QCD calculations of $B \to P$ form factors with subleading power contributions

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Based on: [B-Y Cui, Y-K Huang, Z-H Mi, Y-L Shen, C Wang, Y-M Wang, to appear soon].

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Calculational tools for $B \to P$ form factors

▽ QCD/SCET Factorization:

- Factorization formulae for semeleptonic B-meson decays [BBNS, BPRS, and many others]:

$$F_i^{B\to M}(E) = C_i(E)\xi_a(E) + \int_0^\infty \frac{d\omega}{\omega} \int_0^1 dv\, C_i^{(B1)}(E,\tau) \otimes_\tau J_a(\tau; v, \omega)\phi_B^+(\omega)\phi_M(v).$$

- Perturbative calculations of the hard matching coefficients [Bauer, Fleming, Pirjol, Stewart, 2001].
- Perturbative calculations of the hard-collinear matching coefficients [Becher, Hill, 2004; Hill, Becher, Lee, Neubert, 2004; Beneke, Yang, 2006].

▽ Transverse-Momentum Dependent(TMD) Factorization [Botts, Sterman, 1989; Li, Sterman, 1992]

▽ Lattice QCD Technique:

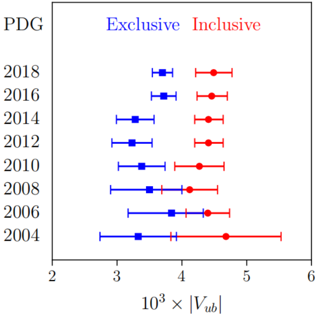
- First-principles calculations numerically at small hadronic recoil.

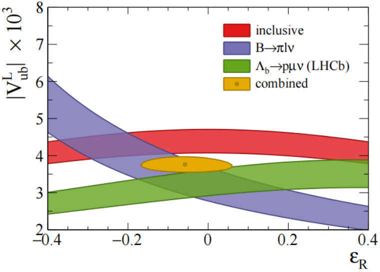
▽ Light-Cone Sum Rules in QCD/SCET(LCSR):

- Light-cone QCD sum rules with the light-meson LCDA. [Ball, Braun, Khodjamirian, etc]
- Light-cone QCD sum rules with the B-meson LCDA. [Khodjamirian, Lü, Shen, Wang, etc]
- Light-cone SCET sum rules with B-meson LCDA. [[Feldmann, Lü, Shen, Wang, etc]
- Light-cone QCD sum rules with the chiral current for the light meson.[Huang, Li, Wu, etc]

Why subleading power corrections

- Understanding the general properties of power expansion in EFTs (HQET, SCET, NRQCD).
- Interesting to understand the strong interaction dynamics of heavy quark decays.
  - Factorization properties of the subleading-power amplitudes.
  - Renormalization and asymptotic properties of the (higher-twist) B-meson DAs.
  - Interplay of different QCD techniques.
- Precision determination of the CKM matrix element $|V_{ub}|$.
  - Power corrections, QED corrections.
  - Long-standing tension between the exclusive and inclusive $|V_{ub}|$.





Improvement

▽ New Improvement I:

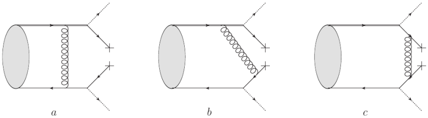
- Subleading power correction from **b-quark field expansion** in HQET as [Mannel,Moreno,Pivovarov, 2020]:

$$b(x) = e^{-im_b v\cdot x}\left[h_v + \frac{i\vec{\not{D}}}{2m_b} h_v + \cdots\right].$$

- Subleading power correction from the **hard-collinear quark propagator**:

$$\underbrace{\frac{i\not{p}}{n\cdot p(\bar{n}\cdot p - \bar{n}\cdot k)}}_{\text{leading power}} + \underbrace{\frac{-i\not{k}}{n\cdot p(\bar{n}\cdot p - \bar{n}\cdot k)} + \frac{\bar{n}\cdot p}{n\cdot p}\frac{in\cdot k}{(\bar{n}\cdot p - \bar{n}\cdot k)^2}\frac{\not{\bar{n}}}{2}}_{\text{subleading power}}.$$

- **Twist-five and -six** four-particle corrections in factorization approximation as two-particle lower-twist LCDAs and quark condensate [Agaev,Braun,Offen,Porkert,2010].



- **Strange-quark mass effect** at leading-order(LO) in α_s.

$$\underbrace{\frac{i\not{p}}{n\cdot p(\bar{n}\cdot p - \bar{n}\cdot k)}}_{\text{leading power}} + \underbrace{\frac{im_q}{l^2 + i0} + \frac{i\not{p}m_q^2}{(l^2+i0)^2}}_{\text{subleading power}}.$$

Improvement

▽ New Improvement II:

- Construct the general and three-parameter model of higher-twist B-meson LCDAs including three-particle twist-five and twist-six LCDAs ($\phi_5(\omega_1, \omega_2, \mu), \psi_5(\omega_1, \omega_2, \mu), \tilde{\psi}_5(\omega_1, \omega_2, \mu)$ and $\phi_6(\omega_1, \omega_2, \mu)$).

- Perform the complete RG evolution for leading power contribution including the lower-twist LCDAs ($\phi_B^+(\omega, \mu)$ and $\phi_B^-(\omega, \mu)$) at leading logarithmic.

B-meson Light-Cone Sum Rules

- The form factors for $B \to P$ are defined as [Beneke,Feldmann,2001]:

$$\langle P(p)|\bar{q}\,\gamma_\mu\, b|\bar{B}(P)\rangle = 2 f^+_{B\to P}(q^2)\, p_\mu + \left[f^+_{B\to P}(q^2) + f^-_{B\to P}(q^2)\right] q_\mu\,,$$

$$\langle P(p)|\bar{q}\,\sigma_{\mu\nu} q^\nu\, b|\bar{B}(P)\rangle = i\,\frac{f^T_{B\to P}(q^2)}{m_B + m_P}\,\left[q^2 (2p+q)_\mu - (m_B^2 - m_P^2)\, q_\mu\right].$$

- Vacuum-to-B-meson correlation function defined as:

$$\Pi_\mu(n\cdot p, \bar{n}\cdot p) = \int d^4x\, e^{ip\cdot x} \langle 0|T\left\{\bar{d}(x)\,\not{n}\gamma_5\, q(x),\ \bar{q}(0)\,\Gamma_\mu\, b(0)\right\}|B(P)\rangle$$

$$= \begin{cases} \Pi(n\cdot p, \bar{n}\cdot p)\, n_\mu + \tilde{\Pi}(n\cdot p, \bar{n}\cdot p)\,\bar{n}_\mu\,, & \Gamma_\mu = \gamma_\mu \\ \\ \Pi_T(n\cdot p, \bar{n}\cdot p)\,\left[\bar{n}\cdot q\, n_\mu - n\cdot q\,\bar{n}_\mu\right], & \Gamma_\mu = \sigma_{\mu\nu} q^\nu \end{cases}.$$

- Power counting: $\quad n\cdot p \sim \mathcal{O}(m_b), \qquad \bar{n}\cdot p \sim \mathcal{O}(\Lambda)$
- The hadronic representation and dispersion relation:

$$\Pi_{\mu,\mathrm{had}}(p,q) = \frac{\langle 0|\bar{d}\,\not{n}\gamma_5\, q|P(p)\rangle\langle P(p)|\bar{q}\,\Gamma^\mu\, b|B(P)\rangle}{m_P^2 - p^2} + \text{continuum},$$

$$= \frac{i f_P}{m_P^2/n\cdot p - \bar{n}\cdot p}\,\langle P(p)|\bar{q}\,\Gamma^\mu\, b|B(P)\rangle + \frac{1}{\pi}\int_{\omega_s}^\infty \frac{d\omega'}{\omega' - \bar{n}\cdot p - i0}\,\mathrm{Im}'_{\omega}\,\Pi_\mu(n\cdot p, \omega')$$

B-meson Light-Cone Sum Rules

- LCSR for $B \to P$ form factors:

$$f^{+}_{B\to P}(q^2) = -i\frac{1}{f_P\, n\cdot p}\, e^{\frac{m_p^2}{(n\cdot p\,\omega_M)}} \int_0^{\omega_s} \frac{d\omega'}{\pi} e^{\frac{\omega'}{\omega_M}}\, \mathrm{Im}'_{\omega} \left[\tilde{\Pi}(q^2) + \frac{n\cdot p - m_B}{m_B}\Pi(q^2)\right],$$

$$f^{0}_{B\to P}(q^2) = -i\frac{1}{f_P\, m_B}\, e^{\frac{m_p^2}{(n\cdot p\,\omega_M)}} \int_0^{\omega_s} \frac{d\omega'}{\pi} e^{\frac{\omega'}{\omega_M}}\, \mathrm{Im}'_{\omega} \left[\tilde{\Pi}(q^2) - \frac{n\cdot p - m_B}{m_B}\Pi(q^2)\right],$$

$$f^{T}_{B\to P}(q^2) = \frac{2(m_B + m_P)}{f_P\, m_B\, n\cdot p}\, e^{\frac{m_p^2}{(n\cdot p\,\omega_M)}} \int_0^{\omega_s} \frac{d\omega'}{\pi} e^{\frac{\omega'}{\omega_M}}\, \mathrm{Im}'_{\omega}\Pi_T(q^2).$$

- In HQET limit:

$$\sigma_{\mu\nu}q^{\nu}h_v = \frac{i}{4}\left[\bar{n}\cdot q\, n_\mu - n\cdot q\bar{n}_\mu\right](\not{\bar{n}} - \not{n})\, h_v, \qquad \to \qquad \Pi_T = -\frac{i}{2}\left(\tilde{\Pi} - \Pi\right).$$

- Large recoil symmetry with LP contribution at LO [Beneke,Feldmann,2001]:

$$f^{+}_{B\to P} = \frac{m_B}{n\cdot p} f^{0}_{B\to P} = \frac{m_B}{m_B - m_P} f^{T}_{B\to P}.$$

Leading power and higher-twist corrections

- Leading power contribution for $B \to \pi, K$ form factors at NLL:

$$f_P\, e^{-\frac{m_P^2}{n\cdot p\,\omega_M}} \left\{\frac{n\cdot p}{m_B} f_{B\to P}^{+,\mathrm{NLL,LP}}(q^2),\quad f_{B\to P}^{0,\mathrm{NLL,LP}}(q^2)\right\} = \left[U_2(\mu_{h2},\mu)\tilde{f}_B(\mu_{h2})\right]$$
$$\times \int_0^{\omega_s} d\omega\, e^{-\frac{\omega}{\omega_M}} \left\{\tilde{\phi}^+_{B,\mathrm{eff}}(\omega,\mu) + \left[U_1(n\cdot p,\mu_{h1},\mu)\tilde{C}^{(-)}(n\cdot p,\mu_{h1})\right]\tilde{\phi}^-_{B,\mathrm{eff}}(\omega,\mu)\right.$$
$$\left.\pm \frac{n\cdot p - m_B}{m_B}\left[\phi^+_{B,\mathrm{eff}}(\omega,\mu) + C^{(-)}(n\cdot p,\mu_{h1})\phi^-_{B,\mathrm{eff}}(\omega,\mu)\right]\right\};$$
$$f_P\, e^{-\frac{m_P^2}{n\cdot p\,\omega_M}} \frac{n\cdot p}{m_B+m_P} f_{B\to P}^{T,\mathrm{NLL,LP}}(q^2) = \left[U_2(\mu_{h2},\mu)\tilde{f}_B(\mu_{h2})\right]\int_0^{\omega_s} d\omega\, e^{-\frac{\omega}{\omega_M}}\left\{\tilde{\phi}^+_{B,\mathrm{eff}}(\omega,\mu)\right.$$
$$\left.+\left[U_1(n\cdot p,\mu_{h1},\mu)U_3(\nu_{h1},\nu)C_T^{(-)}(n\cdot p,\mu_{h1},\nu_h)\right]\tilde{\phi}^-_{B,\mathrm{eff}}(\omega,\mu)\right\}.$$

  - The effective distribution amplitudes $\phi^\pm_{B,\mathrm{eff}}(\omega,\mu)$ are given by [Shen,Wang,2015].
  - The improved definition of $\phi^\pm_{B,\mathrm{eff}}(\omega,\mu)$ including s-quark effect at NLO[Lü,Shen,Wang,Wei,2018].
- Two-particle and three-particle higher-twist B-meson LCDAs corrections for $B \to \pi, K$ form factors at tree level[Lü,Shen,Wang,Wei,2018]:

$$f_P\, n\cdot p\, e^{-\frac{m_P^2}{n\cdot p\,\omega_M}}\left\{f_{B\to P}^{+,\mathrm{hc,NLP}}(q^2),\quad \frac{m_B}{n\cdot p}f_{B\to P}^{0,\mathrm{hc,NLP}}(q^2),\quad \frac{m_B}{m_B+m_P}f_{B\to P}^{T,\mathrm{hc,NLP}}(q^2)\right\}$$
$$= \frac{\tilde{f}_B m_B}{n\cdot p}\int_{0_-}^{\omega_s} d\omega\, e^{\frac{\omega}{\omega_M}}\left\{\frac{d}{d\omega}H_{\bar{n}}^{\mathrm{2PHT}}(\omega,\mu)\theta(\omega) + \frac{d}{d\omega}\int_0^{\omega} d\omega_1\int_{\omega-\omega_1}^{\infty}\frac{d\omega_2}{\omega_2}\right.$$
$$\left.\times\left[H_{\bar{n}}^{\mathrm{3PHT}}\left(\frac{\omega-\omega_1}{\omega_2},\omega_1,\omega_2,\mu\right) + \mathcal{C}_i H_n^{\mathrm{3PHT}}\left(\frac{\omega-\omega_1}{\omega_2},\omega_1,\omega_2,\mu\right)\right]\theta(\omega)\right\}.$$

NLP corrections to $B \to P$ form factors

- Compute the correlation function including the subleading effective current $\bar{q}\gamma^\mu \frac{i\vec{\not{D}}}{2m_b} h_v$ at LO.

$$\Pi_{\mu,V}(P,p) = -\int d^4x \int \frac{d^4l}{(2\pi)^4} \frac{e^{i(p-l)\cdot x}}{l^2} \frac{n\cdot p}{2} \langle 0|\bar{d}(x)\, \not{n}\, \gamma_5\, \not{\bar{n}}\, \gamma_\mu \frac{i\not{D}}{2m_b} h_v(0)|\bar{B}(P)\rangle$$
$$= \frac{i\tilde{f}_B m_B \bar{n}_\mu}{2m_b}\Big\{\int_0^\infty d\omega_1\, d\omega_2 \frac{\rho^{3p}_{\rm A2}(\omega_1,\omega_2)}{(\omega_1-\bar{n}\cdot p)(\omega_1+\omega_2-\bar{n}\cdot p)} + \int_0^\infty d\omega \frac{\rho^{2p}_{\rm A2}(\omega)}{\omega-\bar{n}\cdot p}\Big\}.$$

- Equations of operators derived from the QCD equation of motion [Kawamura,Kodaira,Qiao,Tanaka,2001]:

$$\bar{q}(x)\,\Gamma D_\rho\, h_v(0) = \partial_\rho[\bar{q}(x)\,\Gamma h_v(0)] + i\int_0^1 du(1-u)[\bar{q}(x)x^\lambda g_s G_{\lambda\rho}\Gamma\, h_v(0)] - \frac{\partial}{\partial x_\rho}[\bar{q}(x)\,\Gamma\, h_v(0)],$$
$$v_\rho \frac{\partial}{\partial x_\rho}[\bar{q}(x)\,\Gamma\, h_v(0)] = i\int_0^1 du(1-u)[\bar{q}(x)\,x^\lambda g_s G_{\lambda\rho} v^\rho \Gamma\, h_v(0)] + \partial_\rho[\bar{q}(x)\,v^\rho\Gamma\, h_v(0)],$$
$$\frac{\partial}{\partial x_\rho}[\bar{q}(x)\,\gamma_\rho\Gamma\, h_v(0)] = -i\int_0^1 du\, u[\bar{q}(x)\,x^\lambda g_s G_{\lambda\rho}\gamma^\rho\Gamma\, h_v(0)].$$

- Large recoil symmetry breaking effect:

$$\not{v}\vec{\not{D}} h_v = -\vec{\not{D}} h_v \qquad \to \qquad \Pi_T^{\rm HQE}(q^2) = \frac{i}{2}[\tilde{\Pi}^{\rm HQE}(q^2) - \Pi^{\rm HQE}(q^2)].$$

NLP corrections to $B \to P$ form factors

- Subleading power corrections from hard-collinear propagator:

$$f_P\, n\cdot p\, e^{-\frac{m_P^2}{n\cdot p\,\omega_M}} \Big\{ f_{B\to P}^{+,\mathrm{hc,NLP}}(q^2), \quad \frac{m_B}{n\cdot p} f_{B\to P}^{0,\mathrm{hc,NLP}}(q^2), \quad \frac{m_B}{m_B+m_P} f_{B\to P}^{T,\mathrm{hc,NLP}}(q^2) \Big\}$$
$$= \frac{\tilde{f}_B m_B}{n\cdot p} \int_{0_-}^{\omega_s} d\omega \Big\{ \left(-\frac{1}{2}\right) \frac{d^2}{d\omega^2} \int_0^{\omega} d\omega_1 \int_{\omega-\omega_1}^{\infty} \frac{d\omega_2}{\omega_2}\, \tilde{\rho}_{\mathrm{hc}}^{3p2}\Big(\frac{\omega-\omega_1}{\omega_2}, \omega_1, \omega_2\Big)\theta(\omega)$$
$$+ \frac{d}{d\omega} \int_0^{\omega} d\omega_1 \int_{\omega-\omega_1}^{\infty} d\omega_2\, \frac{1}{\omega_2} \tilde{\rho}_{\mathrm{hc}}^{3p1}\Big(\frac{\omega-\omega_1}{\omega_2}, \omega_1, \omega_2\Big)\theta(\omega) + \frac{d}{d\omega} \tilde{\rho}_{\mathrm{hc}}^{2p}(\omega)\theta(\omega)$$
$$+ \mathcal{C}^i \frac{d}{d\omega} \int_0^{\infty} \frac{d\omega_2}{\omega_2} \int_{\omega-\omega_2}^{\omega} d\omega_1\, \rho_{\mathrm{hc}}^{3p}\Big(\frac{\omega-\omega_1}{\omega_2}, \omega_1, \omega_2\Big)\theta(\omega) + \mathcal{C}^i \rho_{\mathrm{hc}}^{2p}(\omega) \Big\}. \tag{1}$$

- Symmetry breaking effects of the three form factors:

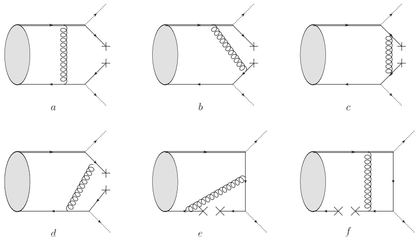
$$\mathcal{C}^+ = -\mathcal{C}^0 = \frac{n\cdot p - m_B}{m_B}, \qquad \mathcal{C}^T = -1. \tag{2}$$

NLP corrections to $B \to P$ form factors

- Four-particle correction in factorization limit:

$$\Pi^{a}_{4p}(n\cdot p, \bar{n}\cdot p) = \frac{i}{6} g_s^2 C_F \langle \bar{q} q\rangle \frac{\tilde{f}_B m_B}{(n\cdot p)^2} \int_0^\infty d\omega\, \frac{\phi_B^-(\omega)}{\bar{n}\cdot p(\omega - \bar{n}\cdot p)},$$

$$\Pi^{T,a}_{4p}(n\cdot p, \bar{n}\cdot p) = \frac{i}{2} \Pi^{a}_{4p}(n\cdot p, \bar{n}\cdot p) \left\{ \phi_B^-(\omega) \to \left(\phi_B^-(\omega) + \frac{n\cdot p}{mb} \phi_B^+(\omega) \right) \right\},$$



- A similar calculation for $B \to \gamma\ell\nu$ form factors [Beneke,Braun,Ji,Wei,2018].
- A complete parameterization of four-particle B-meson LCDA is necessary.

- Strange-quark mass effect:

$$\begin{aligned}\Pi_{\mu,V}(P,p) =& i\langle 0|\bar{q}(k)\, \not{n}\, \gamma_5 \left[\frac{i m_q}{l^2 + i0} + \frac{i \not{p} m_q^2}{(l^2+i0)^2} \right] \gamma_\mu\, h_v(0)|B(P)\rangle \\ =& i \frac{\tilde{f}_B m_B}{n\cdot p} \int_0^\infty d\omega \left\{ \frac{-m_q\, \phi_B^-(\omega)}{\omega - \bar{n}\cdot p} n_\mu + \frac{m_q^2\, \phi_B^-(\omega)}{(\omega - \bar{n}\cdot p)^2} \bar{n}_\mu \right\}.\end{aligned}$$

B-meson LCDAs and RGE

- Two-particle B-meson LCDAs:
  - Definition [Beneke,Feldmann,2001;Beneke,Braun,Ji,Wei,2018]:

$$\langle 0|\bar{q}(x)\,\Gamma[x,0]\,h_v(0)|\bar{B}(v)\rangle = -\frac{i\tilde{f}_B(\mu)m_B}{4}\,\text{Tr}\Big[\frac{1+\not{v}}{2}\Big\{2\left(\Phi_B^+(v\cdot x)+x^2G_B^+(v\cdot x)\right) - \frac{\not{x}}{v\cdot x}\left[\left(\Phi_B^+(v\cdot x)-\Phi_B^-(v\cdot x)\right)+x^2\left(G_B^+(v\cdot x)-G_B^-(v\cdot x)\right)\right]\Big\}\gamma_5\Gamma\Big].$$

  - Shape parameters of $\phi_B^+(\omega,\mu)$:

$$\hat{\sigma}_n(\mu) = \int_0^\infty d\omega\,\frac{\lambda_B}{\omega}\ln^n\frac{\lambda_B e^{-\gamma_E}}{\omega}\phi_B^+(\omega,\,\mu).$$

  - The equations of motion for the light-quark field yield[Kawamura,Kodaira,Qiao,Tanaka,2001]:

$$\phi_B^-(\omega) = \underbrace{\int_\omega^\infty \frac{\phi_B^+(\omega')}{\omega'}d\omega'}_{\text{WW approximation}} - \underbrace{\int_\omega^\infty \frac{d\omega'}{\omega'}\frac{d}{d\omega'}\int_0^\omega d\omega_1\int_{\omega'-\omega_1}^\infty \frac{d\omega_2}{\omega_2}\frac{d}{d\omega_2}\phi_3(\omega_1,\omega_2)}_{\text{three-particle LCDAs correction}}.$$

  - Twist-five LCDA[Beneke,Braun,Ji,Wei,2018]:

$$g_B^-(\omega) = \frac{1}{4}\int_\omega^\infty dx\Big\{(x-\omega)\left[\phi_B^+(\omega)-\phi_B^-(\omega)\right]-2(\bar{\Lambda}-x)\phi_B^-(\omega)\Big\} - \frac{1}{2}\int_0^\omega d\omega_1\int_{\omega-\omega_1}^\infty d\omega_2\,\frac{1}{\omega_2}\left(1-\frac{\omega-\omega_1}{\omega_2}\right)\psi_5(\omega_1,\omega_2).$$

B-meson LCDAs and RGE

- Three-particle higher-twist B-meson LCDAs:
  - Three-particle B-meson LCDAs[Kawamura,Kodaira,Qiao,Tanaka,2001; Braun,Ji,Manashov,2017]:

$$\langle 0|\bar{q}_\alpha(z_1\bar{n})g_s G_{\mu\nu}(z_2\bar{n})h_{\nu\beta}(0)|\bar{B}(v)\rangle$$

$$=\frac{\tilde{f}_B(\mu)m_B}{4}\Big[(1+\not{v})\Big\{(v_\mu\gamma_\nu-v_\nu\gamma_\mu)\left[\Psi_A(z_1,z_2)-\Psi_V(z_1,z_2)\right]-i\sigma_{\mu\nu}\Psi_V(z_1,z_2)$$
$$-(\bar{n}_\mu v_\nu-\bar{n}_\nu v_\mu)X_A(z_1,z_2)+(\bar{n}_\mu\gamma_\nu-\bar{n}_\nu\gamma_\mu)\left[W(z_1,z_2)+Y_A(z_1,z_2)\right]$$
$$+i\epsilon_{\mu\nu\alpha\beta}\bar{n}^\alpha v^\beta\gamma_5\tilde{X}_A(z_1,z_2)-i\epsilon_{\mu\nu\alpha\beta}\bar{n}^\alpha\gamma^\beta\gamma_5\tilde{Y}_A(z_1,z_2)$$
$$-(\bar{n}_\mu v_\nu-\bar{n}_\nu v_\mu)\not{\bar{n}}\,W(z_1,z_2)+(\bar{n}_\mu\gamma_\nu-\bar{n}_\nu\gamma_\mu)\not{\bar{n}}\,Z(z_1,z_2)\Big\}\gamma_5\Big]_{\beta\alpha}.$$

  - Those eight invariant functions are related to the B-meson higher-twist LCDAs:

$$\Phi_3=\Psi_A-\Psi_V, \qquad \Phi_4=\Psi_A+\Psi_V.$$
$$\Psi_4=\Psi_A+X_V, \qquad \tilde{\Psi}_4=\Phi_A-\tilde{X}_A.$$
$$\Psi_5=-\Psi_A+X_A-2Y_A, \qquad \Phi_5=\Psi_A+\Psi_V+2Y_A-2\tilde{Y}_A+2W.$$
$$\tilde{\Psi}_5=-\Psi_V-\tilde{X}_A+2\tilde{Y}_A, \qquad \Phi_6=\Psi_A-\Psi_V+2Y_A+2\tilde{Y}_A+2W-4Z.$$

  - Asymptotic behavior at small quark and gluon momenta [Braun,Filyanov,1990]:

$$\phi(\omega_1,\omega_2)\sim\omega_1^{2j_1-1}\omega_2^{2j_2-1}, \quad \phi\in\{\phi_3,\phi_4,\psi_4\cdots\}.$$

  - Normalization conditions:

$$\Psi_V(z=0)=\frac{1}{3}\lambda_H^2, \qquad \Psi_A(z=0)=\frac{1}{3}\lambda_E^2.$$

B-meson LCDAs and RGE

- The general model of B-meson LCDAs:

$$
\begin{aligned}
\phi_B^+(\omega) &= \omega f(\omega), \\
\phi_B^-(\omega) &= \int_\omega^\infty d\rho\, f(\rho) + \frac{1}{6}\varkappa(\lambda_E^2 - \lambda_H^2)\left[\omega^2 f'(\omega) + 4\omega f(\omega) - 2\int_\omega^\infty d\rho\, f(\rho)\right], \\
g_-(\omega) &= \frac{1}{4}\int_\omega^\infty dx \left\{(x-\omega)\left[\phi_B^+(\omega) - \phi_B^-(\omega)\right] - 2(\bar\Lambda_q - x)\phi_B^-(\omega)\right\} \\
&\quad -\frac{1}{2}\int_0^\omega d\omega_1 \int_{\omega-\omega_1}^\infty d\omega_2\, \frac{1}{\omega_2}(1 - \frac{\omega-\omega_1}{\omega_2})\psi_5(\omega_1,\omega_2), \\
\phi_3(\omega_1,\omega_2) &= -\frac{1}{2}\varkappa(\lambda_E^2 - \lambda_H^2)\omega_1\omega_2^2 f'(\omega_1+\omega_2), \\
\phi_4(\omega_1,\omega_2) &= \frac{1}{2}\varkappa(\lambda_E^2 + \lambda_H^2)\omega_2^2 f(\omega_1+\omega_2), \\
\psi_4(\omega_1,\omega_2) &= \varkappa\lambda_E^2\omega_1\omega_2 f(\omega_1+\omega_2), \\
\phi_5(\omega_1,\omega_2) &= \varkappa(\lambda_E^2 + \lambda_H^2)\omega_1 \int_{\omega_1+\omega_2}^\infty d\omega\, f(\omega), \\
\psi_5(\omega_1,\omega_2) &= \varkappa\lambda_E^2\omega_2 \int_{\omega_1+\omega_2}^\infty d\omega\, f(\omega), \\
\tilde\psi_5(\omega_1,\omega_2) &= \varkappa\lambda_H^2\omega_2 \int_{\omega_1+\omega_2}^\infty d\omega\, f(\omega), \\
\phi_6(\omega_1,\omega_2) &= \varkappa(\lambda_E^2 - \lambda_H^2) \int_{\omega_1+\omega_2}^\infty d\omega \int_\omega^\infty d\omega'\, f(\omega').
\end{aligned}
$$

B-meson LCDAs and RGE

- Exponential model:

$$f(\omega) = \frac{1}{\omega_0^2} e^{-\frac{\omega}{\omega_0}}.$$

- Local duality model:

$$f(\omega) = \frac{5}{8\omega_0^5}(2\omega_0 - \omega)^3 \theta(2\omega_0 - \omega).$$

- Three-parameter model of $\phi_B^+(\omega, \mu_0)$ [Beneke,Braun,Ji,Wei,2018]:

$$\phi_B^+(\omega, \mu_0) = \frac{\Gamma(\beta)}{\Gamma(\alpha)} \frac{\omega}{\omega_0^2} e^{-\omega/\omega_0} \mathsf{U}\left(\beta - \alpha, 3 - \alpha, \omega/\omega_0\right).$$

where the three parameters are related to the shape parameters of $\phi_B^+(\omega, \mu_0)$

$$\lambda_B = \frac{\alpha - 1}{\beta - 1}\omega_0, \qquad \hat{\sigma}_1 = \psi(\beta - 1) - \psi(\alpha - 1) + \ln\left(\frac{\alpha - 1}{\beta - 1}\right),$$

$$\hat{\sigma}_2 = \frac{\pi^2}{6} + \left[\psi(\beta - 1) - \psi(\alpha - 1) + \ln\left(\frac{\alpha - 1}{\beta - 1}\right)\right]^2 - \left[\psi'(\beta - 1) - \psi'(\alpha - 1)\right].$$

- The representation of $\phi_B^+(\omega, \mu)$ and $\phi_B^-(\omega, \mu)$ in dual space [Bell,Feldmann,Wang,Yip,2013]:

$$\phi_B^+(\omega, \mu) = \int_0^\infty ds\, \sqrt{\omega s} J_1(2\sqrt{\omega s}) \eta_+(s, \mu)$$

$$\phi_B^-(\omega, \mu) = \int_\omega^\infty \frac{d\omega'}{\omega'} \int_0^\infty ds\, \sqrt{\omega' s} J_1(2\sqrt{\omega' s}) \left(\eta_+(s, \mu) + \eta_3^{(0)}(s, \mu)\right).$$

- RG equation at one-loop in the dual space [Bell,Feldmann,Wang,Yip,2013;Beneke,Braun,Ji,Wei,2018]:

$$\eta_+(s,\mu) = U_+(s;\mu,\mu_0)\eta_+(s,\mu_0);$$
$$\eta_3^{(0)}(s,\mu) = r^{N_c/\beta_0} U_+(s;\mu,\mu_0)\eta_3^{(0)}(s,\mu_0).$$

- Scale-dependence LCDAs:

$$\phi_B^+(\omega,\mu) = C(\mu,\mu_0)\,\frac{\omega}{\omega_0^{2+q(\mu)}}\Bigg\{\frac{\Gamma(\beta)\Gamma(2+q(\mu))\Gamma(\alpha-q(\mu)-2)}{\Gamma(\alpha)\Gamma(\beta-q(\mu)-2)}$$
$$\times\,{}_2\mathsf{F}_2\left(q(\mu)+2, q(\mu)+3-\beta; 2, q(\mu)+3-\alpha, -\frac{\omega}{\omega_0}\right)+\left(\frac{\omega}{\omega_0}\right)^{\alpha-q(\mu)-1}$$
$$\times\,\frac{\Gamma(\beta)\Gamma(q(\mu)+2-\alpha)}{\Gamma(\alpha-\beta)\Gamma(\alpha-q(\mu))}\,{}_2\mathsf{F}_2\left(\alpha,\alpha-\beta+1;\alpha-q(\mu)-1,\alpha-q(\mu),-\frac{\omega}{\omega_0}\right)\Bigg\},$$

$$\phi_B^-(\omega,\mu) = \phi_-^{\mathsf{WW}}(\alpha,\beta,\omega_0,\omega,\mu) - \frac{\lambda_E^2-\lambda_H^2}{18\omega_0^2}\,\frac{\beta(\beta+1)}{\alpha(\alpha+1)}\,r^{N_c/\beta_0}\Big\{\beta(\beta-1)\phi_-^{\mathsf{WW}}(\alpha,\beta,\omega_0,\omega,\mu)$$
$$-4(\beta-1)\phi_-^{\mathsf{WW}}(\alpha,\beta-1,\omega_0,\omega,\mu)+(\beta-2)(\beta-1)\phi_-^{\mathsf{WW}}(\alpha,\beta-2,\omega_0,\omega,\mu)\Big\}.$$

Numerical results

- The LCSR for $B \to P$ form factors:

$$f^i_{B\to P}(q^2) = f^{i,\mathrm{NLL,LP}}_{B\to P}(q^2) + f^{i,\mathrm{HT}}_{B\to P}(q^2) + f^{i,\mathrm{HQE,NLP}}_{B\to P}(q^2) + f^{i,\mathrm{hc,NLP}}_{B\to P}(q^2) + f^{i,\mathrm{twist-5,6}}_{B\to P}(q^2) + f^{i,m_q}_{B\to P}(q^2).$$

- Input:

$$\hat\sigma_1 = 0.7, \hat\sigma_2 = 6 \quad \text{for model I},$$
$$\hat\sigma_1 = 0, \hat\sigma_2 = \pi^2/6 \quad \text{for model II},$$
$$\hat\sigma_1 = -0.7, \hat\sigma_2 = -6 \quad \text{for model III},$$

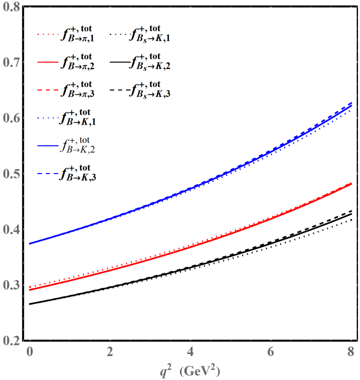
- Fitting $f^+_{B\to K}(0) = 0.37 \pm 0.03$ (Lattice QCD)

$$\lambda_B(\mu_0) = \begin{cases} 146^{+41}_{-45}\ \text{MeV}, & \text{(model I)} \\ 203^{+55}_{-61}\ \text{MeV}, & \text{(model II)} \\ 234^{+63}_{-70}\ \text{MeV}. & \text{(model III)} \end{cases}$$

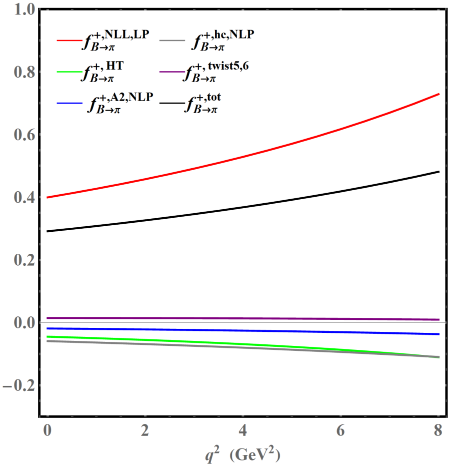
- Fitting $f^+_{B_s\to K}(0) = 0.265 \pm 0.025$ (Lattice QCD)

$$\lambda_{B_s}(\mu_0) = \begin{cases} 211^{+50}_{-57}\ \text{MeV}, & \text{(model I)} \\ 285^{+67}_{-75}\ \text{MeV}, & \text{(model II)} \\ 325^{+76}_{-86}\ \text{MeV}, & \text{(model III)} \end{cases}$$

- The shape of three form factors less model dependent.



Numerical results



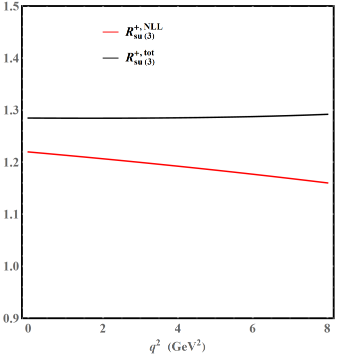
▽ Subleading power corrections from:

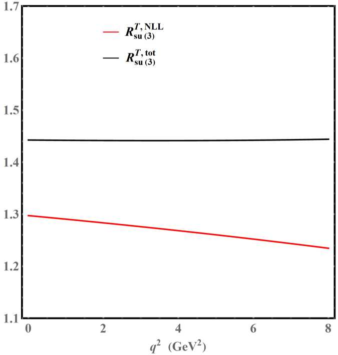
- higher-twist LCDAs - 11%
- b-quark field in HQET - 5%
- hard-collinear propagator - 15%
- four-particle LCDA 4%
- total -27%

Numerical results

- SU(3)-flavour symmetry breaking

$$R^{i}_{\mathrm{SU(3)}}(q^2) = \frac{f^{i}_{B\to K}(q^2)}{f^{i}_{B\to\pi}(q^2)}, \qquad (i = +, 0, T).$$

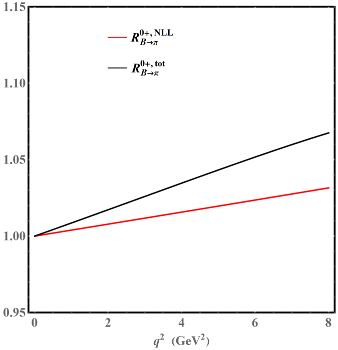


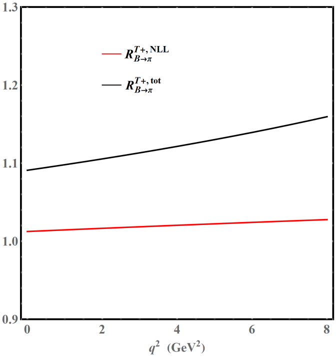


Numerical results

- Large recoil symmetry breaking for $B \to \pi$ form factors:

$$R^{0,+}_{B\to\pi}(q^2) = \frac{m_B}{n\cdot p}\frac{f^0_{B\to\pi}(q^2)}{f^+_{B\to\pi}(q^2)}, \qquad R^{T,+}_{B\to\pi}(q^2) = \frac{m_B}{m_B+m_\pi}\frac{f^T_{B\to\pi}(q^2)}{f^+_{B\to\pi}(q^2)}.$$





- Dominant breaking effect for $R^{T,+}_{B\to\pi}$ due to the subleading term of b-quark field.
- Dominant breaking effect for $R^{0,+}_{B\to\pi}$ duo to the contribution of n_μ direction from subleading power corrections.

Numerical results

- Extrapolation toward larger q^2 with the z-series expansion:

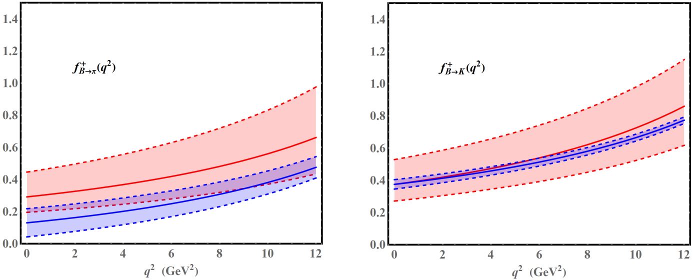
$$z(q^2, t_0) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}.$$

- BCL-parameterize [Bourrely,Caprini and Lellouch,2010]

$$f^{+,T}_{B\to P}(q^2) = \frac{f^{+,T}_{B\to P}(0)}{1 - q^2/m^2_{B^*_q}} \left\{1 + \sum_{n=1}^{N} b^{+,T}_{n,B\to P}\left[z(q^2,t_0)^n - z(0,t_0)^n\right]\right\},$$

$$f^{0}_{B\to P}(q^2) = f^{0}_{B\to P}(0)\left\{1 + \sum_{n=1}^{N} b^{0}_{n,B\to P}\left[z(q^2,t_0)^n - z(0,t_0)^n\right]\right\}.$$

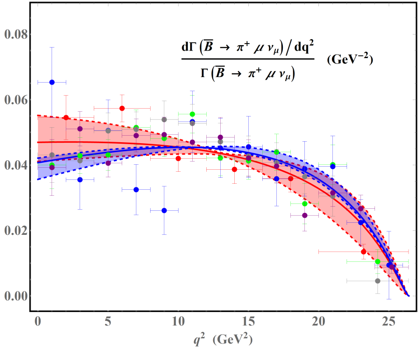
- The predicted form factors(LCSR,Lattice QCD [Aoki *et al*, 2020])



▷ The main uncertainties are generated from variation of the model of the B-meson LCDA ϕ_B^+.

Numerical results

- Normalized differential decay rate of the $\bar{B} \to \pi^+ \mu \nu_\mu$ decay



- Lattice QCD (blue bands)
- LCSR (red bands)
- Experimental results (points)

- Determined of $|V_{ub}|$ by taking experimental data from BaBar and Belle Collaboration:

$$|V_{ub}| = \left(3.24^{+1.44}_{-1.14}\big|_{\text{theo}}\,{}^{+0.11}_{-0.11}\big|_{\text{exp}}\right) \times 10^{-3}.$$

Confronted with $|V_{ub}|_{\text{Lat}} = (3.70 \pm 0.10|_{\text{exp}} \pm 0.12|_{\text{theo}}) \times 10^{-3}$ from $B \to \pi \ell \nu_\ell$.

Conclusions and Outlook

- Subleading power contributions to the form factors reduce the leading power contribution 27% numerically.
- Dominant large recoil symmetry breaking from subleading power corrections instead of Perturbative corrections.
- One-loop level resummation for subleading power contributions is required to improve the precision and reduce the scale dependence.
- Best understanding the RG evolution of the higher-twist LCDAs is necessary.
- A global analysis on the determination of $|V_{ub}|$ is required.

Thanks!