Possible Topics for BEPCIII at $\sqrt{s} = 4.0 - 5.6$ GeV

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IHEP, April 25, 2021

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I. Introduction

Quark model for heavy quarkonium:

Generalized Breit-Fermi Hamiltonian

$$H = 2m + \frac{p^2}{m} - \frac{p^4}{4m^3} + V(r) + H_{SI} + H_{LS} + H_{SS} + H_T$$

$$H_{SS} = \frac{2}{3m^2} S_1 \cdot S_2 \, \nabla^2 V_V(r)$$

$$H_{LS} = \frac{1}{2m^2 r} \left(3 \frac{d}{dr} V_V(r) - \frac{d}{dr} V_s(r) \right) L \cdot S$$

$$H_T = \frac{1}{12m^2} \left(\frac{1}{r} \frac{d}{dr} V_V(r) - \frac{d^2}{dr^2} V_V(r) \right) S_{12}$$

$$S_{12} \equiv 12\left(\frac{(S_1 \cdot \vec{r})(S_2 \cdot \vec{r})}{r^2} - \frac{1}{3} S_1 \cdot S_2\right)$$

$$\langle L \cdot S \rangle = \frac{1}{2}[J(J+1) - L(L+1) - S(S+1)]$$

$$S_{12} = \frac{4}{(2L+3)(2L-1)}\left[S^2 L^2 - \frac{3}{2} L \cdot S - 3(L \cdot S)^2\right]$$

For $S = 1, L \neq 0$

J	$L-1$	L	$L+1$
$\langle L \cdot S \rangle_J$	$-(L+1)$	-1	L
$\langle S_{12} \rangle_J$	$-\frac{2(L+1)}{2L-1}$	2	$-\frac{2L}{2L+3}$

$$\langle L\cdot S\rangle_{avg} = \frac{1}{3(2L+1)}\sum_J \frac{1}{2J+1}\langle L\cdot S\rangle_J = 0$$

$$\langle S_{12}\rangle_{avg} = \frac{1}{3(2L+1)}\sum_J \frac{1}{2J+1}\langle S_{12}\rangle_J = 0$$

$$\langle H_{SS}\rangle \propto |\phi_{nL}(0)|^2 \;\; \text{for} \;\; V_V(r) \propto \frac{1}{r}: \qquad \nabla^2\frac{1}{r} = 4\pi\delta(r)$$

Center-of-gravity mass: M_{COG}

$$M_{COG}(nP) = \frac{1}{9}\left(M_{\chi_{c0}} + 3M_{\chi_{c1}} + 5M_{\chi_{c2}}\right) = M_{h_c}(nP)$$

$$M_{COG}(nD) = \frac{1}{15}\left(3M_{\psi(nD)} + 5M_{\psi_2} + 7M_{\psi_3}\right) = M_{\eta_{c2}}(nD)$$

Experimental results:

$$M_{h_c}(1P) = 3525.4(1)\ \text{MeV}$$

$$M_{COG}(1P) = \frac{1}{9}\left(M_{\chi_{c0}} + 3M_{\chi_{c1}} + 5M_{\chi_{c2}}\right) \approx 3525.3(1)\ \text{MeV}$$

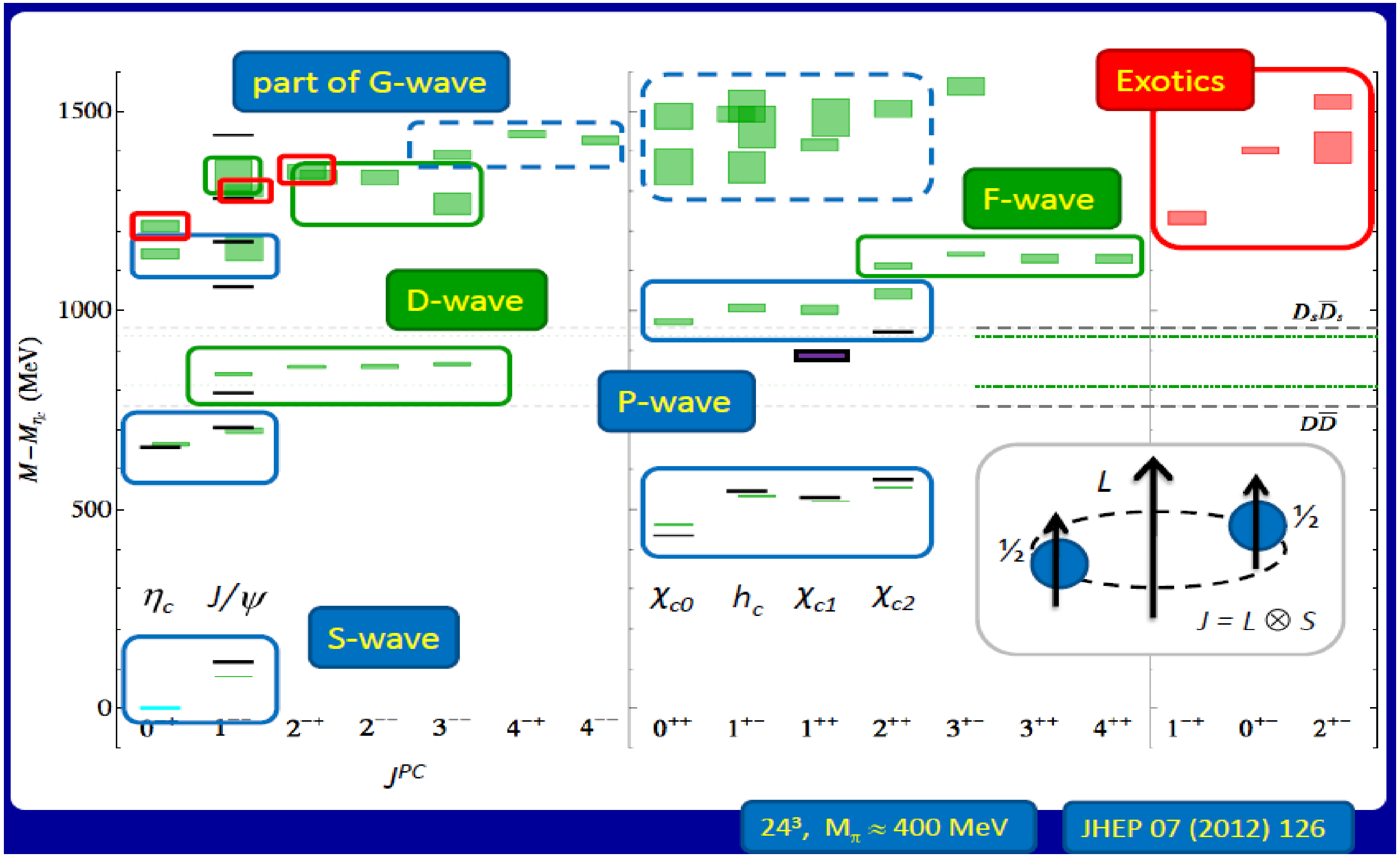
$$M_{h_b}(1P) = 9899.3(8)\ \text{MeV}$$

$$M_{COG}(1P) = \frac{1}{9}\left(M_{\chi_{b0}} + 3M_{\chi_{b1}} + 5M_{\chi_{b2}}\right) \approx 9899.9(5)\ \text{MeV}$$

$$M_{h_b}(2P) = 10259.8(1.2)\ \text{MeV}$$

$$M_{COG}(2P) = \frac{1}{9}\left(M_{\chi_{b0}} + 3M_{\chi_{b1}} + 5M_{\chi_{b2}}\right) \approx 10260.3(6)\ \text{MeV}$$

II. Missing conventional charmonia



Latest lattice QCD results of charmonium

$1D$ states:

	η_{c2}	$\psi(3770)$	$\psi_2(3823)$	$\psi_3(3842)$
J^{PC}	2^{-+}	1^{--}	2^{--}	3^{--}
$^{2S+1}L_J$	$^1D_2$	$^3D_1$	$^3D_2$	$^3D_3$
M(MeV)	?	3773.3(4)	3822.2(1.2)	3842.7(2)
Γ(MeV)	?	87.6(4)	< 16	2.8(6)

$$M_{COG} = \frac{1}{15}\left(3M_{\psi(3770)} + 5M_{\psi_2} + 7M_{\psi_3}\right) \approx 3822 \text{ MeV}$$

$$M_{\eta_{c2}} \approx M_{COG}?$$

Lattice results of $M_{\eta_{c2}}$:

- Quenched approximation: $M_{\eta_{c2}} \approx 3800(30)$ MeV.
- $N_f = 2 + 1$ lattice QCD: $M_{\eta_{c2}} - M_{\eta_c} \approx 860$ MeV.

Decay of η_{c2}: almost no open-charm decay, small width

- $D\bar{D}$ decay is prohibited!
- $M < E_{th}(D\bar{D}^*) \approx 3870$

- Production on e^+e^- collider:

$$e^+e^- \to \psi(3S,4S) \to \gamma + \eta_{c2}$$
$$e^+e^- \to \psi(4160) \to \gamma + \eta_{c2}$$

- The former is the hindered M_1 transition in the nonrelativistic limit , while the later can be suppressed by the orthogonality of the wave functions.

$$\Gamma_{M_1}\left(n^{2S+1}L_J \to n'^{\,2S'+1}L'_{J'} + \gamma\right) = \frac{4}{3} e_c^2 \frac{\alpha}{m_c^2} E_\gamma^3 \frac{E_f}{M_i}$$
$$\times \frac{2J'+1}{2L+1}\,\delta_{LL'}\delta_{S,S'\pm 1}\left|\left\langle n'^{\,2S'+1}L'_{J'}\middle| n^{2S+1}L_J\right\rangle\right|^2$$

- Hadronic transition:

$$e^+e^- \to \psi(nS,nD) \to (\omega,\phi)\eta_{c2}(\mathrm{P-wave})$$

more?

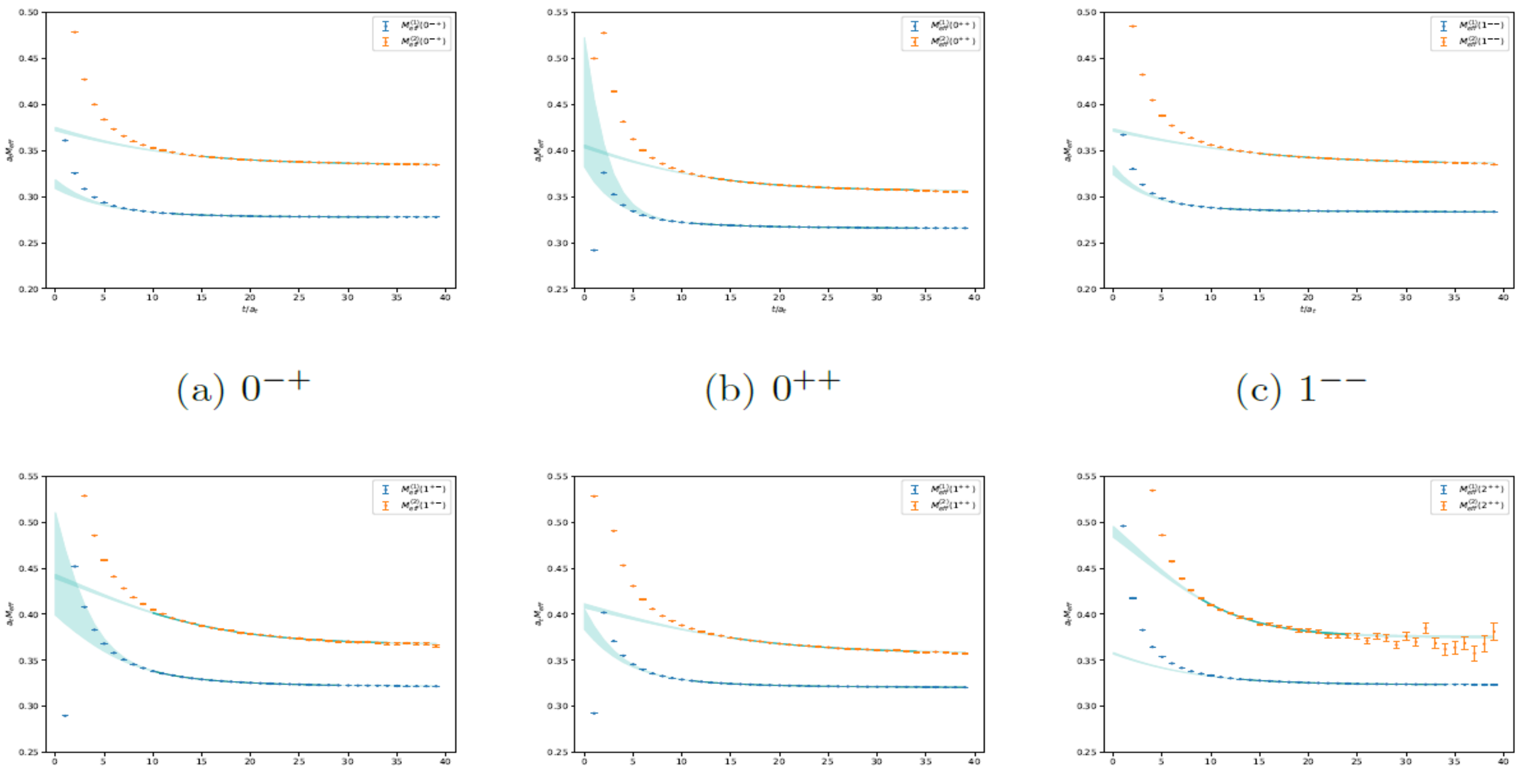
$2P$ states:

	h_c	χ_{c0}	χ_{c1}	χ_{c2}
J^{PC}	1^{+-}	0^{++}	1^{++}	2^{++}
$^{2S+1}L_J$	$^1P_1$	$^3P_0$	$^3P_1$	$^3P_2$
M(1P)(MeV)	3525.4(1)	3414.7(3)	3510.7(1)	3556.2(1)
Γ(1P)(MeV)	0.70(36)	10.8(6)	0.84(4)	1.97(9)
$M(2P?)$(MeV)	?	~3860(50)	3871.7(2)	3922(1)
$\Gamma(2P?)$ (MeV)	?	~ 200	<1.2	35(3)

$$M_{COG}(1P) = \frac{1}{9}\left(M_{\chi_{c0}} + 3M_{\chi_{c1}} + 5M_{\chi_{c2}}\right) \approx 3525.3(1)\ \text{MeV}$$

$$M_{COG}(2P) = \frac{1}{9}\left(M_{\chi'_{c0}} + 3M_{\chi'_{c1}} + 5M_{\chi'_{c2}}\right) \approx 3898(6)\ \text{MeV}$$

J^{PC}	$0^{++}(\chi_{c0})$	$1^{++}(\chi_{c1})$	$2^{++}(\chi_{c2})$	$1^{+-}(h_c)$	COG
$aM(1P)$	0.31586(6)	0.32043(9)	0.32347(6)	0.32166(23)	0.32161(5)
$aM(2P)$	0.3562(3)	0.3571(5)	0.3752(10)	0.3670(5)	0.3670(6)



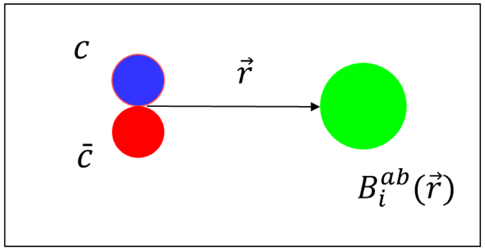
(a) 0^{-+} (b) 0^{++} (c) 1^{--}

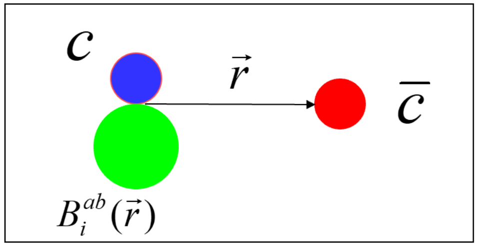
(d) 1^{+-} (e) 1^{++} (f) 2^{++}

III. Possible $(1^{--}, (0,1,2)^{-+})$ $c\bar{c}g$ hybrids

1. Lattice observations:

A. Spectrum from two-types of spatially extended operators:

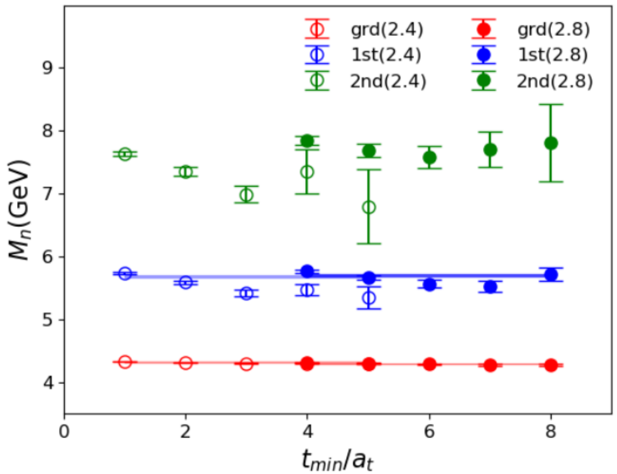


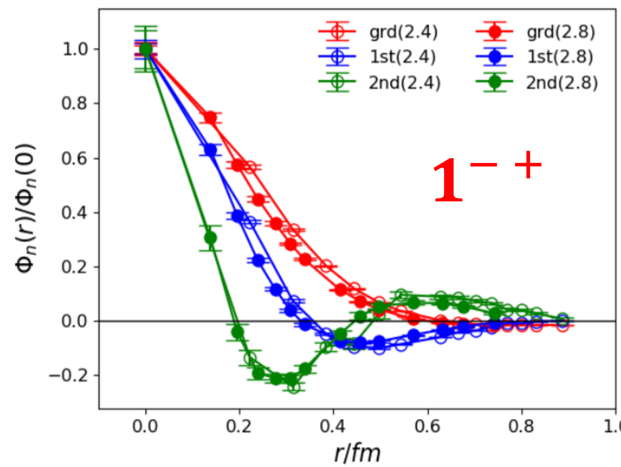


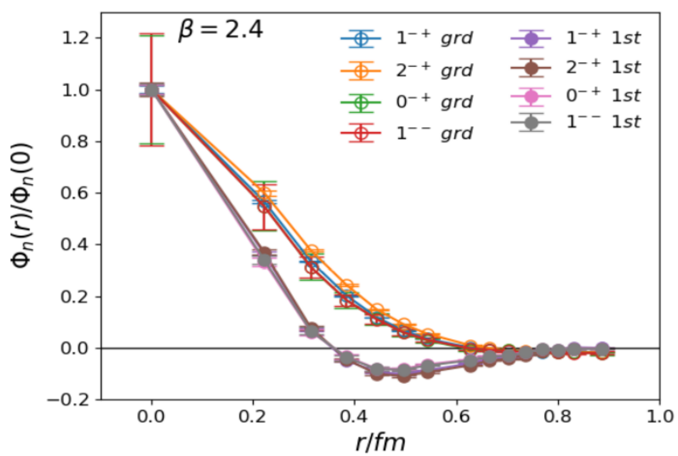
#node	$m(1^{--})$ (GeV)	$m(0^{-+})$ (GeV)	$m(1^{-+})$ (GeV)	$m(2^{-+})$ (GeV)
0	3.109(5)	3.010(4)	-	-
0	3.703(82)	3.672(76)	-	-
0	4.591(69)	4.551(63)	4.309(2)	4.419(3)
1	5.460(31)	5.393(28)	5.693(12)	5.779(12)
2	8.226(99)	8.286(109)	7.661(31)	7.708(29)

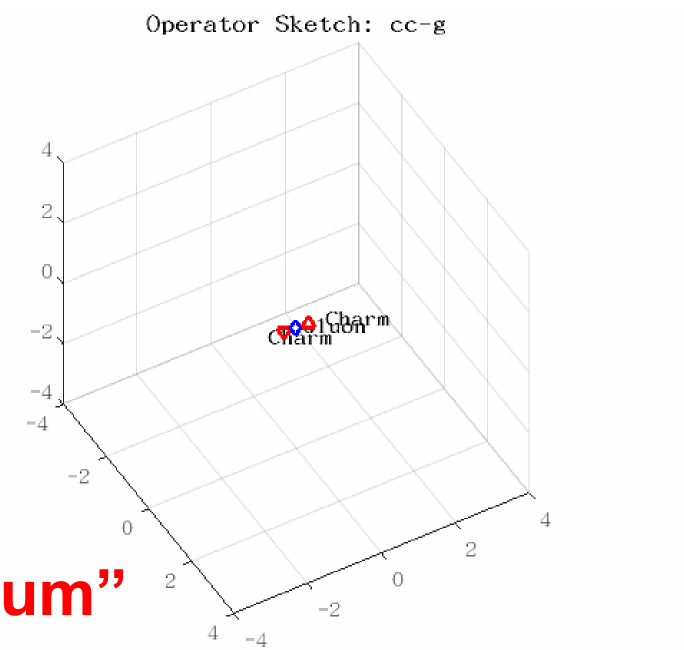
B. BS wave functions (Y. Ma et al., arXiv:1910.09819 (hep-lat))

Results of type-I operator



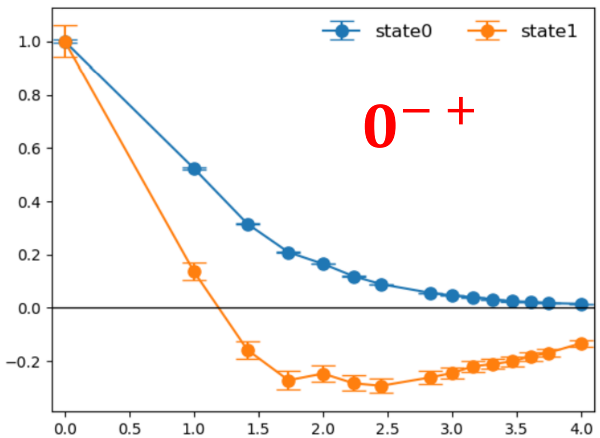


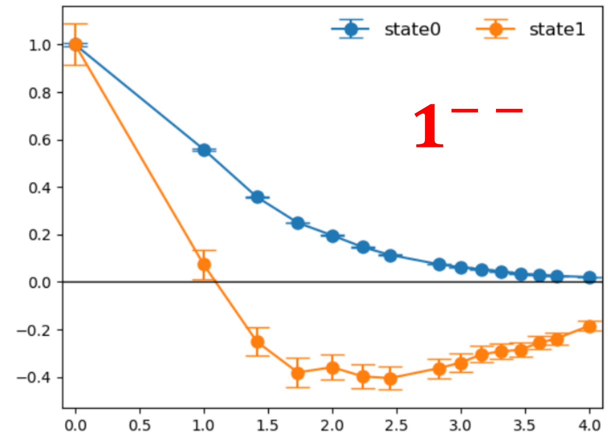


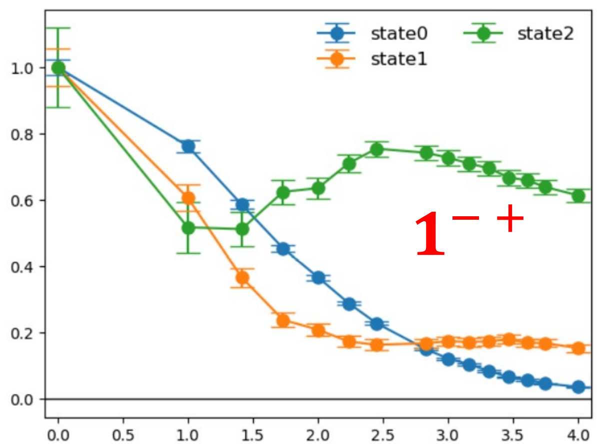


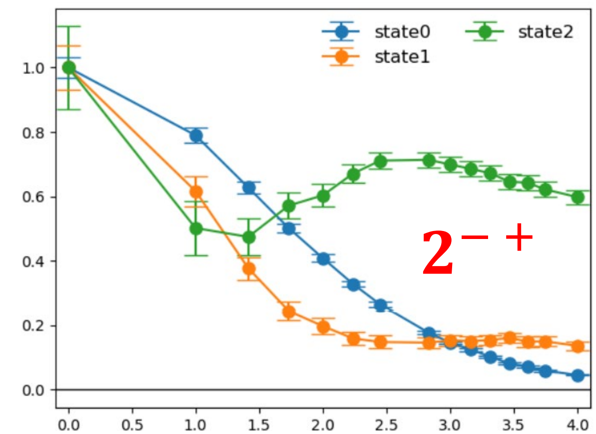
“halo-charmonium”

Results of type-II operator









Discussion in the "halo-charmonium" picture

$J/\psi\pi^+\pi^-$ mode:

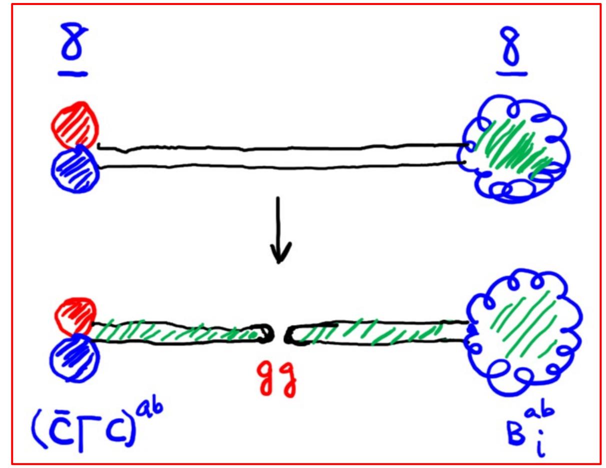
relative S-wave between J/ψ and $\pi^+\pi^-$

$\chi_{c0}\omega$ mode:

relative S-wave between χ_{c0} and ω

$h_c\pi^+\pi^-$ mode:

relative P-wave between h_c and $\pi^+\pi^-$



The $c\bar{c}$ in the halo charmonium is spin singlet (S=0),

$J/\psi\pi^+\pi^-$	mode:	J/ψ (S=1), spin flipping, m_c suppressed, no refugal barrier
$\chi_{c0}\omega$	mode:	χ_{c0} (S=1), spin flipping, m_c suppressed, no refugal barrier
$h_c\pi^+\pi^-$	mode:	h_c (S=0), no spin flipping, but suppressed by the refugal barrier .

In this picture, it is understandable that the above three modes have similar cross section at $\sqrt{s} \sim 4.22\ GeV$

C. 1^{-+} charmonium-like hybrid from $N_f = 2+1$ lattice QCD

(W. Sun et al., arXiv:2012.06228 (hep-lat) , to appear in PRD)

$$H_{QCD} \equiv -\int d^3\vec{x}\, T_{44}(\vec{x}) = H_q + H_g + \frac{1}{4}\left(H_g^a + H_m^\gamma\right)$$

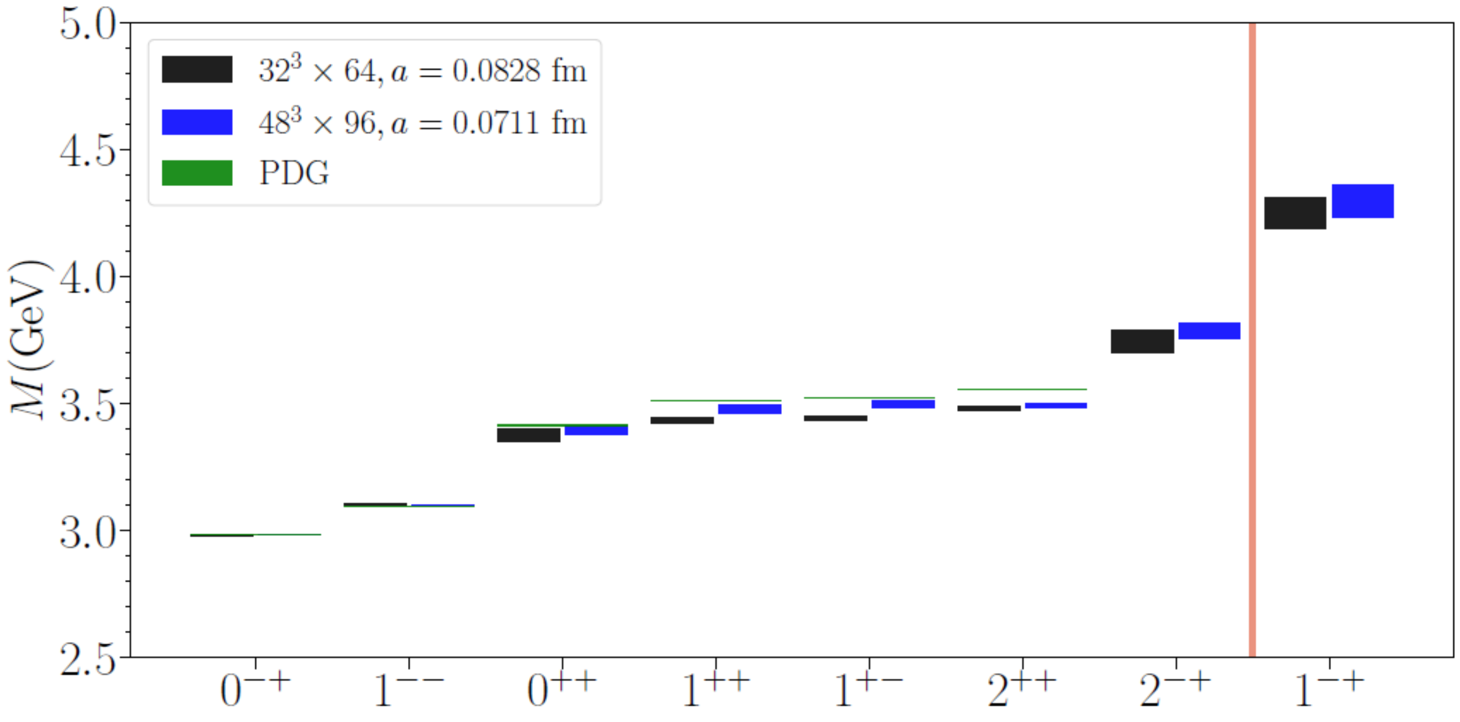
$$= H_m + H_E + H_g + \frac{1}{4} H_a$$

$$\bar{T}_{44}^c = \frac{1}{2}\int d^3\vec{x}\ \bar{c}(x)\left(\gamma_4 \overleftrightarrow{D}_4 - \frac{1}{4}\gamma_\mu \overleftrightarrow{D}_\mu\right) c(x)$$

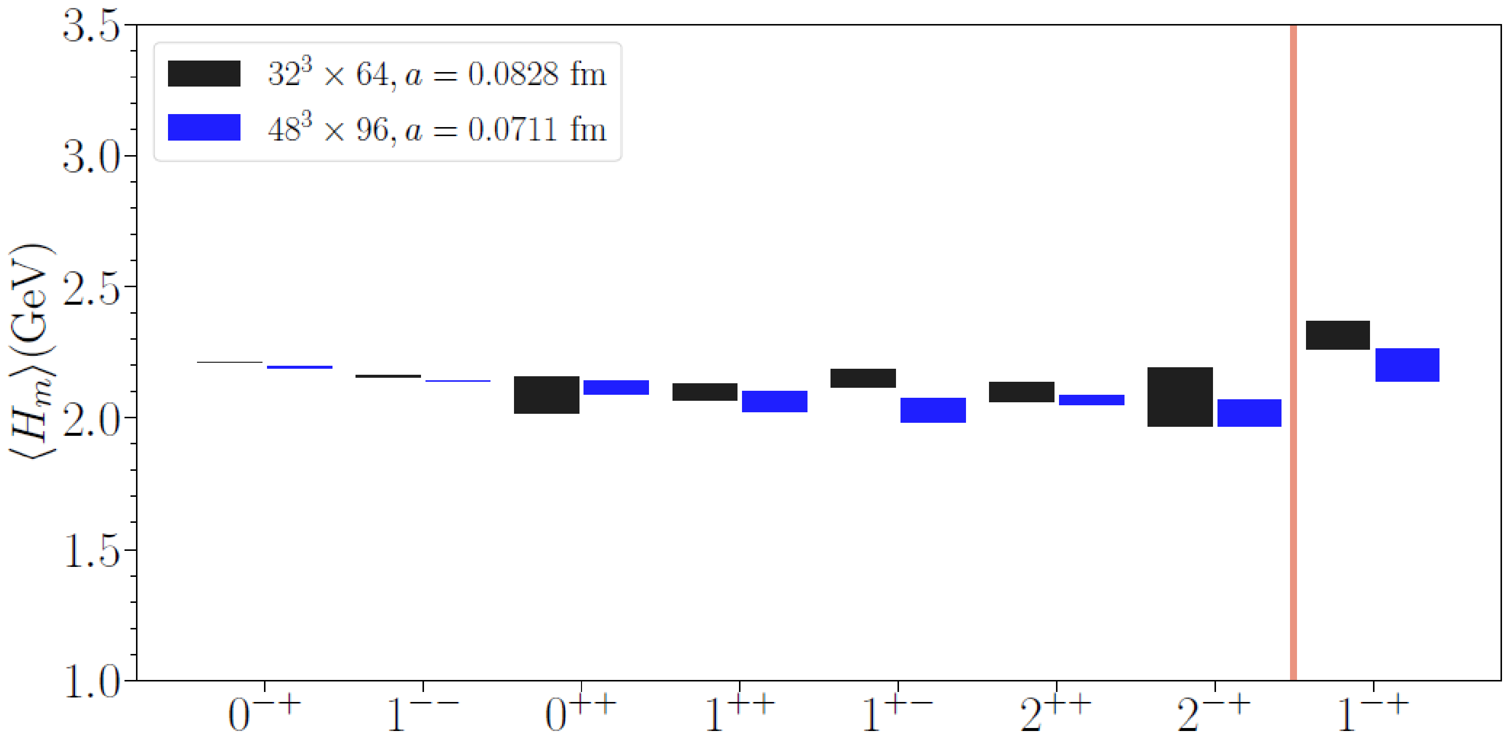
$$\langle x\rangle_c = \left(\frac{4}{3}\langle H|\bar{T}_{44}^c|H\rangle\right) / \left(M_H \langle H|H\rangle\right)$$

ensemble	$L^3 \times T$	a (fm)	m_π (MeV)	$N_{\rm cfg}$
32I	$32^3 \times 64$	0.0828(3)	300	200
48IF	$48^3 \times 96$	0.0711(3)	278	100

- **Mass spectrum**



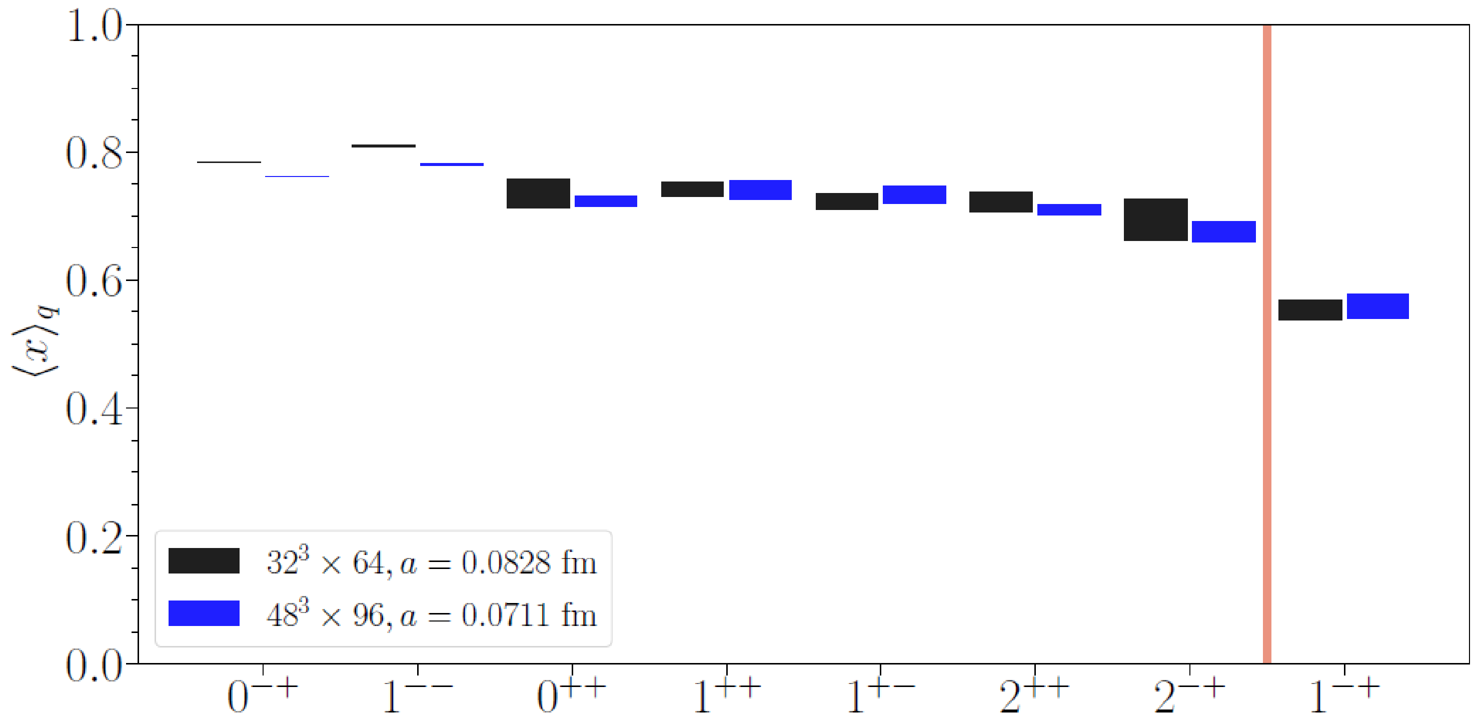
- **Quark mass contribution to charmonium(-like) states $\langle H_m \rangle$;**



For all these states,

$$\langle H_m \rangle \approx 2\left(\bar{m}_c(3GeV) \sim \bar{m}_c(\bar{m}_c)\right)$$

- **Charm quark momentum fraction $\langle x \rangle_c^R$**



- $\langle x \rangle_c^R$ is **almost the same for conventional charmonia**;
- $\langle x \rangle_c^R$ of $(1,2)^{-+}$ **charmonium-like hybrids is obviously smaller .**

2. Search for $c\bar{c}g$ hybrids on e^+e^- collider with $\sqrt{s} \sim 4.0 - 5.6$ GeV

$$e^+e^- \to \psi(nS) \to \gamma + (0,1,2)^{-+}$$

- M_1 transition dominates,
- Partial width is of order $O(1)$ keV for transitions between conventional charmonia.

$$\Gamma(J/\psi \to \gamma\eta_c) = 1.6(4)\text{ keV} \quad \text{(exp.)}$$

$$\Gamma(\eta_{c2} \to \gamma J/\psi) = 3.8(9)\text{ keV} \quad \text{(th.)}$$

- A lattice calculation gives a much larger partial width for

$$\Gamma(\eta_{c1}(1^{-+}) \to \gamma J/\psi) = 115(16)\text{ keV}$$

- The partial width is proportional to $(E_\gamma)^3$:

  for $\eta_{c1}(1^{-+}) \to \gamma J/\psi$, $E_\gamma \sim 1$ GeV;

  for $\psi(4415) \to \gamma\eta_{c1}$, $E_\gamma \sim 0.1$ GeV for $M_{\eta_{c1}} \sim 4.3$ GeV

$$\Gamma(\psi(4415) \to \gamma\eta_{c1}) \sim O(0.1) - O(1)\text{ keV}$$

- **Therefore**

$$\boldsymbol{\Gamma(\psi(4415) = 60(20)}\ \textbf{MeV}$$

$$\boldsymbol{Br(\psi(4415) \to \gamma + (0,1,2)^{-+}) \sim O(10^{-6}) - O(10^{-5})}$$

- **Decay modes of $\boldsymbol{(0,1,2)^{-+}}$ $\boldsymbol{c\bar{c}g}$ hybrids**

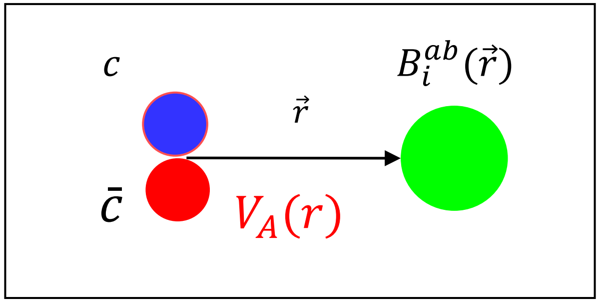
  $\boldsymbol{\chi_{cJ}\eta(\eta'),\ J/\psi\ \omega(\phi)}$**, etc.**

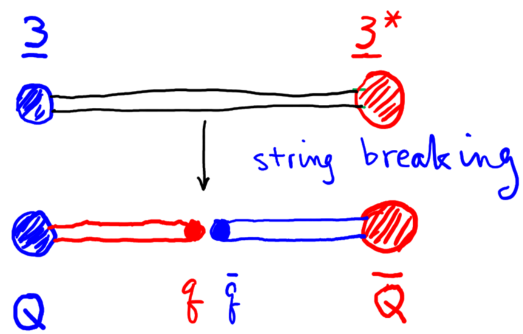
V. Summary

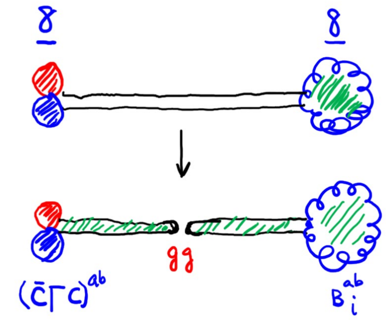
- **Conventional charmonium states to be found**
  $\boldsymbol{\eta_{c2}(1^1D_2)}$, $\boldsymbol{h_c(2^1P_1)}$
- $(\mathbf{0,1,2})^{-+}$ **Charmonium-like hybrids** $(\mathbf{0,1,2})^{-+}$

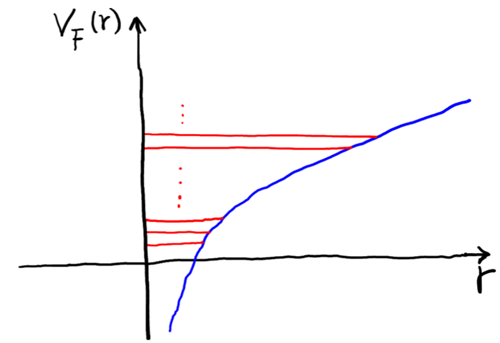
Thanks!

B. Energy levels of hybrid charmonia









If the excitation of hybrid charmonium can be understood as the bound state energy levels in the adjoint potential $V_A(r)$,

then the number of the energy levels is much less than that of the fundamental potential.