NRQCD factorization for the fully-charmed tetraquark production

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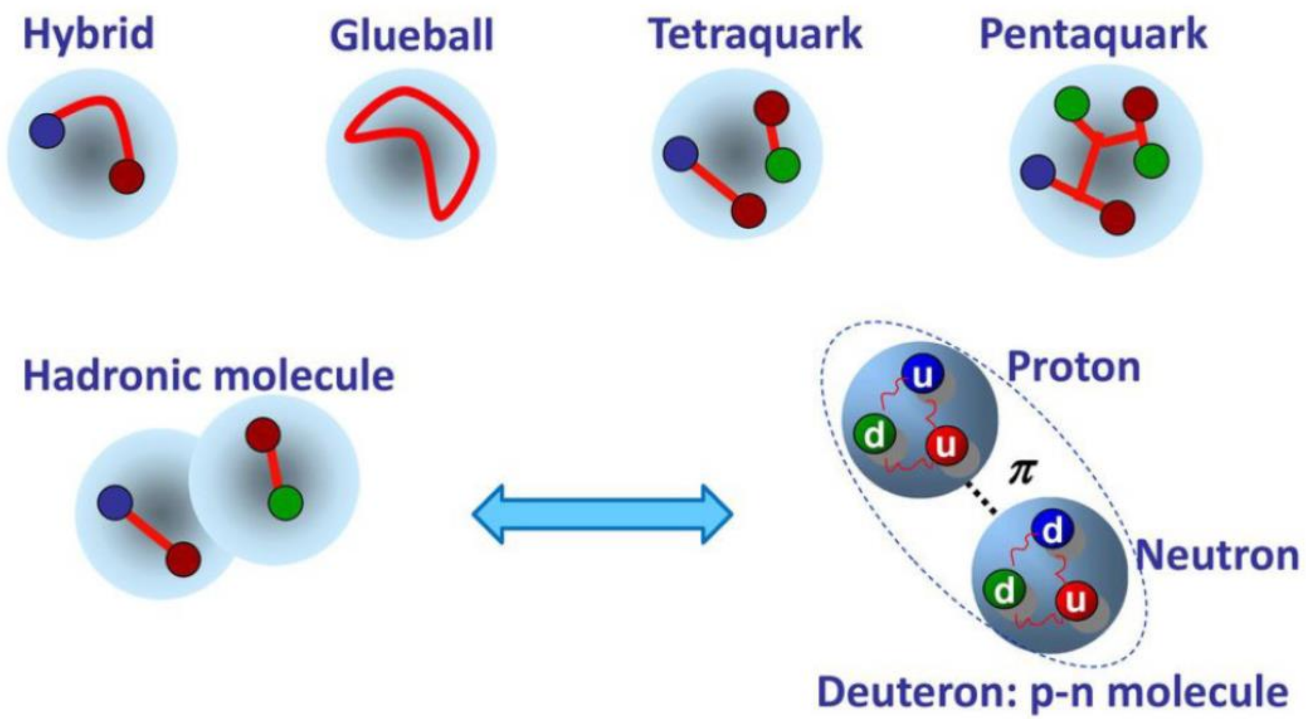
- **Brief review of X(6900) and fully-charmed tetra-quark**
- **Why NRQCD factorization approach can be applied to T4c production – a model-independent framework**
- **T4c inclusive production at LHC and B factory (compute the gluon-to-T4c fragmentation function)**
- **T4c exclusive production at super B factory**
- **Summary**

Exotic states

- Exotic hadrons are composed of quarks and gluons beyond the conventional meson and baryon pictures

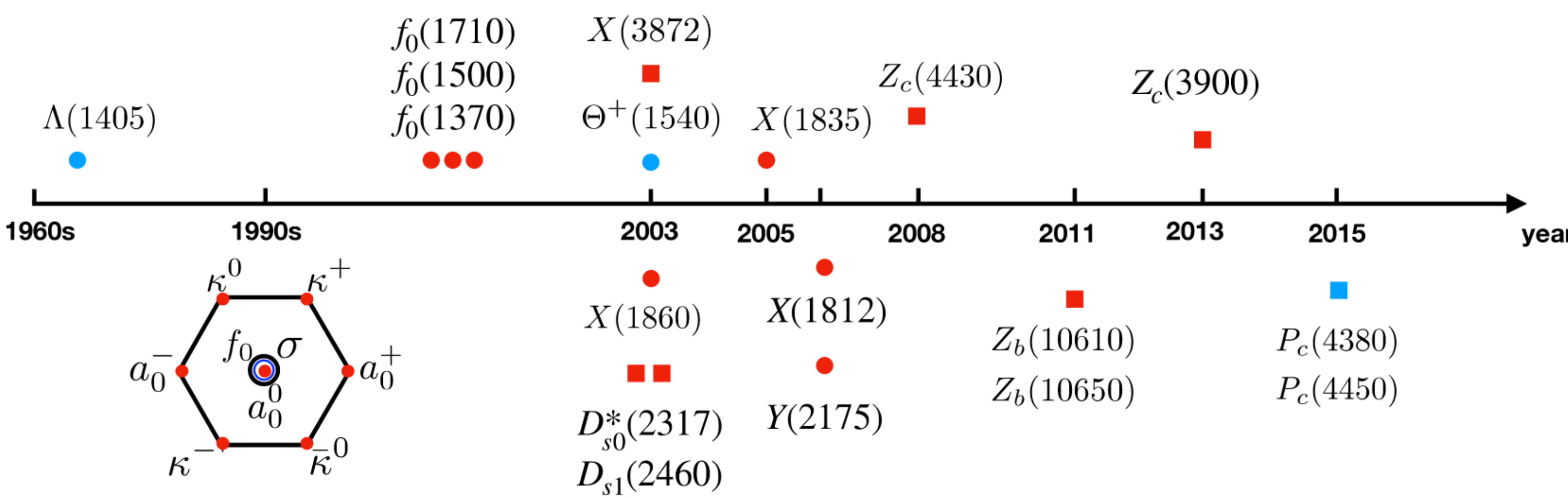


Pictures in courtesy to Qiang Zhao

Exotic multi-quark states

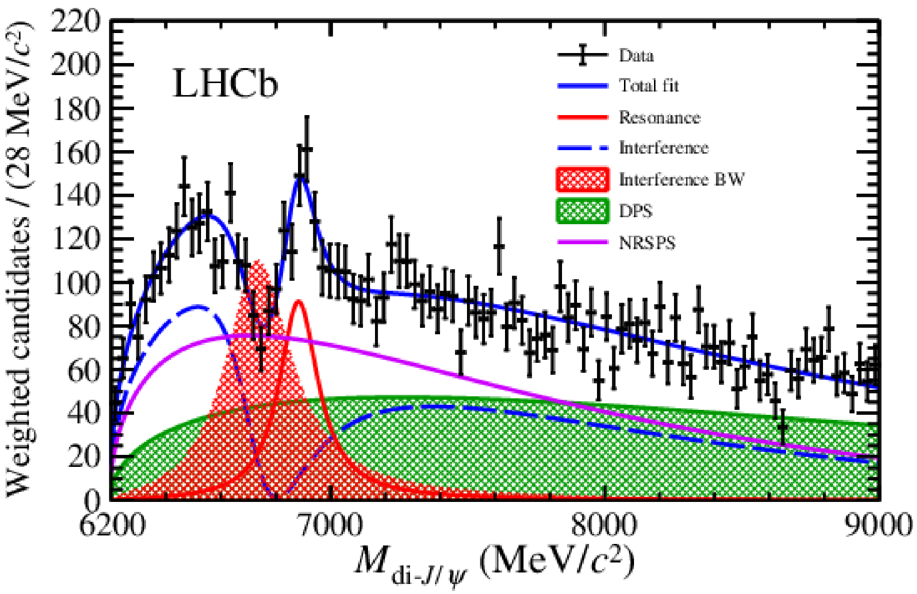
See Xiang Liu's talk

- Since 2003, there has been continuous and unexpected progress in the field.
- Many charmonium-like XYZ states are observed.
- How to interpret all of these states unambigously?



(Unexpected) discovery of X(6900) in summer of 2020

Invariant mass spectrum of J/ψ-pair candidates (LHCb, 2020)



LHCb discovered a narrow structure near 6.9 GeV in the di-J/ψ invariant mass spectrum:X(6900).

Strong candidate for compact fully-charmed tetraquark

There's also a board structure above threshold ranging from 6.2 to 6.8 GeV

An additional vague structure near 7.2 GeV

What X(6900) deserves being studied

- Why is X(6900) important?

- X(6900) is just above di-J/ threshold, likely to be comprised of four charm quarks

- With no light valance quarks, we could have approximate charm/anti-charm quark number conservation, thus a meaningful $|cc\bar{c}\bar{c}\rangle$ Fock component

- Many phenomenological studies about fully-heavy tetraquark: potential models

Interpretation of nature of the X(6900) meson

- **P-wave tetraquark** (M.-S. Liu et al., 2020; H.-X. Chen et al., 2020, R. Zhu 2020).
- **Radial excitation of 0++** (Z.-G. Wang, 2020; Lü et al., 2020; Giron, Lebed, 2020; Karliner & Rosner, 2020; J. Zhao et al., 2020; R. Zhu, 2020; B.-C. Yang et al., 2020; Z. Zhao, 2020; H.-W. Ke et al., 2020),
- **Ground state S-wave tetraquark** (Gordillo et al., 2020).
- $\chi_{c0}\chi_{c0}$ **or** P_cP_c **molecular state** (Albuquerque et al., 2020)
- **0^{++} hybrid** (B.-D. Wan, C.-F.Qiao, 2020),
- **Resonance formed in charmonium-charmonium scattering** (G. Yang et al., 2020; X. Jin et al., 2020), or the kinematic cusp arising from final-state interaction(J.-Z.Wang et al., 2020; X.-K. Dong et al., 2020; Z.-H. Guo 2021; C. Gong et al. 2020).
- **Beyond Standard Model scenario** (J.-W.Zhu et al., 2020; Dosch et al., 2020)

Early day studies of fully-heavy tetraquark states

- Earliest papers regarding T4c: (Ader et al., 1982; Chao, 1981; Iwasaki, 1976)

Z. Physik C, Particles and Fields 7, 317–320 (1981)

Zeitschrift für Physik C Particles and Fields

The (cc)–$(\overline{cc})$ (Diquark-Antidiquark) States in e^+e^- Annihilation

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Popular phenomenological tools handling fully-heavy tetraquark

- Theoretical investigations on the fully heavy tetraquarks dateback to late 1970s (Iwasaki, 1976; Chao, 1981).

- **Potential models** studying T4c spectra and decay properties: (Badalian et al., 1987; Barnea et al., 2006; Liu et al., 2019; Wu et al., 2018), etc.

- **QCD sum rule**: (H.-X. Chen et al., 2020; W. Chen et al., 2017, 2018; Wang, 2017, 2020; Wang & Di, 2019)

- Search for the fully-bottom tetraquark on **Lattice NRQCD**: found no indication of any states below $2\eta_b$ threshold in the 0^{++},1^{+-} and 2^{++} channels (Hugheset al., 2018).

T4c production mechanisms in the market

- In contrast to mass spectra and decay channels, there is relatively sparse studies for T_{4c} production in various enviorment.

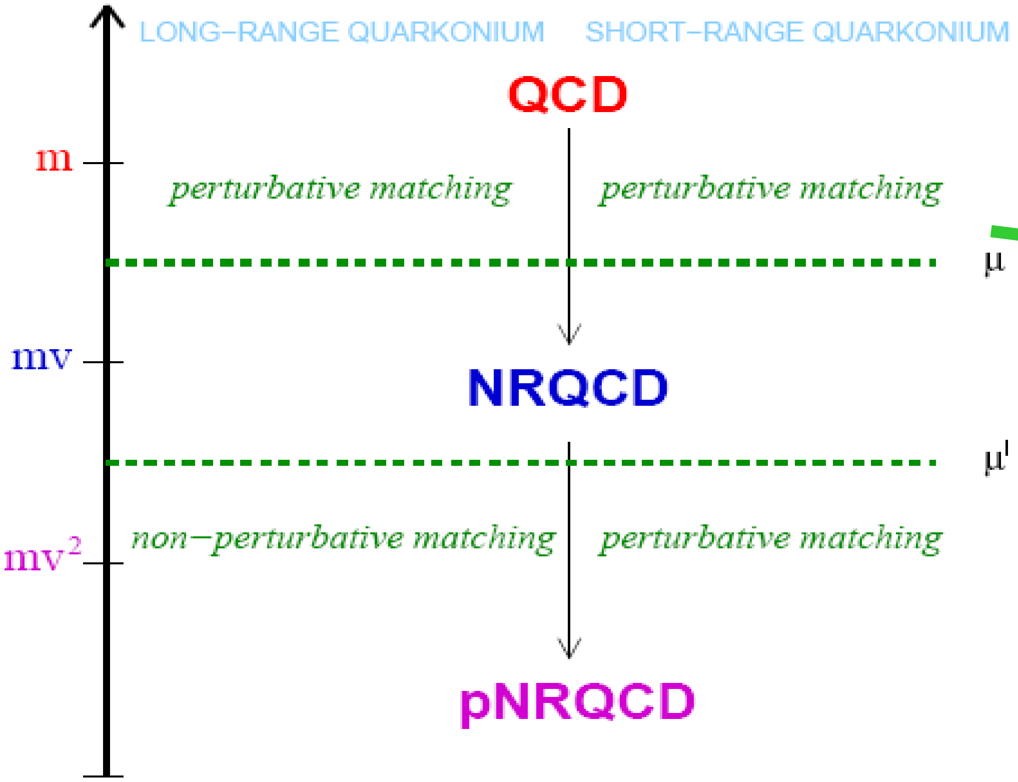
- **Duality relation** model: comparing di-J/psi production cross section with (Berezhnoy et al., 2011; Berezhnoy et al., 2012) and without resonance, rough estimate was achieved (Karliner et al., 2017)
- Treat hadronization within **color evaporation model:** (Carvalho et al., 2016; Maciuła et al., 2020)

- **NRQCD-inspired factorization**: F. Feng, Y.-S. Huang, YJ, W.-L.Sang, X.-N. Xiong, J.-Y.Zhang and D.-S. Yang, 2020-2021

Also see Y.-Q. Ma, Zhang, 2020; R.-L. Zhu, 2020.

Nonrelativistic QCD (NRQCD): Paradigm of EFT, tailored for describing heavy quarkonium dynamics: exploiting NR nature of quarkonium

Caswell, Lepage (1986); Bodwin, Braaten, Lepage (1995)



NRQCD factorization is viewed as being first principle of QCD

This scale separation is usually referred to as **NRQCD factorization.**

The NRQCD short-dist. coefficients can be computed in perturbation theory, order by order

NRQCD Lagrangian (characterized by velocity expansion)

$$\mathcal{L}_{\mathrm{NRQCD}} = \mathcal{L}_{\mathrm{light}} + \mathcal{L}_{\mathrm{heavy}} + \delta\mathcal{L}.$$

$$\mathcal{L}_{\mathrm{light}} = -\frac{1}{2}\mathrm{tr}\, G_{\mu\nu}G^{\mu\nu} + \sum \bar{q}\, i\not{D} q,$$

$$\mathcal{L}_{\mathrm{heavy}} = \psi^\dagger \left(iD_t + \frac{\mathbf{D}^2}{2M} \right) \psi + \chi^\dagger \left(iD_t - \frac{\mathbf{D}^2}{2M} \right) \chi,$$

$$\begin{aligned}
\delta\mathcal{L}_{\mathrm{bilinear}} = & \frac{c_1}{8M^3}\left(\psi^\dagger(\mathbf{D}^2)^2\psi - \chi^\dagger(\mathbf{D}^2)^2\chi\right) \\
& + \frac{c_2}{8M^2}\left(\psi^\dagger(\mathbf{D}\cdot g\mathbf{E} - g\mathbf{E}\cdot\mathbf{D})\psi + \chi^\dagger(\mathbf{D}\cdot g\mathbf{E} - g\mathbf{E}\cdot\mathbf{D})\chi\right) \\
& + \frac{c_3}{8M^2}\left(\psi^\dagger(i\mathbf{D}\times g\mathbf{E} - g\mathbf{E}\times i\mathbf{D})\cdot\boldsymbol{\sigma}\psi + \chi^\dagger(i\mathbf{D}\times g\mathbf{E} - g\mathbf{E}\times i\mathbf{D})\cdot\boldsymbol{\sigma}\chi\right) \\
& + \frac{c_4}{2M}\left(\psi^\dagger(g\mathbf{B}\cdot\boldsymbol{\sigma})\psi - \chi^\dagger(g\mathbf{B}\cdot\boldsymbol{\sigma})\chi\right),
\end{aligned}$$

Identical to HQET, but with different power counting

NRQCD is the mainstream tool in studying quarkonium (see Brambilla et al. EPJC 2011 for a review)

Nowadays, NRQCD becomes standard approach to tackle various quarkonium production and decay processes:

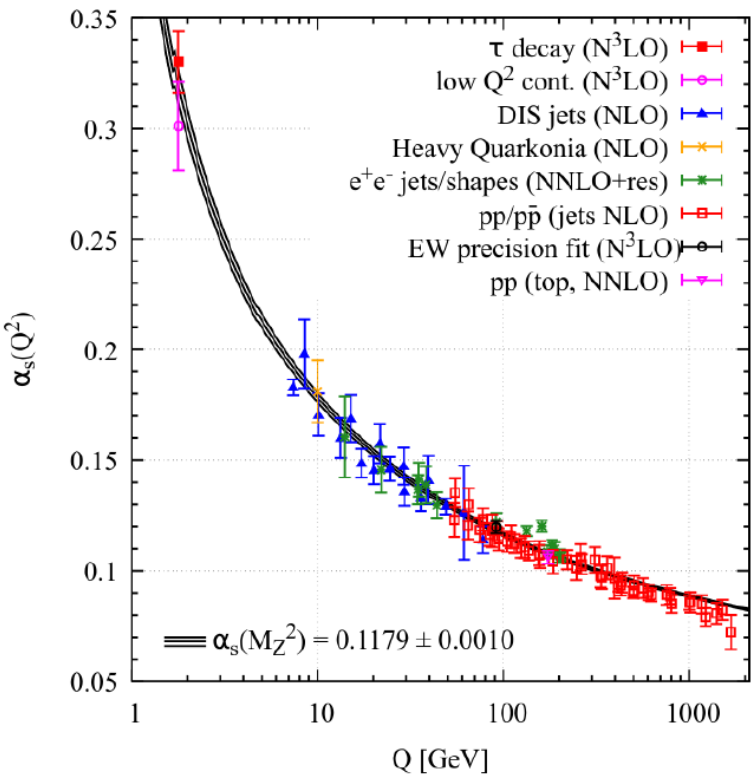
Charmonium: $v^2/c^2 \sim 0.3$ not truly non-relativistic to some extent

Bottomonium: $v^2/c^2 \sim 0.1$ a better "non-relativistic" system

Intuitively, one expects that NRQCD works better for bottomonium than charmonium

However, the data for charmonium production is much more copious than bottomonium!

QCD Factorization Theorem



- Asymptotic freedom
- Scale separation: UV ~ Q (pT for production), IR~ Λ_{QCD}
- QCD Collinear Factorization:

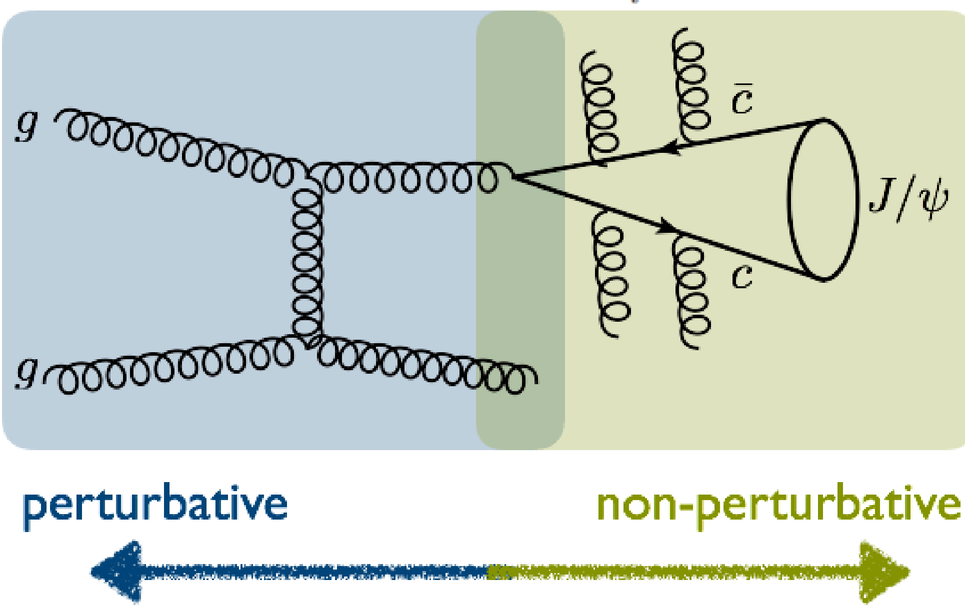
$$\sigma(Q, \Lambda_{\text{QCD}}) = \hat{\sigma}_{\text{pert}}(Q, \mu_F) \otimes \varphi(\mu_F, \Lambda_{\text{QCD}}) + \cdots$$

- This allows perturbative calculation to produce sensible phenomenological results

QCD factorization theorem for the identified hadron inclusive production at large Pt

Collins, Soper, Sterman (1982)

$$d\sigma[A+B\to H(P_\perp)+X]=\sum_i d\hat{\sigma}[A+B\to i(P_\perp/z)+X]\otimes D_{i\to H}(z,\mu)+\mathcal{O}(1/P_\perp^2)$$



The inclusive production of high-transverse-momentum hadrons in the high-energy hadron collision experiments is dominated by the fragmentation mechanism.

Collin-Soper definition of fragmentation function

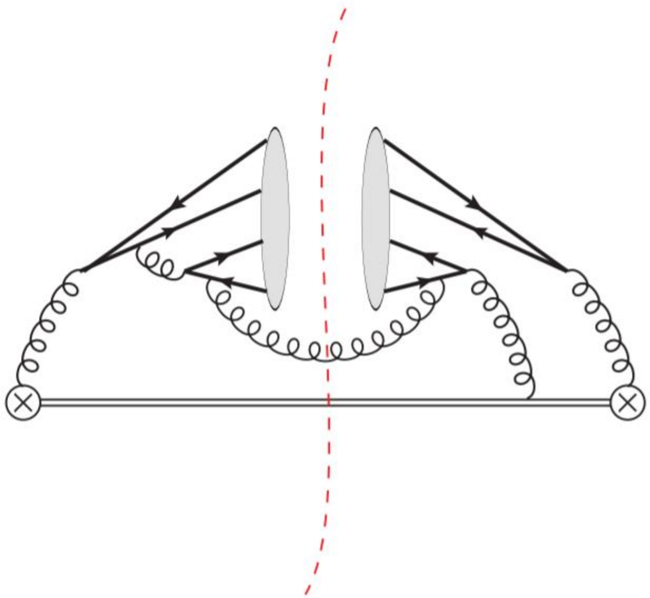
Gauge-invariant operator definition for the gluon fragmentation function(Collins, Soper, 1982):

$$D_{g\to H}(z,\mu) = \frac{-g_{\mu\nu} z^{D-3}}{2\pi k^+ (N_c^2-1)(D-2)} \int_{-\infty}^{+\infty} dx^- e^{-ik^+ x^-}$$
$$\times \langle 0|G_c^{+\mu}(0)\Phi^\dagger(0,0,\mathbf{0}_\perp)_{cb} \sum_X |H(P,\lambda)+X\rangle\langle H(P,\lambda)+X|\Phi(0,x^-,\mathbf{0}_\perp)_{ba} G_a^{+\nu}(0,x^-,\mathbf{0}_\perp)|0\rangle$$

Fragmentation function obeys Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution equation:

$$\frac{d}{d\ln\mu^2} D_{g\to H}(z,\mu) = \sum_i \int_z^1 \frac{d\xi}{\xi} P_{ig}(\xi,\alpha_s(\mu)) D_{i\to H}\left(\frac{z}{\xi},\mu\right)$$

Feynman Diagrams for the fragmentation function of gluon into fully-heavy tetraquark



Using self-written program HepLib, which uses Qgraf and GiNaC to generate Feynman diagram and amplitudes

About 100 diagrams for amplitude in one side of the cut line

Key insight: to create a T4c state, one has to first produce four charm quarks with small velocity before hadronization. Therefore, one can invoke perturbation theory to compute the hard partonic process owing to asymptotic freedom of QCD

NRQCD Factorization for the gluon-to-0^{++} (2^{++}) T4c fragmentation function

For the gluon-to-tetraquark fragmentation function:

$$D_{g\to T_{4c}}(z,\mu_\Lambda) = \frac{d_{3,3}\left[g \to cc\bar{c}\bar{c}^{(J)}\right]}{m^9}\left|\left\langle 0\left|\mathcal{O}^{(J)}_{\mathbf{3}\otimes\bar{\mathbf{3}}}\right|T^{(J)}_{4c}\right\rangle\right|^2 + \frac{d_{6,6}\left[g \to cc\bar{c}\bar{c}^{(J)}\right]}{m^9}\left|\left\langle 0\left|\mathcal{O}^{(J)}_{\mathbf{6}\otimes\bar{\mathbf{6}}}\right|T^{(J)}_{4c}\right\rangle\right|^2$$
$$+ \frac{d_{3,6}\left[g \to cc\bar{c}\bar{c}^{(J)}\right]}{m^9} 2\mathrm{Re}\left[\left\langle 0\left|\mathcal{O}^{(J)}_{\mathbf{3}\otimes\bar{\mathbf{3}}}\right|T^{(J)}_{4c}\right\rangle\left\langle T^{(J)}_{4c}\left|\mathcal{O}^{(J)\dagger}_{\mathbf{6}\otimes\bar{\mathbf{6}}}\right|0\right\rangle\right] + \cdots,$$

Lowest order in v;
Vacuum saturation approximation invoked

Local NRQCD production operators (using diquark-antidiquark color-spin basis)

We construct the NRQCD local operators for the S-wave tetraquark with all possible quantum numbers:

$$\mathcal{O}^{(0)}_{\bar{\mathbf{3}}\otimes\mathbf{3}} = -\frac{1}{\sqrt{3}}[\psi_a^\dagger \sigma^i (i\sigma^2)\psi_b^*][\chi_c^T (i\sigma^2)\sigma^i \chi_d]\, C^{ab;cd}_{\bar{\mathbf{3}}\otimes\mathbf{3}},$$

$$\mathcal{O}^{(0)}_{\mathbf{6}\otimes\bar{\mathbf{6}}} = \frac{1}{\sqrt{6}}[\psi_a^\dagger (i\sigma^2)\psi_b^*][\chi_c^T (i\sigma^2)\chi_d]\, C^{ab;cd}_{\mathbf{6}\otimes\bar{\mathbf{6}}},$$

$$\mathcal{O}^{(2)kl}_{\bar{\mathbf{3}}\otimes\mathbf{3}} = [\psi_a^\dagger \sigma^m (i\sigma^2)\psi_b^*][\chi_c^T (i\sigma^2)\sigma^n \chi_d]\, \Gamma^{kl;mn}\, C^{ab;cd}_{\bar{\mathbf{3}}\otimes\mathbf{3}},$$

$$\mathcal{O}^{i}_{\bar{\mathbf{3}}\otimes\mathbf{3}} = \frac{i}{\sqrt{2}}\epsilon^{ijk} C^{ab;cd}_{\bar{\mathbf{3}}\otimes\mathbf{3}} \left(\psi_a^\dagger \sigma^j i\sigma^2 \psi_b^*\right)\left(\chi_c^T i\sigma^2 \sigma^k \chi_d\right)$$

$$C^{ab;cd}_{\mathbf{3}\otimes\bar{\mathbf{3}}} \equiv \frac{1}{(\sqrt{2})^2}\epsilon^{abm}\epsilon^{cdn}\frac{\delta^{mn}}{\sqrt{3}} = \frac{1}{2\sqrt{N_c}}(\delta^{ac}\delta^{bd} - \delta^{ad}\delta^{bc})$$

$$C^{ab;cd}_{\bar{\mathbf{6}}\otimes\mathbf{6}} \equiv \frac{1}{2\sqrt{6}}(\delta^{ac}\delta^{bd} + \delta^{ad}\delta^{bc}). \quad \Gamma^{kl;mn} \equiv \frac{1}{2}(\delta^{km}\delta^{ln} + \delta^{kn}\delta^{lm} - \frac{2}{3}\delta^{kl}\delta^{mn})$$

NRQCD Operators

The operators manifest the correct C-parity under the charge conjugation transformations.

$$\psi \to i\left(\chi^{\dagger}\sigma_2\right)^{t}, \quad \chi \to -i\left(\psi^{\dagger}\sigma_2\right)^{t}$$

We use the basis in which the quark and anti-quark pairs in the color-triplet and color-sexet, respectively. The operators can also be constructed from quark-antiquark pairs in the color-singlet and color-octet.

These NRQCD operators in can also be inferred by performing the Foldy-Wouthuysen-Tani transformation from the QCD interpolating currents (H.-X. Chen et al.,2020).

Determine short-distance coefficients (SDC): Perturbative Matching

The SDCs are state-independent quantities, insensitive to non-perturbative (long-distance) physics

Use free quark states instead of hadron (tetraquark) state in both FF and LDMEs

Perturbatively calculate both FF and LDMEs; solve the matching equation and deduce the SDCs

Adopt angular momentum eigenstates and project perturbative FF to specific color state in order to deduce each SDCs in question

Free tetraquark States

For convenience, we use the eigenstates of the angular momentum, manifesting the same quantum numbers as the physical tetraquark states.

$$\left|\mathcal{T}^{J,m_j}_{\bar{\mathbf{3}}\otimes\mathbf{3}}(Q)\right\rangle = \frac{1}{2}\sum_{s_*,\lambda_*}\left\langle\frac{1}{2}\lambda_1\frac{1}{2}\lambda_2\middle|1s_1\right\rangle\left\langle\frac{1}{2}\lambda_3\frac{1}{2}\lambda_4\middle|1s_2\right\rangle\langle 1s_1 1s_2|Jm_j\rangle$$

$$\mathcal{C}^{ab;cd}_{\bar{\mathbf{3}}\otimes\mathbf{3}}\left|c^{\lambda_1}_a(q_1)c^{\lambda_2}_b(P-q_1)\bar{c}^{\lambda_3}_c(q_2)\bar{c}^{\lambda_4}_d(Q-P-q_2)\right\rangle$$

$$\left|\mathcal{T}^{0,0}_{\mathbf{6}\otimes\bar{\mathbf{6}}}(Q)\right\rangle = \frac{1}{2}\sum_{\lambda_*}\left\langle\frac{1}{2}\lambda_1\frac{1}{2}\lambda_2\middle|00\right\rangle\left\langle\frac{1}{2}\lambda_3\frac{1}{2}\lambda_4\middle|00\right\rangle$$

$$\mathcal{C}^{ab;cd}_{\mathbf{6}\otimes\bar{\mathbf{6}}}\left|c^{\lambda_1}_a(q_1)c^{\lambda_2}_b(P-q_1)\bar{c}^{\lambda_3}_c(q_2)\bar{c}^{\lambda_4}_d(Q-P-q_2)\right\rangle$$

Determine the SDCs

To project the $QQ\bar{Q}\bar{Q}$ into correct spin/color quantum number oftetraquark, we use the following projector

$$\bar{u}_i^a \bar{u}_j^b v_k^c v_l^d \to (C\Pi_\mu)^{ij} (\Pi_\nu C)^{lk} \mathcal{C}_{\bar{\mathbf{3}}\otimes\mathbf{3}}^{abcd} J_{0,1,2}^{\mu\nu}$$

$$\bar{u}_i^a \bar{u}_j^b v_k^c v_l^d \to (C\Pi_0)^{ij} (\Pi_0 C)^{lk} \mathcal{C}_{\mathbf{6}\otimes\bar{\mathbf{6}}}^{abcd}$$

C is the charge conjugate operator, $\Pi_\mu(\Pi_0)$ is thespin-triplet(singlet) projector of quarks (Petrelliet al., 1997), $J_{0,1,2}^{\mu\nu}$ are the spin projectors of quark and anti-quark pairs(Braaten, Lee, 2003).

$$J_0^{\mu\nu} = \frac{1}{\sqrt{3}} \eta^{\mu\nu}(P)$$

$$J_1^{\mu\nu}(\epsilon) = -\frac{i}{\sqrt{2P^2}} \varepsilon^{\mu\nu\rho\sigma} \epsilon_\rho P_\sigma$$

$$J_2^{\mu\nu}(\epsilon) = \epsilon_{\rho\sigma} \left\{ \frac{1}{2} \left[\eta^{\mu\rho}(P)\eta^{\nu\sigma}(P) + \eta^{\mu\sigma}(P)\eta^{\nu\rho}(P) \right] - \frac{1}{3} \eta^{\mu\nu}(P)\eta^{\rho\sigma}(P) \right\}$$

SDCs

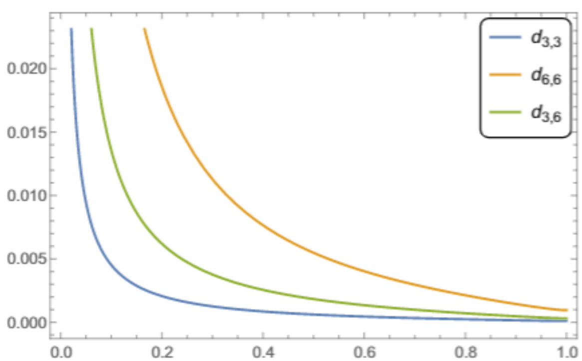
$$d_{3\times3}\left(g\to 0^{++}\right)=\frac{\pi^2\alpha_s^4}{497664z(2-z)^2(3-z)}\Big[186624-430272z+511072z^2-425814z^3$$
$$+217337z^4-61915z^5+7466z^6+42(1-z)(2-z)(3-z)(-144+634z$$
$$-385z^2+70z^3)\log(1-z)+36(2-z)(3-z)\left(144-634z+749z^2-364z^3\right.$$
$$\left.+74z^4\right)\log\left(1-\frac{z}{2}\right)+12(2-z)(3-z)\left(72-362z+361z^2-136z^3+23z^4\right)$$
$$\times\log\left(1-\frac{z}{3}\right)\Big].$$

$$d_{6\times6}\left(g\to 0^{++}\right)=\frac{\pi^2\alpha_s^4}{55296z(2-z)^2(3-z)}\Big[186624-430272z+617824z^2-634902z^3$$
$$+374489z^4-115387z^5+14378z^6-6(1-z)(2-z)(3-z)(-144-2166z$$
$$+1015z^2+70z^3)\log(1-z)-156(2-z)(3-z)\left(144-1242z+1693z^2-876z^3\right.$$
$$\left.+170z^4\right)\log\left(1-\frac{z}{2}\right)+300(2-z)(3-z)\left(72-714z+953z^2-472z^3+87z^4\right)$$
$$\times\log\left(1-\frac{z}{3}\right)\Big].$$

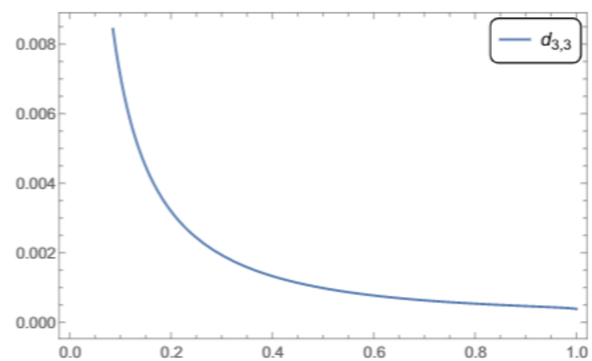
$$d_{3\times6}\left(g\to 0^{++}\right)=-\frac{\pi^2\alpha_s^4}{165888z(2-z)^2(3-z)}\Big[186624-430272z+490720z^2-394422z^3$$
$$+199529z^4-57547z^5+7082z^6+6(1-z)(2-z)(3-z)(-432+3302z$$
$$-1855z^2+210z^3)\log(1-z)-12(2-z)(3-z)\left(720-2258z+2329z^2-1052z^3\right.$$
$$\left.+226z^4\right)\log\left(1-\frac{z}{2}\right)+12(2-z)(3-z)\left(936-4882z+4989z^2-1936z^3+331z^4\right)$$
$$\times\log\left(1-\frac{z}{3}\right)\Big].$$

SDCs

$$
\begin{aligned}
d_{3\times 3}\left(g \to 2^{++}\right) =& \frac{\pi^2\alpha_s^4}{622080 z^2(2-z)^2(3-z)}\left[2\left(46656 - 490536 z + 1162552 z^2\right.\right. \\
& -1156308 z^3 + 595421 z^4 - 170578 z^5 + 21212 z^6\big) z \\
& +3(1-z)(2-z)(3-z)(-20304 - 31788 z)(1296 + 1044 z \\
& +73036 z^2 - 36574 z^3 + 7975 z^4\big)\log(1-z)\big] + 33(2-z)(3-z)(1296+25) \\
& -9224 z^2 + 9598 z^3 - 3943 z^4 + 725 z^5\big)\log\left(1-\frac{z}{3}\right)\Big], \\
d_{6\times 6}\left(g \to 2^{++}\right) =& d_{3\times 6}\left(g \to 2^{++}\right) = 0.
\end{aligned}
$$



SDCs for T_{4c}^{0++}



SDC for T_{4c}^{2++}

Long-Distance Matrix Elements(LDMEs)

- The NRQCD LDMEs should be calculated in lattice QCD in principle since they are non-perturbative.
- We use the diquark model to calculate the LDMEs, resulting in the product of wave functions at the origin.

$$\left|\left\langle T_{4c}^{0}\left|\mathcal{O}_{\bar{\mathbf{3}}\otimes\mathbf{3}}^{(0)}\right|0\right\rangle\right|^2 = \frac{1}{4\pi^3}\left|R_{\mathcal{D}}(0)\right|^4\left|R_{\mathcal{T}}(0)\right|^2,$$

$$\sum_{m_j}\left|\left\langle T_{4c}^{2,m_j}\left|\mathcal{O}_{\bar{\mathbf{3}}\otimes\mathbf{3}}^{(2)kl}\right|0\right\rangle\right|^2 = \frac{5}{4\pi^3}\left|R_{\mathcal{D}}(0)\right|^4\left|R_{\mathcal{T}}(0)\right|^2.$$

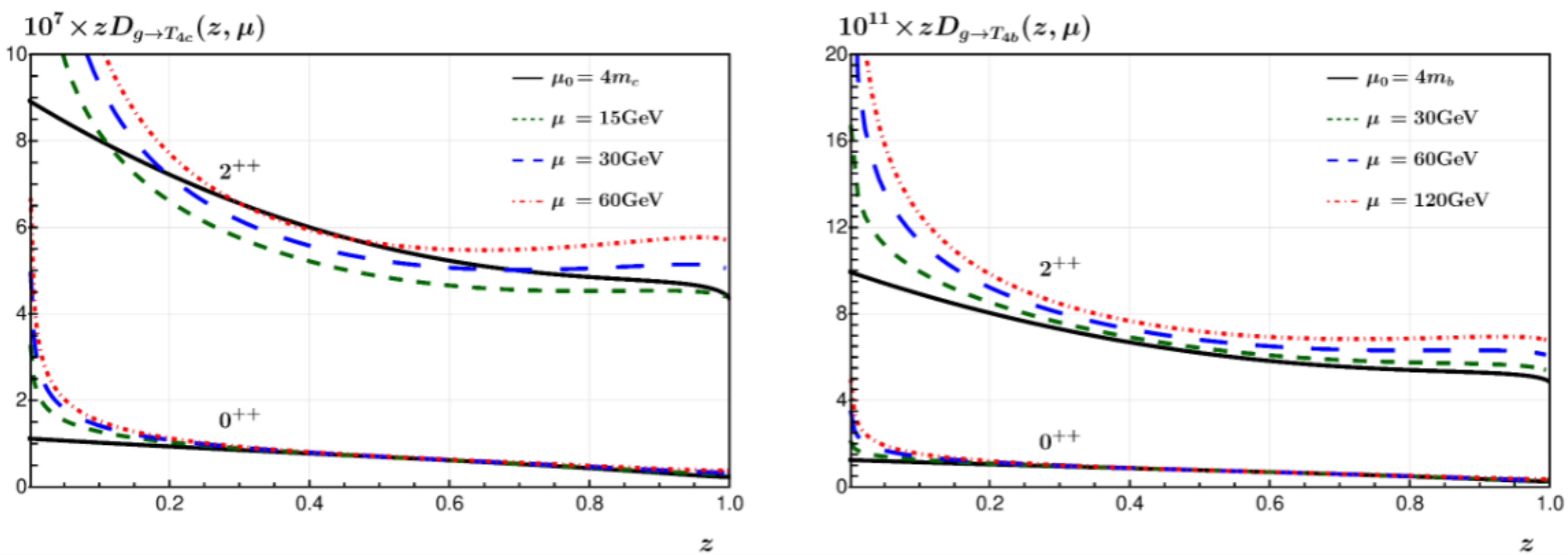
- The fock component of $\mathbf{6}\otimes\bar{\mathbf{6}}$ is neglected for simplicity while there are some results in literature(L ü,et al., 2020; Zhao,etal., 2020).
- The phenomenological results we use (GeV$^{3/2}$)(Kiselevetal., 2002; Debastiani, Navarra,

	$R_{\mathcal{D}}(0)$	$R_{\mathcal{T}^0}(0)$	$R_{\mathcal{T}^2}(0)$
T_{4c}	0.523	2.902	2.583
T_{4b}(Coulomb)	0.703	5.57909	5.57909

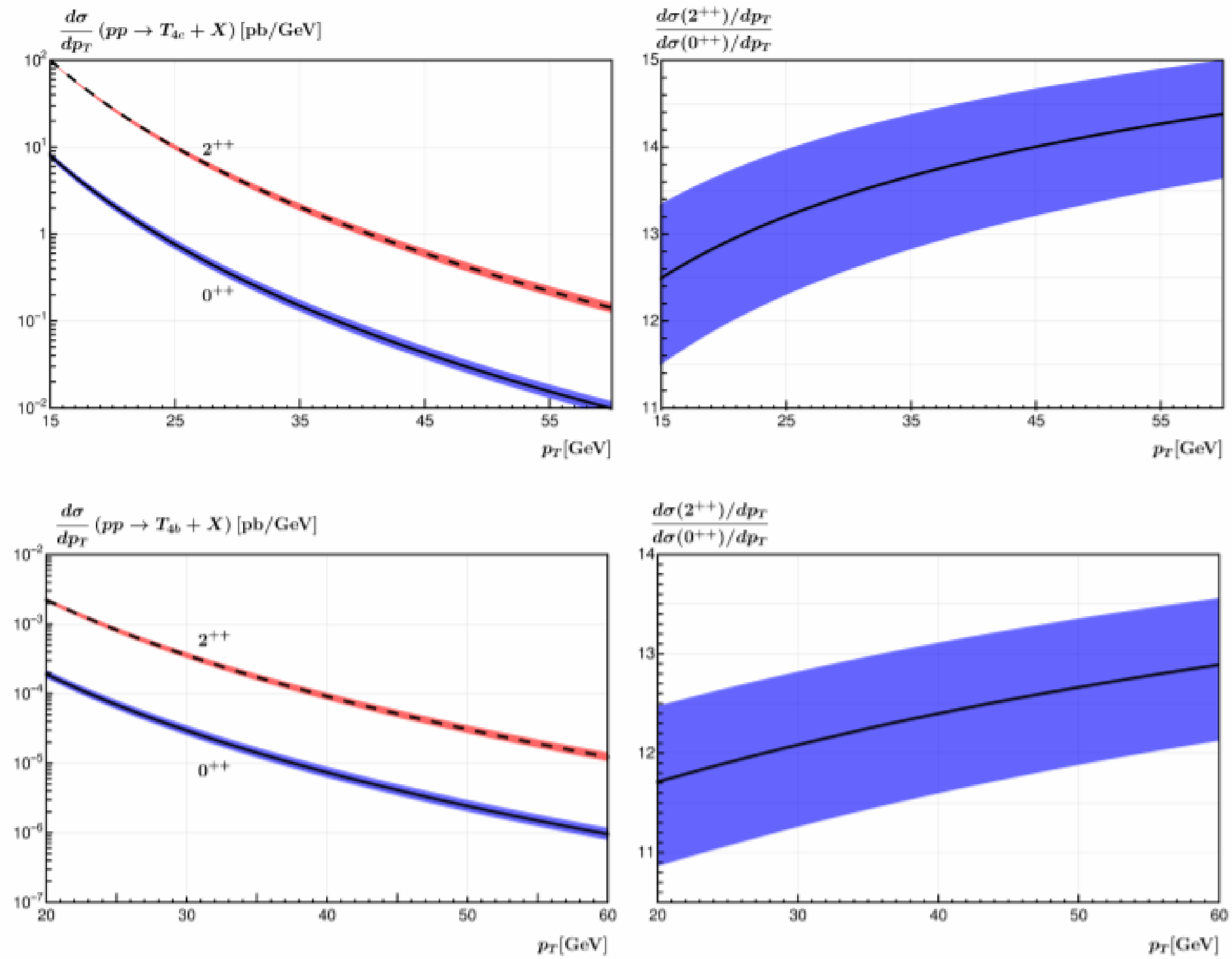
Evolution of Fragmentation Function under DGLAP

- Since the process is gluon dominance, the leading order splitting kernels read (nf: number of active light quark flavors):

$$P_{g\leftarrow g}(z) = 6\left[\frac{(1-z)}{z} + \frac{z}{(1-z)_+} + z(1-z) + \left(\frac{11}{12} - \frac{n_f}{18}\right)\delta(1-z)\right]$$



Phenomenology at LHC



Phenomenology at LHC

- $\sqrt{s} = 13\,\mathrm{TeV}$
- CTEQ14PDF sets
- Factorization scale $\mu \in [p_T/2, 2p_T]$

	p_T range	J^{PC}	σ	N_{events}
T_{4c}	$15\text{GeV} \leq p_T \leq 60\text{GeV}$	0^{++}	33^{+4}_{-4}pb	$9.9^{+1.2}_{-1.2} \times 10^7$
		2^{++}	424^{+13}_{-21}pb	$1.27^{+0.04}_{-0.06} \times 10^9$
T_{4b}	$20\text{GeV} \leq p_T \leq 60\text{GeV}$	0^{++}	$1.04^{+0.17}_{-0.15} \times 10^{-3}\text{pb}$	$3.12^{+0.51}_{-0.45} \times 10^3$
		2^{++}	$1.24^{+0.11}_{-0.11} \times 10^{-2}\text{pb}$	$3.72^{+0.33}_{-0.33} \times 10^4$

NRQCD Factorization for inclusive production of 1^{+-} tetraquark at B factory

For the Inclusive production of fully-charmed 1^{+-} tetraquark at B factory

$$d\sigma(e^+e^- \to T_{4c}(E) + X) = \sum_n \frac{\mathrm{d}F_n(E)}{m_c^8}(2M_{T_{4c}})\langle 0|\mathcal{O}_n^{T_{4c}}|0\rangle,$$

$$\mathcal{O}_{\bar{\mathbf{3}}\otimes\mathbf{3}}^{T_{4c}} = \sum_{m_j,X} \mathcal{O}_{\bar{\mathbf{3}}\otimes\mathbf{3}}^{i\dagger}|T_{4c}(m_j)+X\rangle\langle T_{4c}(m_j)+X|\mathcal{O}_{\bar{\mathbf{3}}\otimes\mathbf{3}}^{i},$$

Vacuum-saturation approximation

$$\left\langle 0\left|\mathcal{O}_0^T\right|0\right\rangle \to \sum_X \left\langle 0\left|\mathcal{O}_{\text{color}}^{(J)}\right|T_{4c}^{(J)}+X\right\rangle \left\langle T_{4c}^{(J)}+X\left|\mathcal{O}_{\text{color}}^{(J)\dagger}\right|0\right\rangle \to \left|\left\langle 0\left|\mathcal{O}_{\text{color}}^{(J)}\right|T_{4c}^{(J)}\right\rangle\right|^2$$

Inclusive production of fully-charmed 1^{+-} tetraquark at B factory

There are roughly **400** Feynman diagrams in total.

C -parity conservation requires that two additional gluons emitted accompanied with C-odd T4c

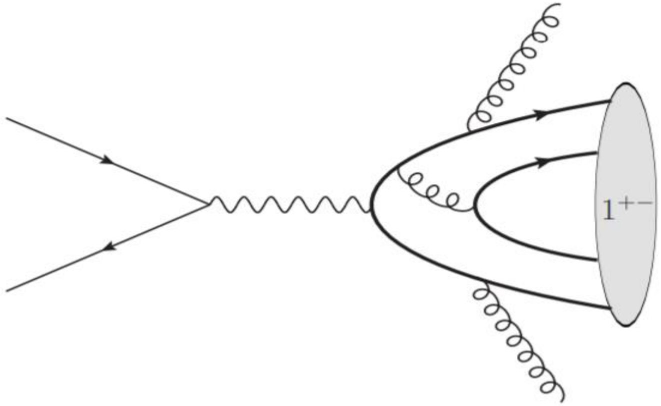


FIG. 1: One of 392 Feynman diagrams for $e^+e^- \to T_{4c}(1^{+-}) + gg$ at $\mathcal{O}(\alpha_s^4)$.

Kinematics

To deduce the SDC with the differential production rate, we need further integrate over the phase space integration of the gluons recoiling against tetraquark.

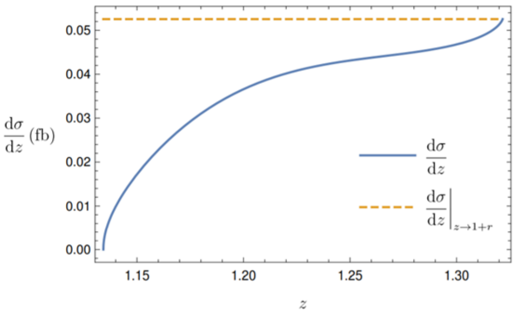
$$\int d\Phi_3 = \frac{s}{2(4\pi)^3} \int_{2\sqrt{r}}^{1+r} \mathrm{d}z \int_{x_1^-}^{x_1^+} \mathrm{d}x_1$$

$$x_1^{\pm} = \frac{1}{2}(2-z) \pm \frac{1}{2}\sqrt{z^2 - 4r}.$$

Differential cross section (asymptotic)

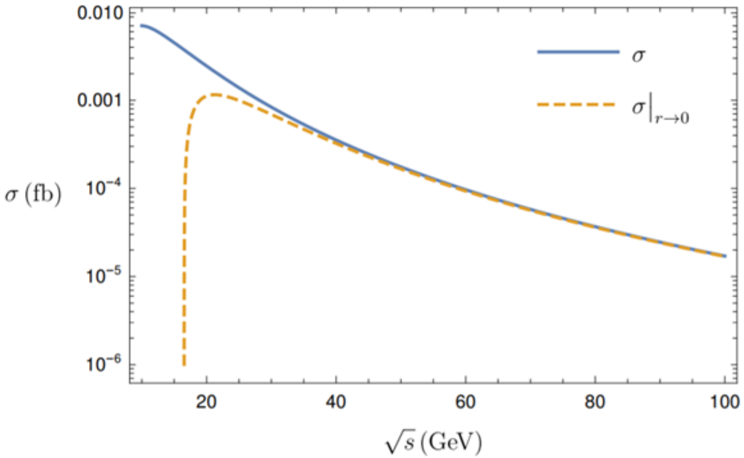
The full analytical expression is too lengthy to be presented here. We choose to present its limiting value near the upper endpoint:

$$\left.\frac{\mathrm{d}F_{\bar{\mathbf{3}}\otimes\mathbf{3}}}{\mathrm{d}z}\right|_{z\to 1+r} = \frac{2^2\pi^3\alpha^2\alpha_s^4}{3^8 s^2(3-r)^2(2-r)^2(3+r)(6+r)} \tag{11}$$
$$\times\Big(550800 + 482112\ln 2 - 803628r - 183168r\ln 2 + 275616r\ln r$$
$$+ 27\left(17856 - 16992r - 844r^2 + 4764r^3 - 779r^4 - 336r^5 + 70r^6 + r^7\right)\ln(2-r)$$
$$+ 16\left(-30132 + 11448r - 3897r^2 + 8403r^3 - 2489r^4 - 475r^5 + 166r^6\right)\ln(3-r)$$
$$+ 235854r^2 + 62352r^2\ln 2 + 85140r^2\ln r + 62742r^3 - 134448r^3\ln 2 - 263076r^3\ln r$$
$$- 50316r^4 + 39824r^4\ln 2 + 60857r^4\ln r + 2706r^5 + 7600r^5\ln 2 + 16672r^5\ln r$$
$$+ 1842r^6 - 2656r^6\ln 2 - 4546r^6\ln r - 27r^7\ln r\Big)$$



Integrated cross section (asymptotic)

$$F_{\bar{\mathbf{3}}\otimes\mathbf{3}}\Big|_{r\to 0} = \frac{\pi^3\alpha^2\alpha_s^4}{2^2 3^8 s^2}\Bigg[48(288\ln 3 - 167)\ln\left(\frac{s}{16m_c^2}\right) - 417996\mathrm{Li}_2\left(\frac{1}{3}\right) - 3744\mathrm{Li}_2\left(\frac{3}{8}\right)$$
$$+ 43005\pi^2 - 386712 + 98082\ln^2 3 + 34128\ln^2 2 + 486032\ln 2 + 55296\ln 2\ln 3$$
$$- 218456\ln 3 + 11232\ln 2\ln 5 - 3744\ln 3\ln 5 + 167920\coth^{-1} 2\Bigg]. \qquad (12)$$



Phenomenology at B Factory

- Unfortunately, this cross section is extreme tiny. Pessimistic to observe at Belle 2

- Nevertheless, it should be cautioned that nonperturbative factors using diquark-antidiqaurk model, very crude approximation)

$$\sigma(e^+e^- \to T_{4c}(1^{+-}) + X) \approx 0.0069\,\mathrm{fb}.$$

Summary and outlook

- We develop a model-independent approach to study production of fully heavy tetraquark based on NRQCD factorization.
- The production rates appears to be significant on the LHC due to the huge luminosity.
- The production rates appear to be too small to be observed at Belle 2 experiment; numerics very sensitive to LDME
- Nevertheless, NRQCD matrix elements are poorly known to date; more reliable phenomenological estimate beyond the naïve diquark model will be useful
- P-wave fully-charmed tetraquark production

Thanks for your attention!