## Class－I $B_{q}^{0} \rightarrow D_{q}^{(*)-} L^{+}$decays at NNLO

## and possible New Physics

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第三届重味物理和量子色动力学研讨会，南开，2021／05／01

## Outline

$\square$ Introduction
Class－I $\overline{\boldsymbol{B}}_{q}^{\mathbf{0}} \rightarrow \mathbf{D}_{q}^{(*)+} \boldsymbol{L}^{-}$
$b \rightarrow c \bar{u} d(s)$

$\square$ NNLO predictions at leading power in QCDF／SCET
－Possible New Physics effects from four－quark operators
－Summary

## Introduction

## Why hadronic B decays

$\square$ direct access to the CKM parameters， especially to the three angles of UT．

$\square$ further insight into strong－interaction effects involved in these decays．

$\square$ Thanks to exp．\＆theo．，entering a precision flavor era！


## Effective Hamiltonian for B decays

$\square$ For hadronic decays：simplicity of weak interactions overshadowed by complex QCD effects！

$\square$ Starting point $\mathcal{H}_{\text {eff }}=-\mathcal{L}_{\text {eff }}$ ：obtained after

$$
\mathcal{L}_{\mathrm{eff}}=-\frac{G_{F}}{\sqrt{2}} \sum_{p=u, c} V_{p b} V_{p D}^{*}\left(C_{1} \mathcal{O}_{1}+C_{2} \mathcal{O}_{2}+\sum_{i=\mathrm{pen}} C_{i} \mathcal{O}_{i, \mathrm{pen}}\right)
$$ integrating out the heavy d．o．f．$\left(m_{W, Z, t} \gg m_{b}\right)$ ；

［Buras，Buchalla，Lautenbacher＇96；Chetyrkin，Misiak，Munz＇98］
$\square$ Wilson coefficients $\boldsymbol{C}_{\boldsymbol{i}}$ ：all physics above $m_{b}$ ；perturbatively
 calculable，and NNLL program now complete；［Gorbahn，Haisch＇04］

## Hadronic matrix elements

$\square$ Decay amplitude for a given decay mode:

$$
\mathcal{A}(\bar{B} \rightarrow f)=\sum_{i}\left[\lambda_{\mathrm{CKM}} \times C \times\langle f| \mathcal{O}|\bar{B}\rangle_{\mathrm{QCD}+\mathrm{QED}}\right]_{i}
$$

$\boldsymbol{\square}\left\langle\boldsymbol{M}_{\mathbf{1}} \boldsymbol{M}_{\mathbf{2}}\right| \boldsymbol{\mathcal { O }}_{\boldsymbol{i}}|\overline{\boldsymbol{B}}\rangle$ : depend on spin and parity of $M_{1,2}$; final-state re-scattering introduces strong phases, and hence non-zero direct CPV; $\quad$ A quite difficult, multi-scale, strong-interaction problem!
$\square$ Different methods for dealing with $\left\langle M_{1} M_{2}\right| \mathcal{O}_{i}|\bar{B}\rangle$ :

- Dynamical approaches based on factorization theorems: PQCD, QCDF, SCET,

Symmetries of QCD: Isospin, U-Spin, V-Spin, and flavour SU(3) symmetries,
[ Zeppenfeld, ' 81
London, Gronau, Rosner, He, Chiang, Cheng et al.]

- QCDF: systematic framework to all orders in $\alpha_{s}$, but limited by $1 / m_{b}$ corrections. [BBNs '99-03]



$+O\left(1 / m_{\mathrm{b}}\right)$
Class-I B decays into heavy-light final states and possible New Physics


## Soft－collinear factorization from SCET

－QCDF formula：based on diagrammatic factorization（method of regions， combining $1 / m_{b}$ expansion with light－cone expansion for hard processes；
［Lepage，Brodsky＇80］
－SCET：a suitable framework for studying factorization and re－summation for processes involving light but energetic particles；［Bauer etal．＇00；Beneke etal．＇02；Becher，Broggio，Ferroglia＇14］

ㅁ For a two－body decay：simple kinematics，but complicated dynamics with several typical scales；

－low－virtuality modes：
＊HQET fields：$p-m_{b} v \sim \mathcal{O}(\Lambda)$
＊soft spectators in $B$ meson：
$p_{s}^{\mu} \sim \Lambda \ll m_{b}, \quad p_{s}^{2} \sim \mathcal{O}\left(\Lambda^{2}\right)$
＊collinear quarks and gluons in pion： $E_{c} \sim m_{b}, \quad p_{c}^{2} \sim \mathcal{O}\left(\Lambda^{2}\right)$
－high－virtuality modes：
＊hard modes： （heavy quark + collinear）${ }^{2} \sim \mathcal{O}\left(m_{b}^{2}\right)$
＊hard－collinear modes： （soft + collinear $)^{2} \sim \mathcal{O}\left(m_{b} \Lambda\right)$
$\pi^{\pi^{+}} \square$ SCET point of view：introduce different fields for different momentum regions；
$\longrightarrow$ achieve soft－collinear factorization via QFT machinery！［Beneke，1501．07374］

## Soft－collinear factorization from SCET

$\square$ SCET diagrams reproduce precisely QCD diagrams in collinear \＆soft momentum regions

$\square$ For hard kernel $\boldsymbol{T}^{\boldsymbol{I}}$ ：one－step matching， $\mathrm{QCD} \rightarrow \operatorname{SCET}_{\mathrm{I}}(\mathrm{hc}, \mathrm{c}, \mathrm{s})$ ！

$\square$ For hard kernel $\boldsymbol{T}^{I I}$ ：two－step matching， $\mathrm{QCD} \rightarrow \operatorname{SCET}_{\mathrm{I}}(\mathrm{hc}, \mathrm{c}, \mathrm{s}) \rightarrow \operatorname{SCET}_{\mathrm{II}}(\mathrm{c}, \mathrm{s})$ ！


ㅁ SCET result exactly the same as QCDF，but more apparent \＆efficient；［Beneke，1501．07374］

## Status of the NNLO calculation of $T^{I} \& T^{I I}$

$\square$ For each $Q_{i}$ insertion，both tree $\&$ penguin topologies，and contribute to both $T^{I} \& T^{I I}$ ．

$$
\left\langle M_{1} M_{2}\right| Q_{i}|B\rangle \simeq F^{B M_{1}} T_{i}^{\prime} \otimes \phi_{M_{2}}+T_{i}^{\prime \prime} \otimes \phi_{B} \otimes \phi_{M_{1}} \otimes \phi_{M_{2}}
$$



## Status of the NNLO calculation of $T^{I} \& T^{I I}$

$\square$ Complete NNLO calculation for $T^{I} \& T^{I I}$ at leading power in QCDF／SCET now complete；
$\square$ Soft－collinear factorization at 2－loop established via explicit calculations；
$\square$ For tree amplitudes，cancellation between $T^{I} \& T^{I I}$ ；


$$
\begin{aligned}
\alpha_{1}(\pi \pi)= & 1.009+[0.023+0.010 i]_{\mathrm{NLO}}+[0.026+0.028 i]_{\mathrm{NNLO}} \\
& -\left[\frac{r_{\mathrm{sp}}}{0.445}\right]\left\{[0.014]_{\mathrm{LOsp}}+[0.034+0.027 i]_{\mathrm{NLOsp}}+[0 .\right. \\
= & 1.000_{-0.069}^{+0.029}+\left(0.011_{-0.050}^{+0.023}\right) i
\end{aligned}
$$

$$
-\left[\frac{r_{\mathrm{sp}}}{0.445}\right]\left\{[0.014]_{\mathrm{LOsp}}+[0.034+0.027 i]_{\mathrm{NLOsp}}+[0.008]_{\mathrm{tw} 3}\right\} \alpha_{2}(\pi \pi)=0.220-[0.179+0.077 i]_{\mathrm{NLO}}-[0.031+0.050 i]_{\mathrm{NNLO}}
$$

$$
+\left[\frac{r_{\mathrm{sp}}}{0.445}\right]\left\{[0.114]_{\mathrm{LOsp}}+[0.049+0.051 i]_{\mathrm{NLOsp}}+[0.067]_{\mathrm{ww} 3}\right\}
$$

$$
=0.240_{-0.125}^{+0.217}+\left(-0.077_{-0.078}^{+0.115}\right) i
$$

$\square$ For leading－power QCD penguin amplitudes，cancellation between $\boldsymbol{Q}_{1,2}^{p} \boldsymbol{\&} \boldsymbol{Q}_{3-6,8 g}$


$$
\begin{aligned}
a_{4}^{u}(\pi \bar{K}) / 10^{-2}= & -2.87-[0.09+0.09 i]_{\mathrm{v}_{1}}+[0.49-1.32 i]_{\mathrm{P}_{1}}-[0.32+0.71 i]_{\mathrm{P}_{2}, \mathrm{Q}_{1,2}}+[0.33+0.38 i]_{\mathrm{P}_{2}, \mathrm{Q}_{3-6,8}} \\
& +\left[\frac{r_{\mathrm{sp}}}{0.434}\right]\left\{[0.13]_{\mathrm{LO}}+[0.14+0.12 i]_{\mathrm{HV}}-[0.01-0.05 i]_{\mathrm{HP}}+[0.07]_{\mathrm{tw} 3}\right\} \\
= & \left(-2.12_{-0.29}^{+0.48}\right)+\left(-1.56_{-0.15}^{+0.29}\right) i,
\end{aligned}
$$

# Class－I $\bar{B}_{q}^{0} \rightarrow D_{q}^{(*)+} \boldsymbol{L}^{-}$decays at NNLO in QCDF／SCET 

$B \rightarrow D^{(*)} L$ decays
$\square$ At quark－level：mediated by $b \rightarrow c \bar{u} d(s)$
all four flavors different from each other，no penguin operators \＆no penguin topologies！

$\square$ For class－I decays：QCDF formula much simpler；
［Beneke，Buchalla，Neubert，Sachrajda＇99－＇03；Bauer，Pirjol，Stewart＇01］

$$
\begin{aligned}
& \mathcal{Q}_{2}=\bar{d} \gamma_{\mu}\left(1-\gamma_{5}\right) u \bar{c} \gamma^{\mu}\left(1-\gamma_{5}\right) b \\
& \mathcal{Q}_{1}=\bar{d} \gamma_{\mu}\left(1-\gamma_{5}\right) T^{A} u \bar{c} \gamma^{\mu}\left(1-\gamma_{5}\right) T^{A} b
\end{aligned}
$$

$$
\begin{aligned}
\left\langle D_{q}^{(*)+} L^{-}\right| \mathcal{Q}_{i}\left|\bar{B}_{q}^{0}\right\rangle & =\sum_{j} F_{j}^{\bar{B}_{q} \rightarrow D_{q}^{(*)}}\left(M_{L}^{2}\right) \\
& \times \int_{0}^{1} d u T_{i j}(u) \phi_{L}(u)+\mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}}{m_{b}}\right)
\end{aligned}
$$

i）only color－allowed tree topology $a_{1}$ ；
ii）spectator \＆annihilation are power－suppressed；
iii）annihilation absent in $B_{d(s)}^{0} \rightarrow D_{d(s)}^{-} K(\pi)^{+}$etal；
iv）they are theoretically simpler and cleaner！
－Hard kernel T：both NLO and NNLO results known；
［Beneke，Buchalla，Neubert，Sachrajda＇01；Huber，Kränkl，Li＇16］

$$
T=T^{(0)}+\alpha_{s} T^{(1)}+\alpha_{s}^{2} T^{(2)}+O\left(\alpha_{s}^{3}\right)
$$

## Calculation of $T$ ：

$\square$ Matching QCD onto $\mathbf{S C E T}_{\mathbf{I}}$ ：［Huber，Kränkl，Li＇16］
$m_{c}$ is also heavy，keep $m_{c} / m_{b}$ fixed as $m_{b} \rightarrow \infty$ ， thus needing two sets of SCET operator basis．
$\left\langle\mathcal{Q}_{i}\right\rangle=\hat{T}_{i}\left\langle\mathcal{Q}^{\mathrm{QCD}}\right\rangle+\hat{T}_{i}^{\prime}\left\langle\mathcal{Q}^{\prime \mathrm{QCD}}\right\rangle+\sum_{a>1}\left[H_{i a}\left\langle\mathcal{O}_{a}\right\rangle+H_{i a}^{\prime}\left\langle\mathcal{O}_{a}^{\prime}\right\rangle\right]$
$\square$ Renormalized on－shell QCD amplitudes：

$$
\begin{aligned}
\left\langle\mathcal{Q}_{i}\right\rangle= & \left\{A_{i a}^{(0)}+\frac{\alpha_{s}}{4 \pi}\left[A_{i a}^{(1)}+Z_{e x t}^{(1)} A_{i a}^{(0)}+Z_{i j}^{(1)} A_{j a}^{(0)}\right] \quad\right. \text { on QCD side } \\
& +\left(\frac{\alpha_{s}}{4 \pi}\right)^{2}\left[A_{i a}^{(2)}+Z_{i j}^{(1)} A_{j a}^{(1)}+Z_{i j}^{(2)} A_{j a}^{(0)}+Z_{e x t}^{(1)} A_{i a}^{(1)}+Z_{e x t}^{(2)} A_{i a}^{(0)}+Z_{e x t}^{(1)} Z_{i j}^{(1)} A_{j a}^{(0)}\right. \\
& \left.+(-i) \delta m_{b}^{(1)} A_{i a}^{*(1)}+(-i) \delta m_{c}^{(1)} A_{i a}^{*(1)}\right) \\
& +\left(A \leftrightarrow A^{\prime}\right)\left\langle\mathcal{O}_{a}^{\prime}\right\rangle^{(0)} .
\end{aligned}
$$

## Renormalized on－shell SCET amplitudes：

$$
\begin{aligned}
\left\langle\mathcal{O}_{a}\right\rangle=\left\{\delta_{a b}\right. & +\frac{\hat{\alpha}_{s}}{4 \pi}\left[M_{a b}^{(1)}+Y_{e x t}^{(1)} \delta_{a b}+Y_{a b}^{(1)}\right] \quad \text { on SCET side } \\
& +\left(\frac{\hat{\alpha}_{s}}{4 \pi}\right)^{2}\left[M_{a b}^{(2)}+Y_{e x t}^{(1)} M_{a b}^{(1)}+Y_{a c}^{(1)} M_{c b}^{(1)}+\hat{Z}_{\alpha}^{(1)} M_{a b}^{(1)}+Y_{e x t}^{(2)} \delta_{a b}\right. \\
& \left.\left.+Y_{e x t}^{(1)} Y_{a b}^{(1)}+Y_{a b}^{(2)}\right]+\mathcal{O}\left(\hat{\alpha}_{s}^{3}\right)\right\}\left\langle\mathcal{O}_{b}\right\rangle^{(0)},
\end{aligned}
$$

## Calculation of $T$ ：

$$
T=T^{(0)}+\alpha_{s} T^{(1)}+\alpha_{s}^{2} T^{(2)}+O\left(\alpha_{s}^{3}\right)
$$

ㅁ $A_{i 1}^{(0)}$ ：

－ $\boldsymbol{A}_{\boldsymbol{i 1}}^{(\mathbf{1}) \boldsymbol{n f}}$ ：［Beneke，Buchalla，Neubert，Sachrajda＇01］


ㅁ $\boldsymbol{A}_{\boldsymbol{i 1}}^{(\mathbf{2}) \boldsymbol{n f}}$ ：［Huber，Kränkl，Li＇16］



$\boldsymbol{\mathcal { O }}(70)$ two－loop two－scale non－factorizable QCD diagrams；their calculations need advanced analytical techniques！［Huber，Kränkl＇15］


## Calculation of $T$ ：

$\square$ Master formulas for hard kernels：
Complete operator basis under renormalization：

$\square$ Factorizable QCD and SCET diagrams：

$$
\begin{aligned}
& \mathcal{Q}_{1}^{p}=\bar{p} \gamma^{\mu}\left(1-\gamma_{5}\right) T^{A} b \bar{d} \gamma_{\mu}\left(1-\gamma_{5}\right) T^{A} u, \\
& \mathcal{Q}_{2}^{p}=\bar{p} \gamma^{\mu}\left(1-\gamma_{5}\right) b \bar{d} \gamma_{\mu}\left(1-\gamma_{5}\right) u, \\
& E_{1}^{(1)}=\left[\bar{c} \gamma^{\mu} \gamma^{\nu} \gamma^{\rho}\left(1-\gamma_{5}\right) T^{A} b\right]\left[\bar{u} \gamma_{\mu} \gamma_{\nu} \gamma_{\rho}\left(1-\gamma_{5}\right) T^{A} d\right]-16 Q_{1}^{c}, \\
& E_{2}^{(1)}=\left[\bar{c} \gamma^{\mu} \gamma^{\nu} \gamma^{\rho}\left(1-\gamma_{5}\right) b\right]\left[\bar{\psi} \gamma_{\mu} \gamma_{\nu} \gamma_{\rho}\left(1-\gamma_{5}\right) d\right]-16 Q_{2}^{c} \text {, } \\
& E_{1}^{(2)}=\left[\bar{c} \gamma^{\mu} \gamma^{\mu} \gamma^{\rho} \gamma^{\sigma} \gamma^{\lambda}\left(1-\gamma_{5}\right) T^{A} b\right]\left[\overline{[ } \gamma_{\mu} \gamma_{\nu} \gamma_{\rho} \gamma_{\sigma} \gamma_{\lambda}\left(1-\gamma_{5}\right) T^{A} d\right]-20 E_{1}^{(1)}-256 Q_{1}^{c} \\
& E_{1}^{(2)}=\left[\bar{c} \gamma^{\mu} \gamma^{\nu} \gamma^{\rho} \gamma^{\sigma} \gamma^{1}\left(1-\gamma_{5}\right) b\right]\left[\overline{[ } \gamma_{\mu} \gamma_{\nu} \gamma_{\rho} \gamma_{\sigma} \gamma_{\lambda}\left(1-\gamma_{5}\right) d\right]-20 E_{2}^{(1)}-256 Q_{2}^{c} \text {. } \\
& \text { evanescent operators }
\end{aligned}
$$


fact．2－loop

fact．1－loop
$\square$ Renormalization constants of SCET operators：


## Decay amplitudes for $B_{q}^{0} \rightarrow D_{q}^{-} L^{+}$

－Color－allowed tree amplitude：

$$
\begin{aligned}
& a_{1}\left(D^{+} L^{-}\right)=\sum_{i=1}^{2} C_{i}(\mu) \int_{0}^{1} d u\left[\hat{T}_{i}(u, \mu)+\hat{T}_{i}^{\prime}(u, \mu)\right] \Phi_{L}(u, \mu), \\
& a_{1}\left(D^{*+} L^{-}\right)=\sum_{i=1}^{2} C_{i}(\mu) \int_{0}^{1} d u\left[\hat{T}_{i}(u, \mu)-\hat{T}_{i}^{\prime}(u, \mu)\right] \Phi_{L}(u, \mu),
\end{aligned}
$$

Numerical result：

$$
\begin{aligned}
a_{1}\left(D^{+} K^{-}\right) & =1.025+[0.029+0.018 i]_{\mathrm{NLO}}+[0.016+0.028 i]_{\mathrm{NNLO}} \\
& =\left(1.069_{-0.012}^{+0.009}\right)+\left(0.046_{-0.015}^{+0.023}\right) i,
\end{aligned}
$$


－both NLO and NNLO add always constructively to LO result！
$\bullet$ NNLO corrections quite small in real（2\％），but rather large in imaginary part（60\％）．
within QCDF／SCET，imaginary part appears firstly at NLO term and the NLO result is
color－suppressed and $\propto$ small $C_{1}=-0.29$ ，while the NNLO term $\propto C_{2}=1.01$ ．

## Scя

Due to perturbative truncation，$a_{1}$ depends on the renormalization scale．


－blue：pole scheme for $\boldsymbol{m}_{\boldsymbol{c}}$ and $\boldsymbol{m}_{\boldsymbol{b}}$
－red：$\overline{\mathrm{MS}}$ scheme for $m_{c}$ and $\boldsymbol{m}_{b}$
＞scale dependence＠NNLO reduced for the real part，but not so obvious for the imaginary part．
＞dependence on the b －and c －quark mass scheme is quite small，especially for the real part．

$$
\begin{array}{|l|}
\hline a_{1}\left(D^{+} K^{-}\right)=\left(1.069_{-0.012}^{+0.009}\right)+\left(0.046_{-0.015}^{+0.023}\right) i, \\
a_{1}\left(D^{+} \pi^{-}\right)=\left(1.072_{-0.011}^{+0.011}\right)+\left(0.043_{-0.0024}^{+0.02}\right) i, \\
a_{1}\left(D^{*+} K^{-}\right)=\left(1.068_{-0.012}^{+0.010}\right)+\left(0.034_{-0.011}^{+0.011}\right) i \\
a_{1}\left(D^{*+} \pi^{-}\right)=\left(1.071_{-0.013}^{+0.012}\right)+\left(0.032_{-0.010}^{+0.016}\right) i . \\
\hline
\end{array}
$$

$\square$ For different decay modes：quasi－universal，with small process－dep．from non－fact．correction．

## Absolute branching ratios for $B_{q}^{0} \rightarrow D_{q}^{-} L^{+}$

## $\square B \rightarrow D^{(*)}$ transition form factors：

Precision results available based on LQCD \＆LCSR
calculations，together with data on $B_{q}^{0} \rightarrow D_{q}^{-} l^{+} v$ ；

|  |
| :---: |
|  |  |

［Bernlochner，Ligeti，Papucci，Robinson＇17；Bordone，Gubernari，Jung，van Dyk＇19

| －35－ $3 \rightarrow D I \bar{v}_{1}$ | Decay mode | LO | NLO | NNLO | Ref．［36］ | Exp．［7，8］ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| ＂ $30{ }^{30} \quad B \rightarrow D \tau \bar{v}_{\tau}$ | $\bar{B}^{0} \rightarrow D^{+} \pi^{-}$ | 4.07 | $4.32_{-0.42}^{+0.23}$ | $4.43_{-0.41}^{+0.20}$ | $3.93_{-0.42}^{+0.43}$ | $2.65 \pm 0.15$ |
| 3 <br>  | $\bar{B}^{0} \rightarrow D^{*+} \pi^{-}$ | 3.65 | $3.88{ }_{-0.41}^{+0.27}$ | $4.00_{-0.41}^{+0.25}$ | $3.45_{-0.50}^{+0.53}$ | $2.58 \pm 0.13$ |
| E15 | $\bar{B}^{0} \rightarrow D^{+} \rho^{-}$ | 10.63 | $11.28_{-1.23}^{+0.84}$ | $11.59_{-1.21}^{+0.79}$ | $10.42_{-1.20}^{+1.24}$ | $7.6 \pm 1.2$ |
| ${ }_{\sim}^{\circ} 10$ | $\bar{B}^{0} \rightarrow D^{*+} \rho^{-}$ | 9.99 | $10.61_{-1.56}^{+1.35}$ | $10.93_{-1.57}^{+1.35}$ | $9.24_{-0.71}^{+0.72}$ | $6.0 \pm 0.8$ |
| －Bernlochner，Ligeti，Papucti，Robinson＇17 | ${ }^{-\bar{B}^{0} \rightarrow D^{+} K^{-}}$ | 3.09 | $3.28_{-0.31}^{+0.16}$ | $3.38_{-0.30}^{+0.13}$ | $3.01-0.31$ | $2.19 \pm 0.13$ |
| $\begin{array}{llllll}1.0 & 1.1 & 1.2 & 1.3 & 1.4 & 1.5 \\ & & & w & & \end{array}$ | $\bar{B}^{0} \rightarrow D^{*+} K^{-}$ | 2.75 | $2.92{ }_{-0.30}^{+0.19}$ | $3.02_{-0.30}^{+0.18}$ | $2.59_{-0.37}^{+0.39}$ | $2.04 \pm 0.47$ |
| $\square$ Updated predictions vs data： | $\bar{B}^{0} \rightarrow D^{+} K^{*-}$ | 5.33 | $5.65{ }_{-0.64}^{+0.47}$ | $5.788_{-0.63}^{+0.44}$ | $5.25_{-0.63}^{+0.65}$ | $4.6 \pm 0.8$ |
| ［Huber，Kränkl，Li＇16；Cai，Deng，Li，Yang＇21］ | ${ }^{+\cdots \cdots \cdots \cdots}{ }^{0}$ | 4.10 | 4．35－0．4． | 4．47－0．0．21 | 4．3－1．36 | － |
| $\left\|V_{c b}\right\|$ and $B_{d, s} \rightarrow D_{d, s}^{(*)}$ form factors | $\bar{B}_{s}^{0} \rightarrow D_{s}^{+} K^{-}$ | 3.12 | $3.322_{-0.32}^{+0.17}$ | $3.42_{-0.31}^{+0.14}$ | $3.344_{-0.90}^{+1.04}$ | $1.92 \pm 0.22$ |

## Non－leptonic／semi－leptonic ratios

$\square$ Non－leptonic／semi－leptonic ratios ：［Bjorken＇89；Neubert，Stech＇97；Beneke，Buchalla，Neubert，Sachrajda＇01］

$$
R_{(s) L}^{(*)} \equiv \frac{\Gamma\left(\bar{B}_{(s)}^{0} \rightarrow D_{(s)}^{(*)+} L^{-}\right)}{d \Gamma\left(\bar{B}_{(s)}^{0} \rightarrow D_{(s)}^{(*)+} \ell^{-} \bar{\nu}_{\ell}\right) / d q^{2} \frac{1 q^{2}=\ldots .2}{2}}=6 \pi^{2}\left|V_{u q}\right|^{2} f_{L}^{2}\left|a_{1}\left(D_{(s)}^{(*)+} L^{-}\right)\right|^{2} X_{L}^{(*)}
$$

－Updated predictions vs data：［Huber，Kränkl，Li＇16；Cai，Deng，Li，Yang＇21］

| $R_{(s) L}^{(*)}$ | LO | NLO | NNLO | Exp． | Deviation（ $\sigma$ ） |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $R_{\pi}$ | 1.01 | $1.07_{-0.04}^{+0.04}$ | $1.10_{-0.03}^{+0.03}$ | $0.74 \pm 0.06$ | 5.4 |
| $R_{\pi}^{*}$ | 1.00 | $1.06{ }_{-0.04}^{+0.04}$ | $1.10_{-0.03}^{+0.03}$ | $0.80 \pm 0.06$ | 4.5 |
| $R_{\rho}$ | 2.77 | $2.94{ }_{-0.19}^{+0.19}$ | $3.02_{-0.18}^{+0.17}$ | $2.23 \pm 0.37$ | 1.9 |
| $R_{K}$ | 0.78 | 0．83－0．033 | 0.85 | 0．62 | 4.4 |
| $R_{K}^{*}$ | 0.72 | $0.76{ }_{-0.03}^{+0.03}$ | $0.79_{-0.02}^{+0.01}$ | $0.60 \pm 0.14$ | 1.3 |
| $R_{K^{*}}$ | 1.41 | $1.50{ }_{-0.11}^{+0.11}$ | $1.53_{-0.10}^{+0.10}$ | $1.38 \pm 0.25$ | 0.6 |
| ${ }^{\sim}$ | 1.01 | $1.07^{10.04}$ |  |  | 4.4 |
| $R_{s K}$ | 0.78 | $0.83{ }_{-0.03}^{+0.03}$ | $0.85_{-0.02}^{+0.01}$ | $0.46 \pm 0.06$ | 6.3 |

free from uncertainties from
$\left|V_{c b}\right| \& B_{d, s} \rightarrow D_{d, s}^{(*)}$ form factors．
$\square$ For a rough estimate：

$$
\begin{aligned}
& \frac{B r^{E x p}}{B r^{S M}} \simeq \frac{\left|a_{1}^{B S M}\right|^{2}}{\left|a_{1}^{S M}\right|^{2}}=0.6 \\
\Rightarrow & \frac{a_{1}^{B S M}}{a_{1}^{S M}} \simeq 0.77=1-0.23 \\
= & \frac{a_{1}^{S M}+\delta a_{1}^{B S M}}{a_{1}^{S M}} \simeq 1+\frac{\delta a_{1}^{B S M}}{a_{1}^{S M}} \\
\Rightarrow & \delta a_{1}^{B S M} \simeq-0.2
\end{aligned}
$$

## Power corrections

$\square$ Sources of sub－leading power corrections：［Beneke，
Buchalla，Neubert，Sachrajda＇01；Bordone，Gubernari，Huber，Jung，van Dyk＇20］

$$
\begin{aligned}
\left\langle D_{q}^{(*)+} L^{-}\right| \mathcal{Q}_{i}\left|\bar{B}_{q}^{0}\right\rangle= & \sum_{j} F_{j}^{\bar{B}_{q} \rightarrow D_{q}^{(*)}}\left(M_{L}^{2}\right) \\
& \times \int_{0}^{1} d u T_{i j}(u) \phi_{L}(u)+\mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}}{m_{b}}\right)
\end{aligned}
$$

$\square$ Scaling of the leading－power contribution：［BBNS＇01］

＞Annihilation topologies；



$$
\mathcal{A}\left(\bar{B}_{d} \rightarrow D^{+} \pi^{-}\right) \sim G_{F} m_{b}^{2} F^{B \rightarrow D}(0) f_{\pi} \sim G_{F} m_{b}^{2} \Lambda_{\mathrm{QCD}}
$$

$>\propto \frac{C_{1}}{a_{1}} \simeq-\frac{1}{3}$ ，all are ESTIMATED to be power－suppressed；not chirality－ enhanced due to $(V-A)(V-A)$ structure
＞Current exp．data could not be easily explained within the SM，at least within
＞Non－leading Fock－state contributions；

estimated

the QCDF／SCET framework．

$\left(\frac{\Lambda_{\mathrm{QCD}}}{m_{b}}\right)^{2}$

# Possible New Physics effects 

 from four－quark operators
## Possible NP in $B_{q}^{0} \rightarrow D_{q}^{-} L^{+}$？

$\square$ Possible NP four－quark operators with different Dirac structures：［Buras，Misiak，Urban＇00］

$$
\begin{array}{rlr}
\mathcal{L}_{\mathrm{WET}}=- & \frac{4 G_{F}}{\sqrt{2}} V_{c b} V_{u q}^{*}\left[\mathcal{C}_{1}^{S M}(\mu) \mathcal{Q}_{1}^{S M}+\mathcal{C}_{2}^{S M}(\mu) \mathcal{Q}_{2}^{S M}\right. & \text { SM current-current operators } \\
& \left.+\sum_{\substack{i=1,2 ; \\
j=1,2,3,4 .}}\left(\mathcal{C}_{i}^{V L L} \mathcal{Q}_{i}^{V L L}+\mathcal{C}_{i}^{V L R} \mathcal{Q}_{i}^{V L R}+\mathcal{C}_{i}^{S L R} \mathcal{Q}_{i}^{S L R}+\mathcal{C}_{j}^{S L L} \mathcal{Q}_{j}^{S L L}\right)\right]+L \leftrightarrow R \\
\mathcal{Q}_{1}^{V L L}=\left(\bar{c}_{\alpha} \gamma^{\mu} P_{L} b_{\beta}\right)\left(\bar{q}_{\beta} \gamma_{\mu} P_{L} u_{\alpha}\right) & \text { NP four-quark operators } \\
\mathcal{Q}_{2}^{V L L}=\left(\bar{c}_{\alpha} \gamma^{\mu} P_{L} b_{\alpha}\right)\left(\bar{q}_{\beta} \gamma_{\mu} P_{L} u_{\beta}\right) & \mathcal{Q}_{1}^{V L R}=\left(\bar{c}_{\alpha} \gamma^{\mu} P_{L} b_{\beta}\right)\left(\bar{q}_{\beta} \gamma_{\mu} P_{R} u_{\alpha}\right) \\
\mathcal{Q}_{1}^{S L L}=\left(\bar{c}_{\alpha} P_{L} b_{\beta}\right)\left(\bar{q}_{\beta} P_{L} u_{\alpha}\right) & \mathcal{Q}_{2}^{V L R}=\left(\bar{c}_{\alpha} \gamma^{\mu} P_{L} b_{\alpha}\right)\left(\bar{q}_{\beta} \gamma_{\mu} P_{R} u_{\beta}\right) \\
\mathcal{Q}_{2}^{S L L}=\left(\bar{c}_{\alpha} P_{L} b_{\alpha}\right)\left(\bar{q}_{\beta} P_{L} u_{\beta}\right) & \mathcal{Q}_{1}^{S L R}=\left(\bar{c}_{\alpha} P_{L} b_{\beta}\right)\left(\bar{q}_{\beta} P_{R} u_{\alpha}\right) \\
\mathcal{Q}_{3}^{S L L}=\left(\bar{c}_{\alpha} \sigma^{\mu \nu} P_{L} b_{\beta}\right)\left(\bar{q}_{\beta} \sigma_{\mu \nu} P_{L} u_{\alpha}\right) & \mathcal{Q}_{2}^{S L R}=\left(\bar{c}_{\alpha} P_{L} b_{\alpha}\right)\left(\bar{q}_{\beta} P_{R} u_{\beta}\right) \\
\mathcal{Q}_{4}^{S L L} & =\left(\bar{c}_{\alpha} \sigma^{\mu \nu} P_{L} b_{\alpha}\right)\left(\bar{q}_{\beta} \sigma_{\mu \nu} P_{L} u_{\beta}\right) &
\end{array}
$$

totally 20 linearly－independent operators，and can be further split into 8 separate sectors！

## Possible sources of these NP operators

$\square$ Possible tree－level mediators：
＞For neutral mediators，necessarily couple to FCNC at tree level；
 Mediators $\square$ excluded by FCNC processes！
$>$ For charged mediators：colorless or colored（limited by di－jet resonance searches）
$\square$ For VLL，VRR，VLR，VRL sectors： generated by a colorless charged gauge boson $\mathrm{A}^{+}($spin－1）；


Charged


Neutral

［Bordone，Greljo，Marzocca，2103．10332］
－For SLL，SRR，SLR，SRL sectors： generated by a colorless charged scalar H＋（spin－0）；


## Possible sources of these NP operators

－Both 1－loop matching conditions \＆2－Ioop QCD ADMs known；［Buras，Misiak，Urban＇00；Buras，Girrbach＇12］

$$
\begin{array}{ll}
C_{1}^{\mathrm{SLR}}(\mu)=3 \frac{\alpha_{s}}{4 \pi}, \\
C_{2}^{\mathrm{SLR}}(\mu)=1-\frac{\alpha_{s}}{4 \pi} \frac{3}{N}=1-\frac{\alpha_{s}}{4 \pi}, & \text { a colorless charged } \\
C_{1}^{\mathrm{SLL}}(\mu)=0, & \text { scalar } \mathrm{H}^{+} . \\
C_{2}^{\mathrm{SLL}}(\mu)=1, & \\
C_{3}^{\mathrm{SLL}}(\mu)=\frac{\alpha_{s}}{4 \pi}\left(-\frac{1}{2} \log \frac{M_{H}^{2}}{\mu^{2}}+\frac{3}{4}\right), & \\
C_{4}^{\mathrm{SLL}}(\mu)=\frac{\alpha_{s}}{4 \pi}\left(\frac{1}{2 N} \log \frac{M_{H}^{2}}{\mu^{2}}-\frac{3}{4 N}\right)=\frac{\alpha_{s}}{4 \pi}\left(\frac{1}{6} \log \frac{M_{H}^{2}}{\mu^{2}}-\frac{1}{4}\right) .
\end{array}
$$

$$
\begin{aligned}
& C_{1}^{\mathrm{VLL}}(\mu)=\frac{\alpha_{s}}{4 \pi}\left(-3 \log \frac{M_{A}^{2}}{\mu^{2}}+\frac{11}{2}\right) \\
& C_{2}^{\mathrm{VLL}}(\mu)=1+\frac{\alpha_{s}}{4 \pi}\left(\frac{3}{N} \log \frac{M_{A}^{2}}{\mu^{2}}-\frac{11}{2 N}\right)=1+\frac{\alpha_{s}}{4 \pi}\left(\log \frac{M_{A}^{2}}{\mu^{2}}-\frac{11}{6}\right)
\end{aligned}
$$

$$
C_{1}^{\mathrm{VLR}}(\mu)=\frac{\alpha_{s}}{4 \pi}\left(3 \log \frac{M_{A}^{2}}{\mu^{2}}+\frac{1}{2}\right)
$$ a colorless charged gauge boson $\mathrm{A}^{+}$．

$C_{2}^{\mathrm{VLR}}(\mu)=1+\frac{\alpha_{s}}{4 \pi}\left(-\frac{3}{N} \log \frac{M_{A}^{2}}{\mu^{2}}-\frac{1}{2 N}\right)=1+\frac{\alpha_{s}}{4 \pi}\left(-\log \frac{M_{A}^{2}}{\mu^{2}}-\frac{1}{6}\right)$
$\square$ RG evolution from down $M_{A, H}$ to $m_{b}$ at NLL；
［Buras，Misiak，Urban＇00；Buras，Girrbach＇12］

$$
\begin{gathered}
\vec{C}\left(\mu_{b}\right)=\left(\mathbb{1}+\frac{\alpha_{s}\left(\mu_{b}\right)}{4 \pi} \hat{J}\right) \hat{U}^{(0)}\left(\mu_{b}, \mu_{\text {in }}\right)\left(\mathbb{1}-\frac{\alpha_{s}\left(\mu_{\text {in }}\right)}{4 \pi}\left(\vec{C}_{1}+\hat{J} \vec{C}_{0}\right)\right) \\
\hat{U}^{(0)}\left(\mu_{b}, \mu_{\text {in }}\right)=\hat{V}\left(\left[\frac{\alpha_{s}\left(\mu_{\text {in }}\right)}{\alpha_{s}\left(\mu_{b}\right)}\right]^{\frac{\hat{z}^{(0)}}{2 \beta_{0}}}\right)_{D} \hat{V}^{-1}
\end{gathered}
$$

## Matrix elements of NP operators

$\square\left\langle D^{+} L^{-}\right| \mathcal{O}_{i}\left|\bar{B}^{\mathbf{0}}\right\rangle$ ：calculated in QCDF at leading－power in $1 / m_{b}$ ，but including $\mathcal{O}\left(\alpha_{s}\right)$ vertex correction．


NLO non－factorizable vertex corrections


＞With NLL Wilson coefficients and NLO matrix elements， un－physical scale－\＆scheme－dependences cancelled in the final decay amplitude．

## Model－independent analysis

$\square$ NP $C_{i}^{N P}$ ：real and take a CKM－like flavor structure for $b \rightarrow c \bar{u} d$ and $b \rightarrow c \bar{u} s$ transitions．

$$
\begin{aligned}
\mathcal{L}_{\mathrm{WET}}= & -\frac{4 G_{F_{i}}}{\sqrt{2}} \begin{aligned}
& V_{c b} V_{u q}^{*} V_{u}^{*} \\
&\left.+\sum_{\substack{i=1,2 ; \\
j=1,2,3,4}}\left(\mathcal{C}_{i}^{V L L}(\mu) \mathcal{Q}_{1}^{S M}+\mathcal{C}_{2}^{S M}(\mu) \mathcal{Q}_{2}^{S M}+\mathcal{C}_{i}^{V L R} \mathcal{Q}_{i}^{V L R}+\mathcal{C}_{i}^{S L R} \mathcal{Q}_{i}^{S L R}+\mathcal{C}_{j}^{S L L} \mathcal{Q}_{j}^{S L L}\right)\right]+L \leftrightarrow R
\end{aligned}
\end{aligned}
$$

$\square$ Use 8 ratios to constrain allowed $C_{i}^{N P}$ ；
Note：different modes show different

| $R_{(s) L}^{(*)}$ | LO | NLO | NNLO | Exp． | Deviation $(\sigma)$ |
| :--- | :--- | :--- | :--- | :---: | :---: |
| $R_{\pi}$ | 1.01 | $1.07_{-0.04}^{+0.04}$ | $1.10_{-0.03}^{+0.03}$ | $0.74 \pm 0.06$ | 5.4 |
| $R_{\pi}^{*}$ | 1.00 | $1.06_{-0.04}^{+0.04}$ | $1.10_{-0.03}^{+0.03}$ | $0.80 \pm 0.06$ | 4.5 |
| $R_{\rho}$ | 2.77 | $2.99_{-0.19}^{+0.19}$ | $3.02_{-0.18}^{+0.17}$ | $2.23 \pm 0.37$ | 1.9 |
| $R_{K}$ | 0.78 | $0.83_{-0.03}^{+0.03}$ | $0.85_{-0.02}^{+0.01}$ | $0.62 \pm 0.05$ | 4.4 |
| $R_{K}^{*}$ | 0.72 | $0.76_{-0.03}^{+0.03}$ | $0.79_{-0.01}^{+0.01}$ | $0.60 \pm 0.14$ | 1.3 |
| $R_{K^{*}}$ | 1.41 | $1.50_{-0.11}^{+0.11}$ | $1.53_{-0.10}^{+0.10}$ | $1.38 \pm 0.25$ | 0.6 |
| $R_{s \pi}$ | 1.01 | $1.07_{-0.04}^{+0.04}$ | $1.10_{-0.03}^{+0.03}$ | $0.72 \pm 0.08$ | 4.4 |
| $R_{s K}$ | 0.78 | $0.83_{-0.03}^{+0.03}$ | $0.85_{-0.02}^{+0.01}$ | $0.46 \pm 0.06$ | 6.3 |

## dependences on NP WCs！

$$
\begin{aligned}
& \left\langle\pi^{-}(q)\right| \bar{d} \gamma_{\mu} \gamma_{5} u|0\rangle=-i f_{\pi} q_{\mu} \\
& \left\langle\rho^{-}(q)\right| \bar{d} \gamma_{\mu} u|0\rangle=-i f_{\rho} m_{\rho} \epsilon_{\mu}^{*}
\end{aligned}
$$

$$
\left\langle D^{+}\right| \bar{c} q b\left|\bar{B}^{0}\right\rangle=\left(m_{B}^{2}-m_{D}^{2}\right) F_{0}^{B \rightarrow D}\left(q^{2}\right)
$$

$$
\left\langle D^{*+}\right| \bar{c} q \gamma_{5} b\left|\bar{B}^{0}\right\rangle=2 m_{D^{*}}\left(\epsilon^{*} \cdot q\right) A_{0}^{B \rightarrow D^{*}}\left(q^{2}\right)
$$

\section*{Analysis at $m_{b}$ scale <br> | $R_{K}$ | 0.78 | $0.83_{-0.03}^{+0.03}$ | $0.85_{-0.02}^{+0.01}$ | $0.62 \pm 0.05$ | 4.4 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $R_{s K}$ | 0.78 | $0.83_{-0.03}^{+0.03}$ | $0.85_{-0.02}^{+0.01}$ | $0.46 \pm 0.06$ | 6.3 |}

$\square$ With only one NP $C_{i}^{N P}$ in each time，NP four－quark operators with three Dirac structures；

| NP Coeff． | C．L． | $R_{\pi}$ | $R_{\pi}^{*}$ | $R_{\rho}$ |  | $R_{K}^{*}$ | $R_{K^{*}}$ | $R_{s \pi}$ |  | Combined |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $C_{1}^{V L L}$ | $1 \sigma$ | ［－1．40，－0．847］ | ［－1．18，－0．626］ | ［－1．50，－0．267］ | ［－1．18，－0．662］ | ［－1．54，－0．145］ | ［－1．05，0．392］ | ［－1．57，－0．835］ | ［－2．12，－1．31］ | $\varnothing$ |
|  | $2 \sigma$ | ［－1．63，－0．656］ | ［－1．41，－0．426］ | ［－2．06，0．135］ | ［－1．42，－0．462］ | ［－2．41，0．402］ | ［－1．70，0．856］ | ［－1．92，－0．567］ | ［－2．55，－1．02］ | ［－1．41，－1．02］ |
| $C_{2}^{V L L}$ | $1 \sigma$ | ［－0．237，－0．148］ | ［－0．205，－0．111］ | ［－0．254，－0．047］ | ［－0．198，－0．116］ | ［－0．261，－0．026］ | ［－0．183，0．070］ | ［－0．264，－0．146］ | ［－0．345，－0．226］ | $\varnothing$ |
|  | $2 \sigma$ | ［－0．273，－0．115］ | ［－0．244，－0．075］ | ［－0．340，0．024］ | ［－0．237，－0．081］ | ［－0．401，0．071］ | ［－0．288，0．155］ | ［－0．318，－0．099］ | ［－0．406，－0．176］ | ［－0．237，－0．176］ |
| $C_{1}^{S R R}$ | $1 \sigma$ | ［－0．748，－0．418］ | ［－1．03，－0．502］ | $\varnothing$ | ［－0．711，－0．368］ | ［－1．50，－0．133］ | R | ［－0．839，－0．412］ | ［－1．25，－0．712］ | $\varnothing$ |
|  | $2 \sigma$ | ［－0．867，－0．326］ | ［－1．23，－0．344］ | R | ［－0．854，－0．259］ | ［－2．32，0．395］ | R | ［－1．02，－0．283］ | ［－1．48，－0．556］ | ［－0．854，－0．556］ |
| $C_{2}^{S R R}$ | $1 \sigma$ | ［－0．249，－0．139］ | ［－0．343，－0．167］ | $\varnothing$ | ［－0．237，－0．123］ | ［－0．500，－0．044］ | R | ［－0．280，－0．137］ | ［－0．417，－0．237］ | $\varnothing$ |
|  | $2 \sigma$ | ［－0．289，－0．109］ | ［－0．410，－0．115］ | R | ［－0．285，－0．086］ | ［－0．773，0．132］ | R | ［－0．339，－0．094］ | ［－0．492，－0．185］ | ［－0．285，－0．185］ |
| $C_{1}^{S R L}$ | $1 \sigma$ | ［0．487，0．873］ | ［0．585，1．20］ | $\varnothing$ | ［0．429，0．829］ | ［0．155，1．75］ | R | ［0．480，0．979］ | ［0．830，1．46］ | $\varnothing$ |
|  | $2 \sigma$ | ［0．381，1．01］ | ［0．401，1．44］ | R | ［0．302，0．996］ | ［－0．460，2．71］ | R | ［0．330，1．18］ | ［0．648，1．72］ | ［0．648，0．996］ |
| $C_{2}^{S R L}$ | $1 \sigma$ | ［0．139，0．249］ | ［0．167，0．343］ | $\varnothing$ | ［0．123，0．237］ | ［0．044，0．500］ | R | ［0．137，0．280］ | ［0．237，0．416］ | $\varnothing$ |
|  | $2 \sigma$ | ［0．109，0．289］ | ［0．115，0．410］ | R | ［0．086， 0.285 ］ | ［－0．132，0．773］ | R | ［0．094，0．339］ | ［0．185，0．492］ | ［0．185， 0.285$]$ |

$$
\begin{aligned}
& \mathcal{Q}_{1,2}^{V L L}=\bar{c}_{\alpha} \gamma_{\mu}\left(1-\gamma_{5}\right) b_{\beta(\alpha)} \bar{q}_{\beta} \gamma^{\mu}\left(1-\gamma_{5}\right) u_{\alpha(\beta)} \\
&(\boldsymbol{V}-\boldsymbol{A}) \otimes(\boldsymbol{V}-\boldsymbol{A}) \\
& \mathcal{Q}_{1,2}^{S R L}= \bar{c}_{\alpha}\left(1+\gamma_{5}\right) b_{\beta(\alpha)} \bar{q}_{\beta}\left(1-\gamma_{5}\right) u_{\alpha(\beta)} \\
&(\boldsymbol{S}+\boldsymbol{P}) \otimes(\boldsymbol{S}-\boldsymbol{P}) \\
& \mathcal{Q}_{1,2}^{S R R}= \bar{c}_{\alpha}\left(1+\gamma_{5}\right) b_{\beta(\alpha)} \bar{q}_{\beta}\left(1+\gamma_{5}\right) u_{\alpha(\beta)} \\
&(\boldsymbol{S}+\boldsymbol{P}) \otimes(\boldsymbol{S}+\boldsymbol{P})
\end{aligned}
$$

＞Constraints on $C_{2}^{N P}$ much stronger than on $C_{1}^{N P}$ ：
$>C_{1}^{N P}$ suppressed by $1 / N_{C}$ at LO and further by $C_{F} / 4 \pi$ at NLO in QCDF；
$>$（Pseudo－）scalar operators associated with a chirally－enhanced factor $\frac{2 m_{L}^{2}}{\left(m_{b} \pm m_{c}\right)\left(m_{u}+m_{d, s}\right)}$ ；
＞NP operators with other Dirac structures already ruled out by combined constraints from 8 ratios；

## Analysis at $\boldsymbol{m}_{b}$ scale

ㅁ Keep only $C_{2}^{V L L}$ nonzero；
－SM： $\mathcal{C}_{1}\left(m_{b}\right)=-0.143$ and $\mathcal{C}_{2}\left(m_{b}\right)=1.058$


## Analysis at $\boldsymbol{m}_{b}$ scale

ㅁ Keep only $C_{2}^{S R L}$ nonzero；
－SM： $\mathcal{C}_{1}\left(m_{b}\right)=-0.143$ and $\mathcal{C}_{2}\left(m_{b}\right)=1.058$


## Analysis at $\boldsymbol{m}_{\boldsymbol{b}}$ scale

$\square$ Keep only $C_{2}^{S R R}$ nonzero；
－SM： $\mathcal{C}_{1}\left(m_{b}\right)=-0.143$ and $\mathcal{C}_{2}\left(m_{b}\right)=1.058$


## Analysis at $\boldsymbol{m}_{\boldsymbol{b}}$ scale

$\square$ Two NP operators with the same Dirac but different color structures；


$$
C_{2}^{N P}+C_{1}^{N P} / N_{C}
$$

$>$ Due to partial cancellation between $C_{2}^{N P} \& C_{1}^{N P}$ ，allowed regions potentially larger than in previous case．
＞For NP operators with other Dirac structures，no allowed regions even at the $2 \sigma$ level．



## Analysis at $\boldsymbol{m}_{W}$ scale

$\square$ Variable solutions：NP four－quark operators with the following three Dirac structures；
$\mathcal{Q}_{1,2}^{V L L}=\bar{c}_{\alpha} \gamma_{\mu}\left(1-\gamma_{5}\right) b_{\beta(\alpha)} \bar{q}_{\beta} \gamma^{\mu}\left(1-\gamma_{5}\right) u_{\alpha(\beta)}$
$(\boldsymbol{V}-\boldsymbol{A}) \otimes(\boldsymbol{V}-\boldsymbol{A})$
$\mathcal{Q}_{1,2}^{S R L}=\bar{c}_{\alpha}\left(1+\gamma_{5}\right) b_{\beta(\alpha)} \bar{q}_{\beta}\left(1-\gamma_{5}\right) u_{\alpha(\beta)}$
$(\boldsymbol{S}+\boldsymbol{P}) \otimes(\boldsymbol{S}-\boldsymbol{P})$
$\mathcal{Q}_{1,2}^{S R R}=\bar{c}_{\alpha}\left(1+\gamma_{5}\right) b_{\beta(\alpha)} \bar{q}_{\beta}\left(1+\gamma_{5}\right) u_{\alpha(\beta)}$
$(\boldsymbol{S}+\boldsymbol{P}) \otimes(\boldsymbol{S}+\boldsymbol{P})$
$\vec{C}\left(\mu_{b}\right)=\hat{U}\left(\mu_{b}, \mu_{W}\right) \vec{C}\left(\mu_{W}\right)$
What implications for the NP Wilson coefficients at the higher scale $\boldsymbol{m}_{W}$ ？
$\square$ With RG evolutions for $C_{i}^{N P}$ taken into account，the following regions obtained：

$$
C_{2}^{V L L}\left(M_{W}\right) \in[-0.220,-0.164] \quad \text { Vs } \quad C_{2}^{V L L}\left(m_{b}\right) \in[-0.237,-0.176]
$$

small RG evolution effect！

$$
\begin{array}{lll}
C_{2}^{S R L}\left(m_{W}\right) \in[0.091,0.139] & \text { Vs } & C_{2}^{S R L}\left(m_{b}\right) \in[0.185,0.285] \\
C_{2}^{S R R}\left(m_{W}\right) \in[-0.129,-0.084] & \text { VS } & C_{2}^{S R R}\left(m_{b}\right) \in[-0.285,-0.185]
\end{array}
$$



## Case with a colorless gauge boson

$\square$ Heff mediated by a colorless charged gauge boson $\mathrm{A}^{+}$；

$$
\begin{aligned}
\mathcal{H}_{\mathrm{eff}}^{\text {gauge }}= & \frac{G_{F}}{\sqrt{2}} V_{c b} V_{u q}^{*}\left\{\lambda_{L L}(A)\left[C_{1}^{V L L}(\mu) Q_{1}^{V L L}(\mu)+C_{2}^{V L L}(\mu) Q_{2}^{V L L}(\mu)\right]\right. \\
& \left.+\lambda_{L R}(A)\left[C_{1}^{V L R}(\mu) Q_{1}^{V L R}(\mu)+C_{2}^{V L R}(\mu) Q_{2}^{V L R}(\mu)\right]+(L \leftrightarrow R)\right\}
\end{aligned}
$$



$$
i \frac{g_{2}}{\sqrt{2}} V_{i j} \gamma^{\mu} \delta_{\alpha \beta}\left[\Delta_{i j}^{L}(A) P_{L}+\Delta_{i j}^{R}(A) P_{R}\right]
$$

$$
\lambda_{L L}(A)=\frac{m_{W}^{2}}{m_{A}^{2}} \Delta_{c b}^{L}(A)\left(\Delta_{u q}^{L}(A)\right)^{*}, \quad \lambda_{L R}(A)=\frac{m_{W}^{2}}{m_{A}^{2}} \Delta_{c b}^{L}(A)\left(\Delta_{u q}^{R}(A)\right)^{*}
$$

$\square$ With $m_{A}=1 \mathrm{TeV}$ ，1－\＆2－loop ADMs and 1－loop matching conditions：［Buras，Misiak，Urban＇ 00 ；Buras，
Girrbach＇12］

$$
\vec{C}\left(\mu_{b}\right)=\hat{U}\left(\mu_{b}, \mu_{W}\right) \hat{U}\left(\mu_{W}, \mu_{0}\right) \vec{C}\left(\mu_{0}\right)
$$

$\square 4$ NP parameters：$\lambda_{L L}(A), \lambda_{L R}(A), \lambda_{R R}(A), \lambda_{R L}(A)$ ；
＞Scenario I：only one effective coefficient nonzero；

$$
\lambda_{L L}(A) \in[-0.211,-0.154]
$$

$>$ Scenario II：$\Delta_{c b}^{L}(A)=\Delta_{c b}^{R}(A), \Delta_{u q}^{L}(A)=\Delta_{u q}^{R}(A) ;$
$\Longrightarrow \lambda_{L L}(A)=\lambda_{R R}(A)=\lambda_{L R}(A)=\lambda_{R L}(A)$
$>$ Scenario III：$\Delta_{c b}^{L}(A)=-\Delta_{c b}^{R}(A), \Delta_{u q}^{L}(A)=-\Delta_{u q}^{R}(A) ; \Longrightarrow \lambda_{L L}(A)=\lambda_{R R}(A)=-\lambda_{L R}(A)=-\lambda_{R L}(A)$

## Case with a colorless gauge boson

$\square$ Scenario I: only $\lambda_{L L}(A)$ nonzero; need $A^{+}$couplings to quarks being of $V-A$ structure!


## Case with a colorless scalar

－Heff mediated by a colorless charged scalar H＋；


$$
\begin{array}{rlr}
\mathcal{H}_{\text {eff }}^{\text {scalar }}=-\frac{G_{F}}{\sqrt{2}} V_{c b} V_{u q}^{*}\left\{\lambda_{L L}(H)\right. & {\left[C_{1}^{S L L}(\mu) Q_{1}^{S L L}(\mu)+C_{2}^{S L L}(\mu) Q_{2}^{S L L}(\mu)\right.} & i \frac{g_{2}}{\sqrt{2}} V_{i j} \delta_{\alpha \beta}\left[\Delta_{i j}^{L}(H) P_{L}+\Delta_{i j}^{R}(H) P_{R}\right] \\
& \left.+C_{3}^{S L L}(\mu) Q_{3}^{S L L}(\mu)+C_{4}^{S L L}(\mu) Q_{4}^{S L L}(\mu)\right] &
\end{array}
$$

$\square$ With $m_{H}=1 \mathrm{TeV}$ ，1－\＆2－loop ADMs and 1－loop matching conditions：［Buras，Misiak，Urban＇ 00 ；Buras，
Girrbach＇12］

$$
\vec{C}\left(\mu_{b}\right)=\hat{U}\left(\mu_{b}, \mu_{W}\right) \hat{U}\left(\mu_{W}, \mu_{0}\right) \vec{C}\left(\mu_{0}\right)
$$

$$
\lambda_{R L}(H) \in[0.059,0.100]
$$

$\square 4$ NP parameters：$\lambda_{L L}(H), \lambda_{L R}(H), \lambda_{R R}(H), \lambda_{R L}(H)$ ；
＞Scenario I：only one effective coefficient nonzero；


$$
\lambda_{R R}(H) \in[-0.090,-0.054]
$$

$\mathrm{H}^{+}$coupling to quarks either scalar or pseudo－scalar！
$>$ Scenario II：$\Delta_{c b}^{L}(\boldsymbol{H})=\Delta_{c b}^{R}(\boldsymbol{H}), \Delta_{\boldsymbol{u q}}^{L}(\boldsymbol{H})=\Delta_{\boldsymbol{u q}}^{R}(\boldsymbol{H}) ; \Longrightarrow \lambda_{L L}(\boldsymbol{H})=\lambda_{\boldsymbol{R} \boldsymbol{R}}(\boldsymbol{H})=\lambda_{L R}(\boldsymbol{H})=\lambda_{\boldsymbol{R L}}(\boldsymbol{H})$
$>$ Scenario III：$\Delta_{c b}^{L}(\boldsymbol{H})=-\Delta_{c b}^{R}(\boldsymbol{H}), \Delta_{u q}^{L}(\boldsymbol{H})=-\Delta_{u q}^{R}(\boldsymbol{H}) ; \Longrightarrow \lambda_{L L}(\boldsymbol{H})=\lambda_{R R}(\boldsymbol{H})=-\lambda_{L R}(\boldsymbol{H})=-\lambda_{R L}(\boldsymbol{H})$

## Case with a colorless scalar

$\square$ Scenario I：only $\lambda_{R L}(H)$ nonzero；need $H^{+}$couplings to quarks being of $S \pm P$ structure！


## Case with a colorless scalar

$\square$ Scenario I: only $\lambda_{R R}(H)$ nonzero; need $H^{+}$couplings to quarks being of $S \pm P$ structure!


## Summary

－NNLO predictions for class－I $B_{q}^{0} \rightarrow D_{q}^{(*)-} L^{+}$decays at LP in QCDF／SCET complete．
$\square \mathcal{O}(4-5 \sigma)$ discrepancies observed between updated SM predictions and current exp．data； sub－leading power corrections in QCDF／SCET or possible NP beyond the SM？
$\square$ Model－indep．analysis shows that only NP operators with 3 Dirac structures possible：
$\mathcal{Q}_{1,2}^{V L L}=\bar{c}_{\alpha} \gamma_{\mu}\left(1-\gamma_{5}\right) b_{\beta(\alpha)} \bar{q}_{\beta} \gamma^{\mu}\left(1-\gamma_{5}\right) u_{\alpha(\beta)}$

$$
\mathcal{Q}_{1,2}^{S R L}=\bar{c}_{\alpha}\left(1+\gamma_{5}\right) b_{\beta(\alpha)} \bar{q}_{\beta}\left(1-\gamma_{5}\right) u_{\alpha(\beta)}
$$

$(\boldsymbol{S}+\boldsymbol{P}) \otimes(\boldsymbol{S}-\boldsymbol{P})$
$(\boldsymbol{S}+\boldsymbol{P}) \otimes(\boldsymbol{S}+\boldsymbol{P})$
generated by a colorless charged gauge boson or by a colorless charged scalar．

## Thank You for your attention！

