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$\Xi_c \to \Xi$ Form Factors and $\Xi_c \to \Xi l^+ \nu_l$ Decay Rates From Lattice QCD

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* Based on arXiv:hep-lat/2103.07064;
* In collaboration with J.Hua, F.Huang, R.Li, Y.Li, C.D.Lv, P.Sun, W.Sun, W.Wang and Y.B.Yang.

May 2, 2021

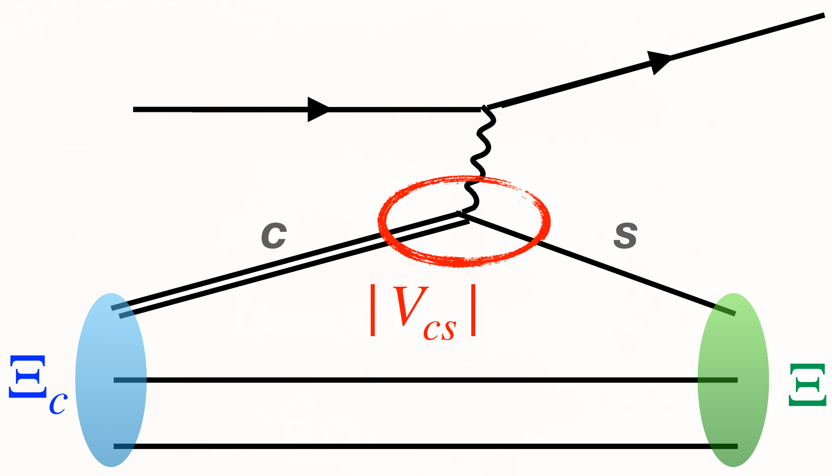
@第三届重味物理与量子色动力学研讨会, 天津

OUTLINE

- **Motivation**
- **Semi-leptonic Ξ_c decays and helicity-based form factors**
- **Lattice calculations of $\Xi_c \to \Xi$ form factors**
- **Differential decay width and branching fractions from lattice results**
- **Determination of $|V_{cs}|$**
- **Summary**

Motivation

- Determine **CKM matrix elements** $|V_{cs}|$;
- Provide a **stringent test and first-principle verification** for theoretical models;

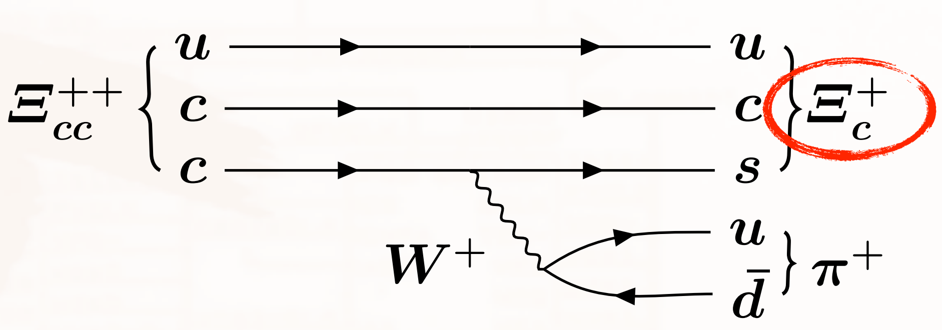


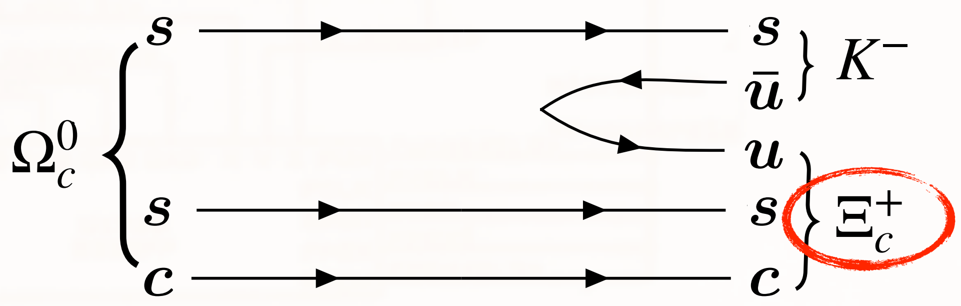
- Important for the experimental researches of heavy baryons:
  - Studies of doubly-charmed baryon Ξ_{cc}^{++} decay

    R. Aaij et al. [LHCb], PRL121, 162002 (2018)
  - Precision measurement of the lifetime of Ξ_b^0

    R. Aaij et al. [LHCb], PRL113, 032001 (2014)
  - Discovery of new exotic hadron candidates Ω_c

    R. Aaij et al. [LHCb], PRL118, 182001 (2017)
  - ......

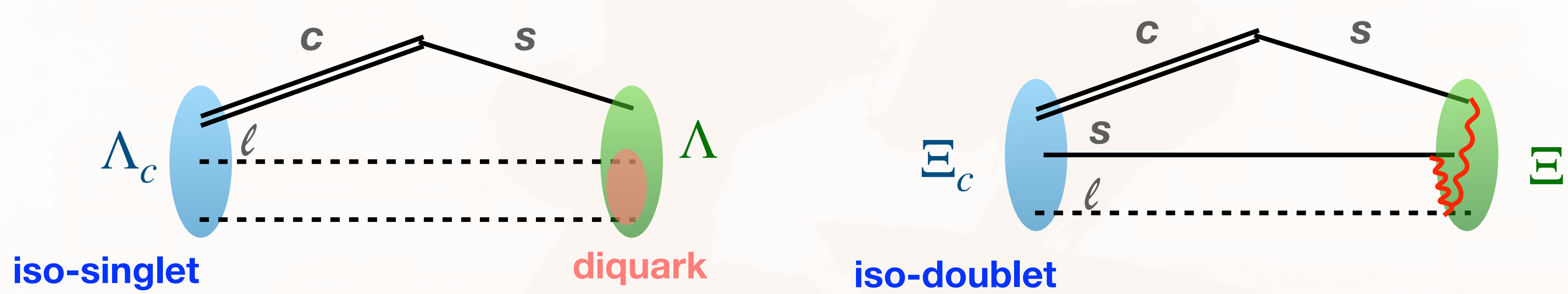




Motivation

- Ξ_c contains more **versatile decay modes**

  - $\Xi_c \to \Xi$ contain different QCD dynamics with $\Lambda_c \to \Lambda$;



  - A **different pattern** between inclusive and exclusive decays of Λ_c and D:

$$\frac{\mathscr{B}(\Lambda_c^+ \to X e^+ \nu_e) = (3.95 \pm 0.34 \pm 0.09)\,\%}{\mathscr{B}(\Lambda_c^+ \to \Lambda e^+ \nu_e) = (3.63 \pm 0.38 \pm 0.20)\,\%} \sim 1 \qquad \frac{\mathscr{B}(D^0 \to X e^+ \nu_e) = (6.49 \pm 0.11)\,\%}{\mathscr{B}(D^0 \to K^- e^+ \nu_e) = (3.542 \pm 0.035)\,\%} \sim 2$$

M. Ablikim et al. [BESIII], PRL121, 251801 (2018)

Motivation

- **Experimental measurements of the branching fractions of semileptonic decays $\Xi_c^0 \to \Xi^- l^+ \nu_l$**

Y. B. Li et al. [Belle], arXiv:2103.06496 [hep-ex].

To optimize the signal selection criteria and calculate the signal reconstruction efficiency, we use Monte Carlo (MC) simulated events. The $e^+e^- \to c\bar{c}$ process is simulated with PYTHIA [55], while the signal events of Ξ_c^0 semileptonic decays are generated using the form factor from Lattice QCD calculation [56], and $\Xi_c^0 \to \Xi^-\pi^+$ decays are generated with EvtGen [57]. The MC events are processed with a detector simulation based

Important input for the signal events generation in experimental simulation!

Motivation

✓ Experimental

Belle $\mathcal{B}\left(\Xi_c^0 \to \Xi^- e^+ \nu_e\right) = (1.72 \pm 0.10 \pm 0.12 \pm 0.50)\,\%$

$\mathcal{B}\left(\Xi_c^0 \to \Xi^- \mu^+ \nu_\mu\right) = (1.71 \pm 0.17 \pm 0.13 \pm 0.50)\,\%$

Y. B. Li et al. [Belle], arXiv:2103.06496 [hep-ex].

ALICE $\mathcal{B}\left(\Xi_c^0 \to \Xi^- e^+ \nu_e\right) = (2.43 \pm 0.25 \pm 0.35 \pm 0.72)\,\%$

J. Zhu on behalf of the ALICE collaboration, PoS ICHEP2020 (2021) 524.

✓ Theoretical

QCD SR $\mathcal{B}\left(\Xi_c^0 \to \Xi^- e^+ \nu_e\right) = (3.4 \pm 1.7)\,\%$ — *Z. X. Zhao, arXiv:2103.09436 [hep-ph].*

LF QM $\mathcal{B}\left(\Xi_c^0 \to \Xi^- e^+ \nu_e\right) = (3.49 \pm 0.95)\,\%$ — *C. Q. Geng et al, arXiv:2012.04147 [hep-ph].*

LCSR $\mathcal{B}\left(\Xi_c^0 \to \Xi^- e^+ \nu_e\right) = (2.4^{+0.9}_{-1.0})\,\%$ — *Y. L. Liu et al, J. Phys. G 37, 115010 (2010).*

……

Lattice ?

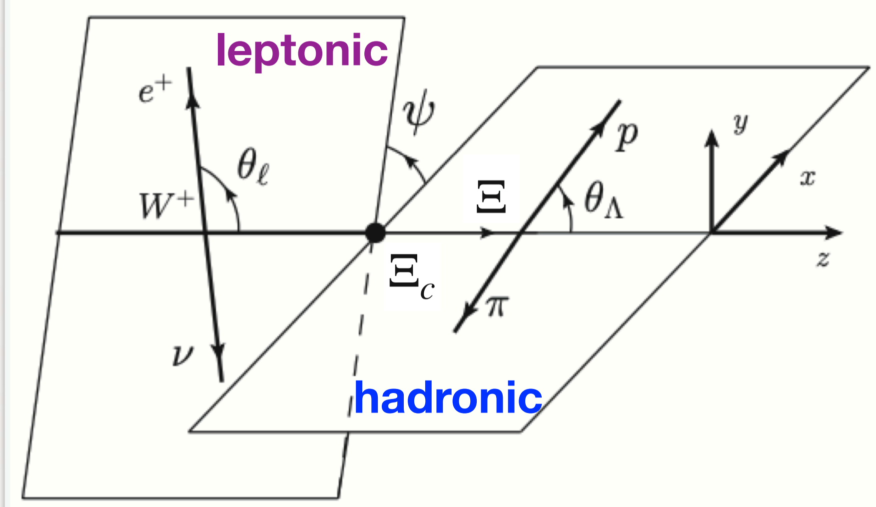
Semileptonic Ξ_c decays:

- **Helicity amplitude**

Hadronic part — Leptonic part

$$\mathscr{M}(\Xi_c \to \Xi l^+ \nu_l) = \frac{G_F}{\sqrt{2}} V_{cs} \langle \Xi | \bar{s}\gamma^\mu (1-\gamma_5) c | \Xi_c \rangle \langle \bar{l}\nu_l | \bar{\nu}\gamma^\nu (1-\gamma_5) l | 0 \rangle g_{\mu\nu}$$

$$= \frac{G_F}{\sqrt{2}} V_{cs} \left(H^\mu \epsilon^*_\mu(t) \times L^\nu \epsilon_\nu(t) - \sum_\lambda H^\mu \epsilon^*_\mu(\lambda) \times L^\nu \epsilon_\nu(\lambda) \right)$$



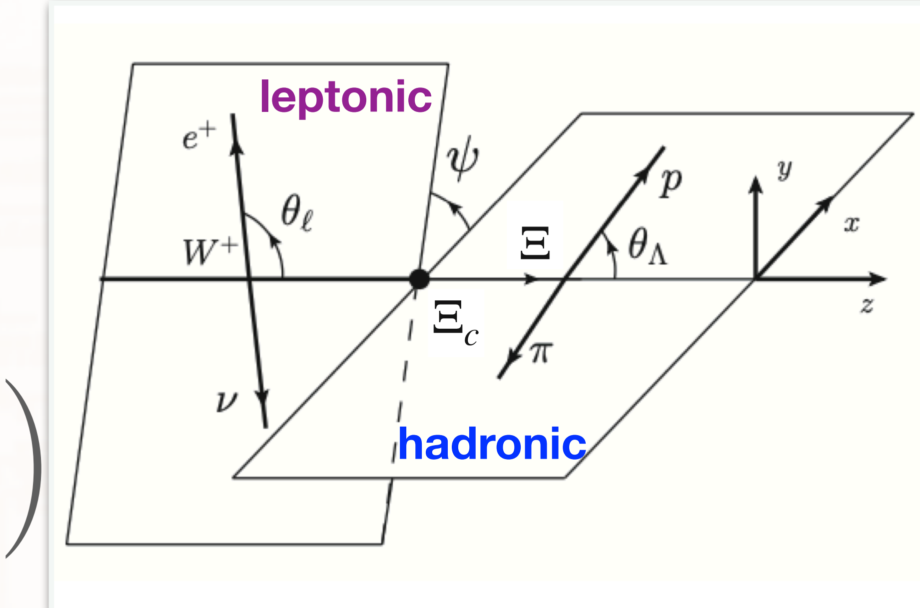
Semileptonic Ξ_c decays:

- **Helicity amplitude**

Hadronic part **Leptonic part**

$$\mathcal{M}(\Xi_c \to \Xi l^+ \nu_l) = \frac{G_F}{\sqrt{2}} V_{cs} \langle \Xi | \bar{s}\gamma^\mu(1-\gamma_5) c | \Xi_c\rangle \langle \bar{l}\nu_l | \bar{\nu}\gamma^\nu(1-\gamma_5) l | 0\rangle g_{\mu\nu}$$

$$= \frac{G_F}{\sqrt{2}} V_{cs}\left(H^\mu \epsilon^*_\mu(t)\times L^\nu \epsilon_\nu(t) - \sum_\lambda H^\mu\epsilon^*_\mu(\lambda)\times L^\nu\epsilon_\nu(\lambda)\right)$$



- **Hadronic matrix element: Helicity-base form factor parametrization**

$$\left\langle \Xi(P_\Xi)\,|V^\mu|\,\Xi_c(P_{\Xi_c})\right\rangle = \bar{u}(P_\Xi,S_2)\Big[(m_{\Xi_c}-m_\Xi)\frac{q^\mu}{q^2} f_0(q^2) + \frac{m_{\Xi_c}+m_\Xi}{s_+}\left(P^\mu_{\Xi_c}+P^\mu_\Xi - \frac{q^\mu}{q^2}(m^2_{\Xi_c}-m^2_\Xi)\right) f_+(q^2) + \left(\gamma^\mu - \frac{2m_\Xi}{s_+}P_1^\mu - \frac{2m_{\Xi_c}}{s_+}P^\mu_\Xi\right) f_\perp(q^2)\Big]\, u(P_{\Xi_c}),$$

vector

$$\left\langle \Xi(P_\Xi)\,|A^\mu|\,\Xi_c(P_{\Xi_c})\right\rangle = -\bar{u}(P_\Xi,S_2)\gamma^5\Big[(m_{\Xi_c}+m_\Xi)\frac{q^\mu}{q^2} g_0(q^2) + \frac{m_{\Xi_c}-m_\Xi}{s_-}\left(P^\mu_{\Xi_c}+P^\mu_\Xi - \frac{q^\mu}{q^2}(m^2_{\Xi_c}-m^2_\Xi)\right) g_+(q^2) + \left(\gamma^\mu + \frac{2m_\Xi}{s_-}P^\mu_{\Xi_c} - \frac{2m_{\Xi_c}}{s_-}P^\mu_\Xi\right) g_\perp(q^2)\Big]\, u(P_{\Xi_c})$$

axial-vector

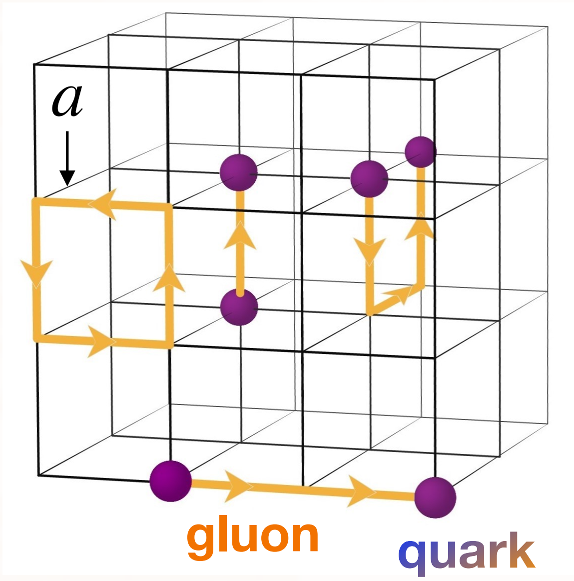
Helicity based form factors

- **Helicity based form factors:** $f_\perp, f_0, f_+, g_\perp, g_0, g_+$
  - Non-perturbative
  - QCD sum rules, light-cone sum rules, light-front quark model, …

    => model dependent

- **Lattice QCD: a non-perturbative theory from the first principle of QCD**
  - Discretization, and wick rotation to Euclidean spacetime;
  - Quark fields on lattice sites, gauge links connecting the sites;
  - Pion mass to benchmark the light quark mass;
  - Able to calculate the non-perturbative observables:

$$\langle O \rangle = \frac{1}{Z} \int D\psi D\bar{\psi} D A e^{-S_E} O$$



Credit: M. Savage@NNPSS2015

Lattice setup

- This work is based on 2+1 flavor ensembles generated with tree level tadpole improved clover fermion action and tadpole improved Symanzik gauge action;

- Basic informations of two ensembles used in this calculation;

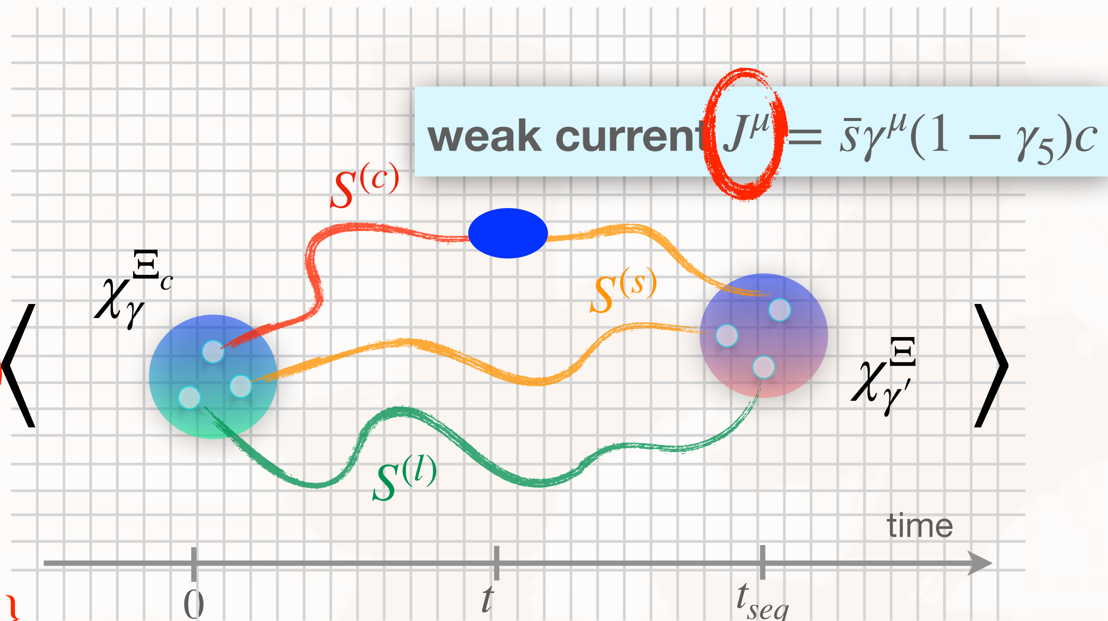
	$\beta = \frac{10}{g^2}$	$L^3 \times T$	a	$c_{\rm sw}$	κ_l	m_π	κ_s	m_{η_s}
s108	6.20	$24^3 \times 72$	0.108	1.161	-0.2770	290	0.1330	640
s080	6.41	$32^3 \times 96$	0.080	1.141	-0.2295	300	0.1318	650

- Determining the charm quark mass by requiring the J/ψ mass to its physical value $m_{J/\psi} = 3.96900(6)$ GeV within 0.3% accuracy: $m_c^{\rm s108} a = 0.485$, $m_c^{\rm s080} a = 0.235$.

Extract bare $\Xi_c \to \Xi$ form factors

- **Correlation functions on lattice**

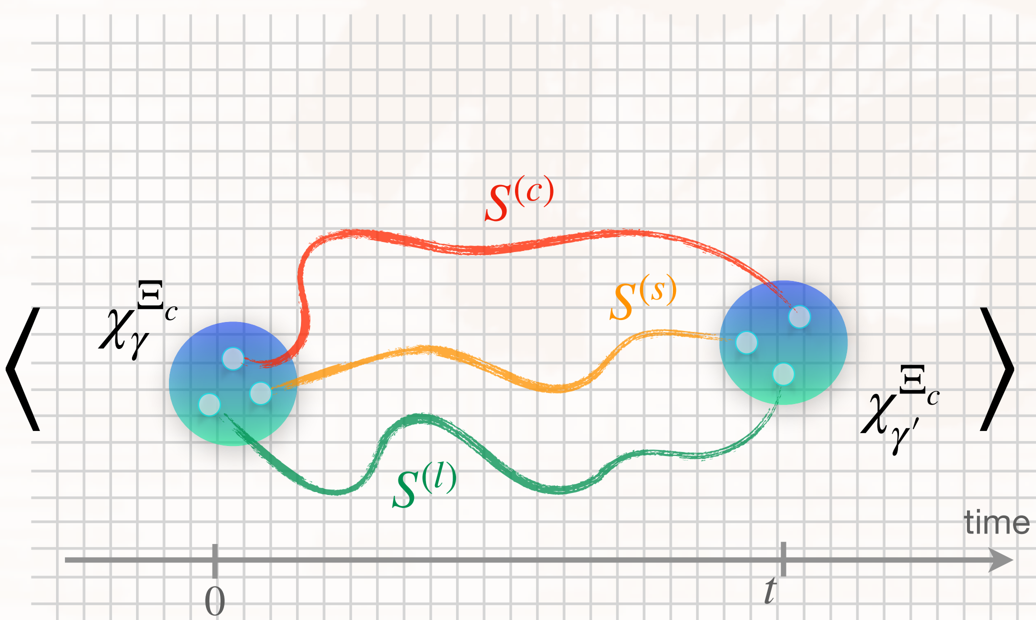
$$C_3(q^2, t, t_{seq}) \sim T^{\gamma\gamma'} \langle \quad \rangle$$



$T = \{I, \gamma_5, \gamma^t, \gamma^x, \dots\}$

$\mu = \{t, x, y, z\}$

$$C_2^{\Xi_c}(t) \sim I^{\gamma\gamma'} \langle \quad \rangle$$



- **Ratios for different projections and currents:**

$$R_{V/A}(T, \mu) = \sqrt{\frac{C_3^{V/A}\left(q^2, t, t_{\rm seq}\right) C_3^{V/A}\left(q^2, t_{\rm seq} - t, t_{\rm seq}\right)}{C_2^{B_1}\left(t_{\rm seq}\right) C_2^{B_2}\left(t_{\rm seq}\right)}}$$

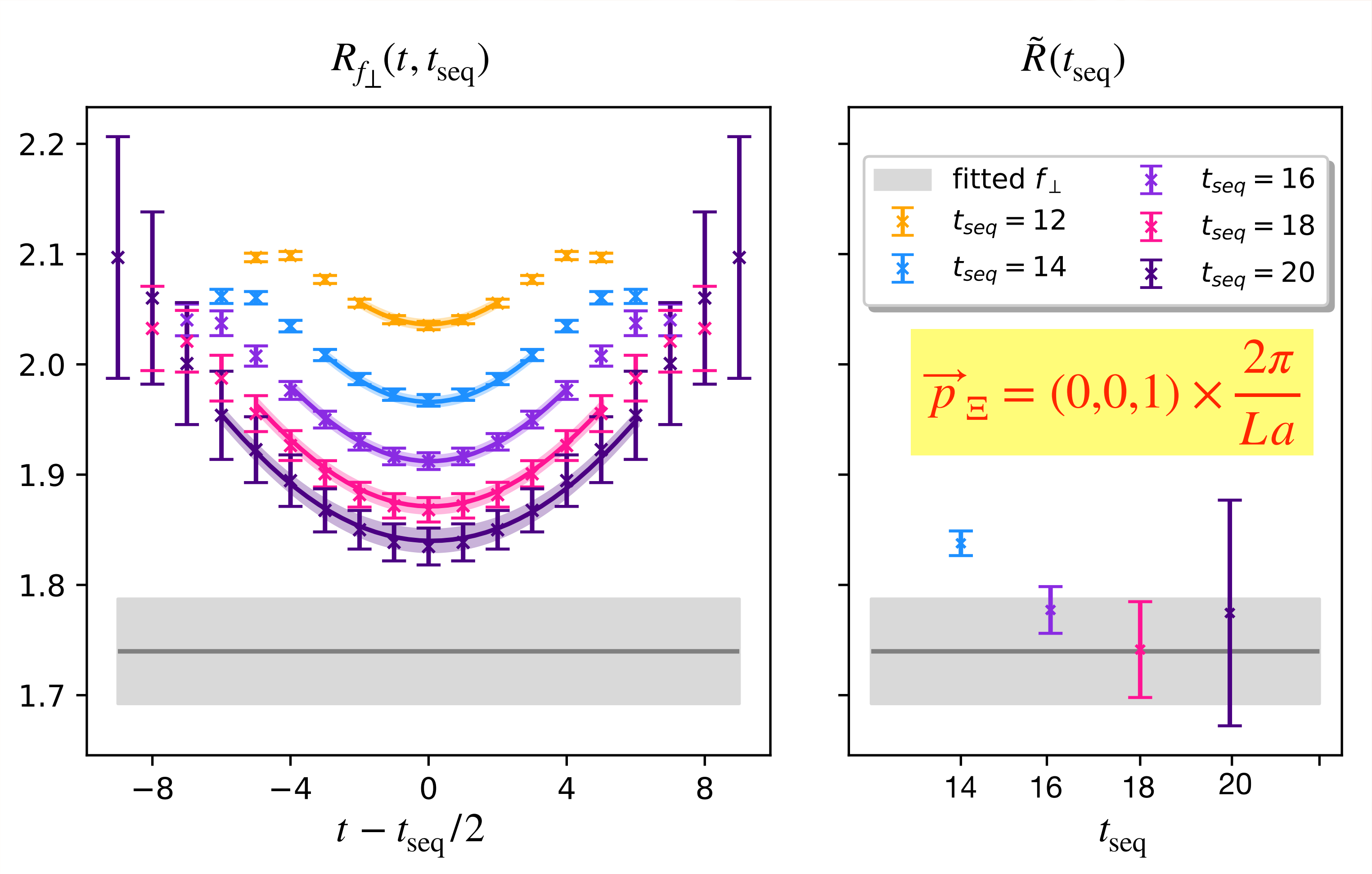
- **Ratios for the six form factors can be constructed by different combinations of $R_{V/A}(T, \mu)$.**

Extract bare $\Xi_c \to \Xi$ form factors

- **Ratios for the six form factors can be constructed by different combinations of $R_{V/A}(T,\mu)$:**

contributions from excited states

$$R_{f_\perp} \equiv \frac{R_V(\gamma_5\gamma^x,\gamma^y)}{4m_{\Xi_c}N_z\hat{p}} = f_\perp\left(\frac{\left(1+c_1e^{-\Delta E_1 t}+c_2e^{-\Delta E_2\left(t_{\rm seq}-t\right)}\right)\left(1+c_1e^{-\Delta E_1\left(t_{\rm seq}-t\right)}+c_2e^{-\Delta E_2 t}\right)}{\left(1+d_1e^{-\Delta E_1 t_{\rm seq}}\right)\left(1+d_2e^{-\Delta E_2 t_{\rm seq}}\right)}\right)^{1/2}$$



- **The differential summed ratio for validity check:**

$$\tilde{R}(t_{\rm seq}) \equiv \frac{SR(t_{\rm seq}) - SR(t_{\rm seq}-\Delta t)}{\Delta t}$$

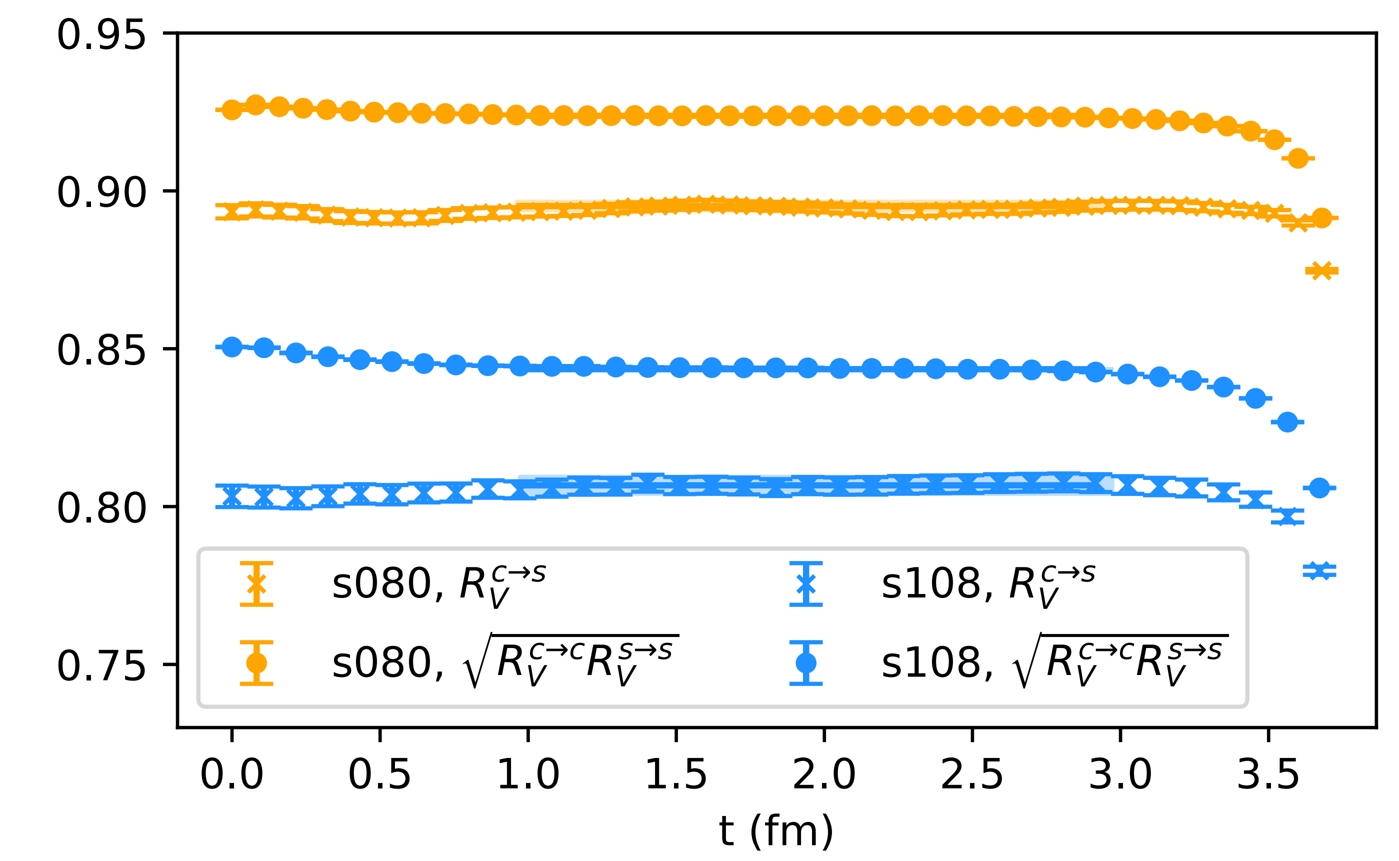
$$SR(t_{\rm seq}) \equiv \sum_{t_c<t<t_{\rm seq}-t_c} R_F(t, t_{\rm seq})$$

Renormalization factors for $c \to s$ current

- Use the ratio of **conserve current** and vector current to determine the renormalization factor Z_V for vector current:

$$\bar{q}_1 V_\mu^{\text{cons.}} q_2 = \bar{q}_1(x) \frac{1-\gamma_\mu}{2} U_{\hat{\mu}}(x) q_2(x+\hat{\mu}) - \bar{q}_1(x+\hat{\mu}) \frac{1+\gamma_\mu}{2} U_{\hat{\mu}}^\dagger(x) q_2(x)$$

$$R_V^{q_1\to q_2}(t) = \frac{\left\langle M_1(T/2) \sum_{\vec{x}} V_{\text{cons.}}^{q_1\to q_2}(\vec{x},t) M_2(0) \right\rangle}{\left\langle M_1(T/2) \sum_{\vec{x}} V^{q_1\to q_2}(\vec{x},t) M_2(0) \right\rangle} = Z_V^{q_1\to q_2} + \mathcal{O}\left(e^{-T/4\Delta E}\right)$$



S. Hashimoto et al., PRD61, 014502 (1999); A. X. El-Khadra et.al., PRD64, 014502 (2001)

- **A "mostly nonperturbative" method:**

$$Z_V^{c\to s} = \sqrt{Z_V^{c\to c} Z_V^{s\to s}}$$

- **Differences arise from discretization effects, and will contribute to the systematic errors.**

Renormalization factors for $c \to s$ current

- **Use the off-shell quark matrix elements:**

$$\frac{Z_q}{Z_A}\mathrm{Tr}[\langle q|A^\mu|q\rangle\gamma_\mu\gamma_5] = \frac{Z_q}{Z_V}\mathrm{Tr}[\langle q|V^\mu|q\rangle\gamma_\mu]$$

- **And the "most nonperturbative" method:**

RI/MOM scheme

$$Z_A^{c\to s} \equiv Z_V^{c\to s}\sqrt{\frac{\mathrm{Tr}[\langle c|V^\mu|c\rangle\gamma_\mu]}{\mathrm{Tr}[\langle c|A^\mu|c\rangle\gamma_\mu\gamma_5]}\frac{\mathrm{Tr}[\langle s|V^\mu|s\rangle\gamma_\mu]}{\mathrm{Tr}[\langle s|A^\mu|s\rangle\gamma_\mu\gamma_5]}}$$

G. Martinelli et al., Nucl. Phys. B445, 81 (1995).

- **With a^2p^2 extrapolation using three values of p^2 in the range of $a^2p^2 = (4,6,8)$, we obtained $Z_A/Z_V = 1.010231(69)$ and $1.020296(68)$ on s108 and s080, respectively.**

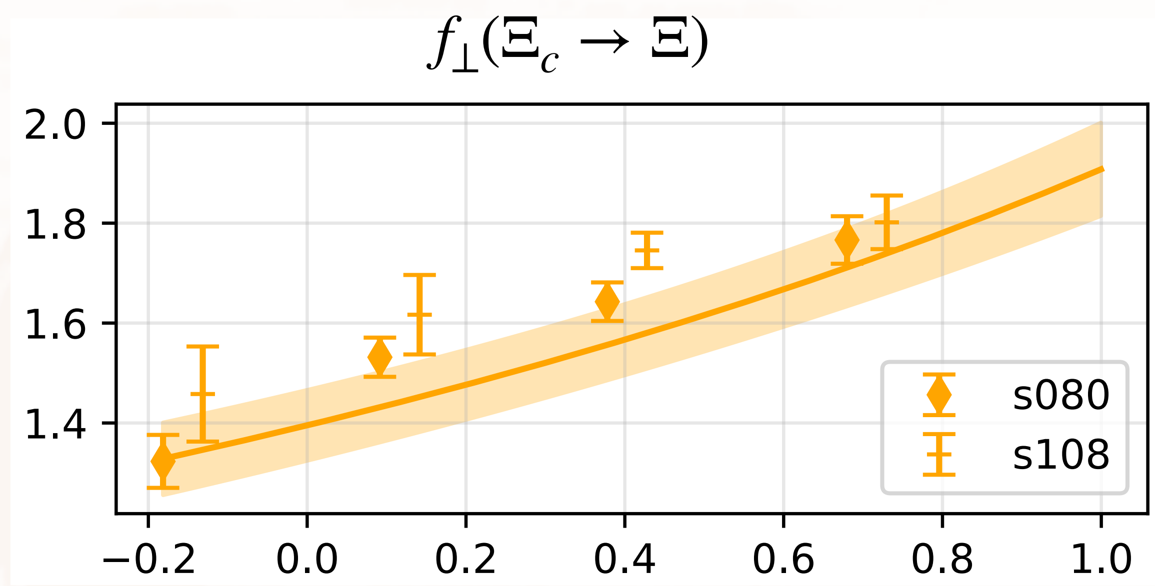
q^2 distribution for $\Xi_c \to \Xi$ form factors

- Extrapolate to the **continuum limit** (shaded regions);

- z-expansion parametrization of form factors to obtain the q^2-distribution:

$$f\left(q^2\right) = \frac{1}{1 - q^2 \Big/ (m_{\rm pole}^f)^2} \sum_{n=0}^{n_{\max}} \left(c_n^f + d_n^f a^2\right) \left[z\left(q^2\right)\right]^n$$

  - Use D_s **meson pole mass** for $m_{\rm pole}^{f_\perp}$, ...
  - Consider the **discretization effects** by estimating the d_n^f terms.



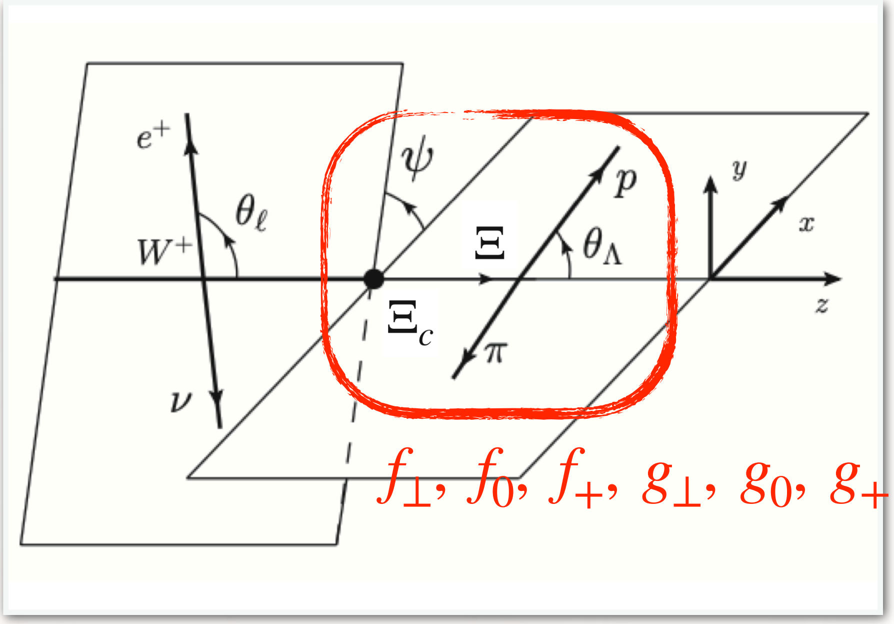
Fit results for the z-expansion parameters

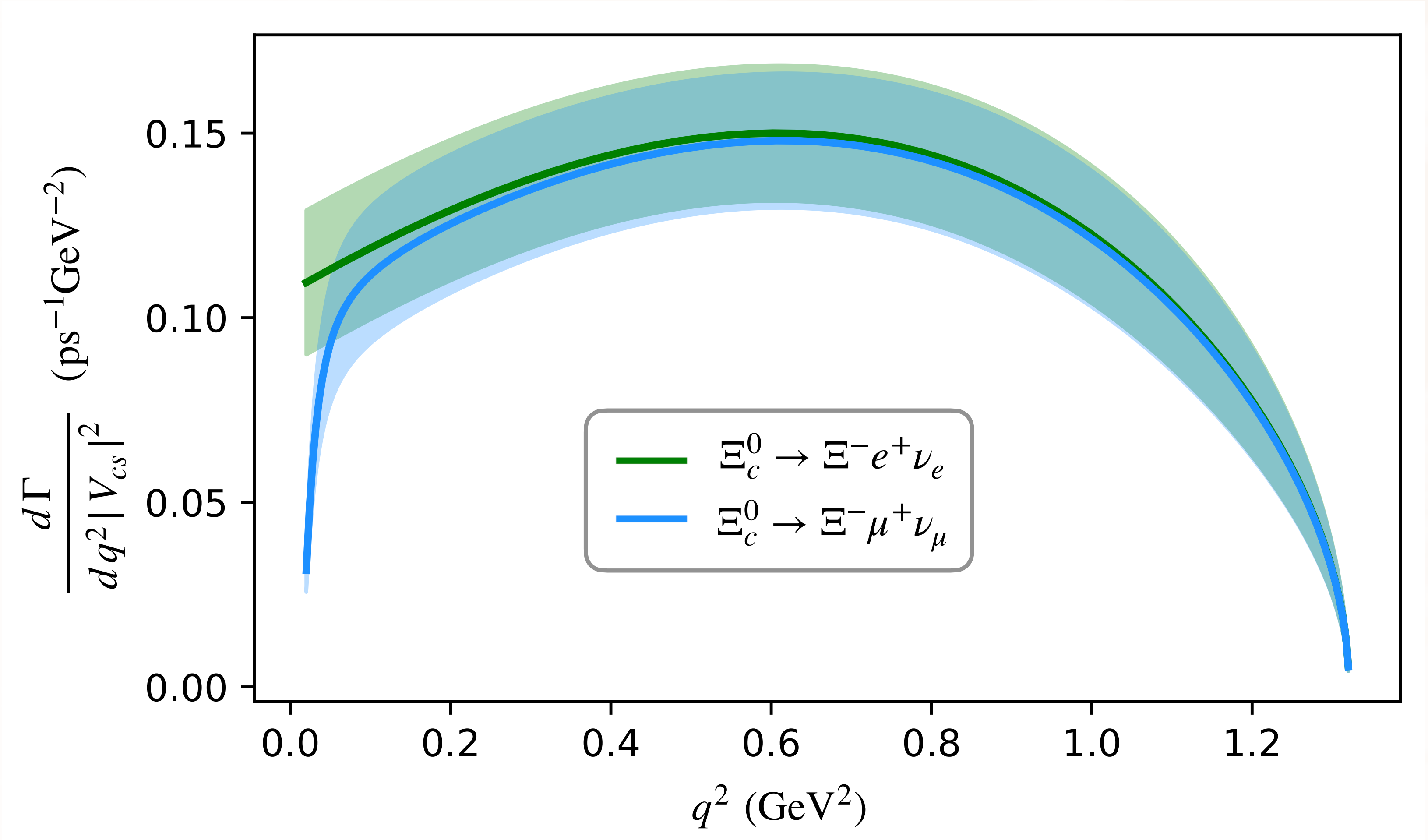
	c_0	c_1	c_2
$f_\perp$	1.51±0.09	-1.88±1.21	1.71±0.49
f_0	0.64±0.09	-1.83±1.22	0.56±0.51
f_+	0.77±0.07	-4.09±1.18	0.35±0.49
$g_\perp$	0.56±0.07	-0.35±1.26	0.15±0.29
g_0	0.63±0.07	-1.37±1.36	0.15±0.29
g_+	0.56±0.08	0.00±1.38	0.14±0.29

Differential decay widths

- **The differential decay widths of $\Xi_c^0 \to \Xi^- l^+ \nu_l$:**

$$\frac{d\Gamma}{dq^2 |V_{cs}|^2} \quad (\mathrm{ps^{-1}GeV^{-2}})$$





- Results for $\Xi_c^+ \to \Xi^0 l^+ \nu_l$ are similar.

Branching fractions and differential decay widths

- **Predicted decay branching fractions:**

$$\mathscr{B}\left(\Xi_c^0 \to \Xi^- e^+ \nu_e\right) = 2.38(0.30)(0.32)(0.07)\,\%$$
$$\mathscr{B}\left(\Xi_c^0 \to \Xi^- \mu^+ \nu_\mu\right) = 2.29(0.29)(0.30)(0.06)\,\%$$
$$\mathscr{B}\left(\Xi_c^+ \to \Xi^0 e^+ \nu_e\right) = 7.18(0.90)(0.96)(0.20)\,\%$$
$$\mathscr{B}\left(\Xi_c^+ \to \Xi^0 \mu^+ \nu_\mu\right) = 6.91(0.87)(0.91)(0.19)\,\%$$

- Statistical errors
- Systematic errors from continuum extrapolation
- Systematic errors from renormalization

$$(2.38 \pm 0.44)\,\%$$

- **Compare with PDG, experiment and theory:**

PDG	$\mathscr{B}\left(\Xi_c^0 \to \Xi^- e^+ \nu_e\right) = (1.8 \pm 1.2)\,\%$	✓
Belle	$\mathscr{B}\left(\Xi_c^0 \to \Xi^- e^+ \nu_e\right) = (1.72 \pm 0.10 \pm 0.12 \pm 0.50)\,\%$	✓
ALICE	$\mathscr{B}\left(\Xi_c^0 \to \Xi^- e^+ \nu_e\right) = (2.43 \pm 0.25 \pm 0.35 \pm 0.72)\,\%$	✓
QCD SR	$\mathscr{B}\left(\Xi_c^0 \to \Xi^- e^+ \nu_e\right) = (3.4 \pm 1.7)\,\%$	✓
LF QM	$\mathscr{B}\left(\Xi_c^0 \to \Xi^- e^+ \nu_e\right) = (3.49 \pm 0.95)\,\%$	✓
LCSR	$\mathscr{B}\left(\Xi_c^0 \to \Xi^- e^+ \nu_e\right) = (2.4^{+0.9}_{-1.0})\,\%$	✓

Our results fit the experimental measurements and theoretical predictions well (within 1-σ) !

Determination of $|V_{cs}|$

- From Belle measurements:

Y. B. Li et al. [Belle], arXiv:2103.06496 [hep-ex].

$$\mathscr{B}\left(\Xi_c^0 \to \Xi^- e^+ \nu_e\right) = (1.72 \pm 0.10 \pm 0.12 \pm 0.50)\,\%$$

$$\mathscr{B}\left(\Xi_c^0 \to \Xi^- \mu^+ \nu_\mu\right) = (1.71 \pm 0.17 \pm 0.13 \pm 0.50)\,\%$$

From the uncertainty of $\Xi_c^0 \to \Xi^- \pi^+$

$$|V_{cs}| = 0.834 \pm (0.051)_{\text{stat.}} \pm (0.56)_{\text{syst.}} \pm (0.127)_{\text{exp.}}$$

Theo. error ~ 8.9%   Exp. error ~ 15.2%

- From ALICE measurements:

J. Zhu on behalf of the ALICE collaboration, PoS ICHEP2020 (2021) 524.

$$\mathscr{B}\left(\Xi_c^0 \to \Xi^- e^+ \nu_e\right) = (2.43 \pm 0.25 \pm 0.35 \pm 0.72)\,\%$$

$$|V_{cs}| = 0.983 \pm (0.060)_{\text{stat.}} \pm (0.065)_{\text{syst.}} \pm (0.167)_{\text{exp.}}$$

Exp. error ~ 17.0%

- Compare with PDG result:

$$|V_{cs}| = 0.97320 \pm 0.00011$$

Theoretical uncertainties:

- total ~ 8.9%
- statistical ~ 6.1%
- systematic from extrapolation ~ 6.5%
- systematic from renormalization ~ 1.5%

Experimental uncertainties:

- Belle ~ 15.2%
- ALICE ~ 17.0%

SUMMARY

- The first lattice QCD calculation of $\Xi_c \to \Xi$ form factors: predicted the differential decay widths, branching fractions, and extracted the CKM matrix element $|V_{cs}|$;
- The lattice calculation was done on two lattice spacings and extrapolated to the continuum, and finite renormalization was done by conserved vector current;
- A more precise experimental measurement will greatly improve the precision in $|V_{cs}|$ and can be achieved in future especially by Belle-II, LHCb and other experiments.

Thank you for your attentions!