夸克准分布函数的次次领头阶(NNLO)计算

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基于如下的工作

- Next-to-next-to-leading order corrections to non-singlet quark Quasi distribution functions

  L.-B. Chen, W. Wang, R. Zhu, Phys.Rev.Lett.126,072002(2021).

- Master Integrals for two-loop QCD corrections to Quasi PDFs

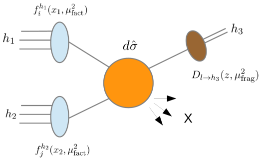
  L.-B. Chen, W. Wang, R. Zhu, JHEP10,079(2020).

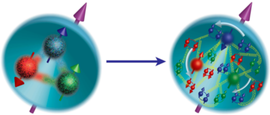
- Quasi parton distribution functions at NNLO: flavor non-diagonal quark contributions

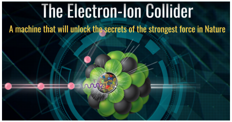
  L.-B. Chen, W. Wang, R. Zhu, Phys.Rev.D102,011503(2020).

部分子分布函数(PDFs)

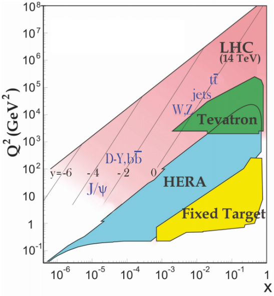
- 最基本的一个物理量
  - 预测强子散射截面的一个基本输入量
  - 长程关联
  - 非微扰相互作用
- 一个重要的问题
  - 部分子在强子内部是如何分布的? (1-D/3-D/5-D 图像)
- 一个重要的物理目标
  EIC, *1602.03922*, EICC, *2102.09222*

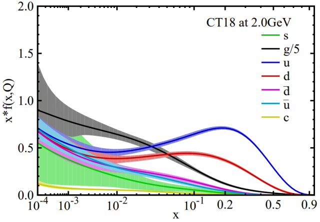


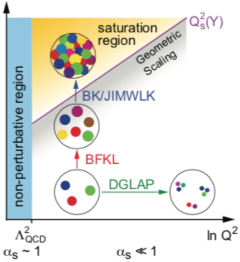




唯象模型PDFs



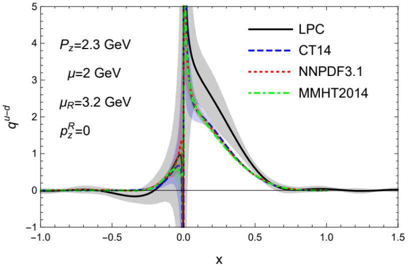




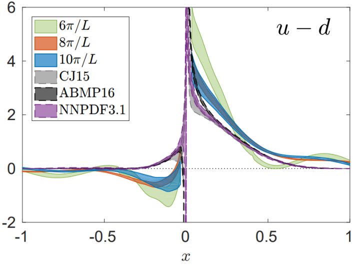
PDG2018 and 1912.10053

在各种高能强子对撞机上，经过50多年的实验测量和拟合，我们对部分子分布函数(1D)有了较为扎实的唯象认识。另一方面我们对部分子分布函数的微扰性质也有了比较深的认识。部分子分布函数是光锥关联的物理量，无法通过格点QCD第一性原理直接计算。

格点QCD+大动量有效理论计算PDFs



LPC,PRD101,034020(2020)



ETMC,PRL121,112001(2018)

目前是QCD第一性原理计算部分子分布函数的一个极具吸引力和活跃的方向

主要内容

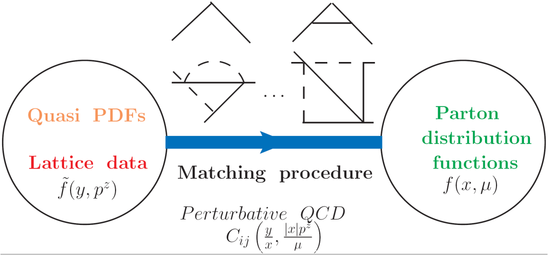
大动量有效理论(LaMET)

- LaMET 因子化公式

$$\tilde{f}_{i/H}(y,p^z)=\int_{-1}^{1}\frac{dx}{|x|}\left[C_{ij}\left(\frac{y}{x},\frac{|x|p^z}{\mu}\right)f_{j/H}(x,\mu)\right]+\mathcal{O}\left(\frac{m_h^2}{p^{z2}},\frac{\Lambda_{\rm QCD}^2}{p^{z2}}\right)$$

$x\in[-1,1]$, $y\in[-\infty,\infty]$

X. Ji, PRL110,262002 (2013), ...



微扰论计算$C_{ij}^{(0)}$, $C_{ij}^{(1)}$, $C_{ij}^{(2)}$, ...

- $C_{ij} = C_{ij}^{(0)} + \frac{\alpha_s}{2\pi} C_{ij}^{(1)} + \left(\frac{\alpha_s}{2\pi}\right)^2 C_{ij}^{(2)} + \ldots$
- 领头阶(LO): $C_{ij}^{(0)}(y) = \delta(1-y)$
- 高阶匹配，重整化方案依赖
- 次领头阶(NLO) $C_{ij}^{(1)}(y, \frac{p^z}{\mu})$

  $\overline{MS}$: Izubuchi,Ji,Jin,Stewart,Zhao,1801.03917;
  $M\overline{MS}$: Alexandrou,Cichy,Constantinou,Jansen,Scapellato,Steffens, 1803.02685;
  RI/MOM: Stewart,Zhao,1709.04933; Wang,Zhang,Zhao,Zhu,1904.00978;
  Others: Ji,Xiong,Zhang,Zhao,1310.7471; Ma,Qiu,1404.6860, ...

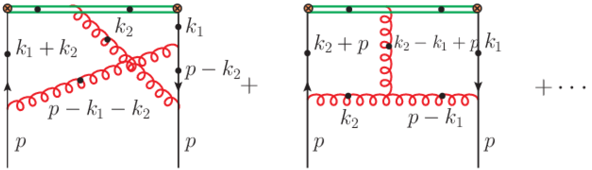
  y有三个区间：$[-\infty, 0], [0, 1], [1, +\infty]$，色因子：$C_F$
- 次次领头阶(NNLO)$C_{ij}^{(2)}$
  - QCD中的高阶修正很重要
  - $\mu = 2GeV$, $\alpha_s(\mu = 2GeV) \sim 0.3$，对于精确预测，$\alpha_s^2$阶修正不能忽略
  - NNLO因子化证明nontrivial

Outline

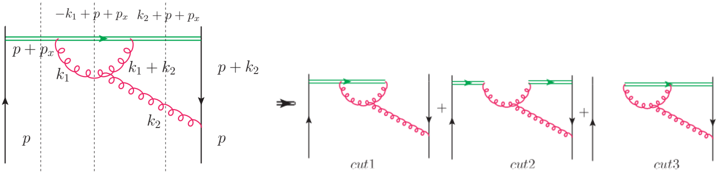
两圈费曼图和费曼振幅

- 高阶修正产生大量费曼图(79^{+} at NNLO)
- 我们把Wilson线处理为辅助夸克场(with linear propagator)
- 使用FeynRules 和FeynArts自动生成两圈费曼图和费曼振幅

Christensen et al,1310.1921, T. Hahn,0012260



Feynman规范下的一个例子(I)



- 从辅助场回到Wilson线相互作用需要做切割(cut). 对于cut1, 我们有$p_x = -p - k_2$

$$\mathcal{M}|_{cut1} = \mu^{4\epsilon} \int\int \frac{d^{4-2\epsilon}k_1}{(2\pi)^{4-2\epsilon}} \frac{d^{4-2\epsilon}k_2}{(2\pi)^{4-2\epsilon}} \text{ampcut1} \times \delta\left(k_2^z + p^z - yp^z\right),$$

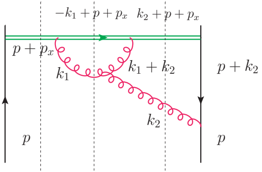
- 对于cut2, 我们有$p_x = -p + k_1$; 两者给出实贡献

$$\mathcal{M}|_{cut2} = \mu^{4\epsilon} \int\int \frac{d^{4-2\epsilon}k_1}{(2\pi)^{4-2\epsilon}} \frac{d^{4-2\epsilon}k_2}{(2\pi)^{4-2\epsilon}} \text{ampcut2} \times \delta\left(-k_1^z + p^z - yp^z\right),$$

- 对于cut3, 我们有$p_x = -p$, 给出虚贡献

$$\mathcal{M}|_{cut3} = \mu^{4\epsilon} \int\int \frac{d^{4-2\epsilon}k_1}{(2\pi)^{4-2\epsilon}} \frac{d^{4-2\epsilon}k_2}{(2\pi)^{4-2\epsilon}} \text{ampcut3} \times \delta\left(1 - y\right),$$

Feynman规范下的一个例子(II)



- 恒等式

$$\frac{1}{k_1\cdot nk_2\cdot n}=\frac{1}{(k_1\cdot n+k_2\cdot n)k_2\cdot n}+\frac{1}{k_1\cdot n(k_1\cdot n+k_2\cdot n)},$$

- 通过动量平移，得到

$$\mathcal{M}|_{cut1+cut2+cut3}=\left[\mu^{4\epsilon}\int\int\frac{d^{3-2\epsilon}k_1}{(2\pi)^{4-2\epsilon}}\frac{d^{4-2\epsilon}k_2}{(2\pi)^{4-2\epsilon}}\mathrm{ampcut1}'|_{k_1^z=yp_z}\right]_+ + \left[\mu^{4\epsilon}\int\int\frac{d^{3-2\epsilon}k_1}{(2\pi)^{4-2\epsilon}}\frac{d^{4-2\epsilon}k_2}{(2\pi)^{4-2\epsilon}}\mathrm{ampcut2}'|_{k_1^z=yp_z}\right]_+$$

- 既包含实图贡献，也包括了虚图贡献

Cutkosky 等式

- Cutkosky 等式, *J.Math.Phys.1,429(1960)*

$$\delta(k_z - xp_z) = \frac{1}{2\pi i}\left(\frac{1}{k_z - xp_z - i0} - \frac{1}{k_z - xp_z + i0}\right).$$

- 所有的实图积分变成了协变积分
- 实图积分的实部⇒ 协变积分的虚部

$$\mu^{4\epsilon}\int\int \frac{d^{3-2\epsilon}k_1}{(2\pi)^{4-2\epsilon}}\frac{d^{4-2\epsilon}k_2}{(2\pi)^{4-2\epsilon}}\mathrm{ampcut1}|_{k_1^z = yp_z}$$

$$\Rightarrow \ Im\left[\mu^{4\epsilon}\int\int \frac{d^{4-2\epsilon}k_1}{(2\pi)^{4-2\epsilon}}\frac{d^{4-2\epsilon}k_2}{(2\pi)^{4-2\epsilon}}\frac{\mathrm{ampcut1}}{4\pi(k_1^z - yp_z)}\right]$$

分部积分(IBP) 约化

- 化简费曼振幅为标量积分
- 将所有标量积分约化为主积分(MIs)

$$\int \frac{d^d k_1}{(2\pi)^d} \cdots \frac{d^d k_L}{(2\pi)^d} \frac{\partial}{\partial k_i^\mu} \left[k_i^\mu I(k_i, L, p_i) \right] = 0$$

$$\int \frac{d^d k_1}{(2\pi)^d} \cdots \frac{d^d k_L}{(2\pi)^d} \frac{\partial}{\partial k_i^\mu} \left[p_i^\mu I(k_i, L, p_i) \right] = 0$$

Chetyrkin, Tkachov, NPB192, 159(1981)

- 在右边的例子中，$F(1)$ 为主积分，其它积分都可以转化为主积分

$$F(a) = \int \frac{d^d k}{(k^2 - m^2)^a}.$$

↓

$$\int d^d k \frac{\partial}{\partial k} \cdot k \frac{1}{(k^2 - m^2)^a} = 0,$$

↓

$$(d - 2a) F(a) - 2am^2 F(a+1) = 0.$$

↓

$$F(a) = \frac{d - 2a + 2}{2(a-1)m^2} F(a-1).$$

Outline

微分方程计算主积分

- 对主积分f_i按照某一个洛伦兹不变量z求导，建立微分方程, 比如可以设置$z = \frac{p^0}{p^z}$
- 当独立的主积分数量大于1, 系数A 是一个$n \times n$ 矩阵，依赖于洛伦兹不变量z 和正规化参数ϵ

$$\frac{d}{dz}\begin{pmatrix} f_1(z,\epsilon) \\ \vdots \\ f_n(z,\epsilon) \end{pmatrix} = \begin{pmatrix} A_{11}(z,\epsilon) & \dots & A_{1n}(z,\epsilon) \\ \vdots & & \vdots \\ A_{n1}(z,\epsilon) & \dots & A_{nn}(z,\epsilon) \end{pmatrix} \begin{pmatrix} f_1(z,\epsilon) \\ \vdots \\ f_n(z,\epsilon) \end{pmatrix}$$

A.V.Kotikov, PLB254,158(1991); PLB267,123(1991)

- 然而对于f_i多元一次微分方程，边界条件不太容易确定

一类特殊的主积分基：正则基

$$\frac{d}{dz}\begin{pmatrix} g_1(z;\epsilon) \\ \vdots \\ g_n(z;\epsilon) \end{pmatrix} = \epsilon \begin{pmatrix} B_{11}(z) & \dots & B_{1n}(z) \\ \vdots & & \vdots \\ B_{n1}(z) & \dots & B_{nn}(z) \end{pmatrix} \begin{pmatrix} g_1(z;\epsilon) \\ \vdots \\ g_n(z;\epsilon) \end{pmatrix}$$

其中

$$\vec{g} = T^{-1}\vec{f}$$
$$B = T^{-1}AT - T^{-1}\partial_z T$$

- 适用于维数正规化方案$D = 4 - 2\epsilon$
- 任一主积分基可以通过线性变换到主积分基
- 系数矩阵B将只依赖于洛伦兹不变量z

  J.M.Henn,PRL110,251601(2013)

计算中采用的主积分基

- 对于夸克PDFs，需要计算三组主积分基
- 比如第一组主积分基

$$I^1_{n_i} = \int\int \frac{\mathcal{D}^D k_1\, \mathcal{D}^D k_2}{(k_1^2)^{n_1}(k_2^2)^{n_2}((k_2-p)^2)^{n_3}((k_1+k_2)^2)^{n_4}} \frac{1}{((k_1+k_2-p)^2)^{n_5}}$$
$$\times \left(\frac{1}{(P_1+i0)^{n_6}} - \frac{1}{(P_1-i0)^{n_6}}\right)\frac{1}{4\pi i}\left(\frac{1}{(Q_1+i0)^{n_7}} + \frac{1}{(Q_1-i0)^{n_7}}\right),$$

其中线性传播子为

$$P_1 = n\cdot k_1 + yn\cdot p, \qquad Q_1 = n\cdot k_2,$$

积分测度为

$$\mathcal{D}^D k_i = \frac{1}{i\pi^{D/2} e^{-\frac{4-D}{2}\gamma_E}} \left(\frac{p_z^2}{\mu^2}\right)^{\left(\frac{4-D}{2}\right)} \mathrm{d}^D k_i\,,$$

确定边界条件

- 直接计算(Feynman/Alpha 参数化方法, 留数定理, ...)
如$g_3^1 = \epsilon(y-1)p_z I^1_{0,2,0,0,2,1,0}$,

$$\begin{aligned} g_3^1 =& \mathrm{Sgn}(y-1)\Big(-2+\epsilon\big[4\ln(4(y-1)^2)\big] \\ & -\frac{1}{3}\epsilon^2[12\ln((y-1)^2)\ln(16(y-1)^2)+5\pi^2+12\ln^2(4)\big]+\mathcal{O}(\epsilon^3)\Big). \end{aligned}$$

- 正规条件

$$\begin{aligned} \frac{\partial g_7^1}{\partial z} =& \frac{\epsilon}{4}\left[\frac{8g_7^1}{z}-\frac{6g_7^1-g_8^1}{z-2y+1}-\frac{6g_7^1+g_8^1}{z+2y-1}\right. \\ & \left.+\frac{2g_3^1-6g_7^1+g_8^1}{z-1}-\frac{2g_3^1+6g_7^1+g_8^1}{z+1}\right]. \end{aligned}$$

对于y位于任一区间，有正规点$z=\frac{p_0}{p_z}=0$ $(p^2=-{p_z}^2)$，从而得到$g_7^1|_{z=0}=0$. 此外我们还有其它类似的正规点，如$z=2y-1$ $(0<y<1)$，$z=1$ $(y>1$ or $y<0)$

主积分的数值检验

- 运用FIESTA程序包，可以对解析计算的主积分进行数值检验
  A.V.Smirnov,1511.03614

$$\boxed{\frac{p^2}{p_z^2} = -\frac{1}{2}, \qquad y = \frac{1}{3}, \qquad \frac{p_z}{\mu} = 1}$$

Analytic:

$$I^1_{1,1,0,0,2,1,0} = \frac{-2.492900960}{\epsilon} + 0.4498613241 + \epsilon(-21.287203876),$$

FIESTA:

$$I^1_{1,1,0,0,2,1,0} = \frac{-2.49290 \pm 0.0000652}{\epsilon} + 0.449836 \pm 0.000847 + \epsilon(-21.2872 \pm 0.004169).$$

Outline

重整化

- 重整化方程

$$\tilde{f}(y,\frac{p^z}{\mu},\epsilon_{\rm IR})=\int\frac{dy_1}{|y_1|}\left[Z_q\tilde{Z}\left(\frac{y}{y_1}\right)\right]\left[Z_q^{-1}\tilde{f}\left(y_1,\frac{p^z}{\mu},\epsilon\right)\right].$$

这里Z_q 是夸克重整化常数, $\tilde{Z}$ 是准分布函数重整化常数

$$\tilde{Z}(\xi)=\delta(1-\xi)\left(1+\frac{\alpha_s}{2\pi}\frac{\tilde{Z}^{(1)}}{\epsilon_{UV}}+\left(\frac{\alpha_s}{2\pi}\right)^2\frac{\tilde{Z}^{(2)}}{\epsilon_{UV}^2}\right),$$

$$\tilde{Z}^{(1)}=-\frac{3C_FS_\epsilon}{2},\quad \tilde{Z}^{(2)}=S_\epsilon^2\left(\frac{a+9C_F^2}{4}+\frac{b}{4}\epsilon\right)$$

X. Ji and J.H. Zhang,1505.07699;

Braun, Chetyrkin and Kniehl,2004.01043

准分布函数的红外行为

- 软发散严格相消
- 剩下的共线发散分两种，一种是“Reducible”

$$\tilde{f}^{(2)}_{q/q}(y, \frac{p^z}{\mu}, \epsilon_{\rm IR})|_{div.part.1} = C^{(1)}_{qq}\left(\frac{y}{x}, \frac{|x|p^z}{\mu}\right) \otimes \left[-\frac{(1+x^2)}{(1-x)}\right]_+ \frac{1}{\epsilon_{\rm IR}}.$$

- 另一种是“Irreducible”
  与光锥分布函数严格相等, 包含有 $\frac{1}{\epsilon_{\rm IR}}$ 和 $\left(\frac{1}{\epsilon_{\rm IR}}\right)^2$ 发散

$$\tilde{f}^{(2)}_{i/j}(y, \frac{p^z}{\mu}, \epsilon_{\rm IR})|_{div.part.2} = f^{(2)}_{i/j}(x, \epsilon_{\rm IR}).$$

$$f^{(2)}_{i/j}(x) = \frac{1}{2\epsilon^2_{\rm IR}}\left[\sum_k P^{(0)}_{ik}(z) \otimes P^{(0)}_{kj}(x) + \beta_0 P^{(0)}_{ij}(z)\right] - \frac{P^{(1)}_{ij}(x)}{\epsilon_{\rm IR}}$$

两圈水平下因子化公式

- 将重整化后的准分布函数，匹配到最小减除方案下的光锥分布函数:

$$\begin{aligned}
\tilde{f}^{(0)}_{i/k}(y,\frac{p^z}{\mu}) =& C^{(0)}_{ij}\left(\frac{y}{x},\frac{|x|p^z}{\mu}\right)\otimes f^{(0)}_{j/k}(x),\\
\tilde{f}^{(1)}_{i/k}(y,\frac{p^z}{\mu},\epsilon_{\rm IR}) =& C^{(1)}_{ij}\left(\frac{y}{x},\frac{|x|p^z}{\mu}\right)\otimes f^{(0)}_{j/k}(x)\\
&+C^{(0)}_{ij}\left(\frac{y}{x},\frac{|x|p^z}{\mu}\right)\otimes f^{(1)}_{j/k}(x,\epsilon_{\rm IR}),\\
\tilde{f}^{(2)}_{i/k}(y,\frac{p^z}{\mu},\epsilon_{\rm IR}) =& C^{(2)}_{ij}\left(\frac{y}{x},\frac{|x|p^z}{\mu}\right)\otimes f^{(0)}_{j/k}(x)\\
&+C^{(1)}_{ij}\left(\frac{y}{x},\frac{|x|p^z}{\mu}\right)\otimes f^{(1)}_{j/k}(x,\epsilon_{\rm IR})\\
&+C^{(0)}_{ij}\left(\frac{y}{x},\frac{|x|p^z}{\mu}\right)\otimes f^{(2)}_{j/k}(x,\epsilon_{\rm IR}).
\end{aligned}$$

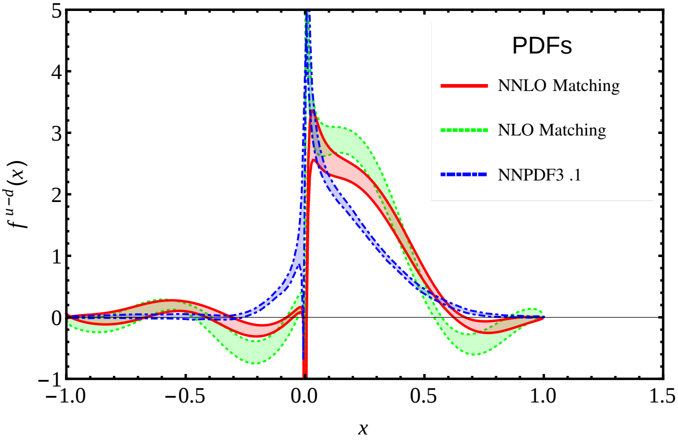
Outline

NNLO阶匹配系数$C_{qq}^{(2)}$

- 与Li-Ma-Qiu在$\overline{\mathrm{MS}}$方案下计算的结果一致
  Phys.Rev.Lett.126,072001(2021)
- 同时我们也得到了RI/MOM和$\mathrm{M}\overline{\mathrm{MS}}$方案下的NNLO阶匹配系数$C_{qq}^{(2)}(y,\frac{p^z}{\mu})$
- 对于准分布动量分数y，我们有四个区间，色结构有三种(C_F，C_A，$nf\ T_F$)C_F
- 在动量分数y趋于无穷大的行为为$C_{qq}^{(2),\mathrm{M}\overline{\mathrm{MS}}}|_{y\to\infty}\propto\frac{1}{y^2}$

$$
\begin{aligned}
&C_{qq}^{(2),\mathrm{M}\overline{\mathrm{MS}}}(y,\frac{p^z}{\mu})\\
=&[C_{qq}^{(2),\mathrm{M}\overline{\mathrm{MS}}}(y,\frac{p^z}{\mu})|_{y>1}]_+ + [C_{qq}^{(2),\mathrm{M}\overline{\mathrm{MS}}}(y,\frac{p^z}{\mu})|_{0<y<1}]_+\\
&+[C_{qq}^{(2),\mathrm{M}\overline{\mathrm{MS}}}(y,\frac{p^z}{\mu})|_{-1<y<0}]_+ + [C_{qq}^{(2),\mathrm{M}\overline{\mathrm{MS}}}(y,\frac{p^z}{\mu})|_{y<-1}]_+
\end{aligned}
$$

NNLO阶匹配，抽取PDFs



采用了ETMC格点组$M\overline{\text{MS}}$重整化方案的数据，其中$z_{cut} = 10a$, $\mu = 2GeV$，不确定性来自于格点QCD数据

总结

- NNLO计算无论在形式理论还是实践应用上都比较重要
- 在NNLO，我们利用格点数据，模型无关地得到了非单态夸克的光锥分布函数，NNLO贡献一定程度缓解PDFs震荡行为
- 红外发散相消得到了证明，进而在两圈水平下验证了LaMET因子化公式

- 展望
    - 胶子准分布函数的*NNLO*计算
    - π介子光锥分布振幅的*NNLO*计算
    - 在*NNLO*精度下，模型无关地抽取其它光锥关联函数

谢谢各位老师和同学！

Back up

Canonical bases for family-1 MIs, here only list 10 of them

$$
\begin{aligned}
g_1^1 &= \epsilon(y+1)p_z I^1_{0,0,2,2,0,1,0}\,,\\
g_2^1 &= \epsilon y p_z I^1_{0,2,0,2,0,1,0}\,,\\
g_3^1 &= \epsilon(y-1)p_z I^1_{0,2,0,0,2,1,0}\,,\\
g_4^1 &= \epsilon y p_z p_1^2 I^1_{2,2,1,0,0,1,0}\,,\\
g_5^1 &= \epsilon^2\sqrt{p_1^2+p_z^2}\,I^1_{0,1,1,0,2,1,0}\,,\\
g_6^1 &= \epsilon(p_1^2-4y(y-1)p_z^2)I^1_{0,1,1,0,2,2,0}+8\epsilon^2(2y-1)p_z I^1_{0,1,1,0,2,1,0}\\
&\quad+\epsilon(y-1)p_z I^1_{0,2,0,0,2,1,0}+\epsilon y p_z I^1_{0,2,0,2,0,1,0}\,,\\
g_7^1 &= \epsilon^2\sqrt{p_1^2+p_z^2}\,I^1_{1,1,0,0,2,1,0}\,,\\
g_8^1 &= \epsilon(p_1^2-4y(y-1)p_z^2)I^1_{1,1,0,0,2,2,0}+6\epsilon^2(2y-1)p_z I^1_{1,1,0,0,2,1,0}\,,\\
g_9^1 &= \epsilon^2\sqrt{p_1^2+p_z^2}\,I^1_{0,1,1,2,0,1,0}\,,\\
g_{10}^1 &= \epsilon(p_1^2-4y(y+1)p_z^2)I^1_{0,1,1,2,0,2,0}+8\epsilon^2(2y+1)p_z I^1_{0,1,1,2,0,1,0}
\end{aligned}
$$

Back up

NLO matching coefficient in $\overline{MS}$ scheme

$$
C_{qq}^{(1),\overline{MS}}\left(y,\frac{\mu}{p^z}\right)|_{\mathcal{O}(\epsilon^0)}
= C_F \begin{cases} \left[\frac{1+y^2}{1-y}\ln\frac{y}{y-1}+1+\frac{3}{2y}\right]_{+(1)}^{[1,\infty]} - \frac{3}{2y}, & y>1 \\ \left[\frac{1+y^2}{1-y}\left(-\ln\frac{\mu^2}{4p^{z2}}+\ln(y(1-y))\right)-\frac{y(1+y)}{1-y}\right]_{+(1)}^{[0,1]}, & 0<y<1 \\ \left[-\frac{1+y^2}{1-y}\ln\frac{y}{y-1}-1+\frac{3}{2(1-y)}\right]_{+(1)}^{[-\infty,0]} - \frac{3}{2(1-y)}, & y<0 \end{cases}
+ C_F\left[\delta(1-y)\left(\frac{3}{2}\ln\frac{\mu^2}{4p^{z2}}+\frac{5}{2}\right)\right]
$$

NLO matching coefficient in $M\overline{MS}$ scheme

$$
C\left(\xi,\frac{\xi\mu}{xP_3}\right) = \delta(1-\xi) + \frac{\alpha_s}{2\pi}C_F \begin{cases} \left[\frac{1+\xi^2}{1-\xi}\ln\frac{\xi}{\xi-1}+1+\frac{3}{2\xi}\right]_+ & \xi>1, \\ \left[\frac{1+\xi^2}{1-\xi}\ln\frac{x^2P_3^2}{\xi^2\mu^2}(4\xi(1-\xi))-\frac{\xi(1+\xi)}{1-\xi}+2\iota(1-\xi)\right]_+ & 0<\xi<1, \\ \left[-\frac{1+\xi^2}{1-\xi}\ln\frac{\xi}{\xi-1}-1+\frac{3}{2(1-\xi)}\right]_+ & \xi<0, \end{cases}
$$

Back up

One loop quasi pdf in RI/MOM scheme

$$\tilde{f}^{(1)}_{q/q,\xi}(y,p^z)/C_F$$

$$= \begin{cases} -\frac{\left((\xi-2)\rho+2y^2-(\xi+1)\rho y+2\right)\log\left(\frac{\sqrt{1-\rho}+2y-1}{-\sqrt{1-\rho}+2y-1}\right)}{2(1-\rho)^{3/2}(y-1)} + \frac{\xi\rho\left(\rho(\rho+1)+16y^4-40y^3+4(\rho+8)y^2-2(3\rho+4)y\right)}{2(\rho-1)(y-1)\left(\rho+4y^2-4y\right)^2} \\ +\frac{y\left((4-5\rho)\rho-16y^4+8(\rho+3)y^3-24\rho y^2+4\left(\rho^2+3\rho-2\right)y\right)}{(\rho-1)(y-1)\left(\rho+4y^2-4y\right)^2} + \frac{3}{2-2y}, & y>1 \\ -\frac{\xi\rho+\xi-3\rho+4y^2-2(\xi+1)y+3}{2(\rho-1)(y-1)} - \frac{\log\left(\frac{\sqrt{1-\rho}+1}{1-\sqrt{1-\rho}}\right)\left((\xi-2)\rho+2y^2-(\xi+1)\rho y+2\right)}{2(1-\rho)^{3/2}(y-1)}, & 0<y<1, \\ \frac{\left((\xi-2)\rho+2y^2-(\xi+1)\rho y+2\right)\log\left(\frac{\sqrt{1-\rho}+2y-1}{-\sqrt{1-\rho}+2y-1}\right)}{2(1-\rho)^{3/2}(y-1)} - \frac{\xi\rho\left(\rho(\rho+1)+16y^4-40y^3+4(\rho+8)y^2-2(3\rho+4)y\right)}{2(\rho-1)(y-1)\left(\rho+4y^2-4y\right)^2} \\ -\frac{y\left((4-5\rho)\rho-16y^4+8(\rho+3)y^3-24\rho y^2+4\left(\rho^2+3\rho-2\right)y\right)}{(\rho-1)(y-1)\left(\rho+4y^2-4y\right)^2} - \frac{3}{2-2y}), & y<0. \end{cases}$$

$$\log\left(\frac{\sqrt{1-\rho}+1}{1-\sqrt{1-\rho}}\right) = -\log\left(\frac{1-\sqrt{z^2}}{\sqrt{2-z^2}+1}\right), \quad \log\left(\frac{\sqrt{1-\rho}+2y-1}{-\sqrt{1-\rho}+2y-1}\right) = -\log\left(\frac{2y-\sqrt{z^2}-1}{2y+\sqrt{2-z^2}-1}\right)$$

$$\int dy[h(y)]_+ g(y) = \int dy h(y)[g(y)-g(1)]$$

$$iD^{\mu\nu}_\xi(k) = -\frac{i}{k^2}\left[g^{\mu\nu}-(1-\xi)\frac{k^\mu k^\nu}{k^2}\right].$$

Back up

NNLO matching coefficients

$$
\begin{aligned}
C_{qq}^{(2),\mathrm{M}\overline{\mathrm{MS}}}(y,\frac{p^z}{\mu})|_{y>1} =& C_{qq}^{(2),\overline{\mathrm{MS}}}(y,\frac{p^z}{\mu})|_{y>1} + \frac{C_F\left(11C_A+9C_F-2n_f\right)}{4y}\log\left(\frac{\mu^2}{p^{z2}}\right) - \frac{C_F n_f(5-4\log(2y))}{4y} \\
&- \frac{C_A C_F\left(132\log(2y)+4\pi^2-159\right)}{24y} + \frac{C_F^2\left(-108\log(2y)+16\pi^2+75\right)}{24y}, \\
C_{qq}^{(2),\mathrm{M}\overline{\mathrm{MS}}}(y,\frac{p^z}{\mu})|_{0<y<1} =& C_{qq}^{(2),\overline{\mathrm{MS}}}(y,\frac{p^z}{\mu})|_{0<y<1}, \\
C_{qq}^{(2),\mathrm{M}\overline{\mathrm{MS}}}(y,\frac{p^z}{\mu})|_{-1<y<0} =& C_{qq}^{(2),\overline{\mathrm{MS}}}(y,\frac{p^z}{\mu})|_{-1<y<0}, \\
C_{qq}^{(2),\mathrm{M}\overline{\mathrm{MS}}}(y,\frac{p^z}{\mu})|_{y<-1} =& C_{qq}^{(2),\overline{\mathrm{MS}}}(y,\frac{p^z}{\mu})|_{y<-1} + \frac{C_F\left(11C_A+9C_F-2n_f\right)}{4(1-y)}\log\left(\frac{\mu^2}{p^{z2}}\right) - \frac{C_F n_f(5-4\log(-2y))}{4(1-y)} \\
&- \frac{C_A C_F\left(132\log(-2y)+4\pi^2-159\right)}{24(1-y)} + \frac{C_F^2\left(-108\log(-2y)+16\pi^2+75\right)}{24(1-y)}.
\end{aligned}
$$

$$
\begin{aligned}
C_{qq}^{(2),\overline{\mathrm{MS}}}(y,\frac{p^z}{\mu})|_{y>1} =& \left(C_F c_1^{C_F} + C_A c_1^{C_A} + 2T_F n_f c_1^{T_F}\right) C_F + \left(\Gamma_1(y)|_{y>1}\right)\log(\frac{\mu^2}{p_z^2}), \\
C_{qq}^{(2),\overline{\mathrm{MS}}}(y,\frac{p^z}{\mu})|_{0<y<1} =& C_F\left(C_F c_2^{C_F} + C_A c_2^{C_A} + 2T_F n_f c_2^{T_F}\right) + \left(\Gamma_2(y)\right)\log^2(\frac{\mu^2}{p_z^2}) \\
&+ \left(\left(\Gamma_1(y)|_{0<y<1}\right) - \left(P_{qq}^{(1),V}(y)|_{0<y<1}\right)\right)\log(\frac{\mu^2}{p_z^2}), \\
C_{qq}^{(2),\overline{\mathrm{MS}}}(y,\frac{p^z}{\mu})|_{-1<y<0} =& C_F\left(C_F c_3^{C_F} + C_A c_3^{C_A} + 2T_F n_f c_3^{T_F}\right) + \left(\left(\Gamma_1(y)|_{-1<y<0}\right) - P_{\bar{q}q}^{(1),V}(-y)\right)\log(\frac{\mu^2}{p_z^2}),
\end{aligned}
$$

$$
C_{qq}^{(2),\overline{\mathrm{MS}}}(y,\frac{p^z}{\mu})|_{y<-1} = -C_{qq}^{(2),\overline{\mathrm{MS}}}(y,\frac{p^z}{\mu})|_{y>1}\ .
$$

The complete analytic expression can be found in our paper.