Relativistic effect of double heavy mesons

王国利

河北大学

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- In collaboration with

重庆大学：吴兴刚

河北大学：冯太傅

哈工大：耿子侃、王天鸿、姜越等

Outline

- Motivation
- Heavy Vector Quarkonium Leptonic Widths
- Average speed of quark and its powers in charmonium and bottomonium
- Relativistic effects in Semileptonic Bc decays to charmonium
- Summary

Motivation

- Since $v^2 \sim 0.1 - 0.3$ for b and c, so we believed that relativistic correction for double heavy meson is small. Is this true?
- The annihilations of heavy vector quarkonium are studied in Bethe-Salpeter method, the Bottomnium cases are consistent with EX, but not the Charmonium.
- We have used a relativistic method, the possible reason of the deviation for Charmonium case is that the QCD corrections are ignored. Since in Coulomb potential $\alpha_s \sim v$, or numerically $\alpha_s \sim v^2$, both means large QCD corrections equivalent to large relativistic effects.

Heavy Vector Quarkonium Leptonic Widths

- Decay rate of a nS quarkonium dilepton annihilation

$$\Gamma_{V\to\ell^+\ell^-} = \frac{4\pi\alpha_{em}^2 e_Q^2 F_V^2}{3M_{nS}} \times \left(1+2\frac{m_\ell^2}{M_{nS}^2}\right)\sqrt{1-4\frac{m_\ell^2}{M_{nS}^2}},$$

where F_V is the decay constant, now in literature

$$F_V = F_V^{NR}\cdot\beta_V \qquad F_V^{NR} = \sqrt{\frac{12}{M_{nS}}}|\Psi_V(0)|$$

F_V^{NR} is the result from NRQCD, β_V is the correction from QCD.

- We use the relativistic Bethe-Slapeter method to provide a relativistic result $F_V^{NR} \to F_V^{RE}$

Wave function

- A non-relativistic wave function for a vector meson

$$\Psi_P(\vec{q}) = (\not{P} + M)\not{\epsilon}\psi(\vec{q})$$

- A relativistic one with instantaneous approximation

$$\Psi_P^{1^-}(q_\perp) = q_\perp \cdot \epsilon_\perp \left[\psi_1(q_\perp) + \frac{\not{P}}{M}\psi_2(q_\perp) + \frac{\not{q}_\perp}{M}\psi_3(q_\perp) + \frac{\not{P}\not{q}_\perp}{M^2}\psi_4(q_\perp)\right] + M\not{\epsilon}_\perp\psi_5(q_\perp)$$

$$+\not{\epsilon}_\perp\not{P}\psi_6(q_\perp) + (\not{q}_\perp\not{\epsilon}_\perp - q_\perp \cdot \epsilon_\perp)\psi_7(q_\perp) + \frac{1}{M}(\not{P}\not{\epsilon}_\perp\not{q}_\perp - \not{P}q_\perp \cdot \epsilon_\perp)\psi_8(q_\perp)$$

If we delete the relativistic terms (q dependent), then only two terms remained, if set $\psi_5 = -\psi_6 = \psi$ then the wave function become to the non-relativistic one.

The radial wave functions can be obtained by solving the instantaneous Bethe-Salpeter equation.

Solve the full Salpeter equation

- The wave function for vector quarkonium

$$\Psi_P^{1^{--}}(q_\perp) = q_\perp \cdot \epsilon_\perp \left[\psi_1(q_\perp) + \frac{\not{q}_\perp}{M}\psi_3(q_\perp) + \frac{\not{P}\not{q}_\perp}{M^2}\psi_4(q_\perp)\right]$$
$$+ M\not{\epsilon}_\perp\psi_5(q_\perp) + \not{\epsilon}_\perp\not{P}\psi_6(q_\perp) + \frac{1}{M}(\not{P}\not{\epsilon}_\perp\not{q}_\perp - \not{P}q_\perp \cdot \epsilon_\perp)\psi_8(q_\perp)$$

- The last two equations in Salpeter equations result in

$$\psi_1(q_\perp) = \frac{q_\perp^2\psi_3(q_\perp) + M^2\psi_5(q_\perp)}{Mm_1}, \quad \psi_8(q_\perp) = -\frac{\psi_6(q_\perp)M}{m_1}$$

- We obtained 4 coupled equation from the first two equations, and the unknown numbers of radial wave functions are also 4: $\psi_3(q_\perp), \psi_4(q_\perp), \psi_5(q_\perp), \psi_6(q_\perp)$

Decay constant

- Definition of the vector decay constant

$$F_V^{RE} M \epsilon_\mu = \sqrt{N_c} \int \frac{d^4 q}{(2\pi)^4} \mathrm{Tr}[\chi_P(q)\gamma_\mu] = i\sqrt{N_c} \int \frac{d^3 \vec{q}}{(2\pi)^3} \mathrm{Tr}[\Psi_P(\vec{q})\gamma_\mu]$$

- In this realtivistic method

$$F_V^{RE} = 4\sqrt{3} \int \frac{d^3 \vec{q}}{(2\pi)^3} \left[\psi_5(\vec{q}) - \frac{\vec{q}^2}{3M^2} \psi_3(\vec{q}) \right]$$

- Non-relativistic method

$$F_V = 4\sqrt{N_c} \int \frac{d\vec{q}}{(2\pi)^3} \psi_5(\vec{q})$$

粲偶素衰变到双轻子结果

modes	NR	Re	$\frac{\text{NR}-\text{Re}}{\text{Re}}$	Exp
$J/\psi \to e^+e^-$	$10.95^{+2.20}_{-1.86}$	$8.95^{+1.57}_{-1.38}$	$22.3^{+2.7}_{-2.2}\%$	5.55 ± 0.16
$\psi(2S) \to e^+e^-$	$5.92^{+1.05}_{-0.89}$	$4.43^{+0.60}_{-0.54}$	$33.6^{+5.0}_{-4.3}\%$	2.33 ± 0.04
$\psi(2S) \to \tau^+\tau^-$	$2.31^{+1.68}_{-2.31}$	$1.73^{+1.29}_{-1.73}$	$33.6^{+3.2}_{-4.1}\%$	0.91 ± 0.14
$\psi(3S) \to e^+e^-$	$4.30^{+0.69}_{-0.66}$	$3.04^{+0.35}_{-0.35}$	$41.4^{+5.6}_{-6.1}\%$	0.86 ± 0.07
$\psi(3S) \to \tau^+\tau^-$	$2.87^{+0.61}_{-1.23}$	$2.03^{+0.48}_{-0.87}$	$41.4^{+6.9}_{-6.3}\%$	–
$\psi(4S) \to e^+e^-$	$3.53^{+0.65}_{-0.66}$	$2.32^{+0.24}_{-0.26}$	$52.2^{+11.1}_{-12.9}\%$	0.48 ± 0.22
$\psi(4S) \to \tau^+\tau^-$	$2.70^{+0.26}_{-0.60}$	$1.78^{+0.25}_{-0.43}$	$52.2^{+15.4}_{-13.0}\%$	–
$\psi(5S) \to e^+e^-$	$3.05^{+0.55}_{-0.52}$	$1.88^{+0.16}_{-0.19}$	$62.2^{+14.3}_{-12.5}\%$	0.58 ± 0.07
$\psi(5S) \to \tau^+\tau^-$	$2.49^{+0.30}_{-0.58}$	$1.54^{+0.21}_{-0.31}$	$62.2^{+16.4}_{-23.1}\%$	–

Table 5. Ratio $\Gamma(\psi(nS) \to e^+e^-)/\Gamma(J/\psi \to e^+e^-)$.

	$\frac{\Gamma(\psi(2S))}{\Gamma(J/\psi)}$	$\frac{\Gamma(\psi(3S))}{\Gamma(J/\psi)}$	$\frac{\Gamma(\psi(4S))}{\Gamma(J/\psi)}$	$\frac{\Gamma(\psi(5S))}{\Gamma(J/\psi)}$
Ours	$0.495^{+0.019}_{-0.017}$	$0.340^{+0.016}_{-0.017}$	$0.259^{+0.013}_{-0.016}$	$0.210^{+0.013}_{-0.016}$
Exp [50]	0.42 ± 0.02	0.15 ± 0.02	0.086 ± 0.042	0.10 ± 0.02

底偶素结果

modes	NR	Re	$\frac{\text{NR}-\text{Re}}{\text{Re}}$	Exp
$\Upsilon(1S)\to e^+e^-$	$1.47^{+0.23}_{-0.20}$	$1.29^{+0.19}_{-0.16}$	$14.0^{+0.9}_{-1.6}\%$	1.340±0.018 (1.29±0.09)
$\Upsilon(1S)\to \tau^+\tau^-$	$1.46^{+0.22}_{-0.20}$	$1.28^{+0.18}_{-0.16}$	$14.0^{+1.1}_{-1.5}\%$	1.40±0.09
$\Upsilon(2S)\to e^+e^-$	$0.771^{+0.123}_{-0.125}$	$0.629^{+0.104}_{-0.088}$	$22.6^{+0.0}_{-3.2}\%$	0.612±0.011
$\Upsilon(2S)\to \tau^+\tau^-$	$0.766^{+0.120}_{-0.123}$	$0.625^{+0.101}_{-0.086}$	$22.6^{+0.0}_{-3.3}\%$	0.64±0.12
$\Upsilon(3S)\to e^+e^-$	$0.541^{+0.088}_{-0.088}$	$0.450^{+0.070}_{-0.065}$	$20.2^{+7.8}_{-2.5}\%$	0.443±0.008
$\Upsilon(3S)\to \tau^+\tau^-$	$0.538^{+0.086}_{-0.087}$	$0.448^{+0.068}_{-0.064}$	$20.2^{+7.7}_{-2.8}\%$	0.47±0.10
$\Upsilon(4S)\to e^+e^-$	$0.429^{+0.083}_{-0.059}$	$0.355^{+0.058}_{-0.050}$	$20.8^{+6.6}_{-7.2}\%$	0.272±0.029 (0.322±0.056)
$\Upsilon(4S)\to \tau^+\tau^-$	$0.427^{+0.081}_{-0.057}$	$0.353^{+0.056}_{-0.050}$	$20.8^{+6.5}_{-7.1}\%$	–
$\Upsilon(5S)\to e^+e^-$	$0.380^{+0.048}_{-0.069}$	$0.296^{+0.048}_{-0.038}$	$28.4^{+1.1}_{-7.9}\%$	0.31±0.07
$\Upsilon(5S)\to \tau^+\tau^-$	$0.378^{+0.047}_{-0.068}$	$0.295^{+0.047}_{-0.038}$	$28.4^{+1.0}_{-7.8}\%$	–

	$\frac{\Gamma(\Upsilon(2S))}{\Gamma(\Upsilon(1S))}$	$\frac{\Gamma(\Upsilon(3S))}{\Gamma(\Upsilon(1S))}$	$\frac{\Gamma(\Upsilon(4S))}{\Gamma(\Upsilon(1S))}$	$\frac{\Gamma(\Upsilon(5S))}{\Gamma(\Upsilon(1S))}$
Ours	$0.488^{+0.008}_{-0.009}$	$0.349^{+0.003}_{-0.008}$	$0.275^{+0.004}_{-0.000}$	$0.229^{+0.003}_{-0.001}$
Exp1 [50]	0.457±0.014	0.33±0.01	0.203±0.024 (0.240±0.045)	0.23±0.06
Exp2 [50]	0.47±0.04	0.34±0.03	0.21±0.04(0.25±0.06)	0.24±0.07

结果分析

- 粲偶素的结果与实验值差别大
- 底偶素情况与实验值符合很好
- 虽然考虑了相对论效应，但是没有完整考虑QCD修正
- 大的QCD修正同时也意味着大的相对论修正
- 为了验证，计算夸克速率及其幂次

Average speed of quark and its powers in charmonium and bottomonium

Wave Fuctions

- In our method the relativistic wave function for a pseudoscalar 0^{-+} meson

$$\varphi_{0^{-+}}(\boldsymbol{q}) = M\left[\gamma_0 a_1(\boldsymbol{q}) + a_2(\boldsymbol{q}) + \frac{\boldsymbol{q}\cdot\boldsymbol{\gamma}\gamma_0}{M} a_3(\boldsymbol{q})\right]\gamma_5$$

- Wave function for ${}^3P_0$ 0^{++} scalar meson

$$\varphi_{0^{++}}(\boldsymbol{q}) = -\boldsymbol{q}\cdot\boldsymbol{\gamma} f_1(\boldsymbol{q}) + \boldsymbol{q}\cdot\boldsymbol{\gamma}\gamma_0 f_2(\boldsymbol{q}) + M f_3(\boldsymbol{q})$$

Wave functions

- Wave function for ${}^1P_1$ 1^{+-} meson

$$\varphi_{1^{+-}}(\boldsymbol{q}) = \boldsymbol{q}\cdot\boldsymbol{\epsilon}\left[h_1(\boldsymbol{q}) + \gamma_0 h_2(\boldsymbol{q}) + \frac{\boldsymbol{q}\cdot\boldsymbol{\gamma}\gamma_0}{M}h_4(\boldsymbol{q})\right]\gamma_5$$

- Wave function for ${}^3P_1$ 1^{++} meson

$$\varphi_{1^{++}}(\boldsymbol{q}) = i\varepsilon_{0\mu\alpha\beta}q^\alpha\epsilon^\beta[\gamma^\mu g_1(\boldsymbol{q}) + \gamma^0\gamma^\mu g_2(\boldsymbol{q}) + ig_4(\boldsymbol{q})\varepsilon^{0\mu\rho\delta}q_\rho\gamma_\delta\gamma_5/M],$$

- Wave function for $2^{++}({}^3P_2)$ meson

$$\varphi_{2^{++}}(\boldsymbol{q}) = \varepsilon_{\mu\nu}q^\nu\left\{q^\mu\left[j_1(\boldsymbol{q}) - \frac{\boldsymbol{q}\cdot\boldsymbol{\gamma}}{M}j_3(\boldsymbol{q}) + \frac{\boldsymbol{q}\cdot\boldsymbol{\gamma}\gamma_0}{M}j_4(\boldsymbol{q})\right] + M\gamma^\mu[j_5(\boldsymbol{q}) + \gamma^0 j_6(\boldsymbol{q})] - i\epsilon^{0\mu\beta\gamma}q_\beta\gamma_\gamma\gamma_5 j_8(\boldsymbol{q})\right\}$$

动量幂次的平均值

$$\langle q^n\rangle_{0^{-+}} = \int \frac{\mathrm{d}^3q}{(2\pi)^3} 2a_1a_2|\boldsymbol{q}|^n M\left\{\frac{\omega_1}{m_1}+\frac{m_1}{\omega_1}+\frac{\boldsymbol{q}^2}{\omega_1 m_1}\right\}$$

$$\langle q^n\rangle_{1^{--}} = \int \frac{\mathrm{d}^3q}{(2\pi)^3} \frac{4\omega_1|\boldsymbol{q}|^n}{3m_1M}\left[3b_5b_6M^2+b_4b_5\boldsymbol{q}^2-b_3\boldsymbol{q}^2\left(b_4\frac{\boldsymbol{q}^2}{M^2}+b_6\right)\right]$$

$$\langle q^n\rangle_{0^{++}} = \int \frac{\mathrm{d}^3q}{(2\pi)^3} \frac{4f_1f_2\omega_1|\boldsymbol{q}|^{2+n}}{m_1M}$$

$$\langle q^n\rangle_{1^{++}} = \int \frac{\mathrm{d}^3q}{(2\pi)^3} \frac{8g_1g_2\omega_1|\boldsymbol{q}|^{2+n}}{3m_1M}$$

$$\langle q^n\rangle_{1^{+-}} = \int \frac{\mathrm{d}^3q}{(2\pi)^3} \frac{4h_1h_2\omega_1|\boldsymbol{q}|^{2+n}}{3m_1M}$$

$$\langle q^n\rangle_{2^{++}} = \int \frac{\mathrm{d}^3q}{(2\pi)^3} \frac{4\omega_1|\boldsymbol{q}|^{2+n}}{15m_1M}\left\{5j_5j_6M^2+2j_4j_5\boldsymbol{q}^2-2\boldsymbol{q}^2j_3\left(j_4\frac{\boldsymbol{q}^2}{M^2}+j_6\right)\right\}$$

结果

State	Mass	q	q^2	q^3	q^4	v	v^2	v^3	v^4
$\eta_c(1S)$	2980.3	0.728	0.653	0.706	0.915	0.449	0.249	0.166	0.133
$\eta_c(2S)$	3576.4	0.796	0.885	1.17	1.72	0.491	0.337	0.274	0.249
$\eta_c(3S)$	3948.8	0.877	1.06	1.52	2.41	0.541	0.405	0.357	0.350
$\eta_c(4S)$	4224.6	0.939	1.21	1.82	3.03	0.579	0.460	0.428	0.440
J/ψ	3096.9	0.743	0.679	0.744	0.970	0.459	0.259	0.175	0.141
$\psi(2S)$	3688.1	0.810	0.914	1.22	1.81	0.500	0.348	0.286	0.262
$\psi(3S)$	4056.8	0.894	1.10	1.59	2.54	0.552	0.419	0.374	0.369
$\psi(4S)$	4329.4	0.956	1.25	1.90	3.19	0.590	0.476	0.447	0.463

State	Mass	q	q^2	q^3	q^4	v	v^2	v^3	v^4
$\chi_{c0}(1P)$	3414.7	0.838	0.796	0.850	1.02	0.517	0.303	0.200	0.148
$\chi_{c0}(2P)$	3836.8	0.882	0.992	1.30	1.87	0.544	0.378	0.305	0.271
$\chi_{c0}(3P)$	4140.1	0.937	1.15	1.64	2.58	0.579	0.439	0.387	0.375
$\chi_{c0}(4P)$	4376.9	0.985	1.28	1.94	3.21	0.608	0.489	0.455	0.466
$\chi_{c1}(1P)$	3510.3	0.849	0.814	0.874	1.05	0.524	0.310	0.205	0.152
$\chi_{c1}(2P)$	3928.7	0.896	1.02	1.34	1.93	0.553	0.388	0.315	0.280
$\chi_{c1}(3P)$	4228.8	0.953	1.18	1.70	2.68	0.588	0.451	0.401	0.389
$\chi_{c1}(4P)$	4463.1	1.00	1.32	2.01	3.34	0.619	0.503	0.473	0.485
$\chi_{c2}(1P)$	3555.6	0.839	0.791	0.829	0.959	0.518	0.301	0.195	0.139
$\chi_{c2}(2P)$	3971.0	0.896	1.01	1.30	1.84	0.553	0.385	0.307	0.267
$\chi_{c2}(3P)$	4269.3	0.957	1.18	1.68	2.60	0.590	0.451	0.396	0.378
$\chi_{c2}(4P)$	4502.0	1.01	1.33	2.00	3.28	0.622	0.505	0.471	0.476
$h_c(1P)$	3526.0	0.844	0.802	0.851	1.00	0.521	0.306	0.200	0.146
$h_c(2P)$	3943.0	0.896	1.01	1.32	1.89	0.553	0.387	0.311	0.274
$h_c(3P)$	4242.4	0.955	1.18	1.69	2.64	0.589	0.451	0.398	0.384
$h_c(4P)$	4476.2	1.00	1.32	2.01	3.31	0.620	0.504	0.472	0.481

结果

State	v	v^2	v^3	v^4
$\eta_c(1S)$	0.449	0.249	0.166	0.133
$\eta_c(2S)$	0.491	0.337	0.274	0.249
$\eta_c(3S)$	0.541	0.405	0.357	0.350
$\eta_c(4S)$	0.579	0.460	0.428	0.440
J/ψ	0.459	0.259	0.175	0.141
$\psi(2S)$	0.500	0.348	0.286	0.262
$\psi(3S)$	0.552	0.419	0.374	0.369
$\psi(4S)$	0.590	0.476	0.447	0.463

State	v	v^2	v^3	v^4
$\chi_{c0}(1P)$	0.517	0.303	0.200	0.148
$\chi_{c0}(2P)$	0.544	0.378	0.305	0.271
$\chi_{c0}(3P)$	0.579	0.439	0.387	0.375
$\chi_{c0}(4P)$	0.608	0.489	0.455	0.466
$\chi_{c1}(1P)$	0.524	0.310	0.205	0.152
$\chi_{c1}(2P)$	0.553	0.388	0.315	0.280
$\chi_{c1}(3P)$	0.588	0.451	0.401	0.389
$\chi_{c1}(4P)$	0.619	0.503	0.473	0.485
$\chi_{c2}(1P)$	0.518	0.301	0.195	0.139
$\chi_{c2}(2P)$	0.553	0.385	0.307	0.267
$\chi_{c2}(3P)$	0.590	0.451	0.396	0.378
$\chi_{c2}(4P)$	0.622	0.505	0.471	0.476
$h_c(1P)$	0.521	0.306	0.200	0.146
$h_c(2P)$	0.553	0.387	0.311	0.274
$h_c(3P)$	0.589	0.451	0.398	0.384
$h_c(4P)$	0.620	0.504	0.472	0.481

State	Mass	q	q^2	q^3	q^4	v	v^2	v^3	v^4
$\eta_b(1S)$	9390.2	1.19	1.74	3.02	6.00	0.240	0.0708	0.0247	0.00991
$\eta_b(2S)$	9950.0	1.18	2.00	4.04	9.00	0.237	0.0811	0.0331	0.0149
$\eta_b(3S)$	10311.4	1.21	2.11	4.43	10.3	0.244	0.0860	0.0363	0.0170
$\eta_b(4S)$	10554.0	1.30	2.41	5.32	13.0	0.262	0.0981	0.0436	0.0214
$\Upsilon(1S)$	9460.5	1.20	1.76	3.05	6.10	0.241	0.0715	0.0250	0.0101
$\Upsilon(2S)$	10023.1	1.16	1.96	3.95	8.79	0.234	0.0797	0.0324	0.0145
$\Upsilon(3S)$	10368.9	1.25	2.22	4.74	11.1	0.251	0.0904	0.0388	0.0184
$\Upsilon(4S)$	10635.8	1.34	2.52	5.61	13.7	0.270	0.103	0.0460	0.0226

State	Mass	q	q^2	q^3	q^4	v	v^2	v^3	v^4
$\chi_{b0}(1P)$	9859.0	1.29	1.88	3.08	5.62	0.259	0.0764	0.0253	0.00929
$\chi_{b0}(2P)$	10240.6	1.29	2.15	4.22	9.07	0.259	0.0875	0.0346	0.0150
$\chi_{b0}(3P)$	10524.7	1.35	2.44	5.20	12.1	0.271	0.0993	0.0426	0.0200
$\chi_{b0}(4P)$	10757.0	1.39	2.65	5.92	14.5	0.280	0.108	0.0485	0.0239
$\chi_{b1}(1P)$	9892.2	1.28	1.88	3.08	5.60	0.259	0.0763	0.0252	0.00925
$\chi_{b1}(2P)$	10272.7	1.29	2.16	4.25	9.13	0.260	0.0880	0.0348	0.0151
$\chi_{b1}(3P)$	10556.2	1.35	2.45	5.22	12.2	0.271	0.0994	0.0427	0.0201
$\chi_{b1}(4P)$	10787.8	1.38	2.61	5.83	14.2	0.278	0.106	0.0477	0.0235
$\chi_{b2}(1P)$	9913.3	1.26	1.80	2.88	5.13	0.254	0.0731	0.0236	0.00847
$\chi_{b2}(2P)$	10284.0	1.24	2.04	3.94	8.35	0.251	0.0830	0.0323	0.0138
$\chi_{b2}(3P)$	10591.6	1.36	2.49	5.32	12.4	0.275	0.101	0.0436	0.0205
$\chi_{b2}(4P)$	10786.9	1.43	2.76	6.17	15.0	0.289	0.112	0.0506	0.0248
$h_b(1P)$	9900.2	1.27	1.84	2.97	5.32	0.257	0.0747	0.0243	0.00879
$h_b(2P)$	10280.4	1.25	2.05	3.93	8.30	0.252	0.0832	0.0322	0.0137
$h_b(3P)$	10562.0	1.34	2.42	5.11	11.8	0.270	0.0983	0.0419	0.0195
$h_b(4P)$	10793.8	1.39	2.65	5.90	14.4	0.281	0.108	0.0484	0.0237

State	v	v^2	v^3	v^4
$\eta_b(1S)$	0.240	0.0708	0.0247	0.00991
$\eta_b(2S)$	0.237	0.0811	0.0331	0.0149
$\eta_b(3S)$	0.244	0.0860	0.0363	0.0170
$\eta_b(4S)$	0.262	0.0981	0.0436	0.0214
$\Upsilon(1S)$	0.241	0.0715	0.0250	0.0101
$\Upsilon(2S)$	0.234	0.0797	0.0324	0.0145
$\Upsilon(3S)$	0.251	0.0904	0.0388	0.0184
$\Upsilon(4S)$	0.270	0.103	0.0460	0.0226

State	v	v^2	v^3	v^4
$\chi_{b0}(1P)$	0.259	0.0764	0.0253	0.00929
$\chi_{b0}(2P)$	0.259	0.0875	0.0346	0.0150
$\chi_{b0}(3P)$	0.271	0.0993	0.0426	0.0200
$\chi_{b0}(4P)$	0.280	0.108	0.0485	0.0239
$\chi_{b1}(1P)$	0.259	0.0763	0.0252	0.00925
$\chi_{b1}(2P)$	0.260	0.0880	0.0348	0.0151
$\chi_{b1}(3P)$	0.271	0.0994	0.0427	0.0201
$\chi_{b1}(4P)$	0.278	0.106	0.0477	0.0235
$\chi_{b2}(1P)$	0.254	0.0731	0.0236	0.00847
$\chi_{b2}(2P)$	0.251	0.0830	0.0323	0.0138
$\chi_{b2}(3P)$	0.275	0.101	0.0436	0.0205
$\chi_{b2}(4P)$	0.289	0.112	0.0506	0.0248
$h_b(1P)$	0.257	0.0747	0.0243	0.00879
$h_b(2P)$	0.252	0.0832	0.0322	0.0137
$h_b(3P)$	0.270	0.0983	0.0419	0.0195
$h_b(4P)$	0.281	0.108	0.0484	0.0237

结果分析

- 有关系 $v^n_{4S} > v^n_{3S} > v^n_{2S} > v^n_{1S}$

  $v^n_{4P} > v^n_{3P} > v^n_{2P} > v^n_{1P}$

- 粲偶素展开法，收敛很慢。相对论修正和QCD修正大，特别是高激发态，相对论效应很大。

- 底偶素展开法，收敛快。相对论修正和QCD修正小。

Relativistic effects in Semileptonic Bc decays to charmonium

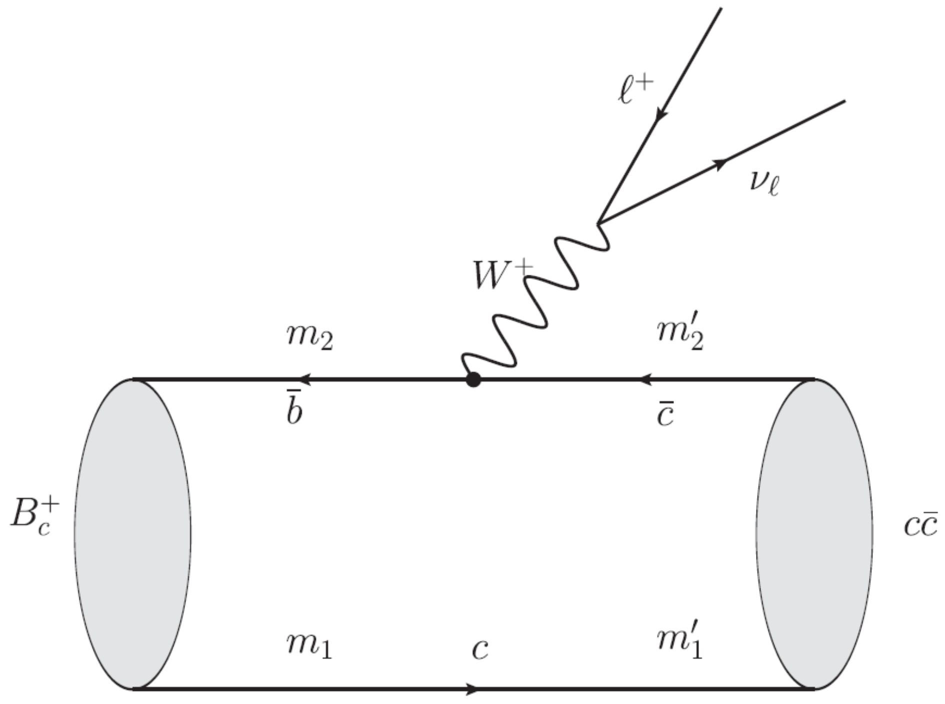

FIG. 1. Feynman diagram corresponding to the semileptonic decays $B_c^+ \to (c\bar{c})\ell^+\nu_\ell$.

Hadronic transition amplitude

- 表示为初末态介子相对论波函数重叠积分

$$\int \frac{\mathrm{d}\vec{q}}{(2\pi)^3} \mathrm{Tr}\left[\bar{\varphi}_{P_f}^{++}(\vec{q}^{\prime})\frac{\not{P}}{M}\varphi_{P}^{++}(\vec{q})\gamma^{\mu}(1-\gamma^5)\right]$$

计算过程中，采用两种方法分离出相对论修正

Relativistic corrections Method I and Method II

Expansion Method I

- Relativistic wave function of pseudoscalar

$$\varphi_{0^-}^{++}(q_\perp) = \left[A_1(q_\perp) + \frac{\not{P}}{M}A_2(q_\perp) + \frac{\not{q}_\perp}{M}A_3(q_\perp) + \frac{\not{P}\not{q}_\perp}{M^2}A_4(q_\perp)\right]\gamma^5,$$

where $A_1 = \frac{M}{2}\left[\frac{\omega_1+\omega_2}{m_1+m_2}f_1 + f_2\right], \qquad A_3 = -\frac{M(\omega_1-\omega_2)}{m_1\omega_2+m_2\omega_1}A_1,$

$$A_2 = \frac{M}{2}\left[f_1 + \frac{m_1+m_2}{\omega_1+\omega_2}f_2\right], \qquad A_4 = -\frac{M(m_1+m_2)}{m_1\omega_2+m_2\omega_1}A_1,$$

and $\omega_i = \sqrt{m_i^2 - q_\perp^2} = \sqrt{m_i^2 + \vec{q}^2}\ (i=1,2)$

- Expansion on $|\vec{q}|/M$ or $|\vec{q}|/m_i\ \ i=1,\ 2$, when q is small, these quantities are small, when q is large, these contribution will be suppressed by wave functions.
- $\vec{q}$ relate to relative velocity between quarks $\vec{q} = \frac{m_1 m_2}{m_1+m_2}\vec{v}$

Expansion Method II

- Expansion according to the wave functions

$$\varphi_{0^-}^{++}(q_\perp) = \varphi_0^{++}(q_\perp) + \varphi_1^{++}(q_\perp),$$

where

$$\varphi_0^{++}(q_\perp) = \left[A_1(q_\perp) + \frac{\not{P}}{M}A_2(q_\perp)\right]\gamma^5$$

$$\varphi_1^{++}(q_\perp) = \left[\frac{\not{q}_\perp}{M}A_3(q_\perp) + \frac{\not{P}\not{q}_\perp}{M^2}A_4(q_\perp)\right]\gamma^5$$

- Without expansion about ω_i on q, if $\omega_i = m_i + \vec{q}^2/2m_i$

and $f_1 = f_2$, then $\varphi_0^{++}(q_\perp) = Mf_1\left[\left(1 + \frac{\vec{q}^2}{4m_1m_2}\right) + \frac{\not{P}}{M}\left(1 - \frac{\vec{q}^2}{4m_1m_2}\right)\right]\gamma^5$

then the difference is leave to the q^2

Expansion Method II

- So we have

$$\langle \eta_c | \bar{c}\gamma^{\mu}(1-\gamma^5) b | B_c^+ \rangle = T_0 + T_1 + T_2$$

$$= \int \frac{d\vec{q}}{(2\pi)^3} \mathrm{Tr}\left[\frac{\not{P}}{M} \overline{\varphi}^{++}_{P_f}(\vec{q}\,') \gamma^{\mu}(1-\gamma^5) \varphi^{++}_{P}(\vec{q}) \right]$$

$$= \int \frac{d\vec{q}}{(2\pi)^3} \mathrm{Tr}\left[\frac{\not{P}}{M} (\overline{\varphi}'^{++}_0 + \overline{\varphi}'^{++}_1) \gamma^{\mu}(1-\gamma^5) (\varphi^{++}_0 + \varphi^{++}_1) \right]$$

$$= \int \frac{d\vec{q}}{(2\pi)^3} \mathrm{Tr}\left[\frac{\not{P}}{M} (\overline{\varphi}'^{++}_0) \gamma^{\mu}(1-\gamma^5) (\varphi^{++}_0) \right] \Leftrightarrow \textit{the leading order } (LO)$$

$$+ \int \frac{d\vec{q}}{(2\pi)^3} \mathrm{Tr}\left[\frac{\not{P}}{M} (\overline{\varphi}'^{++}_1) \gamma^{\mu}(1-\gamma^5) (\varphi^{++}_0) + \frac{\not{P}}{M} (\overline{\varphi}'^{++}_0) \gamma^{\mu}(1-\gamma^5) (\varphi^{++}_1) \right]$$

$$\Leftrightarrow \textit{the first order of relativistic correction } (1stRC)$$

$$+ \int \frac{d\vec{q}}{(2\pi)^3} \mathrm{Tr}\left[\frac{\not{P}}{M} (\overline{\varphi}'^{++}_1) \gamma^{\mu}(1-\gamma^5) (\varphi^{++}_1) \right]$$

$$\Leftrightarrow \textit{the second order of relativistic correction } (2ndRC)$$

Expansion Method II

- Then

$$
\begin{aligned}
|T|^2 &= (T_0 + T_1 + T_2)(T_0^* + T_1^* + T_2^*) \\
&= |T_0|^2 \Leftrightarrow LO \\
&+ T_0 T_1^* + T_0^* T_1 \Leftrightarrow 1stRC \\
&+ |T_1|^2 + (T_0 T_2^* + T_0^* T_2) \Leftrightarrow 2ndRC \\
&+ T_1 T_2^* + T_1^* T_2 \Leftrightarrow 3rdRC \\
&+ |T_2|^2 \Leftrightarrow 4thRC.
\end{aligned}
$$

Branching ratios in Method I

Mode	$\vec{q}^{\,0}$	$\vec{q}^{\,1}$	$\vec{q}^{\,2}$	$\vec{q}^{\,3}$	$\vec{q}^{\,4}$	$\vec{q}^{\,5}$	$\vec{q}^{\,6}$
η_c	47.4	11.0	4.10	-3.49	-0.390	-0.135	0.0811
J/ψ	157	20.2	18.8	0.517	0.150	-0.00270	0.0203
$\eta_c(2S)$	3.66	2.29	1.43	-0.294	-0.122	-0.0951	0.0284
$\psi(2S)$	6.88	1.87	2.74	0.362	0.185	0.0247	0.00849
$\eta_c(3S)$	0.408	0.339	0.293	-0.0100	-0.0133	-0.0287	0.00675
$\psi(3S)$	0.652	0.241	0.510	0.108	0.0809	0.0138	0.00282
Mode	$\vec{q}^{\,2}$	$\vec{q}^{\,3}$	$\vec{q}^{\,4}$	$\vec{q}^{\,5}$	$\vec{q}^{\,6}$	$\vec{q}^{\,7}$	$\vec{q}^{\,8}$
h_c	15.4	11.0	4.20	-0.420	-0.142	0.00354	0.0100
χ_{c0}	5.13	7.90	1.85	-1.12	-0.0765	0.0224	0.00224
χ_{c1}	7.82	1.50	2.30	0.218	-0.480	-0.0189	0.0278
$h_c(2P)$	1.75	1.86	1.11	-0.0110	-0.0810	-0.0168	0.00575
$\chi_{c0}(2P)$	0.508	1.16	0.635	-0.0776	-0.0531	0.00158	0.00121
$\chi_{c1}(2P)$	0.666	0.104	0.444	0.0265	-0.157	-0.00197	0.0159
$h_c(3P)$	0.173	0.249	0.207	0.0205	-0.0136	-0.00470	0.00103
$\chi_{c0}(3P)$	0.0731	0.220	0.170	-0.00436	-0.0183	-0.000377	0.000541
$\chi_{c1}(3P)$	0.0580	0.00784	0.0758	0.00580	-0.0379	-0.000542	0.00539

Table 1. The branch ratios of $B_c^+ \to (c\bar{c}) + e^+ + \nu_e$ in Method I according to the power $\vec{q}^{\,n}$ (in 10^{-4}).

Branching ratios in Method II

Mode	LO	1st	2nd	3rd	4th	Total(BS)
η_c	44.1	8.24	7.32	0.650	0.279	60.7
J/ψ	158	18.2	15.2	1.94	0.219	193
$\eta_c(2S)$	3.24	1.81	1.71	0.420	0.166	7.34
$\psi(2S)$	6.96	1.66	2.00	0.353	0.108	11.1
$\eta_c(3S)$	0.355	0.272	0.311	0.101	0.0475	1.09
$\psi(3S)$	0.651	0.201	0.365	0.0834	0.0388	1.34
h_c	14.7	10.2	5.21	0.688	0.0822	30.9
χ_{c0}	5.20	7.88	2.03	-0.736	0.0453	14.4
χ_{c1}	7.75	1.35	2.31	0.307	0.0292	11.8
$h_c(2P)$	1.61	1.67	1.27	0.288	0.0448	4.88
$\chi_{c0}(2P)$	0.523	1.16	0.664	0.0211	0.000490	2.37
$\chi_{c1}(2P)$	0.650	0.0804	0.406	0.0353	0.00421	1.18
$h_c(3P)$	0.159	0.228	0.222	0.0614	0.0111	0.682
$\chi_{c0}(3P)$	0.0760	0.221	0.175	0.0208	0.000717	0.493
$\chi_{c1}(3P)$	0.0562	0.00589	0.0615	0.00427	0.000765	0.129

Mode	$\vec{q}^{\,0}$	sum	BS	NR	$\frac{\text{BS}-\text{sum}}{\text{BS}}$
η_c	47.4	58.6	60.7	56.7	3.4%
J/ψ	157	197	193	188	-1.8%
$\eta_c(2S)$	3.66	6.90	7.34	4.48	6.0%
$\psi(2S)$	6.88	12.1	11.1	8.40	-8.8%
$\eta_c(3S)$	0.408	0.995	1.09	0.509	8.7%
$\psi(3S)$	0.652	1.61	1.34	0.806	-20%
Mode	$\vec{q}^{\,2}$	sum	BS	NR	$\frac{\text{BS}-\text{sum}}{\text{BS}}$
h_c	15.4	30.0	30.9	18.8	2.9%
χ_{c0}	5.13	13.7	14.4	6.28	4.8%
χ_{c1}	7.82	11.4	11.8	9.60	2.8%
$h_c(2P)$	1.75	4.62	4.88	2.18	5.3%
$\chi_{c0}(2P)$	0.508	2.17	2.37	0.633	8.4%
$\chi_{c1}(2P)$	0.666	1.10	1.18	0.853	7.2%
$h_c(3P)$	0.173	0.633	0.682	0.220	7.1%
$\chi_{c0}(3P)$	0.0731	0.440	0.493	0.0923	11%
$\chi_{c1}(3P)$	0.0580	0.114	0.129	0.0735	11%

Table 3. Comparisons of the branch ratios of $B_c^+ \to (c\bar{c}) + e^+ + \nu_e$ obtained by different ways, where $\vec{q}^{\,0}$ means the leading order result; **sum** means the sum of all of expansion orders; **BS** means the result by BS method without expansion, and **NR** means the result by the non-relativistic wave function and the leading order expansion of the amplitude (in 10^{-4} except the last column).

Relativistic effects

Method	η_c	J/ψ	$\eta_c(2S)$	$\psi(2S)$	$\eta_c(3S)$	$\psi(3S)$
I	21.9	18.8	50.2	38.0	62.5	51.3
II	27.3	18.5	55.8	37.2	67.3	51.5

Table 4. The relativistic effects of $B_c^+ \to (c\bar{c})e^+\nu_e$: $\frac{BS-LO}{BS}$ from two methods (in %).

Method	h_c	χ_{c0}	χ_{c1}	$h_c(2P)$	$\chi_{c0}(2P)$	$\chi_{c1}(2P)$	$h_c(3P)$	$\chi_{c0}(3P)$	$\chi_{c1}(3P)$
I	50.2	64.4	33.7	64.1	78.5	43.3	74.6	85.2	54.9
II	52.5	63.9	34.0	67.0	77.9	44.7	76.7	84.6	56.3

Table 5. The relativistic effects of $B_c^+ \to (c\bar{c})e^+\nu_e$: $\frac{BS-LO}{BS}$ from two methods (in %).

Summary

- Relativistic corrections (RC) in the semileptonic Bc decays to charmonium are large.
- RC of 1S are about 19~22%
- RC of 2S are about 38~50%
- RC of 3S are about 51~62%
- RC of 1P are about 34~64%
- RC of 2P are about 43~79%
- RC of 3P are about 55~85%
- Relativistic corrections are very important for excited heavy mesons.

Thank you!

Mass spectra

- Parameters:

$$a = e = 2.7183\ , \quad \alpha = 0.06\ \text{GeV}, \quad \lambda = 0.21\ \text{GeV}^2,$$

$$\text{and}\ \ m_c = 1.62\ \text{GeV}, \quad m_b = 4.96\ \text{GeV}\ .$$

for charmonium $N_f = 3,\ \Lambda_{QCD} = 0.27\ \text{GeV}$

$$\alpha_s(m_c) = 0.38$$

for bottomonium $N_f = 4,\ \Lambda_{QCD} = 0.20\ \text{GeV}$

$$\alpha_s(m_b) = 0.23$$

n $J^{PC}(^{(2S+1)}L_J)$	Th($c\bar{c}$)	Ex($c\bar{c}$)	Th($b\bar{b}$)	Ex($b\bar{b}$)
1 $0^{-+}(^1S_0)$	2980.3(input)	2980.3	9390.2(input)	9388.9
2 $0^{-+}(^1S_0)$	3576.4	3637	9950.0	
3 $0^{-+}(^1S_0)$	3948.8		10311.4	
1 $1^{--}(^3S_1)$	3096.9(input)	3096.916	9460.5(input)	9460.30
2 $1^{--}(^3S_1)$	3688.1	3686.09	10023.1	10023.26
3 $1^{--}(^3D_1)$	3778.9	3772.92	10129.5	
4 $1^{--}(^3S_1)$	4056.8	4039	10368.9	10355.2
5 $1^{--}(^3D_1)$	4110.7	4153	10434.7	
6 $1^{--}(^3S_1)$	4329.4	4421	10635.8	10579.4
7 $1^{--}(^3S_1)$	4545.9		10852.1	10865

n $J^{PC}(^{2S+1})L_J$	Th($c\bar{c}$)	Ex($c\bar{c}$)	Th($b\bar{b}$)	Ex($b\bar{b}$)
1 $0^{++}(^3P_0)$	3414.7(input)	3414.75	9859.0	9859.44
2 $0^{++}(^3P_0)$	3836.8		10240.6	10232.5
3 $0^{++}(^3P_0)$	4140.1		10524.7	
1 $1^{++}(^3P_1)$	3510.3(input)	3510.66	9892.2	9892.78
2 $1^{++}(^3P_1)$	3928.7		10272.7	10255.46
3 $1^{++}(^3P_1)$	4228.8		10556.2	

n $J^{PC}(^{2S+1})L_J$	Th($c\bar{c}$)	Ex($c\bar{c}$)	Th($b\bar{b}$)	Ex($b\bar{b}$)
1 $2^{++}(^3P_2)$	3556.1(input)	3556.20	9914.4	9912.21
2 $2^{++}(^3P_2)$	3972.4		10293.6	10268.65
3 $2^{++}(^3F_2)$	4037.9		10374.4	
4 $2^{++}(^3P_2)$	4271.0		10561.5	
1 $1^{+-}(^1P_1)$	3526.0(input)	3525.93	9900.2	
2 $1^{+-}(^1P_1)$	3943.0		10280.4	
3 $1^{+-}(^1P_1)$	4242.4		10562.0	