A Brief Introduction to the Cosmology of Sound Speed Resonance

声速共振宇宙学简介

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Collaborations with Chao Chen, Jie Jiang, Chunshan Lin, Xiao-Han Ma, Shi Pi, Misao Sasaki, Xi Tong, Valeri Vardanyan, Bo Wang, Dong-Gang Wang, Sheng-Feng Yan, Zihan Zhou

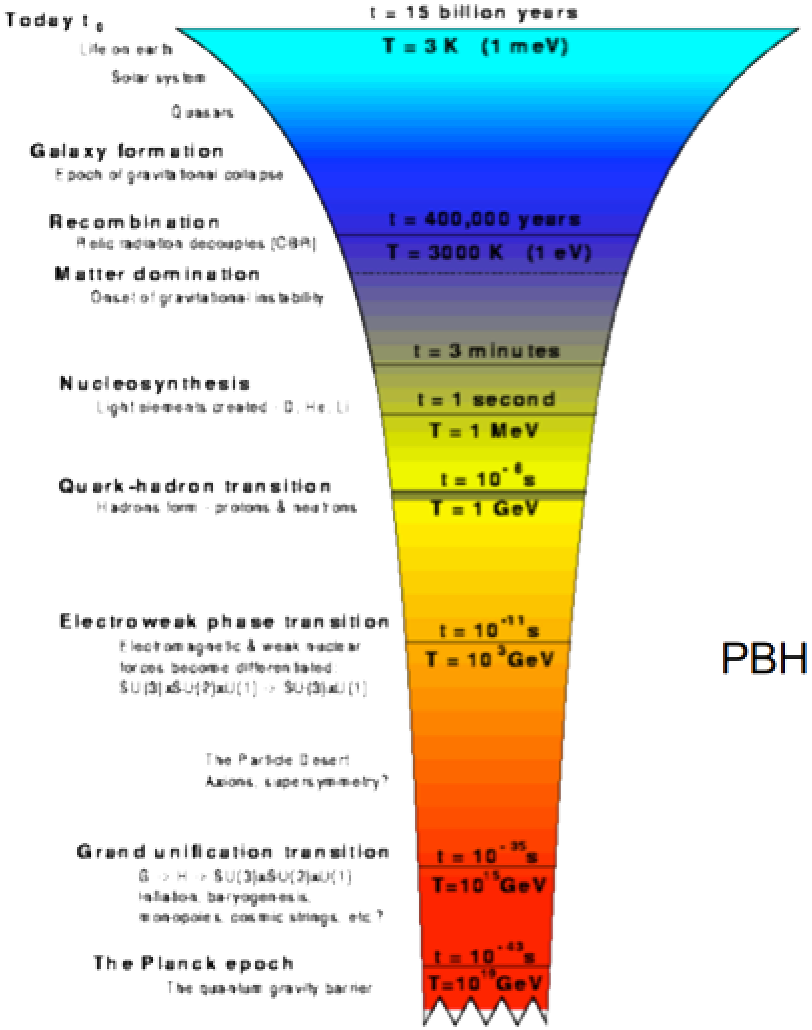
Based on arXiv: 1805.03639, 1902.08187, 1908.03942, 2003.03821, 2009.09833, 2010.03537, 2105.12554

Content

- Primordial black holes
- Sound speed resonance (SSR) mechanism
- Induced GWs: a promising approach to detect PBHs
- Probing new physics via the GW astronomy due to SSR
- Summary

What is a Primordial Black Hole?

WHEN BLACK HOLES FORM



[from Carr]

SMBH

IMBH

Stellar BH

PBH

But still no definite evidence for PBHs

Nonbaryonic

Dark Matter

A PBH is a type of black hole which is not formed through the gravitational collapse of a star, but of the sufficiently high density perturbation in the early Universe.

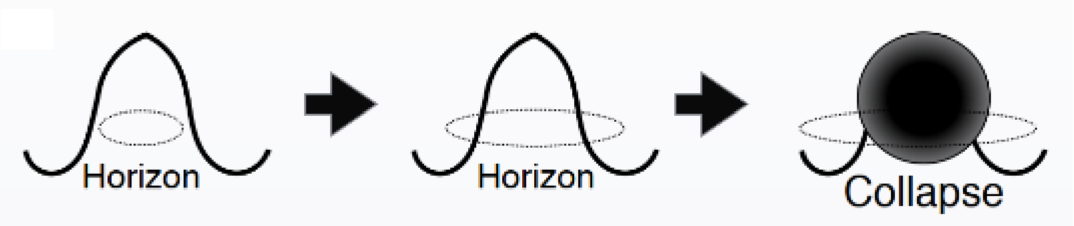
Related to plentiful cosmological and astrophysical phenomena:

- **Dark matter;**
- **LIGO/Virgo event;**
- **Seeds for the SMBHs in galactic nuclei;**
- **Hawking radiation;**
- **…**

[Carr et al., 2002.12778, 2006.02838]

PBH Formation

- **Collapse of overdense region:**



[from Fernandez]

- **Threshold:** $R_H \sim R > R_J$ → Threshold: $\boxed{\delta > \omega}$, for $p = \omega\rho$ [Carr,'75]

Hubble radius

Overdense region $\delta = \delta\rho/\rho$

Jeans length

The precise value of the threshold for PBH formation $\delta_c \sim 0.45$

- **Huge mass range:** $M_{\rm PBH} \sim M_H \sim \frac{c^3 t}{G} \sim \boxed{10^{15}\left(\frac{t}{10^{-23}\rm s}\right)\rm g}$

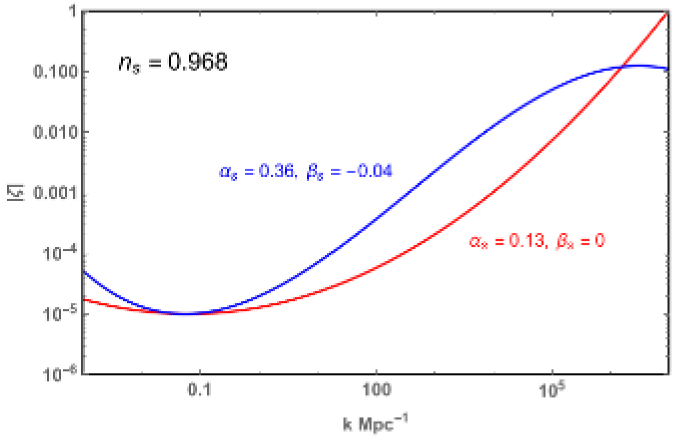
Planck scale	$10^{-43}s$	$\rightarrow$	$10^{-5}g$
EW scale	$10^{-10}s$	$\rightarrow$	$10^{28}g$
QCD scale	$10^{-6}s$	$\rightarrow$	$10^{32}g$
Neutrino decoupling	$1s$	$\rightarrow$	$10^{5}M_{\odot}$

Generating large inhomogeneities: inflationary scenarios

- **Running mass inflation:**

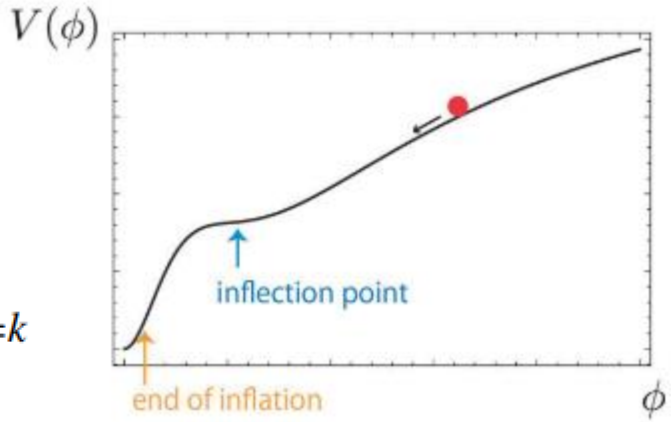
$$\mathcal{P}_{\mathcal{R}_c}(k) := A_{\mathcal{R}_c}\left(\frac{k}{k_*}\right)^{n_s-1+\frac{1}{2!}\alpha_s\ln(k/k_*)+\frac{1}{3!}\beta_s\ln^2(k/k_*)+\cdots}$$

Running parameters



- **Inflection inflation:**

$$\mathcal{P}_{\mathcal{R}_c}(k) = \left(\frac{8\pi GH^2}{\epsilon}\right)_{aH=k}$$

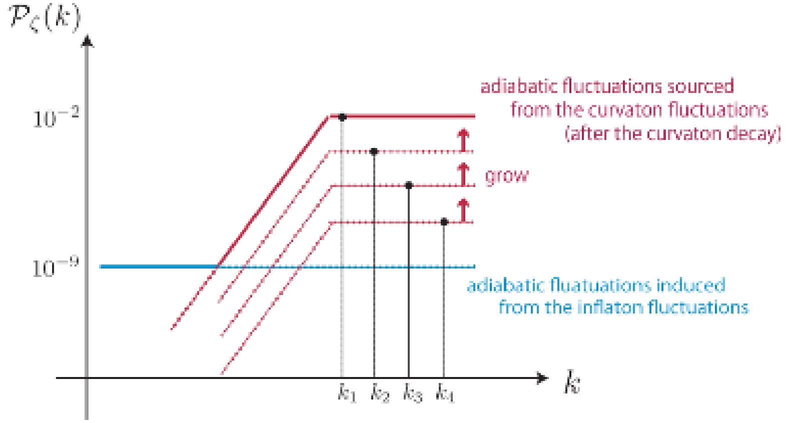


R. Saito, J. Yokoyama, R. Nagata, 0804.3470
J. Garcia-Bellido, E. Ruiz Morales, 1702.03901
S. Pi, Y. Zhang, Q. G. Huang, M. Sasaki, 1712.09896
… too many to track all … For a comprehensive review, see Misao et al., 1801.05235

- **Axion-like curvaton inflation:**

$$\mathcal{P}_\zeta(k) = \mathcal{P}_{\zeta,\mathrm{inf}}(k) + \mathcal{P}_{\zeta,\mathrm{curv}}(k).$$

$$V_\chi = \Lambda^4\left[1-\cos\left(\frac{\chi}{f}\right)\right] \simeq \frac{1}{2}m_\chi^2\chi^2$$



Generating large inhomogeneities: SSR mechanism

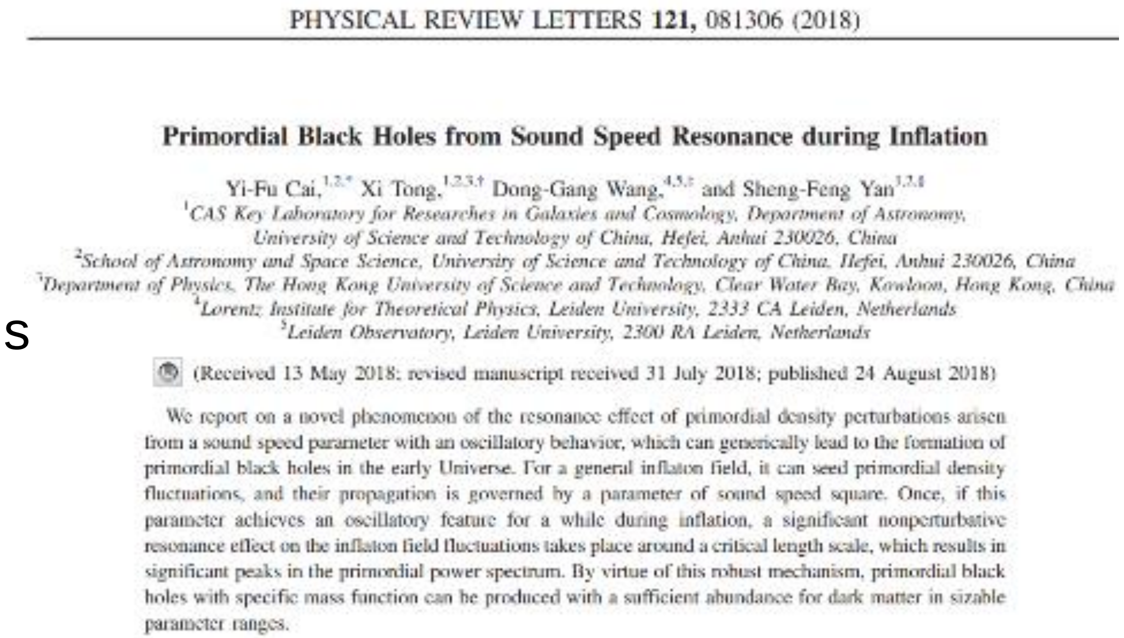
- **Oscillating sound speed:**

$$\boxed{c_s^2 = 1 - 2\xi[1-\cos(2k_*\tau)],\ \text{with}\ \tau > \tau_i}$$

The amplitude: ξ is small and $\xi < 1/4$, such that c_s^2 is positively definite;

The characteristic scale: k_* is the oscillation frequency;

The beginning of oscillation: $|k_*\tau_i| \gg 1$.



PHYSICAL REVIEW LETTERS 121, 081306 (2018)

Primordial Black Holes from Sound Speed Resonance during Inflation

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We report on a novel phenomenon of the resonance effect of primordial density perturbations arisen from a sound speed parameter with an oscillatory behavior, which can generically lead to the formation of primordial black holes in the early Universe. For a general inflaton field, it can seed primordial density fluctuations, and their propagation is governed by a parameter of sound speed square. Once, if this parameter achieves an oscillatory feature for a while during inflation, a significant nonperturbative resonance effect on the inflaton field fluctuations takes place around a critical length scale, which results in significant peaks in the primordial power spectrum. By virtue of this robust mechanism, primordial black holes with specific mass function can be produced with a sufficient abundance for dark matter in sizable parameter ranges.

[CYF, et al., PRL 121 (2018) 8, 081306]

- **Equation of Motion:** $$v_k'' + \left(c_s^2k^2 - \frac{z''}{z}\right)v_k = 0\,,$$

Mukhanov-Sasaki variable: $v = z\zeta \quad z = \sqrt{2\epsilon}a/c_s$ with $\epsilon \equiv -\dot{H}/H^2$

- **Mathieu equation:**

$$\frac{d^2v_k}{dx^2} + (A_k - 2q\cos 2x)v_k = 0$$

narrow resonance

where $x = -k_*\tau$, $A_k = \frac{k^2}{k_*^2}(1-2\xi)$ and $q = \boxed{(2-\frac{k^2}{k_*^2})\xi}$ <<1

Resonance effect

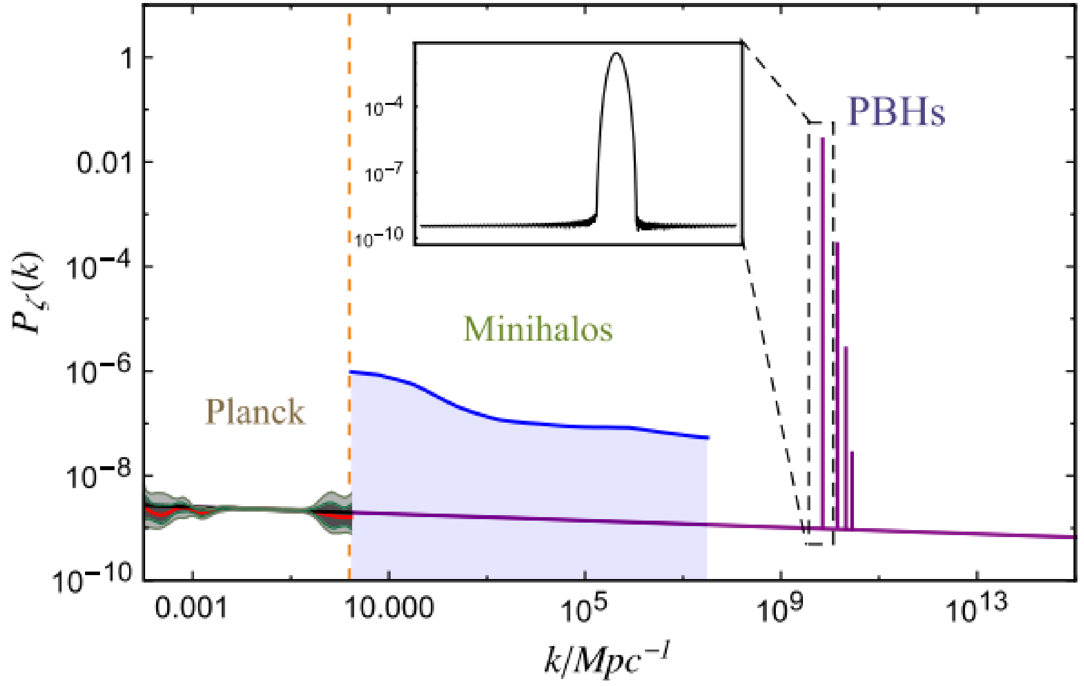
Generating large inhomogeneities: SSR mechanism

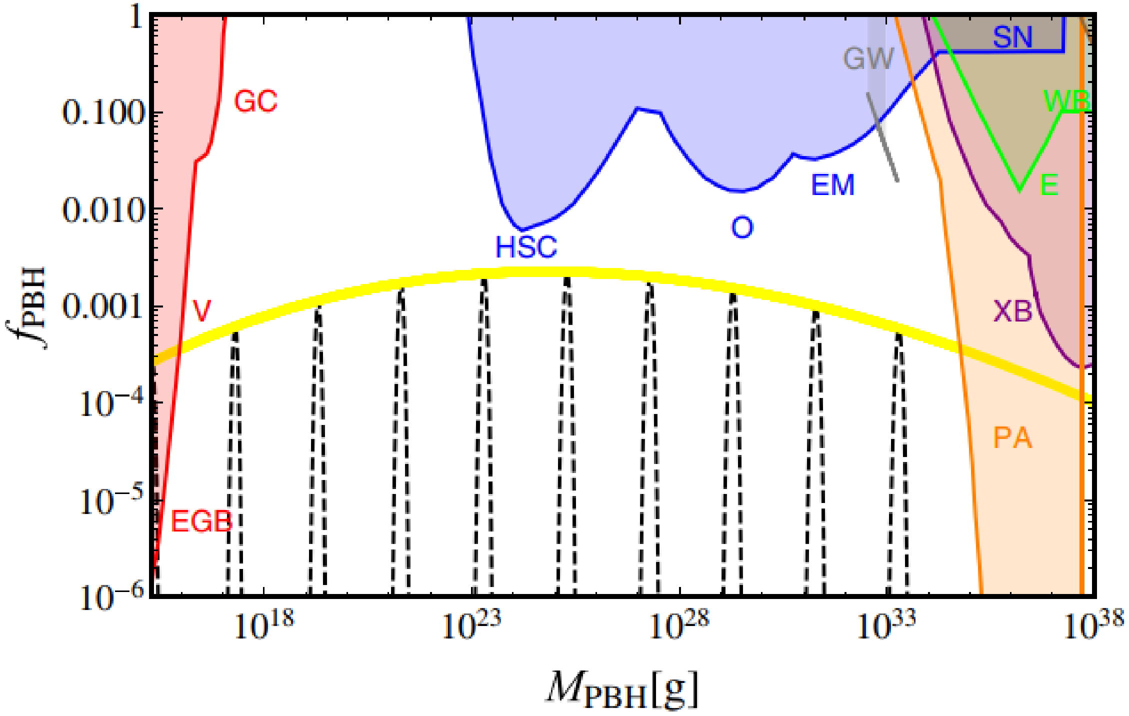
- **Enhanced curvature perturbation:**

$$P_\zeta(p) = A_s \left(\frac{p}{k_p}\right)^{n_s-1} \left\{1 + \frac{\xi p_*}{2} \boxed{e^{-\xi p_* \tau_i}} \left[\delta(p-p_*) + \sum_{n=2}^{\infty} a_n \delta(p - np_*)\right]\right\}$$

Enhancement

$$f_{\rm PBH}(M) \equiv \frac{\Omega_{\rm PBH}}{\Omega_{\rm DM}}$$





Spikes on small scales, scale-invariant on large scales

SSR in the inflaton-curvaton mixed scenario

- **The total curvature perturbation in the mixed scenario**

[Chen & CYF, JCAP 10 (2019) 068]

Symbolically: $\mathcal{P}_\zeta^{\rm tot} = \mathcal{P}_\zeta^\phi + \mathcal{P}_\zeta^\sigma$

inflaton

curvaton

Adiabatic scale-invariant:

$$\mathcal{P}_\zeta^\phi = \frac{1}{8\pi^2\epsilon}\left(\frac{H_*}{M_p}\right)^2$$

Conversion: isocurvature → curvature

SSR: c_s oscillates during inflation

Curvaton scenario

➢ **The primordial density perturbation generated in curvaton mechanism**

The curvaton is assumed to be a second, light, scalar field presents during inflation:

1. has a subdominant energy density compared to the inflaton's, while the inflaton drives inflation.
2. is long lived (i.e. it decays later than the inflaton).
3. generates the entire primordial curvature perturbation.

The conversion between isocurvature perturbation and adiabatic curvature perturbation.

$$\dot{\zeta} = -\frac{H}{\rho + p}\delta p_{\mathrm{nad}} - \frac{1}{3}\nabla^2 (\sigma + v + B) \ .$$

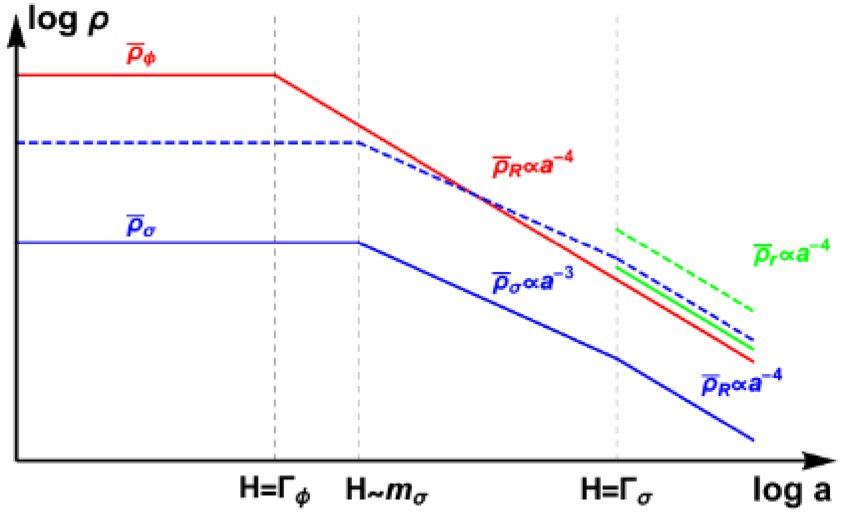
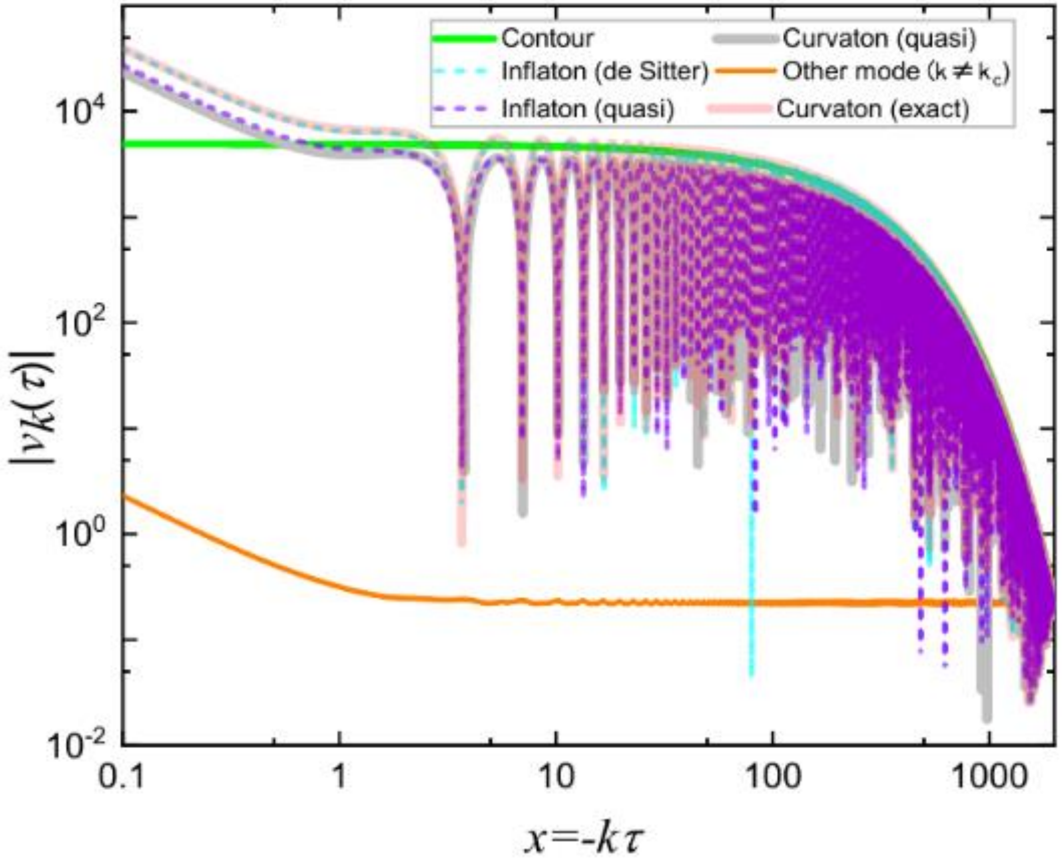


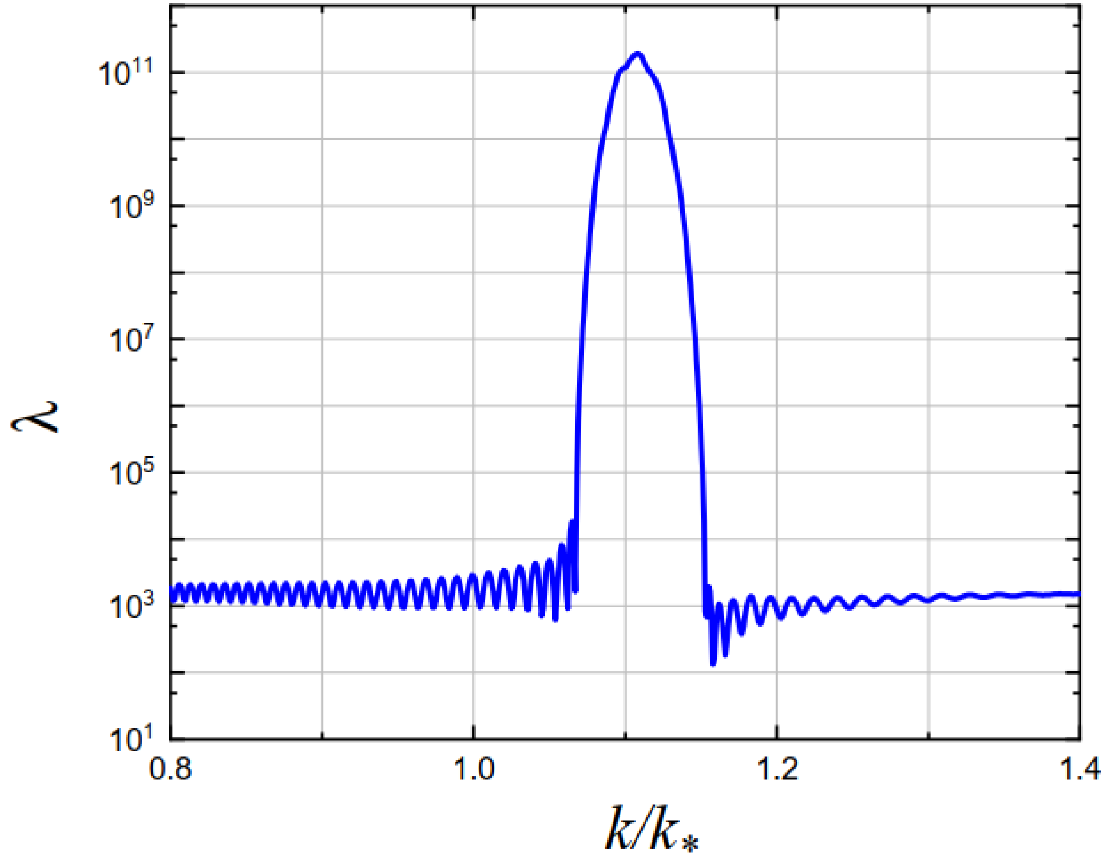
FIG. 3. The diagram of the evolution of the background energy density of inflaton (red line), curvaton (blue line), and their decay products (radiation). The green line denotes the total energy density of radiation after curvaton decay. The solid lines refer to the case that curvaton is still subdominant at its decay, while the dashed lines refer to the case that curvaton becomes dominant before its decay.

[D.H. Lyth, C. Ungarelli, D. Wands, PRD 67, 023503 (2003)]

Enhanced Power Spectrum in the mixed scenario

Enhanced Power spectrum $\mathcal{P}_\zeta^{\rm tot} = \mathcal{P}_{\zeta_r} = (1+\lambda)\mathcal{P}_{\zeta_\phi}$ $\quad \lambda = \frac{16}{9} f^2 \epsilon(\epsilon-1)^2 c_s^2(\tau_*) k\, |v_k(\tau_*)|^2 \left(\frac{M_p}{\bar{\sigma}_*}\right)^2$





[Chen & CYF, JCAP 10 (2019) 068]

SSR is insensitive to the background dynamics

How to realize SSR Mechanism in inflation?

- **DBI inflation:** $S = \int d^4x\sqrt{-g}[f(\phi)^{-1}(1-\sqrt{1+2f(\phi)X})-V(\phi)]$

Sound speed: $c_s^2 = 1 - f(\phi)\dot{\phi}^2$

- **Effective field theory:**

$$S_2 = \frac{1}{2}\int a^3\left[\frac{\dot{\phi}_0^2}{H^2}\dot{\mathcal{R}}^2 - \frac{\dot{\phi}_0^2}{H^2}\frac{(\nabla\mathcal{R})^2}{a^2} + \dot{\mathcal{F}}^2 - \frac{(\nabla\mathcal{F})^2}{a^2} - M_{\rm eff}^2\mathcal{F}^2 \underline{- 4\dot{\theta}\frac{\dot{\phi}_0}{H}\dot{\mathcal{R}}\mathcal{F}}\right]$$

Adiabaticity condition: $\left|\frac{\ddot{\theta}}{\dot{\theta}}\right| \ll M_{\rm eff}$

$$S_{\rm eff} = \frac{1}{2}\int a^3\frac{\dot{\phi}_0^2}{H^2}\left[\frac{\dot{\mathcal{R}}^2}{c_s^2(k)} - \frac{k^2\mathcal{R}^2}{a^2}\right] \qquad c_s^{-2}(k) = 1 + 4\dot{\theta}^2/(k^2/a^2 + M_{\rm eff}^2)$$

SSR Mechanism: Realization in Dirac-Born-Infeld Inflation

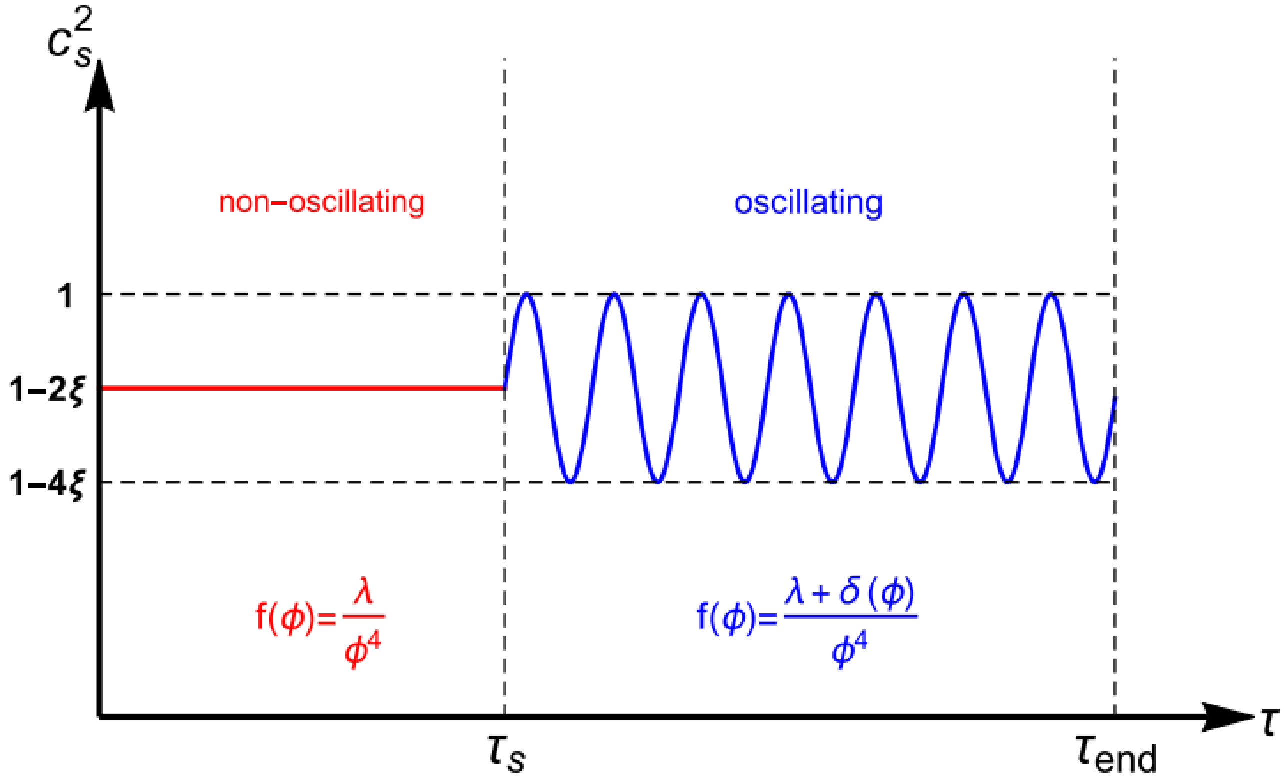
[Chen, Ma, CYF, PRD 102 (2020) 6, 063526]

- **DBI inflation:** $c_s^2 = 1 - f(\phi)\dot{\phi}^2$,

EoM: $$\ddot{\phi} + 3Hc_s^2\dot{\phi} + c_s^3 V'(\phi) + \frac{f'(\phi)}{2f(\phi)}\left(1 - \frac{2c_s^2}{1+c_s}\right)\dot{\phi}^2 = 0,$$

- **Perturbative approach:**

ξ is small:



SSR Mechanism: Realization in Dirac-Born-Infeld Inflation

- **Matching condition:**

$$f(\phi)\left(\frac{d\phi}{d\tau}\right)^2 = 2\xi a(\tau)^2[1-\cos(2k_*\tau)]$$
$$\simeq \frac{2\xi}{(\epsilon-1)^2}\frac{1-\cos(2k_*\tau)}{H^2\tau^2},$$

[Chen, Ma, CYF, PRD 102 (2020) 6, 063526]

- **Dynamics of inflaton (Hamilton-Jacobi formalism):**

$$\phi(\tau) \simeq \left(\frac{1}{\phi_i} \pm \frac{\sqrt{2\xi}}{H(1-\epsilon)\sqrt{\lambda}}\ln\frac{\tau}{\tau_i}\right)^{-1}$$

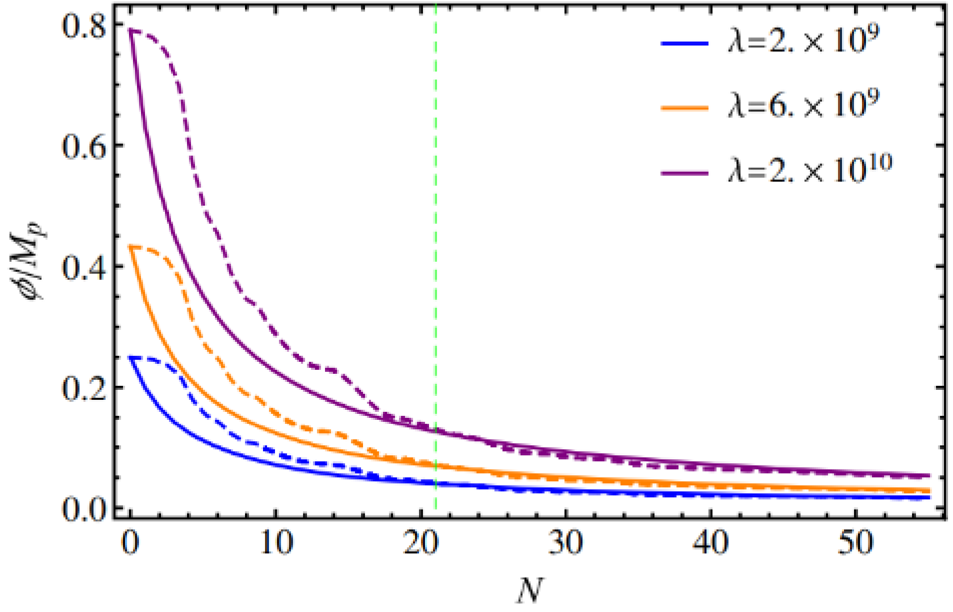


FIG. 4. Numerical (the dashed curves) and semianalytical results (the solid curves) of the evolutions of the inflaton field ϕ with different values of λ. The green dashed line denotes the beginning of the oscillating stage. The parameter values are chosen to be $H_0 = 10^{-5}M_p$, $\xi = 0.1$ and $N_s = 21$.

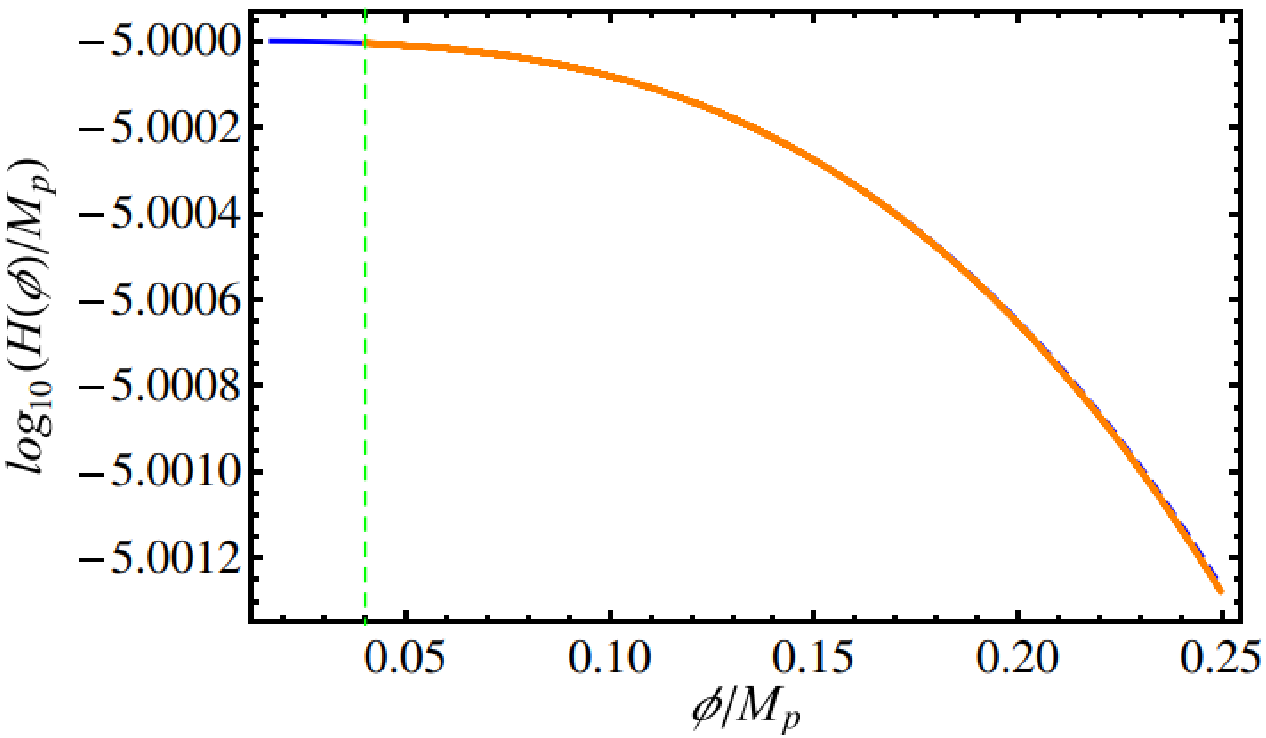
SSR Mechanism: Realization in Dirac-Born-Infeld Inflation

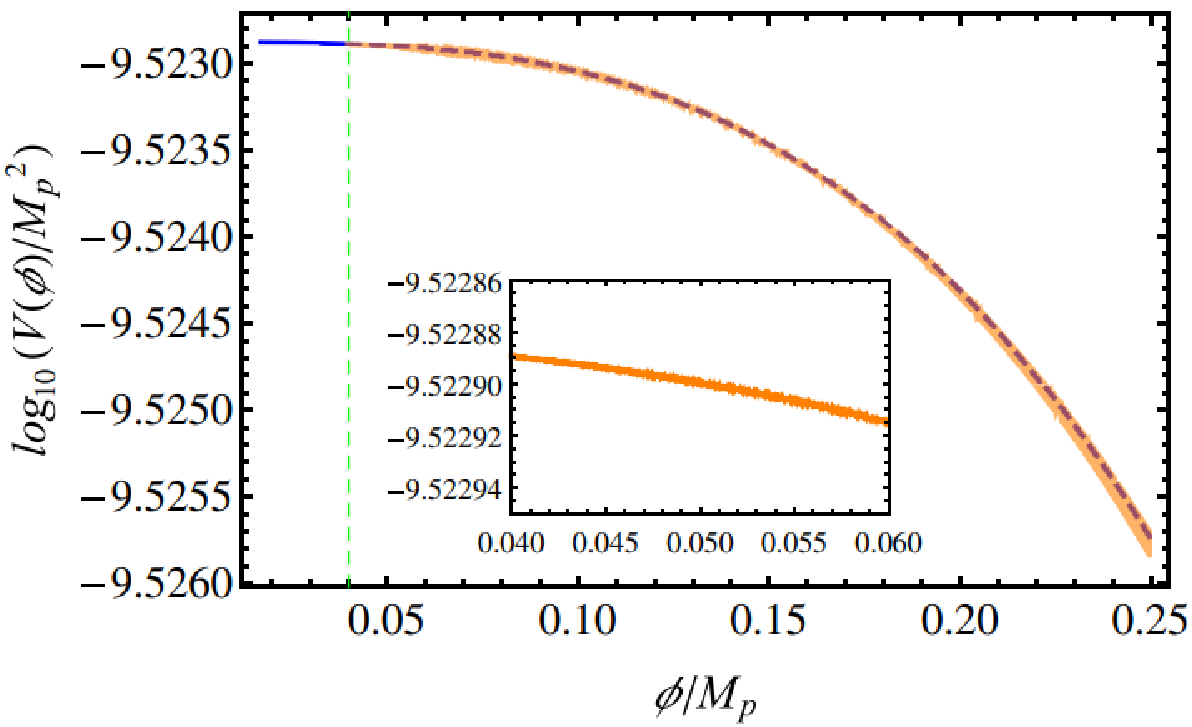
- **Hubble parameter and Potential:**

[Chen, Ma, CYF,
PRD 102 (2020) 6, 063526]

$$H(\phi) = H_0 - \sqrt{\frac{\xi}{2\lambda(1-2\xi)}}\frac{\phi^3}{3M_p^2}$$

$$V(\phi) = 3H_0^2M_p^2 - \sqrt{\frac{2\xi}{\lambda(1-2\xi)}}H_0\phi^3 + \mathcal{O}(\phi^4)$$



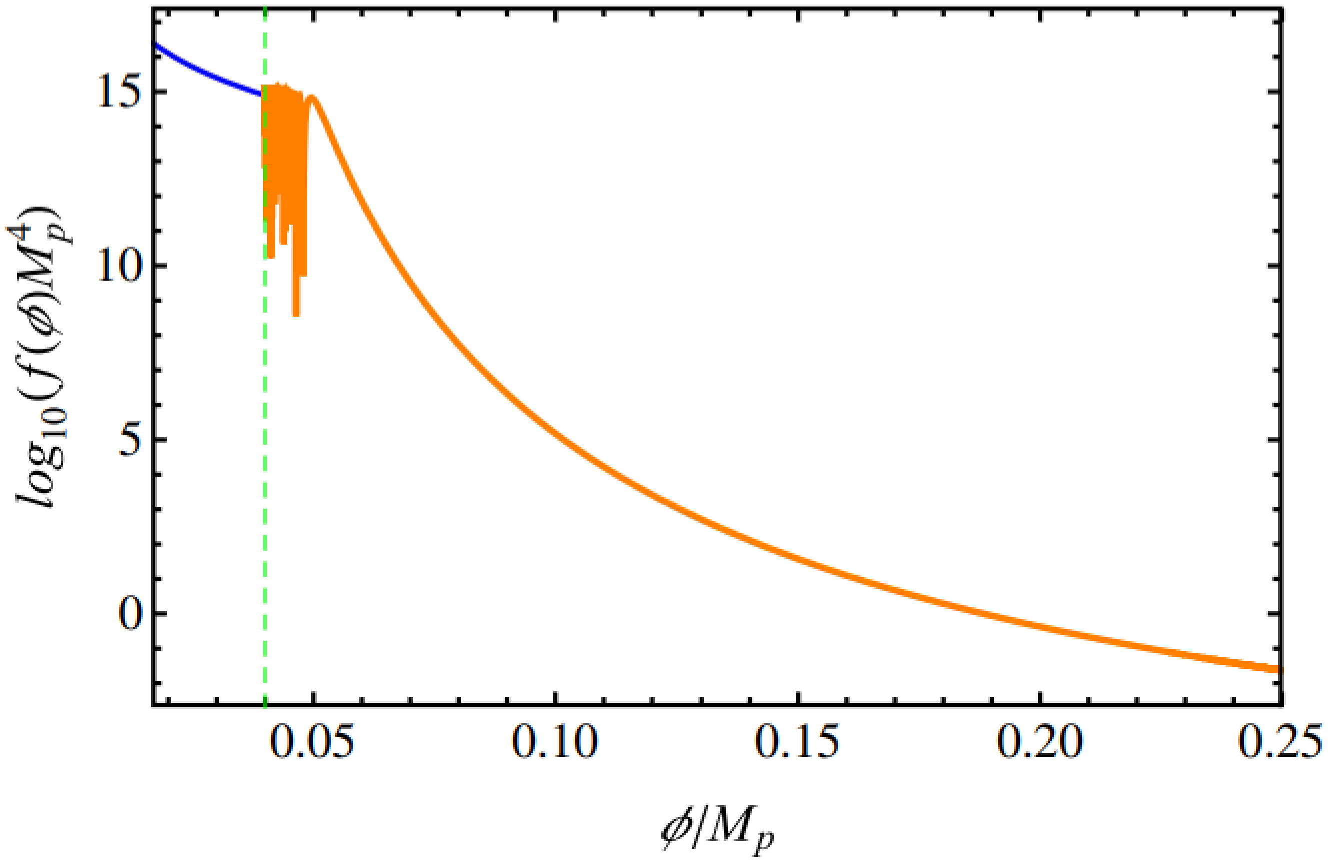


SSR Mechanism: Realization in Dirac-Born-Infeld Inflation

[Chen, Ma, CYF,
PRD 102 (2020) 6, 063526]

- **Warped factor:**

$$f(\phi) = \frac{\lambda[1 - \Theta(\phi - \phi_s)C(\phi)]}{\phi^4} \qquad C(\phi) \equiv \cos\left\{2k_*\tau_s \exp\left[\frac{H(1-\epsilon)\sqrt{\lambda}}{\sqrt{2\xi}}\left(\frac{1}{\phi} - \frac{1}{\phi_s}\right)\right]\right\}$$



Multifield Inflation

Multiple Field Dynamics:

$$S = \frac{1}{2}\int d^4x\left[M_p^2 R - \delta_{IJ} g^{\mu\nu}\partial_\mu\phi^I\partial_\nu\phi^J - 2V(\phi)\right]$$

3+1 Decomposition:

$$ds^2 = -N^2 dt^2 + \gamma_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$

Spatially-flat gauge: $\gamma_{ij} = a^2(t)\delta_{ij}, \quad \phi^I(t,\mathbf{x}) = \phi^I(t) + \delta\phi^I(t,\mathbf{x})$

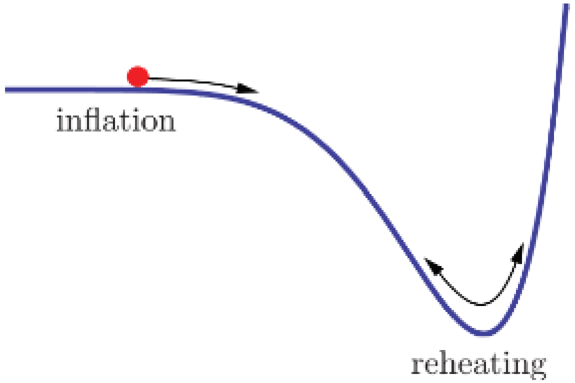
Perturbation evolution:

$$\ddot{\delta\phi}^I + 3H\dot{\delta\phi}^I + \frac{k^2}{a^2}\delta\phi^I + \sum_J\left[V_{,IJ} - \frac{1}{M_p^2 a^3}\frac{d}{dt}\left(\frac{a^3}{H}\dot{\phi}_I\dot{\phi}_J\right)\right]\delta\phi^J = 0$$

Motivation

- For single-field inflation, adiabatic curvature perturbations are conserved on the super-horizon scales.

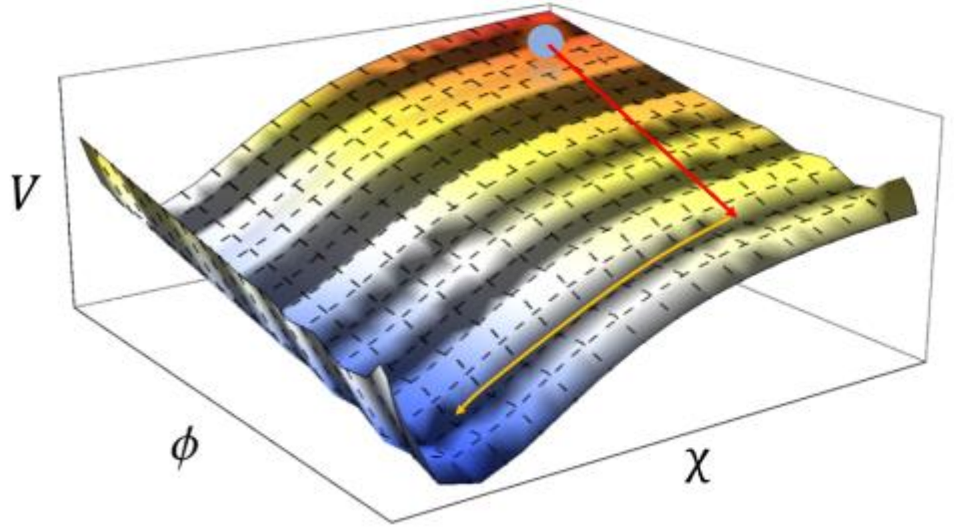
- From preheating, we know that controllable instabilities could be realized by making use of coherent oscillations and adding a second field that plays the role of entropy modes.



$$\ddot{\chi}_k + 3H\dot{\chi}_k + \left(\frac{k^2}{a^2} + g^2\phi^2(t)\right)\chi_k = 0$$

Question: Can we find a field that plays the role of coherently oscillating inflaton?

The model

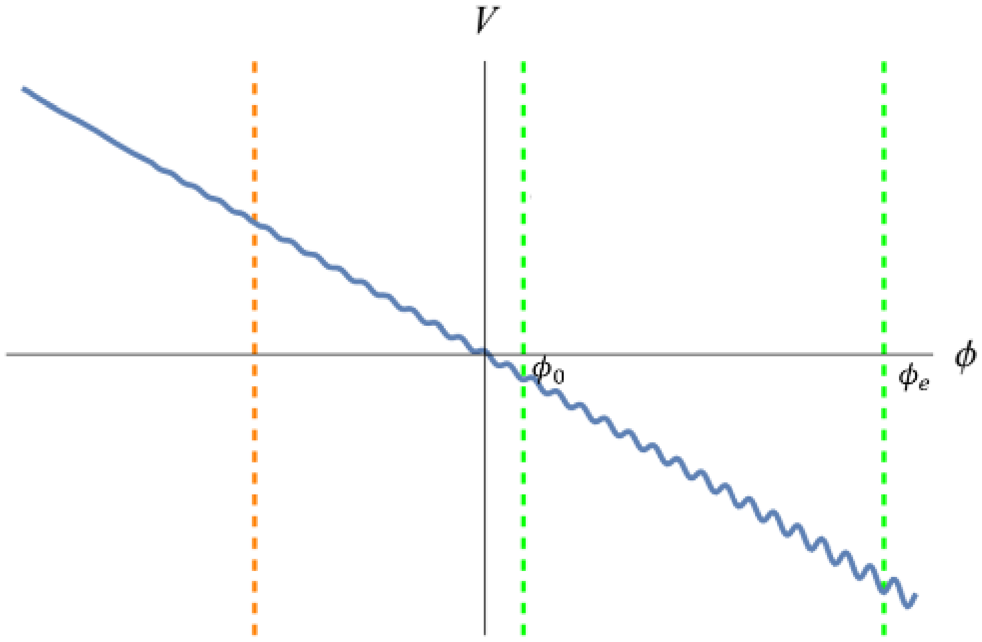


Extended from axion-monodromy:

$$V(\phi,\chi) = g\Lambda_0^3\phi + \Lambda^4(\phi)\cos(\frac{\phi}{f_a}) + \xi\Lambda_0^3\chi + V_0$$

Extended from relaxion:

$$\Lambda(\phi) = \Lambda_0(1+\alpha\frac{\phi}{M_p})$$



Mathieu Instabilities:

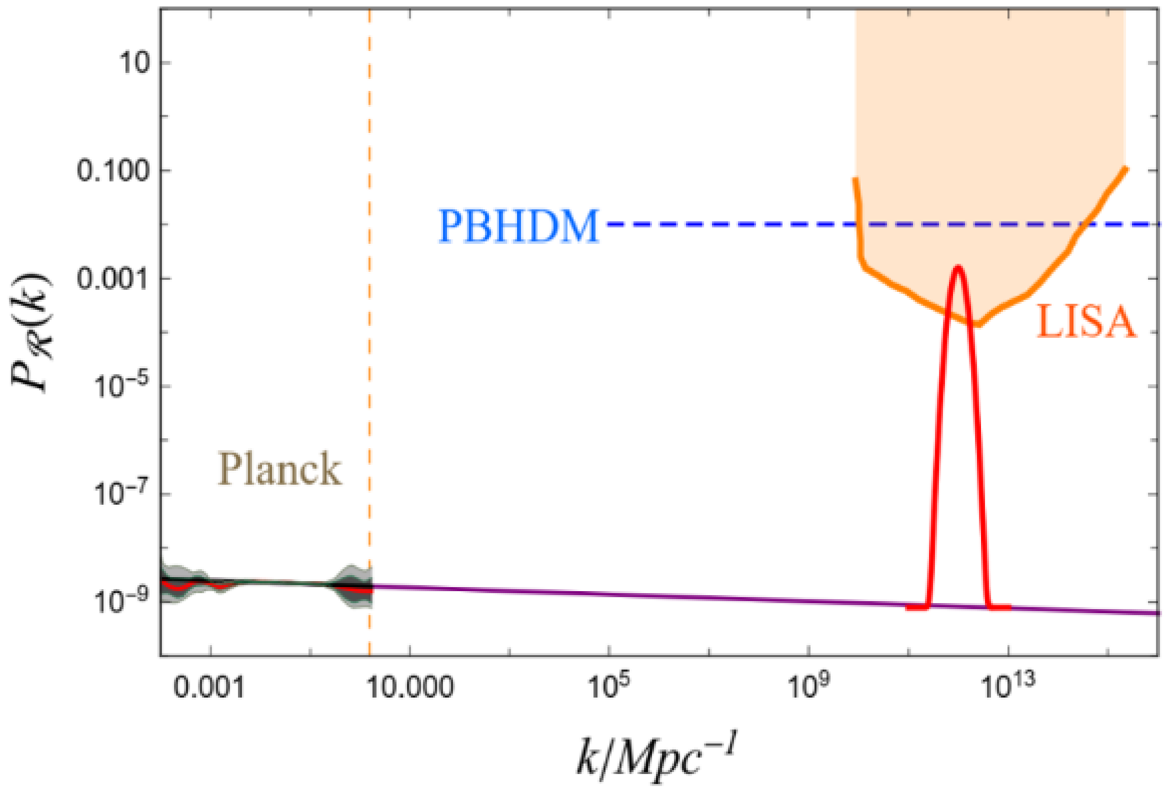
$$\delta\ddot{\chi}_k + 3H\delta\dot{\chi}_k + \frac{k^2}{a^2}\delta\chi_k \simeq \frac{\dot{\chi}\ddot{\phi}}{M_p^2 H}\delta\phi_k\,,$$

$$\delta\ddot{\phi}_k + 3H\delta\dot{\phi}_k + \left(\frac{k^2}{a^2} - \frac{\Lambda^4(\phi)}{f_a^2}\cos(\frac{\phi}{f_a})\right)\delta\phi_k = 0$$

[Zhou, Jiang, CYF, Sasaki, Pi,
PRD 102 (2020) 10, 103527]

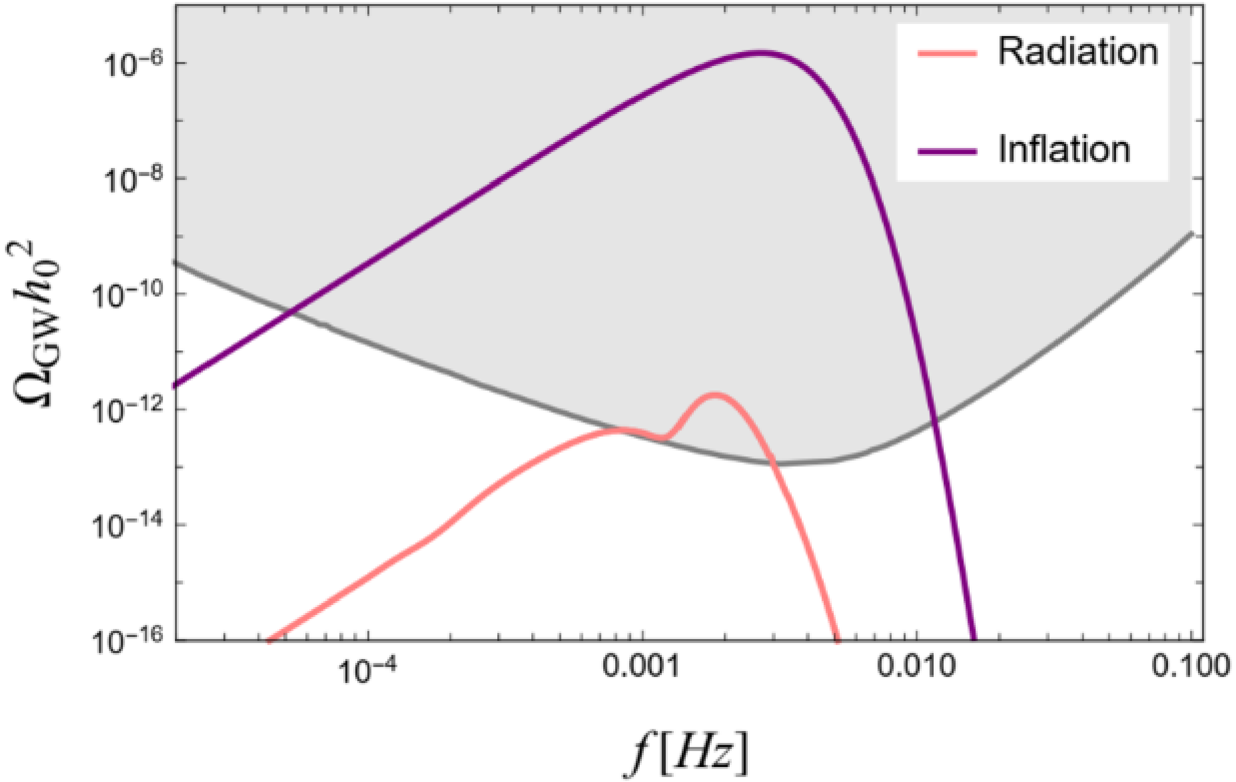
Perturbation Spectrum

Scalar Spectrum



$$P_{\mathcal{R}}(k) = \frac{k^3}{2\pi^2}\left|\frac{\partial N}{\partial \chi}\right|^2 |\delta\chi_k|^2(t_i) \simeq \frac{H^2}{8\pi^2 M_p^2 \epsilon_{\chi\chi}}\mathcal{A}^2(k)$$

Tensor Spectrum



$$\frac{\Omega_{\rm GW}^{\rm Inf}}{\Omega_{\rm GW}^{\rm Rad}} \sim \epsilon_{\chi\chi}^2 \frac{P_\phi^2(k_*)}{P_\chi^2(k_*)}$$

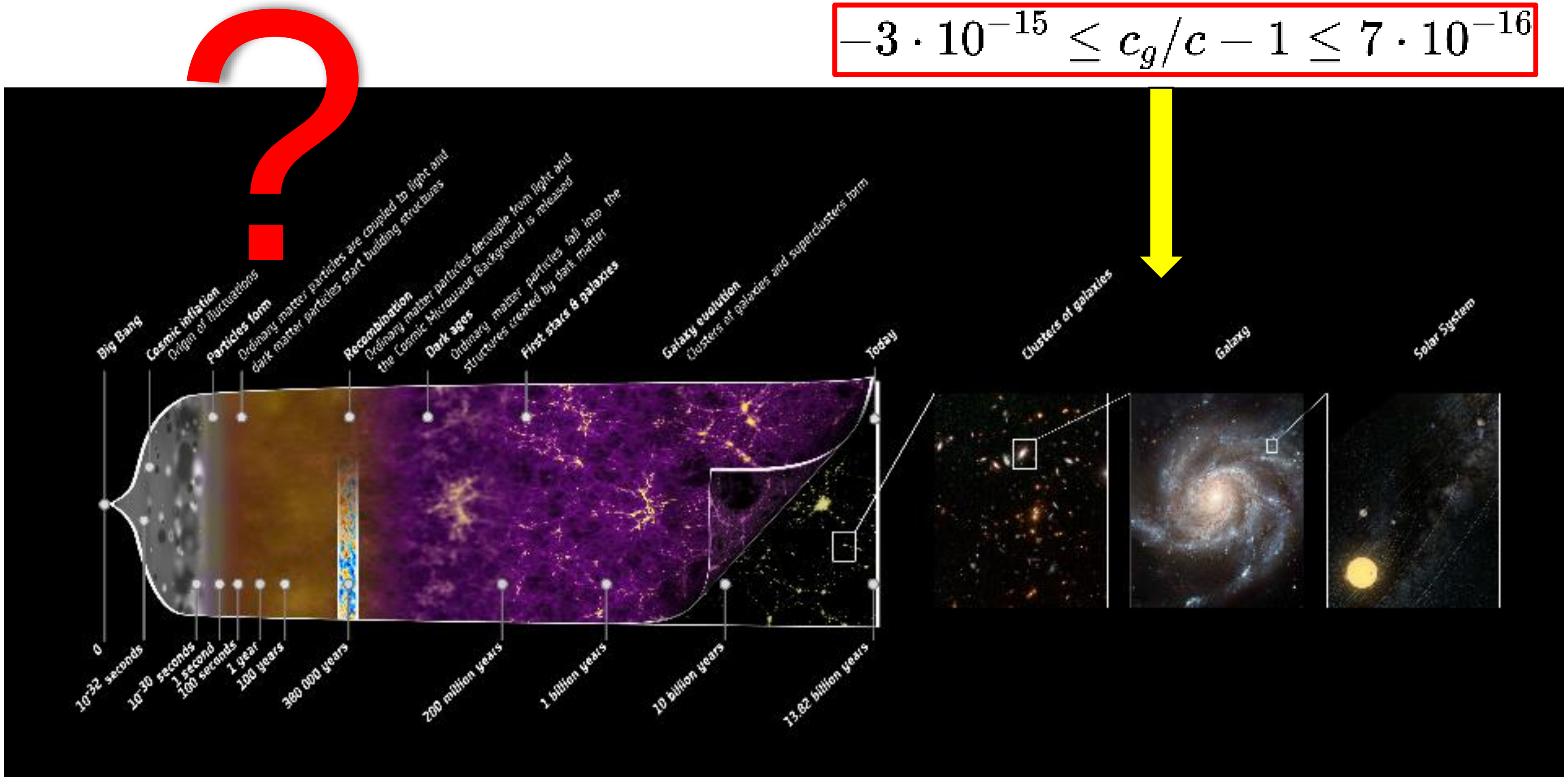
New Physics at Early Universe?

LHC energy scale $\sim 13\mathrm{TeV}$ No signals!

Propagation speed of GWs:

$$z < 1$$

$$\boxed{-3 \cdot 10^{-15} \leq c_g/c - 1 \leq 7 \cdot 10^{-16}}$$



Hidden New Physics in Gravity Theories?

A wide class of MG theories can raise non-trivial GWs speed, for instance, in scalar-tensor theories:

[Horndeski, IJTP(1974)]

$$\mathcal{L} = G_2(\phi, X) - G_3(\phi, X)\Box\phi + G_4(\phi, X)R + G_{4X}\left[(\Box\phi)^2 - \phi^{\mu\nu}\phi_{\mu\nu}\right] + G_5(\phi, X)G^{\mu\nu}\phi_{\mu\nu} - \frac{G_{5X}}{6}\left[(\Box\phi)^3 - 3\Box\phi\phi^{\mu\nu}\phi_{\mu\nu} + 2\phi_{\mu\nu}\phi^{\nu\lambda}\phi^{\mu}_{\lambda}\right]$$

$$\Downarrow$$

Perturbative expansion:

$$c_g^2 = \frac{G_4 - X\left(\ddot{\phi}G_{5X} + G_{5\phi}\right)}{G_4 - 2XG_{4X} - X\left(H\dot{\phi}G_{5X} - G_{5\phi}\right)} \simeq 1 - \frac{\phi^2}{M^2} + \ldots$$

Hidden New Physics in Gravity Theories?

Another class of MG theory can raise non-trivial GWs speed, namely, when we consider 4-D Einstein-Gauss-Bonnet gravity:

[Glavan, Lin, PRL (2019)]

$$S[g_{\mu\nu}] = \int d^D x \sqrt{-g} \left[\frac{M_\mathrm{P}^2}{2} R - \Lambda_0 + \frac{\alpha}{D-4} \mathcal{G} \right]$$

$$\Downarrow$$

Scalar:

$$3M_\mathrm{P}^2 H^2 + 6\alpha H^4 = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$

$$-M_\mathrm{P}^2 \Gamma \dot{H} = \frac{1}{2}\dot{\phi}^2$$

Tensor:

$$\ddot{\gamma}_{ij} + 3H\left(1 - \frac{8\alpha\epsilon H^2}{3M_\mathrm{P}^2\Gamma}\right)\dot{\gamma}_{ij} - c_\mathrm{s}^2 \frac{\partial^2 \gamma_{ij}}{a^2} = 0 \qquad c_\mathrm{s}^2 \equiv 1 - \frac{8\alpha\epsilon H^2}{M_\mathrm{P}^2 \Gamma}$$

Oscillatory Sound Speed

A wide class of MG theories can lead to the non-trivial GWs speed, in particular, oscillation of scalar at reheating triggers on an oscillating sound speed of tensor modes.

PHYSICAL REVIEW LETTERS **126,** 071303 (2021)

Sound Speed Resonance of the Stochastic Gravitational-Wave Background

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We propose a novel mechanism to test time variation of the propagation speed of gravitational waves (GWs) in light of GWs astronomy. As the stochastic GWs experience the whole history of cosmic expansion, they encode potential observational evidence of such variation. We report that, one feature of a varying GWs speed is that the energy spectrum of GWs will present resonantly enhanced peaks if the GWs speed oscillates in time at high-energy scales. Such oscillatory behavior arises in a wide class of modified gravity theories. The amplitude of these peaks can be at reach by current and forthcoming GWs instruments, hence making the underlying theories falsifiable. This mechanism reveals that probing the variation of GWs speed can be a promising way to search for new physics beyond general relativity.

$$h_k''(\tau) + 2\mathcal{H}h_k'(\tau) + c_g^2 k^2 h_k(\tau) = 0$$

$$\boxed{c_g^2 = 1 - \frac{\alpha}{(1+\tau/\tau_0)^2}\cos^2(k_*\tau)}$$

In the very late universe,

$$c_g^2 = 1$$

[CYF, Lin, Wang, Yan, PRL 126 (2021) 7, 071303]

Parametric Resonance

For a general picture of GWs:

$$\boxed{h''_k(\tau) + 2\mathcal{H}h'_k(\tau) + c_g^2 k^2 h_k(\tau) = 0}$$

For sub-Hubble modes

$$k \gg a'/a$$

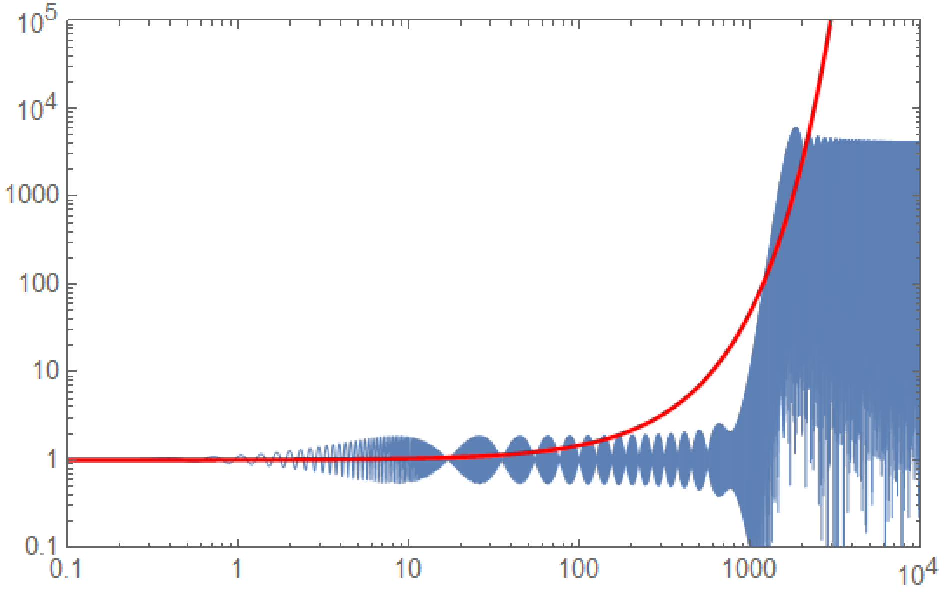
Friction term can be omitted

$$\boxed{\frac{\partial^2 h_k}{\partial x^2} + [A - 2q\cos(2x)]h_k = 0}$$

$$x = k_* \tau \quad A = \frac{k^2}{k_*^2} - 2q \quad q = \frac{\alpha k^2}{4k_*^2\left(1 + \frac{x}{x_0}\right)^2}$$

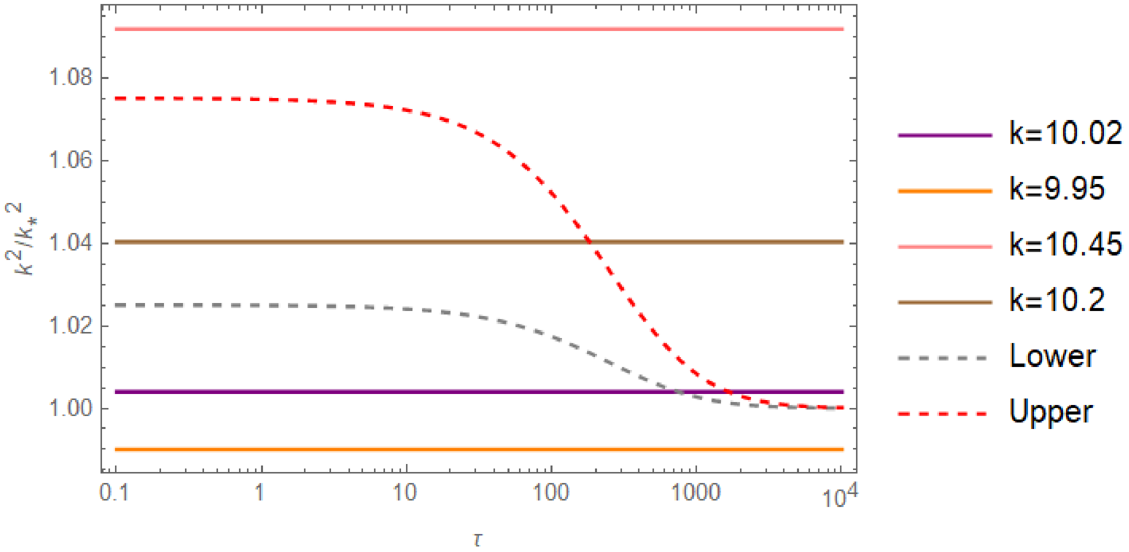
Parametric Resonance

Demo: EoM without Hubble friction term



The resonance started automatically, as well as its exit, due to the evolution of resonance band.

The evolution of resonance band

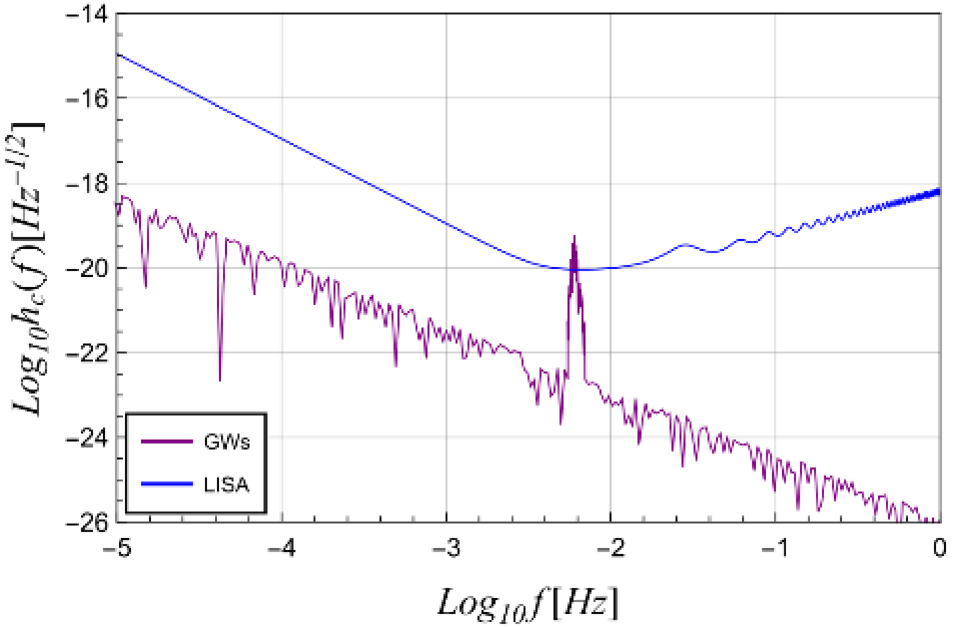


Significant Features

Energy scales:

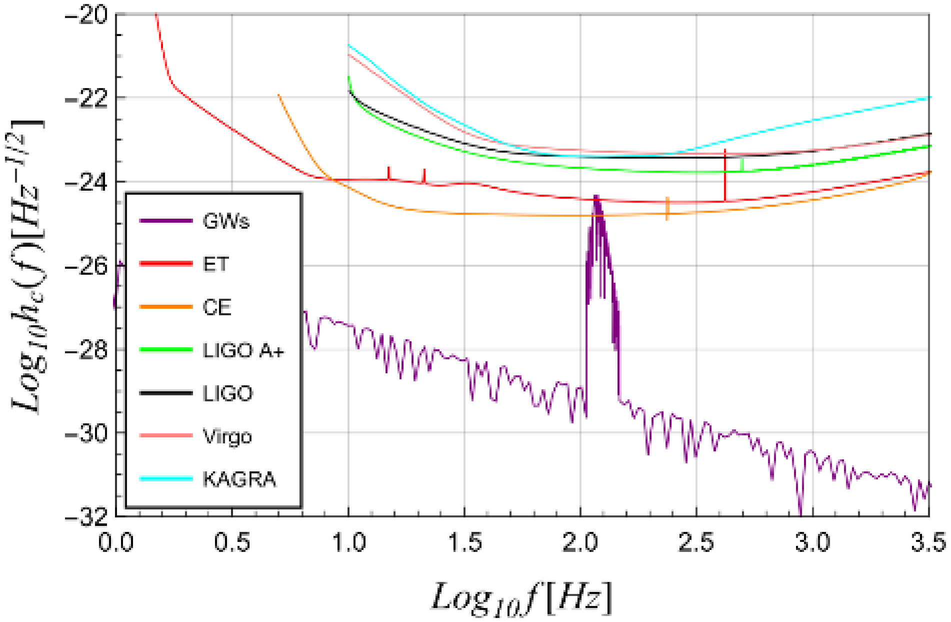
[CYF, Lin, Wang, Yan, PRL 126 (2021) 7, 071303]

$\sim \mathrm{TeV}$



Space-based detectors

$\sim 10^4\mathrm{TeV}$



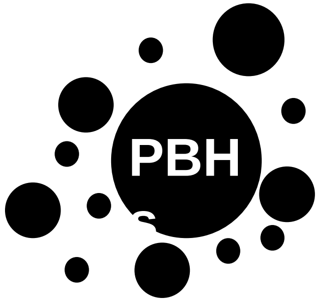
Terrestrial detectors

Bonus:

Nonlinear enhancement of density perturbations.

PBHs and induced GWs

PBH formation



$$\beta \equiv \frac{\rho_{\mathrm{PBH}}}{\rho_{\mathrm{crit}}}\bigg|_{\mathrm{at\ formation}} \sim \mathrm{erfc}\left[\frac{\delta_c}{\sqrt{2}\boxed{\sqrt{\mathcal{P}_{\mathcal{R}}}}}\right]$$

The peaks of primordial density perturbations

Cross correlation

Secondary coupling

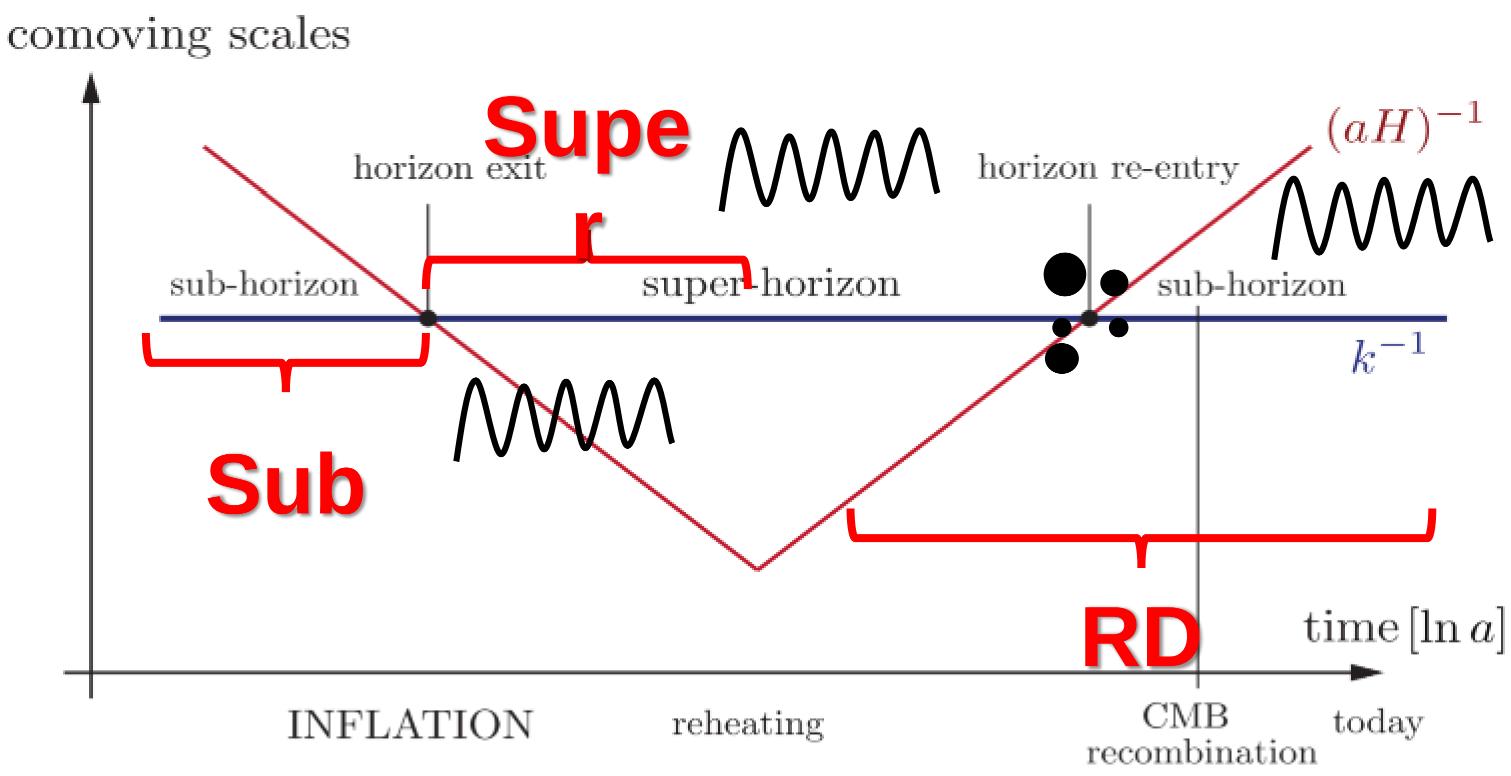
$$\Omega_{\mathrm{IGW}} \sim \langle hh \rangle \sim \langle \mathcal{R}\mathcal{R}\mathcal{R}\mathcal{R} \rangle \sim \boxed{\mathcal{P}_{\mathcal{R}}^2}$$

[Saito & Yokoyama,0812.4339]

Induced Gravitational Waves in SSR Mechanism

[CYF, et al., PRD 100 (2019) 4, 043518]

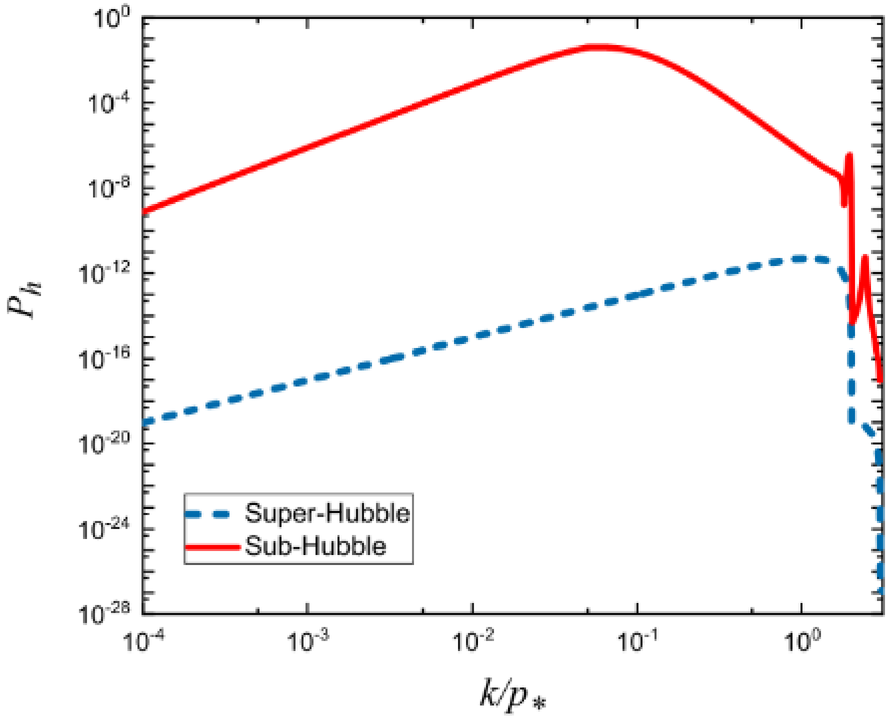
- **In SSR mechanism, we calculated the induced GWs originated from the inflationary stage.**



Induced Gravitational Waves: Inflationary stage

[CYF, et al., PRD 100 (2019) 4, 043518]

➢ IGWs from the inflationary stage



Super-Hubble regime:

$$\mathcal{P}_h^{\rm Super}(k,\tau) = \int_0^\infty \mathrm{d}y \int_{|1-y|}^{1+y} \mathrm{d}x \left[\frac{4y^2-(1+y^2-x^2)^2}{4xy}\right]^2 \mathcal{P}_\zeta(kx)\mathcal{P}_\zeta(ky) F_{\rm Inf}(k,\tau,u)$$

Sub-Hubble regime:

$$\mathcal{P}_h^{\rm Sub}(k,\tau_*) = \begin{cases} 16\xi^2\epsilon^2 A_s^2 \left(1-\frac{u^2}{4}\right)^2 \frac{1}{u^4}|\mathcal{I}(u,v)|^2\,, & u>\frac{\xi}{2}, \\ 32\xi\epsilon^2 A_s^2 \left(1-\frac{u^2}{4}\right)^2 \frac{1}{u^3}|\mathcal{I}(u,v)|^2\,, & u<\frac{\xi}{2}, \end{cases}$$

$$\begin{aligned} \mathcal{I}(u,v) = &\int_1^v ds\,[1-2\xi(1-\cos(2s))]\,e^{ius} \\ &\times \left[e^{-2iu}(1+iu)(-i-us)+e^{-2ius}(1-iu)(i-us)\right] \\ &\times \left[\frac{S(s)\,(iC(v)-C'(v))+C(s)\,(-iS(v)+S'(v))}{-S(v)C'(v)+C(v)S'(v)}\right]^2 . \end{aligned}$$

Induced Gravitational Waves: Energy Spectra

[CYF, et al., PRD 100 (2019) 4, 043518]

- **The energy spectra observed at the present**

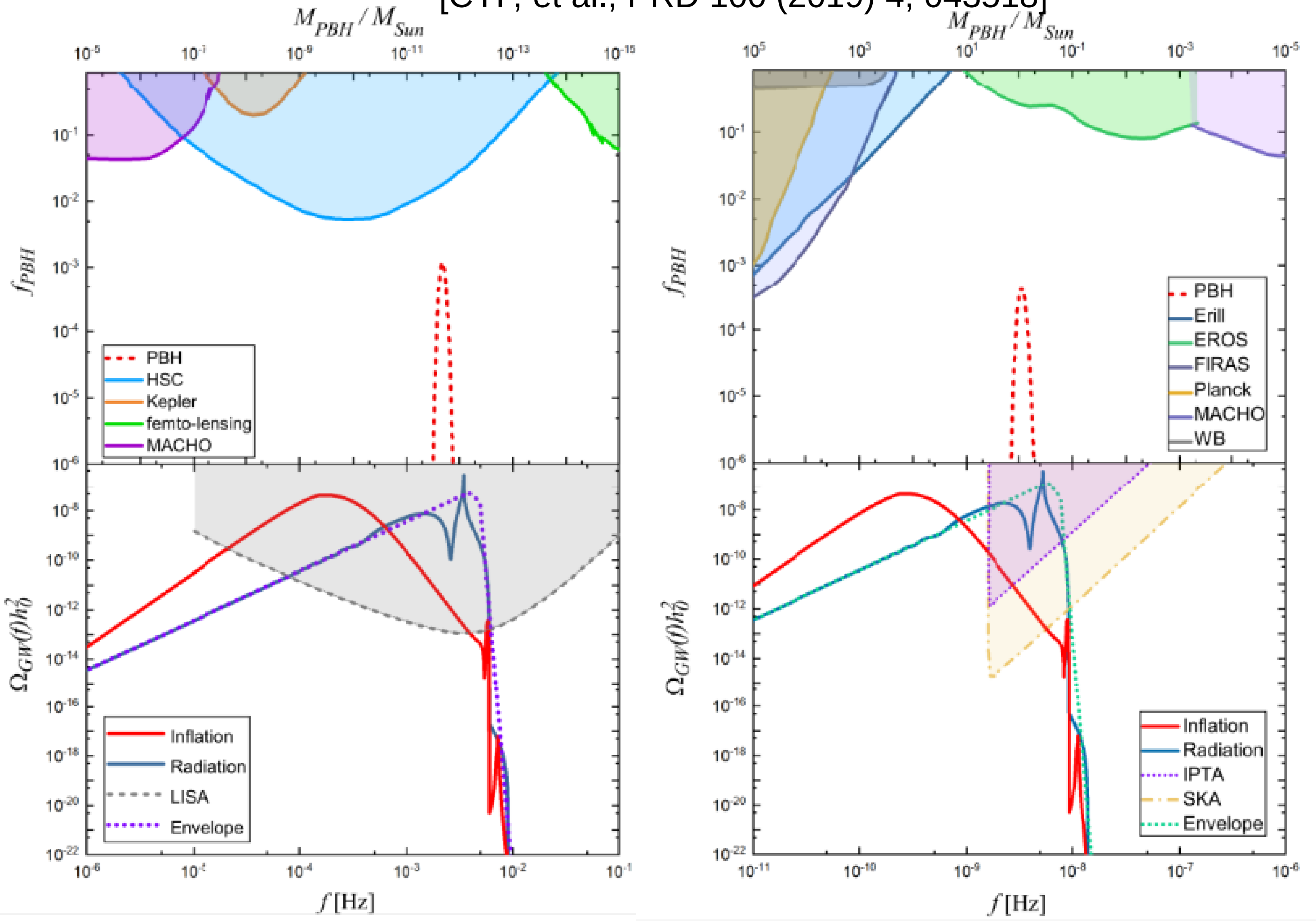
$$\Omega_{\rm GW}(\tau_0,k)\simeq\begin{cases}\Omega_{r,0}\Omega_{\rm GW}(\tau_f,k) & \text{RD}\\ 1.08\times10^{-6}\mathcal{P}_h^{\rm Inf}(k,\tau_{\rm end}) & \text{Inflation}\end{cases}$$

- **Parametrization**

$$\Omega_{\rm GW}(f)h_0^2\sim\begin{cases}A\left(\frac{f}{f_c}\right)^{\alpha}, & f<f_c,\\ A\left(\frac{f}{f_c}\right)^{\beta}, & f>f_c\ ,\end{cases}$$

$$A\simeq4.31\times10^{-7},\quad \alpha\simeq3.0,\quad \beta\simeq-5.4,\quad f_c=0.08f_*\ . \qquad \text{Inflation}$$

$$A\simeq8.41\times10^{-8}\ \text{(LISA)},\quad A\simeq2.05\times10^{-7}\ \text{(SKA\&IPTA)},\quad \alpha\simeq2.0,\quad \beta\simeq-50.4,\quad f_c=1.6f_*\ . \qquad \text{RD}$$



[Saito&Yokoyama, 0812.4339; Baumann, et al., PRD 76 (2007) 084019; Ananda, et al., PRD 75 (2007) 123518]

Summary

- Primordial black holes may be formed in the early Universe. They could account for plentiful cosmological and astrophysical phenomena, e.g., dark matter;
- Sound speed resonance mechanism could produce PBHs efficiently. Abundant underlying physics of SSR needs to explore.
- The Induced GWs associated with PBH formation is becoming a promising approach to detect the physics of PBHs and the early Universe.

Summary

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- Sound speed resonance mechanism could produce PBHs efficiently. Abundant underlying physics of SSR needs to explore.

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Thanks