北京航空航天大學

北航物理学院
SCHOOL OF PHYSICS,BUAA

Study on multi-hadron molecular states

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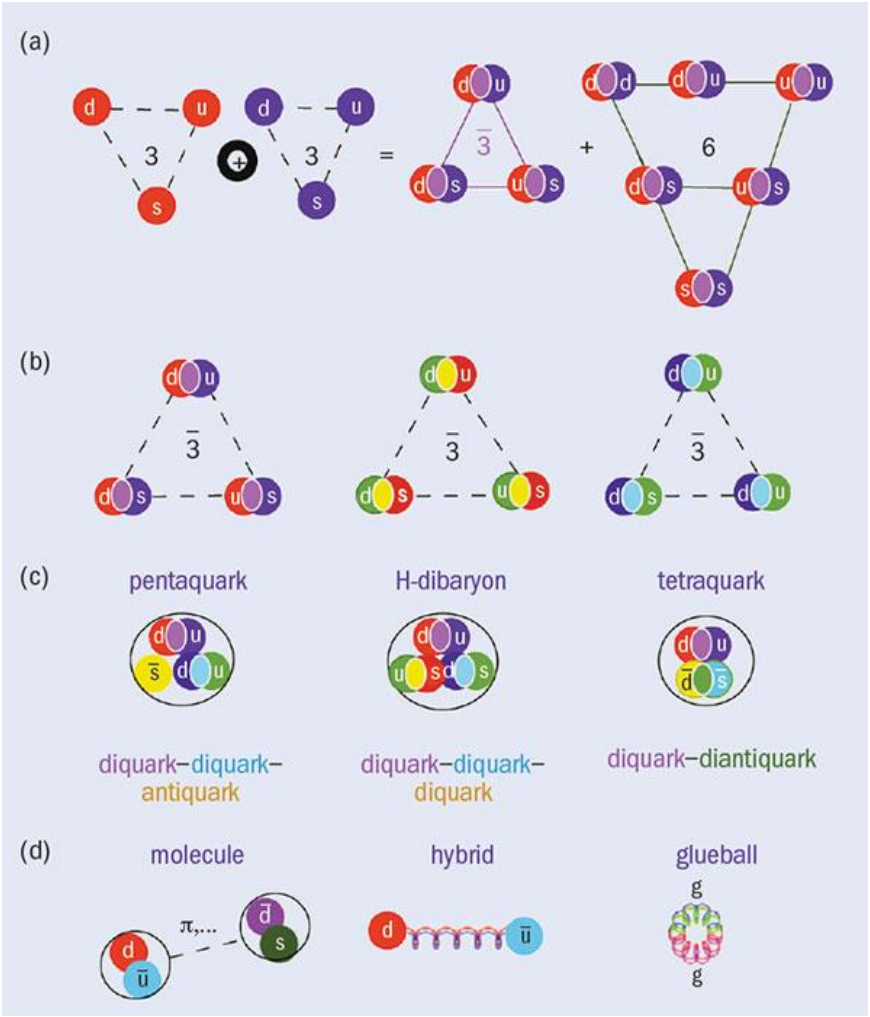
XYZ粒子会议@青岛0518

强子和强子的少体结构



一般强子：

介子：由正反夸克对组成

重子：由三个夸克组成

奇特强子态:

1. 胶球(glueball)

   由胶子构成

2. 混杂态(hybrid)

   由胶子和夸克构成

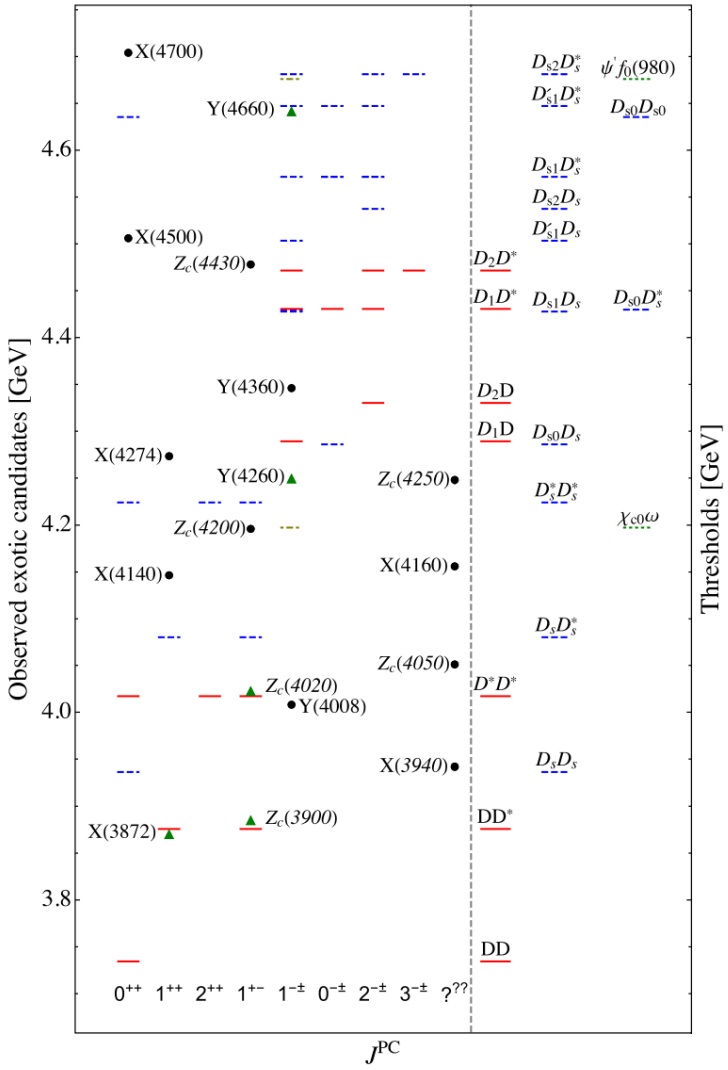
3. 多夸克态(multiquark state)

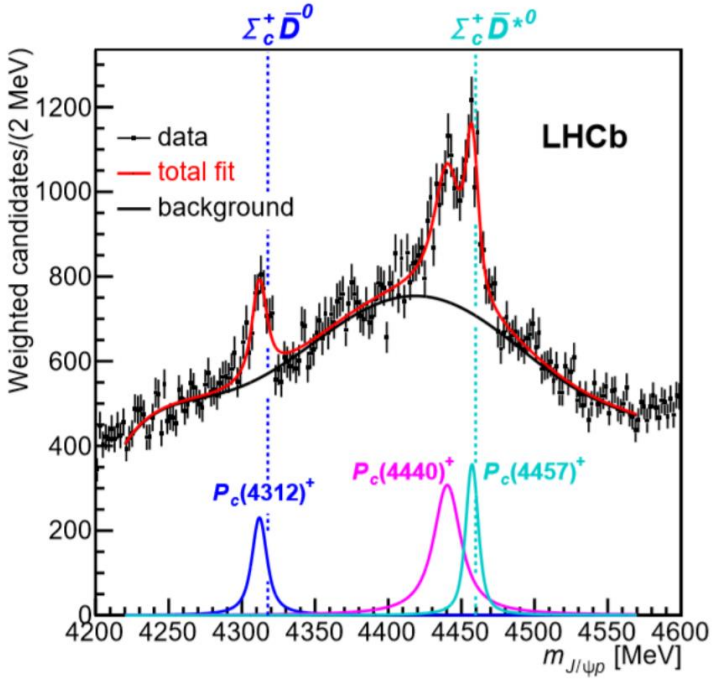
   由超过三个夸克组成

4. 强子分子态(hadronic molecule)

   由两个或多个强子组成的强子态

强子分子态





X(3872)，Pc states，Ds0*(2317) 等许多近阈粒子被认为是强子分子态的有力候选者。

F.K. Guo, etc. Hadronic molecules, Rev.Mod.Phys.90.015004

从两体到多体

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A. Martines, etc. Few-Body Syst (2020) 61:35

Components	States generated	Method used	References
$\bar{K}NN$	$\bar{K}$ bound states	F V FCA	[76–88]
$2PN$	$1/2^+$ Σ, Λ excited $1/2^+$ N^* states ($N^*(1920)$)	χF χF	[53,55]
$\pi\pi N$	$N^*(1710)$	χF	[54]
$K\bar{K}N$	$N^*(1920)$	V, FCA	[56,70,89]
$KK\bar{K}$	$K(1460)$	χF CS F	[90–92]
$\pi K\bar{K}$, $\pi\pi\eta$	$\pi(1300)$, $f_0(1790)$	χF	[93]
$\phi K\bar{K}$, $\phi\pi\pi$	$\phi(2170)$	χF	[50]
$\pi\rho\Delta$	$\Delta_{5/2^+}(2000)$	FCA	[94]
$\pi\bar{K}K^*$	$\pi_1(1600)$	FCA	[95]
$\eta\bar{K}K^*$	$0(1^-)$ state around 1700 MeV	FCA	[96]
$\rho K\bar{K}$	$\rho(1700)$	FCA	[97]
Multi-ρ	$f_2(1270)$, $\rho_3(1690)$, $f_4(2050)$, $\rho_5(2350)$, $f_6(2510)$	FCA	[65]
K^* multi-ρ	$K_2^*(1430)$, $K_3^*(1780)$ $K_4^*(2045)$, $K_5^*(2380)$, K_6^*	FCA	[74]
PVV	$\pi_2(1670)$, $\eta_2(1645)$, $K_2^*(1770)$	FC	[75]
K multi-ρ	several K^* states	FCA	[98]
DNN	D bound state	FCA V	[99,100]
NDK, $ND\bar{K}$, $ND\bar{D}$	bound states of 3050, 3150, 4400 MeV	FC	[100]
DDK, $DD_s\eta$, $DD_s\pi$	$I = 1/2$ state around 4140 MeV	χF	[101,102]
DDK	Bound state, $B \simeq 70$ MeV	GE	[103]
$DDDK$	Bound state, $B \simeq 90 - 110$ MeV	GE	[103]
$J/\psi K\bar{K}$	$Y(4260)$	χF	[104]
$KD\bar{D}^*$	K^* Bound states	FCA	[105]
$DK\bar{K}$	D-like state at 2900 MeV	QSR, χF FC	[71,72]
$\rho D\bar{D}$	$I = 0$, 1 states 4200–4300 MeV	FCA	[106]
$\rho B^*\bar{B}^*$	$J = 3$ state at 10950 MeV	FCA	[107]
$D^{(*)}B^{(*)}\bar{B}^{(*)}$	Several bound states	FCA	[108]
$BD\bar{D}$, BDD	$BD\bar{D}$ bound state $\sim$ 8950 MeV	FCA	[109]
BB^*B^*, $B^*B^*B^*$	Bound $C = 3$ meson	F	[110]
DD^*K, $BB^*\bar{K}$	Bound states 4318 MeV, 11014 MeV	BO	[111]
$\bar{K}^*B\bar{B}$, $\bar{K}^*B^*\bar{B}^*$	Several bound states	FCA	[112]
BBB^*	Probable bound state	BO	[113]
D multi-ρ	Seven D^* states	FCA	[114]

P Pseudoscalar, *F* Faddeev, *FCA* Fixed center approximation, χF Chiral Faddeev, *V* Variational, *GE* Gaussian expansion, *QSR* QCD sum rules, *BO* Born–Oppenheimer, *CS* Complex scaling

Methods:

Faddeev equation

Fixed Center Approximation

χ Faddeev

Variational method

Gaussian Expansion Method

QCD Sum Rules

Born-Oppenheimer Approximation

Complex Scaling

三体强子分子态?

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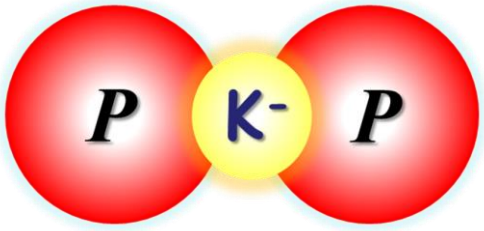
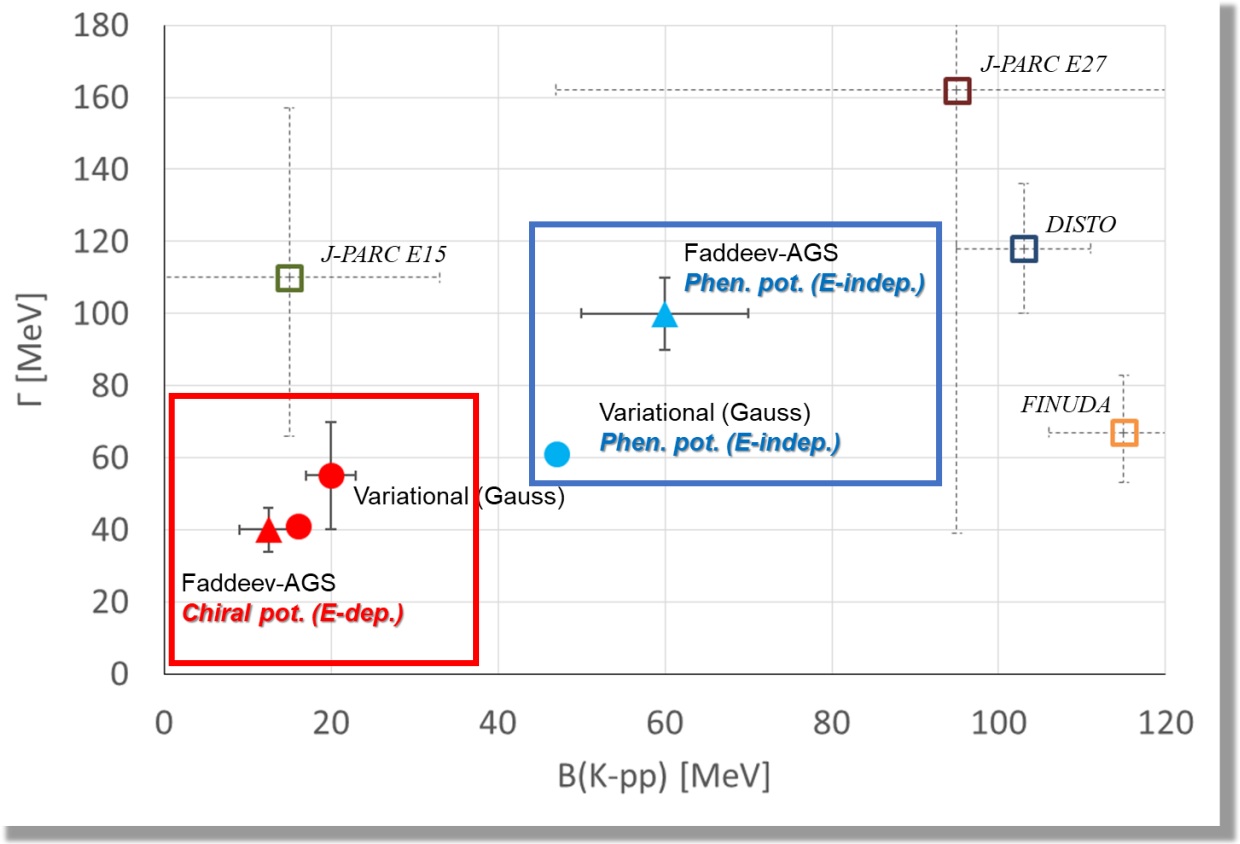
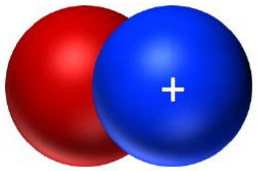
NNK̄的理论和实验研究



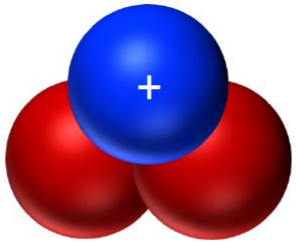


A. Gal, E. V. Hungerford, D. J. Millener, Rev. Mod. Phys. 88, 035004 (2016).

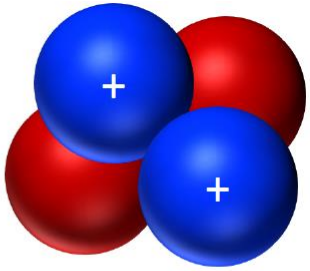
从核子到强子



deuteron



triton

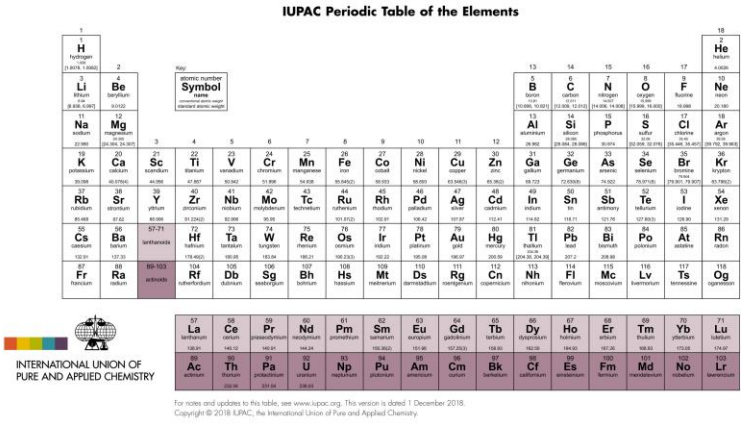


alpha

元素和核素是有限的

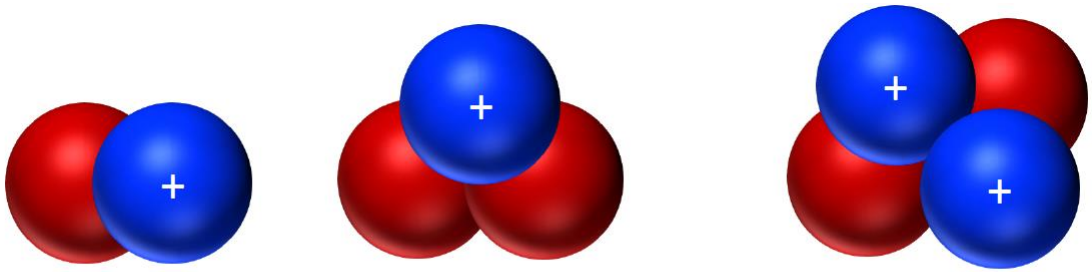
np, nnp, nnpp...

NK, NNK, NNNK...



是否存在类核的多强子分子态?

类核的多强子分子态?



deuteron　　triton　　alpha

np, nnp, nnpp...

NK, NNK, NNNK...

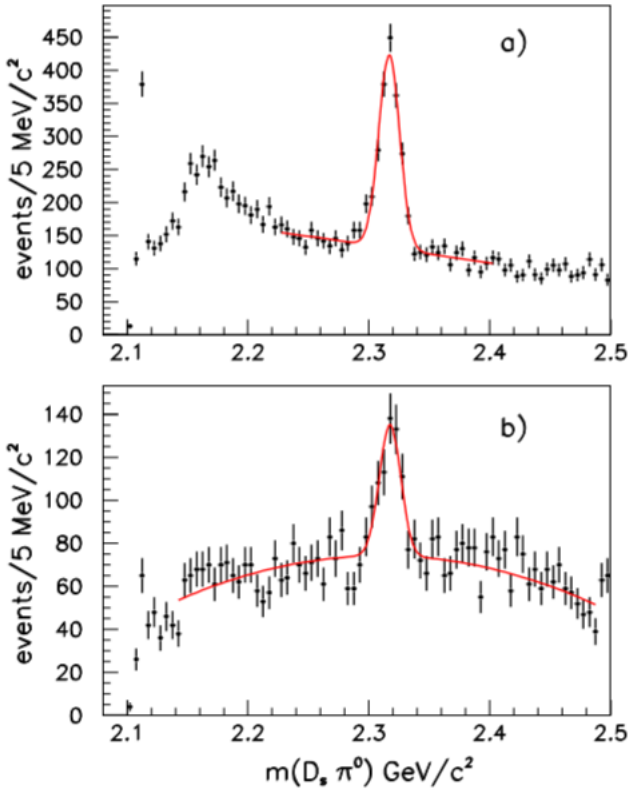
DK, DDK, DDDK？ ...

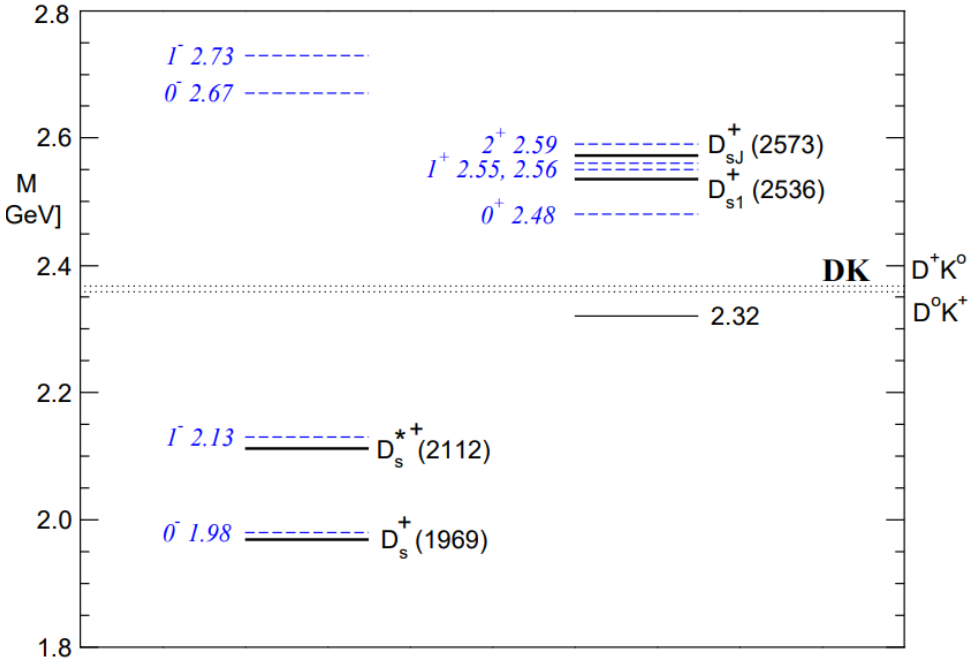
...

特点：

由两种粒子构成;
系统对称性高;
系统内相互作用单一;
理论上较易向多体发展;
介子系统没有泡利原理的限制。

Ds0(2317)





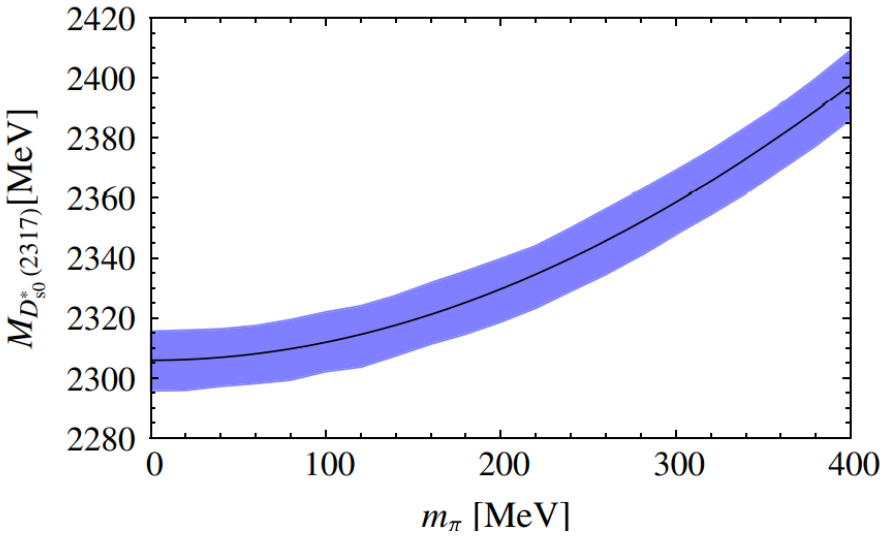
PHYSICAL REVIEW D 68, 054006 (2003)

@PDG
质量：2317.7 MeV
总宽度：< 3.8MeV
伙伴态Ds1: 2460 MeV
M(Ds1)-M(Ds0)=140 MeV

@Quark model
质量：约2480 MeV
总宽度：270-990 MeV
伙伴态Ds1: 2560 MeV
M(Ds1)-M(Ds0)=80 MeV

Ds0(2317)的分子态解释

Phys. Rev. D 89, 014026

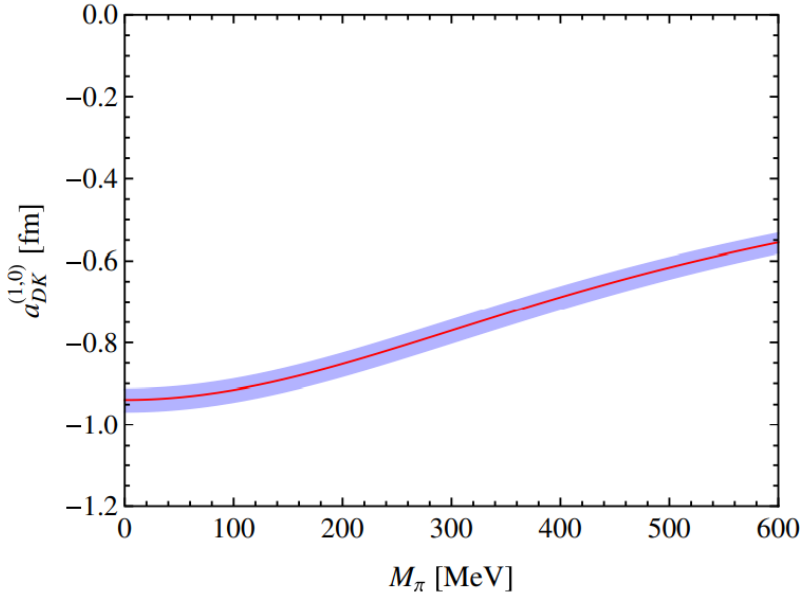


DK pole position@ChPT

TABLE V. Pole positions $\sqrt{s} = M - i\frac{\Gamma}{2}$ (in units of MeV) of charm mesons dynamically generated in the HQS UChPT.

(S, I)	$J^P = 0^+$	$J^P = 1^+$
(1, 0)	2317 ± 10	2457 ± 17
(0, 1/2)	$(2105 \pm 4) - i(103 \pm 7)$	$(2248 \pm 6) - i(106 \pm 13)$

Phys. Rev. D 87, 014508



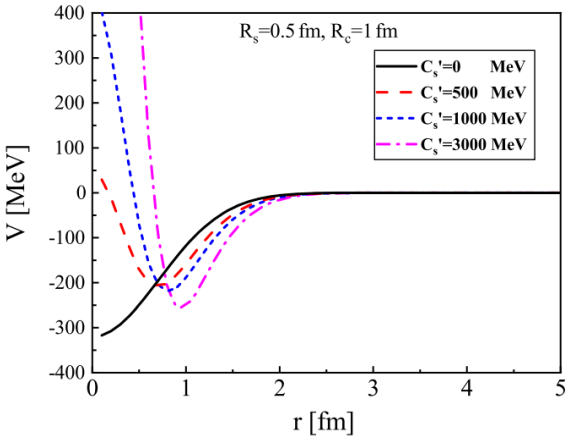
DK 散射长度@LQCD

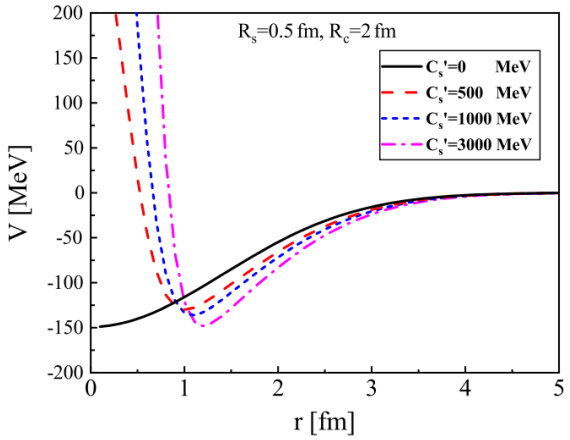
Pole position: 2315^{+18}_{-28} MeV

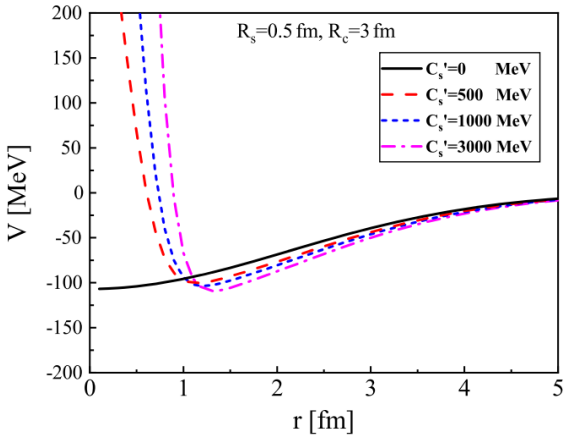
$\Gamma(D_{s0}^*(2317) \to D_s\pi) = (133 \pm 22)$ keV.

手征、格点、实验均支持Ds0(2317)是DK分子态或至少含有很大的DK分子态成分。

DK相互作用







LO Weinberg-Tomozawa (WT) DK interaction

$$V_{DK}(\vec{q}) = -\frac{C_W(I)}{2f_\pi^2},$$

$$C_W(0) = 2 \quad \text{and} \quad C_W(1) = 0,$$

$$V_{DK}(\vec{r}) = -\frac{C_W(I)}{2f_\pi^2}\delta^{(3)}(\vec{r}),$$

$$V_{DK}(r;R_c) = -\frac{C_W(I)}{2f_\pi^2}\frac{e^{-(r/R_c)^2}}{\pi^{3/2}R_c^3},$$

LO attractive main part+NLO repulsive core

$$V_{DK}(\vec{r};R_c) = C_S\frac{e^{-(r/R_S)^2}}{\pi^{3/2}R_S^3} + C(R_C)\frac{e^{-(r/R_c)^2}}{\pi^{3/2}R_c^3}$$
$$= C'_S e^{-(r/R_S)^2} + C'_L e^{-(r/R_c)^2},$$

- **拟合Ds0的束缚能45MeV：**

取固定短程排斥芯, Rs(0.5 fm), Cs'(0-3000 MeV);

取不同的cutoff Rc(1-3fm)，Rc表征有效力程。

DD相互作用

DD单玻色子交换相互作用

$$V_{DD}(r;\Lambda) = V_\rho(r;\Lambda) + V_\omega(r;\Lambda) + V_\sigma(r;\Lambda)$$

各个交换势具体形式

$$V_\sigma(r;\Lambda) = -g_\sigma^2\, m_\sigma\, W_C(m_\sigma r, \frac{\Lambda}{m_\sigma})\,,$$

$$V_\rho(r;\Lambda) = +\vec{\tau}_1\cdot\vec{\tau}_2\, g_\rho^2\, m_\rho\, W_C(m_\rho r, \frac{\Lambda}{m_\rho})\,,$$

$$V_\omega(r;\Lambda) = +g_\omega^2\, m_\omega\, W_C(m_\omega r, \frac{\Lambda}{m_\omega})\,,$$

其中

$$W_C(x,\lambda) = \frac{e^{-x}}{4\pi x} - \lambda\frac{e^{-\lambda x}}{4\pi\lambda x} - \frac{(\lambda^2-1)}{2\lambda}\frac{e^{-\lambda x}}{4\pi}.$$

高斯展开法简介

高斯展开法是一种通过变分原理求解少体系统薛定谔方程的基展开方法。

- 薛定谔方程和基展开

$$(H-E)\Psi_{JM}=0$$

$$\Psi_{JM}=\sum_{n=1}^{n_{max}} C_n^{(J)}\Phi_{JM,n}.$$

- 转化为本征值问题求解

$$\sum_{n'=1}^{n_{max}}(H_{nn'}^{(J)}-EN_{nn'}^{(J)})C_{n'}^{(J)}=0,$$

$$H_{nn'}^{(J)}=\langle\Phi_{JM,n}|H|\Phi_{JM,n'}\rangle,$$

$$N_{nn'}^{(J)}=\langle\Phi_{JM,n}|1|\Phi_{JM,n'}\rangle.$$

- 高斯基

$$\phi_{nlm}^G(r)=\phi_{nl}^G(r)Y_{lm}(\hat{r}),$$

$$\phi_{nl}^G(r)=N_{nl}r^l e^{-\nu_n r^2},$$

$$N_{nl}=\sqrt{\frac{2^{l+2}(2\nu_n)^{l+\frac{3}{2}}}{\sqrt{\pi}(2l+1)!!}}\quad(n=1-n_{max}).$$

- 高斯基参数(变分参数)

$$\nu_n=\frac{1}{r_n^2},$$

$$r_n=r_1a^{n-1}\quad(n=1-n_{max}).$$

变分和基展开使得高斯展开法同时具有精度高和收敛快的特点。

三体系统

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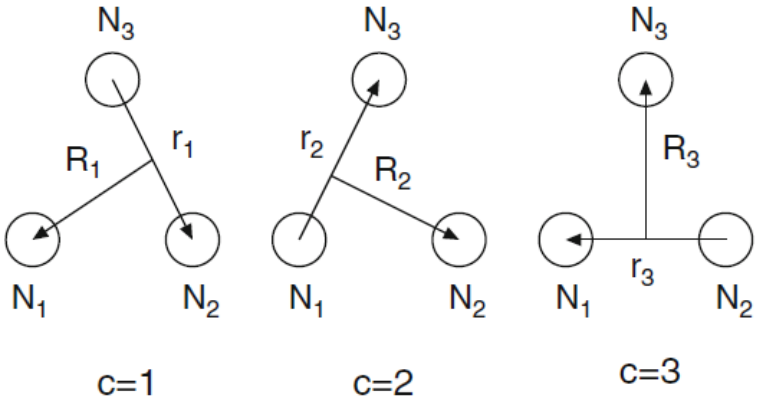
- 三体雅可比坐标系



- 总波函数

$$\Psi_{JM} = \Phi_{JM}^{(c=1)}(\mathbf{r}_1, \mathbf{R}_1) + \Phi_{JM}^{(c=2)}(\mathbf{r}_2, \mathbf{R}_2) + \Phi_{JM}^{(c=3)}(\mathbf{r}_3, \mathbf{R}_3)$$

- 每个雅可比道的波函数

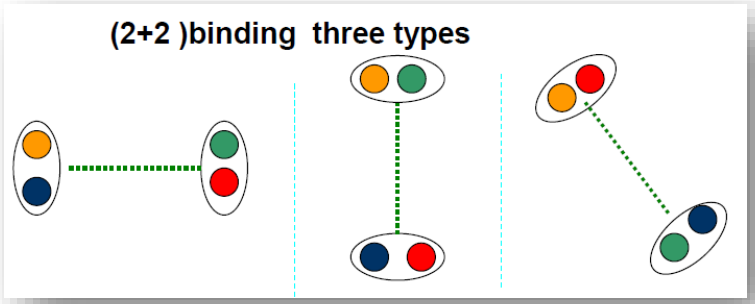
$$\Phi_{JM}^{(c)}(\mathbf{r}_c, \mathbf{R}_c) = \sum_{n_c l_c, N_c L_c} A_{n_c l_c, N_c L_c}^{(c)} [\phi_{n_c l_c}^G(\mathbf{r}_c) \psi_{N_c L_c}^G(\mathbf{R}_c)]_{JM}$$

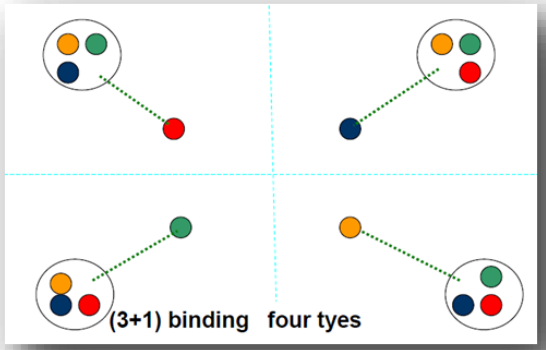
$$\phi_{nlm}^G(\mathbf{r}) = \phi_{nl}^G(r) Y_{lm}(\hat{r}), \phi_{nl}^G(r) = N_{nl} r^l e^{-\nu_n r^2}$$

$$\psi_{NLM}^G(\mathbf{R}) = \psi_{NL}^G(R) Y_{LM}(\hat{R}), \phi_{NL}^G(R) = N_{NL} R^L e^{-\lambda_N R^2}$$

两个两体波函数耦合成三体波函数.

四体系统





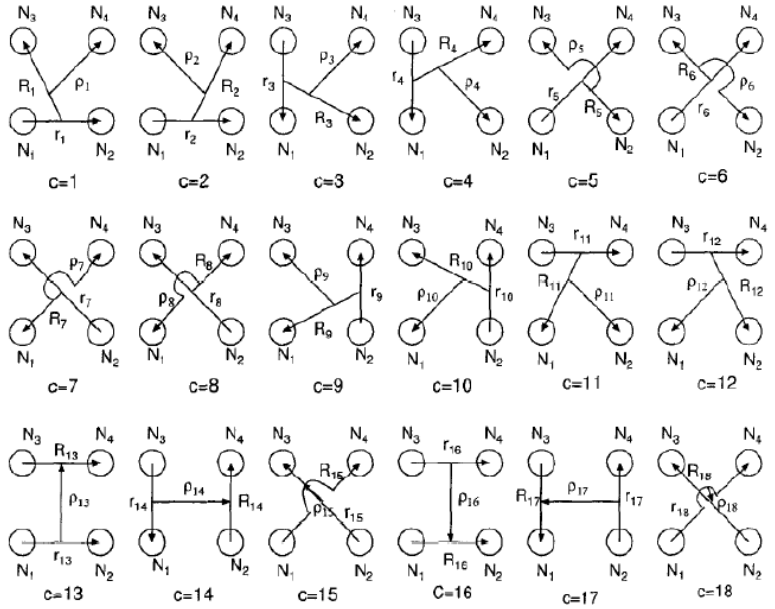


Figure 1: Jacobian coordinates for the 18 rearrangement channels of four-body system

K型耦合	12 个
H型耦合	6 个

2-body: 1 Jacobian channel
3-body: 3 Jacobian channels
4-body: 18 Jacobian channels
5-body: 100+ Jacobian channels

Straightly but not easy!

DK, DDK, DDDK

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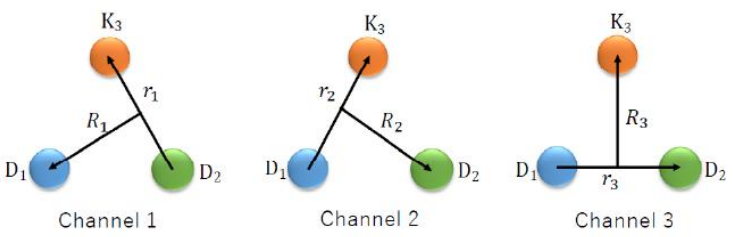
T. W. Wu, M. Z. Liu, L. S. Geng, E. Hiyama and M. P. Valderrama, Phys. Rev. D 100 (2019) 034029



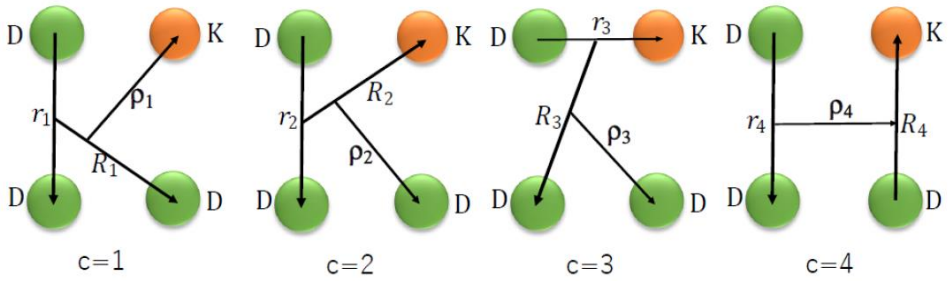
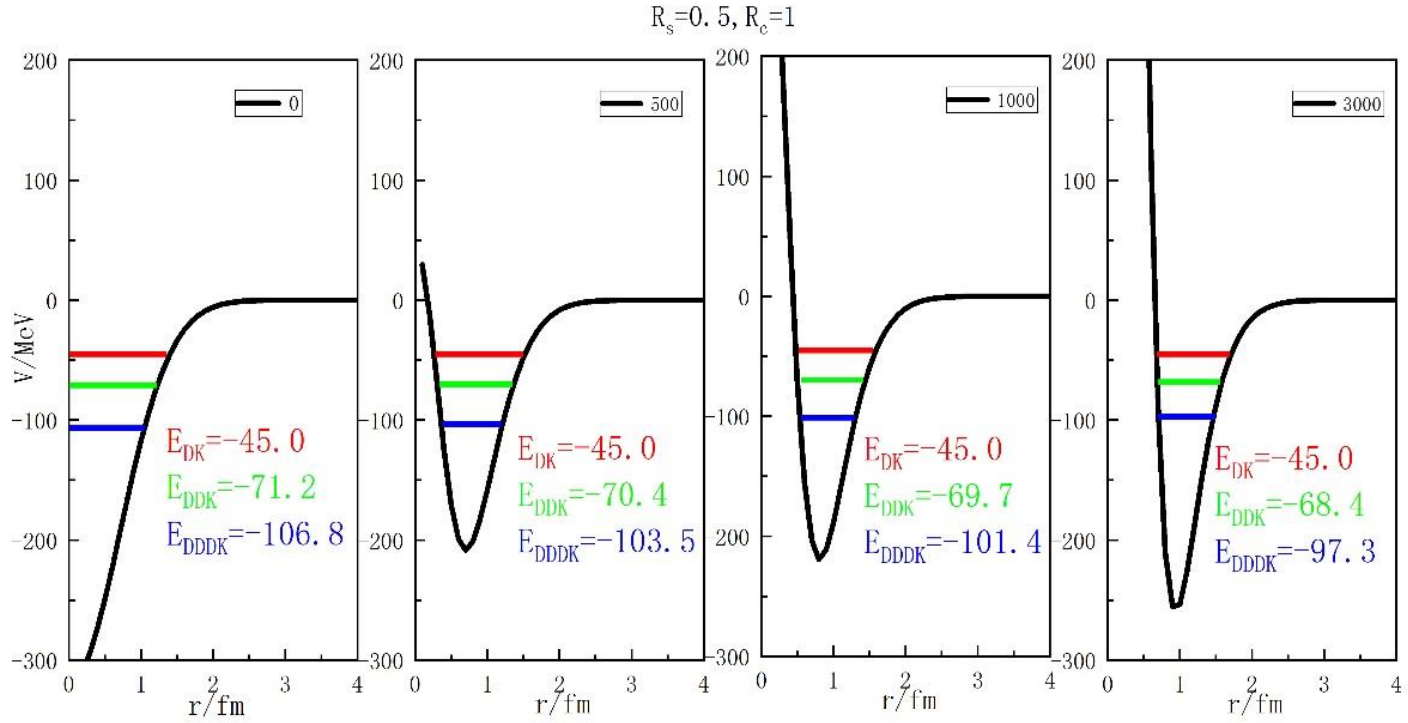


FIG. 2. Three Jacobian coordinates for *DDK* system



我们的研究表明，DK可以形成类核的多强子分子态集团。

DDK分子态的均方根半径与形状

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C'_S	C'_L	$r_2(DK)$	$r_3(DK)$	$r_3(DD)$	$\langle T \rangle$	$\langle V_{DK} \rangle$	$\langle V_{DD} \rangle$
		$R_S = 0.5$ fm		$R_c = 1$ fm			
0	−320.1	1.28	1.32	1.36	124.37	−189.61	−5.98
500	−455.4	1.39	1.44	1.47	99.51	−164.83	−5.03
1000	−562.6	1.46	1.53	1.54	91.43	−156.67	−4.51
3000	−838.7	1.61	1.69	1.68	93.24	−157.80	−3.82
		$R_S = 0.5$ fm		$R_c = 2$ fm			
0	−149.1	1.74	1.80	1.80	60.20	−125.74	−3.23
500	−178.4	1.91	1.98	1.96	51.00	−116.59	−2.64
1000	−195.0	1.99	2.07	2.04	50.63	−116.12	−2.43
3000	−225.9	2.13	2.22	2.15	53.61	−118.59	−2.24
		$R_S = 0.5$ fm		$R_c = 3$ fm			
0	−107.0	2.13	2.19	2.17	39.49	−105.35	−2.13
500	−119.4	2.31	2.38	2.34	34.80	−100.73	−1.77
1000	−125.6	2.37	2.47	2.42	34.90	−100.77	−1.65
3000	−136.2	2.53	2.61	2.53	36.66	−102.24	−1.54

$$r_3(DK) \approx r_3(DD)$$

DK均方根半径依赖于有效力程Rc的取值。DD的均方根半径也依赖于DK的力程。

D介子及其重夸克对称性伙伴

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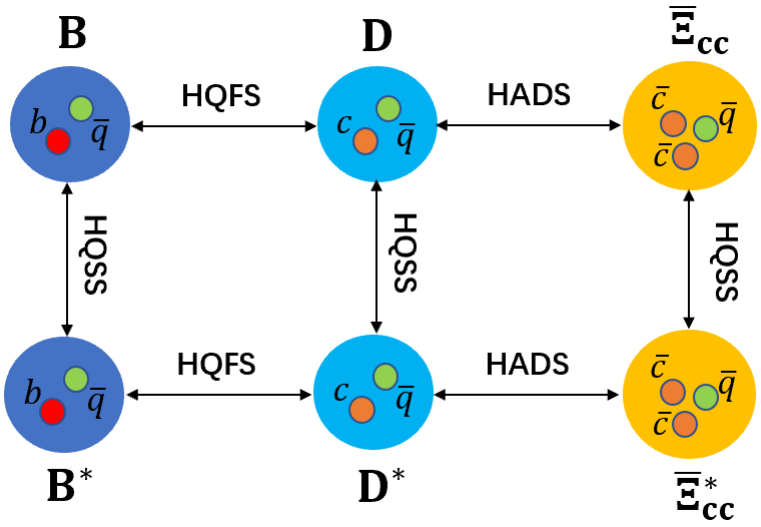
Heavy quark symmetry partners of D meson



重夸克自旋对称性
(Heavy quark spin symmetry, HQSS)

重夸克味道对称性
(Heavy quark flavor symmetry, HQFS)

重反夸克-双夸克对称性
(Heavy antiquark-diquark symmetry, HADS)

原则上只要DDK存在，其重夸克对称性伙伴态也存在。

$\Xi_{cc}\Xi_{cc}\bar{K}$ 系统

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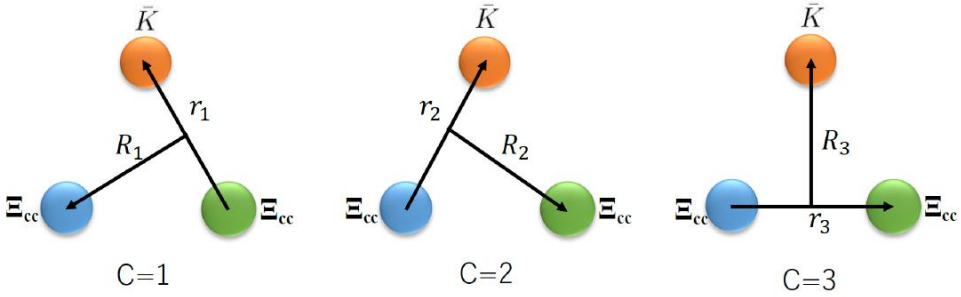
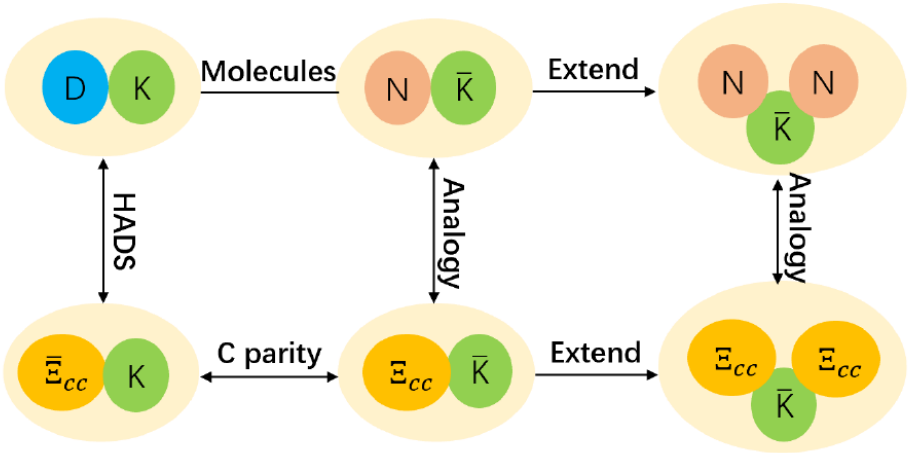
Jacobian 坐标系



与DDK和NN$\bar{K}$的关系

FIG. 2. $\Xi_{cc}\Xi_{cc}\bar{K}$ system reviewed from two kinds of logic.



T.W. WU, etc. Eur.Phys.J. C80 (2020) 901

$C(R_c)$	R_c	$B_2(\Xi_{cc}K)$	$B_3(\Xi_{cc}\Xi_{cc}K)$
	$C_S=0$	$R_s=0.1$	
-862.4	0.5	63.6^{+72}_{-51}	118.4^{+109}_{-78}
-320.1	1.0	53.7^{+37}_{-31}	92.8^{+55}_{-47}
-198.7	1.5	50.6^{+27}_{-24}	$84.1^{+41}_{-36}, 54.2^{+31}_{-26}$
-149.1	2.0	49.1^{+23}_{-21}	$79.6^{+34}_{-31}, 55.5^{+28}_{-24}$
	$C_S=1000$	$R_s=0.1$	
-884.7	0.5	63.2^{+70}_{-50}	117.5^{+106}_{-77}
-324.0	1.0	53.6^{+37}_{-31}	92.5^{+55}_{-41}
-200.2	1.5	50.6^{+27}_{-24}	$83.9^{+41}_{-36}, 54.2^{+31}_{-25}$
-149.9	2.0	49.1^{+23}_{-21}	$79.6^{+34}_{-31}, 55.5^{+28}_{-24}$

双重子+介子

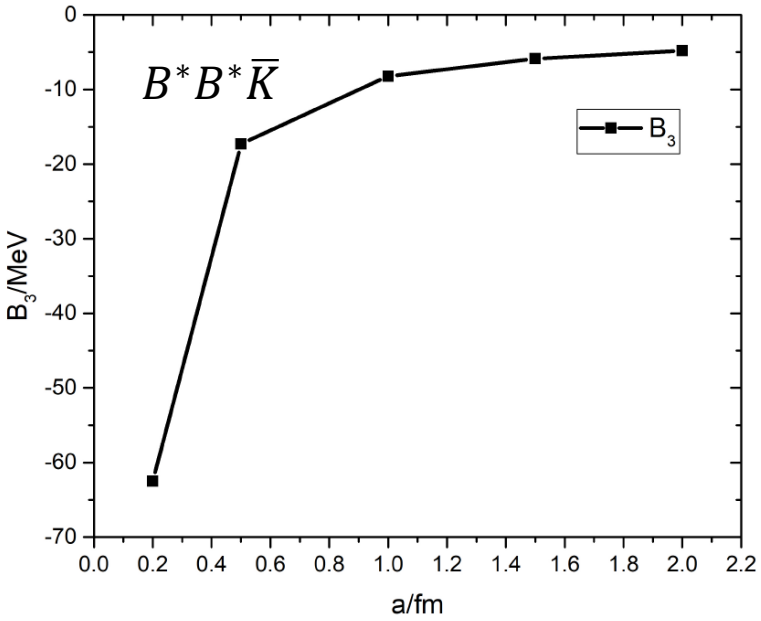
$C(R_c)$	R_c	$B_2(NK)$	$B_3(NNK)$
	$C_S=0$	$R_s=0.1$	
-925.9	0.5	29.4	35.2
-316.4	1.0	29.4	39.3
-132.6	2.0	29.4	41.8
-89.2	3.0	29.4	42.5
	$C_S=1000$	$R_s=0.1$	
-946.6	0.5	29.4	35.4
-319.8	1.0	29.4	39.4
-133.2	2.0	29.4	41.8
-89.4	3.0	29.4	42.5

Thomas Collapse in AAB system

L. Thomas, Phys. Rev. **47**, 903 (1935)

系统AAB，当Mass(A)>>Mass(B)时，若V_{AB}使得AB有固定的束缚能B_2，那么三体束缚能B_3随着V_{AB}的有效作用范围a趋于0而发散。

$B^*\bar{K}$束缚能1MeV时，$B^*B^*\bar{K}$束缚能随a的变化



$$V_{B^*\bar{K}}(r) = C_G e^{-r^2/a^2}$$

$C_G[MeV]$	$a[fm]$	B_2	B_3
−45.0	2.0	−1	−4.79
−74.5	1.5	−1	−5.86
−155.7	1	−1	−8.22
−577.3	0.5	−1	−17.3
−3443	0.2	−1	−62.5

a 减小　B_3 增大

不考虑自旋同位旋时B*B*Kbar的Thomas Collapse。

Thomas Collapse in $\Xi_{cc}\Xi_{cc}\bar{K}$ and $BB\bar{K}$ systems

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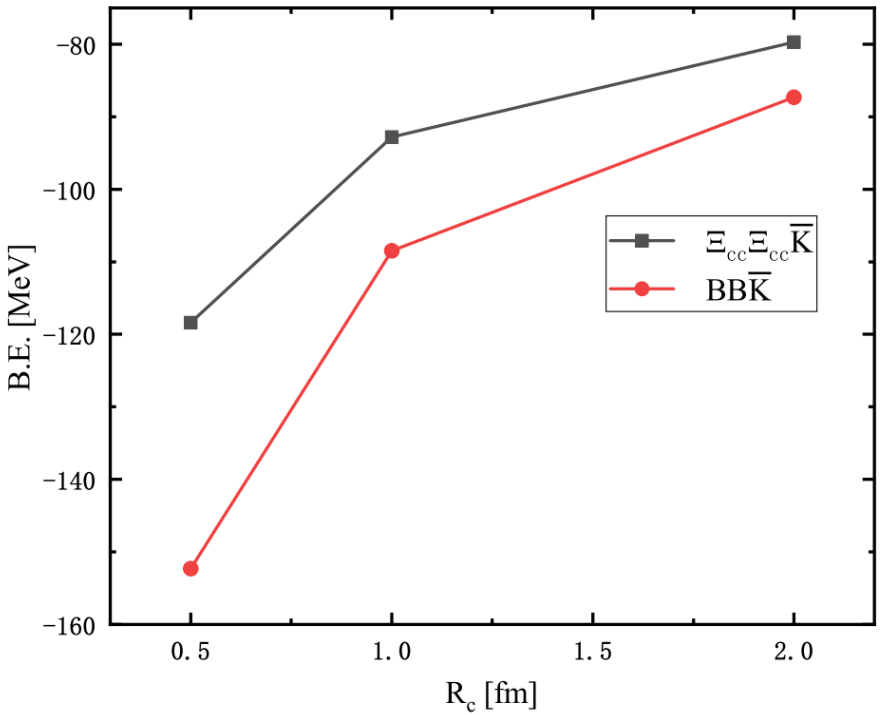

产生条件:

$$\lambda_\alpha = \frac{\sin 2\alpha}{2\alpha} \leq \lambda\,,$$

α与质量比值相关:

$$\alpha = \arcsin\left(\frac{1}{1+\frac{m_B}{m_A}}\right).$$

λ与系统量子数相关:

$$\lambda = \left\langle H^{c=1}_{\frac{1}{2},0} | H^{c=2}_{\frac{1}{2},0} \right\rangle = \frac{1}{2}.$$

$\lambda_\alpha(\Xi_{cc}\Xi_{cc}\bar{K}) = 0.389$ √

$\lambda_\alpha(BB\bar{K}) = 0.321$

$\lambda_\alpha(DDK) = 0.531$ ×

$\lambda_\alpha(NN\bar{K}) = 0.693$

隐粲三强子系统

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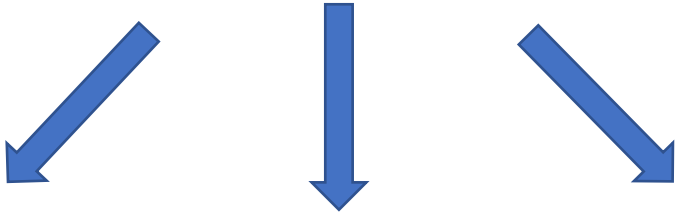
- DDK：双粲系统，实验上较难观测到。

  B(R++→D+Ds*+), **Phys.Rev. D102 (2020) no.11, 112001，@Belle**

- 隐粲系统：XYZ， Zcs， Pcs，实验容易观测到。



$\bar{D}_s^{(*)} D^{(*)} K$　　$D\bar{D}K$　　$\Sigma_c \bar{D} \bar{K}$

非类核多强子系统

隐粲三强子系统$D\bar{D}^{(*)}K$

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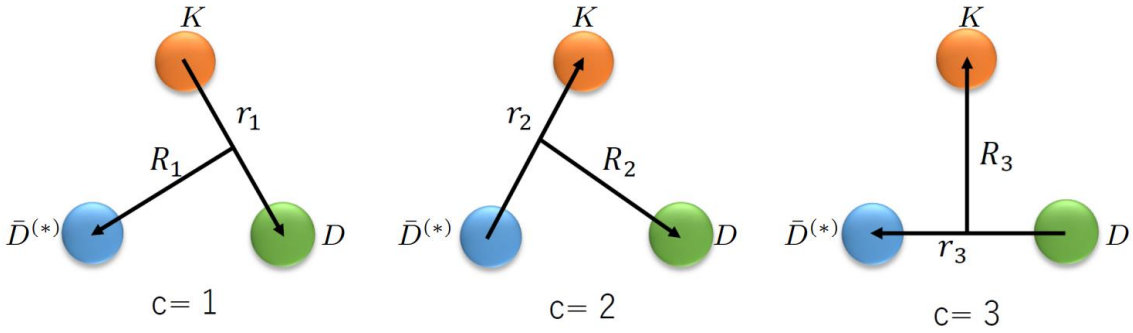
T.W. Wu, M.Z. Liu and L.S. Geng, Phys.Rev. D103 (2021) L031501

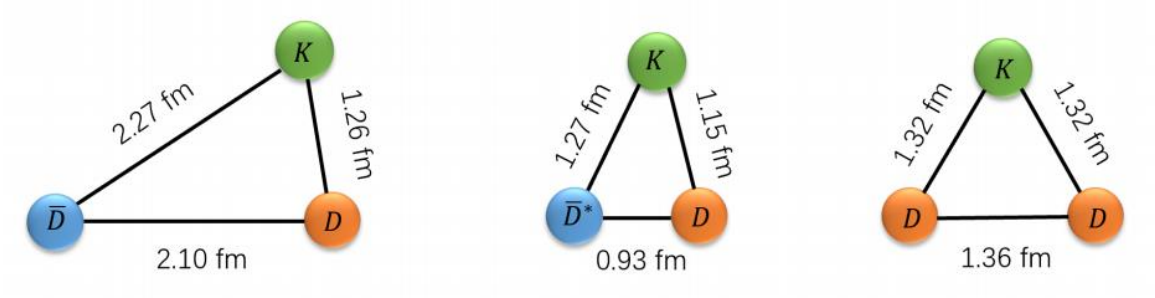


$D\bar{D}K$和$D\bar{D}^*K$可以形成三体束缚态

$$B_3(D\bar{D}K) \simeq 48.9^{+1.4}_{-2.4} \quad \text{MeV}.$$

$$B_3(D\bar{D}^*K) \simeq 77.3^{+3.1}_{-6.6} \quad \text{MeV}.$$

$D\bar{D}K$、$D\bar{D}^*K$和DDK束缚态的均方根半径



约49MeV　　约77MeV　　约70MeV

$D\bar{D}K$三体束缚态K_c的衰变

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Kc的衰变三角图

各个顶点的拉式量

$$\mathcal{L}_{K_c(x)} = g_{K_c D_{s0} \bar{D}} K_c^T(x) \int dy \Phi_{K_c}(y^2) D_{s0}(x + \omega_{\bar{D}} y) \bar{D}(x - \omega_{D_{s0}} y) + H.c.,$$

$$\mathcal{L}_{D_{s0}DK} = g_{D_{s0}DK} D_{s0} D K,$$

$$\mathcal{L}_{D_{s0}D_s\eta} = g_{D_{s0}D_s\eta} D_{s0} D_s \eta,$$

$$\mathcal{L}_{\psi\bar{D}D} = -ig_{\psi\bar{D}D}\psi_\mu(\partial^\mu D D^\dagger - D\partial^\mu D^\dagger),$$

$$\mathcal{L}_{\bar{D}\bar{D}^*\eta} = -ig_{\bar{D}\bar{D}^*\eta}(\bar{D}^\dagger \partial^\mu \eta \bar{D}^* - \bar{D}\partial_\mu \eta \bar{D}^{*\dagger}),$$

$$\Gamma[K_c \to] = \frac{1}{2J+1} \frac{1}{32\pi^2} \frac{|\vec{p}_1|}{m_{K_c}^2} \overline{|\mathcal{M}|^2} d\Omega,$$

分宽带0.5MeV

分宽带0.2MeV

总结和展望

- 我们预言存在DK, DDK 和 DDDK的强子分子态，并指出有可能存在类核的多强子分子态集团。
- 我们研究了$\Xi_{cc}\Xi_{cc}\overline{K}$ 和$BB\overline{K}$系统，并讨论了它们可以产生Thomas collapse现象 。
- 我们预言了$D\overline{D}K$分子态的存在，并计算了它的衰变道和宽度，希望给实验的寻找提供方向。
- 如果这些新的奇特强子态被发现，将可以对强子分子态图像作非平庸检验。

- 如何找：产生过程、反应截面等

  期待实验和理论上的进一步研究。