# X atom

张振华
中国科学院理论物理研究所

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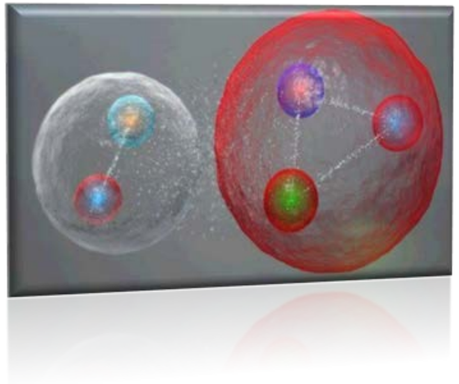
Based on Z.-H. Zhang, F.-K. Guo. arXiv:2012.08281

# X atom: Background

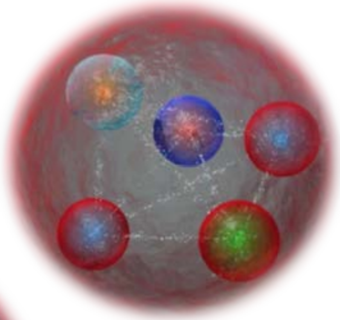


Exotic hadrons are the hadrons beyond the quark model.

XYZ states, Glueballs, Pentaquarks…

$X(3872)$ is one of the most important XYZ states

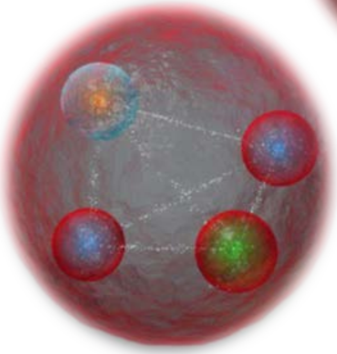
$X(3872)$ is first discovered in the $J/\psi\pi^+\pi^-$ invariant mass distribution by Belle Collaboration in 2003, with $I^G J^{PC}=0^+(1^{++})$ , $m_X=(3871.69\pm0.17)$ MeV



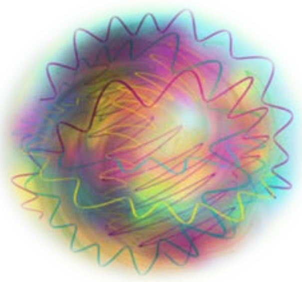
Salient features: (a) $\delta=m_{D^0}+m_{D^{*0}}-m_X=(0.00\pm0.18)$ MeV

(b) $\mathcal{B}\left(X(3872)\to D^0\bar{D}^0\pi^0\right)>40\%$

$\mathcal{B}\left(X(3872)\to D^0\bar{D}^{*0}\right)>30\%$



At long distance, $D^0\bar{D}^{*0}$ is dominant in $X(3872)$



$$|X(3872)\rangle=\frac{1}{\sqrt{2}}\left(|D^0\bar{D}^{*0}\rangle-|\bar{D}^0D^{*0}\rangle\right)\qquad\begin{pmatrix}\mathcal{C}|D\rangle=|\bar{D}\rangle\\ \mathcal{C}|D^*\rangle=-|\bar{D}^*\rangle\end{pmatrix}$$

# X atom: Introduction

Typical size for the $X(3872)$ at long distance: $r_X \simeq \frac{1}{\sqrt{2\mu_c^0\delta}} \gtrsim 10\ \mathrm{fm}$

Typical size (Bohr radius) for the $D^+D^{*-}$ bound state: $r_B = \frac{1}{\alpha\mu_c} = 27.86\ \mathrm{fm}$

$$\mu_0 = \frac{m_{D^0}m_{D^{*0}}}{\Sigma_0} \quad \mu_c = \frac{m_D m_{D^*}}{\Sigma_c} \quad \Sigma_0 = m_{D^0} + m_{D^{*0}} \quad \Sigma_c = m_D + m_{D^*} = (3879.91 \pm 0.07)\ \mathrm{MeV}$$

Coulomb binding energies: $-E_n = -\frac{\alpha^2\mu_c}{2n^2} = \frac{-E_1}{n^2} = -\frac{25.81\ \mathrm{keV}}{n^2}$

X atom: The ground state $\frac{1}{\sqrt{2}}(|D^+D^{*-}\rangle - |D^-D^{*+}\rangle)$ atom with $C = +$

Scale separation: $r_B\Lambda_{\mathrm{QCD}} \gg 1$ , strong interaction between $D^+D^{*-}$ is a correction

Effects of strong interaction at LO:

(a) Energy level shift: $\Delta E_n^{\mathrm{str}} \sim \mathcal{O}(\alpha^3)$ (b) Decay modes: $D^0\bar{D}^{*0}, D^0\bar{D}^0\pi^0, J/\psi\pi\pi, \cdots$

The strong interaction is non-perturbative due to the existence of the $X(3872)$

Only hadronic atoms with light quarks have been studied

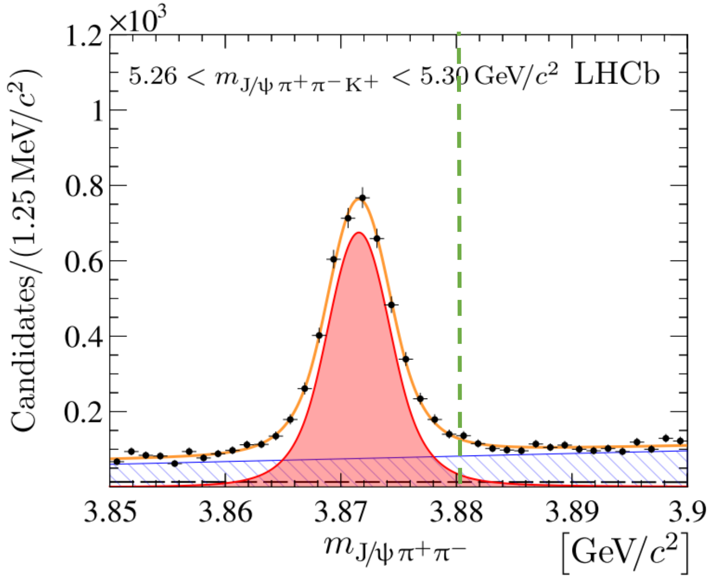
Gasser, Lyubovitskij, Rusetsky, *Phys.Rept.* 456 (2008)

# X atom: Introduction

The X atom is related to the $X(3872)$ (as a hadronic molecule) by isospin symmetry

$D^+D^{*-}$ threshold: $\Sigma_c = m_D + m_{D^*} = (3879.91 \pm 0.07)$ MeV

No Signal near the $D^+D^{*-}$ threshold




LHCb, JHEP 08 (2020) 123

Make use of the zero signal to:

- Put a lower bound on the $X(3872)$ binding energy
- Give a criterion on the $X(3872)$ nature

Scale separation: $r_B\Lambda_{\mathrm{QCD}} \gg 1$; Nonrelativistic effective field theory (NREFT) applicable

Approximation: Isospin-1 strong interaction neglected

- No isovector state was found
- Isospin breaking in the couplings is small $\frac{g_{X\rho}}{g_{X\omega}} = 0.26^{+0.08}_{-0.05}$

Hanhart et al. , *Phys.Rev.D* 85 (2012) 011501

# X atom: NREFT

**Coupled channel:** $\mathbf{CH\,1}: D^+D^{*-} \to D^+D^{*-} \quad \mathbf{CH\,2}: D^0\bar{D}^{*0} \to D^0\bar{D}^{*0}$

**Non-relativistic effective Lagrangian: Galilean, Gauge invariant; C, P, T**

**Around threshold, LO Lagrangian: constant contact terms for strong interactions**

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \sum_{\phi=D^\pm,D^0,\bar{D}^0} \phi^\dagger\left(iD_t - m_\phi + \frac{\nabla^2}{2m_\phi}\right)\phi + \sum_{\phi=D^{*\pm},D^{*0},\bar{D}^{*0}} \phi^\dagger\left(iD_t - m_\phi + i\frac{\Gamma_\phi}{2} + \frac{\nabla^2}{2m_\phi}\right)\phi$$
$$-\frac{C_0}{2}\left(D^+D^{*-} - D^-D^{*+}\right)^\dagger\left(D^+D^{*-} - D^-D^{*+}\right) - \frac{C_0}{2}\left[\left(D^+D^{*-} - D^-D^{*+}\right)^\dagger\left(D^0\bar{D}^{*0} - \bar{D}^0D^{*0}\right) + \text{h.c.}\right]$$
$$-\frac{C_0}{2}\left(D^0\bar{D}^{*0} - \bar{D}^0D^{*0}\right)^\dagger\left(D^0\bar{D}^{*0} - \bar{D}^0D^{*0}\right) + \cdots$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \qquad D_t\phi = \partial_t\phi \mp iQA_0\phi$$

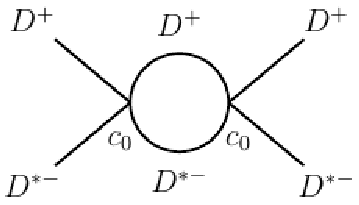
**Constant width approximation for $D^*$**

Hanhart, Kalashnikova, Nefediev, *Phys.Rev.D* 81 (2010) 094028

# X atom: NREFT

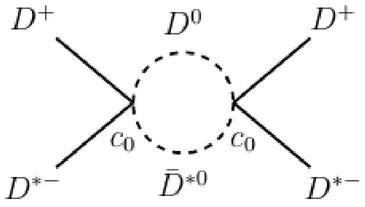
***S*-wave *T*-matrix for** $I^G J^{PC} = 0^+(1^{++})$ **coupled channel:** $T(E) = V[1 - G(E)V]^{-1}$

**Strong contact term:** $V = C_0 \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ **Green's function:** $G(E) = \begin{pmatrix} J_c(E) + J_{|\Psi\rangle}(E) & 0 \\ 0 & J_0(E) \end{pmatrix}$



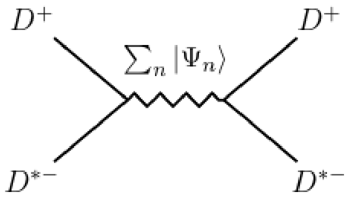

$$J_c(E) = \frac{\mu_c}{2\pi}\left(-\frac{2\Lambda}{\pi} + \sqrt{-2\mu_c(E + i\Gamma_c/2)}\right)$$

$$E = \sqrt{s} - \Sigma_c$$




$$J_0(E) = \frac{\mu_0}{2\pi}\left(-\frac{2\Lambda}{\pi} + \sqrt{-2\mu_0(E + \Delta + i\Gamma_0/2)}\right)$$

$$\Delta = \Sigma_c - \Sigma_0$$



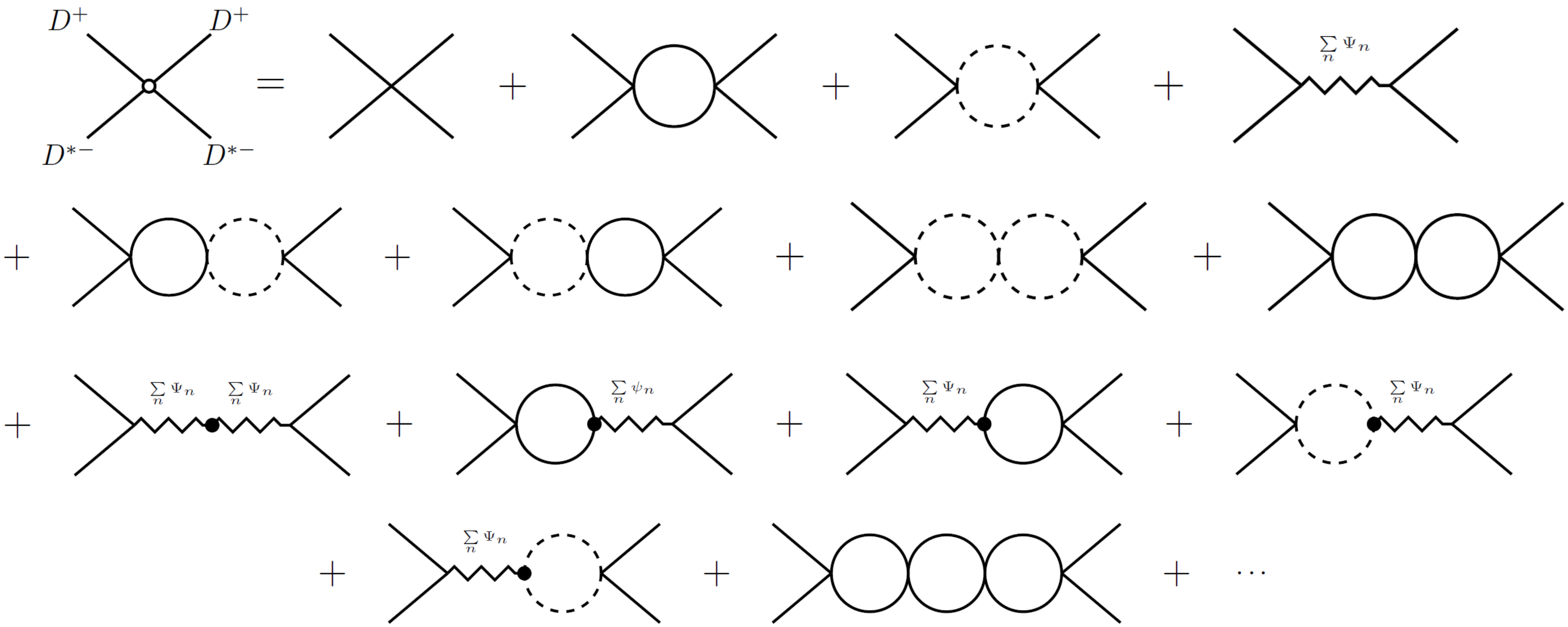

$$J_{|\Psi\rangle}(E) = \sum_{n=1}^{\infty} \frac{\alpha^3 \mu_c^3}{\pi n^3} \frac{1}{E + E_n + i\Gamma_c/2}$$

$$\Gamma_c \equiv \Gamma_{D^*}, \quad \Gamma_0 \equiv \Gamma_{D^{*0}}$$

# X atom: NREFT

***S*-wave *T*-matrix for $I^G J^{PC} = 0^+(1^{++})$ coupled channel:**

$$T(E) = \frac{1}{C_0^{-1} - \left[J_0(E) + J_c(E) + J_{|\Psi\rangle}(E)\right]} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$



# X atom: Strong Energy Level Shift

***S*-wave *T*-matrix for $I^G J^{PC} = 0^+(1^{++})$ coupled channel:**

$$T(E) = \frac{1}{C_0^{-1} - \left[J_0(E) + J_c(E) + J_{|\Psi\rangle}(E)\right]} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

**Renormalization:** $C_{0R}^{-1} = C_0^{-1} + \Lambda(\mu_0 + \mu_c)/\pi^2$

**The $X(3872)$ and hadronic atoms appear as poles of the *T*-matrix**

**$X(3872)$ pole:** $E = -\Delta - \delta - i\dfrac{\Gamma_0}{2}$ $\qquad \delta\Gamma = \Gamma_c - \Gamma_0$

$$C_{0R}^{-1} = \frac{\mu_0}{2\pi}\sqrt{2\mu_0\delta} + \frac{\mu_c}{2\pi}\sqrt{2\mu_c\left(\Delta + \delta - i\frac{\delta\Gamma}{2}\right)} - \sum_{n=1}^{\infty}\frac{\alpha^3\mu_c^3}{\pi n^3}\frac{1}{\Delta + \delta - E_n - i\delta\Gamma/2} = \frac{\mu_c}{2\pi}\sqrt{2\mu_c\Delta}\left[1 + \mathcal{O}\left(\frac{\delta}{\Delta}, \frac{\delta\Gamma}{\Delta}, \frac{\alpha^3\mu_c^{3/2}}{\Delta^{3/2}}\right)\right]$$

***S*-wave hadronic atom poles:** $E = -E_{An} - i\dfrac{\Gamma_c}{2}$

$$0 = C_{0R}^{-1} + i\frac{\mu_0}{2\pi}\sqrt{2\mu_0\left(\Delta - E_{An} - i\frac{\delta\Gamma}{2}\right)} - \frac{\mu_c}{2\pi}\sqrt{2\mu_c E_{An}} - \sum_{n=1}^{\infty}\frac{\alpha^3\mu_c^3}{\pi n^3}\frac{1}{-E_{An} + E_n}$$

# X atom: Strong Energy Level Shift

**Strong energy level shift:** $\Delta E_n = E_{An} - E_n$

$$\Delta E_n = \frac{2\alpha^3\mu_c^2}{n^3\sqrt{2\mu_c\Delta}}\left[-1 - i + \mathcal{O}\left(\alpha\sqrt{\frac{\mu_c}{\Delta}}\right)\right]^{-1}$$

***S*-wave hadronic atom poles:** $E = -E_{An} - i\frac{\Gamma_c}{2} = -E_n - \Delta E_n - i\frac{\Gamma_c}{2}$

**Ground state:** $n = 1$

**Binding energy:** $\mathrm{Re}\, E_{A1} = E_1 - \frac{\alpha^3\mu_c^2}{\sqrt{2\mu_c\Delta}} \simeq 22.92\ \mathrm{keV}$ $\qquad M_{A1} = (3879.89 \pm 0.07)\ \mathrm{MeV}$

**Decay width:** $\Gamma_c + 2\,\mathrm{Im}\, E_{A1} = \Gamma_c + \frac{2\alpha^3\mu_c^2}{\sqrt{2\mu_c\Delta}} = (89.2 \pm 1.8)\ \mathrm{keV}$

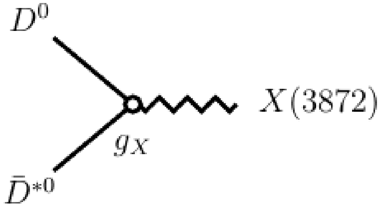
$D^* \to D\pi, D\gamma, \cdots \qquad \Gamma_c = (83.4 \pm 1.8)\ \mathrm{keV}$

$A\ (\mathrm{X\ atom}) \to D^0\bar{D}^{*0}(\bar{D}^0 D^{*0}) \qquad \Gamma_s = 2\mathrm{Im}E_{A1} = 5.8\ \mathrm{keV}$

# X atom: Effective Coupling

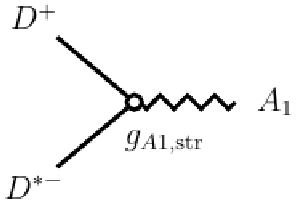
**The effective coupling squared is the residue of the *T*-matrix at the pole**

$D^0\bar{D}^{*0} \to X(3872)$ $X(3872)$ **pole:** $E = -\Delta - \delta - i\frac{\Gamma_0}{2}$




$$g_X^2 = \lim_{E\to -\Delta-\delta-i\frac{\Gamma_0}{2}} \left(E + \Delta + \delta + i\frac{\Gamma_0}{2}\right) T_{22}(E) = \frac{2\pi}{\mu_0^2}\sqrt{2\mu_0\delta}\left[1 + \mathcal{O}\left(\frac{\delta^{1/2}}{\Delta^{1/2}}\right)\right]^{-1}$$

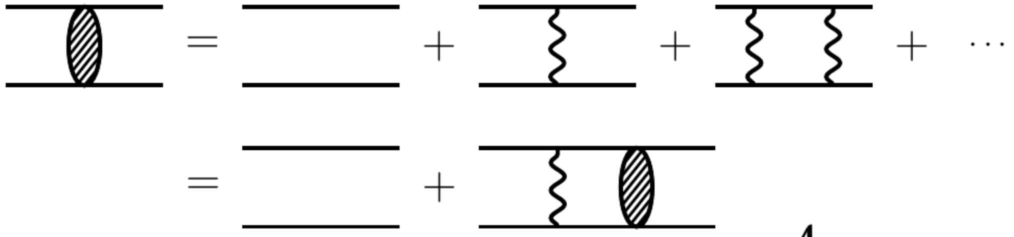
$D^+D^{*-} \to A_1$ **Hadronic atom poles:** $E = -E_{An} - i\frac{\Gamma_c}{2}$

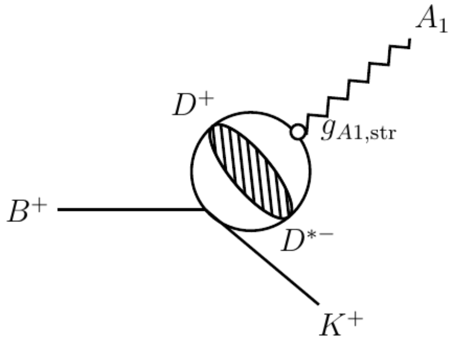


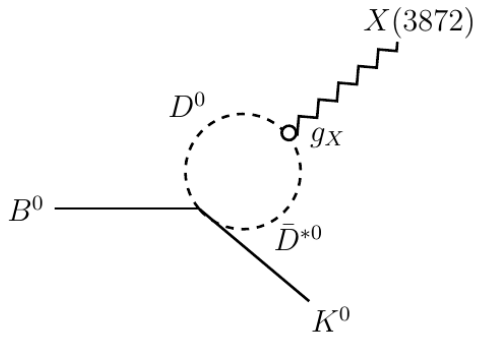

$$g_{A1,\mathrm{str}}^2 = \lim_{E\to -E_{A1}-i\frac{\Gamma_c}{2}} \left(E + E_{A1} + i\frac{\Gamma_c}{2}\right) T_{11}(E) = -i\frac{\pi\alpha^3}{\Delta}\left[1 + \mathcal{O}\left(\frac{\alpha^2\mu_c}{\Delta}\right)\right]^{-1}$$

# X atom: Production

## Production in exclusive *B* decays:






$$B^+ \to (DD^*)_+ K^+ \to A_1 K^+ \qquad B^0 \to (DD^*)^0_+ K^0 \to X K^0$$

$$\mathcal{A}_{B^+\to A_1K^+} = \mathcal{A}^{(\Lambda)}_{B^+\to(DD^*)_+K^+} G_C(\Lambda, E) g_{A_1,\text{str}} \qquad \mathcal{A}_{B^0\to XK^0} = \mathcal{A}^{(\Lambda)}_{B^0\to(DD^*)^0_+K^0} G_0(\Lambda, E) g_X$$

$$G_C(\Lambda, E) = -\frac{\mu_c\Lambda}{\pi^2} - \frac{\alpha\mu_c^2}{\pi}\left[\ln\frac{\Lambda}{\alpha\mu_c} + \ln(ix) + \frac{1}{2ix} - \psi(-ix) - \gamma_E\right] \qquad G_0(\Lambda, E) = -\frac{\mu_c^0\Lambda}{\pi^2} + \frac{\mu_c^0}{2\pi}\left(\sqrt{-2\mu_c^0 E - i\epsilon}\right)$$

$$\left|(DD^*)^0_+\right\rangle = \frac{1}{\sqrt{2}}\left(\left|D^0\bar{D}^{*0}\right\rangle - \left|\bar{D}^0 D^{*0}\right\rangle\right)$$

$$\left|(DD^*)_+\right\rangle = \frac{1}{\sqrt{2}}\left(\left|D^+D^{*-}\right\rangle - \left|D^-D^{*+}\right\rangle\right)$$

$$x = \frac{\alpha\mu_c}{\sqrt{2\mu_c E}} \qquad \psi(x) = \frac{\Gamma'(x)}{\Gamma(x)}$$

**Kong, Ravndal, *Nucl.Phys.A* 665 (2000)**

## Factorized amplitudes:

$$\mathcal{A}_{B^+\to A_1K^+} = \mathcal{A}^{\text{s.d}}_{B^+\to(DD^*)_+K^+} g_{A1,\text{str}} \qquad \mathcal{A}_{B^0\to XK^0} = \mathcal{A}^{\text{s.d.}}_{B^0\to(DD^*)^0_+K^0} g_X$$

**Braaten, Kusunoki, *Phys.Rev.D* 72 (2005) 014012**

## Isospin symmetry:

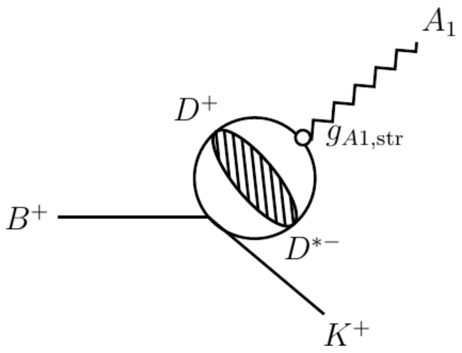
$$\left|\mathcal{A}^{\text{s.d}}_{B^+\to(DD^*)_+K^+}\right| = \left|\mathcal{A}^{\text{s.d}}_{B^0\to(DD^*)^0_+K^0}\right|$$

# X atom: Production

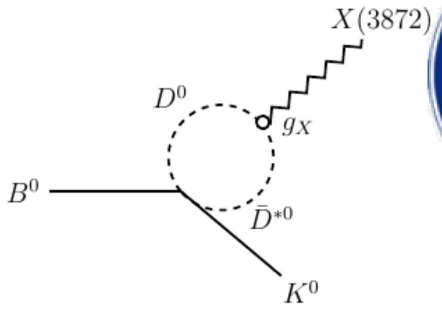
**Production in exclusive $B$ decays:**

$$|(DD^*)^0_+\rangle = \frac{1}{\sqrt{2}}\left(|D^0\bar{D}^{*0}\rangle - |\bar{D}^0 D^{*0}\rangle\right)$$

$$|(DD^*)_+\rangle = \frac{1}{\sqrt{2}}\left(|D^+ D^{*-}\rangle - |D^- D^{*+}\rangle\right)$$




$B^+ \to (DD^*)_+ K^+ \to A_1 K^+$




$B^0 \to (DD^*)^0_+ K^0 \to X K^0$

**Factorization amplitudes:** $\mathcal{A}_{B^+\to A_1 K^+} = \mathcal{A}^{\mathrm{s.d}}_{B^+\to(DD^*)_+K^+} g_{A1,\mathrm{str}}$ $\quad \mathcal{A}_{B^0\to XK^0} = \mathcal{A}^{\mathrm{s.d.}}_{B^0\to(DD^*)^0_+K^0} g_X$

**Isospin symmetry:** $\left|\mathcal{A}^{\mathrm{s.d}}_{B^+\to(DD^*)_+K^+}\right| = \left|\mathcal{A}^{\mathrm{s.d}}_{B^0\to(DD^*)^0_+K^0}\right|$

**Lower bound on the $X(3872)$ binding energy:**

$$R_\Gamma \equiv \frac{\Gamma_{B^+\to A_1K^+}}{\Gamma_{B^0\to XK^0}} = \frac{|g_{A1,\mathrm{str}}|^2}{|g_X|^2} \qquad \delta \simeq \frac{0.25\ \mathrm{eV}}{R_\Gamma^2}$$

**Production in inclusive *pp* collisions:**

$$R_\sigma \equiv \frac{d\sigma_{pp\to A_1+y}}{d\sigma_{pp\to X+y}} = \frac{|g_{A1,\mathrm{str}}|^2}{|g_X|^2} \qquad \delta \simeq \frac{0.25\ \mathrm{eV}}{R_\sigma^2} \qquad R_\Gamma \simeq R_\sigma \gtrsim 1\times 10^{-3}$$

# X atom: Decay

**Constituent $D^*$ decay:** $D^* \to D\pi, D\gamma, \cdots$ $\Gamma_c = (83.4 \pm 1.8)$ keV

**Decay into neutral pair:** $A$ (X atom) $\to D^0\bar{D}^{*0}(\bar{D}^0 D^{*0})$ $\Gamma_s = 2\mathrm{Im}E_{A1} = 5.8$ keV

**Decay into $J/\psi\pi\pi$ & $J/\psi\pi^+\pi^-\pi^0$ (like the $X(3872)$ )** $A \to J/\psi\pi\pi, J/\psi\pi^+\pi^-\pi^0$

**Ratio of branchings for the $X(3872)$:** $\frac{\mathrm{Br}^{\exp}_{[X(3872)\to J/\psi\pi^+\pi^-\pi^0]}}{\mathrm{Br}^{\exp}_{[X(3872)\to J/\psi\pi^+\pi^-]}} = 0.8 \pm 0.3$

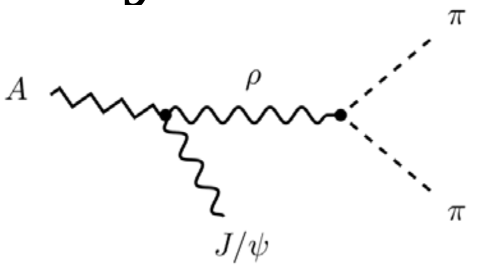
**Isospin breaking:** $R_X = \frac{g_{[X(3872)\to J/\psi\rho]}}{g_{[X(3872)\to J/\psi\omega]}} = 0.26$ C. Hanhart et al. , *Phys. Rev. D* 85 (2012) 011501

**$D^+D^{*-}$ atom (A):** $m_A = 3879.89 \pm 0.07$ MeV

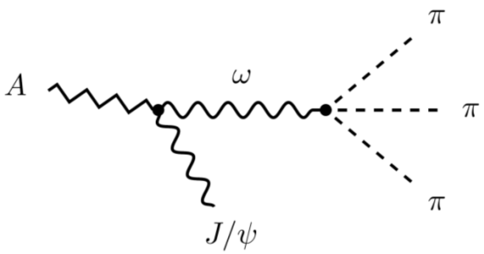
**Isospin breaking negligible:** $|D^+D^{*-}\rangle = \frac{1}{\sqrt{2}}(|I=1\rangle + |I=0\rangle)$ $R_A = \frac{g_{[A\to J/\psi\rho]}}{g_{[A\to J/\psi\omega]}} = 1$

**The phase space of the $D^+D^{*-}$ atom is larger than the phase space of the $X(3872)$**

# X atom: Decay




$$A \to J/\psi\pi\pi$$




$$A \to J/\psi\pi^+\pi^-\pi^0$$

C. Hanhart et al. , *Phys. Rev. D* 85 (2012) 011501

O. Kaymakcalan, S. Rajeev, and J. Schechter, *Phys. Rev. D* 30, 594 (1984)

E. A. Kuraev and Z. K. Silagadze, *Phys. At. Nucl.* 58, 1589 (1995)

**Effective couplings:**

$$R_A = \frac{g_{[A\to J/\psi\rho]}}{g_{[A\to J/\psi\omega]}} = 1 \qquad R_X = \frac{g_{[X(3872)\to J/\psi\rho]}}{g_{[X(3872)\to J/\psi\omega]}} = 0.26$$

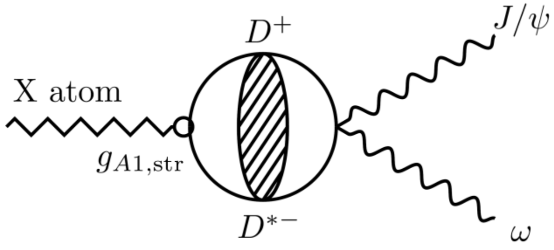
$$\frac{\mathrm{Br}^{\mathrm{exp}}_{[X(3872)\to J/\psi\pi^+\pi^-\pi^0]}}{\mathrm{Br}^{\mathrm{exp}}_{[X(3872)\to J/\psi\pi^+\pi^-]}} = 0.8 \pm 0.3$$

**Ratio of branchings:**

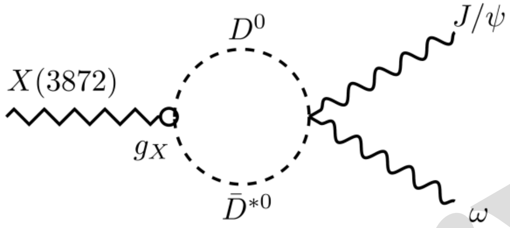
$$\frac{\mathrm{Br}_{[A\to J/\psi\pi\pi]}}{\mathrm{Br}_{[A\to J/\psi\pi^+\pi^0\pi^-]}} = 3.34 \qquad \frac{\mathrm{Br}_{[X(3872)\to J/\psi\pi\pi]}}{\mathrm{Br}_{[X(3872)\to J/\psi\pi^+\pi^0\pi^-]}} = 0.91$$

$$\frac{\mathrm{Br}_{[A\to J/\psi\pi\pi]}}{\mathrm{Br}_{[A\to J/\psi\pi^+\pi^0\pi^-]}} \simeq 3.65\, \frac{\mathrm{Br}_{[X(3872)\to J/\psi\pi\pi]}}{\mathrm{Br}_{[X(3872)\to J/\psi\pi^+\pi^0\pi^-]}}$$

# X atom: Decay




$$A \to J/\psi\omega$$




$$X(3872) \to J/\psi\omega$$

**Factorization amplitudes :** $\mathcal{A}_{[A\to J/\psi\omega]} = g_{A1,\mathrm{str}}\mathcal{A}^{\mathrm{s.d.}}_{[(DD^*)_+\to J/\psi\omega]}$ $\quad \mathcal{A}_{[X(3872)\to J/\psi\omega]} = g_X\mathcal{A}^{\mathrm{s.d.}}_{[(DD^*)^0_+\to J/\psi\omega]}$

**Ratio of phase spaces :** $$\frac{\Phi_{[A\to J/\psi\pi^+\pi^-\pi^0]}}{\Phi_{[X(3872)\to J/\psi\pi^+\pi^-\pi^0]}} = 3.76$$

**Ratio of decay widths :** $$\frac{\Gamma_{[A\to J/\psi\pi^+\pi^-\pi^0]}}{\Gamma_{[X(3872)\to J/\psi\pi^+\pi^-\pi^0]}} = \frac{|g_{A1,\mathrm{str}}|^2}{|g_X|^2}\frac{\Phi_{[A\to J/\psi\pi^+\pi^-\pi^0]}}{\Phi_{[X(3872)\to J/\psi\pi^+\pi^-\pi^0]}} \gtrsim 3.76\times 10^{-3}$$

$$\frac{\mathrm{Br}_{[A\to J/\psi\pi\pi]}}{\mathrm{Br}_{[A\to J/\psi\pi^+\pi^0\pi^-]}} \simeq 3.65\,\frac{\mathrm{Br}_{[X(3872)\to J/\psi\pi\pi]}}{\mathrm{Br}_{[X(3872)\to J/\psi\pi^+\pi^0\pi^-]}} \qquad \frac{\Gamma_{[A\to J/\psi\pi\pi]}}{\Gamma_{[X(3872)\to J/\psi\pi\pi]}} \gtrsim 1.37\times 10^{-2}$$

# X atom: Results

## (a) Binding Energy and Decay Width for the X Atom

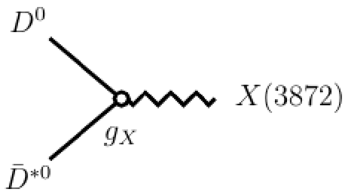
$$\mathrm{Re}\,E_{A1} = E_1 - \frac{\alpha^3\mu_c^2}{\sqrt{2\mu_c\Delta}} \simeq 22.92\ \mathrm{keV}$$

$$M_{A1} = (3879.89 \pm 0.07)\ \mathrm{MeV}$$

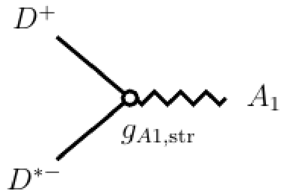
$$\Gamma_c + 2\,\mathrm{Im}\,E_{A1} = \Gamma_c + \frac{2\alpha^3\mu_c^2}{\sqrt{2\mu_c\Delta}} = (89.2 \pm 1.8)\ \mathrm{keV}$$

$$\Gamma_c = (83.4 \pm 1.8)\ \mathrm{keV}$$

## (b) LO Effective Couplings




$$g_X^2 = \frac{2\pi}{\mu_0^2}\sqrt{2\mu_0\delta}$$




$$|g_{A1,\mathrm{str}}|^2 = \frac{\pi\alpha^3}{\Delta}$$

## (c) Lower bound on the $X(3872)$ binding energy

$$\delta \simeq \frac{0.25\ \mathrm{eV}}{R^2_{\Gamma(\sigma)}} \qquad R_\Gamma \equiv \frac{\Gamma_{B^+\to A_1K^+}}{\Gamma_{B^0\to XK^0}} \qquad R_\sigma \equiv \frac{d\sigma_{pp\to A_1+y}}{d\sigma_{pp\to X+y}} \qquad \delta = m_{D^0} + m_{D^{*0}} - m_X \qquad R_\Gamma \simeq R_\sigma \gtrsim 1\times 10^{-3}$$

## (d) Ratio of decay widths

$$\frac{\mathrm{Br}_{[A\to J/\psi\pi\pi]}}{\mathrm{Br}_{[A\to J/\psi\pi^+\pi^0\pi^-]}} \simeq 3.65\,\frac{\mathrm{Br}_{[X(3872)\to J/\psi\pi\pi]}}{\mathrm{Br}_{[X(3872)\to J/\psi\pi^+\pi^0\pi^-]}}$$

$$\frac{\Gamma_{[A\to J/\psi\pi^+\pi^-\pi^0]}}{\Gamma_{[X(3872)\to J/\psi\pi^+\pi^-\pi^0]}} \gtrsim 3.76\times 10^{-3} \qquad \frac{\Gamma_{[A\to J/\psi\pi\pi]}}{\Gamma_{[X(3872)\to J/\psi\pi\pi]}} \gtrsim 1.37\times 10^{-2}$$

# X atom: Summary

- **We show that a null signal of the X atom can be used to put a lower limit on the binding energy of the $X(3872)$.**
- **If the binding energy of the $X(3872)$ is measured, the lower limit gives a criterion on the $X(3872)$ nature.**
- **From more and more events collected at the PANDA and LHCb experiments for the $X(3872)$, we can except the signal from the X atom.**

# Thanks!