Investigation of meson-meson interaction

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Outline

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2. Theoretical framework
3. Results and Discussion
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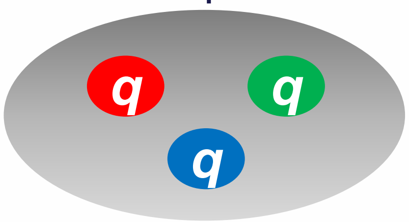
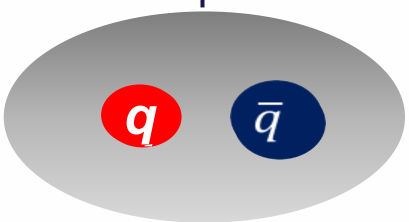
1. Introduction

In the conventional quark model since 1964:

Bayron



Meson

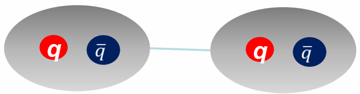


Other exotica states outside conventional quark model:

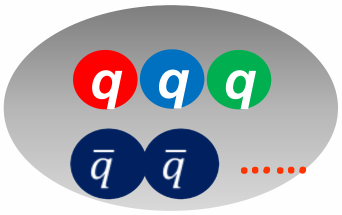
Glueball

Hybrid

Molecule



Muti-quark:
- Tetraquark
- Pentaquark
- Dibaryon



Introduction

The study of hadron-hadron

- Nucleon-nucleon (NN) interaction
- ππ interaction is also a classical subject in the field of strong interactions

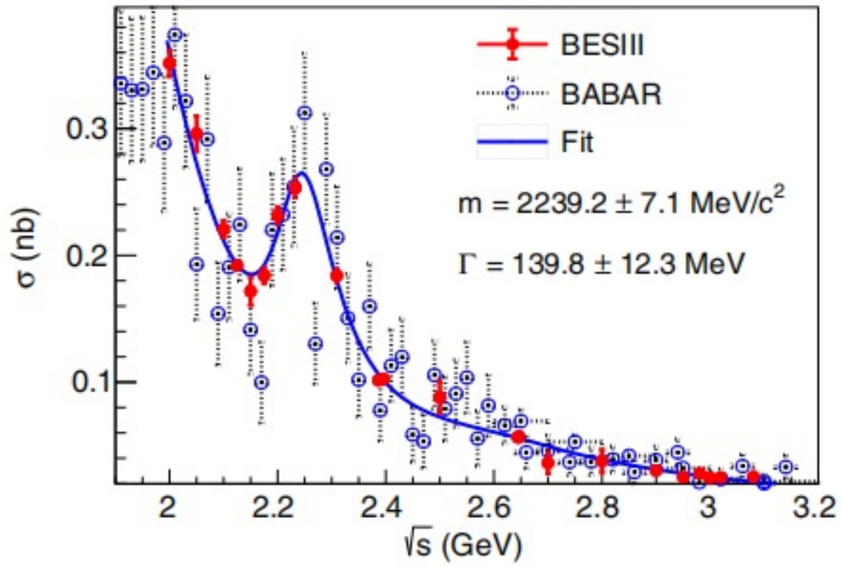
Controversy:

f0(980) and a0(980) →

- molecule state
- qqbar meson
- compact tetraquark

Introduction

Different explanations for the X（2239）



Molecule state

Jun-Tao Zhu, Yi Liu, Dian-Yong Chen, Longyu Jiang, Jun He, Chin.Phys. C44 (2020) no.12, 123103.

Compact state

Qi-Fang Lv, Kai-Lei Wang, Yu-Bing Dong, arXiv:1903.05007.

Introduction

baryon-baryon interaction

decuplet-decuplet interaction is almost deeply attractive

octet-decuplet interaction is weakly attractive

octet-octet interaction is more weakly attractive or repulsive

meson-meson interaction

2. Theoretical framework

Different research methods

- Lattice QCD
- QCD sum rule
- Triangle singularity
- Quark model
- Heavy chiral unitary approach

...

Theoretical framework

(1) Quark delocalization color screening model

- QDCSM was developed by Nanjing-Los Alamos collaboration in1990s aimed to multi-quark study. (PRL 69, 2901, 1992)

- Two new ingredients
  quark delocalization (orbital excitation)
  color screening (color structure)
- Apply to the study of baryon-baryon interaction and dibaryons

  deuteron, d*, NN, NΛ, NΩ, …

- Apply to the study of baryon-meson interaction and pentaquarks

  NK, Nπ, Pc, …

Theoretical framework

$$H = \sum_{i=1}^{4}\left(m_i+\frac{p_i^2}{2m_i}\right)-T_{CM}+\sum_{j>i=1}^{4}\left(V_{ij}^{CON}+V_{ij}^{OGE}+V_{ij}^{OBE}\right),$$

$$V_{ij}^{CON} = \begin{cases} -a_c\boldsymbol{\lambda}_i^c\cdot\boldsymbol{\lambda}_j^c\ \left(r_{ij}^2+a_{ij}^0\right), & \text{if } i,j \text{ in the same baron orbit} \\ -a_c\boldsymbol{\lambda}_i^c\cdot\boldsymbol{\lambda}_j^c\ \left(\frac{1-e^{-\mu_{ij}\mathbf{r}_{ij}^2}}{\mu_{ij}}+a_{ij}^0\right), & \text{otherwise} \end{cases}$$

$$V_{ij}^{OGE} = \frac{1}{4}\alpha_s\boldsymbol{\lambda}_i^c\cdot\boldsymbol{\lambda}_j^c\left[\frac{1}{r_{ij}}-\frac{\pi}{2}\delta(\boldsymbol{r}_{ij})\left(\frac{1}{m_i^2}+\frac{1}{m_j^2}+\frac{4\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j}{3m_im_j}\right)-\frac{3}{4m_im_jr_{ij}^3}S_{ij}\right]$$

$$V_{ij}^{OBE} = V_\pi(\boldsymbol{r}_{ij})\sum_{a=1}^{3}\lambda_i^a\cdot\lambda_j^a+V_K(\boldsymbol{r}_{ij})\sum_{a=4}^{7}\lambda_i^a\cdot\lambda_j^a+V_\eta(\boldsymbol{r}_{ij})\left[\left(\lambda_i^8\cdot\lambda_j^8\right)\cos\theta_P-\left(\lambda_i^0\cdot\lambda_j^0\right)\sin\theta_P\right]$$

$$V_\chi(\boldsymbol{r}_{ij}) = \frac{g_{ch}^2}{4\pi}\frac{m_\chi^2}{12m_im_j}\frac{\Lambda_\chi^2}{\Lambda_\chi^2-m_\chi^2}m_\chi\left\{(\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j)\left[Y(m_\chi r_{ij})-\frac{\Lambda_\chi^3}{m_\chi^3}Y(\Lambda_\chi r_{ij})\right]\right.$$
$$\left.+\left[H(m_\chi r_{ij})-\frac{\Lambda_\chi^3}{m_\chi^3}H(\Lambda_\chi r_{ij})\right]S_{ij}\right\},\qquad \chi=\pi,K,\eta,$$

$$S_{ij} = \left\{3\frac{(\boldsymbol{\sigma}_i\cdot\boldsymbol{r}_{ij})(\boldsymbol{\sigma}_j\cdot\boldsymbol{r}_{ij})}{r_{ij}^2}-\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j\right\},$$

$$H(x) = (1+3/x+3/x^2)Y(x),\qquad Y(x)=e^{-x}/x.$$

Theoretical framework

（2）The wave function of the four-quark system

$$\Psi = \mathcal{A}\left[[\psi^L \psi^\sigma]_{JM} \psi^f \psi^c\right].$$

The meson-meson structure: $\mathcal{A} = 1 - P_{13} - P_{24} + P_{13}P_{24}.$

$$\psi^L = \psi_1(\boldsymbol{R}_1)\psi_2(\boldsymbol{R}_2)\chi_L(\boldsymbol{R}).$$

$$\chi_L(\boldsymbol{R}) = \frac{1}{\sqrt{4\pi}}\left(\frac{3}{2\pi b^2}\right)\sum_{i=1}^{n} C_i \times \int \exp\left[-\frac{3}{4b^2}(\boldsymbol{R}-\boldsymbol{S}_i)^2\right] Y_{LM}(\hat{\boldsymbol{S}}_i)d\hat{\boldsymbol{S}}_i.$$

Theoretical framework

- **The flavor wave function**

The flavor wave functions of the meson cluster:

$$\chi^1_{I_{11}} = u\bar{d}, \quad \chi^2_{I_{1-1}} = -d\bar{u}, \quad \chi^3_{I_{10}} = \sqrt{\frac{1}{2}}(d\bar{d} - u\bar{u}),$$

$$\chi^4_{I_{\frac{1}{2}\frac{1}{2}}} = s\bar{d}, \quad \chi^5_{I_{\frac{1}{2}\frac{1}{2}}} = u\bar{s}, \quad \chi^6_{I_{00}} = s\bar{s}.$$

The flavor wave functions of the four quark cluster

$$\psi^{f_1}_{22} = \chi^1_{I_{11}}\chi^1_{I_{11}}, \quad \psi^{f_2}_{\frac{3}{2}\frac{3}{2}} = \chi^1_{I_{11}}\chi^5_{I_{\frac{1}{2}\frac{1}{2}}}, \quad \psi^{f_3}_{\frac{1}{2}\frac{1}{2}} = \chi^6_{I_{00}}\chi^5_{I_{\frac{1}{2}\frac{1}{2}}}$$

$$\psi^{f_4}_{11} = \sqrt{\frac{1}{2}}\left[\chi^1_{I_{11}}\chi^3_{I_{10}} - \chi^3_{I_{10}}\chi^1_{I_{11}}\right]$$

$$\psi^{f_5}_{00} = \sqrt{\frac{1}{3}}\left[\chi^1_{I_{11}}\chi^2_{I_{1-1}} - \chi^3_{I_{10}}\chi^3_{I_{10}} + \chi^2_{I_{1-1}}\chi^1_{I_{11}}\right].$$

Theoretical framework

- **The spin wave function**

The spin wave functions of the meson cluster:

$$\chi^1_{\sigma 11} = \alpha\alpha, \quad \chi^2_{\sigma 10} = \sqrt{\frac{1}{2}}(\alpha\beta + \beta\alpha),$$
$$\chi^3_{\sigma 1-1} = \beta\beta, \quad \chi^4_{\sigma 00} = \sqrt{\frac{1}{2}}(\alpha\beta - \beta\alpha).$$

The spin wave functions of the four quark cluster

$$\psi^{\sigma_1}_{00} = \chi^4_{\sigma 00}\chi^4_{\sigma 00},$$
$$\psi^{\sigma_2}_{00} = \sqrt{\frac{1}{3}}\left[\chi^1_{\sigma 11}\chi^3_{\sigma 1-1} - \chi^2_{\sigma 10}\chi^2_{\sigma 10} + \chi^3_{\sigma 1-1}\chi^1_{\sigma 11}\right]$$
$$\psi^{\sigma_3}_{11} = \chi^4_{\sigma 00}\chi^1_{\sigma 11}, \quad \psi^{\sigma_4}_{11} = \chi^1_{\sigma 11}\chi^4_{\sigma 00},$$
$$\psi^{\sigma_5}_{11} = \sqrt{\frac{1}{2}}\left[\chi^1_{\sigma 11}\chi^2_{\sigma 10} - \chi^2_{\sigma 10}\chi^1_{\sigma 11}\right].$$

Theoretical framework

- **The color wave function**

The color wave functions of the meson cluster:

$$\chi^1_{[111]} = \sqrt{\frac{1}{3}}(r\bar{r} + g\bar{g} + b\bar{b}).$$

The color wave functions of the four quark cluster

$$\psi^{c_1} = \chi^1_{[111]}\chi^1_{[111]}.$$

3. Results and Discussion

- **parameters :**

TABLE I: Model parameters: $m_\pi = 0.7$ fm^{-1}, $m_K = 2.51$ fm^{-1}, $m_\eta = 2.77$ fm^{-1}, $\Lambda_\pi = 4.2$ fm^{-1}, $\Lambda_K = \Lambda_\eta = 5.2$ fm^{-1}, $g^2_{ch}/(4\pi) = 0.54$, $\theta_p = -15^0$.

b (fm)	m_u (MeV)	m_d (MeV)	m_s (MeV)	
0.518	313	313	470	
a_c (MeV fm^{-2})	a^0_{uu} (fm^2)	a^0_{ud} (fm^2)	a^0_{us} (fm^2)	a^0_{ss} (fm^2)
58.03	-0.733	-0.733	-0.309	-0.943
$\alpha_{s_{uu}}$	$\alpha_{s_{ud}}$	$\alpha_{s_{us}}$	$\alpha_{s_{ss}}$	
1.50	1.50	1.46	0.352	

Results and Discussion

- **the masses of the mesons:**

TABLE II: The masses (in MeV) of the mesons obtained from QDCSM. Experimental values are taken from the Particle Data Group (PDG) [22].

Meson	M_{the}	M_{exp}
π	140	140
ρ	772	770
K	495	495
K^*	892	892
η	284	547
η'	781	958
ω	724	782
ϕ	1020	1020

Results and Discussion

- **the possible quantum number:**

TABLE III: The coupling channels of each quantum number.

$I=0$	$J=0$	$\pi\pi,\ \eta\eta,\ \eta'\eta',\ \eta\eta',\ \phi\phi,\ \omega\omega,\ \omega\phi,\ \rho\rho$
$I=0$	$J=1$	$\eta\phi,\ \eta'\phi,\ \eta\omega,\ \eta'\omega,\ \pi\rho,\ K^*K^*,\ K^*\bar{K}^*$
$I=0$	$J=2$	$\phi\phi,\ \omega\omega,\ \rho\rho,\ \omega\phi$
$I=1$	$J=0$	$KK,\ \pi\eta',\ \omega\rho,\ \phi\rho,\ K^*K^*,\ K\bar{K},\ K^*\bar{K}^*$
$I=1$	$J=1$	$KK^*,\ \pi\phi,\ \pi\omega,\ \eta\rho,\ \eta'\rho,\ \rho\rho$
$I=1$	$J=2$	$K^*K^*,\ \omega\rho,\ \phi\rho,\ K^*\bar{K}^*$
$I=2$	$J=0$	$\pi\pi,\ \rho\rho$
$I=2$	$J=1$	$\pi\rho$
$I=2$	$J=2$	$\rho\rho$
$I=\frac{1}{2}$	$J=0$	$\eta K,\ \eta'K,\ \phi K^*\ ,\ \omega K^*$
$I=\frac{1}{2}$	$J=1$	$\eta K^*,\ \eta'K^*,\ \phi K,\ \omega K,\ \rho K^*$
$I=\frac{1}{2}$	$J=2$	$\phi K^*,\ \omega K^*$
$I=\frac{3}{2}$	$J=0$	$\pi K,\ \rho K^*$
$I=\frac{3}{2}$	$J=1$	$\pi K^*,\ \rho K$
$I=\frac{3}{2}$	$J=2$	ρK^*

Results and Discussion

- **The effctive potential:**

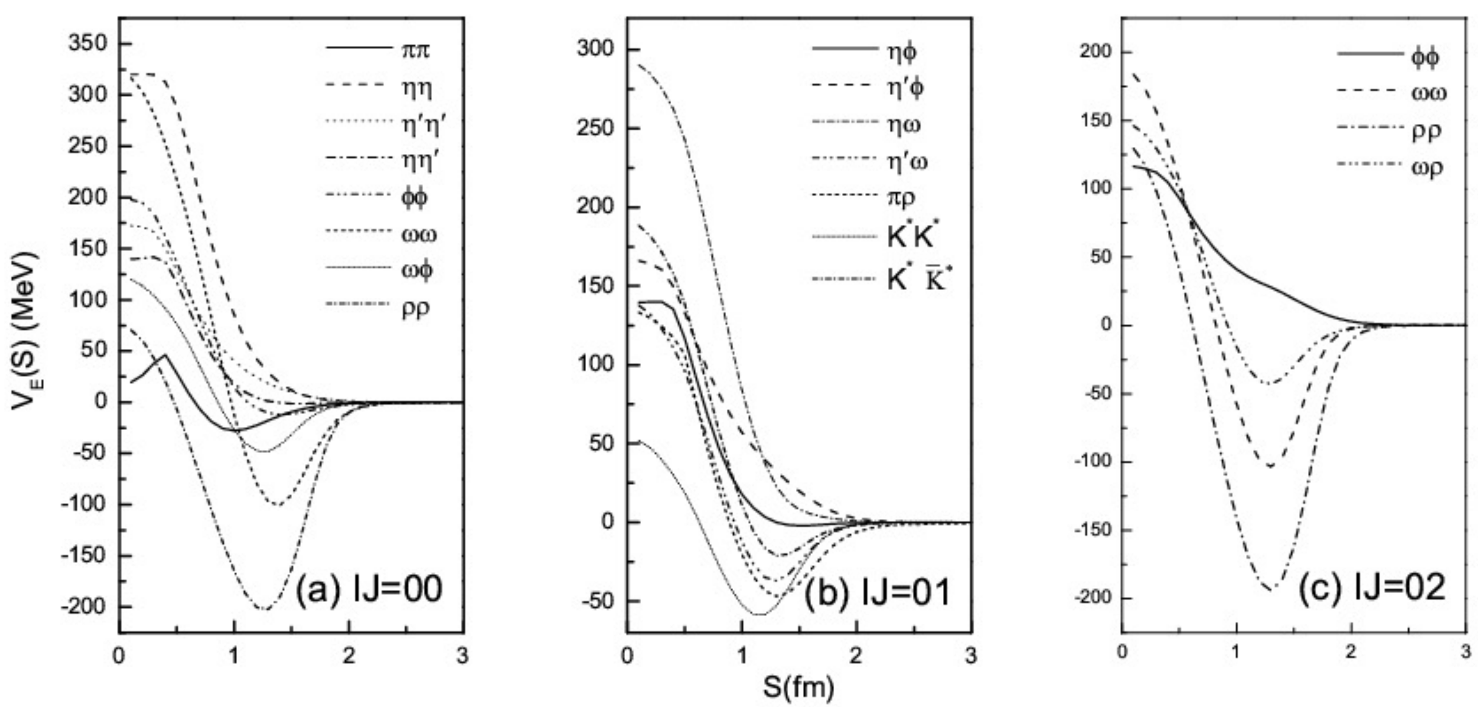


Figure 1: I = 0 states

Results and Discussion

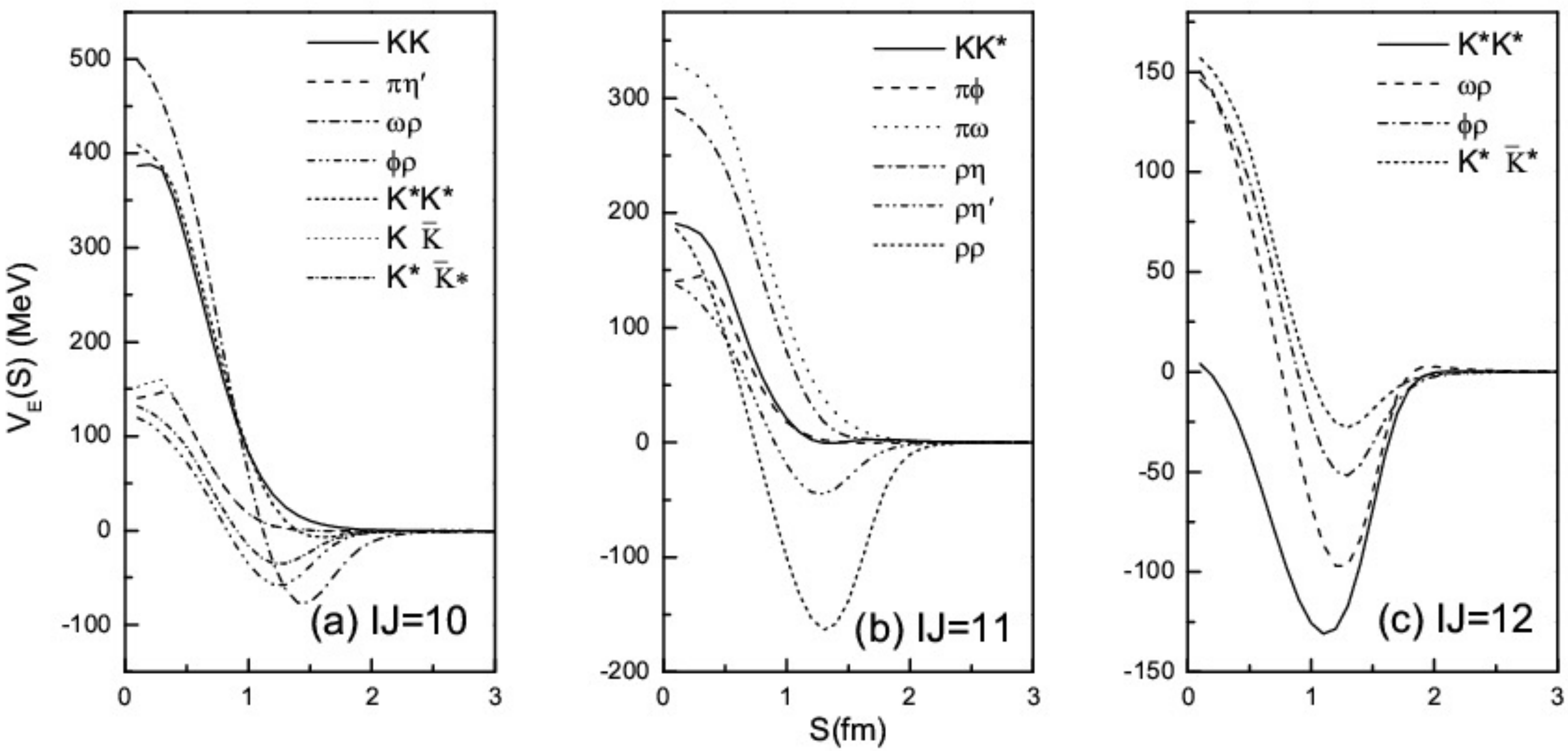


Figure 2: I = 1 states.

Results and Discussion

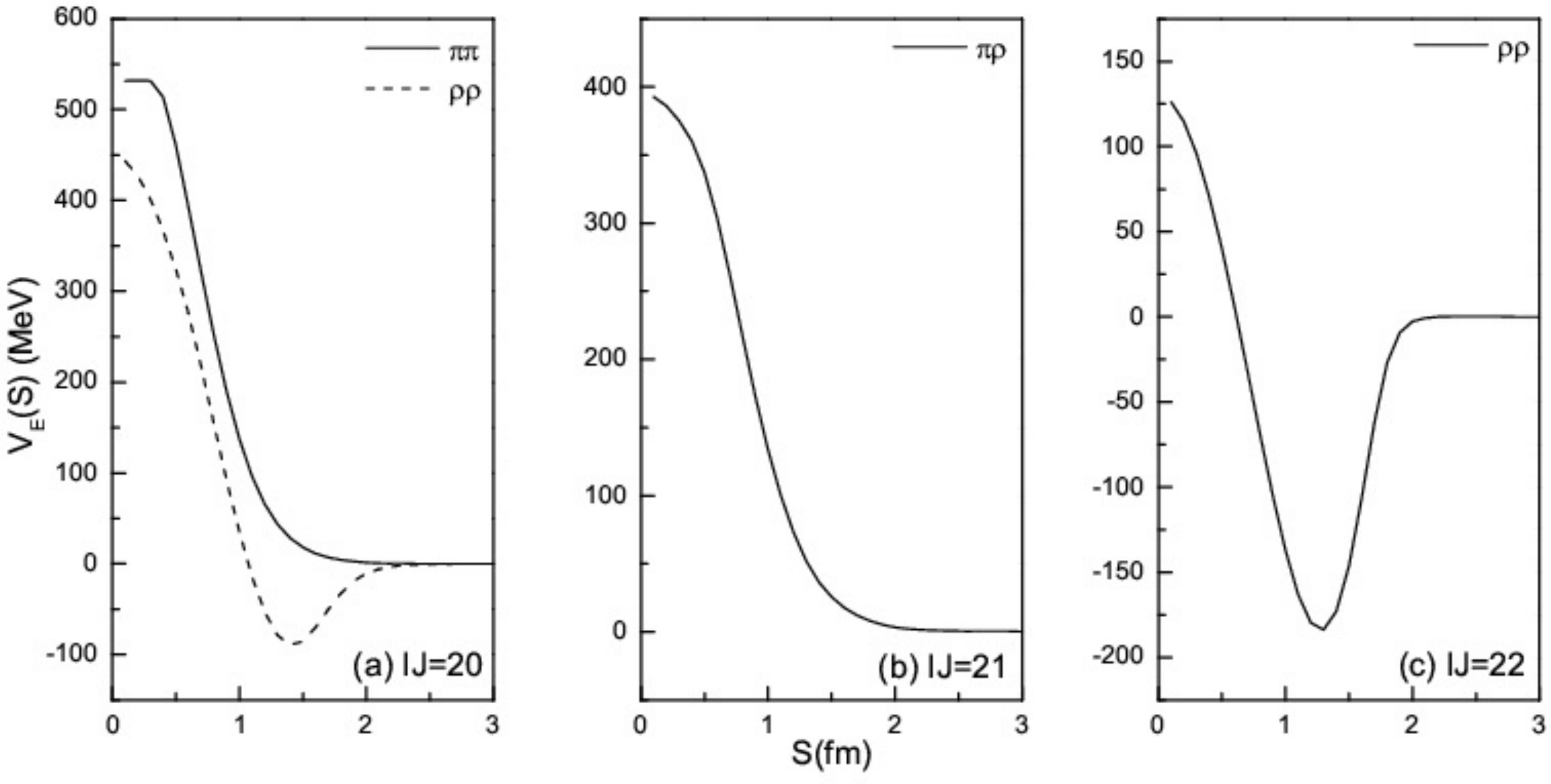


Figure 3: I = 2states.

Results and Discussion

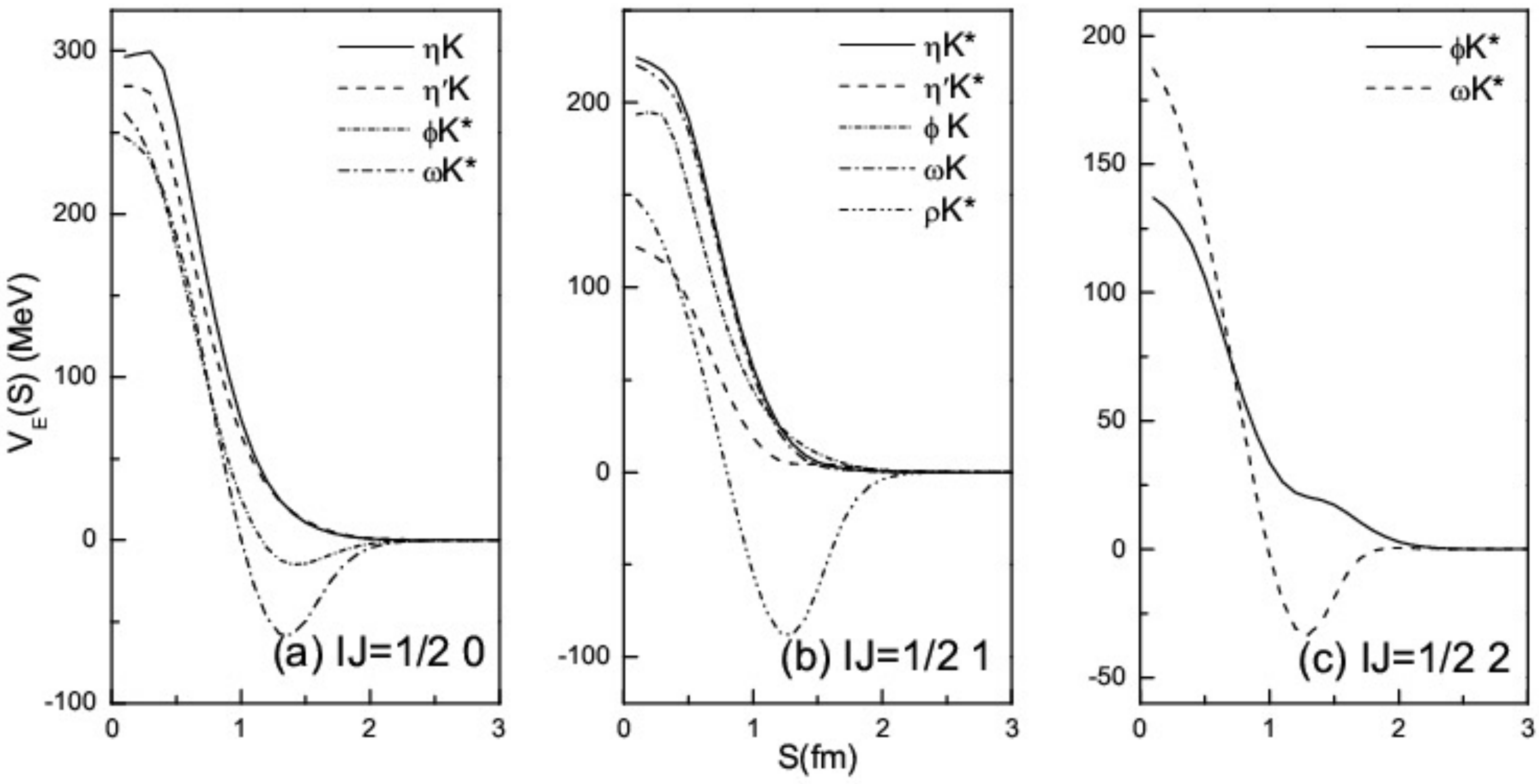


Figure 4: I = 1/2states.

Results and Discussion

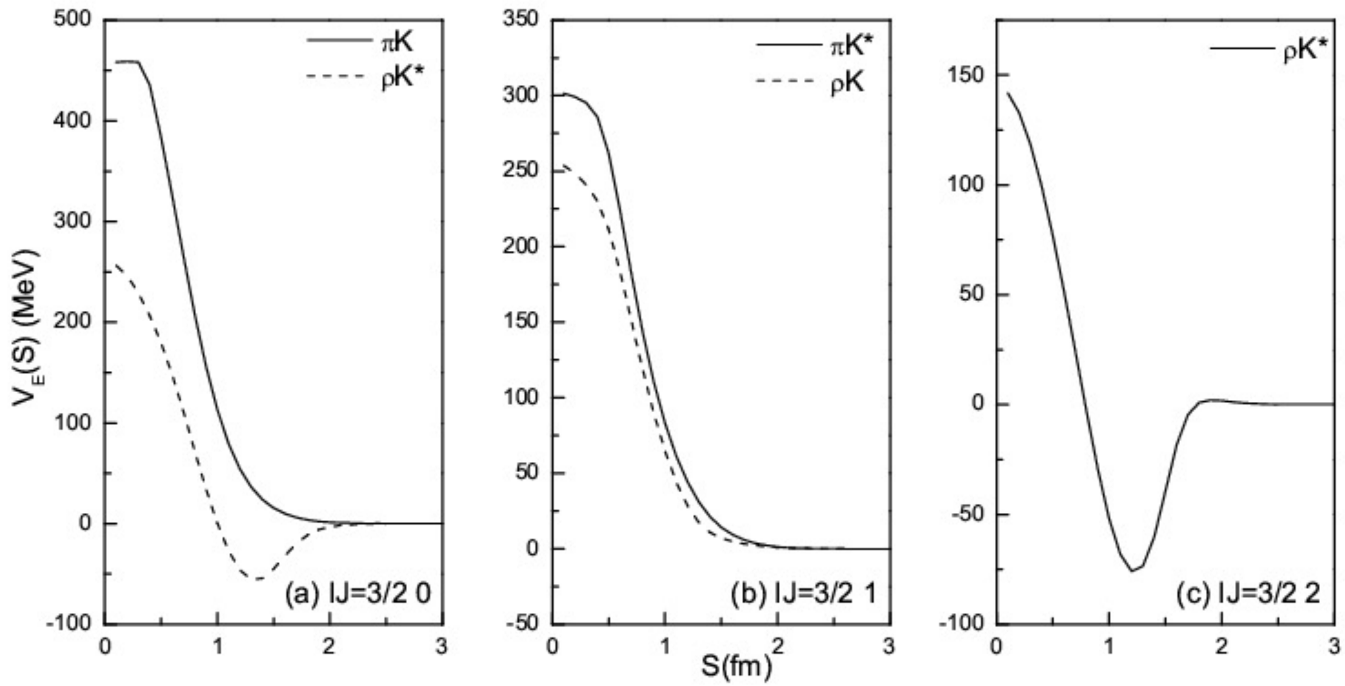


Figure 5: I = 3/2states.

- The interaction between two vector mesons is almost deeply attractive;
- The one between a pseudoscalar meson and a vector meson is repulsive or weakly attractive;
- The one between two pseudoscalar mesons is almost repulsive.

Results and Discussion

- The contributions to the effective potentials

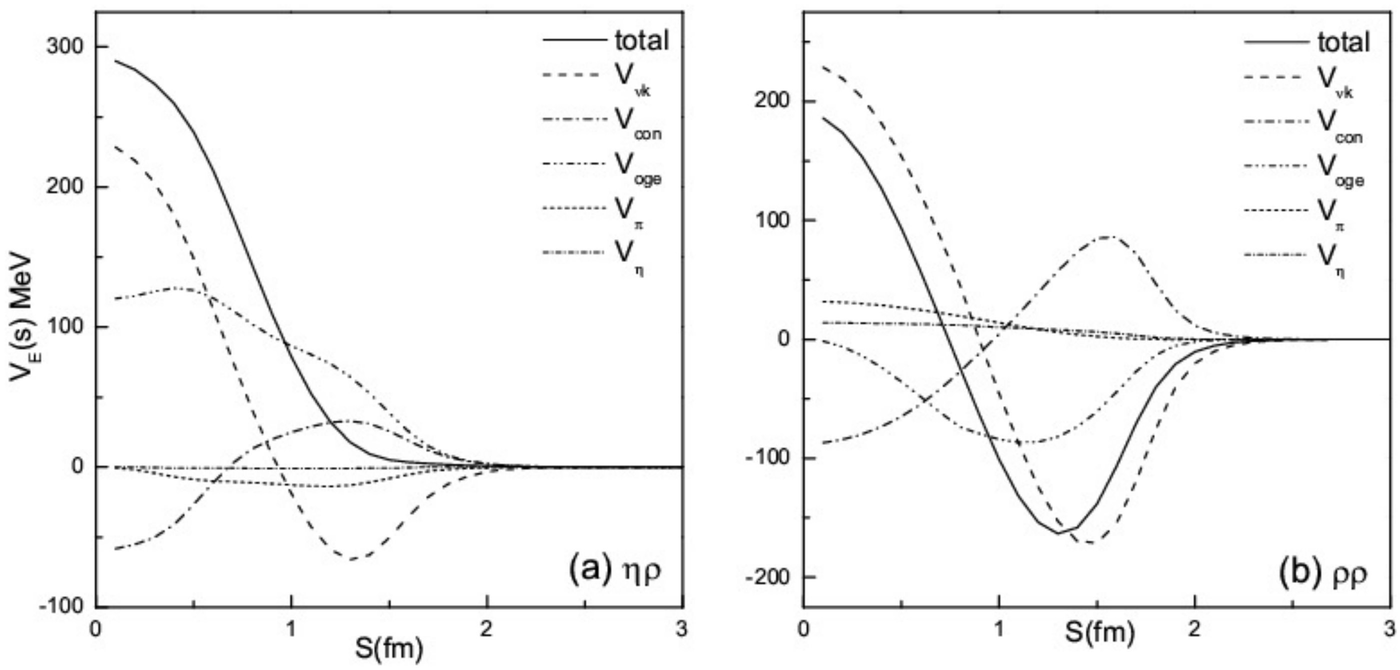


Figure 6: IJ = 11.

- The kinetic energy term provides an attractive interaction
- The one-gluon exchange provides deep attraction for the ρρ state,while it is repulsive for the ηρ state

Results and Discussion

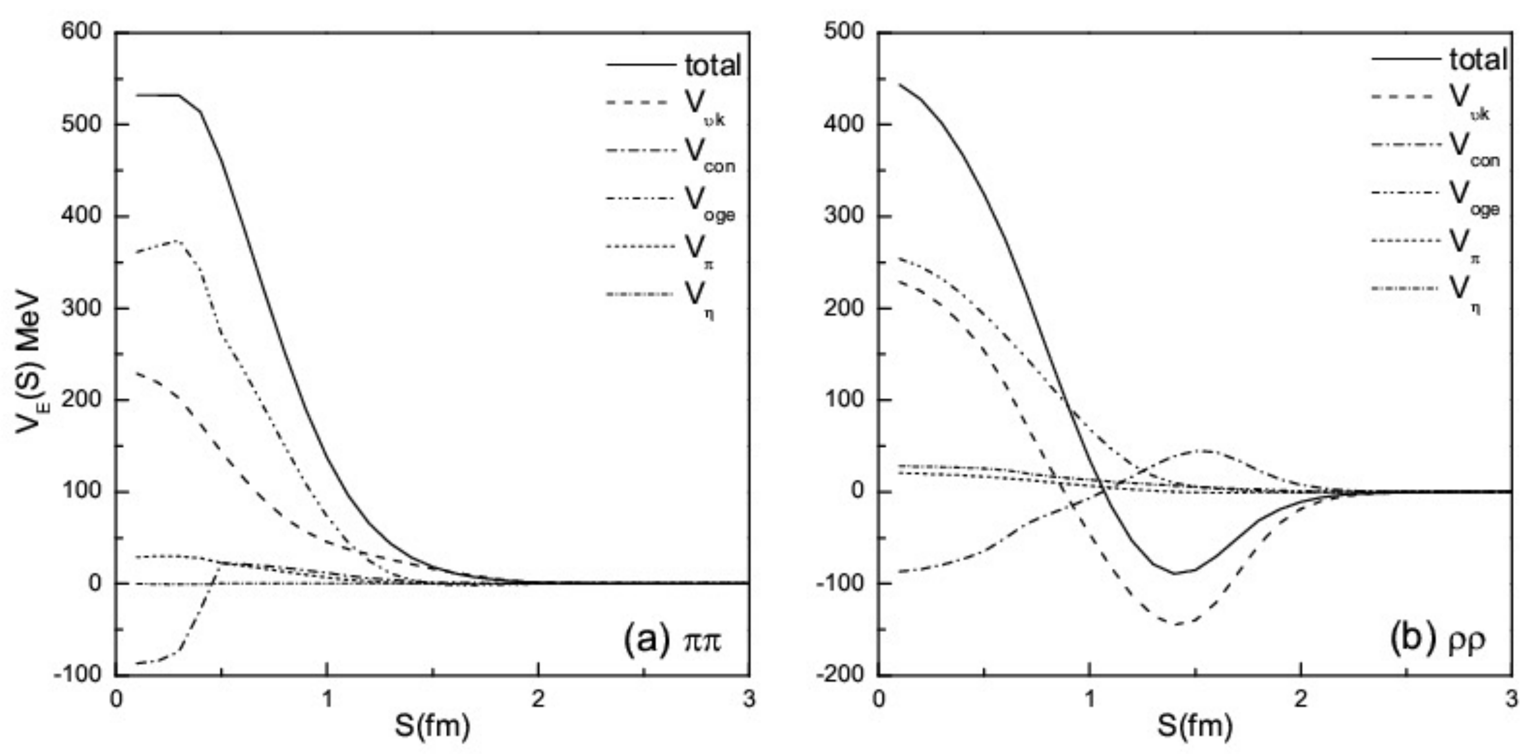


Figure 7: IJ = 20.

- The kinetic energy interaction is attractive for the ρρ state,while it is repulsive for the ππ state
- The one-gluon exchange provides repulsive interaction for both ππ and ρρ states

Results and Discussion

- The delocalization parameter ε

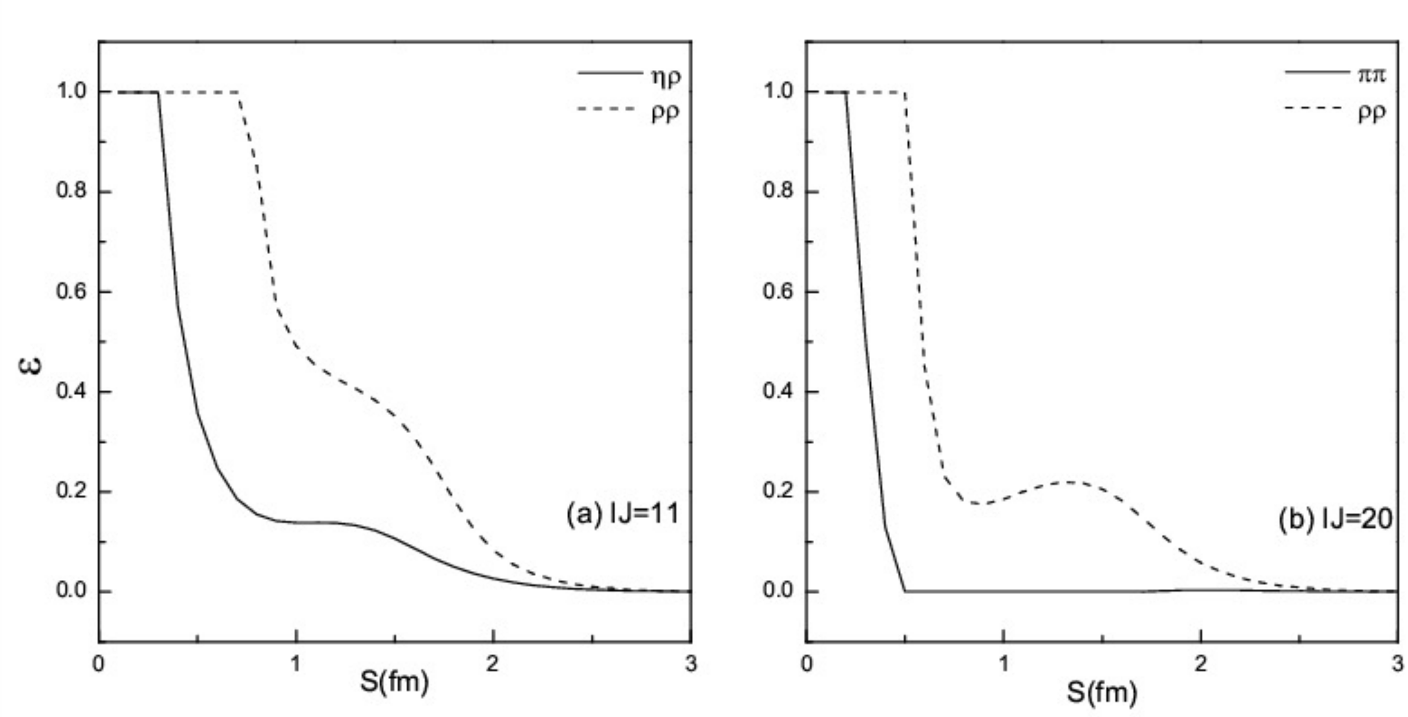


Figure 8: IJ =11 and IJ = 20

- **The delocalization parameter approaching to 1 means that the quarks are more willing to run between the two mesons, thereby reduce the kinetic energy and introduce the attractive interaction.**

Results and Discussion

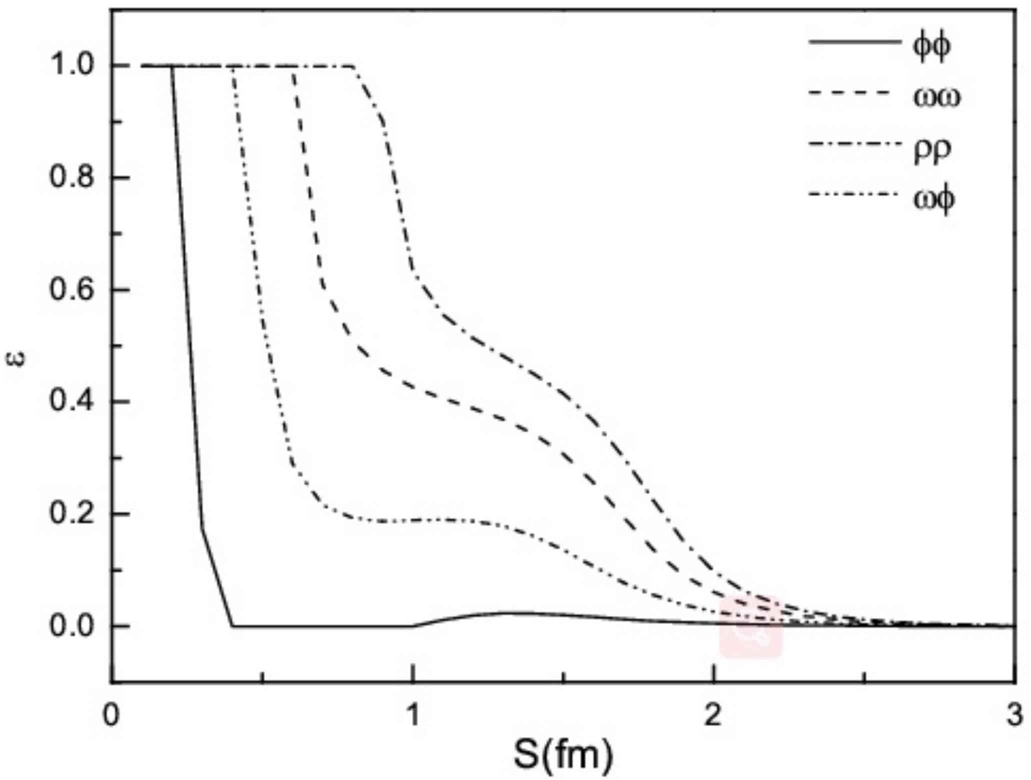


Figure 9: IJ = 02

the same reason with Figure 8

4. Summary

- The interaction between two vector mesons is almost deeply attractive;the one between a pseudoscalar meson and a vector meson is repulsive or weakly attractive;the one between two pseudoscalar mesons is always repulsive.
- The meson-meson interaction is similar to that of baryon-baryon interaction.
- The kinetic energy relates to the intermediate-range attraction mechanism in QDCSM.

Thanks for your attention！