

Exploring the opportunities of the 3HDMs

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Non-minimal Higgs sectors

Non-minimal Higgs sectors: a conservative approach to New Physics.

SM

u up	c charm	t top	γ photon
d down	s strange	b bottom	Z Z boson
ν_e electron neutrino	ν_μ muon neutrino	ν_τ tau neutrino	W W boson
e electron	μ muon	τ tau	g gluon

+



Multi-Higgs models

u up	c charm	t top	γ photon
d down	s strange	b bottom	Z Z boson
ν_e electron neutrino	ν_μ muon neutrino	ν_τ tau neutrino	W W boson
e electron	μ muon	τ tau	g gluon

+



Several Higgs generations

Higgses can come in **generations** → **N -Higgs-doublet models** (NHDMs).

- **T.D. Lee, 1973**: 2HDM as a new source of CP -violation (CPV);
- **Weinberg, 1976**: 3HDM with natural flavour conservation (NFC) and CPV;
- Intense activity in **70–80's**: trying to reconstruct hierarchical quark and lepton **masses and mixing** patterns from **symmetries** and their breaking;
- **1990–2000's**: **MSSM** requires two Higgs doublets;
- Around **2000's**: cosmological consequences of extra scalar fields: scalar **dark matter candidates** (protected by symmetries), strong first-order phase transitions leading to **baryogenesis**.
- In total, $\mathcal{O}(10^4)$ papers over 40 years; overview e.g. in [[Ivanov, 1702.03776](#)].

3HDM vs 2HDM

- more options for model-building (scalar and fermion) \Rightarrow richer pheno;
 - ▶ many symmetry options [classic papers];
 - ▶ automatic scalar alignment from large symmetry groups;
 - ▶ new options for CP violation [Branco, Gerard, Grimus, 1984];
 - ▶ exotic CP symmetry of order 4 [Ivanov, Silva, 2015];
 - ▶ combining features of 2HDM: NFC + CPV [Weinberg, 1976; Branco, 1979], scalar DM + CPV [Grzadkowski et al, 2009].
- astroparticle consequences:
 - ▶ more options for dark sectors [Cordero et al, 2017];
 - ▶ new options for baryon asymmetry [Davoudiasl, Lewis, Sullivan, 2019];
 - ▶ many minima \rightarrow multi-step phase transitions \rightarrow GW signals.

3HDMs bring numerous **model-building opportunities** which are barely explored.

But they also involve **technical challenges** which require new tools beyond straightforward numerical scans.

3HDM in the vicinity of a large symmetry group

Curse of dimensionality

If you are given all numerical values of free parameters ([single point](#) in the parameter space), calculating phenomenology consequences is straightforward.

But how to explore the [54-dimensional parameter space](#) (general 3HDM scalars sector)? Or 100+ dimensional parameter space if Yukawas are added?

No chance to cover the full 3HDM parameter space with a random scan!

If one wants to go [beyond isolated examples](#), one must learn how to navigate in the entire parameter space. This task [precedes](#) collider predictions!

The goal

learn where to look for benchmark models with [desired phenomenological features](#) — and then explore them in detail.

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Symmetries in 3HDM

The 3HDM potential with ϕ_a , $a = 1, 2, 3$,

$$V = Y_{ab}(\phi_a^\dagger \phi_b) + Z_{ab,cd}(\phi_a^\dagger \phi_b)(\phi_c^\dagger \phi_d)$$

can be invariant under global symmetries $\phi_a \rightarrow U_{ab}\phi_b$ forming a group G .

- General 3HDM \rightarrow 54 free parameters in the scalar sector alone.
- Impose group $G \rightarrow$ reduce free parameters in scalar and Yukawa sectors.
- Pick up a minimum \rightarrow deduce scalar and fermion properties, explore pheno consequences.

Symmetries in 3HDM

Which **symmetry groups** G are possible within 3HDM?

- **abelian** groups: [Ferreira, Silva, 1012.2874; Ivanov, Keus, Vdovin, 1112.1660]

$$\mathbb{Z}_2, \quad \mathbb{Z}_3, \quad \mathbb{Z}_4, \quad \mathbb{Z}_2 \times \mathbb{Z}_2, \quad U(1), \quad U(1) \times \mathbb{Z}_2, \quad U(1) \times U(1).$$

- discrete **non-abelian** groups: [Ivanov, Vdovin, 1210.6553]:

$$S_3, \quad D_4, \quad A_4, \quad S_4, \quad \Delta(54), \quad \Sigma(36).$$

- The classification is **exhaustive**: imposing any other discrete group in the 3HDM scalar sector will produce an accidental continuous symmetry.
- symmetry breaking patterns $G \rightarrow G_V$: [Ivanov, Nishi, 1410.6139]
- interplay between G and CP [many classical works].
- accidental symmetries of the potential: [Darvishi, Pilaftsis, 1912.00887].

The symmetry dilemma of the 3HDM

Some history:

- The original idea from 1970's: pick up a **large G** , extend it to the fermion sector, observe $G \rightarrow G_\nu$ at the minimum \rightarrow **derive masses/mixing/CPV**.
- Many combinations of **$G + \text{irreps} + \text{vevs}$** were tested \rightarrow severe problems in the quark sector; A_4, S_4 illustrations in [Gonzales Felipe et al, 2013].
- The fundamental obstacle [Leurer, Nir, Seiberg, 1993]:
If the (active) Higgs sector is equipped with G , **vevs must break G completely** in order to produce physical m_q 's and CKM.
- For large G , this is **algebraically impossible** [Gonzales Felipe et al, 2014]
- For small G , **too many free parameters** \rightarrow poor predictive power.

3HDMs with **approximate symmetries** seem to be perfectly viable candidates.

But we need to learn how to **work efficiently in the entire parameter space**.

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An illustration: 3HDM with softly broken $\Sigma(36)$

$\Sigma(36)$ is the largest discrete group for the 3HDM scalar sector:

$$V = -m^2(\phi_a^\dagger \phi_a) + V_4,$$

where V_4 contains just $\lambda_1, \lambda_2, \lambda_3$ [Ivanov, Vdovin, 2013]. The model leads to

- rigid vev alignments: $(1, 0, 0), (1, 1, 1)$, etc.;
- pairwise mass degenerate Higgses with the extra relation $m_H^2 = 3m_h^2$;
- exact scalar alignment;
- no CP violation in the scalar sector;
- scalar DM candidates.

What if we add soft breaking terms to explore the vicinity of the exact symmetry?

An illustration: 3HDM with softly broken $\Sigma(36)$

Soft breaking terms $m_{ab}^2(\phi_a^\dagger\phi_b)$ can violate all these features.

But are these 9 free parameters m_{ab}^2 on equal footing?

5 free parameters preserve the vev alignment:

- can be found explicitly for any choice of the minimum;
- induce mass splitting of degenerate Higgses;
- control the choice of local vs. global minimum;
- lead to loop-induced decays of previously stable scalars;
- do not spoil the scalar alignment.

More details in [Varzielas, Ivanov, Levy, 2107.08227].

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CP4 3HDM

The freedom of defining CP

General CP transformation:

$$J: \phi_i \xrightarrow{CP} X_{ij} \phi_j^*, \quad X \in U(N),$$

[Grimus, Rebelo, 1997; Branco, Lavoura, Silva, 1999].

Applying J twice leads to family transformation $J^2 = XX^*$ which may be non-trivial. It may happen that only $J^k = \mathbb{I}$ ($k = \text{power of } 2$).

CP-symmetry does not have to be of order 2

The usual CP = CP2, the first non-trivial is CP4, then CP8, CP16, etc.

CP4 3HDM [Ivanov, Silva, 2015], which is physically distinct from the usual CP [Haber, OGREID, Osland, Rebelo, 2018].

Consider 3HDM with the following potential $V = V_0 + V_1$ (notation: $i \equiv \phi_i$):

$$V_0 = -m_{11}^2(1^\dagger 1) - m_{22}^2(2^\dagger 2 + 3^\dagger 3) + \lambda_1(1^\dagger 1)^2 + \lambda_2 \left[(2^\dagger 2)^2 + (3^\dagger 3)^2 \right] \\ + \lambda_3(1^\dagger 1)(2^\dagger 2 + 3^\dagger 3) + \lambda'_3(2^\dagger 2)(3^\dagger 3) + \lambda_4 \left[(1^\dagger 2)(2^\dagger 1) + (1^\dagger 3)(3^\dagger 1) \right] + \lambda'_4(2^\dagger 3)(3^\dagger 2),$$

with all parameters real, and

$$V_1 = \frac{\lambda_6}{2} \left[(2^\dagger 1)^2 - (3^\dagger 1)^2 \right] + \lambda_8(2^\dagger 3)^2 + \lambda_9(2^\dagger 3) \left[(2^\dagger 2) - (3^\dagger 3) \right] + h.c.$$

with real λ_6 and complex $\lambda_{8,9}$. It is invariant under CP4 $\phi_i \xrightarrow{CP} X_{ij} \phi_j^*$ with

$$X = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & i \\ 0 & -i & 0 \end{pmatrix}, \quad CP4^2 = \text{diag}(1, -1, -1), \quad CP4^4 = \mathbb{I}.$$

Phenomenology of CP4 3HDM

If vevs conserve CP4 \rightarrow scalar DM candidates stabilized by exotic CP [Koepke, 2018; Ivanov, Laletin, 2018].

flavored CP4 3HDM:

- CP4 can be extended to the Yukawa sector, four realizations found [Aranda, Ivanov, Jimenez, 2017; Ferreira et al, 2017];
- CP4 must be spontaneously broken \rightarrow peculiar patterns in the flavor sector;
- Parameter space scan of [Ferreira et al, 2017] identified many points compatible with theory constraints, EWPT, fermion masses and mixing, meson oscillation parameters.
- However, the scan of [Ferreira et al, 2017] produced many points with H_i^\pm lighter than top, leading to

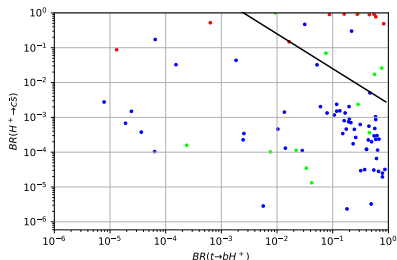
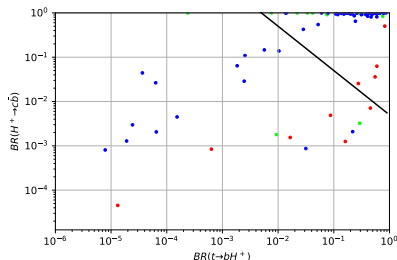
$$t \rightarrow H^+ d_i, \quad H^+ \rightarrow \bar{d}_i u_j,$$

with a variety of $H^+ d_i u_j$ coupling patterns.

Phenomenology of CP4 3HDM

In [Ivanov, Obodenko, 2021] we took all these points and checked for

- the total $\Gamma_t = 1.42^{+0.19}_{-0.15}$ GeV [PDG];
- $Br(t \rightarrow H^+ b) \times Br(H^+ \rightarrow c\bar{b}) < 0.5\%$ based on [CMS, 2018];
- $Br(t \rightarrow H^+ b) \times Br(H^+ \rightarrow c\bar{s}) < 0.25\%$ based on [CMS, 2020].



Almost all points were excluded. Exotic cases survived: $H^+ \rightarrow u\bar{b}$ as the dominant decay mode or $t \rightarrow H^+ s$ as the main production mode.

Single assumption \rightarrow numerous consequences \rightarrow requires further study.

Conclusions

- In terms of phenomenological signals, **3HDMs** can offer much more than 2HDMs. But they were mostly explored for isolated 3HDM examples.
- Navigating the huge parameter space of the general 3HDM is **challenging** and requires methods beyond straightforward numerical scans.
- The complete classification of 3HDM symmetries and efficient basis-invariant tools open up a way to **systematic exploration** of phenomenologically distinct situations of the **general** 3HDM, leading to **collider and astroparticle predictions**.
- A team in **SYSU, Zhuhai**, is being set up to investigate these issues in detail.

We are open to all sorts of collaboration.

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