上海交通大學
SHANGHAI JIAO TONG UNIVERSITY

TMD Soft Function From Lattice QCD

Wei Wang

(Lattice Parton Collaboration)

上海交通大学

第三届重味物理与量子色动力学研讨会

02.05.2021

OUTLINE

- PDF and LaMET
- TMD Factorization and TMD Soft Function
- Calculate the TMD Soft Function on lattice
- Lattice Calculation and Numerical Results
- Summary and outlook

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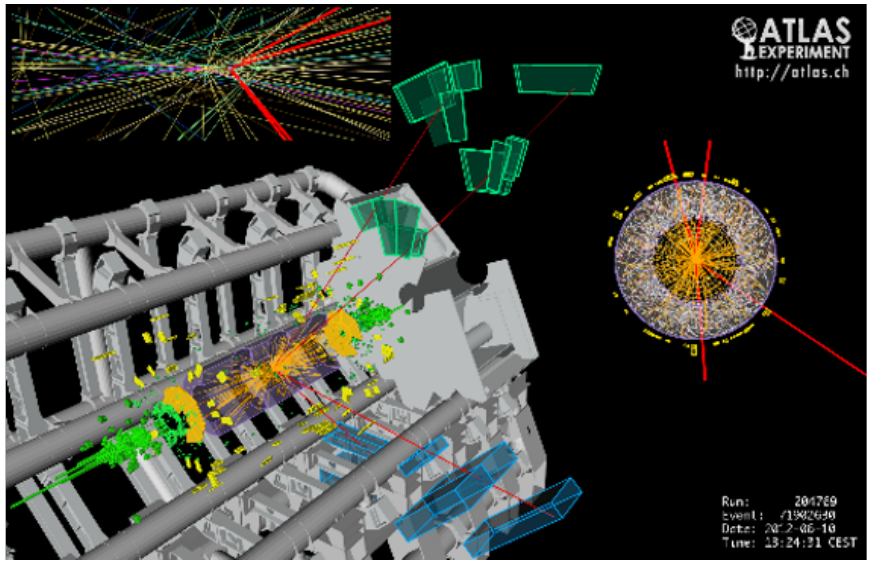
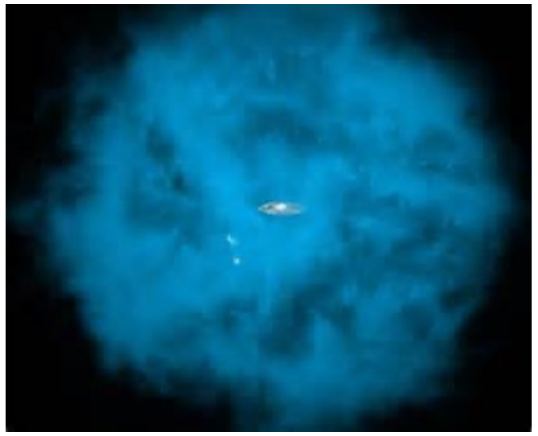
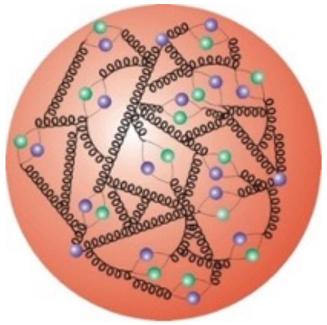
Parton Distribution Functions



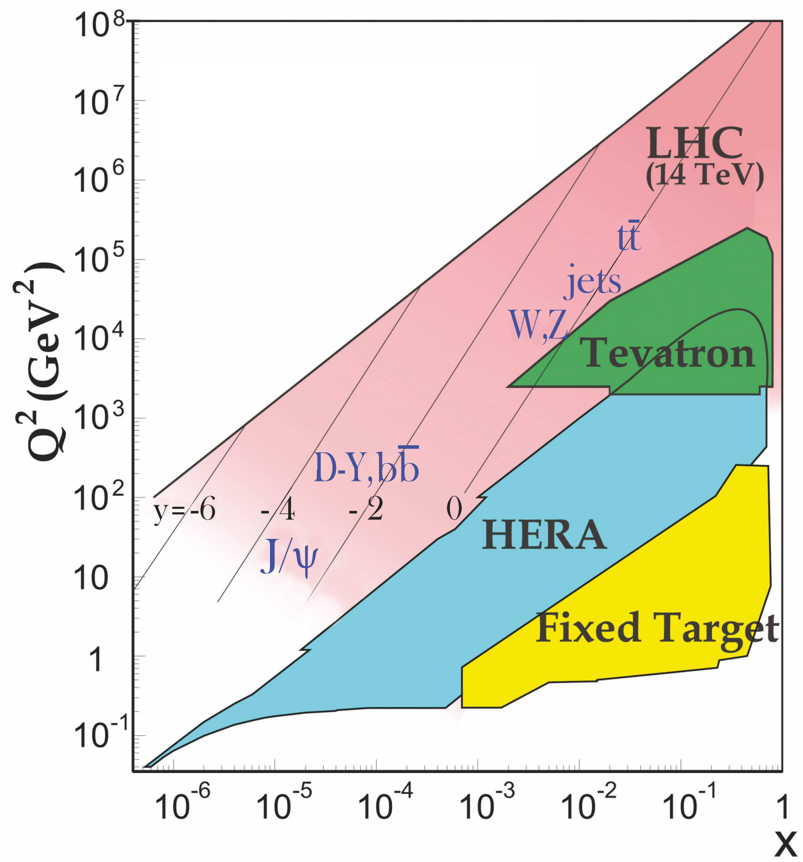


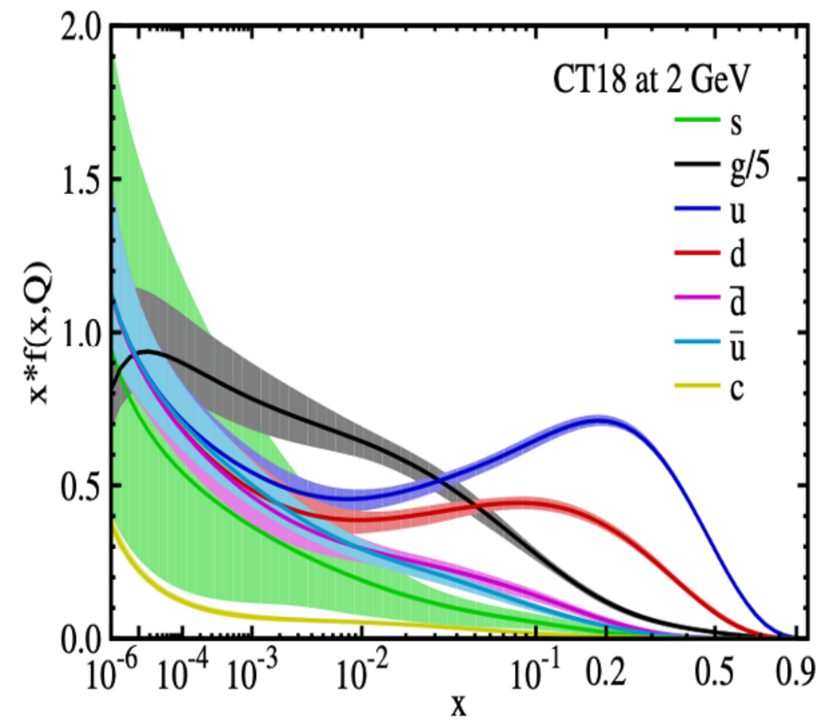


basic inputs for particle physics
at hadron colliders

$$\sigma \sim \int dx_1 dx_2 f(x_1) \times f(x_2) \times C(x_1, x_2, Q)$$

Parton Distribution Functions





CTEQ
MSTW
NNPDF
ABM
…

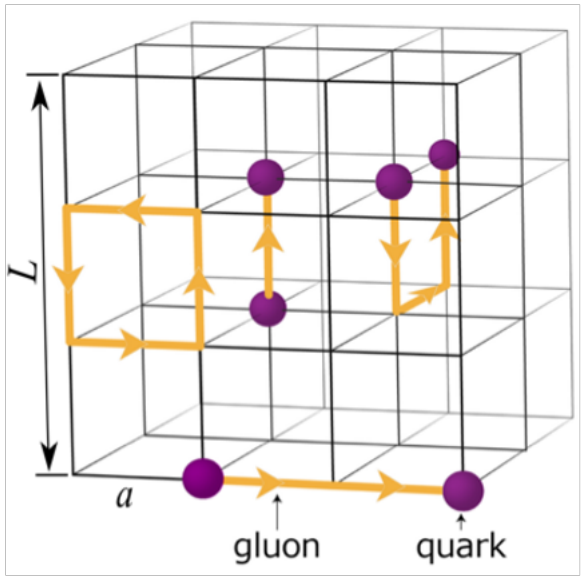
*1912.10053, **J.Gao**, et.al*

Lattice QCD

Numerical simulation in discretized Euclidean space-time

Finite volume (L should be large)

Finite lattice spacing (a should be small)



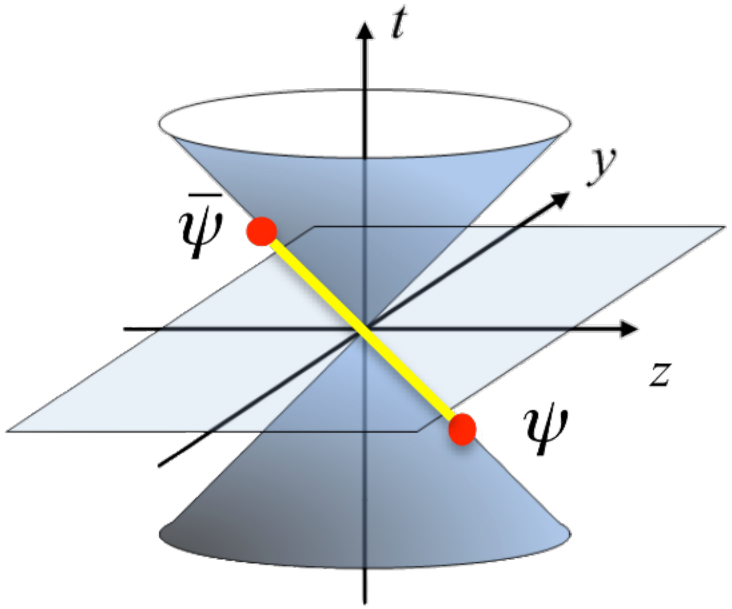
PDF (or more general parton physics):
Minkowski space, real time
infinite momentum frame, on the light-cone

Lattice QCD:

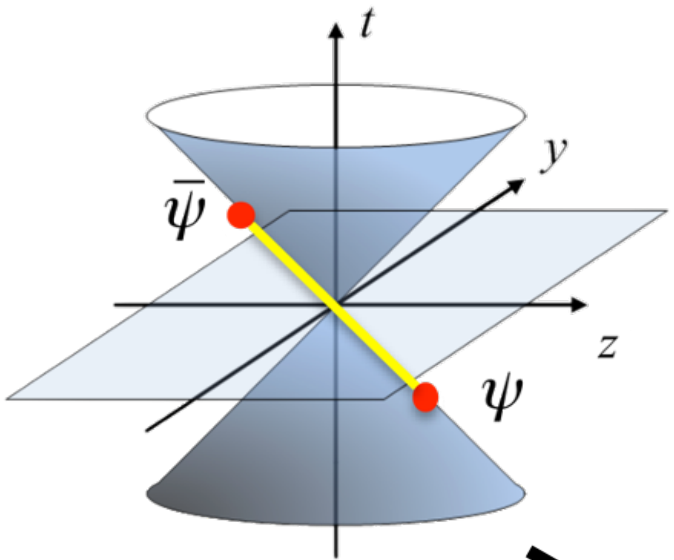
Euclidean space, imaginary time ($t = i\tau$)
Difficulty in time

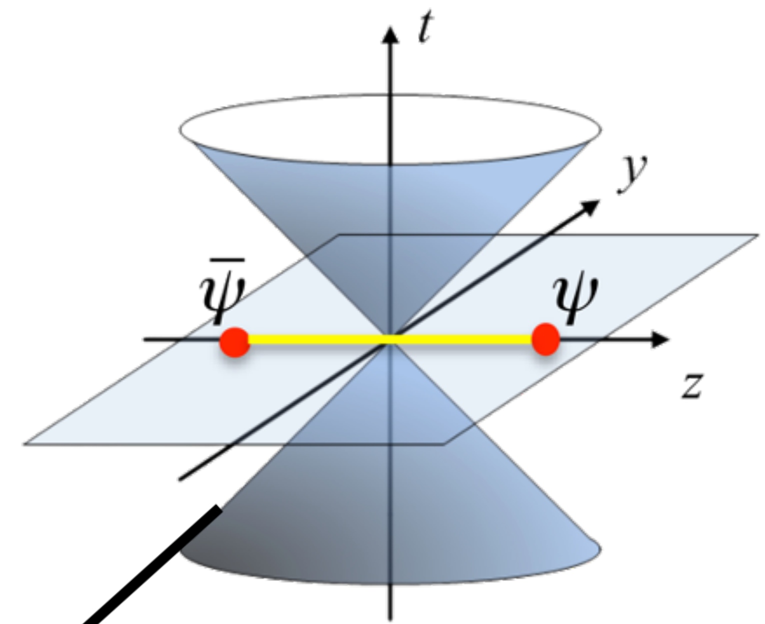
$$x_E^\mu x_E^\mu = 0 \to x_E^\mu = (0,0,0,0)$$

Unable to distinguish local operator and light-cone operator
Sign problem in simulating real-time dynamics.

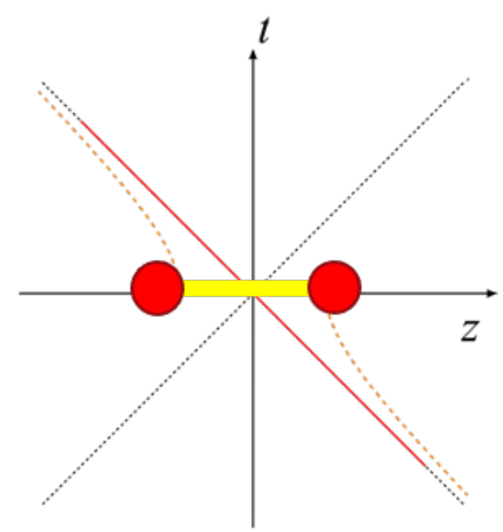


Large Momentum Effective Theory (LaMET)





Lorentz boost



PDF:
light-cone separation;
Cannot be calculated on the lattice

Quasi-PDF :
Equal-time correlation;
Directly calculable on the lattice

Ji,PRL110, 262002(2013)

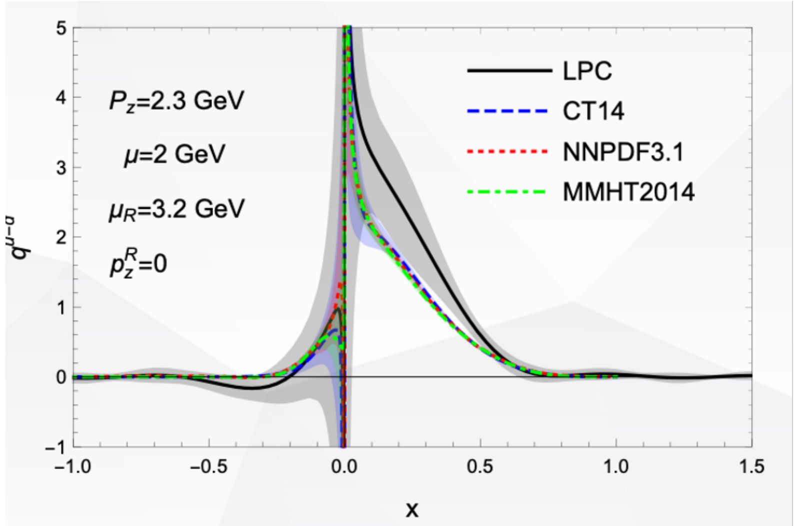
$$\tilde{q}(x,\mu^2,P^z)=\int_{-1}^{1}\frac{dy}{|y|}Z\left(\frac{x}{y},\frac{\mu}{P^z}\right)q(y,\mu^2)+\mathcal{O}\left(\Lambda^2/(P^z)^2,M^2/(P^z)^2\right)\ ,$$

Large Momentum Effective Theory (LaMET)

$$\tilde{q}(x,\mu^2,P^z) = \int_{-1}^{1} \frac{dy}{|y|} Z\left(\frac{x}{y}, \frac{\mu}{P^z}\right) q(y,\mu^2) + \mathcal{O}\left(\Lambda^2/(P^z)^2, M^2/(P^z)^2\right) ,$$

1. Formalism: factorization, renormalization, power corrections
2. Perturbative Matching: QCD corrections to Z
3. Lattice QCD simulations

Many Progress has been made on quasi PDFs in the three directions, see Reviews:
Alexandrou et al., 1811.07248, Adv.High Energy Phys. 2019 ,3036904 (2019)
Ji, et al. 2004.03543, to be published in Rev.Mod.Phy.



LPC:PRD 101,034020(2020)

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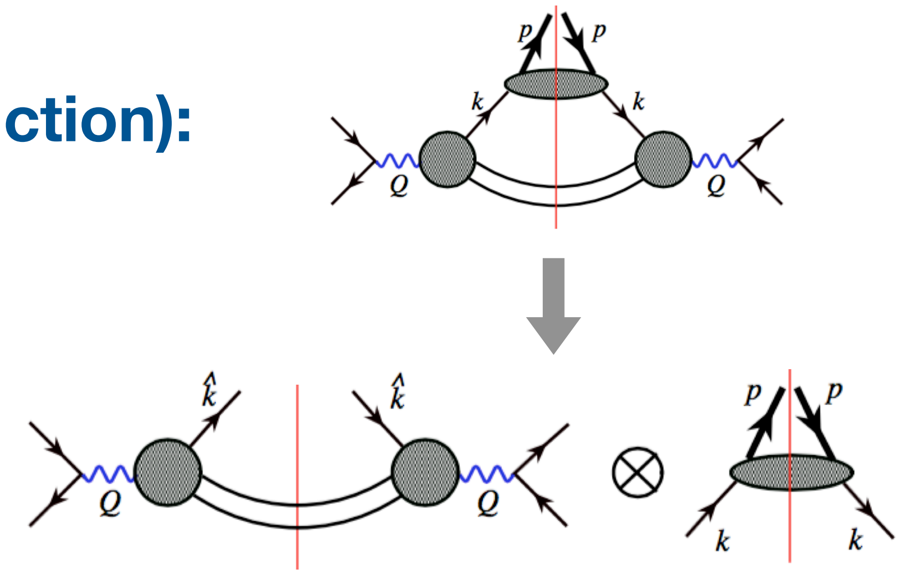
TMD Factorization and Soft Function

- **Collinear factorization (e.g., for the DIS structure function):**

$$F_a(x,Q) = \sum_i \int_0^1 \frac{d\xi}{\xi} H_i(x/\xi, Q, \mu) f_{i/N}(\xi,\mu) + \mathcal{O}\left(\frac{\Lambda_{QCD}^2}{Q^2}\right)$$



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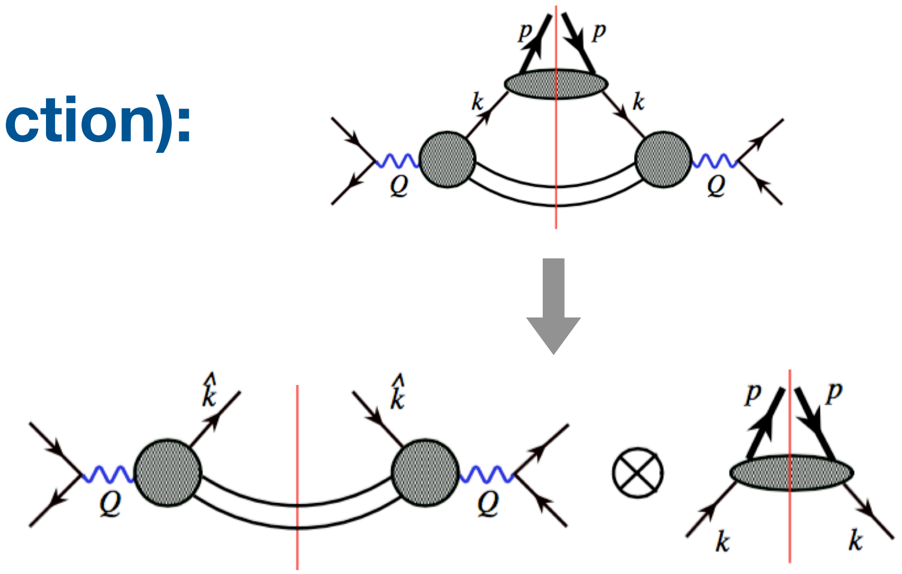
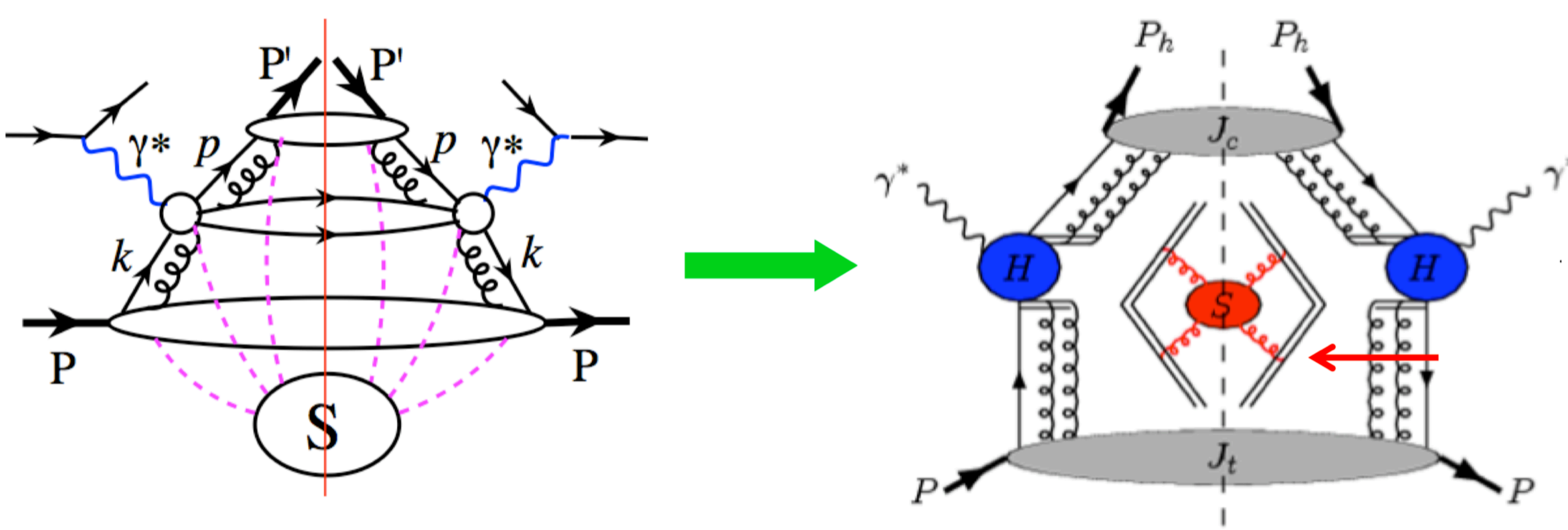


$$F_a(x,Q) = \sum_i \int_0^1 \frac{d\xi}{\xi} H_i(x/\xi, Q, \mu) f_{i/N}(\xi,\mu) + \mathcal{O}\left(\frac{\Lambda^2_{QCD}}{Q^2}\right)$$

- **TMD factorization (e.g., for SIDIS cross section, $P_{h\perp} \ll Q$):**

$$\sigma_{SIDIS} = \sum_i \hat{H}(Q,\mu) \otimes f_i^{TMD}(x, k_\perp, \mu, \zeta) \otimes D_{i/N}(x', p_\perp, \mu, \zeta') \otimes S(k_{s\perp}, \mu, Y, Y') + \mathcal{O}\left(\frac{P^2_{h\perp}}{Q^2}, \frac{\Lambda^2_{QCD}}{Q^2}\right)$$



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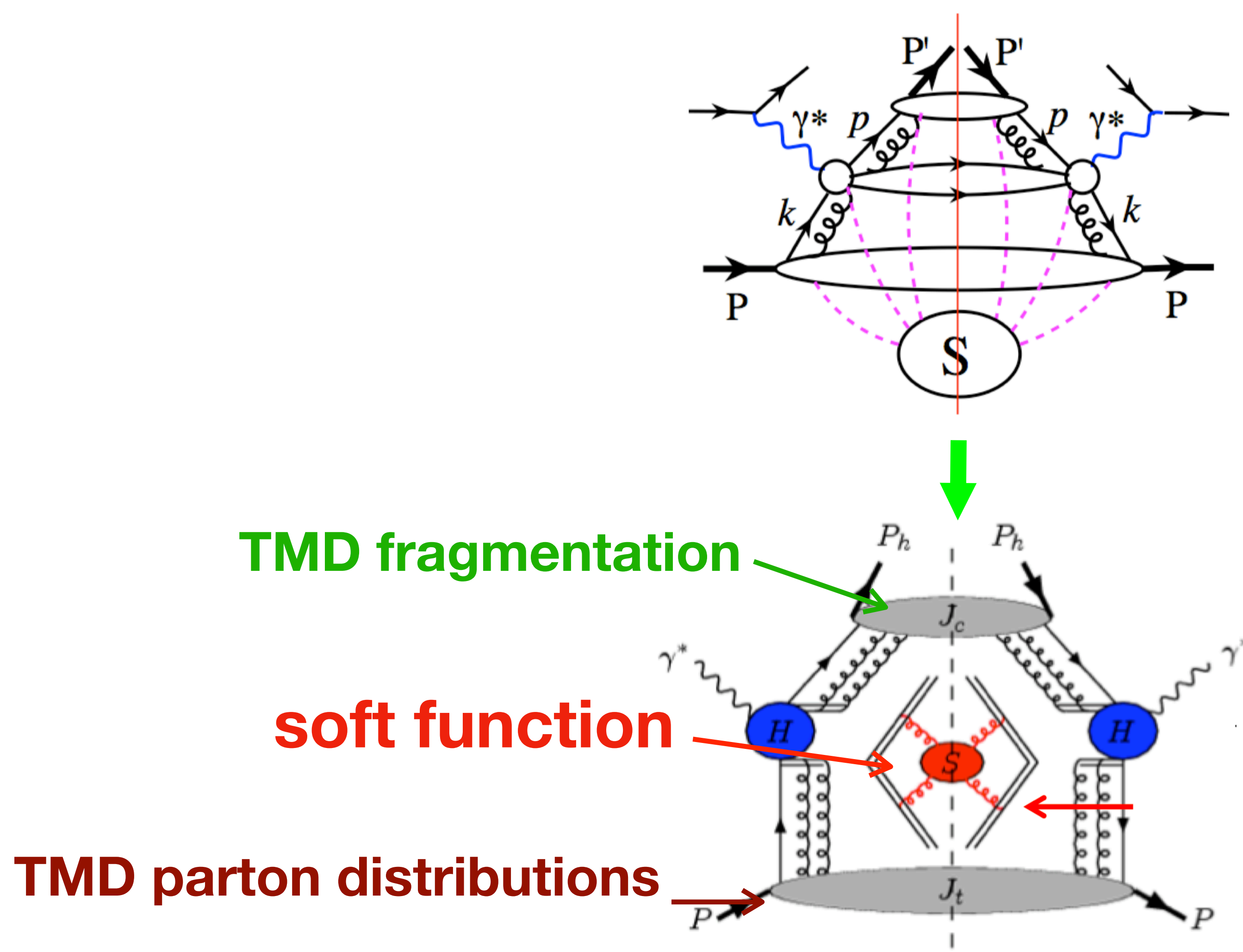

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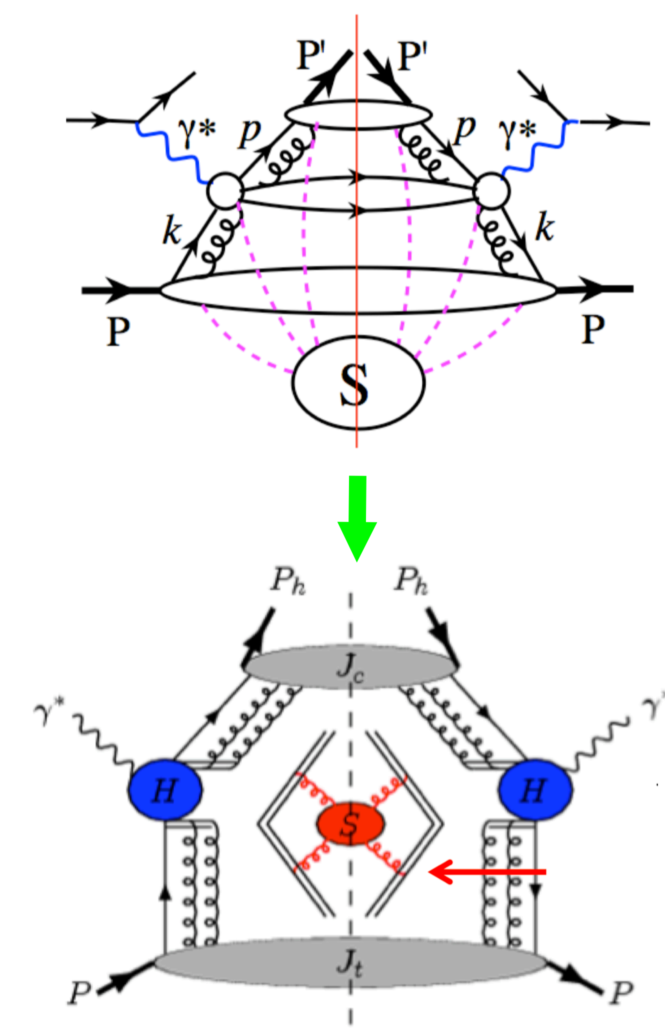
The definition of TMDPDF:

$$f_i^{TMD}(x, b_\perp, \mu, \zeta) = \frac{f(x, b_\perp, \mu, Y)}{\sqrt{S(b_\perp, \mu, Y, Y')}}$$

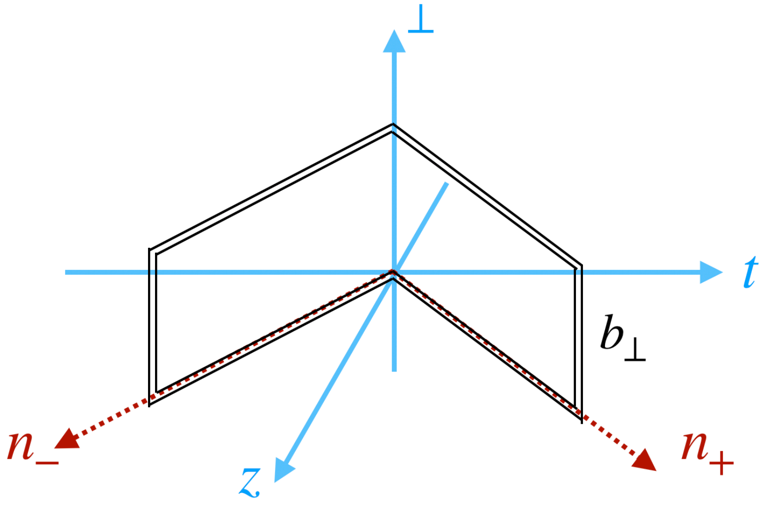
$$f(x, b_\perp, \mu, Y) = \frac{1}{2}\int \frac{d\xi^-}{2\pi} e^{-ix\xi^- P^+} \langle PS | \bar{\psi}_n(\xi^-, 0, b_\perp) \gamma^+ \psi_n(0) | PS \rangle \Big|_Y$$



TMD Soft Function

- **The TMD soft function is defined by two conjugate light-like Wilson lines:**

$$S(b_\perp, \mu, Y, Y') = \frac{1}{N_c} \mathrm{tr} \langle 0 | \bar{\mathcal{T}} \left[U^\dagger_{n^+}(-\infty, \vec{b}_\perp)_{Y'} U^\dagger_{n^-}(\pm\infty, \vec{b}_\perp)_Y \right]$$
$$\mathcal{T} \left[U_{n^-}(\pm\infty, \vec{0}_\perp)_Y U_{n^+}(-\infty, \vec{0}_\perp)_{Y'} \right] | 0 \rangle$$



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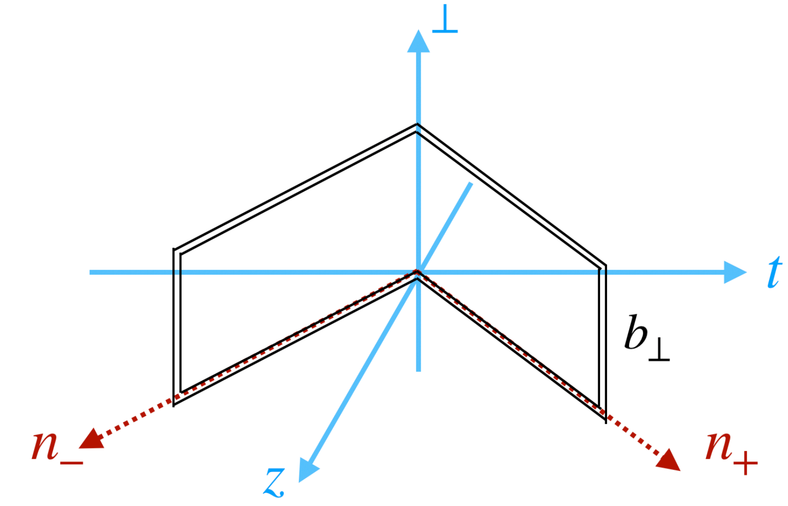

- **The intrinsic, rapidity independent, soft function $S_I(b_\perp,\mu)$:**

$$S(b_\perp,\mu,Y,Y') = e^{(Y+Y')K(b_\perp,\mu)}S_I^{-1}(b_\perp,\mu)$$

- **X. Ji, Y. Liu, and Y.-S. Liu, (2019), arXiv:1910.11415 [hep-ph];**
- **J. Collins, Camb. Monogr. Part. Phys. Nucl. Phys. Cosmol. 32, 1 (2011).**

Rapidity evolution can be described by the Collins-Soper kernel $K(b_\perp,\mu)$

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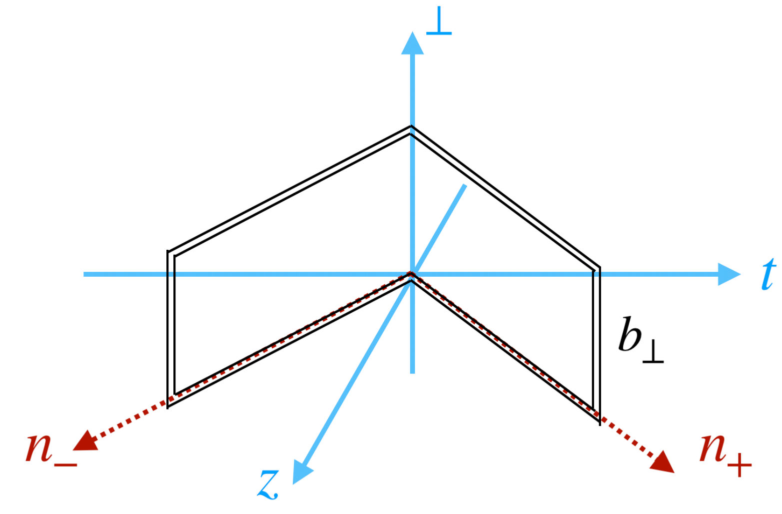

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Rapidity evolution can be described by the Collins-Soper kernel $K(b_\perp, \mu)$

The calculation of the intrinsic soft function and Collins-Super kernel paves the way towards the **first principle predictions** of physical cross sections for, e.g., SIDIS and Drell-Yan processes at small transverse momentum.

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Calculate the TMD Soft Function on lattice

QCD Soft Function from Large-Momentum Effective Theory on Lattice

Xiangdong Ji,[1,2] Yizhuang Liu,[1,*] and Yu-Sheng Liu[1]

[1] *Tsung-Dao Lee Institute, Shanghai Jiao Tong University, Shanghai 200240, China*
[2] *Department of Physics, University of Maryland, College Park, MD 20742, USA*

(Dated: October 28, 2019)

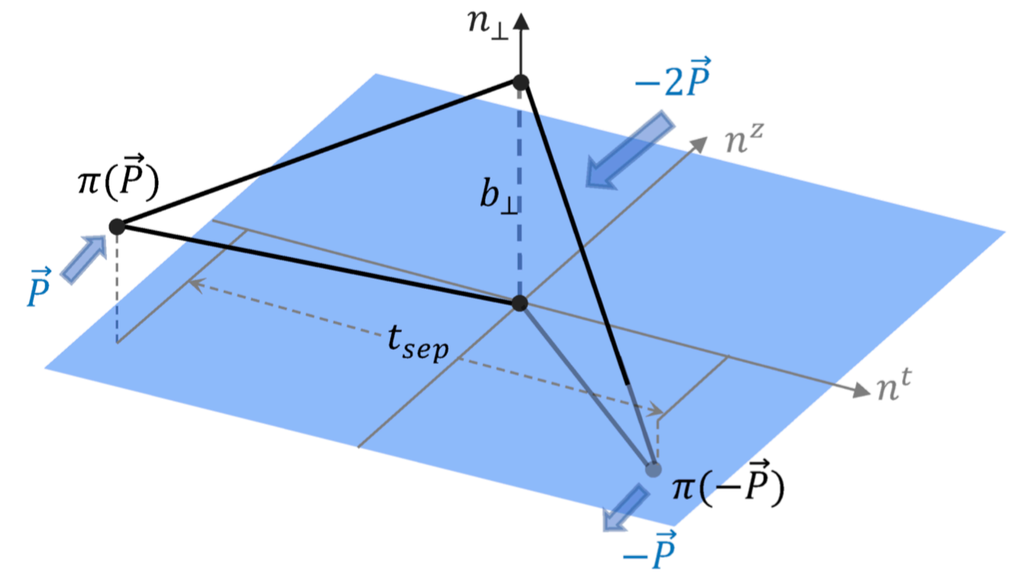
We study Euclidean lattice calculation of the QCD soft function, which involves two conjugate lightcone directions in the framework of large-momentum effective theory. We find that the transverse momentum dependent (TMD) soft function required by TMD factorization can be formulated as the form factor of a pair of color sources traveling with nearly-lightlike velocities, and thus can be calculated using lattice heavy-quark effective theory. A simple generalization shows that the factorization of a large-momentum light-meson form factor combining with quasi-TMD wave function can also be used to extract the soft function on lattice.

- **Ji, Xiangdong and Liu, Yizhuang and Liu, Yu–Sheng, Nucl. Phys. B 955 (2020), 115054**

Calculate the TMD Soft Function on lattice

1. Define a large-momentum **form factor** of a non-singlet light pseudo-scalar meson:

$$F(b_\perp, P^z) = \langle \pi(-P^z) \,|\, \left(\bar{q}_1 \Gamma q_1\right)(b_\perp)\left(\bar{q}_2 \Gamma q_2\right)(0) \,|\, \pi(P^z)\rangle$$



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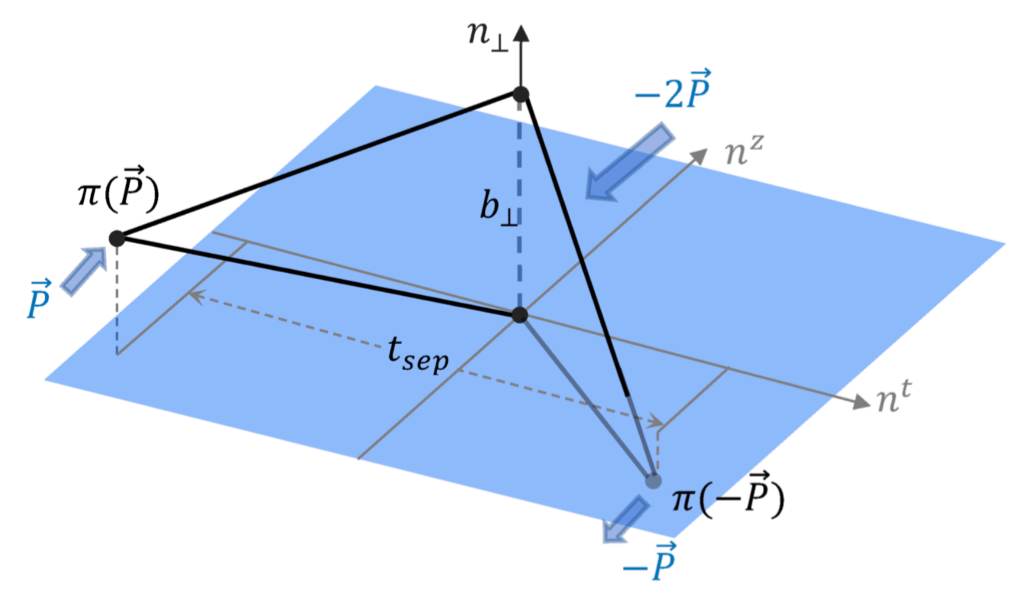


2. For large P^z, the **form factor** can be factorized into the **quasi-TMDWF** and the **intrinsic soft function** in the framework of **LaMET**:

$$F(b_\perp, P^z) = S_I(b_\perp) \int_0^1 dx dx' H(x, x', P^z) \Phi^\dagger(x', b_\perp, -P^z) \Phi(x, b_\perp, P^z)$$

The perturbative kernel $H(x, x', P^z) = \frac{1}{2N_c} + \mathcal{O}(\alpha_s)$

Calculate the TMD Soft Function on lattice

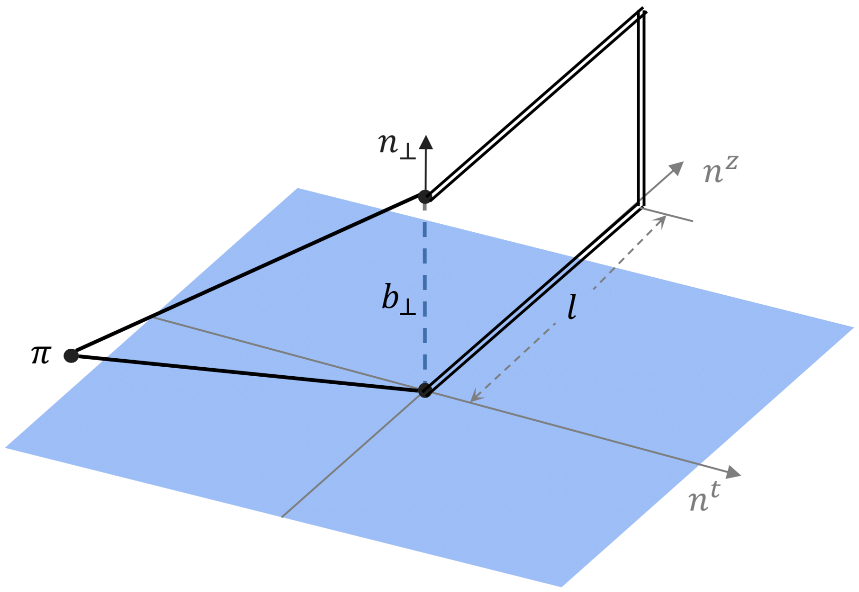
3. The subtracted quasi-TMDWF in coordinate space:

$\Gamma_\Phi = \gamma^t \gamma_5$

l: **length of the Wilson line**

$$\phi\left(z, b_\perp, P^z\right) = \lim_{\ell\to\infty} \frac{\phi_\ell\left(z, b_\perp, P^z, \ell\right)}{\sqrt{Z_E\left(2\ell, b_\perp\right)}}$$

$$= \lim_{\ell\to\infty} \frac{\left\langle 0 \left| \bar{q}_1\left(\frac{z}{2}n^z + \vec{b}\right) \Gamma_\Phi \mathscr{W}(\vec{b}, \ell) q_2\left(-\frac{z}{2}n^z\right) \right| \pi(\vec{P}) \right\rangle}{\sqrt{Z_E\left(2\ell, b_\perp\right)}}$$



- $\mathscr{W}(\vec{b}, \ell)$ **is the spacelike staple-shaped gauge link.**
- $Z_E(2l, b_\perp)$ is the vacuum expectation value of a rectangular spacelike Wilson loop, which removes the **pinch-pole singularity** and **Wilson-line self-energy** in quasi-TMDWF.

Calculate the TMD Soft Function on lattice

4. The intrinsic soft function at leading order:

$$S_I(b_\perp) = \frac{2N_c F(b_\perp, P^z)}{\left|\phi(0, b_\perp, P^z)\right|^2} + \mathcal{O}\left(\alpha_s, (1/P^z)^2\right)$$

Calculate the TMD Soft Function on lattice

4. The intrinsic soft function at leading order in α_s:

$$S_I\left(b_\perp\right) = \frac{2N_c F\left(b_\perp, P^z\right)}{\left|\phi\left(0, b_\perp, P^z\right)\right|^2} + \mathcal{O}\left(\alpha_s, (1/P^z)^2\right)$$

The intrinsic soft function in the $\overline{\text{MS}}$ **scheme:**

$$S_{I,\overline{\text{MS}}}\left(b_\perp, \mu\right) = \left(\frac{S_I\left(b_\perp, 1/a\right)}{S_I\left(b_{\perp,0}, 1/a\right)}\right) S_{I,\overline{\text{MS}}}\left(b_{\perp,0}, \mu\right) = \frac{F\left(b_\perp, P^z\right)}{F\left(b_{\perp,0}, P^z\right)} \frac{\left|\phi\left(0, b_{\perp,0}, P^z\right)\right|^2}{\left|\phi\left(0, b_\perp, P^z\right)\right|^2} + \mathcal{O}\left(\alpha_s, (1/P^z)^2\right)$$

$b_{\perp,0}$ is a normalization reference

calculable on lattice

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Lattice Set Up

We do the lattice QCD calculation of the intrinsic soft function on the **A654 configurations** (generated by CLS collaboration):

β	$L^3 \times T$	a (fm)	c_{sw}	κ_l^{sea}	m_π^{sea}(MeV)
3.34	$24^3 \times 48$	0.098	2.06686	0.13675	333
			N_{cfg}	κ_l^v	m_π^v (MeV)
			868	0.13622	547

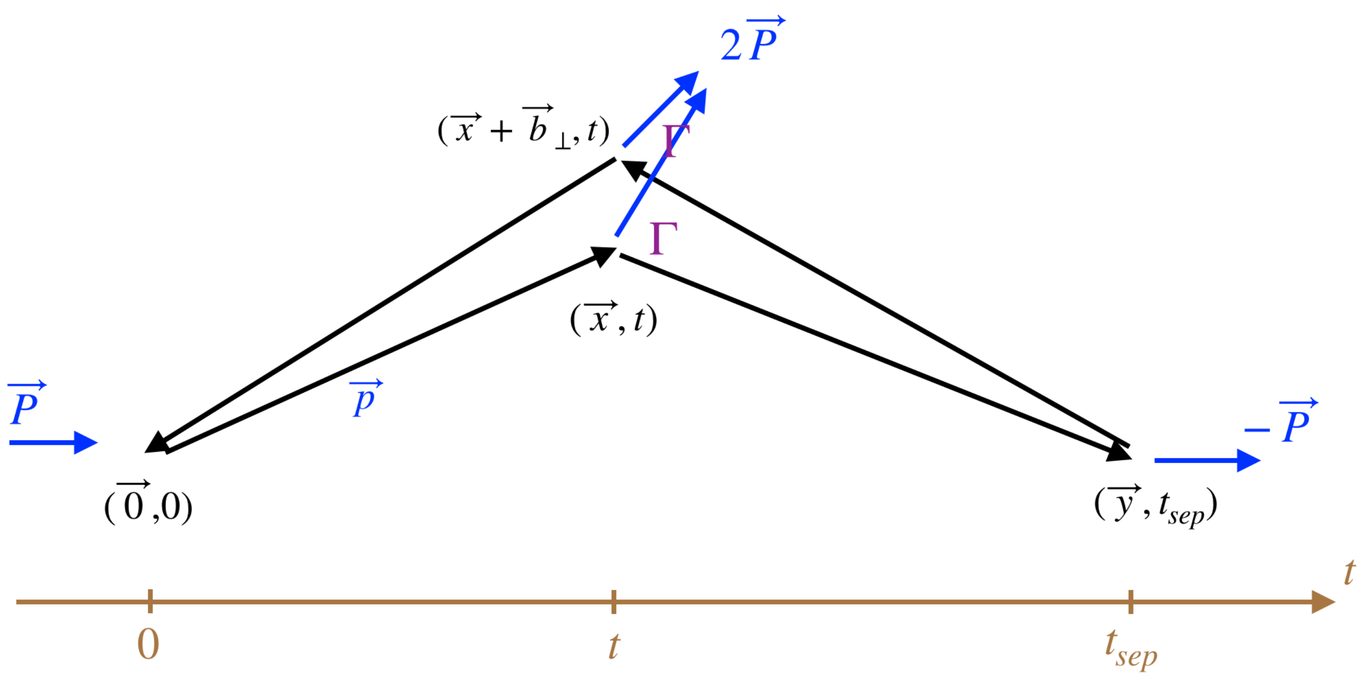
- 2+1flavor clover fermions and tree-level Symanzik gauge action;
- Coulomb gauge fixed **wall source propagators** for both initial and final states;
- $P^z = 1.05,\ 1.58,\ 2.11\mathrm{GeV}$;
- Use $m_\pi = 547\mathrm{MeV}$ instead of $333\mathrm{MeV}$ to get a **better signal**;
- Physically, the soft function becomes **independent** of the meson mass for large momentum P^z.

Numerical Results

The 2-point and 3-point correlation functions can be calculated on lattice:

$$C_2(b_\perp, P^z; p^z, l, t) = \frac{A_w(p^z)A_p}{2E} e^{-Et} \phi_l(0, b_\perp, P^z, l)\left(1 + c_0 e^{-\Delta E t}\right)$$

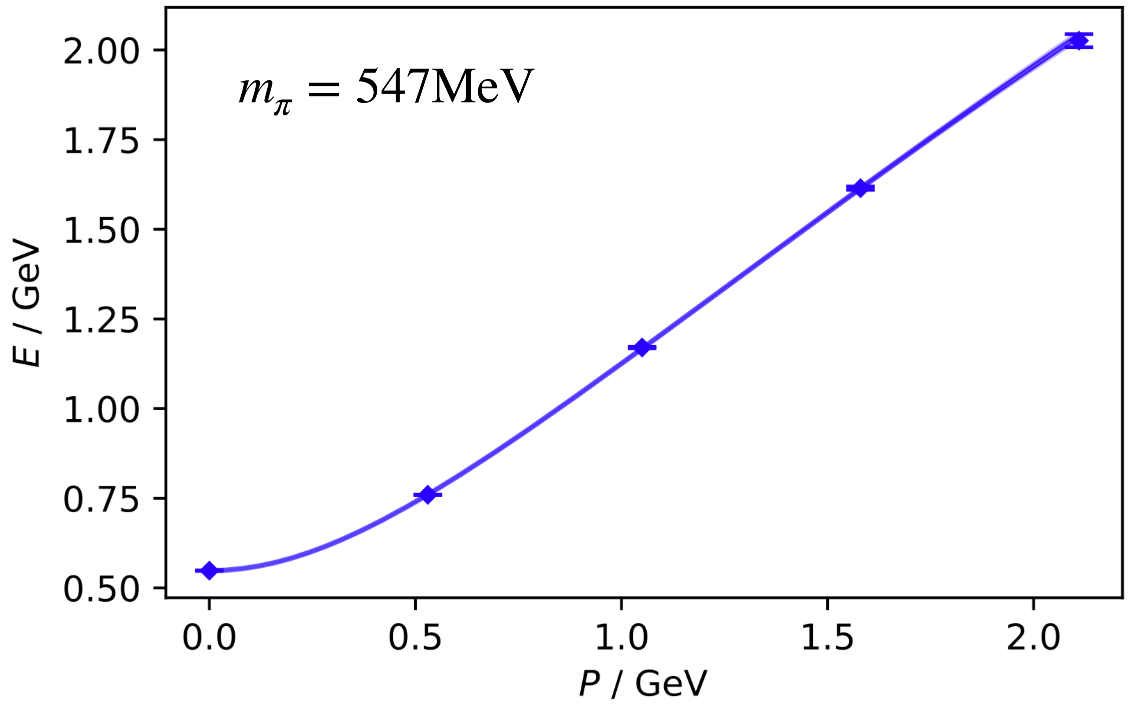
$$C_3(b_\perp, P^z; p^z, t_{sep}, t) = \frac{A_w(p^z)^2}{(2E)^2} e^{-Et_{sep}} \left[F(b_\perp, P^z) + c_1\left(e^{-\Delta E t} + e^{-\Delta E(t_{sep}-t)}\right) + c_2 e^{-\Delta E t_{sep}}\right]$$



The quasi-TMDWF and form factor can be obtained from a **joint fit** of 2-point and 3-point correlation functions.

Numerical Results

- **Simulation check:** the dispersion relation of the pion

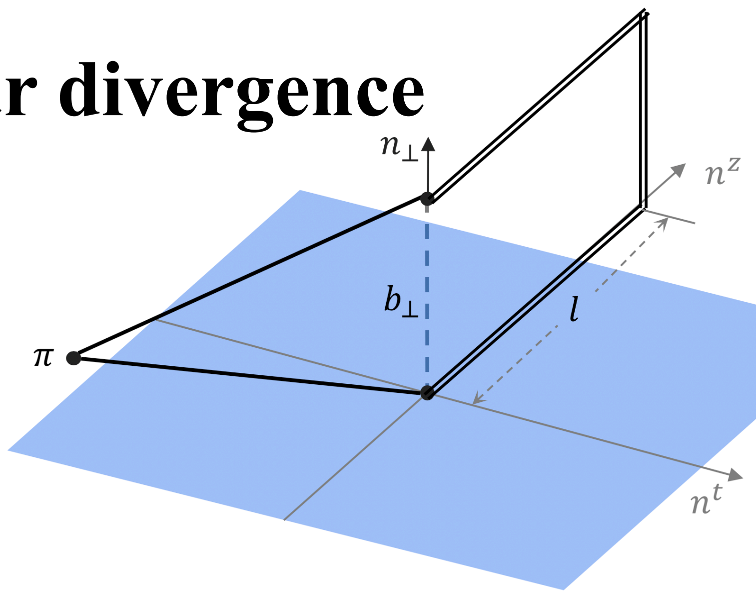


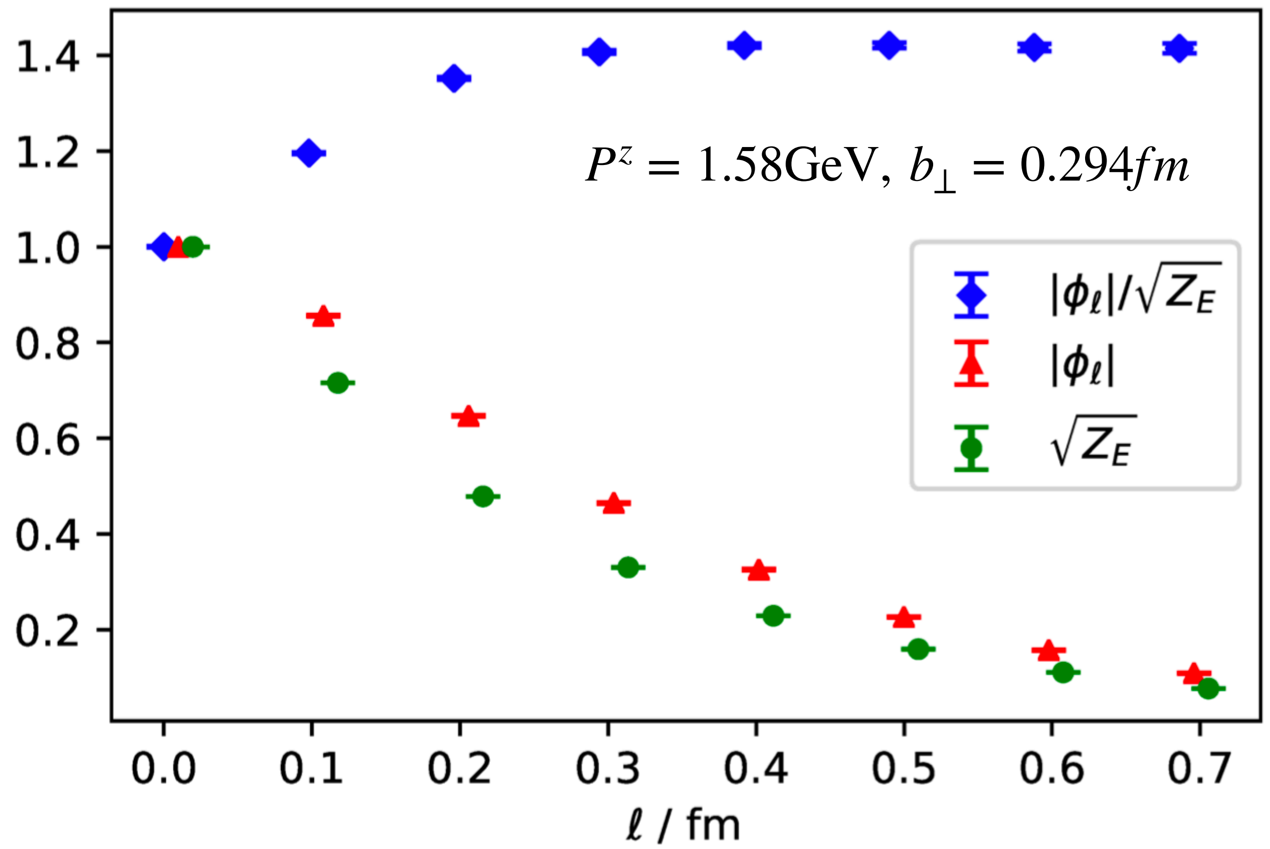
- $E_\pi = \sqrt{m_\pi^2 + c_1 P^2 + c_2 a^2 P^4}$;
- Fit results: $c_1 = 0.9945(40)$ and $c_2 = -0.0053(5)$
- **Consistent** with the dispersion relation of pion in the continuum limit with error.

Numerical Results

- **l-dependence of quasi-TMDWF and Linear divergence**

$$\phi\left(z, b_\perp, P^z\right) = \lim_{\ell \to \infty} \frac{\phi_\ell\left(z, b_\perp, P^z, \ell\right)}{\sqrt{Z_E\left(2\ell, b_\perp\right)}}$$





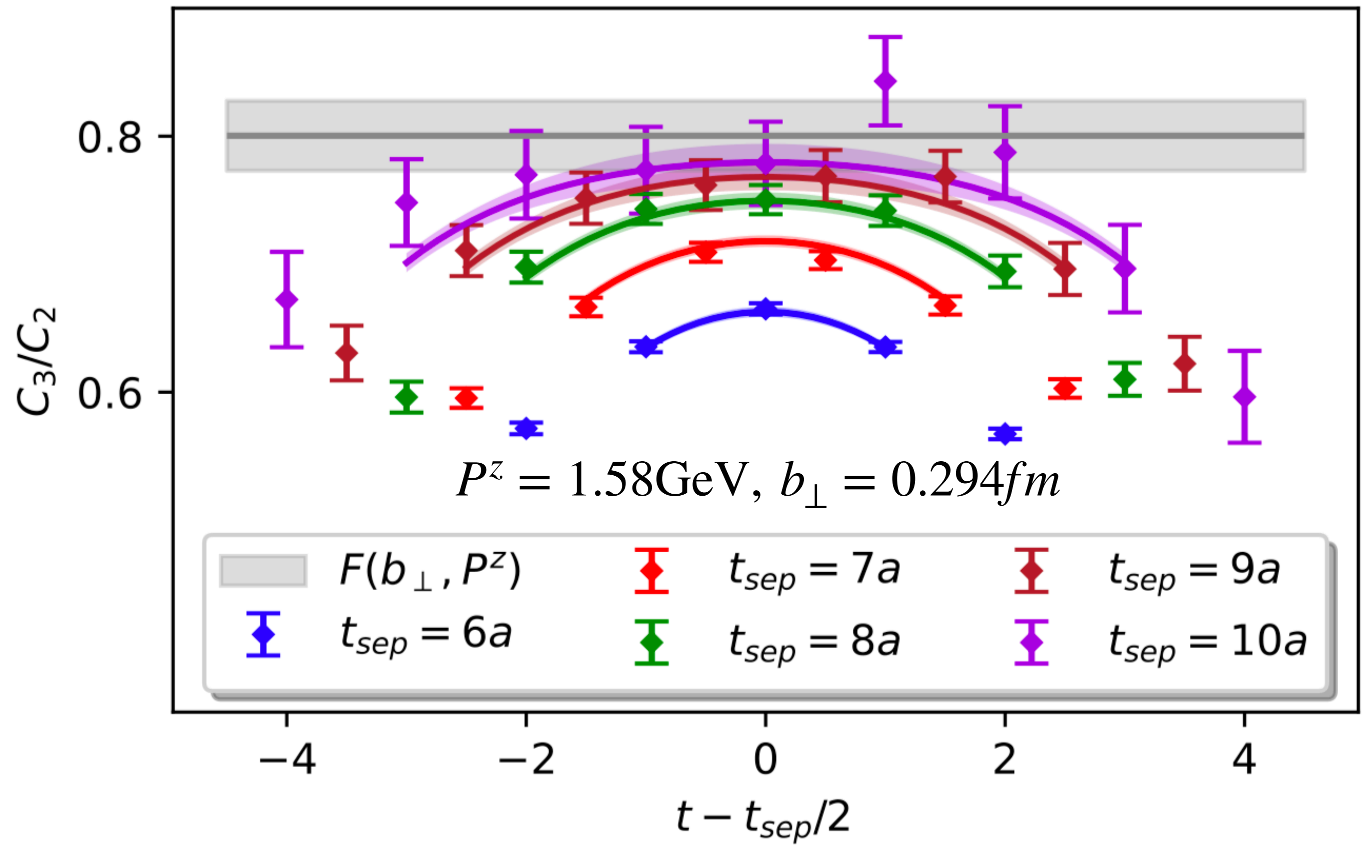
- Both $|\phi_l|$ and $\sqrt{Z_E}$ contain **linear divergence**
- Linear divergence **cancelled** at large-l
- ϕ is **length independent** when $l > 0.4fm$
- We use $l = 7a = 0.686fm$ as asymptotic results for all cases in our calculation to **remove the linear divergence**

Numerical Results

- **Joint fit results of the form factor**

$$C_2(b_\perp, t) \sim \langle O_\pi(t_{sep}) \bar{O}_\pi(0) \rangle = \frac{A_w A_p}{2E} e^{-Et} \phi_l \left(1 + c_0 e^{-\Delta E t}\right)$$

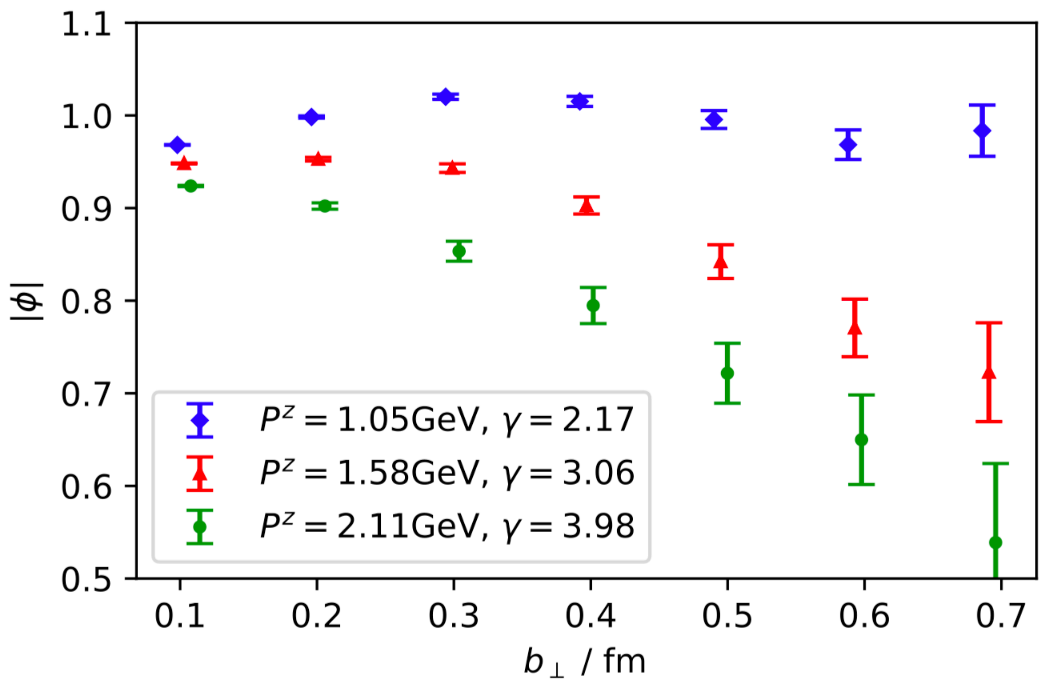
$$C_3(b_\perp, t) \sim \langle O_\pi(t_{sep}) O_\Gamma(t) \bar{O}_\pi(0) \rangle = \frac{A_w^2}{(2E)^2} e^{-Et_{sep}} \left[F + c_1 \left(e^{-\Delta E t} + e^{-\Delta E(t_{sep}-t)}\right) + c_2 e^{-\Delta E t_{sep}}\right]$$



- Fitted data at $t_{sep} = 6 \sim 10a$;
- $F(b_\perp, P^z)$ corresponds to the **ground state contribution at $t_{sep} \to \infty$**;
- $\chi^2/\mathrm{d.o.f} = 0.6$, our data in general **agree with** the predicted fit function.

Numerical Results

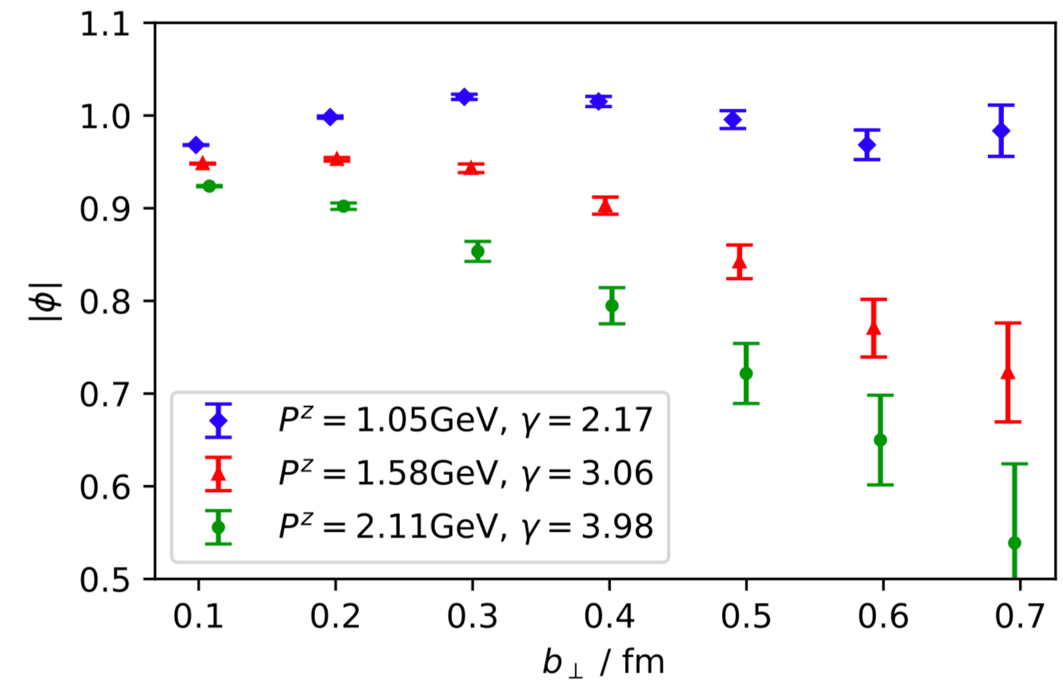
- **Joint fit results of the quasi-TMDWF**

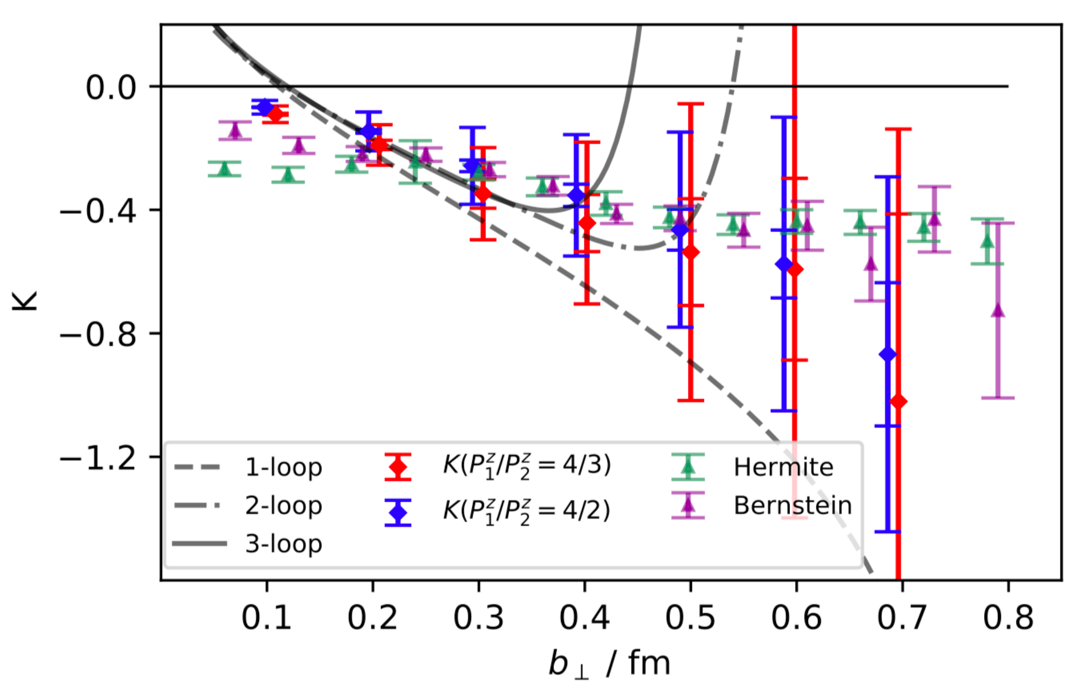


- Upper panel is the P^z-dependence of quasi-TMDWF;
- The P^z-dependence in $|\phi|$ is related to the **Collins-Soper kernel**

Numerical Results

- **Joint fit results of the quasi-TMDWF**





- Upper panel is the P^z-dependence of quasi-TMDWF;
- The P^z-dependence in $|\phi|$ is related to the **Collins-Soper kernel**

$$K(b_\perp, \mu) = \frac{1}{\ln(P_1^z/P_2^z)} \ln \left| \frac{\phi(0, b_\perp, P_1^z)}{\phi(0, b_\perp, P_2^z)} \right| + \mathcal{O}(\alpha_s, \gamma^{-2})$$

- Lower panel shows the extracted Collins-Soper kernel, compare with **perturbative calculations up to 3-loops*** and results from **quenched lattice calculations of TMDPDF****.
- Our results are **consistent with** perturbative calculations (at small $b_\perp$) and results from the TMDPDF.

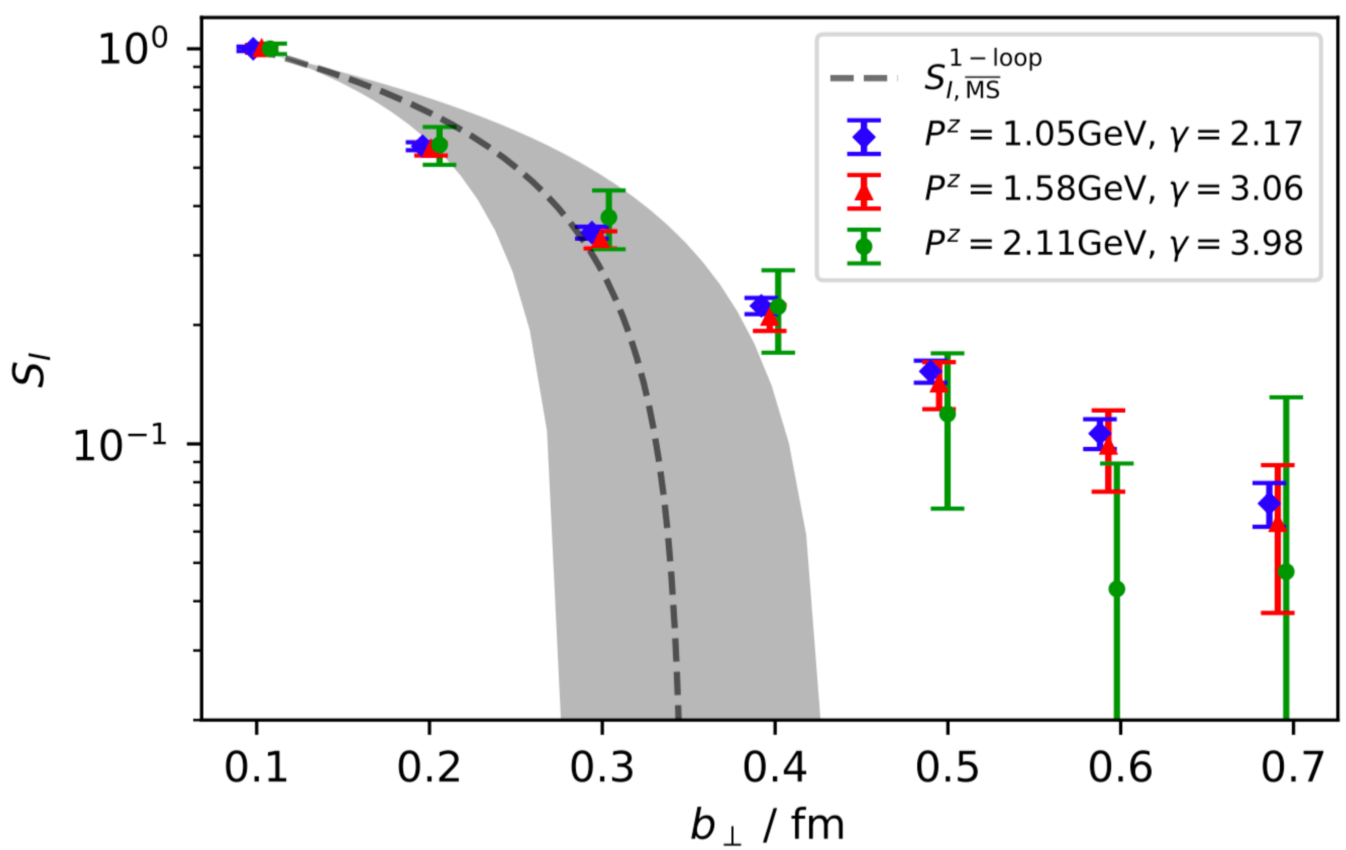
*** Li and Zhu, Phys. Rev. Lett.118(2017)2, 022004;**

**** Shanahan and Wagman and Zhao, arXiv:hep-lat/2003.06063.**

Numerical Results

- **Joint fit results of the intrinsic soft function**

$$S_{I,\overline{\mathrm{MS}}}\left(b_\perp,\mu\right)=\frac{F\left(b_\perp,P^z\right)}{F\left(b_{\perp,0},P^z\right)}\frac{\left|\phi\left(0,b_{\perp,0},P^z\right)\right|^2}{\left|\phi\left(0,b_\perp,P^z\right)\right|^2}+\mathcal{O}\left(\alpha_s,(1/P^z)^2\right)$$



- With different P^z , the results are **consistent** with each other, demonstrating that the **asymptotic limit** is stable within errors;
- The systematic uncertainty from the **operator mixing** has been taken into account.
- The dashed curve shows the result of the **1-loop calculation** with the strong coupling constant $\alpha_s\left(1/b_\perp\right)$, and the shaded band corresponds to the scale uncertainty of α_s:

$$S_{I,\overline{\mathrm{MS}}}\left(b_\perp,\mu\right)=1-\frac{\alpha_s C_F}{\pi}\ln\frac{\mu^2 b_\perp^2}{4e^{-2\gamma_E}}+\mathcal{O}\left(\alpha_s\right)$$

Summary and Outlook

- This work present an **exploratory** lattice calculation of the intrinsic soft function;
- The Collins-Soper kernel obtained from our quasi-TMDWF agrees with the perturbative results and previous quenched lattice calculations;
- Our results of the intrinsic soft function are almost **independent** of the hadron momentum, and **consistent with** the 1-loop perturbative calculation;
- This work paves the way towards the **first principle predictions** of physical cross sections for, e.g., Drell-Yan and Higgs productions at small transverse momentum.

Thank you very much!

Backup

Heatmap for the NPR factor and operator mixing effective of our result with $P^z = 1.58\text{GeV}$, $b_\perp = \{0, 0.294, 0.588\}\text{fm}$