Precision Predictions to $e^+e^- \to HZ$ (toward NNLO EW)

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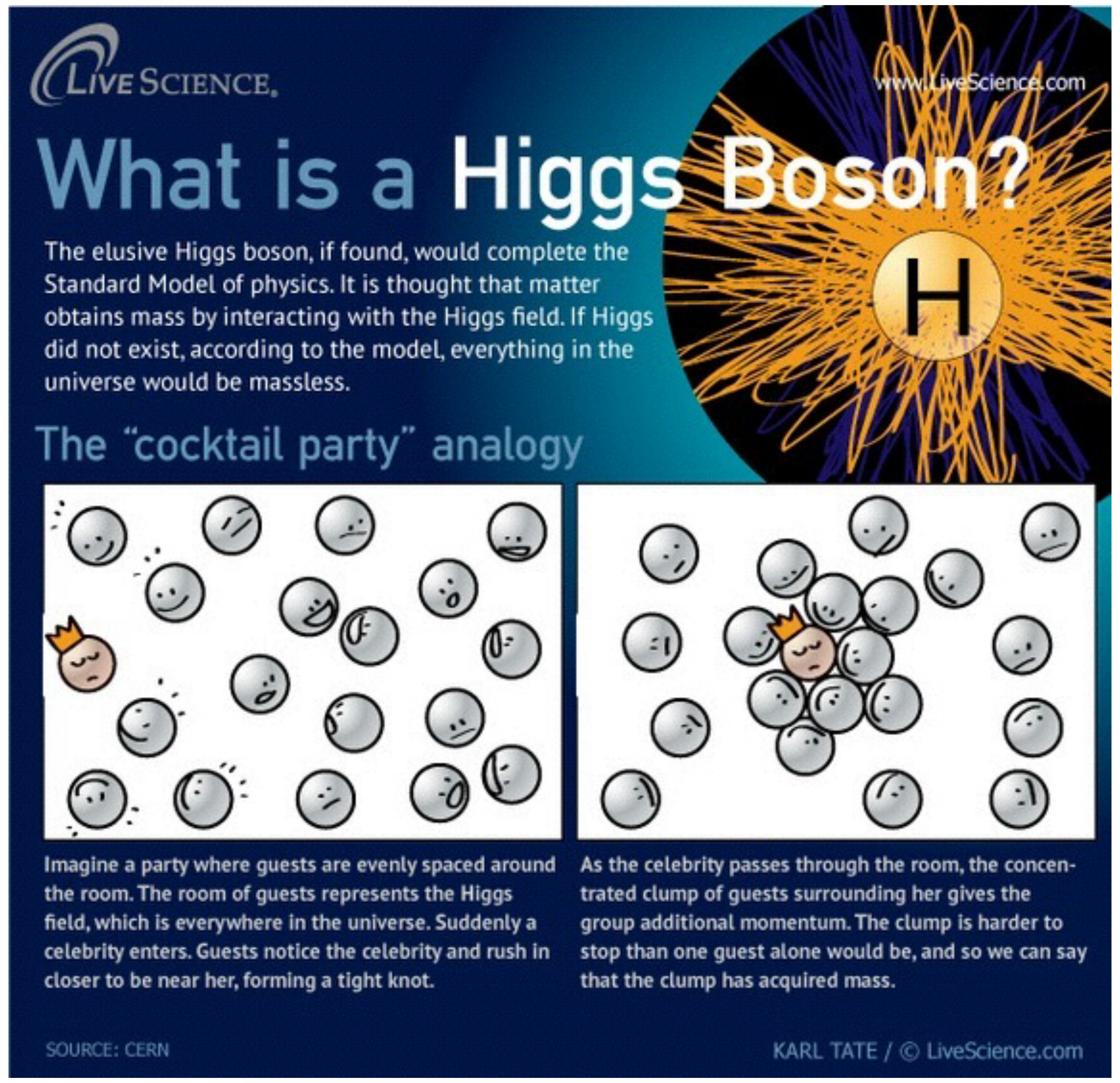
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What is a Higgs Boson?

The elusive Higgs boson, if found, would complete the Standard Model of physics. It is thought that matter obtains mass by interacting with the Higgs field. If Higgs did not exist, according to the model, everything in the universe would be massless.

The "cocktail party" analogy

Imagine a party where guests are evenly spaced around the room. The room of guests represents the Higgs field, which is everywhere in the universe. Suddenly a celebrity enters. Guests notice the celebrity and rush in closer to be near her, forming a tight knot.

As the celebrity passes through the room, the concentrated clump of guests surrounding her gives the group additional momentum. The clump is harder to stop than one guest alone would be, and so we can say that the clump has acquired mass.

SOURCE: CERN

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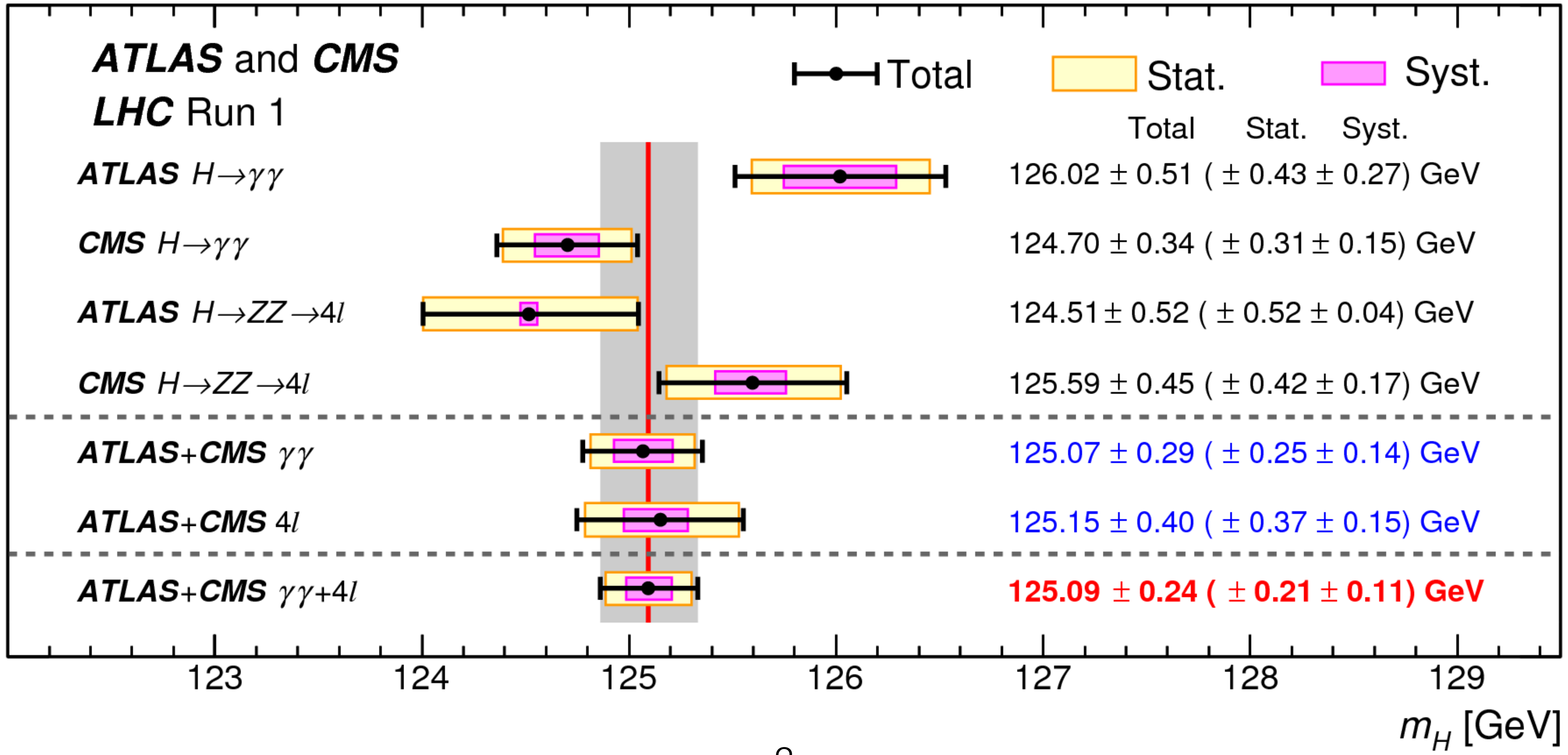
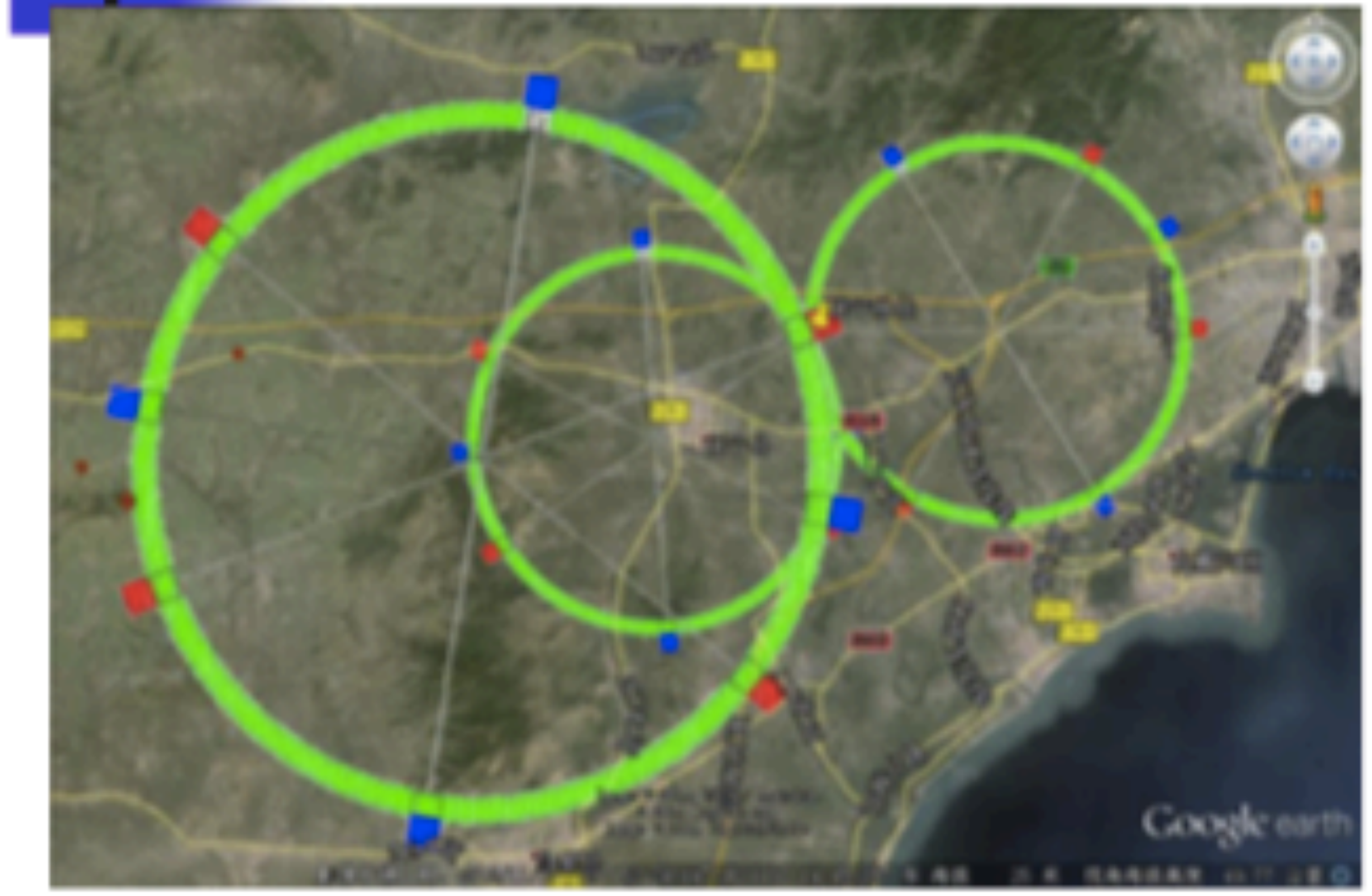
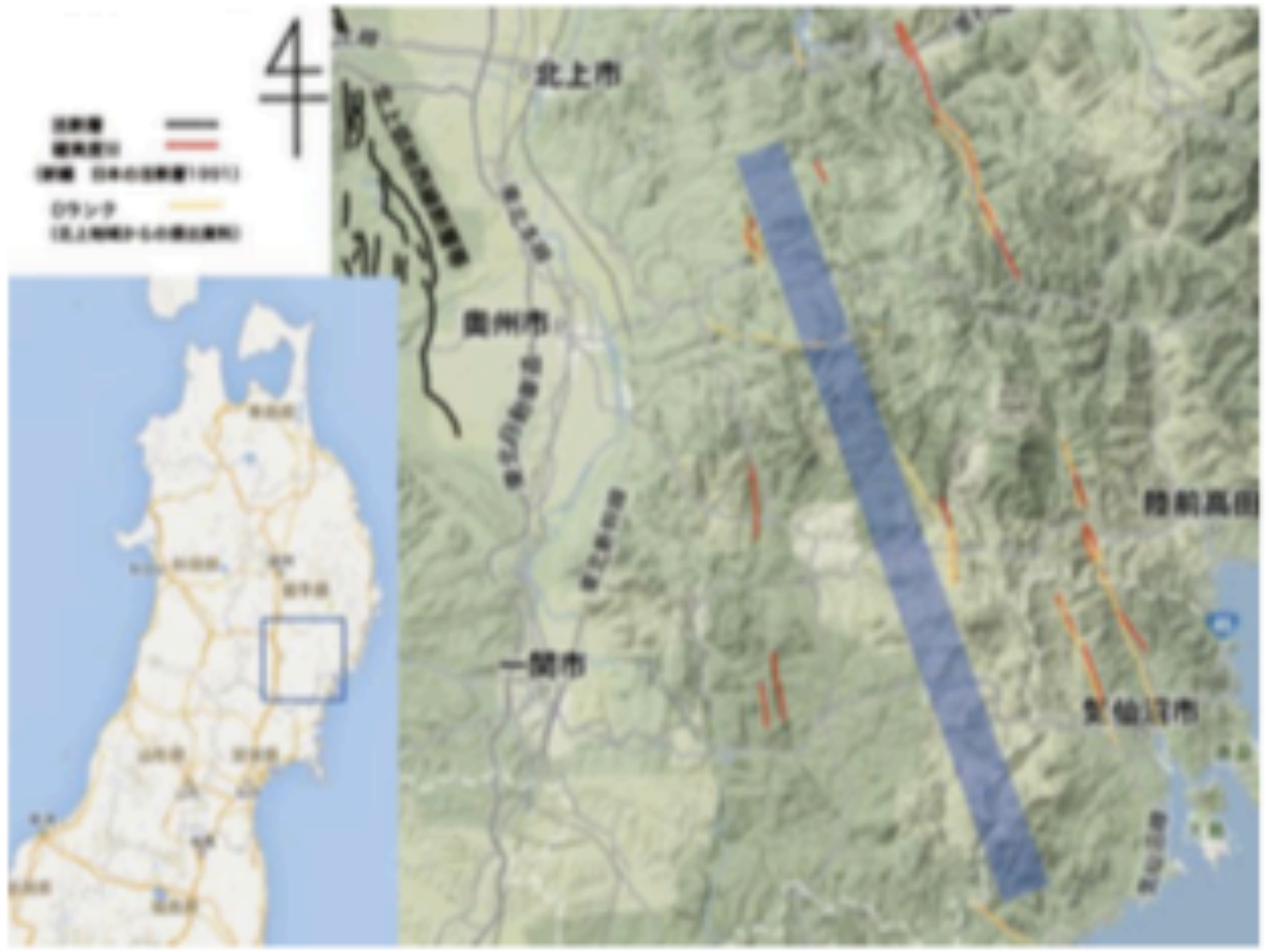


Several Higgs factories under plan



CEPC@90-240 GeV (China)
秦皇岛 or 雄安?

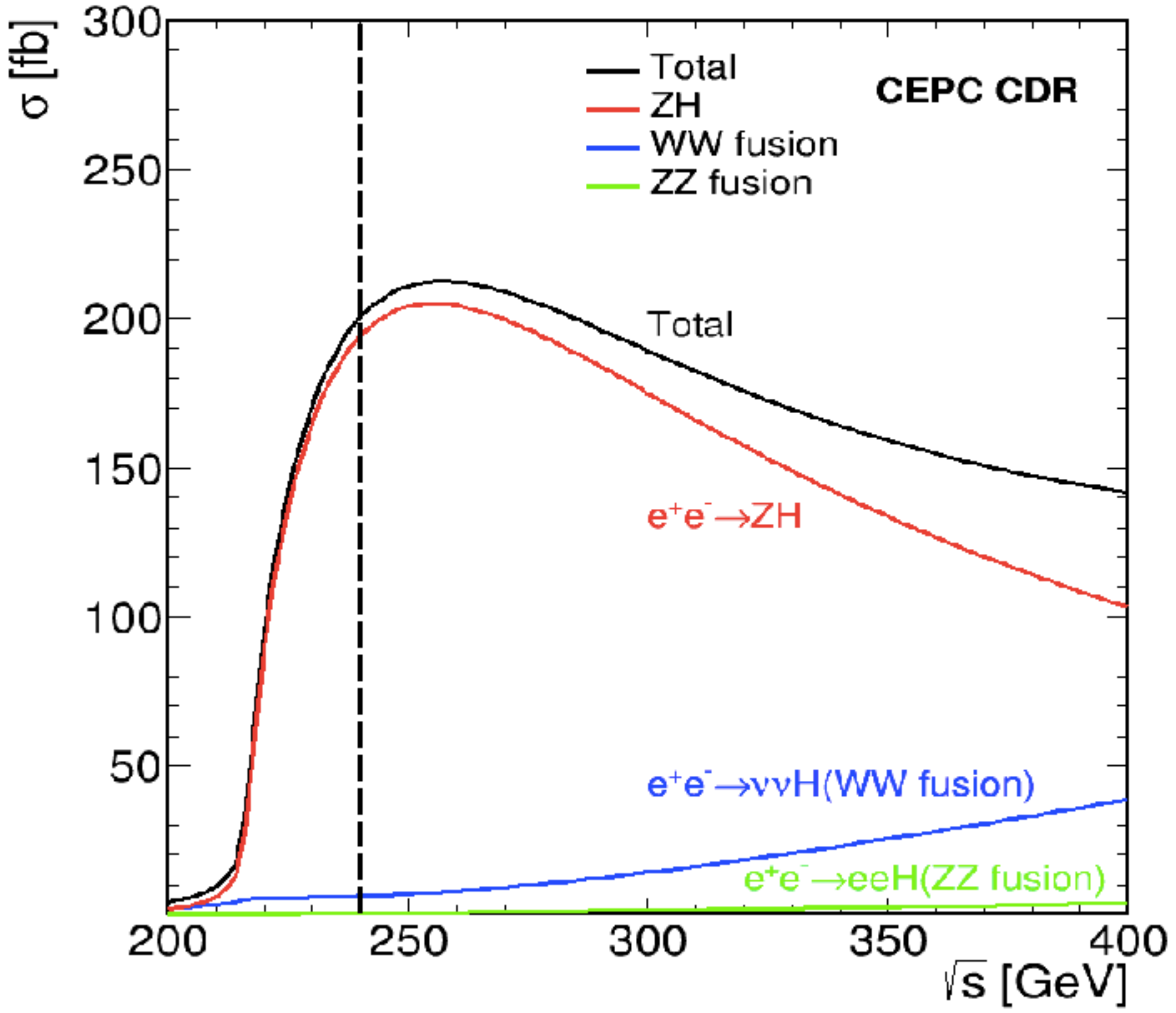


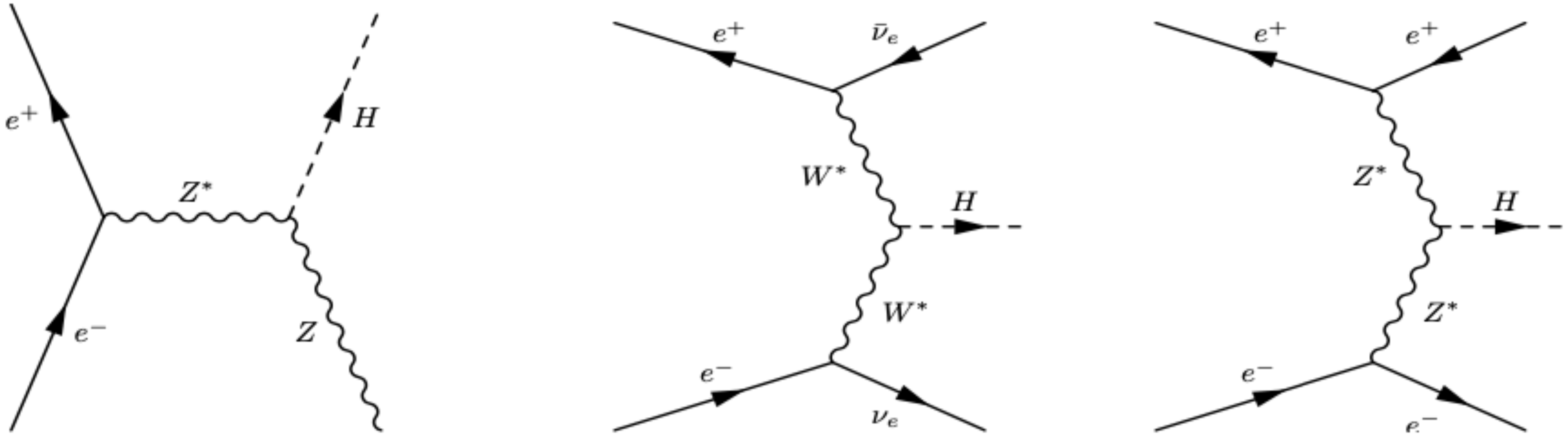
ILC@500,350,250 GeV (Japan)
Kitakami Candidate Site

FCC-ee @ 90-400 GeV (Geneva, EU)

Next Generation e^+e^- Collider (Higgs Factory)

- ILC (Japan)
- CEPC (China)
- FCC-ee (Europe)



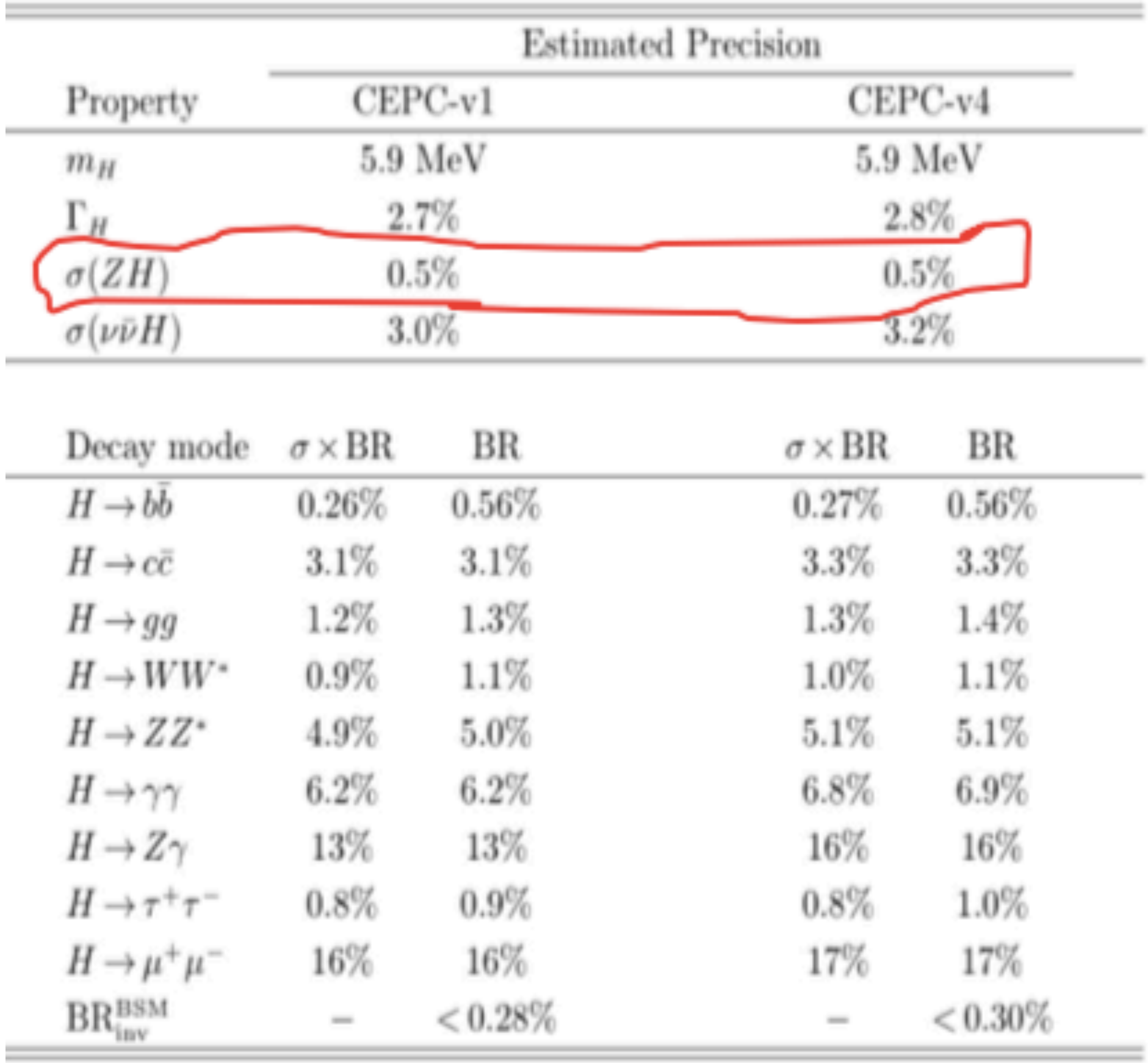


(a) (b) (c)

Process	Cross section	Events in 5.6 ab^{-1}
Higgs boson production, cross section in fb		
$e^+e^- \to ZH$	196.2	1.10×10^6
$e^+e^- \to \nu_e\bar{\nu}_e H$	6.19	3.47×10^4
$e^+e^- \to e^+e^- H$	0.28	1.57×10^3
Total	203.7	1.14×10^6

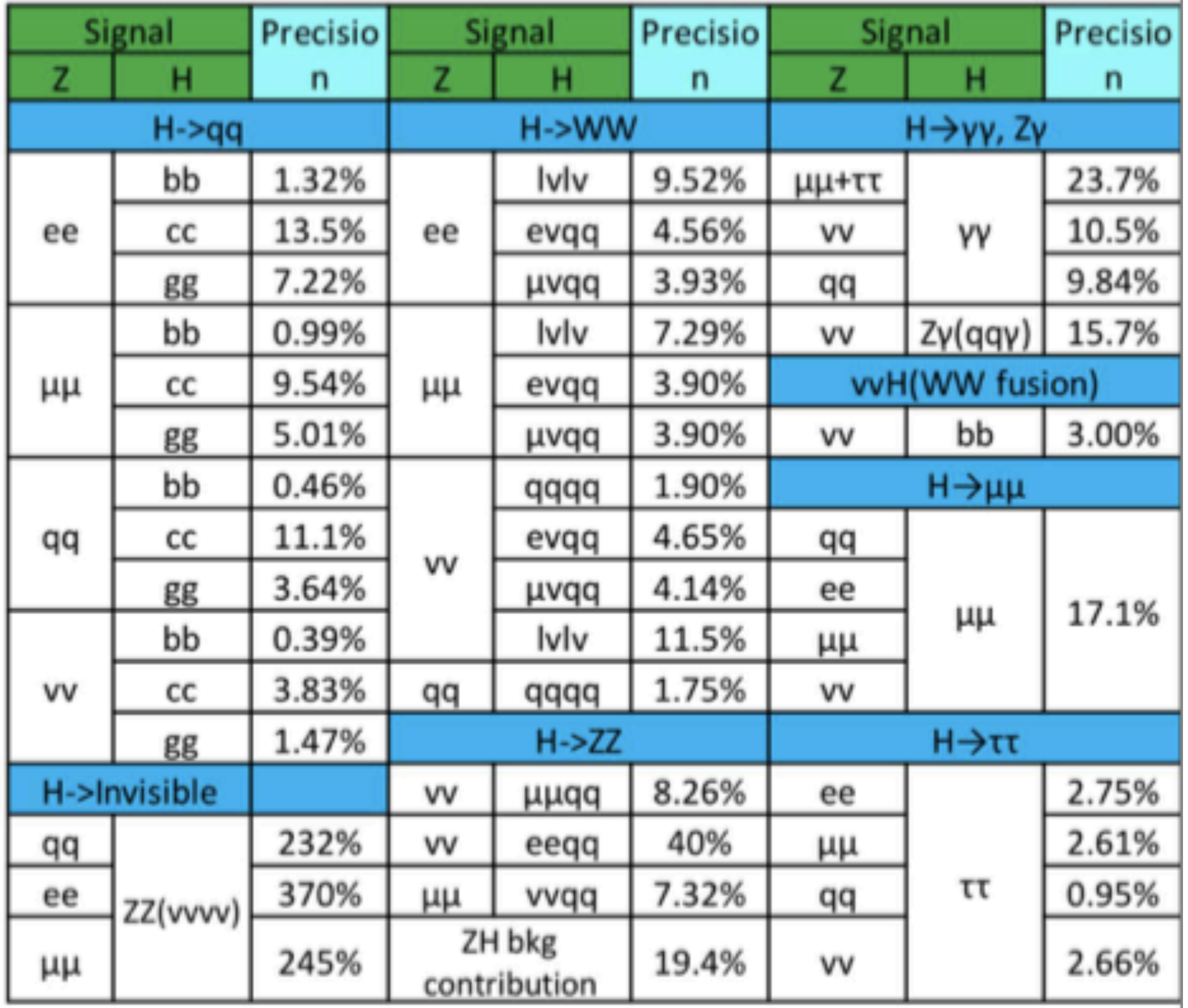
Results in CDR (2018.11)

All scaled to 240 GeV, 5.6ab^{-1}

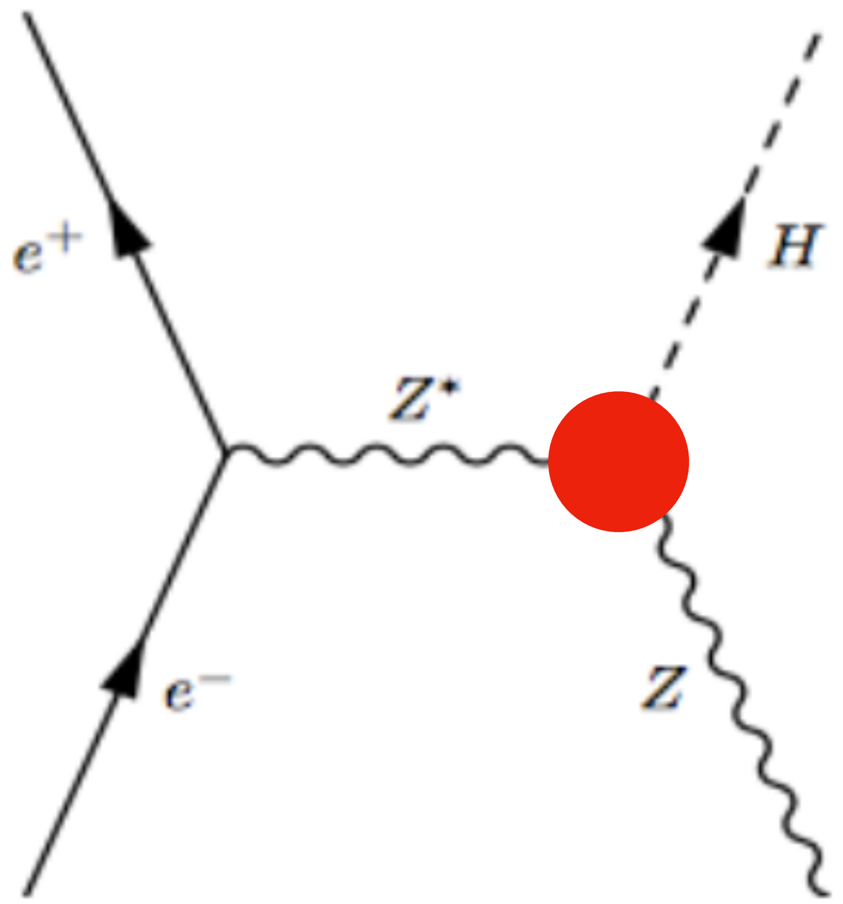


Property	Estimated Precision CEPC-v1	Estimated Precision CEPC-v4
m_H	5.9 MeV	5.9 MeV
Γ_H	2.7%	2.8%
$\sigma(ZH)$	0.5%	0.5%
$\sigma(\nu\bar{\nu}H)$	3.0%	3.2%

Decay mode	CEPC-v1 $\sigma \times \mathrm{BR}$	CEPC-v1 BR	CEPC-v4 $\sigma \times \mathrm{BR}$	CEPC-v4 BR
$H \to b\bar{b}$	0.26%	0.56%	0.27%	0.56%
$H \to c\bar{c}$	3.1%	3.1%	3.3%	3.3%
$H \to gg$	1.2%	1.3%	1.3%	1.4%
$H \to WW^*$	0.9%	1.1%	1.0%	1.1%
$H \to ZZ^*$	4.9%	5.0%	5.1%	5.1%
$H \to \gamma\gamma$	6.2%	6.2%	6.8%	6.9%
$H \to Z\gamma$	13%	13%	16%	16%
$H \to \tau^+\tau^-$	0.8%	0.9%	0.8%	1.0%
$H \to \mu^+\mu^-$	16%	16%	17%	17%
$\mathrm{BR}_{\mathrm{inv}}^{\mathrm{BSM}}$	–	$< 0.28\%$	–	$< 0.30\%$



Signal		Precision	Signal		Precision	Signal		Precision
Z	H		Z	H		Z	H	
H->qq			H->WW			H→γγ, Zγ		
ee	bb	1.32%	ee	lvlv	9.52%	μμ+ττ	γγ	23.7%
	cc	13.5%		evqq	4.56%	vv		10.5%
	gg	7.22%		μvqq	3.93%	qq		9.84%
μμ	bb	0.99%	μμ	lvlv	7.29%	vv	Zγ(qqγ)	15.7%
	cc	9.54%		evqq	3.90%	vvH(WW fusion)		
	gg	5.01%		μvqq	3.90%	vv	bb	3.00%
qq	bb	0.46%	vv	qqqq	1.90%	H→μμ		
	cc	11.1%		evqq	4.65%	qq	μμ	17.1%
	gg	3.64%		μvqq	4.14%	ee		
vv	bb	0.39%		lvlv	11.5%	μμ		
	cc	3.83%	qq	qqqq	1.75%	vv		
	gg	1.47%	H->ZZ			H→ττ		
H->Invisible			vv	μμqq	8.26%	ee	ττ	2.75%
qq	ZZ(vvvv)	232%	vv	eeqq	40%	μμ		2.61%
ee		370%	μμ	vvqq	7.32%	qq		0.95%
μμ		245%	ZH bkg contribution		19.4%	vv		2.66%



The relative precision of $\sigma(ZH)$ can be 0.5% ($\mathscr{L} = 5.6\ \mathrm{ab}^{-1}$)

Model independent analysis gives $\delta\kappa_Z = 0.25\ \%$, which however did not include **theoretical uncertainty**.

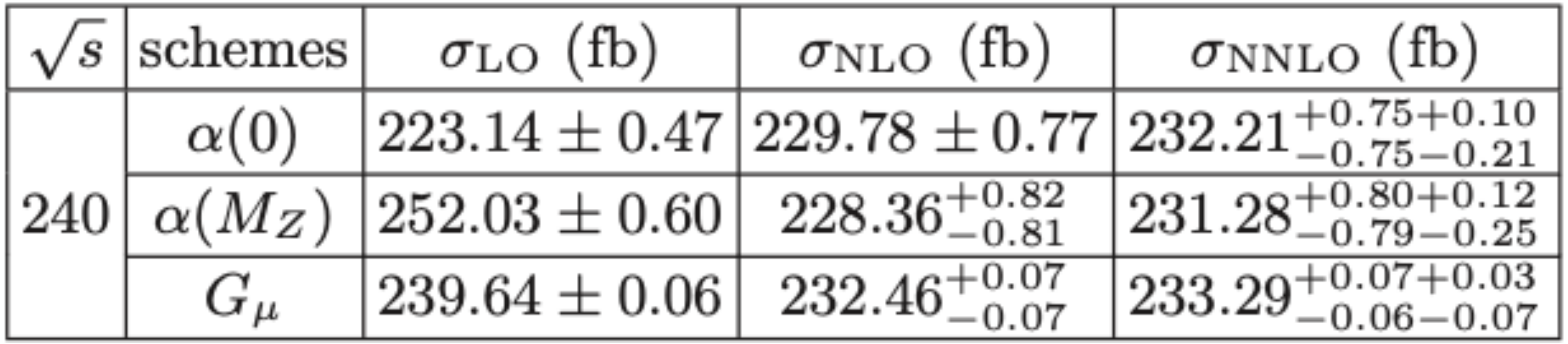
- Precision of $\delta\kappa_Z$ can affect the studies of EW phase transition.
Phys. Lett. B 317, 385 (1993); Phys. Lett. B 323, 339 (1994); JHEP 08, 010 (2007)
- Precision measurement on $\sigma(ZH)$ may shed light on new physics search, e.g. *JHEP 1503 (2015) 146; Phys.Rept. 496 (2010) 1-77; JHEP 08 (2015) 152*

Theoretical prediction uncert. involves variation of scale and scheme.

- EW NLO $\mathcal{O}(\alpha)$ (~6% uncert.)
  Nucl. Phys. B 216, 469 (1983); Z. Phys. C 55, 605 (1992); Phys. Rev. D 100, 073002 (2019); Phys. Rev. D90, 073007 (2014); Phys. Rev. D 96, 075044 (2017); etc.

- EW+QCD mixed NNLO $\mathcal{O}(\alpha\alpha_s)$ (>1% uncert.)
  Phys. Rev. D 95, 093003 (2017); Phys. Rev. D 96, 051301 (2017)

- **Therefore, NNLO EW correction must be included for future physics analysis.**



$\sqrt{s}$	schemes	σ_{LO} (fb)	σ_{NLO} (fb)	σ_{NNLO} (fb)
240	$\alpha(0)$	223.14 ± 0.47	229.78 ± 0.77	$232.21^{+0.75+0.10}_{-0.75-0.21}$
	$\alpha(M_Z)$	252.03 ± 0.60	$228.36^{+0.82}_{-0.81}$	$231.28^{+0.80+0.12}_{-0.79-0.25}$
	G_μ	239.64 ± 0.06	$232.46^{+0.07}_{-0.07}$	$233.29^{+0.07+0.03}_{-0.06-0.07}$

Difficulties on NNLO EW correction

- **Real correction (QED):**

  1. double QED radiation should be handled easily (Abelian case compared to well studied QCD radiation)

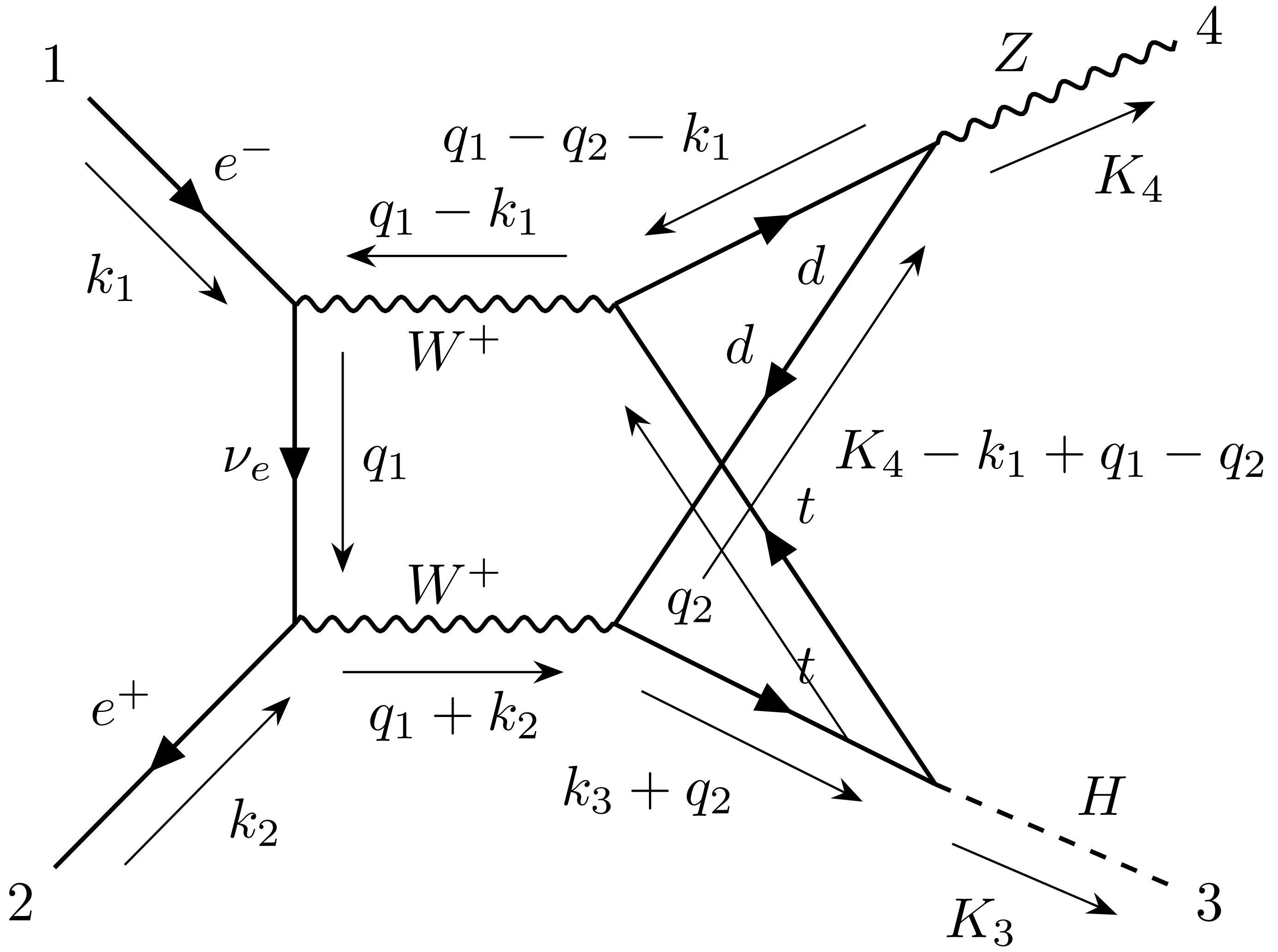
- **Virtual correction:**

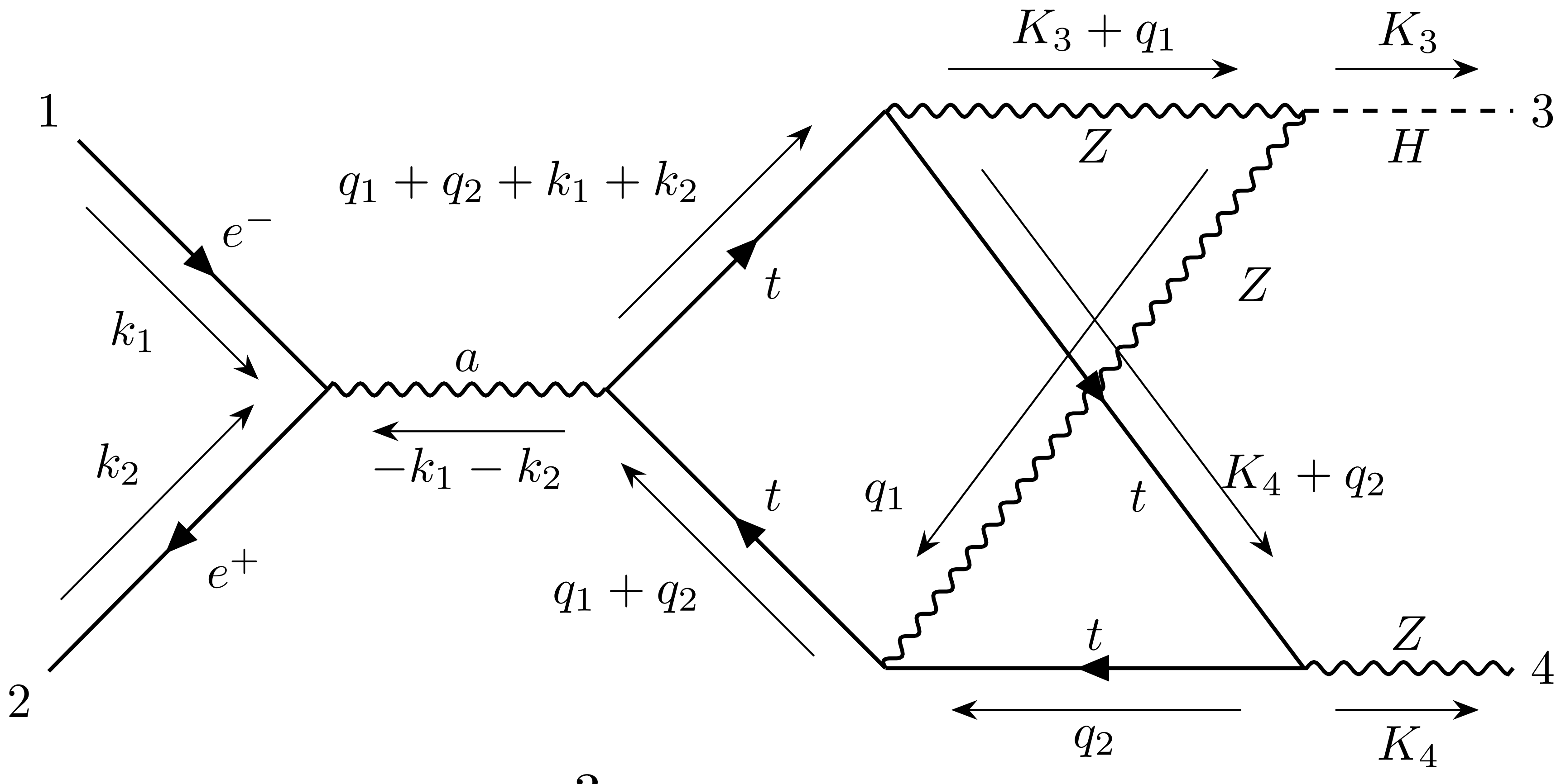
  1. Multi-scale two-loop Feynman diagrams, esp. non-planar double-box ⇒ reduction of amplitude, reduction to master integrals, calculation of master integrals

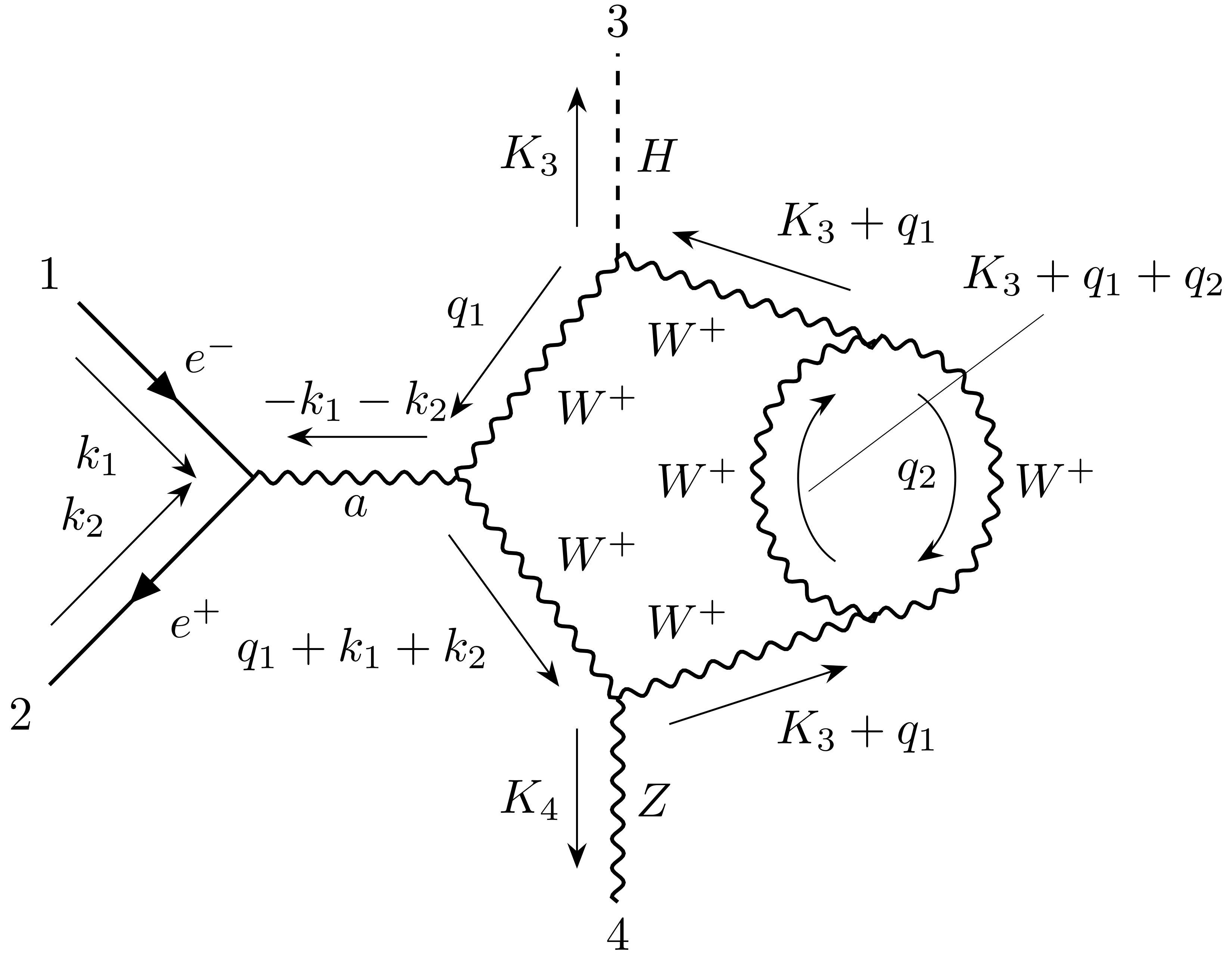
  2. 25377 Feynman diagrams (Feynman gauge), where only 2250 diagrams are very challenging, i.e. with 7 denominators.

Virtual NNLO EW correction

Virtual NNLO EW correction

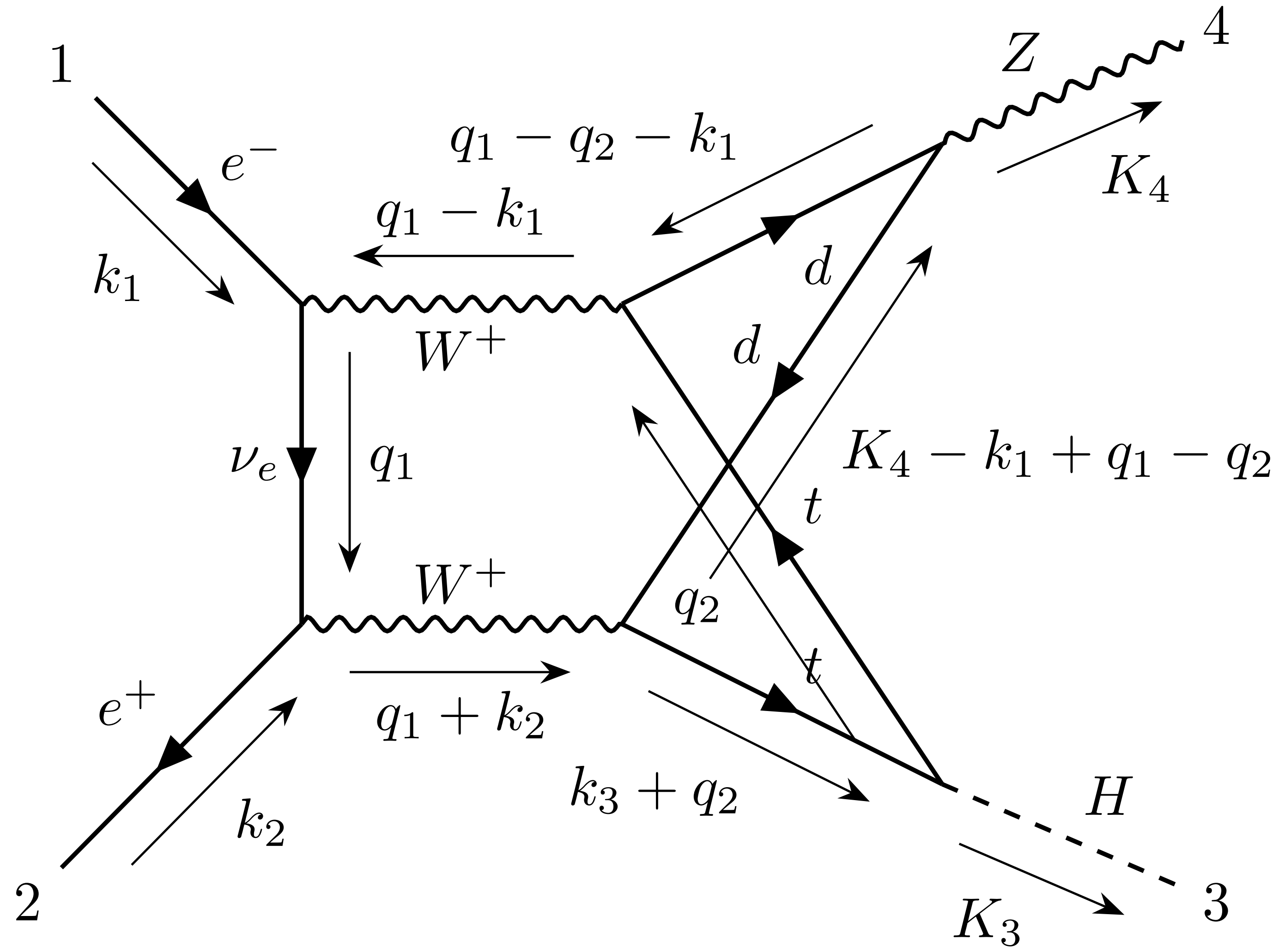


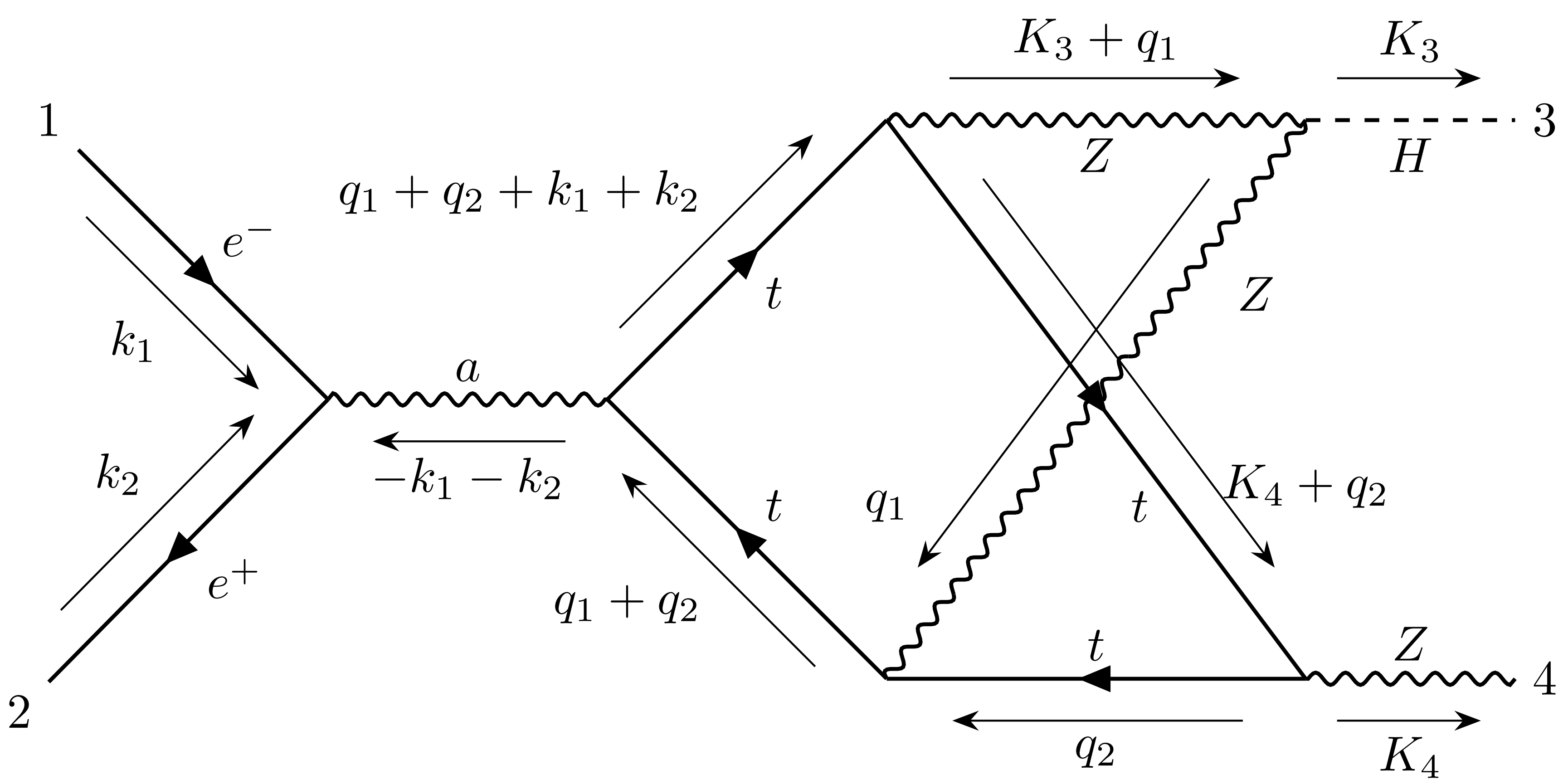


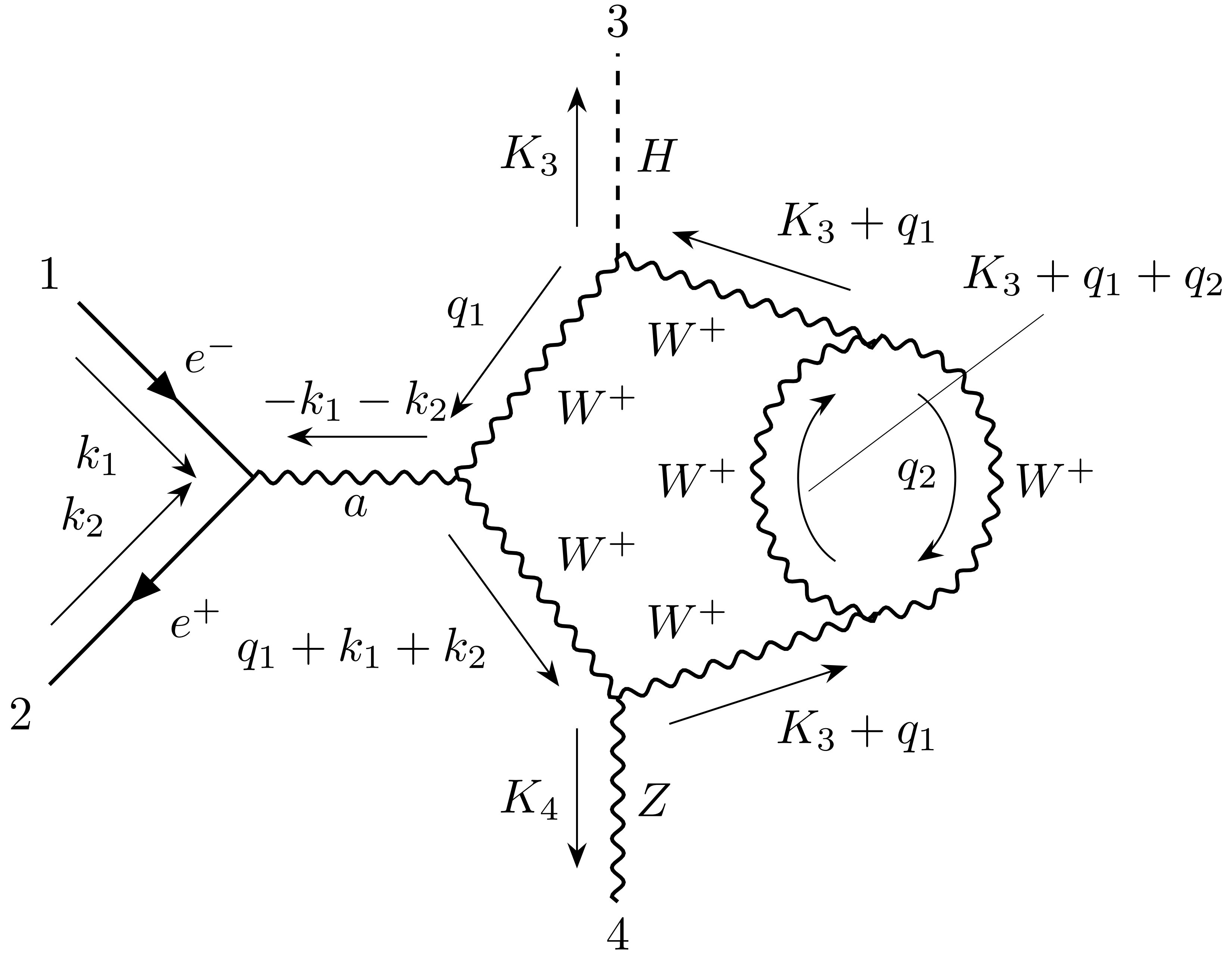


Virtual NNLO EW correction

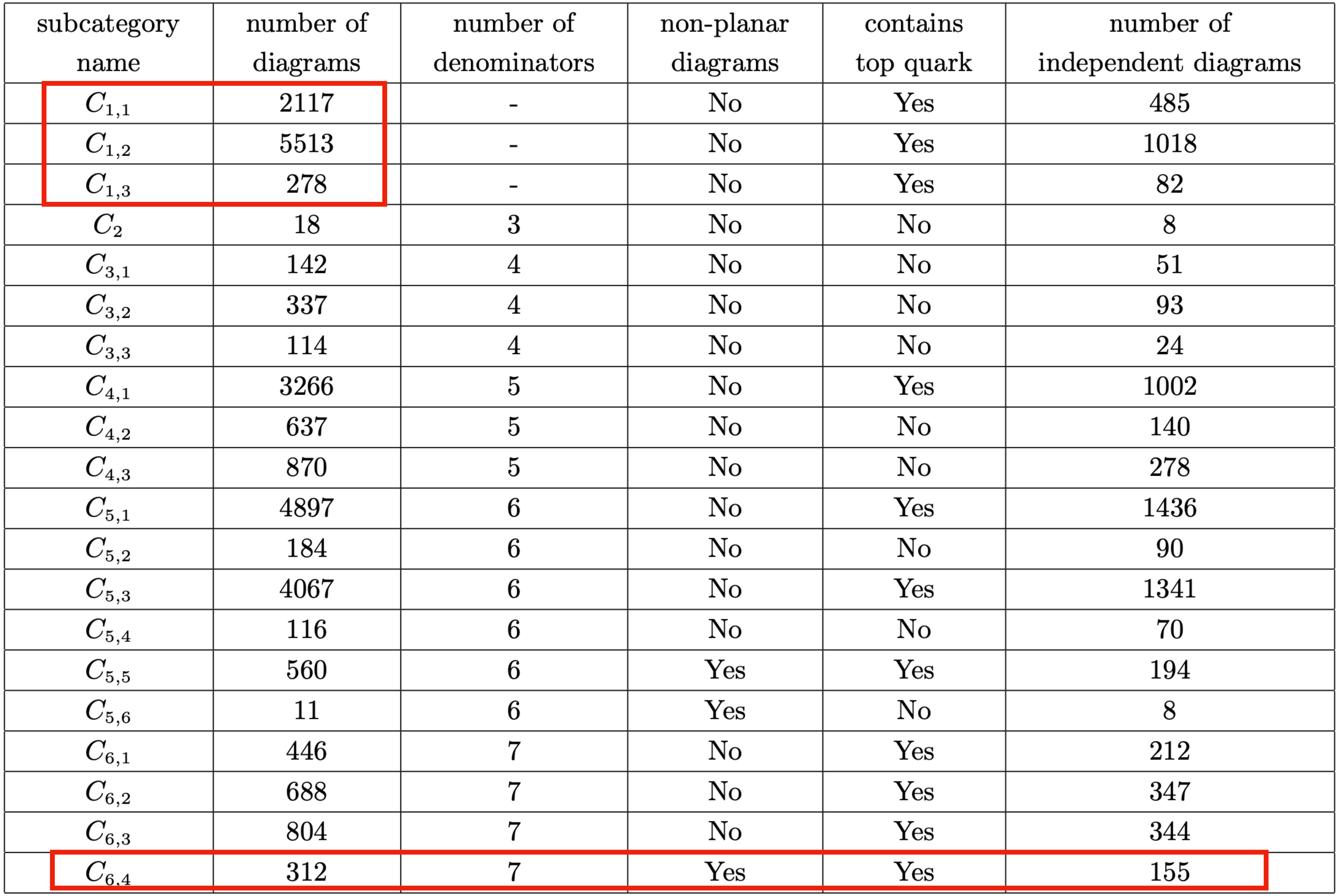
Total: 25377







Z. Li, Y. Wang and Q-f. Wu, Chin. Phys. C ***45*** *No.5 053102 (2021)*



subcategory name	number of diagrams	number of denominators	non-planar diagrams	contains top quark	number of independent diagrams
$C_{1,1}$	2117	-	No	Yes	485
$C_{1,2}$	5513	-	No	Yes	1018
$C_{1,3}$	278	-	No	Yes	82
C_2	18	3	No	No	8
$C_{3,1}$	142	4	No	No	51
$C_{3,2}$	337	4	No	No	93
$C_{3,3}$	114	4	No	No	24
$C_{4,1}$	3266	5	No	Yes	1002
$C_{4,2}$	637	5	No	No	140
$C_{4,3}$	870	5	No	No	278
$C_{5,1}$	4897	6	No	Yes	1436
$C_{5,2}$	184	6	No	No	90
$C_{5,3}$	4067	6	No	Yes	1341
$C_{5,4}$	116	6	No	No	70
$C_{5,5}$	560	6	Yes	Yes	194
$C_{5,6}$	11	6	Yes	No	8
$C_{6,1}$	446	7	No	Yes	212
$C_{6,2}$	688	7	No	Yes	347
$C_{6,3}$	804	7	No	Yes	344
$C_{6,4}$	312	7	Yes	Yes	155

Choices for amplitude reduction

- Projection method

  JHEP, 05: 060 (2002); JHEP, 04: 021 (2004)

- IBP method

  Int. J. Mod. Phys., A15: 5087–5159 (2000)

- AMF method

  Phys. Rev., D99(7): 071501 (2019); Phys.Rev.D 101 (2020) 7, 076023

Choices for reduction to master integrals

- IBP reduction
  Int. J. Mod. Phys., A15: 5087–5159 (2000)
- AMF method
  Phys. Rev., D99(7): 071501 (2019)

Calculation of master integrals

- Analytical method:
  Most conventional approach, but one may confront difficult integrals, e.g. elliptic integral, in the complicated amplitudes.

- Differential Equations (analytical or numerical) ~ construction of canonical integrals etc.
  Phys. Rev. Lett. 110, 251601 (2013); etc.

- Numerical method:

  1. Sector decomposition
     Nucl. Phys. B585 (2000) 741; Int.J.Mod.Phys.A 23 (2008) 1457-1486; etc.

  2. Mellin-Barnes
     Phys. Lett. B460 (1999) 397; Phys.Lett.B469 (1999) 225; etc.

  3. AMF
     Phys. Rev., D99(7): 071501 (2019); etc.

- Semi-numerical method
  JHEP 11 (2006) 048

Finally

- NNLO EW correction to $e^+e^- \to HZ$ must be done to match the precision of experiment data.
- The complete solution may take years.
- Call for international collaboration or working group for this big project.
  - ✦ Ayres Freitas group @ University of Pittsburgh
  - ✦ Zhao Li group @ IHEP-CAS

Thank you!