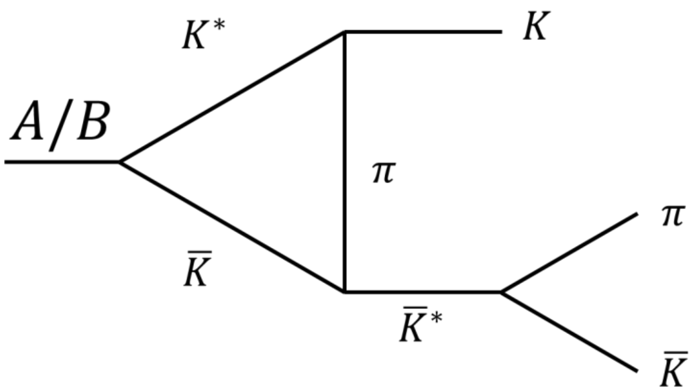
Loop integral for $B \to VP$



$$M^{loop} \sim \epsilon_\mu I^{\mu\alpha} * \frac{\left(-g_{\alpha\beta} + \frac{p_{c\alpha}p_{c\beta}}{p_c^2}\right)}{p_c^2 - m_\phi^2 + i m_\phi \Gamma_\phi}(p_a - p_d)_\beta$$

$$I^{\mu\alpha} = \int \frac{d^4q}{(2\pi)^4} * \frac{\left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right)(q - 2p_b)_\nu (p_c + 2p_b - 2q)_\alpha F(q^2)}{(q^2 - m_1^2 + i m_1 \Gamma_1)[(q - p_b)^2 - m_2^2][(p_0 - q)^2 - m_3^2]}$$

The same loop contributes to the vertex correction of $A/B \to K^*\overline{K} + c.c.$



- In NR limit

  Num.$\sim 4p_b^\mu p_b^\alpha - 4p_b^\mu q^\alpha - 2q^\alpha p_b^\alpha + 2q^\mu q^\alpha$

- Vertex correction to $A/B \to K^*\overline{K} + c.c.$ is weak due to insufficient phases space

Numerical Results

Partial widths of f_1 and f_1'

Channels	$f_1(1285)$	$f_1(1420)$
$a_0\pi \to \eta\pi\pi$ tree	7.5 ± 1.7	$(1.7 \pm 0.4) \times 10^{-1}$
$a_0\pi \to \eta\pi\pi$ tri.	$(2.0 \pm 0.6) \times 10^{-1}$	9.2 ± 3.0
$a_0\pi \to \eta\pi\pi$ total	$9.7 \pm 2.4\ (8.6 \pm 1.3)$	10 ± 3.5
$a_0\pi \to K\bar{K}\pi$ tree	$(3.7 \pm 0.4) \times 10^{-1}$	$(1.7 \pm 0.2) \times 10^{-2}$
$a_0\pi \to K\bar{K}\pi$ tri.	$(2.4 \pm 0.5) \times 10^{-2}$	1.2 ± 0.2
$a_0\pi \to K\bar{K}\pi$ total	$(5.0 \pm 0.5) \times 10^{-1}$	1.0 ± 0.1
$K^*\bar{K} \to K\bar{K}\pi$	$(3.0 \pm 0.1) \times 10^{-1}$	49 ± 1
$K\bar{K}\pi$ total	$1.5 \pm 0.2 (2.1 \pm 0.2)$	52 ± 3
$f_0\pi \to 3\pi$ mixing	$(3.6 \pm 1.2) \times 10^{-3}$	$(9.0 \pm 2.9) \times 10^{-5}$
$f_0\pi \to 3\pi$ tri.	$(1.0 \pm 0.1) \times 10^{-3}$	$(1.1 \pm 0.1) \times 10^{-1}$
$f_0\pi \to 3\pi$ mixing via tri.	$(1.4 \pm 0.4) \times 10^{-4}$	$(8.0 \pm 2.6) \times 10^{-3}$
$f_0\pi \to 3\pi$ total	$(1.0 \pm 0.3) \times 10^{-2}$	$(1.6 \pm 0.3) \times 10^{-1}$

Tab.1 Partial width in MeV, at $g_{AVP} = 2.04$ GeV , $\beta = 2$. Only the uncertainties of g_{VPP} and g_{SPP} couplings are considered. The PDG averaged value are in round brackets.

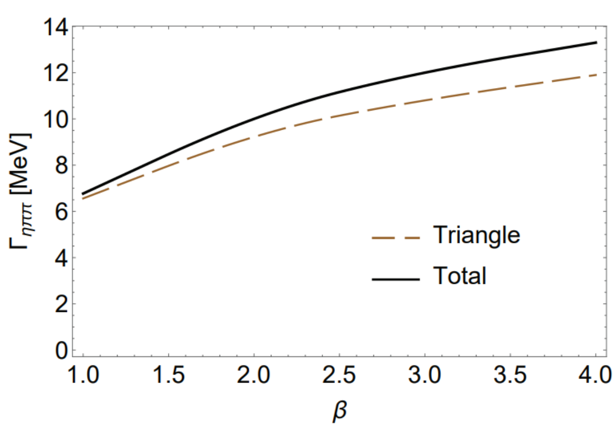


Fig.2 β dependence of $\Gamma(f_1' \to a_0\pi \to \eta\pi\pi)$

Couplings	Value
$g_{f_0K^+K^-}$	5.96 ± 0.13 GeV
$g_{f_0\pi^+\pi^-}$	2.96 ± 0.12 GeV
$g_{a_0K^+K^-}$	2.24 ± 0.11 GeV
$g_{a_0\eta\pi}$	3.02 ± 0.35 GeV
g_{AVP}	2.04 GeV
$g_{f_1a_0\pi}$	2.93
$g_{f_1'a_0\pi}$	0.282
$g_{f_1K^*\bar{K}}$	1.24 GeV
$g_{f_1'K^*\bar{K}}$	2.17 GeV
$g_{a_1K^*\bar{K}}$	2.04 GeV
$g_{a_1f_0\pi}$	2.93

Tab.2 Coupling constants in 1^{++} sector, where the SPP couplings are adopted from Ref. [1,2].

Couplings	Value
g_{BVP}	3.03 GeV
$g_{h_1\rho\pi}$	4.28 GeV
$g_{h_1'\rho\pi}$	0.13 GeV
$g_{h_1'K^*\bar{K}}$	3.09 GeV
$g_{b_1K^*\bar{K}}$	3.03 GeV
$g_{b_1\omega\pi}$	4.28 GeV
$g_{\psi BP}$	4.35×10^{-3} GeV

Tab.3 Coupling constants in 1^{+-} sector

[1] A.Aloisio et al. [KLOE], PLB537,21 (2002);
[2] A.Aloisio et al. [KLOE], PLB536,209(2002)

Spectra in J/ψ decay

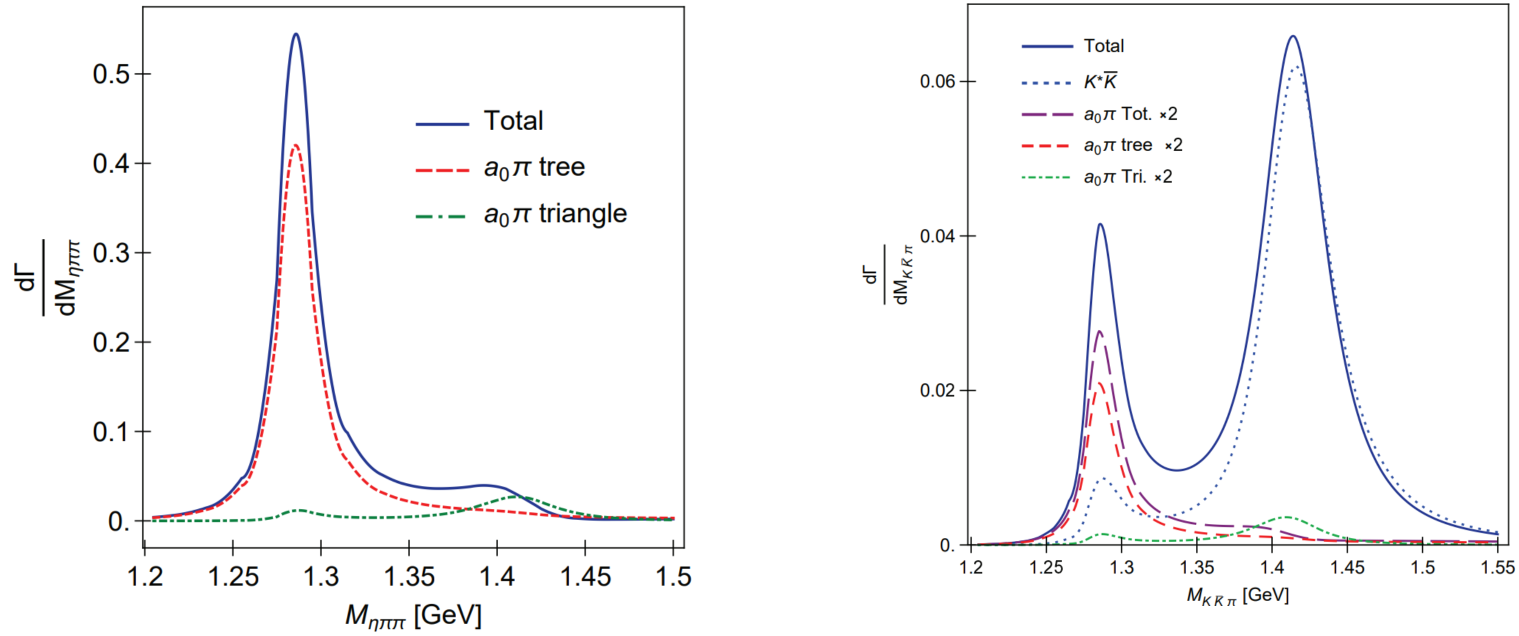


Fig.3 $\eta\pi\pi$ (left) and $K\bar{K}\pi$ spectra of $J/\psi \to \gamma(f_1 + f_1') \to \gamma\eta\pi\pi, \gamma K\bar{K}\pi$.

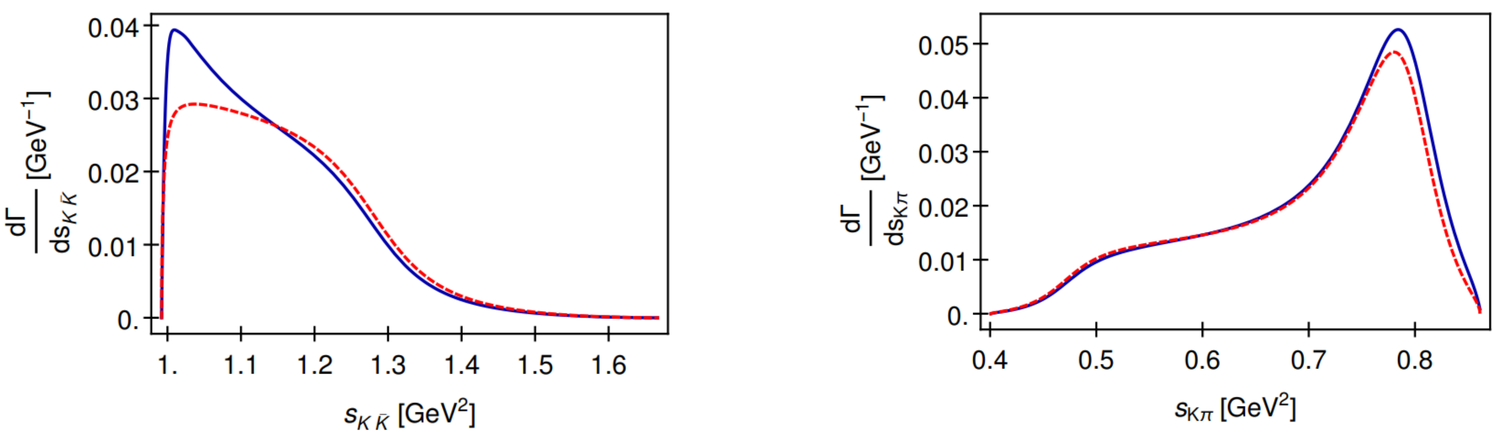


Fig.4 $K\bar{K}$ (left) and $K\pi$ (right) spectra of $f_1' \to K\bar{K}\pi$

$f_1(f_1') \to 3\pi$

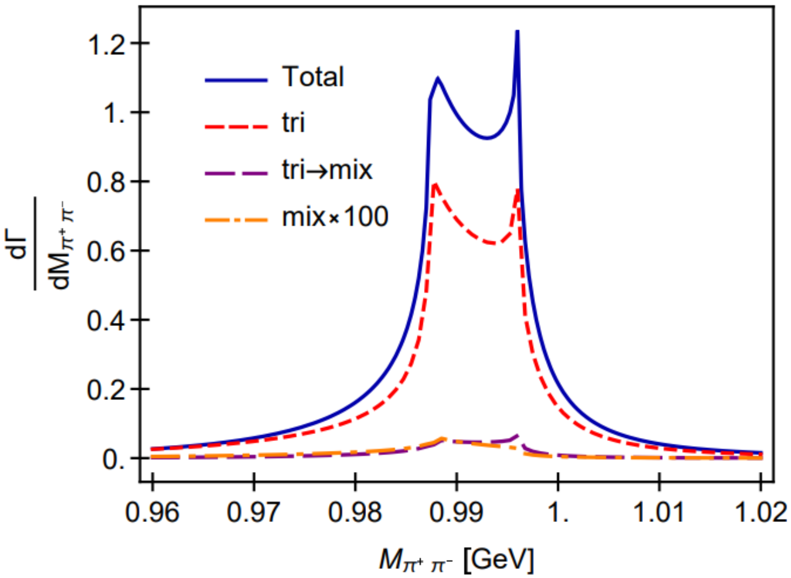


Fig.5 $\pi^+\pi^-$ spectrum in $f_1' \to \pi^+\pi^-\pi^0$.

$$R_{f_1'\to 3\pi} = \frac{B.R.(f_1'\to f_0\pi\to\pi^+\pi^-\pi^0)}{B.R.(f_1'\to a_0\pi\to\eta\pi^0\pi^0)} \sim 5\%.$$

$$R^{total}_{f_1\to 3\pi} = \frac{B.R.(f_1\to f_0\pi\to\pi^+\pi^-\pi^0)}{B.R.(f_1\to a_0\pi\to\eta\pi^0\pi^0)} \sim 0.3\%$$

$$R^{mix}_{f_1\to 3\pi} = \frac{B.R.(f_1\to f_0\pi\to\pi^+\pi^-\pi^0)}{B.R.(f_1\to a_0\pi\to\eta\pi^0\pi^0)} \sim 0.1\%$$

Our result is consistent with BESIII measurement [1]

$$R_{\chi_{c1}\to 3\pi} = \frac{B.R.(\chi_{c1}\to a_0\pi\to f_0\pi\to\pi^+\pi^-\pi^0)}{B.R.(\chi_{c1}\to a_0\pi\to\eta\pi^0\pi^0)}$$
$$= \left(0.31 \pm 0.16(stat.) \pm 0.14(sys.) \pm 0.03(para.)\right)\%$$

The large isospin violation from Ref.[2] requires

$$R_{f_1\to 3\pi} = \frac{B.R.(f_1\to\pi^+\pi^-\pi^0)}{B.R.(f_1\to a_0\pi\to\eta\pi^0\pi^0)} = (2.5 \pm 0.9)\%$$

Single specific mechanism ($a_0 - f_0$ mixing, $\kappa\bar{K}$, $K^*\bar{K}$ rescattering) cannot explain the large isospin violation of f_1 [3].

[1] M. Ablikim et al., PRD 83, 032003 (2011). [2] V. Dorofeev et al., EPJ A47,68(2011)
[3] N.N. Achasov, G.N.Shestakov, Nucl.Part.Phys.Proc.287,89(2017)

$a_1 \to 3\pi$

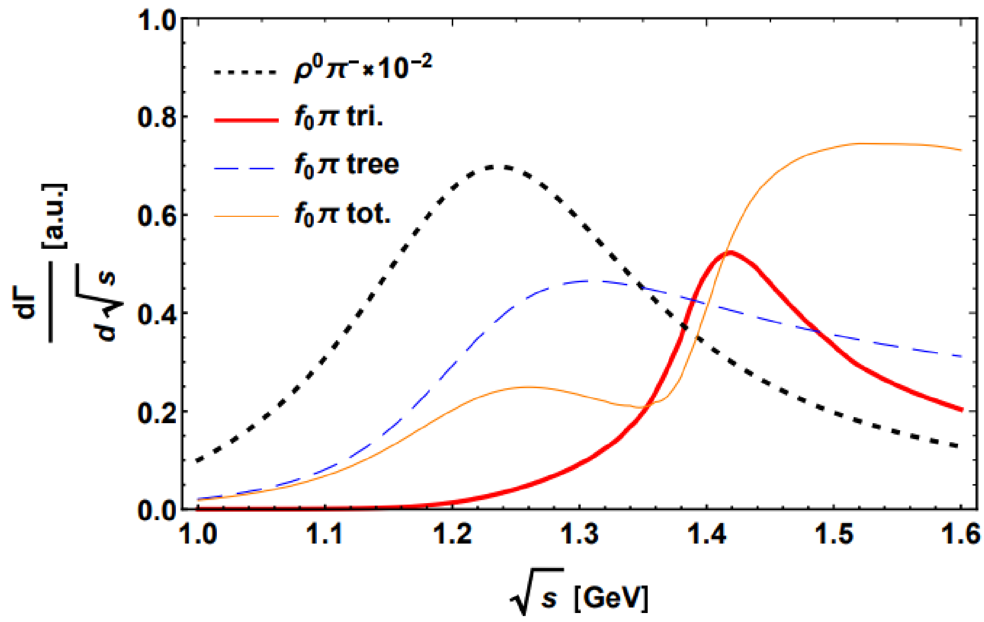


Fig.6 a_1 spectrum of $\chi_{c1} \to \pi a_1$

$\chi_{c1} \to \pi a_1 \to 4\pi$ can be factorized for the large momentum of the pion recoiling against a_1. The intensity and width of $a_1 \to f_0\pi$ through triangle mechanism are consistent with Ref. [1][2]

$$\frac{a_1 \to f_0\pi}{a_1 \to \rho\pi} \simeq 0.6\% \qquad \Gamma \simeq 150 \text{ MeV}$$

[1] M. Mikhasenko, B. Ketzer and A. Sarantsev, PRD 91,094015 (2015)

[2] F. Aceti, L.R.Dai and E. Oset, PRD 94,096015(2016)

$h_1' \to \phi\pi$

$$B.R.(J/\psi \to h_1'\eta \to \eta\phi\pi) = 6.3 \times 10^{-8}$$

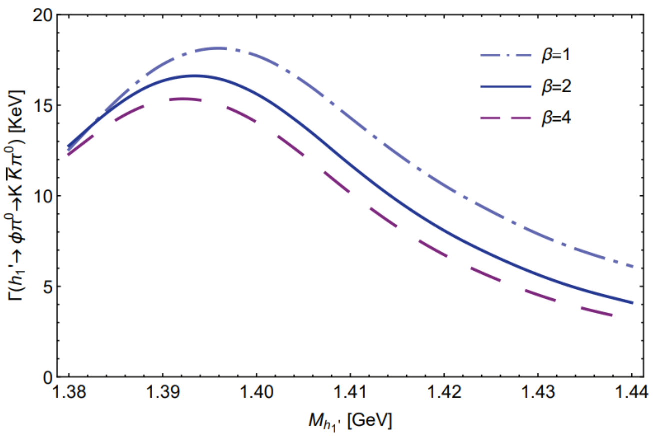


Fig.7 Partial width of $h_1' \to \phi\pi^0 \to K\bar{K}\pi^0$ in unit of KeV

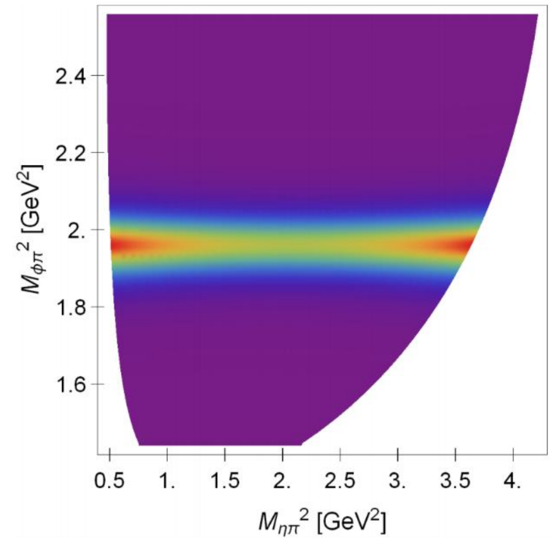


Fig.8 Dalitz plot of $J/\psi \to \eta h_1' \to \eta\phi\pi$ with the resonance of h_1'

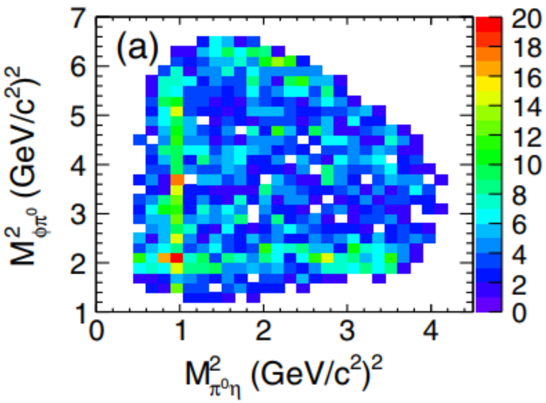


Fig.9 Dalitz plot of $J/\psi \to \eta\phi\pi$ from Ref.[1]

[1] M. Ablikim, et al. PRL,121,022001(2018)
[2] H. J. Jing, F. K. Guo, B. S. Zou, PRD 100,114010(2019)

$b_1 \to \phi\pi$

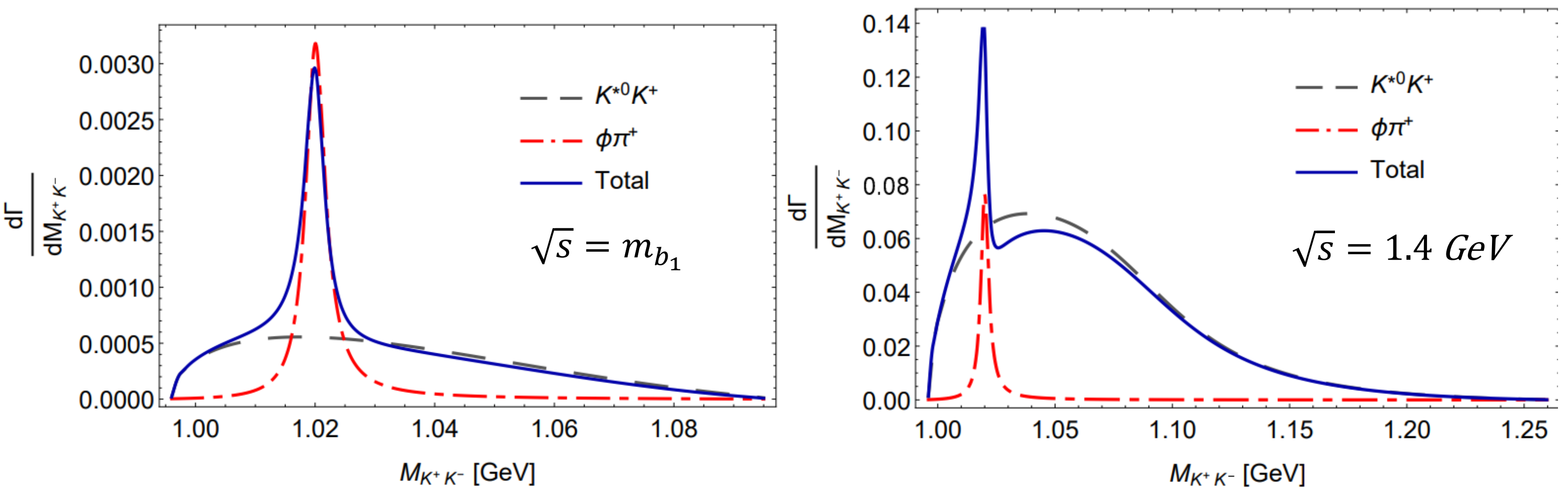
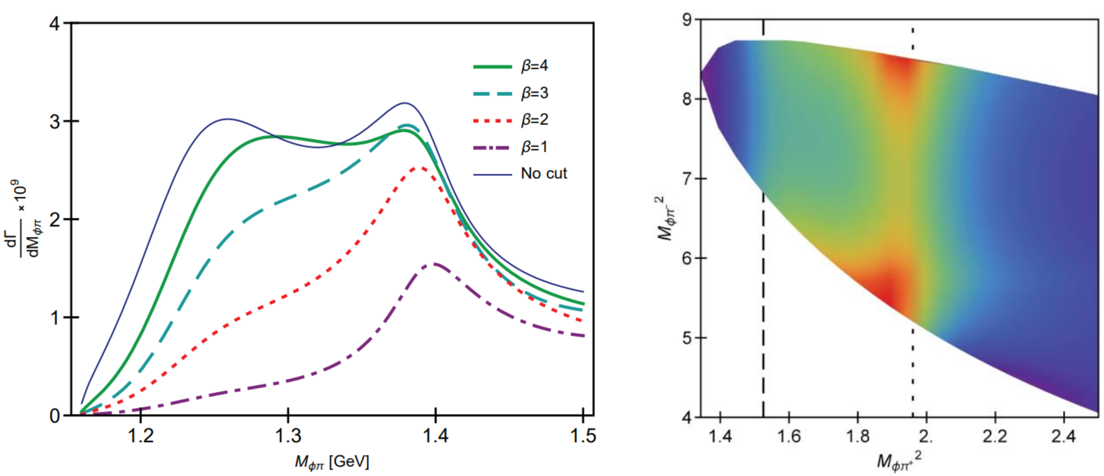


Fig.10 K^+K^- spectra of $b_1^+ \to K^+K^-\pi^+$ evaluated at $\sqrt{s} = m_{b_1}$ (left) and $\sqrt{s} = 1.4$ GeV (right). The $\phi\pi^+$ signal is produced by OZI evading triangle mechanism.



On-shell branching ratio (at $\beta = 2$)

$$\frac{B.R.(b_1^+ \to \phi\pi^+ \to K\bar{K}\pi^+)}{B.R.(b_1^+ \to \omega\pi^+)(S-wave)} = 1.8 \times 10^{-4}$$

Estimation without TS:

$$B.R.^{est.}\left(J/\psi \to \pi^\pm b_1^\mp \to \phi\pi^+\pi^-\right) \simeq 1 \times 10^{-6}$$

Actual branching ratio:

$$B.R.\left(J/\psi \to \pi^\pm b_1^\mp \to \phi\pi^+\pi^-\right) \simeq 1 \times 10^{-5}$$

Fig.11 $\phi\pi^+$ spectrum (left) and the Dalitz plot (right) of $J/\psi \to \pi^\pm b_1^\mp \to \phi\pi^+\pi^-$

$b_1 \to \phi\pi$

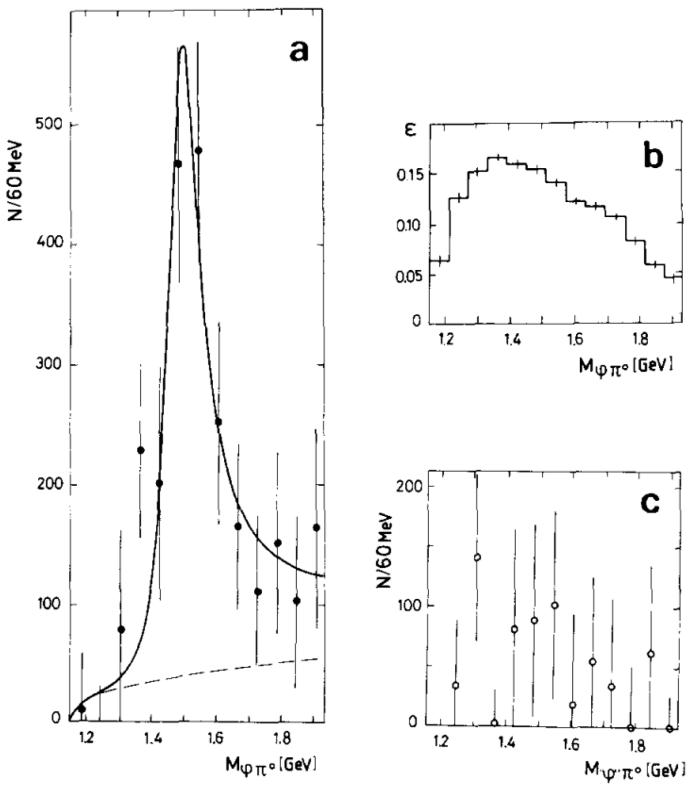


Fig.12 (a) The acceptance-corrected $\phi\pi^0$ mass spectrum in $\pi^- p \to \phi\pi^0 n$. (b) The acceptance of the Lepton-F spectrometer for $\phi\pi^0$ events. (c) The false "ϕ"π^0 mass spectrum *[1]*.

"C(1480)" is reported with $I = 1$ $J^{PC} = 1^{--}$, by OPE.
$M = 1480 \pm 40$ MeV, $\Gamma = 130 \pm 60$ MeV *[1]*

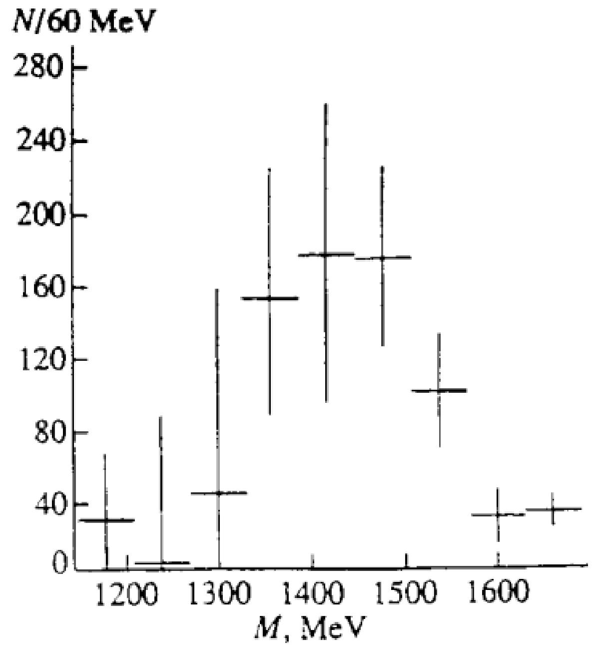


Fig.13 Effective mass spectrum of the $\phi\pi^0$ system in the reaction $\pi^- p \to \phi\pi^0 n$ (the results are weighted with detector efficiency) [2,3]. The "anti-OPE" selection $|t'| > 0.1$ GeV^2 is applied, which affects only slightly the efficiency for $\pi^- p \to b_1 n$ (not proceeding via OPE exchange), but which reduces the background from the OPE-mediated reaction $\pi^- p \to C(1480) n$ by a factor of 5 [2,3].

[1] Bityukov S.I. et al. Phys. Lett. B 118,383 (1987)
[2] V. A. Viktorov et al. Phys. Atom. Nucl. 59,1184 (1996)
[3] S.V. Golovkin et al. Z. Phys. A 359,435 (1997)

Conclusions

Conclusion

- Our combined study taken into account the TSM is consistent with taking the axial vector mesons as quark model states
- Particularly, in our study, only TSM of $f_1(1285)$ cannot account for $f_1(1420)$, but the TSM of $a_1(1260)$ can produce $a_1(1420)$.
- TSM produces non-trivial features in the spectra of $J/\psi \to \gamma(f_1 + f_1') \to \gamma\eta\pi\pi$ and $\gamma K\bar{K}\pi$, in both the three-body and the two-body distributions.
- TSM also plays a role in $C = -1$ sector, i.e. b_1 and h_1, where we predicted the branching ratio of OZI suppressed $J/\psi \to b_1\pi \to \phi\pi\pi$ and Isospin violated $J/\psi \to h_1(1425)\eta \to \phi\pi\eta$ induced by TSM.

Thank you for your attention