Two-phase flow model for energetic proton beam induced pressure waves in mercury target systems in the planned European Spallation Source

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Outline

- *The idea of a thermo-hydraulical model,*

  gentle introduction into shock-waves (ideal gas)

  a general thermo-hydraulics model (for water)

  Equation of states for Hg, proton – Hg interaction

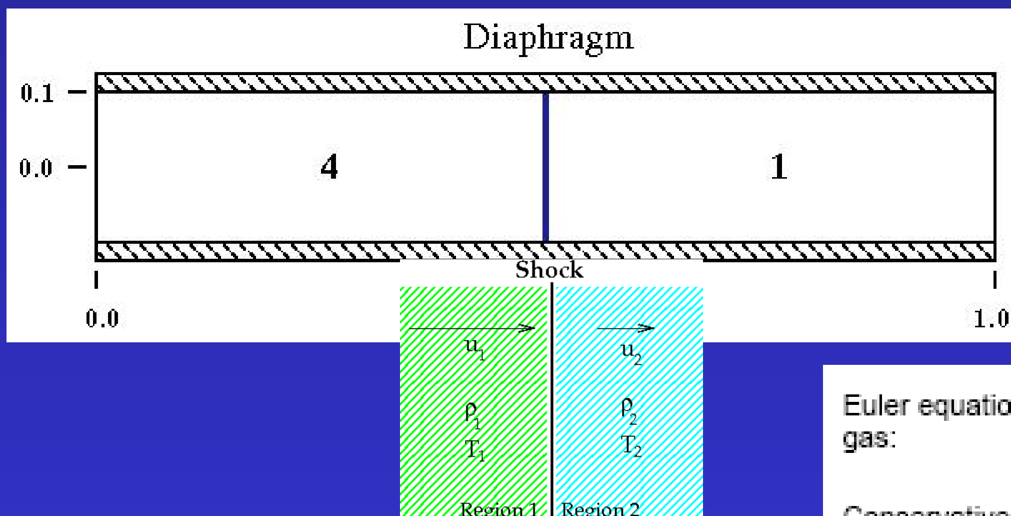
  the model for ESS and some results

- *Summary and Outlook*

Gentle introduction on shock waves, two-phase flow

Def: a surface of discontinuity propagating in a gas at which density and velocity experience abrupt changes

→ **we call it a shock wave**



basic experimental setup

(shock tube)

mathemetical description:

conservation laws,

continuity, momentum(Euler)

energy equation + Equation of state

Euler equations of single-phase compressible quasi-1D flow of ideal gas:

Conservative form

$$\begin{bmatrix} A\rho \\ A\rho v \\ AE \end{bmatrix}_t + \begin{bmatrix} A\rho v \\ A(\rho v^2 + p) \\ Av(E+p) \end{bmatrix}_x = \begin{bmatrix} 0 \\ p \\ 0 \end{bmatrix} \frac{dA}{dx}$$

Non-conservative vectorial form:

$$\frac{\partial \vec{\psi}}{\partial t} + \underline{\underline{C}} \frac{\partial \vec{\psi}}{\partial x} = \vec{S}$$

Conservative variables are used in vector

$$\vec{\psi} = [A\rho \ , A\rho \ v \ , A\rho \ e] \quad \left(E = \rho \ e = \rho \ u + \frac{1}{2}\rho v^2 \right)$$

Equation of state (ideal gas):

$$E = \frac{p}{\gamma - 1} + \frac{1}{2}\rho v^2 \qquad \gamma = \frac{c_p}{c_v}$$

Gentle introduction on shock waves, two-phase flow

- *the measurable physical quantities change in time non-continously, (non-continous solutions for partial differential equations)*

- *in a real shock wave physical quantities grow up in approx. 2-4 free mean path of a particle*

(10^-2 mm at normal p, T, quick phenomena no time for diffusion, heat exchange)

- *the given equations have differents wave solutions with different wave propagation velocities*

Jacobian matrix:

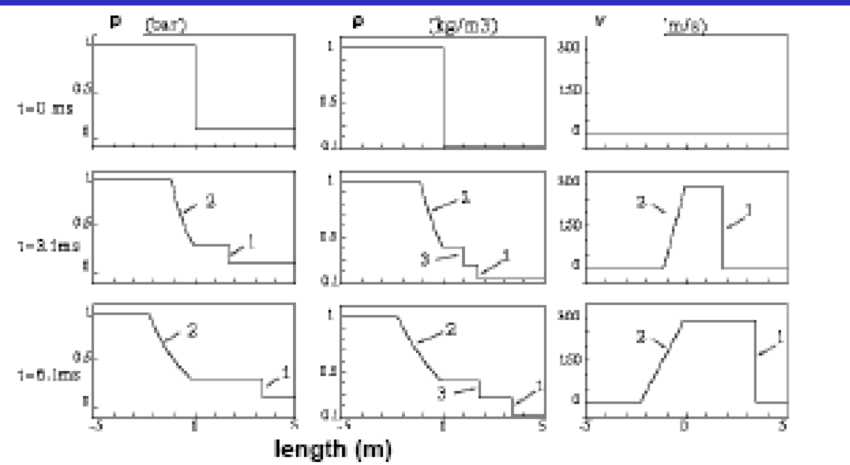
$$\underline{C} = \begin{bmatrix} 0 & 1 & 0 \\ (\gamma-3)v^2/2 & (3-\gamma)v & \gamma-1 \\ (\gamma-1)v^3/2 - vh & h-(\gamma-1)v^2 & \gamma v \end{bmatrix}$$

Diagonalized:

$$\underline{C} = \underline{L} \cdot \underline{\Lambda} \cdot \underline{L}^{-1} \qquad \left(c^2 = \frac{\gamma p}{\rho}\right) \qquad h = e + p/\rho$$

Eigenvalues: $\underline{\Lambda} = \begin{bmatrix} v+c & 0 & 0 \\ 0 & v-c & 0 \\ 0 & 0 & v \end{bmatrix}$ Eigenvectors $\underline{L} = \begin{bmatrix} 1 & 1 & 1 \\ v+c & v-c & v \\ h+cv & h-cv & v^2/2 \end{bmatrix}$

characteristic-upwind schemes 8



1- shock wave, 2- rarefaction wave, 3 - contact discontinuity

Gentle introduction on shock waves, two-phase flow

- ***1D single phase flow can be generalised for 2 phases,*** $\longrightarrow$ ***averaging over the volume***
  (void fraction, α)

- ***Different models, with diferent number of equ.s***
  from 3 up to 7 equation models are available, with different physical backgrounds

$$\frac{\partial \vec{\psi}}{\partial t} + \underline{\underline{C}} \frac{\partial \vec{\psi}}{\partial x} = \vec{S}$$

$$\vec{\psi} = (\rho_m, v_m, p_m)$$

$$\vec{\psi} = (\rho_m, \rho_g, \rho_m v_m, \rho_m u_m)$$

$$\vec{\psi} = (\rho_g, \rho_f, \rho_m v_m, \rho_f u_f, \rho_g u_g)$$

mixture density, velocity, energy is defined

$$\vec{\psi} = (\rho_g, \rho_f, \rho_g v_g, \rho_f v_f, \rho_g u_g, \rho_f u_f)$$

we use this one, well tested, single pressure for both phase phases

$$\vec{\psi} = (\rho_g, \rho_f, \rho_g v_g, \rho_f v_f, \rho_g u_g, \rho_f u_f, 7^{th}\ \text{variable})$$

could be a two pressure model

Introduction of the WAHA model

- *1Dim surface averaged, 6 equation model*
- *mass, momentum, energy balance equ. for both phases*

$$\frac{\partial \vec{\psi}}{\partial t} + \underline{\underline{C}}\,\frac{\partial \vec{\psi}}{\partial x} = \vec{S}$$

$$\vec{\psi} = (\, p, \alpha, v_f, v_g, u_f, u_g \,)$$

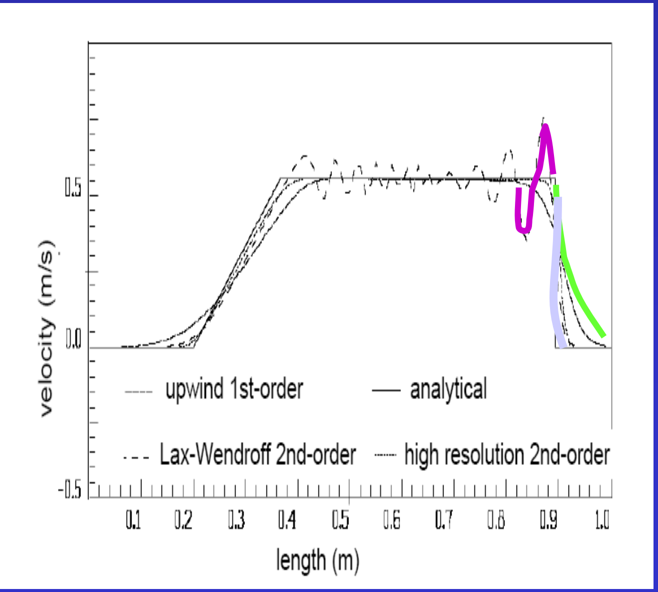
- *Single pressure model*
- *Relap5 based Model, with different numerics, for quick processes*
- *6 Equ. Model Widely used in the nuclear industry*
- *Realistic water-steam table, correlations for both phases*
- *Simplified Flow Maps*
- *Source Code available Fortran 90* J. Comp. Phys. **136**, 503 (1997)

Idea of the numerical method

- *Hyperbolic partial dif. equ. systems* → *non-continous solutions*
-
- *(jump initial condition is conserved in time)*
- *Special numerical method is needed*

$$\frac{\partial \vec{\psi}}{\partial t} + \underline{\underline{C}}\,\frac{\partial \vec{\psi}}{\partial x} = \vec{S}$$

(example: ideal gas shock wave)



- ***Pure 1st order* method *smears* discontinuity**
- ***Pure 2nd order* creates unphysical *oscillations***
- ***Mixed method* gives physicaly *correct* answer (flux limiters)**

Flow maps, correlations

two-phase flows have very complex flow maps

→ these are the main uncertanities in the theory

we use simplified flow maps

stratified, bubbly, droplet flow *with different correlations*

Heat-mass, energy, impulse transfer

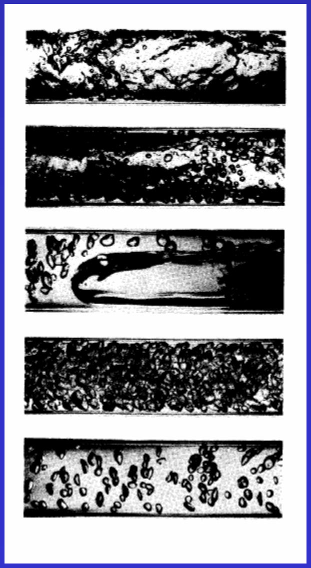
between phases

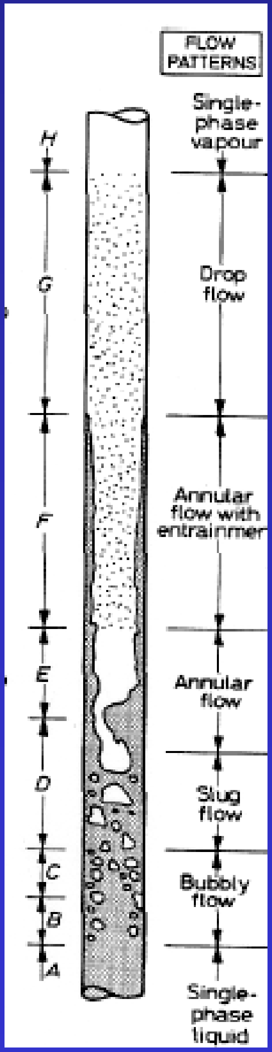
Wall friction *steady state/dynamical*

Interphase friction

All can be swtiched on/off for different regimes

For steam induced water hammer all correlations are needed



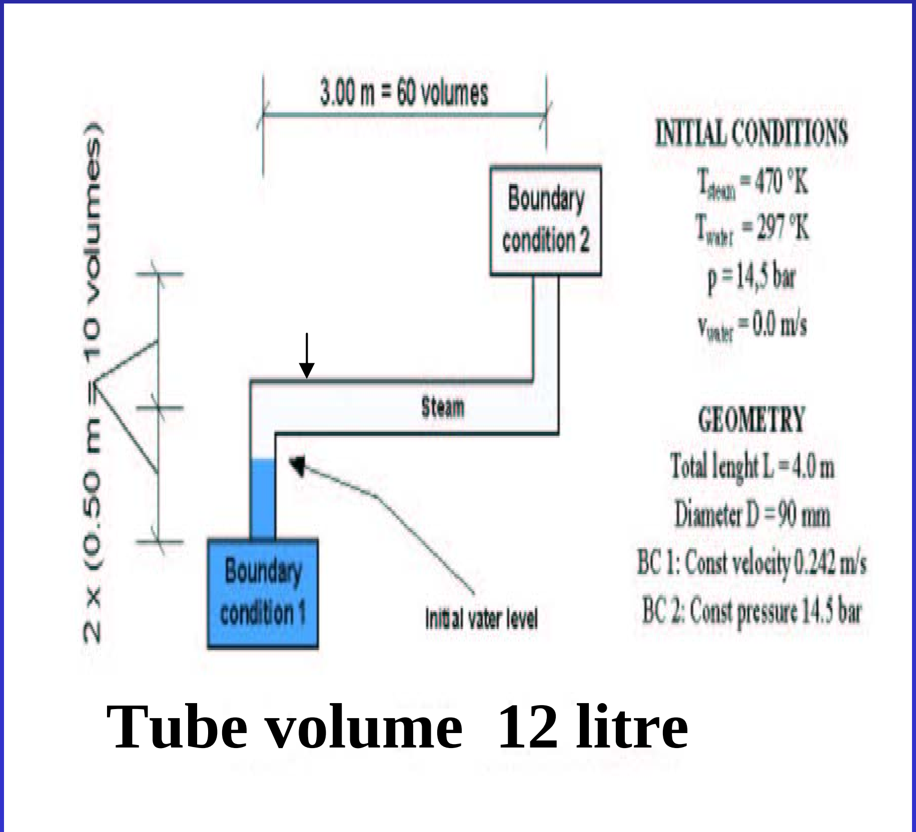


Our former aim/experience with WAHA

study of the steam condensation induced water hammer phenomena,

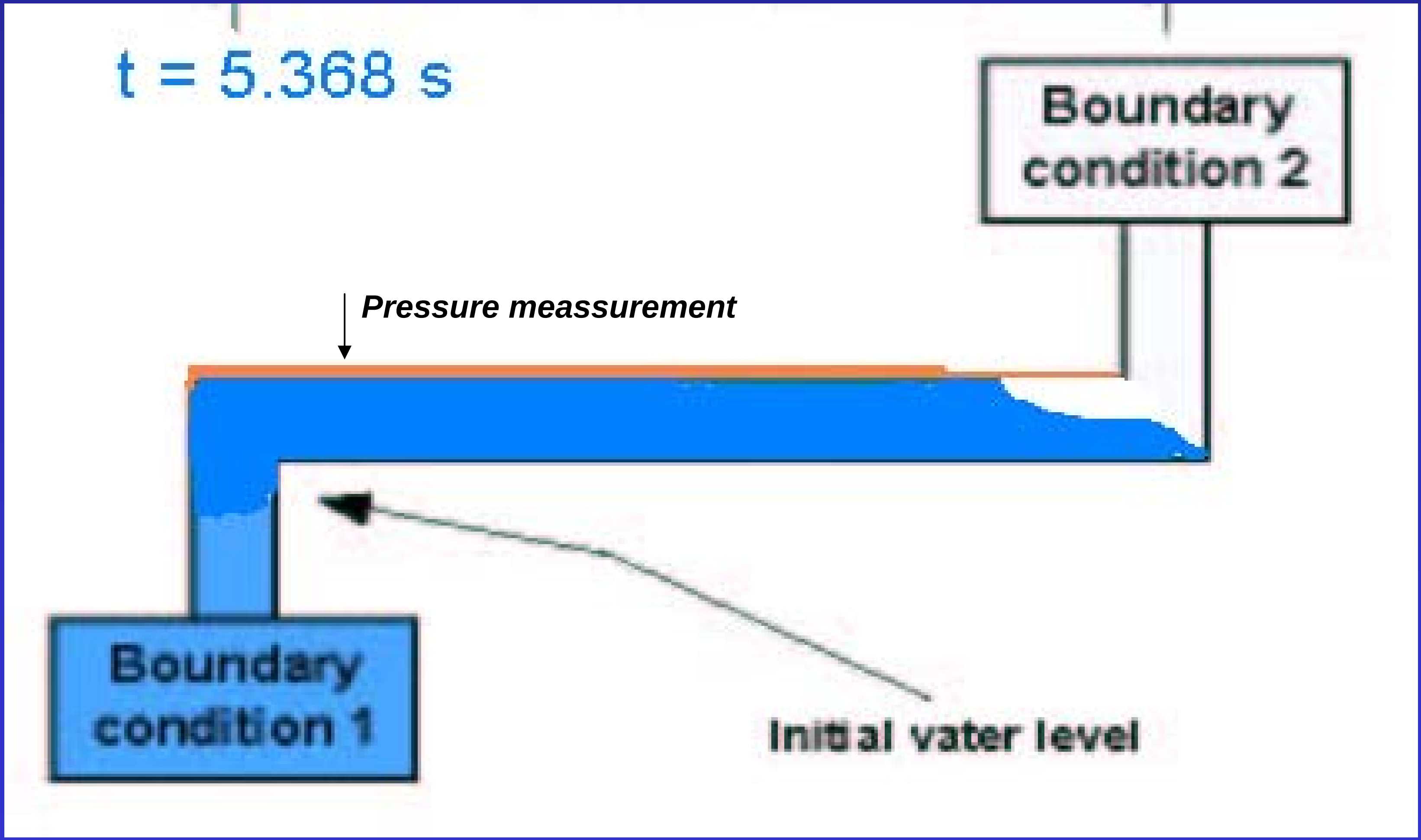
the most complex two-phase flow phenomena

(experimental setup and the model scheme in numerical simulation)



The physical mechanism for steam condensation induced water hammer

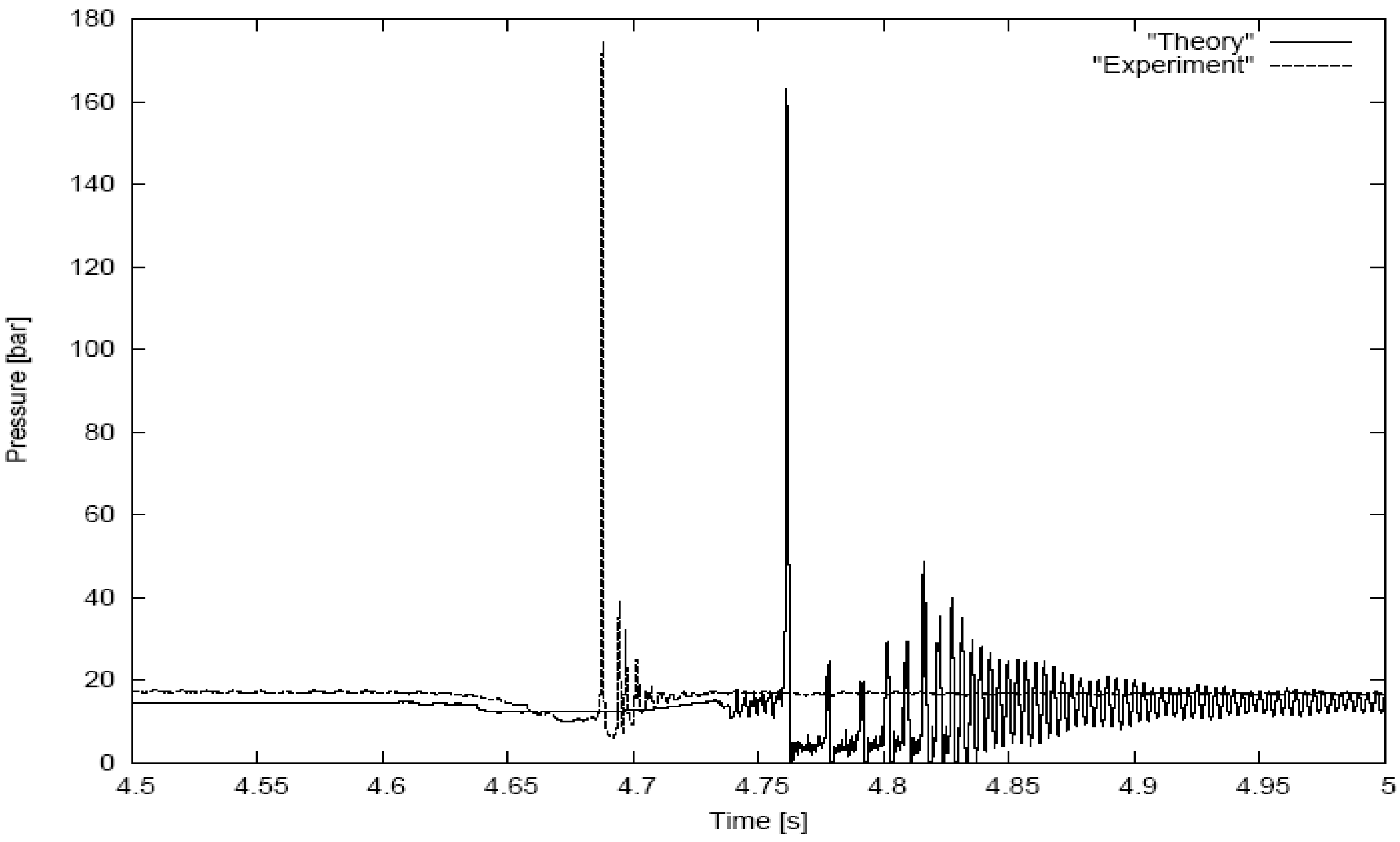
(some figures, poore man's animation)



Analysis of the pressure peaks

(all the correlations and all know-hows are needed)

I.F. Barna et al. Nucl. Eng. and Des. 240, 146, (2010)



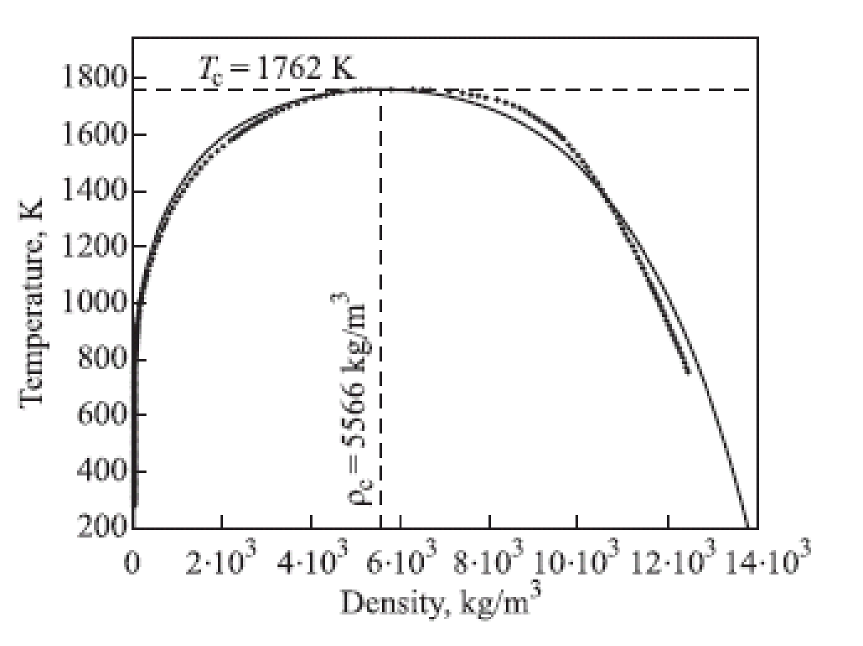
Idea, use the modified 6equ. model for proton – Hg interaction

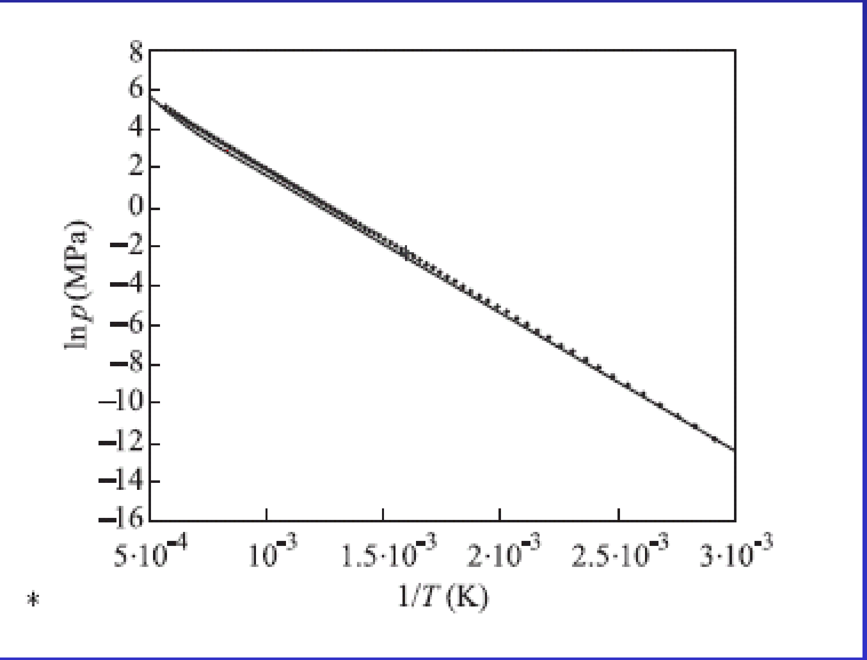
- ***Basicaly two new points***
- ***Not water but mercury (new & complete liquid- steam table), equation of state (EOS)***
  (Subbotin correlation, surface tension, heat conduction, viscosity is known)
- ***Periodic driving from absorbed proton pulses, a new source term in both energy equations***

How to create a steam table for Hg

For standard 2 phase-flow calculation a 6-fold table is needed: T

T (K), p (Pa), ρ_vap (kg/m^3), u_vap (J/kg), ρ _liq (kg/m^3), u_liq (J/kg)



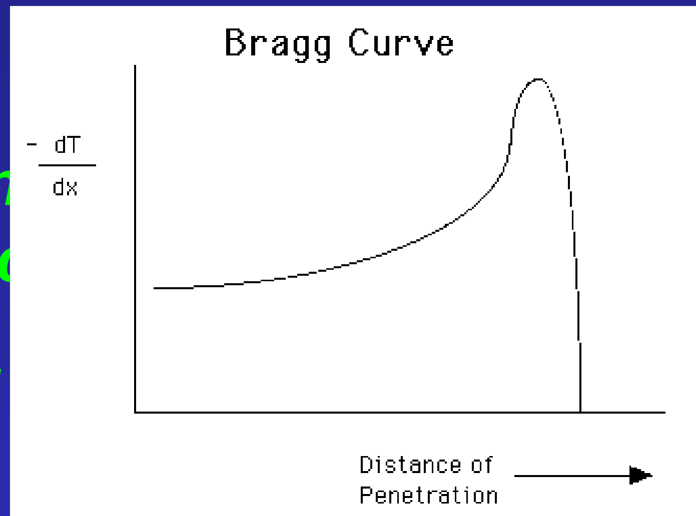


Our Van-der Waals fit for vapour-liquid coexistence and vapour pressure

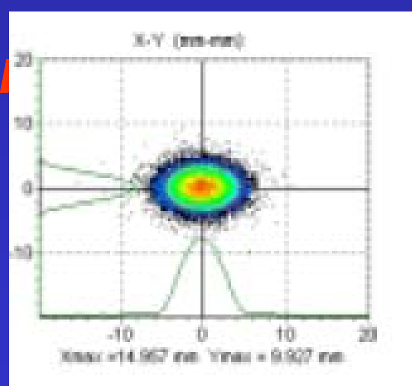
N. B. Vargaftik, Y. K. Vinogradov, and V. S. Yargin, *Handbook of Physical Properties of Liquid and Gases*, 3rd ed. Begell House, New York, 1996.

Proton beam–Hg target interaction

- Accurate en… g peak) of proton in Hg can be c… LUKA, MARS, GEANT etc. … e heat deposition

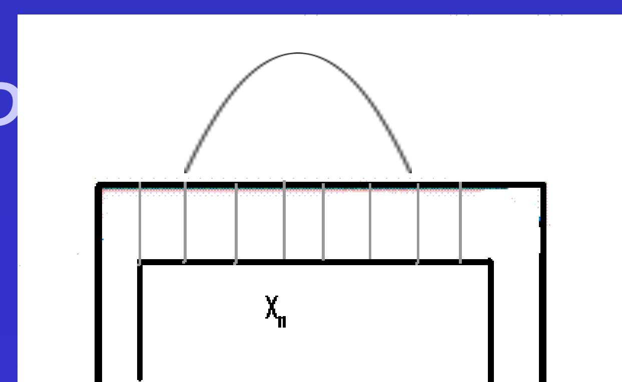


- beam pr… atic (measured)



$$\rho(x,y) = \frac{3\lambda}{2\pi ab}\left[1-\frac{x^2}{a^2}-\frac{y^2}{b^2}\right]^{1/2}$$

- We have 1D model  only 1D



The new source terms

- *We consider proton beam-Hg interaction as a periodic sudden heat shock in the energy equation of the liquid and gas phase*

$$\frac{\partial \vec{\psi}}{\partial t} + \underline{\underline{C}} \frac{\partial \vec{\psi}}{\partial x} = \vec{S}$$

$$\frac{\partial A(1-\alpha)\rho_f u_f}{\partial t} + \frac{\partial A(1-\alpha)\rho_f u_f v_f}{\partial x} - p\frac{\partial A\alpha}{\partial t} + p\frac{\partial A(1-\alpha)v_f}{\partial x} = A\left(Q_{if} - \Gamma_g h_f^* + v_f F_{f,wall}\right) + E_{f,pulse}(x,t)$$

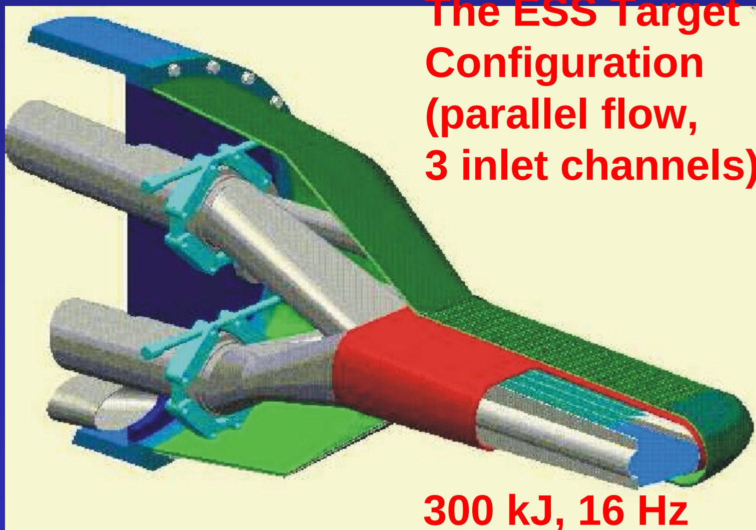
$$\frac{\partial A\alpha\rho_g u_g}{\partial t} + \frac{\partial A\alpha\rho_g u_g v_g}{\partial x} + p\frac{\partial A\alpha}{\partial t} + p\frac{\partial A\alpha v_g}{\partial x} = A\left(Q_{ig} + \Gamma_g h_g^* + v_g F_{g,wall}\right) + E_{g,pulse}(x,t)$$

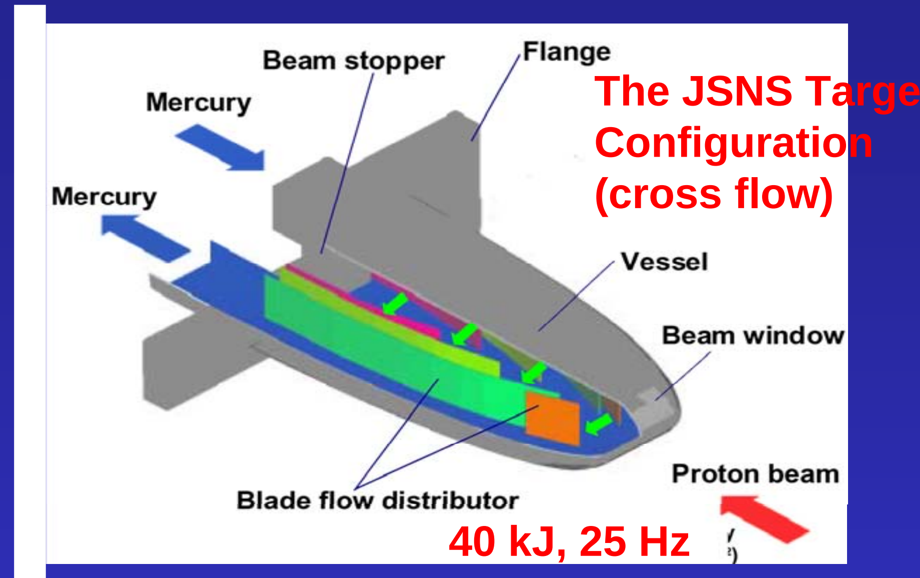
$$E_{g,pulse}(x,t) = \frac{\rho_g \alpha}{\rho_m} E_0 sin^2\left[\frac{\Pi t}{\tau}\right](1-(x/x_s)^2)$$

$$E_{f,pulse}(x,t) = \frac{\rho_f(1-\alpha)}{\rho_m} E_0 sin^2\left[\frac{\Pi t}{\tau}\right](1-(x/x_s)^2)$$

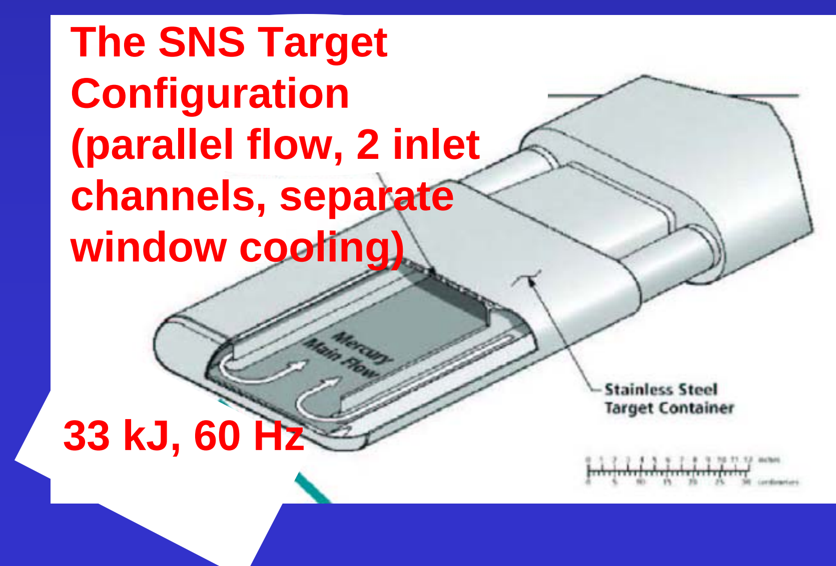
continuity, momentum equations will not be changed

The ESS-SNS-JSNS Hg Target Concepts



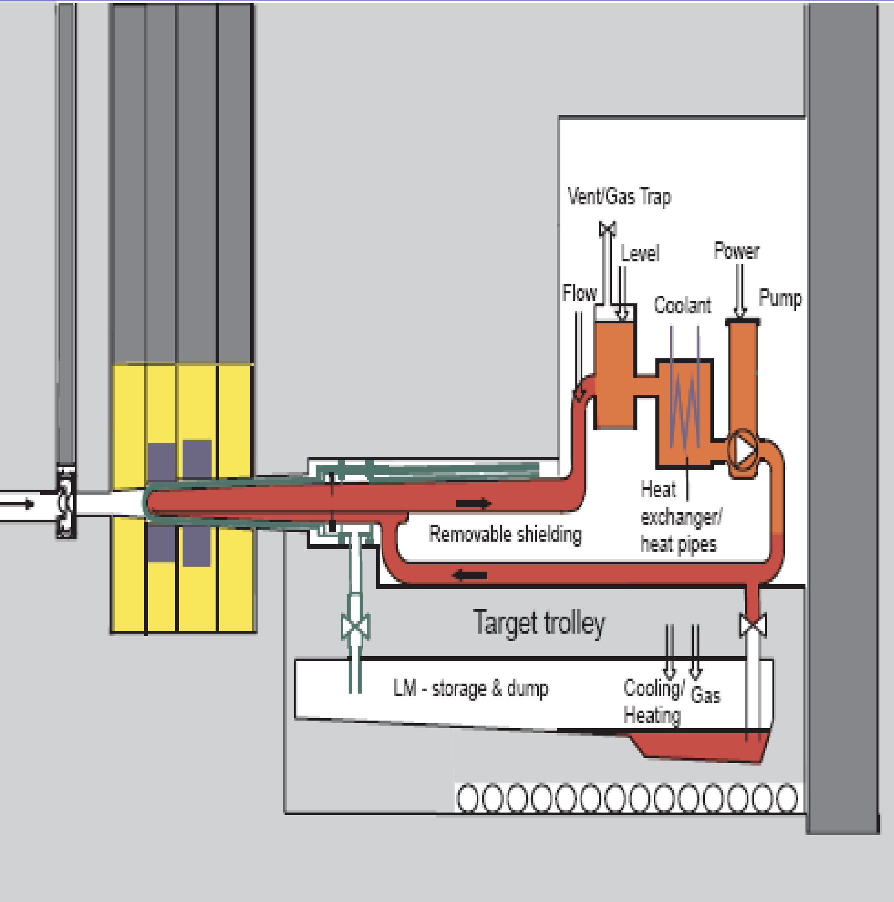


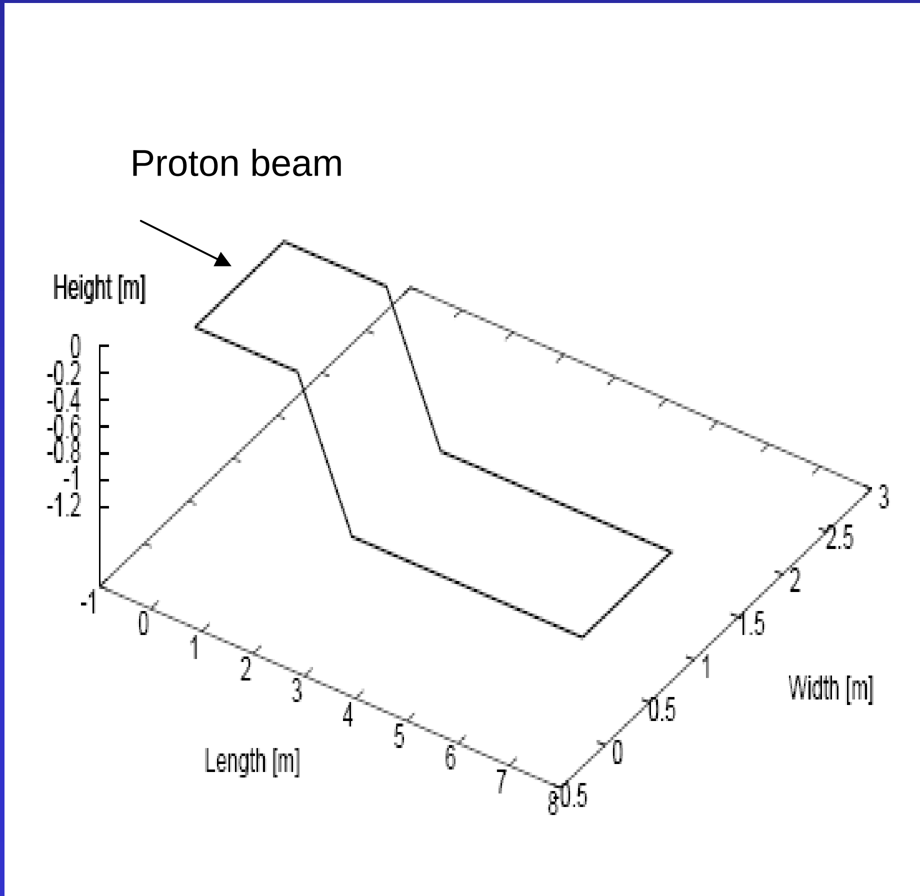
ESS (5 MW), SNS (2 MW) and JSNS (1 MW) use liquid mercury targets enclosed in steel shells, albeit with different internal flow distributions.



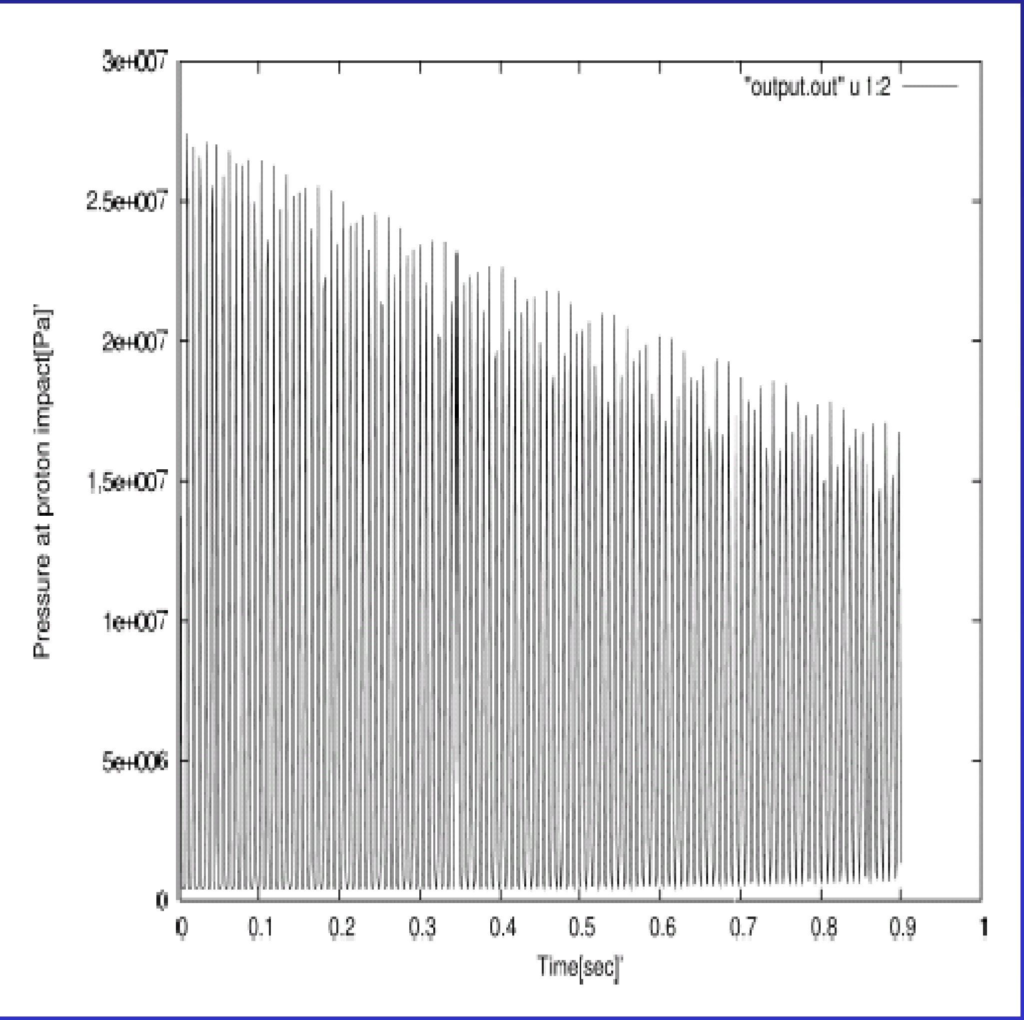
The scematic scheme of the target

No sophisticated 3D model, like Fluent, Ansys of CFX NO ENGINEERING
But better phyiscs for boiling-condensation, 2 phase flow





Our results till now



Total Length = 18 m
Tube diameter = 15 cm
V_Hg = 4 m/s
P_0 = 4.5 bar
T_0 = 385 K
T_max = 475 K
P_max = 280 bar

Only a single pulse is taken at t = 0 sec which is 2 ms long

E = 300 KJ/puls ESS Energy

If Hg begins to cavitate, calculation Breaks down, EOS must be more Accurate

Summary and Outlook

We gave a short/gentle introduction into shock waves/2phase flow ☺

introduced the WAHA3 model, which is feasible to describe shock waves, quick transients in two phase-flows

presented a model which is hopefully a good choice to understand some new physics in proton-Hg system

further work is in progress to clear out the dark points and present reasonable results

I.F. Barna et al. Eur. Phys. J. B. 66 (2008) 419
Or: arXiv:0805.3618v1 [cond-mat.other]

There are liquid-metal (eq. Li) or liquid helium cooled systems as well… ☺
(work for the next 20-30 years)

Thank you for your attention!

Questions, comments, remarks?…