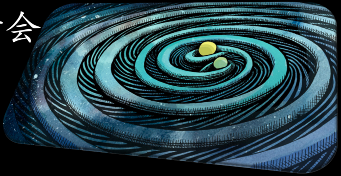
第十五届TeV物理工作组学术研讨会



18-21 July 2021, Beijing China

北京大學
PEKING UNIVERSITY

Testing Spacetime Symmetries via GWs

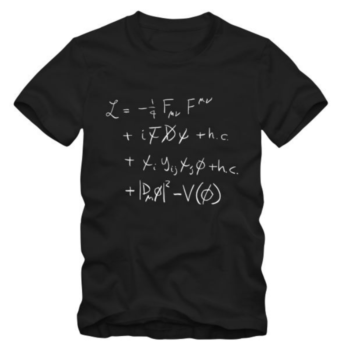
Kavli Institute for Astronomy and Astrophysics

Lijing Shao (邵立晶)

Modern Physics Landscape

Standard Model

quantum field theory



General Relativity

gravitation and spacetime

How the Universe is Ruled

- **Particles of strong, weak, electromagnetic interactions**

$$\mathcal{L}_{\text{lepton}} = \frac{1}{2} i e e_a^{\mu} \left[\bar{L}_A \gamma^a \overleftrightarrow{D}_\mu L_A + \bar{R}_A \gamma^a \overleftrightarrow{D}_\mu R_A \right]$$

$$\mathcal{L}_{\text{quark}} = \frac{1}{2} i e e_a^{\mu} \left[\bar{Q}_A \gamma^a \overleftrightarrow{D}_\mu Q_A + \bar{U}_A \gamma^a \overleftrightarrow{D}_\mu U_A + \bar{D}_A \gamma^a \overleftrightarrow{D}_\mu D_A \right]$$

$$\mathcal{L}_{\text{Yukawa}} = -e \left[(G_L)_{AB} \bar{L}_A \varphi R_B + (G_U)_{AB} \bar{Q}_A \varphi^c U_B + (G_D)_{AB} \bar{Q}_A \varphi D_B \right] + \text{h.c.}$$

$$\mathcal{L}_{\text{Higgs}} = -e \left[(D_\mu \varphi)^\dagger D^\mu \varphi - \mu^2 \varphi^\dagger \varphi + \frac{\lambda}{3!} \left(\varphi^\dagger \varphi \right)^2 \right]$$

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{2} e \left[\text{Tr} \left(G_{\mu\nu} G^{\mu\nu} \right) + \text{Tr} \left(W_{\mu\nu} W^{\mu\nu} \right) + \frac{1}{2} B_{\mu\nu} B^{\mu\nu} \right]$$

- **Spacetime of gravitational interaction**

$$S_{\text{gravity}} = \frac{1}{2\kappa} \int \mathrm{d}^4 x \, e (R - 2\Lambda + \cdots)$$

Absence of Quantum Gravity

- On one hand, we have **Quantum Field Theory** to describe the electromagnetic, strong, and weak interactions
- On the other hand, we have **General Relativity** to describe the gravity, as the dynamics of curved spacetime

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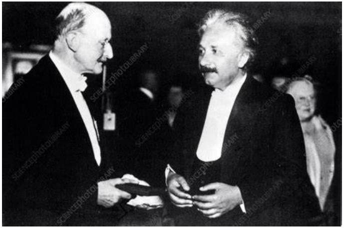
- On one hand, we have **Quantum Field Theory** to describe the electromagnetic, strong, and weak interactions
- On the other hand, we have **General Relativity** to describe the gravity, as the dynamics of curved spacetime
- However, QFT and GR are **Not Compatible** at their face values!



[Planck & Einstein]

Theoretical physics is beautiful,

but not yet complete

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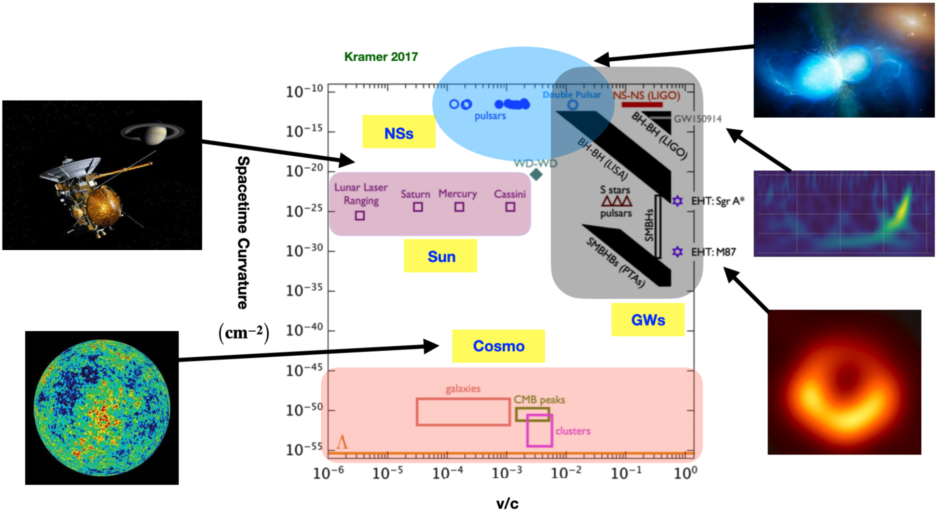
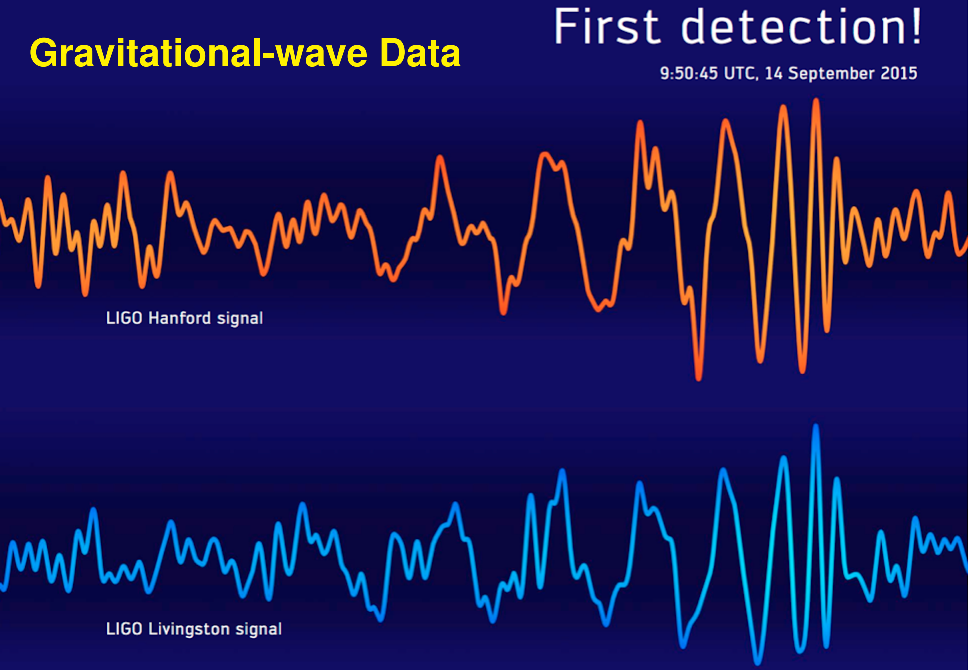
Gravity may be holding the key

Parameter Space in Gravity Tests



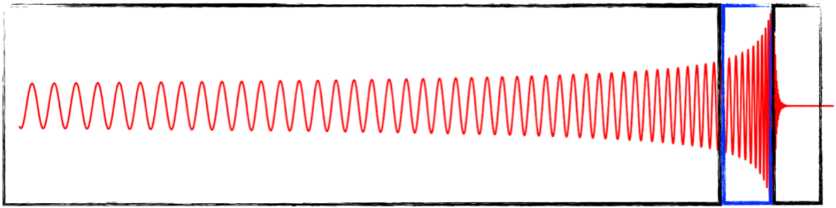
Gravitational-wave Data



Gravitational Waveform (Time Domain)

"Inspiral"
post-Newtonian method

"Ringdown"
BH perturbation

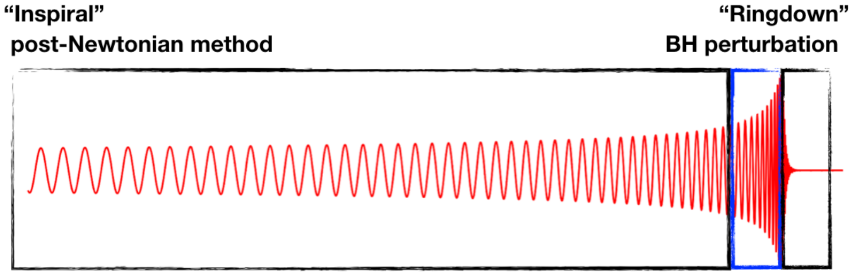


"Merge"
Numerical relativity

Effective-one body (EOB): Buonanno & Damour 1999, 2000
Bohé, Shao, Taracchini et al. 2017

Gravitational Waveform (Time Domain)

- **Inspiral**: post-Newtonian expansion



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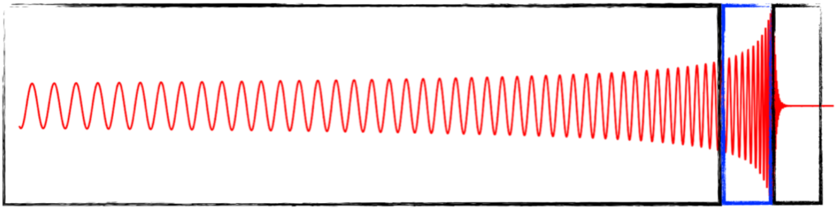

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“Merge”
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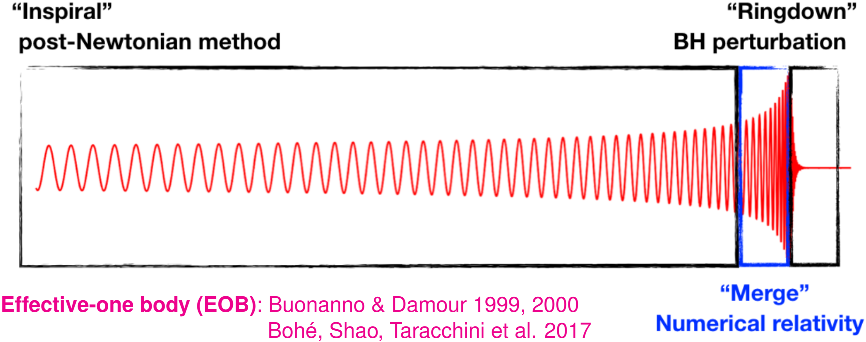
Gravitational Waveform (Time Domain)

- **Inspiral**: post-Newtonian expansion
- **Merger**: numerical relativity
- **Ringdown**: black hole perturbation



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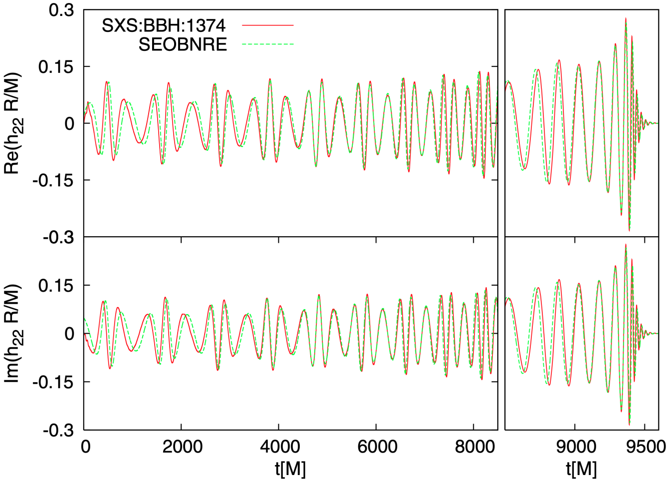
Eccentric Waveform (Time Domain)

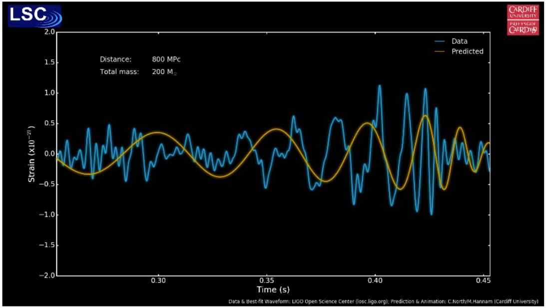


SEOBNRE: Cao & Han 2017; Liu, Cao, Shao 2020; Liu, Cao, Zhu 2021

Matched Filter

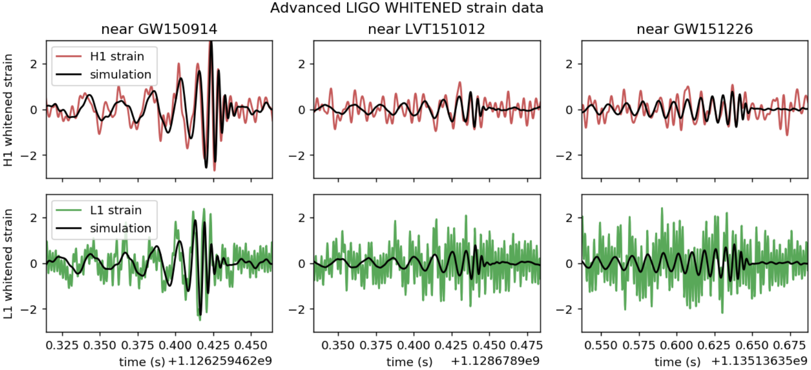
- **Matched fitlering** is a standard analysis method for **wideband** time series data [Finn 1992]

$$(\mathbf{g}|\mathbf{k}) \equiv 2\int_0^\infty \frac{\tilde{g}^*(f)\tilde{k}(f)+\tilde{g}(f)\tilde{k}^*(f)}{S_n(f)}\mathrm{d}f$$



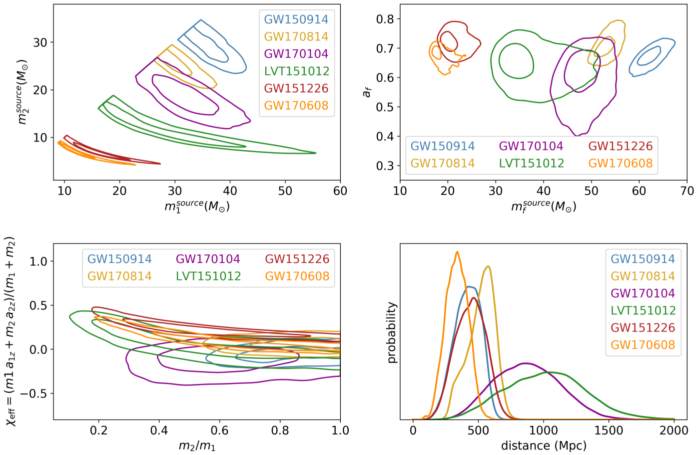
Matched Filter

- The power of **matched fitlering** lays in its ability/sensitivity to the **phase** of time-series data



Credit: Vivien Raymond / Cardiff U.

Parameter Estimation



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Parameter Estimation: GW150914

- GW data encode plenty of information of GW sources
  - Apply matched filter to data & theory

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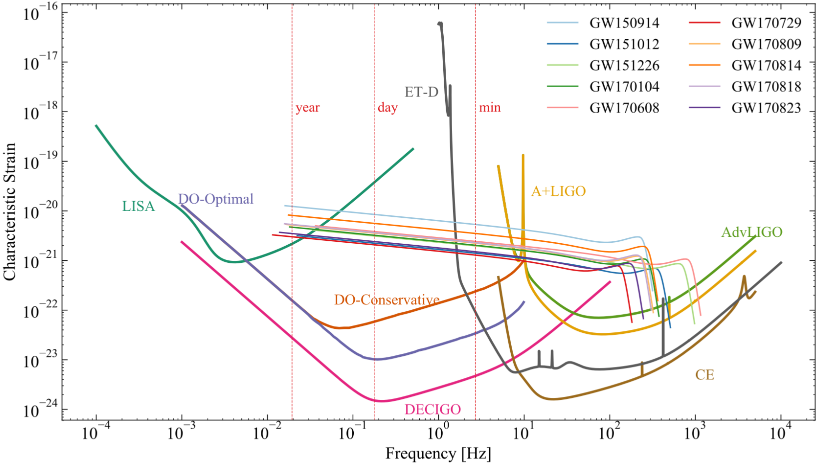
Primary black hole mass	$36^{+5}_{-4} M_\odot$
Secondary black hole mass	$29^{+4}_{-4} M_\odot$
Final black hole mass	$62^{+4}_{-4} M_\odot$
Final black hole spin	$0.67^{+0.05}_{-0.07}$
Luminosity distance	410^{+160}_{-180} Mpc
Source redshift z	$0.09^{+0.03}_{-0.04}$

LIGO/Virgo 2016, PRL

GW Transient Catalog GWTC-1 (LIGO/Virgo 2019)

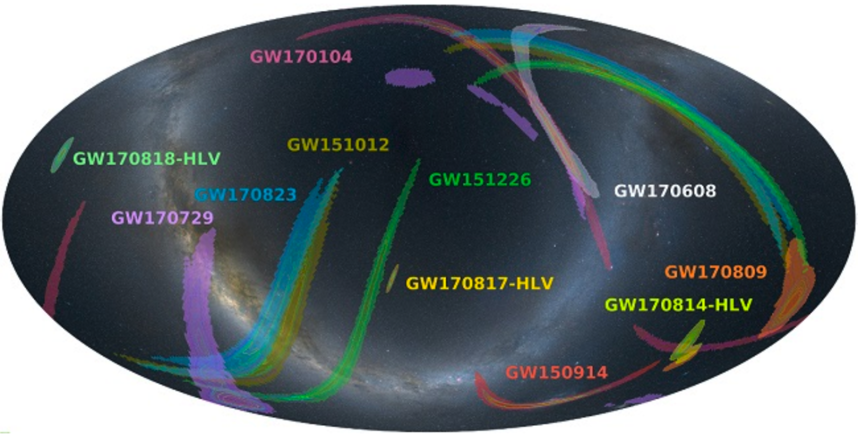
	Type	m_1 $[M_\odot]$	m_2 $[M_\odot]$	d_L [Mpc]	Redshift z
GW150914	BBH	$35.6^{+4.8}_{-3.0}$	$30.6^{+3.0}_{-4.4}$	430^{+150}_{-170}	$0.09^{+0.03}_{-0.03}$
GW151012	BBH	$23.3^{+14.0}_{-5.5}$	$13.6^{+4.1}_{-4.8}$	1060^{+540}_{-480}	$0.21^{+0.09}_{-0.09}$
GW151226	BBH	$13.7^{+8.8}_{-3.2}$	$7.7^{+2.2}_{-2.6}$	440^{+180}_{-190}	$0.09^{+0.04}_{-0.04}$
GW170104	BBH	$31.0^{+7.2}_{-5.6}$	$20.1^{+4.9}_{-4.5}$	960^{+430}_{-410}	$0.19^{+0.07}_{-0.08}$
GW170608	BBH	$10.9^{+5.3}_{-1.7}$	$7.6^{+1.3}_{-2.1}$	320^{+120}_{-110}	$0.07^{+0.02}_{-0.02}$
GW170729	BBH	$50.6^{+16.6}_{-10.2}$	$34.3^{+9.1}_{-10.1}$	2750^{+1350}_{-1320}	$0.48^{+0.19}_{-0.20}$
GW170809	BBH	$35.2^{+8.3}_{-6.0}$	$23.8^{+5.2}_{-5.1}$	990^{+320}_{-380}	$0.20^{+0.05}_{-0.07}$
GW170814	BBH	$30.7^{+5.7}_{-3.0}$	$25.3^{+2.9}_{-4.1}$	580^{+160}_{-210}	$0.12^{+0.03}_{-0.04}$
GW170817	**BNS**	$1.46^{+0.12}_{-0.10}$	$1.27^{+0.09}_{-0.09}$	40^{+10}_{-10}	$0.01^{+0.00}_{-0.00}$
GW170818	BBH	$35.5^{+7.5}_{-4.7}$	$26.8^{+4.3}_{-5.2}$	1020^{+430}_{-360}	$0.20^{+0.07}_{-0.07}$
GW170823	BBH	$39.6^{+10.0}_{-6.6}$	$29.4^{+6.3}_{-7.1}$	1850^{+840}_{-840}	$0.34^{+0.13}_{-0.14}$

Signals of GW Events (Frequency Domain)



Liu, Shao, Zhao, Gao 2020, MNRAS [arXiv:2004.12096]

GWTC-1: Sky Position (LIGO/Virgo 2019)



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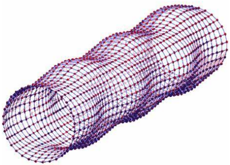
  - GWs gain *frequency-dependent* time delays when they arrive at the Earth



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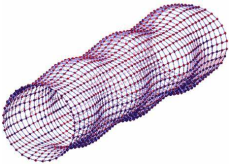
  - GWs gain *frequency-dependent* time delays when they arrive at the Earth
  - In a FRW spacetime, one has [Will 1998, PRD57:2061]

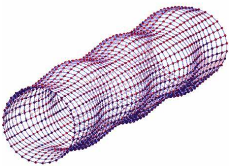
$$\Delta t_a = (1+z)\left[\Delta t_e + \frac{D}{2\lambda_g^2}\left(\frac{1}{f_e^2} - \frac{1}{f_e'^2}\right)\right]$$



Propagation of GWs

- The extra time delay results in a phase shift in $h(f) \propto e^{i\Psi(f)}$

$$\Psi(f) = \Psi_{\rm GR}(f) - \frac{\pi^2 D \mathcal{M}}{\lambda_g^2 (1+z)} (\pi \mathcal{M} f)^{-1}$$



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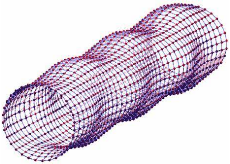

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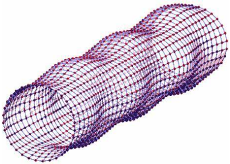
- On the other hand, the waveform is *totally* calculable and deterministic in GR
- Therefore, GWs provide an observational window to the dispersion relation of graviton



Propagation of GWs with Lorentz Violation

- Lorentz violation occurs in a few quantum gravity candidate theories [Kostelecký & Samuel 1989; Amelino-Camelia 2013]

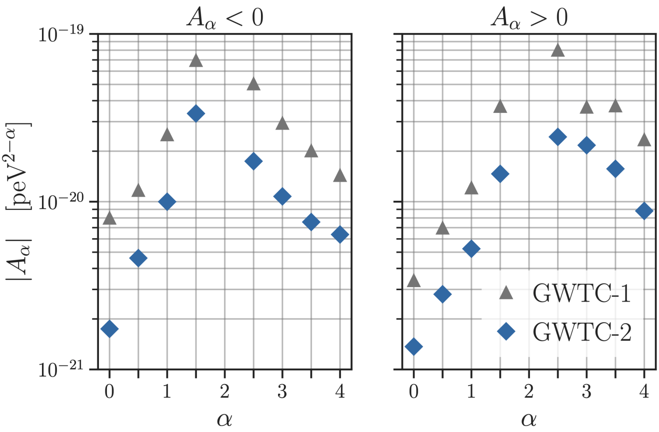
Propagation of GWs with Lorentz Violation

- Lorentz violation occurs in a few quantum gravity candidate theories [Kostelecký & Samuel 1989; Amelino-Camelia 2013]
- Dispersion relation of GWs with isotropic Lorentz violation [Mirshekari, Yunes, Will 2012]

$$E^2 = p^2c^2 + m_g^2c^4 + \mathbb{A}p^\alpha c^\alpha$$

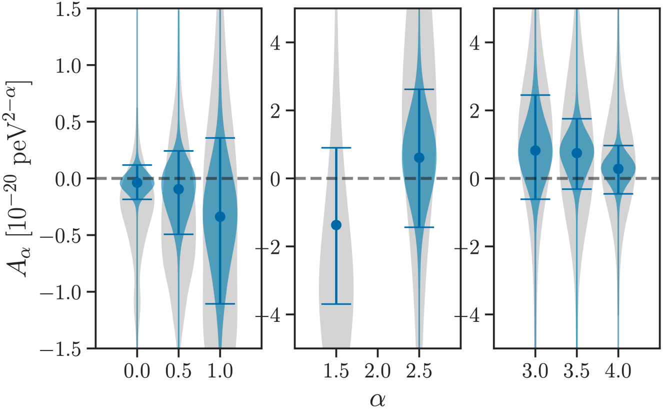
where m_g is the graviton mass; $\mathbb{A}$ and α are two Lorentz-violating parameters

Lorentz-violating Propagation of GWs



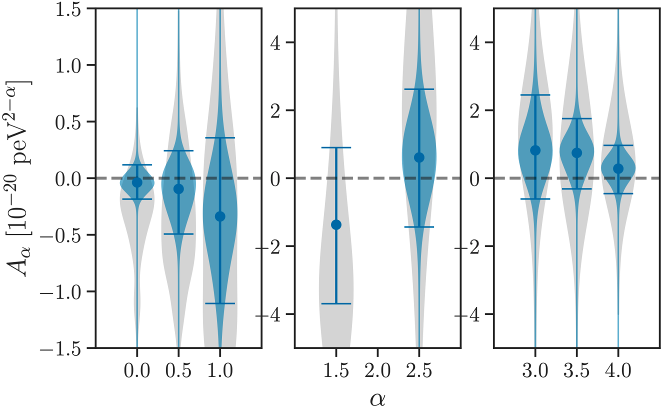
LIGO/Virgo 2021

Lorentz-violating Propagation of GWs



LIGO/Virgo 2021

Lorentz-violating Propagation of GWs



But... such a combination is **problematic** in general

LIGO/Virgo 2021

Standard-model Extension

- The most generic linearized gravity has the Lagrangian

  [Kostelecký & Mewes 2018]

$$\mathcal{L}_{\mathcal{K}^{(d)}} = \frac{1}{4} h_{\mu\nu} \hat{\mathcal{K}}^{(d)\mu\nu\rho\sigma} h_{\rho\sigma}$$

where $\hat{\mathcal{K}}^{(d)\mu\nu\rho\sigma} = \mathcal{K}^{(d)\mu\nu\rho\sigma i_1 i_2 \cdots i_{d-2}} \partial_{i_1} \partial_{i_2} \cdots \partial_{i_{d-2}}$

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- It predicts a modified dispersion relation for GWs

$$\omega = \left(1 - \zeta^0 \pm \sqrt{\left(\zeta^1\right)^2 + \left(\zeta^2\right)^2 + \left(\zeta^3\right)^2}\right) p$$

Standard-model Extension

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$$\zeta^0 = \sum_{djm} \omega^{d-4} Y_{jm}(\hat{\boldsymbol{n}}) k^{(d)}_{(I)jm}$$

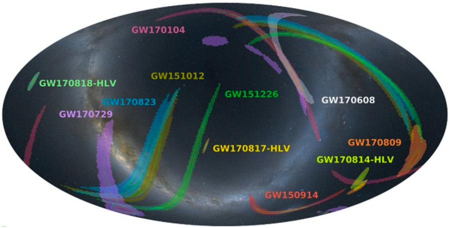
$$\zeta^1 \mp i\zeta^2 = \sum_{djm} \omega^{d-4}_{\pm 4} Y_{jm}(\hat{\boldsymbol{n}}) \left[k^{(d)}_{(E)jm} \pm i k^{(d)}_{(B)jm}\right]$$

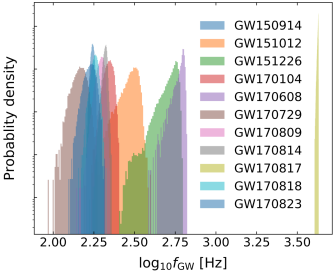
$$\zeta^3 = \sum_{djm} \omega^{d-4} Y_{jm}(\hat{\boldsymbol{n}}) k^{(d)}_{(V)jm}$$

- Therefore, gravitons of different **polarization** or **frequency**, coming from different **directions** have different velocity

GWTC-1 Events

A simplified/naive approach: $|\omega_{\rm GW} \Delta t| \leq 2\pi/\rho$

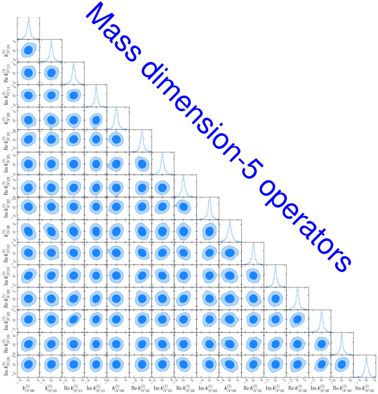


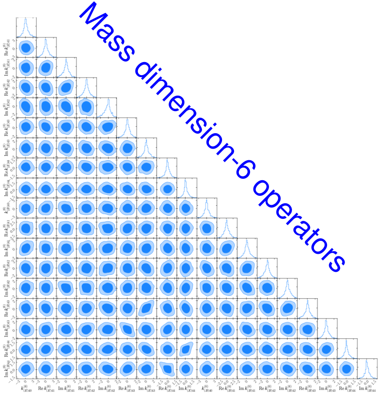


We have all the information available to perform the test

Shao 2020, PRD101:104019

Anisotropic Birefringence Combined Search





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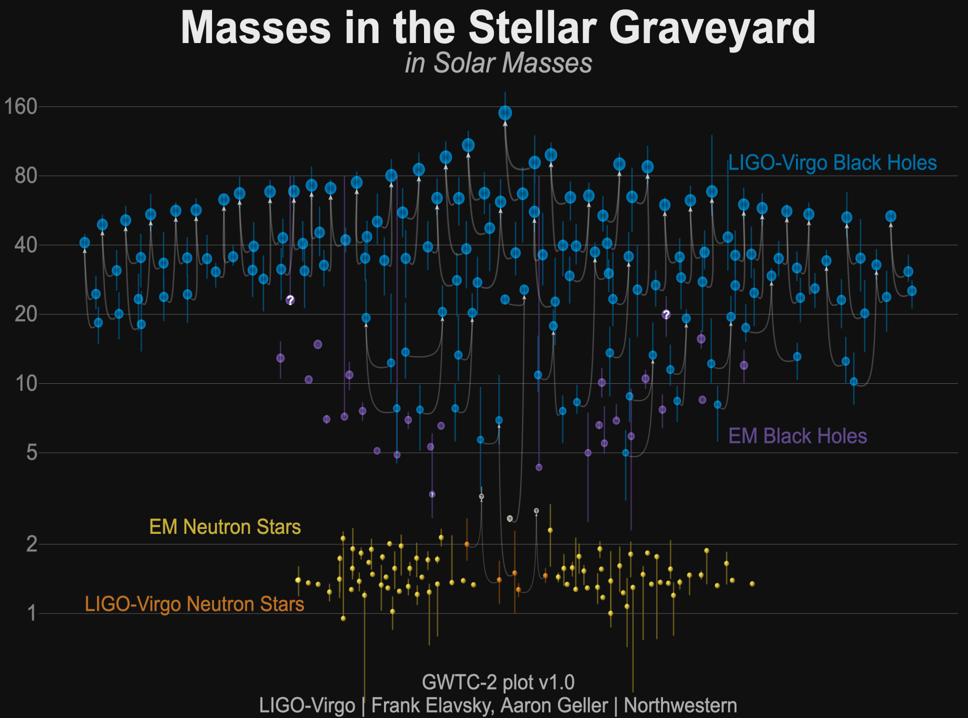
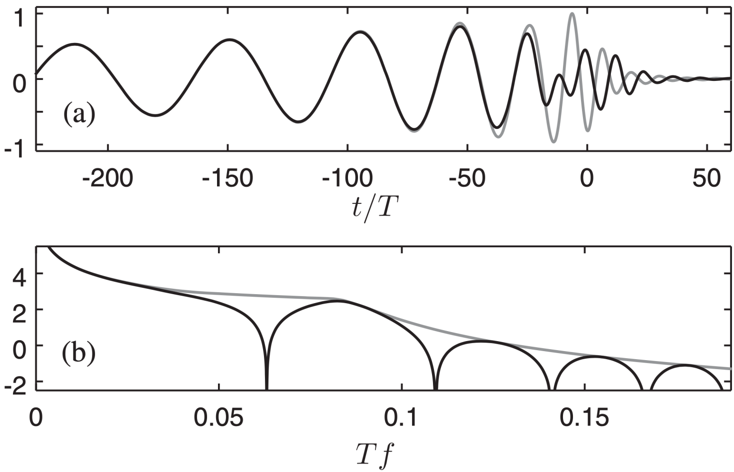
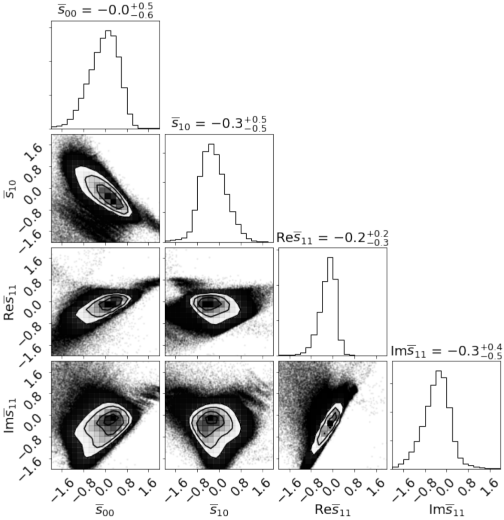

Masses in the Stellar Graveyard
in Solar Masses
160
80
40
20
10
5
2
1
LIGO-Virgo Black Holes
EM Black Holes
EM Neutron Stars
LIGO-Virgo Neutron Stars
GWTC-2 plot v1.0
LIGO-Virgo | Frank Elavsky, Aaron Geller | Northwestern

Matched Filter Analysis



Mewes, PRD99:104062

Monte Carlo Markov-Chain Runs



Liu *et al.*, PRD102:024028

Summary

- The **most generic propagation** of GWs depends on
  - GW **frequency**
  - GW **polarization**
  - GW source **direction**

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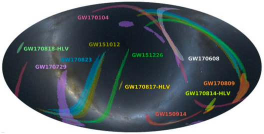
- For the first time, we combined 11 GW events in the catalog **GWTC-1**, and bounded *multiple* coefficients for generic Lorentz/CPT violation



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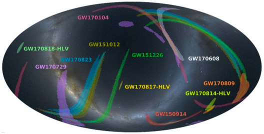
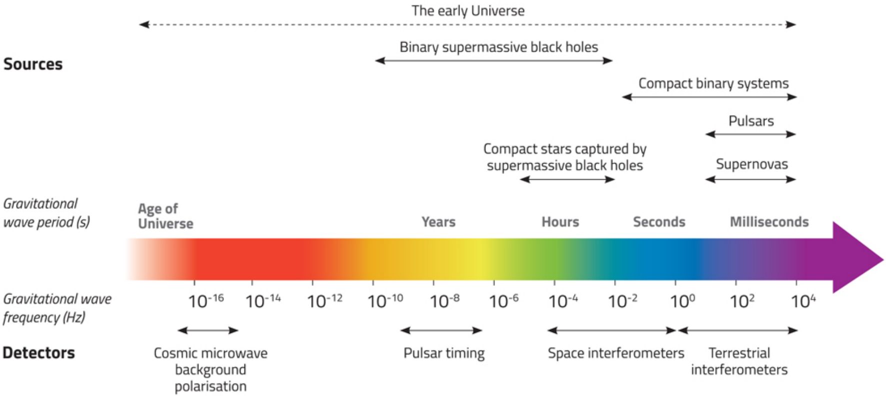
- For the first time, we combined 11 GW events in the catalog **GWTC-1**, and bounded *multiple* coefficients for generic Lorentz/CPT violation
- **GWTC-2** improves Lorentz/CPT-violating bounds by another factor of 5 [Wang, Shao, Liu, in prep.]





Only a tiny part of GW spectrum was revealed by now

Stay tuned!

Thank you!