Joint Workshop of
the CEPC Physics, Software and New
Detector Concept

Higgs–Top couplings

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University of Minnesota

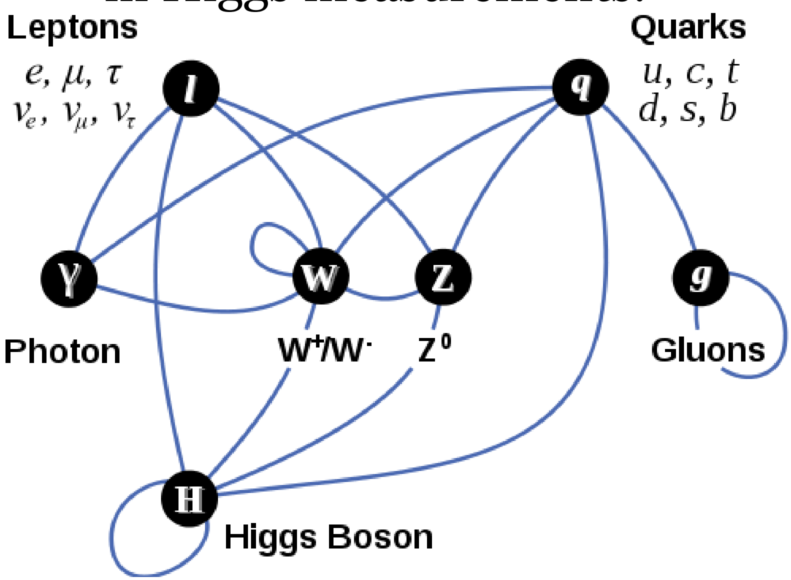
04/14/2021

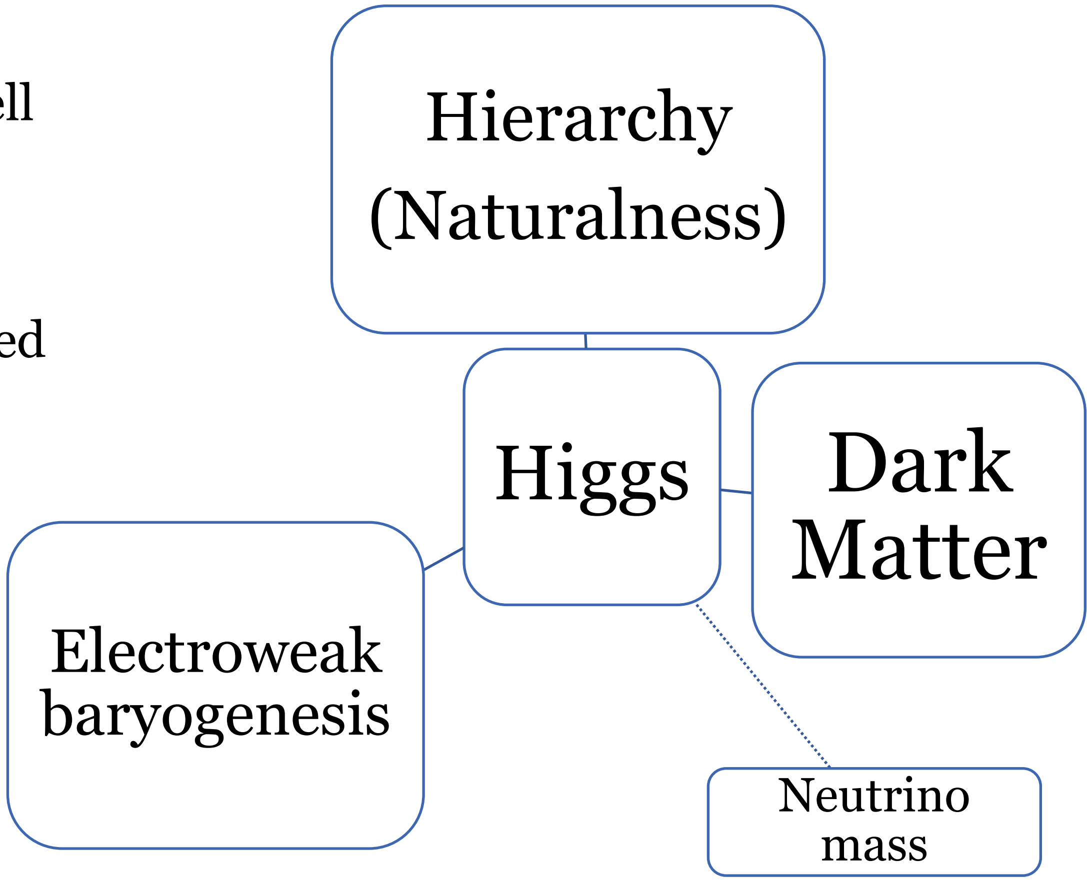
Key to many Puzzles

Top quark plays special roles in Higgs physics

Higgs boson discovery substantiates (more) many big questions in nature. It could well be the key to unlock some of nature's secrets.

All connections could be revealed in Higgs measurements.





Top quark and Higgs EFT Overview

Top-quark and Higgs couplings are the key driver of the hierarchy problem (and subsequent naturalness problem). Solutions to such problem are likely to induce corrections to these couplings.

Important to consider the CEPC sensitivity to Higgs and top EFT, even though the operational energy is below ttbar+Higgs threshold.

Top quark and Higgs EFT Overview

$$\mathcal{O}_{tH} = \frac{1}{\Lambda^2}(H^\dagger H)(\bar{q}_L \tilde{H} t_R),$$
$$\mathcal{O}_{bH} = \frac{1}{\Lambda^2}(H^\dagger H)(\bar{q}_L H b_R),$$
$$\mathcal{O}_{Hq}^{(1)} = \frac{i}{\Lambda^2}(H^\dagger \overleftrightarrow{D}_\mu H)(\bar{q}_L \gamma^\mu q_L),$$
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Here we choose a (minimal-)complete set of relevant operators, can be obtain by integrating out heavy particles and EOM.
J. Aguilar-Saavedra, arXiv:0811.3842, arXiv:0904.2387

C. Degrande, J. Gerard, C. Grojean, F. Maltoni, and G. Servant arXiv:1205.1065, B. A. Kniehl and O. L. Veretin arXiv:1206.7110, A. Hayreter and G. Valencia arXiv:1304.6976

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Important to consider the CEPC sensitivity to Higgs and top EFT, even though the operational energy is far below ttbar+Higgs threshold.

I will go through the physics probes for these operators individually and by groups

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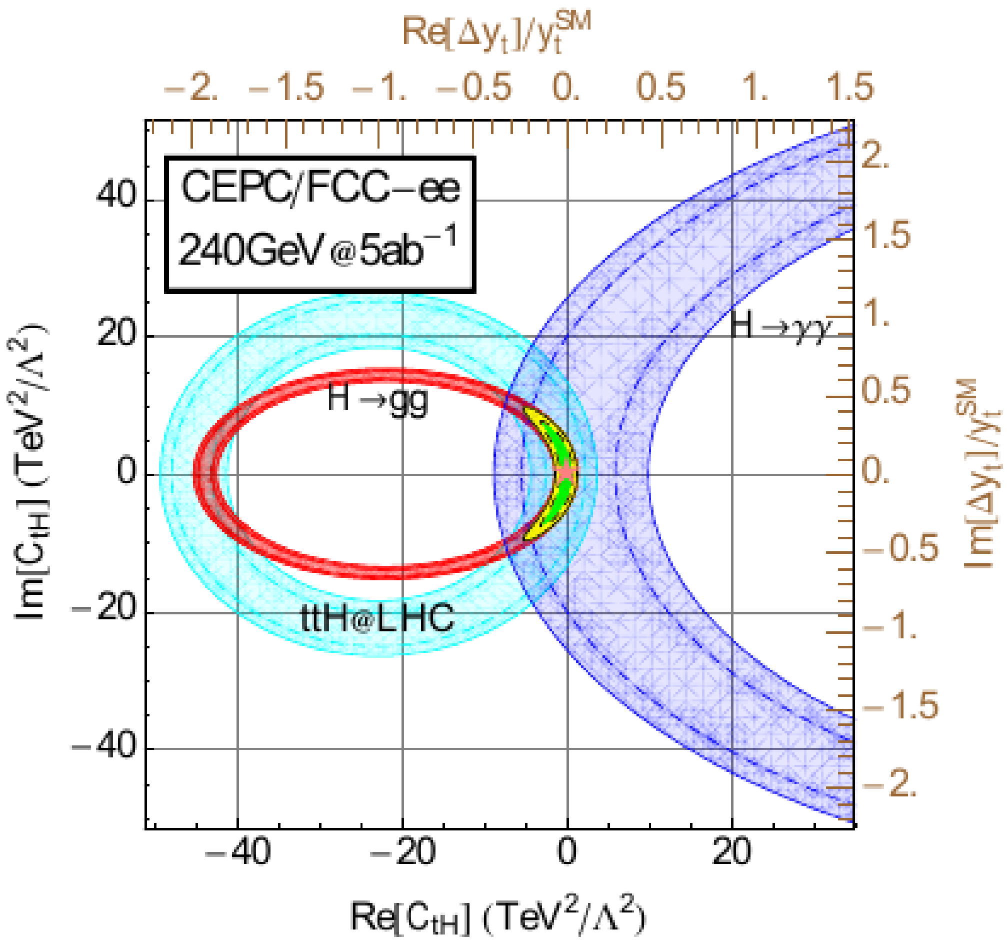
Top quark and Higgs EFT O_{tH}

$$\mathcal{O}_{tH} = \frac{1}{\Lambda^2}(H^\dagger H)(\bar{q}_L \tilde{H} t_R),$$

CP-even and CP-odd type of Yukawas, asymmetries too tiny below ttbar threshold.

Sensitivity from loop process. Gluon-gluon and diphoton drives the limits, though the precision of corresponding coupling is worse than κ_Z measurement.

Better than HL-LHC tth direct production.



Assuming no new HGG and HFF operators;
In cases where HGG and HFF are of the same order, e.g., top partners with mixing, a correlation presents and the constraints on new physics scales are generically still be the same order

$$\Delta y_t \approx \mathrm{Re}[C_{tH}]\frac{v^2}{\Lambda^2} + i\,\mathrm{Im}[C_{tH}]\frac{v^2}{\Lambda^2} + \mathcal{O}\left(\frac{v^4}{\Lambda^4}\right)$$

Top quark and Higgs EFT O_{bH}

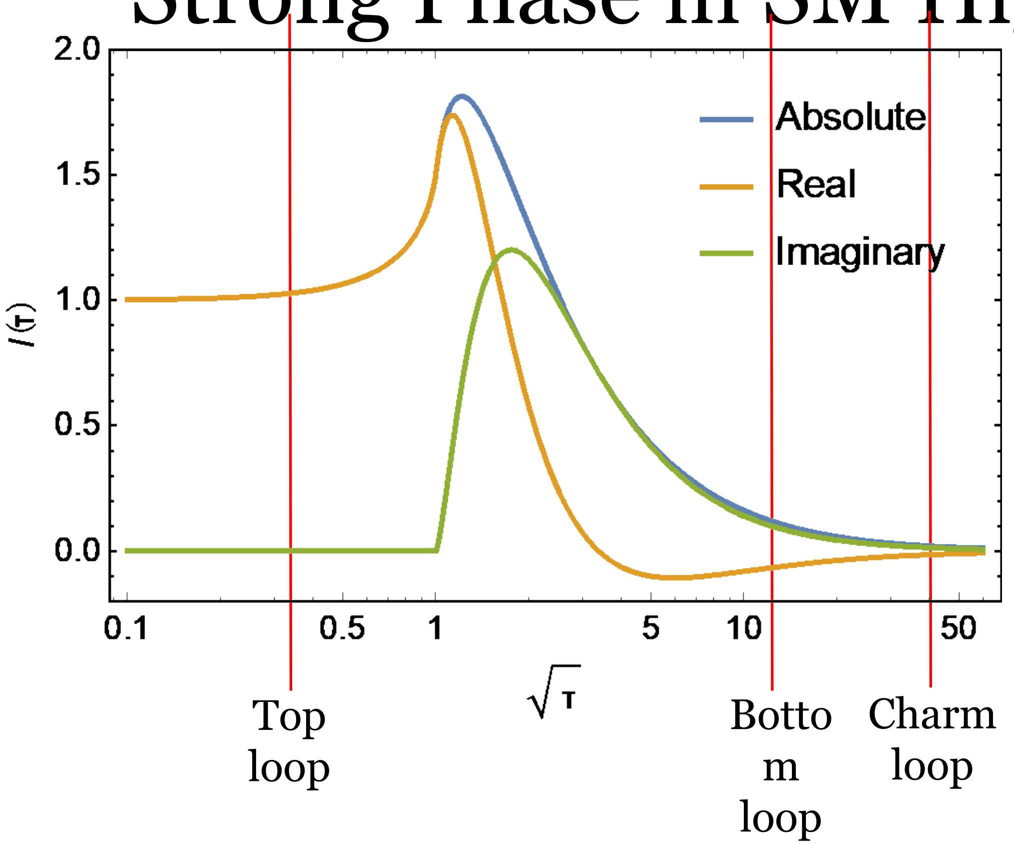
$$\mathcal{O}_{bH} = \frac{1}{\Lambda^2}(H^\dagger H)(\bar{q}_L H b_R),$$

Direct constraints from $H \to \bar{b}b$ precision.

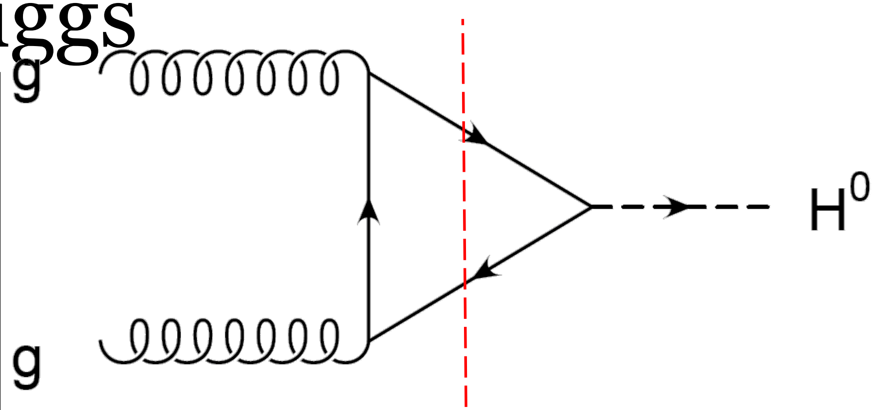
CEPC projection of 1.5% on bottom Yukawa~9 TeV on Λ

But that is not all the story.

Strong Phase in SM Higgs



A strong phase in the gluon-gluon fusion production at hadron colliders (imaginary part)



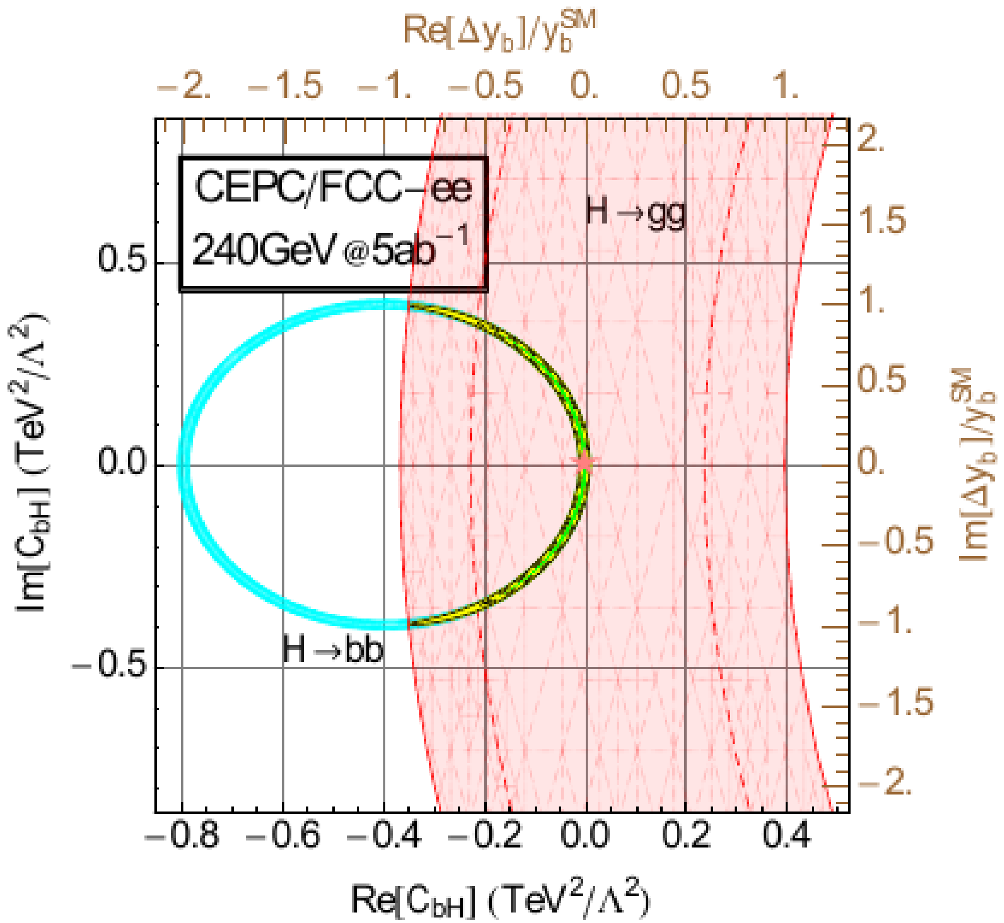
- All quark contributions normalized the same way, the plot represents the relative contributions
- Numerically:
  - t-loop $+1.034$
  - b-loop $-0.035 + 0.039i$
  - c-loop $-0.004 + 0.002i$

Top quark and Higgs EFT O_{bH}

$$\mathcal{O}_{bH} = \frac{1}{\Lambda^2}(H^\dagger H)(\bar{q}_L H b_R),$$

Direct constraints from $H \to \bar{b}b$ precision.

Sensitivity to CP-phase phase through interference with top loop for the gluon-gluon-Higgs coupling.



Joint Analysis
O_{bH} and O_{tH}

Key measurements (CEPC):
Higgs to diphoton
Higgs to digluon
Higgs to bb
(ttH@LHC)

Four d.o.f., bottom and top Yukawa:
Strengths (x-axes) and phases (y-axes)

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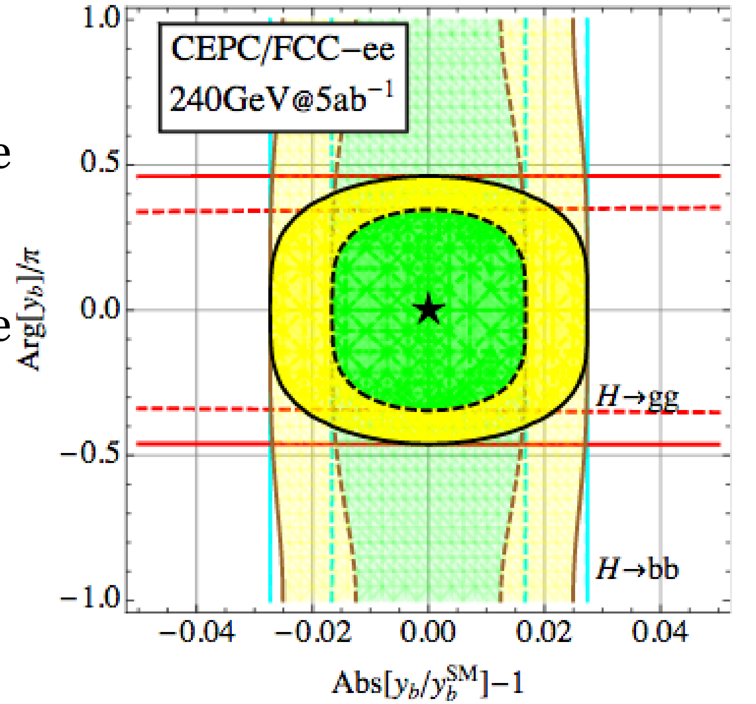
Bottom Yukawa:

- H to bb constraints the strength
- H to digluon constraints the phase through interference



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Key measurements (CEPC):
Higgs to diphoton
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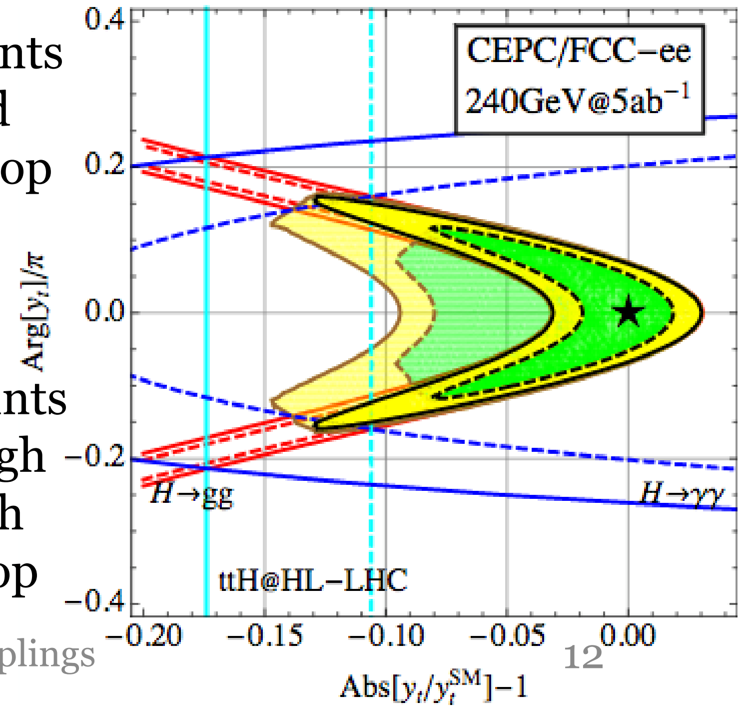
(ttH@LHC)

Four d.o.f., bottom and top Yukawa:
Strengths (x-axes) and phases (y-axes)

Top Yukawa:

- H to gg constraints the strength and phase (due to loop function differences)
- H to gamma gamma constraints the phase through interference with dominant W-loop



Top quark and Higgs EFT

$$O_{Hq}^{(1)}, O_{Hq}^{(3)}, O_{Ht}, O_{Hb}$$

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$q_L = (t_L, b_L)$, $H^\dagger \overleftrightarrow{D}_\mu H = H^\dagger(D_\mu H) - (D_\mu H)^\dagger H$, and $\tilde{H} = i\sigma^2 H$.

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Three-point functions only qqV

$$Z_\mu \bar{b}_R\gamma^\mu b_R: \quad -g_Z\frac{v^2}{2\Lambda^2}C_{Hb}^{(1)}$$
$$Z_\mu \bar{b}_L\gamma^\mu b_L: \quad -g_Z\frac{v^2}{2\Lambda^2}(C_{Hq}^{(1)} + C_{Hq}^{(3)})$$
$$Z_\mu \bar{t}_R\gamma^\mu t_R: \quad -g_Z\frac{v^2}{2\Lambda^2}C_{Ht}^{(1)}$$
$$Z_\mu \bar{t}_L\gamma^\mu t_L: \quad -g_Z\frac{v^2}{2\Lambda^2}(C_{Hq}^{(1)} - C_{Hq}^{(3)})$$
$$W_\mu^+ \bar{t}_L\gamma^\mu b_L: \quad g_2\frac{v^2}{\sqrt{2}\Lambda^2}C_{Hq}^{(3)},$$

Higgs modification starting at the four-point function qqVH

*little impact on the Higgs coupling precision fits at tree-level (since most Higgs decay are two-body)

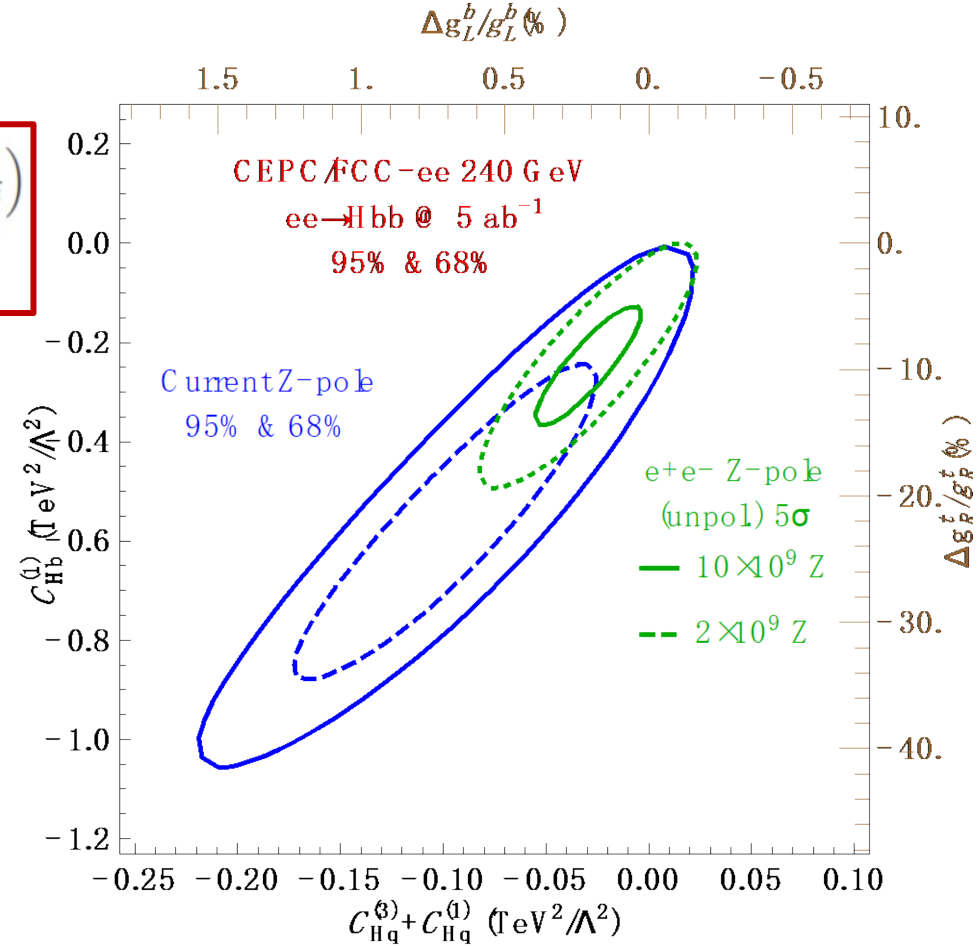
**No photon, only Z and W

$q_L = (t_L, b_L)$, $H^\dagger \overleftrightarrow{D}_\mu H = H^\dagger(D_\mu H) - (D_\mu H)^\dagger H$, and $\tilde{H} = i\sigma^2 H$.

Top quark and Higgs EFT $O_{Hq}^{(1)}, O_{Hq}^{(3)}, O_{Hb}$

$$\delta\varepsilon_1 = \frac{3m_t^2 G_F}{2\sqrt{2}\pi^2} \mathrm{Re}\left[C_{\phi q}^{(3,33)} - C_{\phi q}^{(1,33)} + C_{\phi u}^{33} + \mathcal{O}\left(\frac{v^2}{\Lambda^2}\right)\right]\left(\frac{v^2}{\Lambda^2}\right)\log\left(\frac{\Lambda^2}{m_t^2}\right)$$

$$\delta\varepsilon_b = -\frac{m_t^2 G_F}{2\sqrt{2}\pi^2} \mathrm{Re}\left[C_{\phi q}^{(3,33)} - C_{\phi q}^{(1,33)} + \frac{1}{4}C_{\phi u}^{33}\right]\left(\frac{v^2}{\Lambda^2}\right)\log\left(\frac{\Lambda^2}{m_t^2}\right).$$



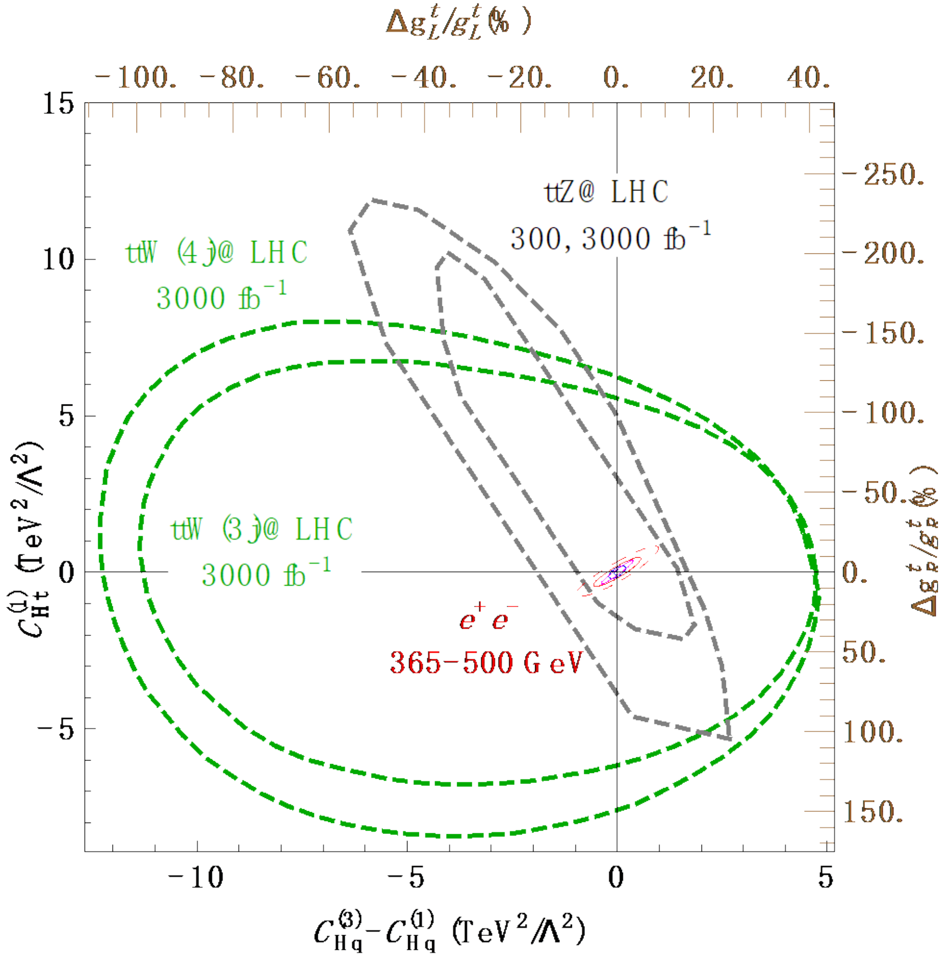
Z-pole provides very high precision on these coupling.

Z-pole result from S. Gori, J. Gu, and L.-T. Wang 1508.07010

Top quark and Higgs EFT $O_{Hq}^{(1)}, O_{Hq}^{(3)}, O_{Ht}$

LHC-DY-ttbar is buried under the QCD ttbar production

Need ttZ, ttW final states

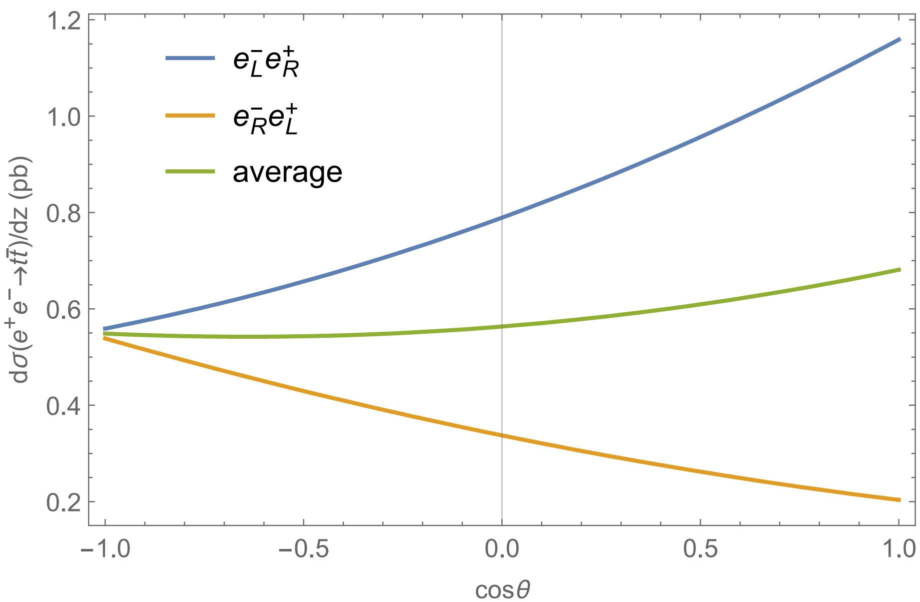


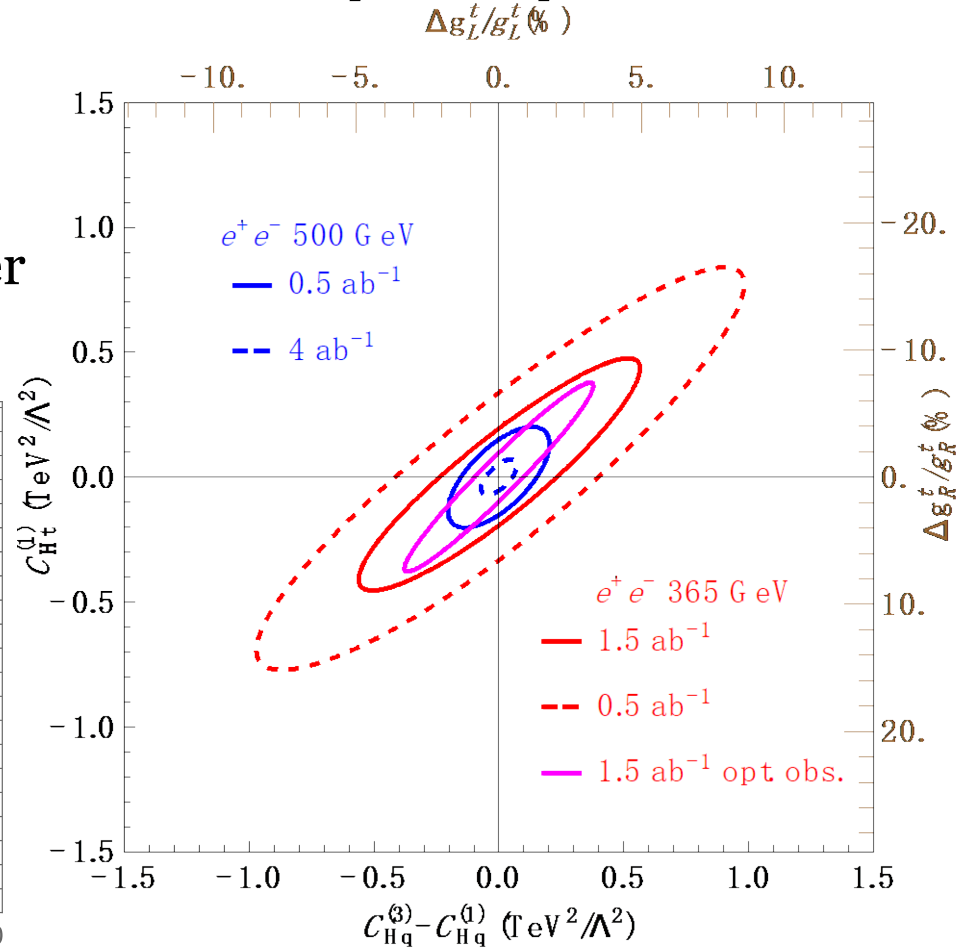
R. Rontsch and M. Schulze, 1404.1005 , J. Dror, M. Farina, E. Salvioni, J. Serra, 1511.03674

Top quark and Higgs EFT $O_{Hq}^{(1)}, O_{Hq}^{(3)}, O_{Ht}$

Cross section and forward-backward asymmetry

ILC 0.5 ab^{-1} with polarization better than 1.5 ab^{-1} unpolarized





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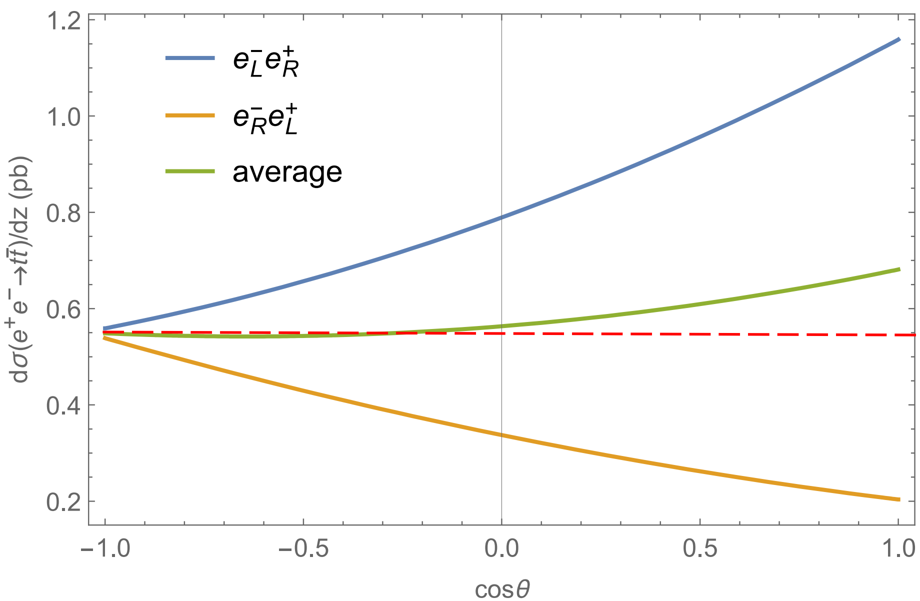
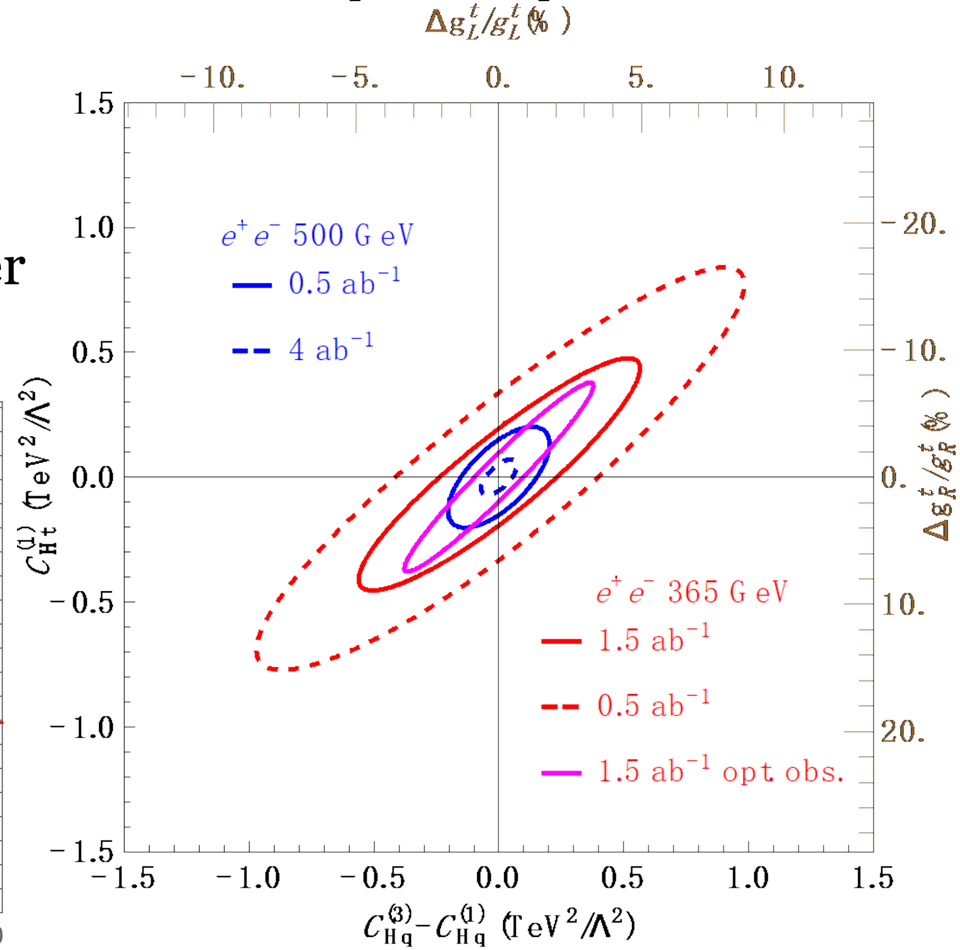
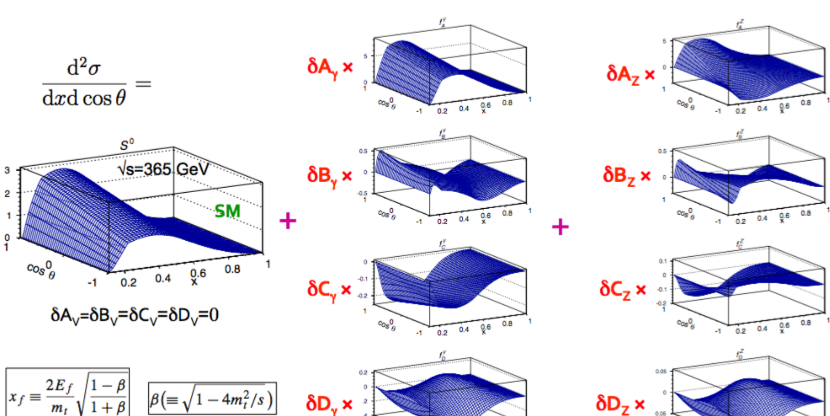




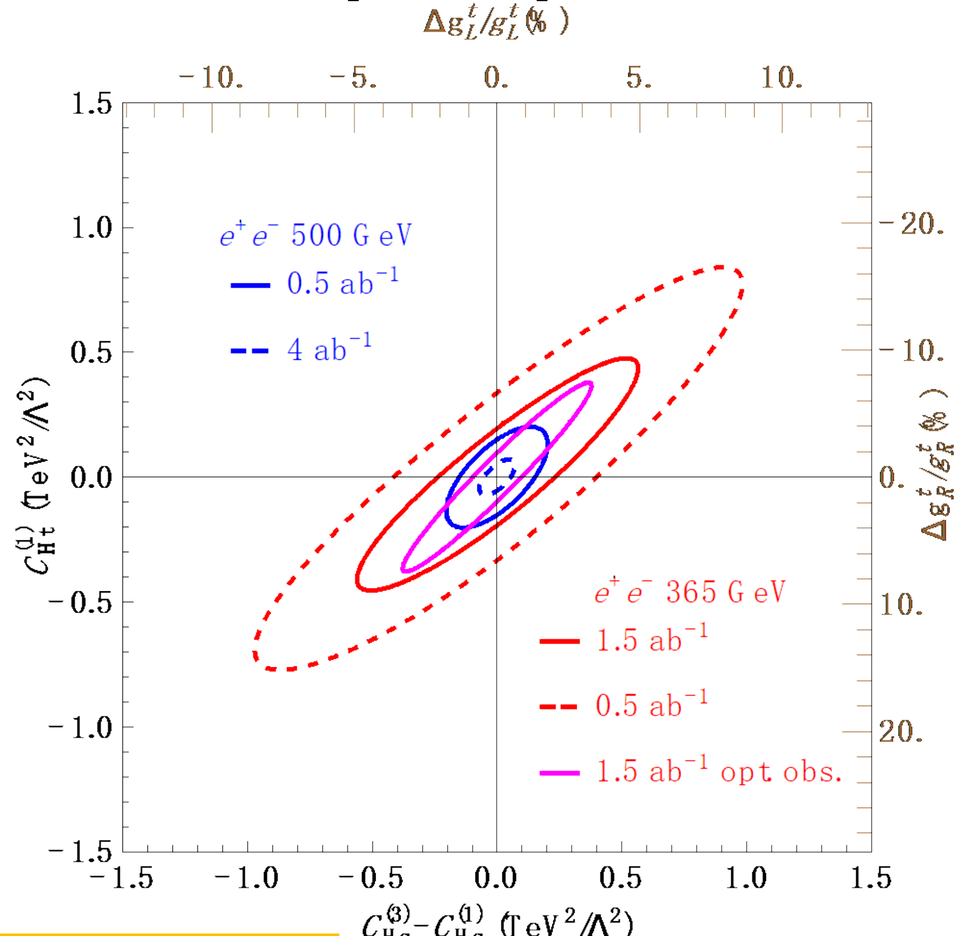
Little asymmetry for unpolarized beam

Top quark and Higgs EFT $O_{Hq}^{(1)}, O_{Hq}^{(3)}, O_{Ht}$

At or above $t\bar{t}$ threshold at lepton colliders, one immediately again great sensitivities to the top gauge couplings.

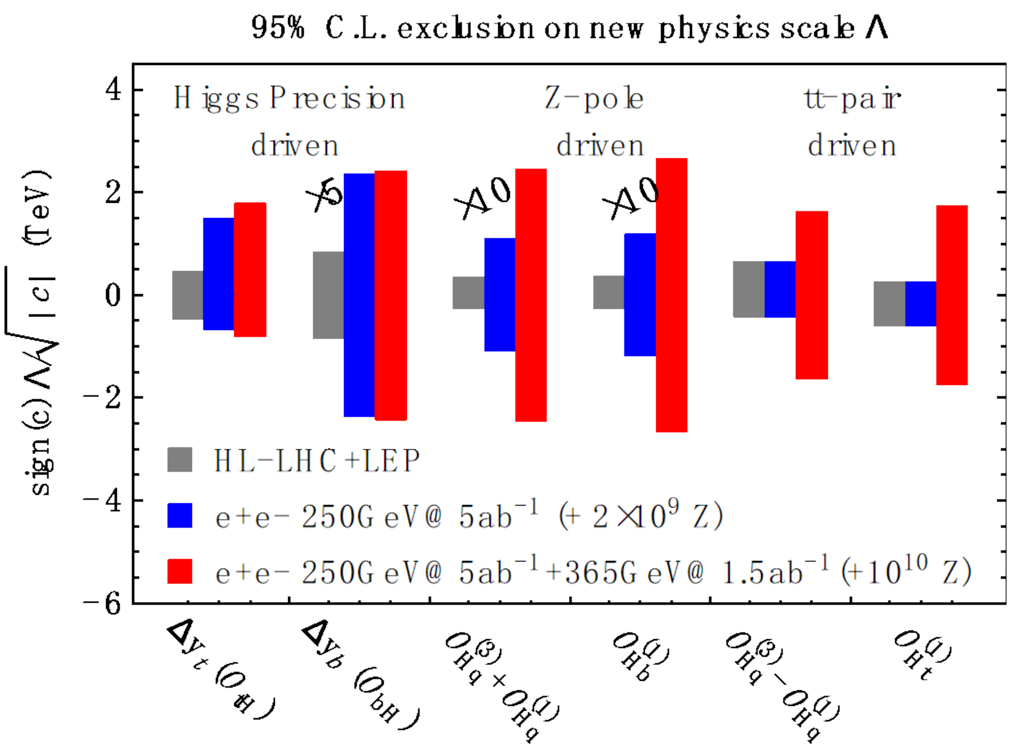


Patrick Janot



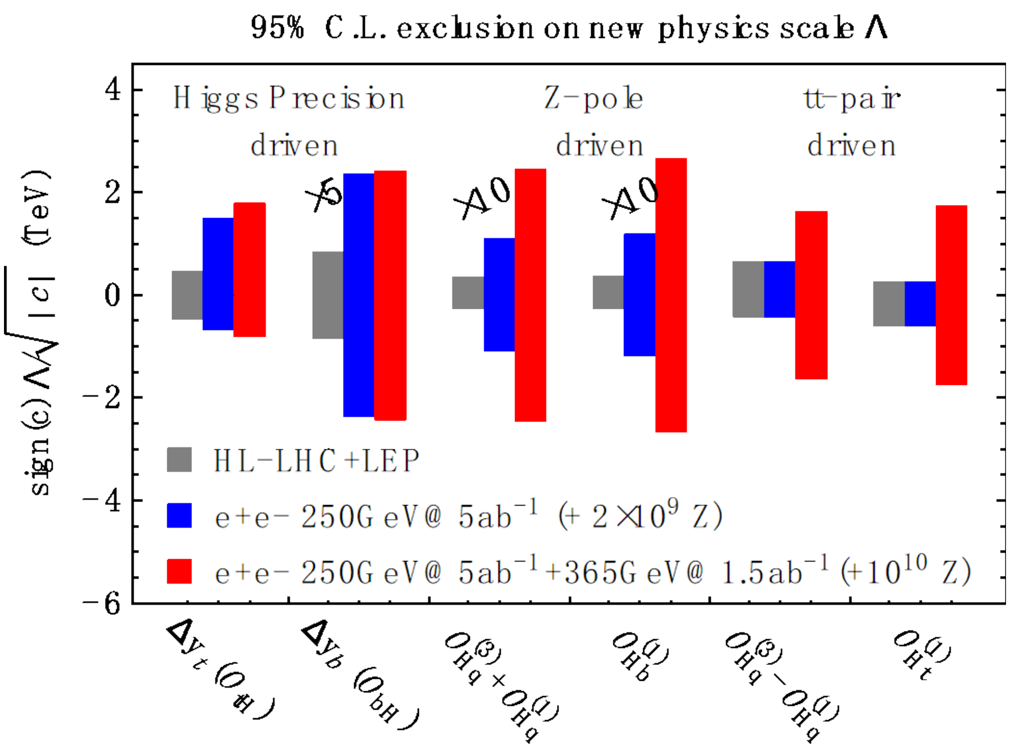
Note that the opt. obs. Analysis is a rescaling of Janot's study, we probably want to confirm that ourselves.

Top quark and Higgs EFT summary



Naturally divide into groups, where the correlations between the measurements are not large at linear level.

Top quark and Higgs EFT summary



Higgs-Top couplings important and interesting.

We try to develop some comprehensive understanding of the minimal Higgs Top anomalous coupling EFT set.

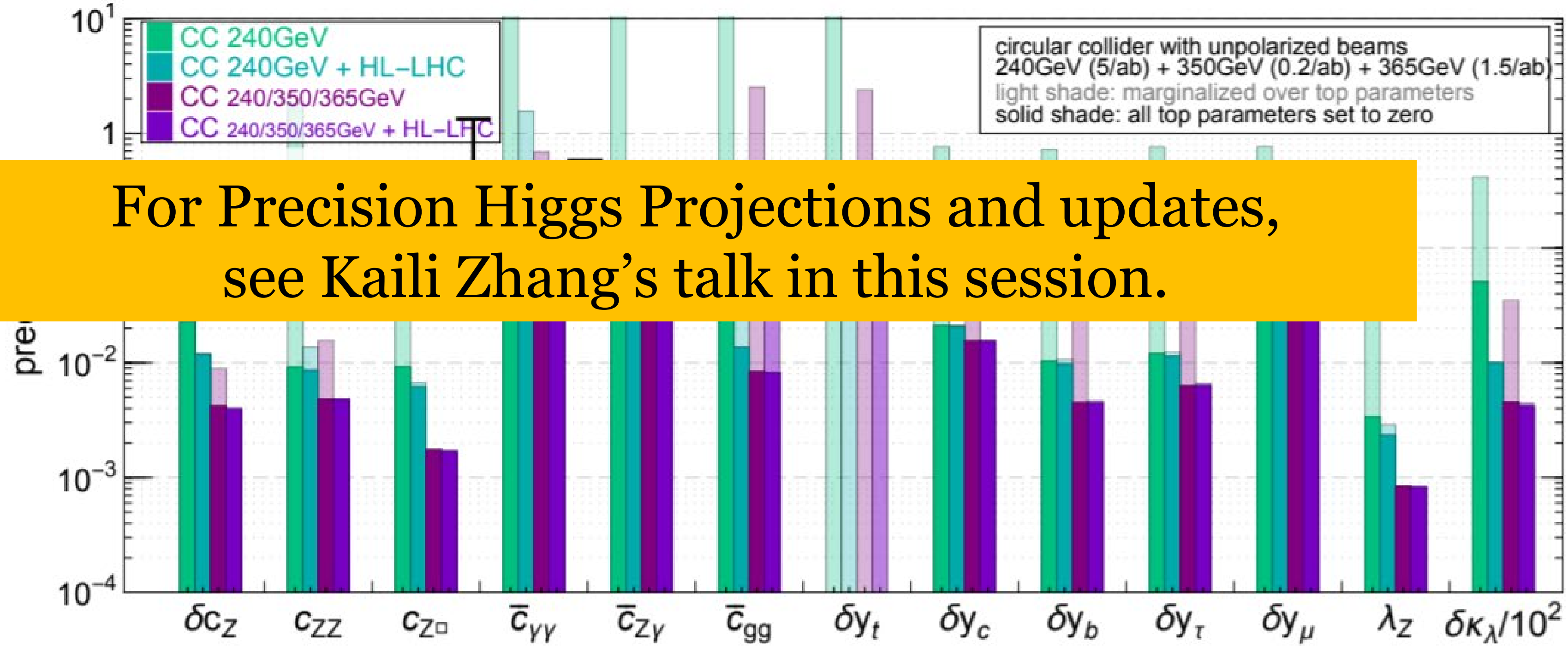
Higgs precision, Z-pole precision, ttbar (365 GeV) all needed to complete the picture.

Might be interesting to consider the synergy and physics outcome of larger ring (100 km) and larger energy (350~400 GeV).

Naturally divide into groups, where the correlations between the measurements are not large at linear level.

Q2: Contamination in Higgs operators

light shades: 12 Higgs op. floated + 6 top op. floated
dark shades: 12 Higgs op. floated + 6 top op. $\to 0$



For Precision Higgs Projections and updates, see Kaili Zhang's talk in this session.

Uncertainties on the top have a big effect on the Higgs

- Higgsstr. run: insufficient
- Higgsstr. run $\oplus$ top@HL-LHC: large top contaminations in $\bar{c}_{\gamma\gamma,gg,Z\gamma,ZZ}$
- Higgsstr. run $\oplus$ $e^+e^- \to t\bar{t}$: large y_t contaminations in various coefficients
- Higgsstr. run $\oplus$ $e^+e^- \to t\bar{t}$ $\oplus$ top@HL-LHC: top contam. in $\bar{c}_{gg}$ only

Results

CEPC

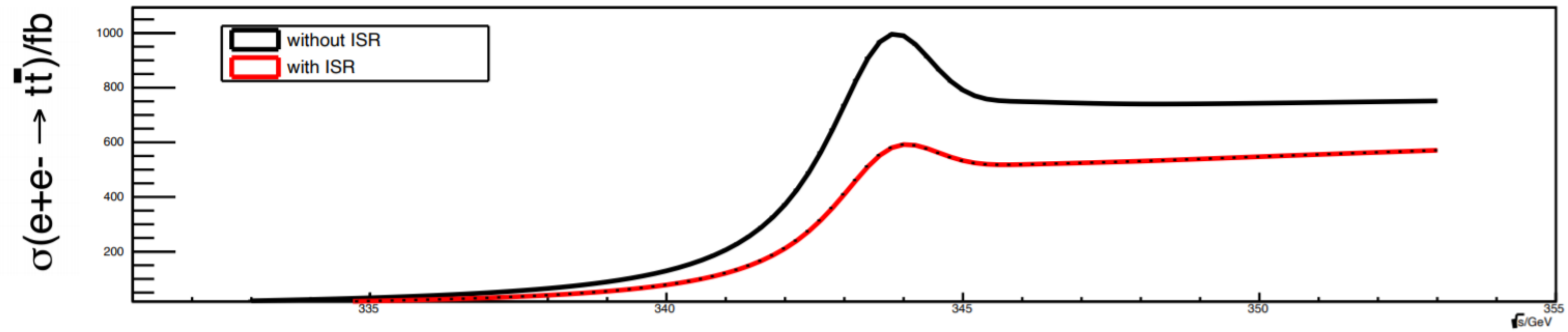
	240GeV, 5.6ab^{-1}	360GeV, 2ab^{-1}	
	ZH	ZH	ννH
any	**0.50%**	**1%**	\
$H \to bb$	**0.27%**	**0.63%**	**0.76%**
$H \to cc$	**3.3%**	**6.2%**	**11%**
$H \to gg$	**1.3%**	**2.4%**	**3.2%**
$H \to WW$	**1.0%**	**2.0%**	**3.1%**
$H \to ZZ$	**7.9%**	**14%**	**15%**
$H \to \tau\tau$	**0.8%**	**1.5%**	**3%**
$H \to \gamma\gamma$	**5.4%**	**8%**	**11%**
$H \to \mu\mu$	**12%**	**29%**	**40%**
$\mathrm{Br_{upper}}(H \to inv.)$	**0.2%**	\	\
$\sigma(ZH) * \mathrm{Br}(H \to Z\gamma)$	**16%**	**25%**	\
Width	**2.9%**	**1.4%**	

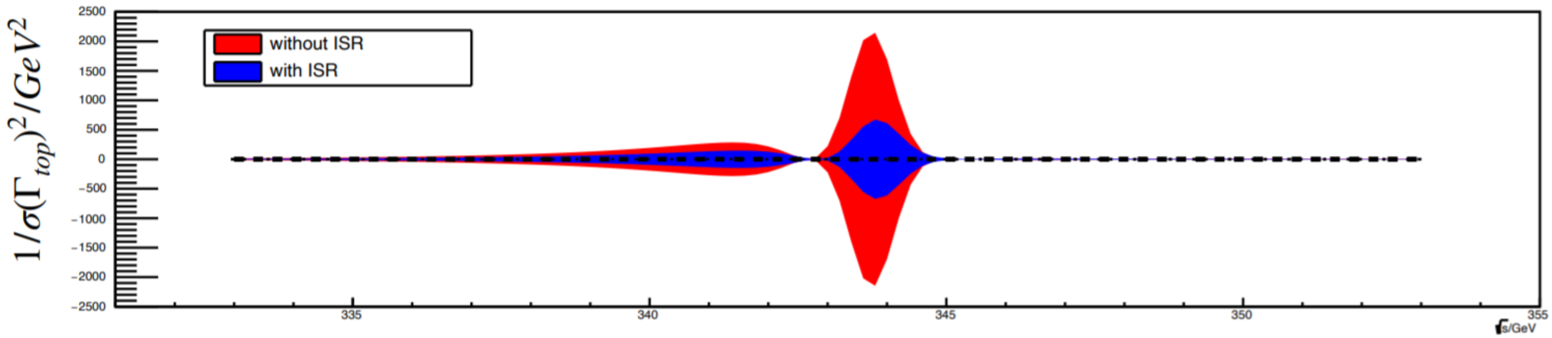
Generally, since the extrapolation is not so accurate, results are comparable.
For Higgs coupling, also similar performance could be expected.

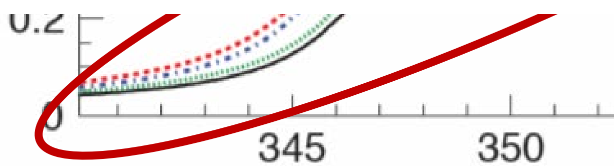
Fcc:

$\sqrt{s}$ (GeV)	240		365	
Luminosity (ab^{-1})	5		1.5	
$\delta(\sigma \mathrm{BR})/\sigma \mathrm{BR}$ (%)	HZ	$\nu\bar{\nu}$ H	HZ	$\nu\bar{\nu}$ H
$H \to$ any	±0.5		±0.9	
$H \to b\bar{b}$	±0.3	±3.1	±0.5	±0.9
$H \to c\bar{c}$	±2.2		±6.5	±10
$H \to gg$	±1.9		±3.5	±4.5
$H \to W^+W^-$	±1.2		±2.6	±3.0
$H \to ZZ$	±4.4		±12	±10
$H \to \tau\tau$	±0.9		±1.8	±8
$H \to \gamma\gamma$	±9.0		±18	±22
$H \to \mu^+\mu^-$	±19		±40	
$H \to$ invisible	< 0.3		< 0.6	

Fisher information







This solid black curve (much less feature) is what we are to measure, do we really expect simple statistical scaling on the sensitivities to these free parameters from this single curve?

- Effects of some parameters are corr… dependence on Yukawa coupling rather weak - precise external α_s helps

Frank Sir André H. Hoang
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Top Mass from Direct Reconstruction

Why bother given that we have the top threshold?

- For lepton collision is it much easier to understand the MC top mass interpretation problem and we can use the consistency with the threshold mass measurements as a benchmark to improve the intrinsic precision of MC generators and make them into much more reliable tools.

$$m_t^{\rm MC} = m_t^{\rm pole} + \Delta_m^{\rm pert} + \Delta_m^{\rm non-pert} + \Delta_m^{\rm MC}$$

pQCD contribution:

- Perturbative correction
- Depends on MC parton shower setup

Was analyzed in
Plätzer, Samitz, AHH '18

Non-perturbative contribution:

- Effects of hadronization model
- May depend on parton shower setup

Monte Carlo shift:

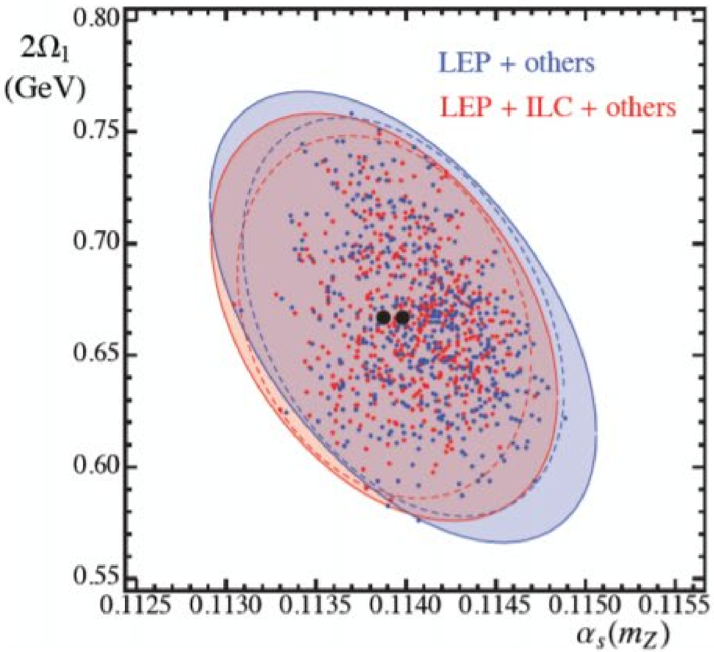
- Contribution arising from systematic MC uncertainties not related to top
- E.g. b-jet modeling, finite width, ...

André H. Hoang
University of Vienna

What can a future lepton collider help?

What would a precise measurement of event shapes at higher Q values contribute?

Exercise: Make up fictitious ILC data at 500 GeV, with assumed 1% statistical and 1% systematical uncertainties. Repeat fits.



- Limited impact concerning precision because high-energy uncertainties blown up in the evolution to Z mass
- Nevertheless important impact in lifting degeneracy between α_s and Ω_1.

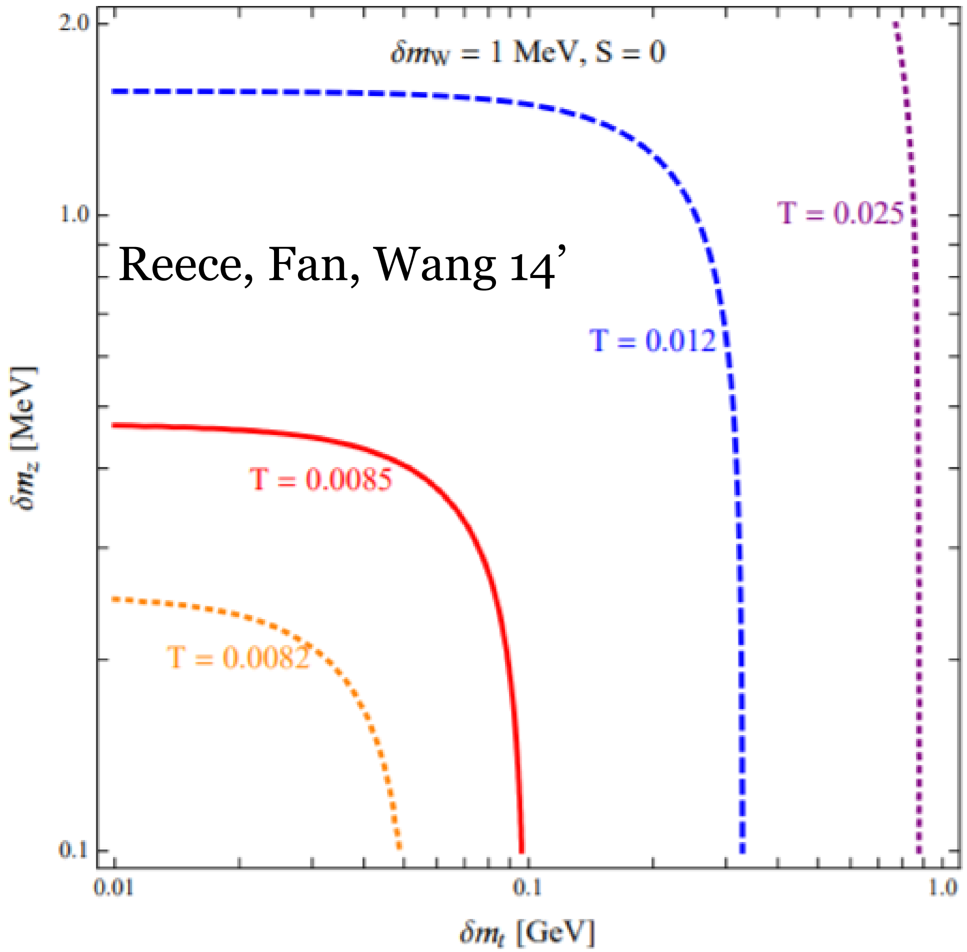
Taken from my talk at the FCCee α_S-Workshop 2015

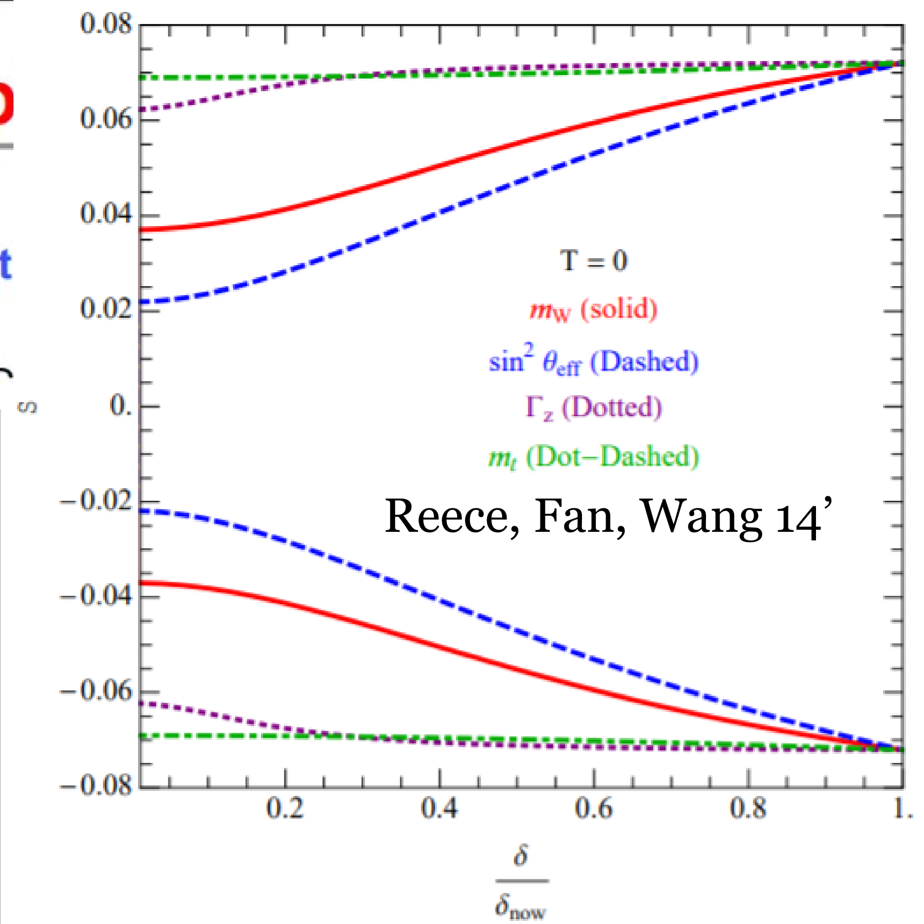
André H. Hoang
University of Vienna

What can a future lep

What would a precise measurement of event

Exercise: Make up fictitious ILC data at 500 G





Cee α_S-Workshop 2015

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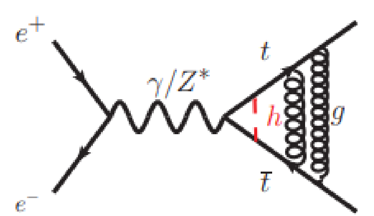
Physics outputs from ttbar run

ttbar run defined as O(0.x ab^{-1})(run1) for the threshold scan near 346 and O(1.x ab^{-1})(run2) at 360~365 where ever the maximum amount of ttbar events can be accumulated.

With the interests from flavor/muon anomalies, which indicates possible new physics near the weak scale, CEPC 360 GeV run will provide additional added value in probing new these new physics.

Physics ou

1. Precisi
   - Fo
   - Fo
2. Precision Higgs Physics (run2)
   - Leading order study with 360 statistics (even plus 346) well underway;
   - Higher order fit needs more theorists for consistency;
3. Precision Top mass measurement (run1)
   - Physics case: help with EWPO interpretation (under universal theory);
   - Non-Universal case, worthies a consistent theory study;
4. Get Yukawa coupling from the ttbar radiative corrections (run1+run2)
   - There is 4.2% projection from 1310.0563, but later on the theory uncertainties is estimated to be ~30% by 1506.06865
5. Get a bit constraint on the double Higgs production (run2+, 400 GeV needed)…
   - Far fetching, leave this in the back of our mind if our accelerator friends make some breal



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$$
\begin{aligned}
hGG(\pm\pm):&\ \ +1.035\,\mathrm{Re}\left[\frac{y_t}{y_t^{\rm SM}}\right]+0.053e^{i0.732\pi}\,\mathrm{Re}\left[\frac{y_b}{y_b^{\rm SM}}\right]\\
hG\tilde{G}(\pm\pm):&\ \ \pm1.575\,\mathrm{Im}\left[\frac{y_t}{y_t^{\rm SM}}\right]\pm0.055e^{i0.747\pi}\,\mathrm{Im}\left[\frac{y_b}{y_b^{\rm SM}}\right],
\end{aligned}
\tag{3.5}
$$

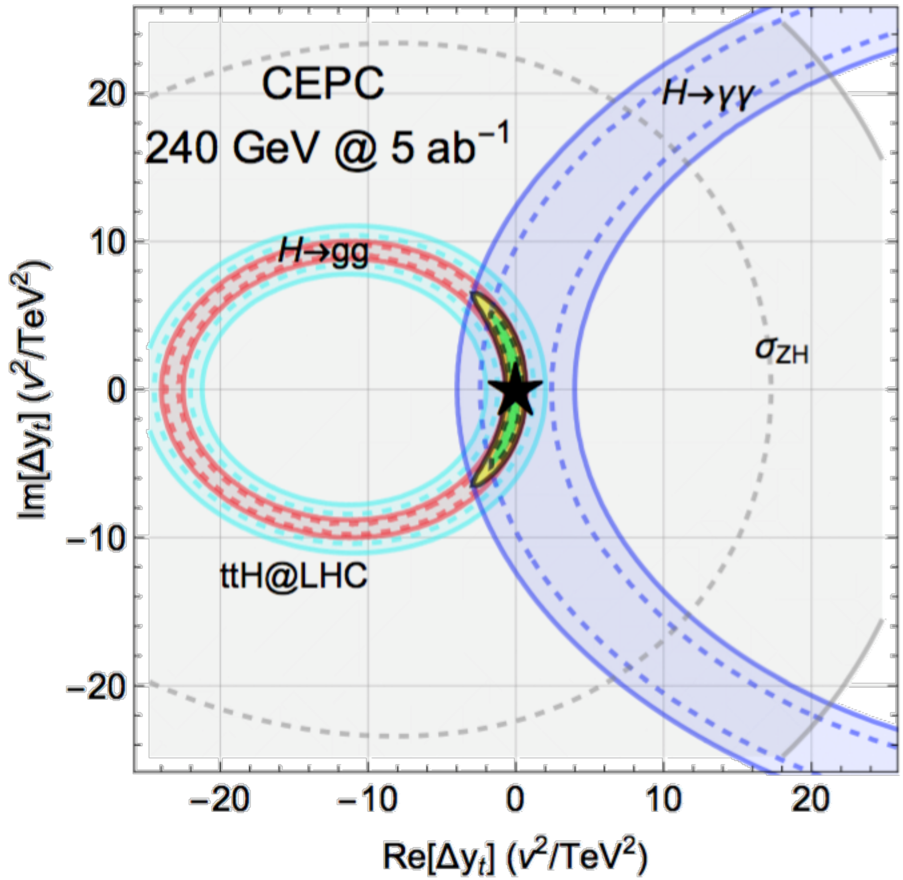
where θ_t and θ_b are the CP phases (weak phase) for the top Yukawa and bottom Yukawa, respectively, and the phase $\sim 0.7\pi$ is the phase of the bottom loop-function evaluate for an on-shell Higgs. The analytic expressions are listed in the Appendix. After squaring and averaging over helicity states, we can obtain the parametric dependence of the $H\to gg$ partial width (which is directly related to measurements) to be,

$$
\begin{aligned}
\frac{\Gamma(h\to gg)}{\Gamma(h\to gg)^{\rm SM}} =&\ \ 1.070\left|\frac{y_t}{y_t^{\rm SM}}\right|^2-0.073\left|\frac{y_t}{y_t^{\rm SM}}\frac{y_b}{y_b^{\rm SM}}\right|\cos\theta_t^{\rm CP}\cos\theta_b^{\rm CP}+0.03\left|\frac{y_b}{y_b^{\rm SM}}\right|^2\\
&+1.410\left|\frac{y_t}{y_t^{\rm SM}}\right|^2\sin^2\theta_t^{\rm CP}-0.122\left|\frac{y_t}{y_t^{\rm SM}}\frac{y_b}{y_b^{\rm SM}}\right|\sin\theta_t^{\rm CP}\sin\theta_b^{\rm CP}\\
&+O(0.0001;\left|\frac{y_t}{y_t^{\rm SM}}\right|,\left|\frac{y_b}{y_b^{\rm SM}}\right|),
\end{aligned}
\tag{3.6}
$$

where $\theta_t^{\rm CP}$ is the CP angle in the top Yukawa, relating to the Yukawa modifications and thus Wilson coefficients of operator $\mathcal{O}_{tH}$ as shown in Eq. 3.3,

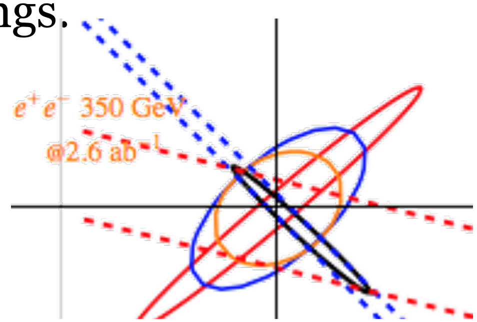
$$
\mathrm{Re}\left[\frac{\Delta y_t}{y_t^{\rm SM}}\right]=\left|\frac{y_t}{y_t^{\rm SM}}\right|\cos\theta_t^{CP}\quad\text{and}\quad\mathrm{Im}\left[\frac{\Delta y_t}{y_t^{\rm SM}}\right]=\left|\frac{y_t}{y_t^{\rm SM}}\right|\sin\theta_t^{CP},
\tag{3.7}
$$

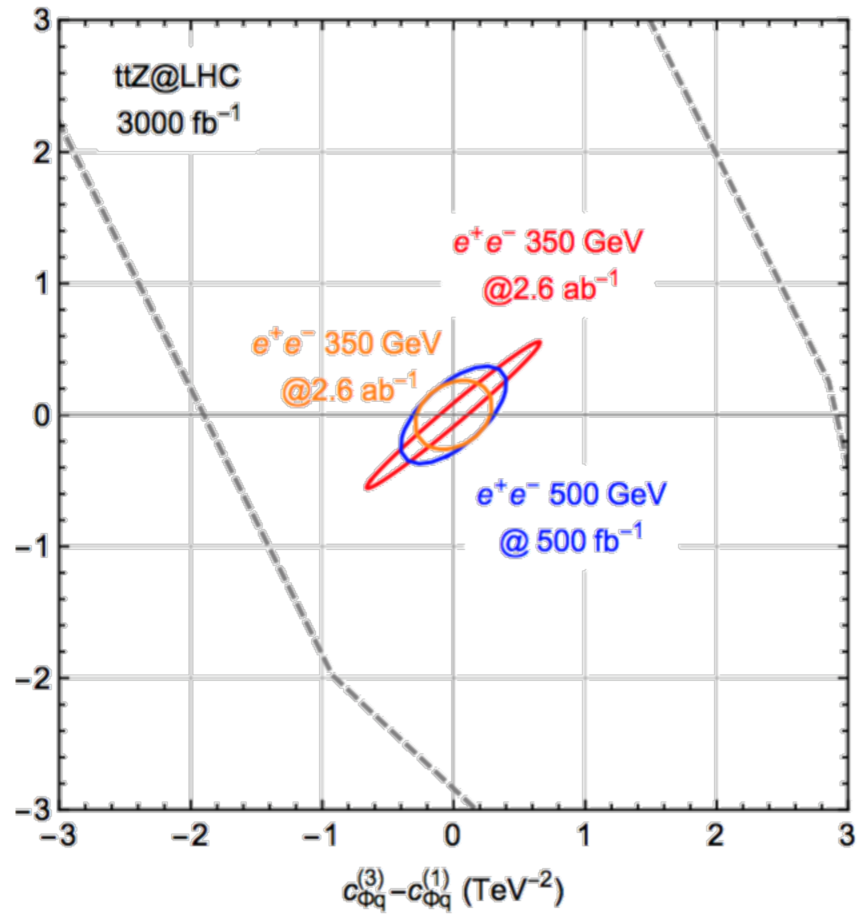
Loop-level constraints from precision Zh measurements



Top quark and Higgs EFT DHq-DHq(3), DHt

At or above $t\bar{t}$ threshold at lepton colliders, one immediately again huge sensitivities to the top gauge couplings.





Top quark loop can also induce some operator mixing and enter the Z-pole precisions (Altarelli, Barbieri, Caravaglios, 93') ϵ_1, ϵ_b

Top quark and Higgs EFT DHq-DHq(3), DHt

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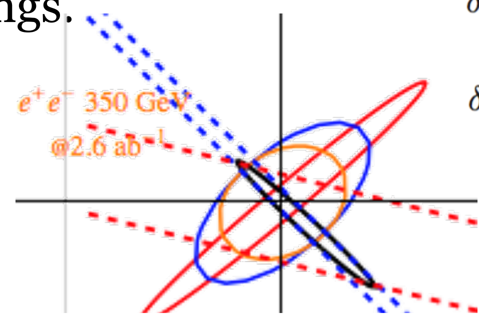

$$\delta\varepsilon_1 = \frac{3 m_t^2 G_{\rm F}}{2\sqrt{2}\pi^2}\,\mathrm{Re}\left[C_{\phi q}^{(3,33)} - C_{\phi q}^{(1,33)} + C_{\phi u}^{33} + \mathcal{O}\left(\frac{v^2}{\Lambda^2}\right)\right]\left(\frac{v^2}{\Lambda^2}\right)\log\left(\frac{\Lambda^2}{m_t^2}\right)$$

$$\delta\varepsilon_b = -\frac{m_t^2 G_{\rm F}}{2\sqrt{2}\pi^2}\,\mathrm{Re}\left[C_{\phi q}^{(3,33)} - C_{\phi q}^{(1,33)} + \frac{1}{4}C_{\phi u}^{33}\right]\left(\frac{v^2}{\Lambda^2}\right)\log\left(\frac{\Lambda^2}{m_t^2}\right).$$

Top quark loop can also induce some operator mixing and enter the Z-pole precisions (Altarelli, Barbieri, Caravaglios, 93') ϵ_1, ϵ_b

However, these are essentially R_b and A_{FB} . To use them, one have to assume extreme cases of DHq+DHq(3) and DHb both are zero at the same time. Only known example is custodial Zbb Agashe, Contino, De Rold, Pomarol, 06'.

In addition, there are some controversies about finite pieces in these relations.