Recent Progress in Precision Calculation for the LHC

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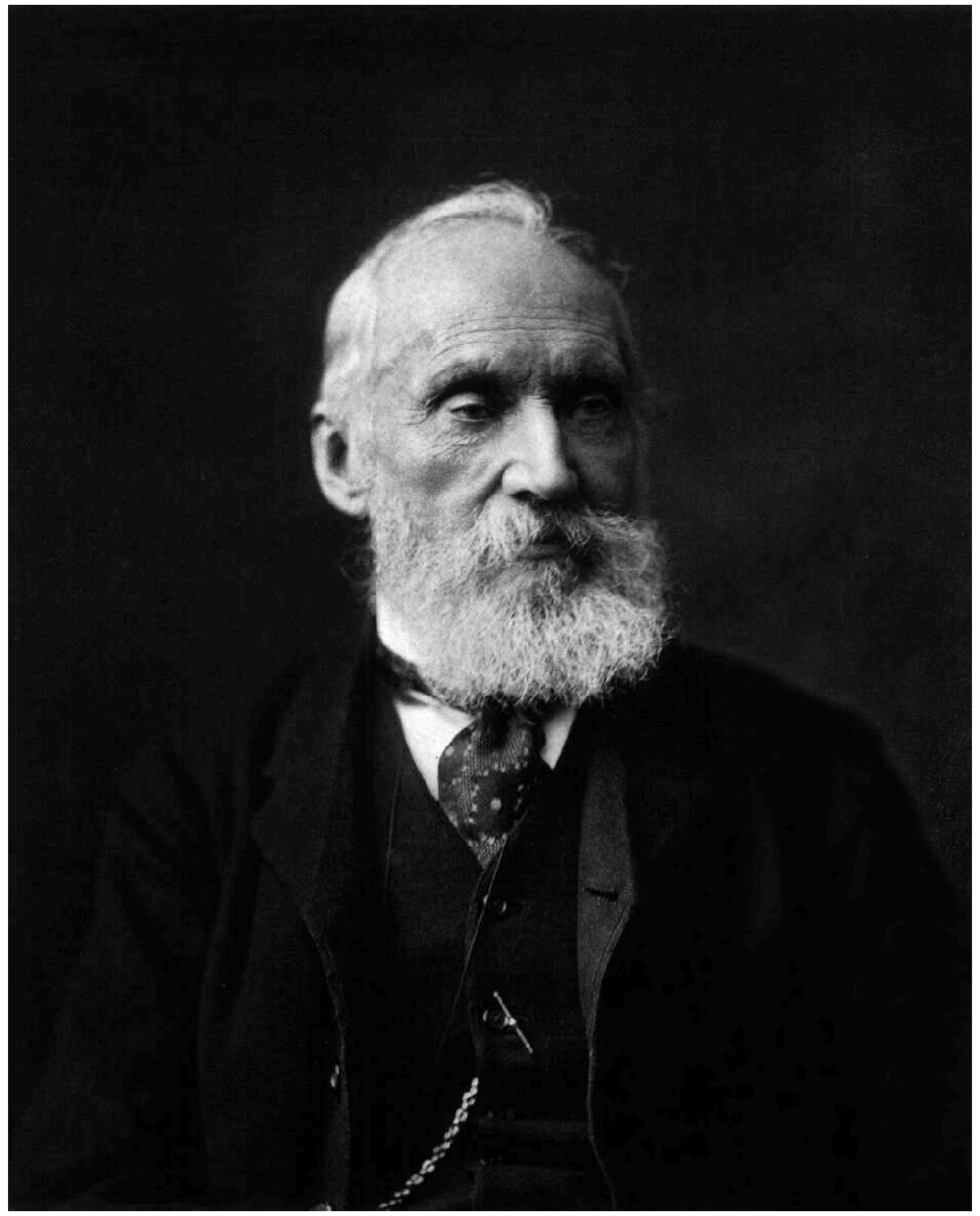


Lord Kelvin

"Now, there is nothing new to discover in physics. It only remains to be measured more and more precisely ."

This pessimistic viewpoint about fundamental physics has drastically changed after the discovery of **Quantum Field Theory**:

1. **Fundamental physical processes indeed can be calculated and measured to very high precision**

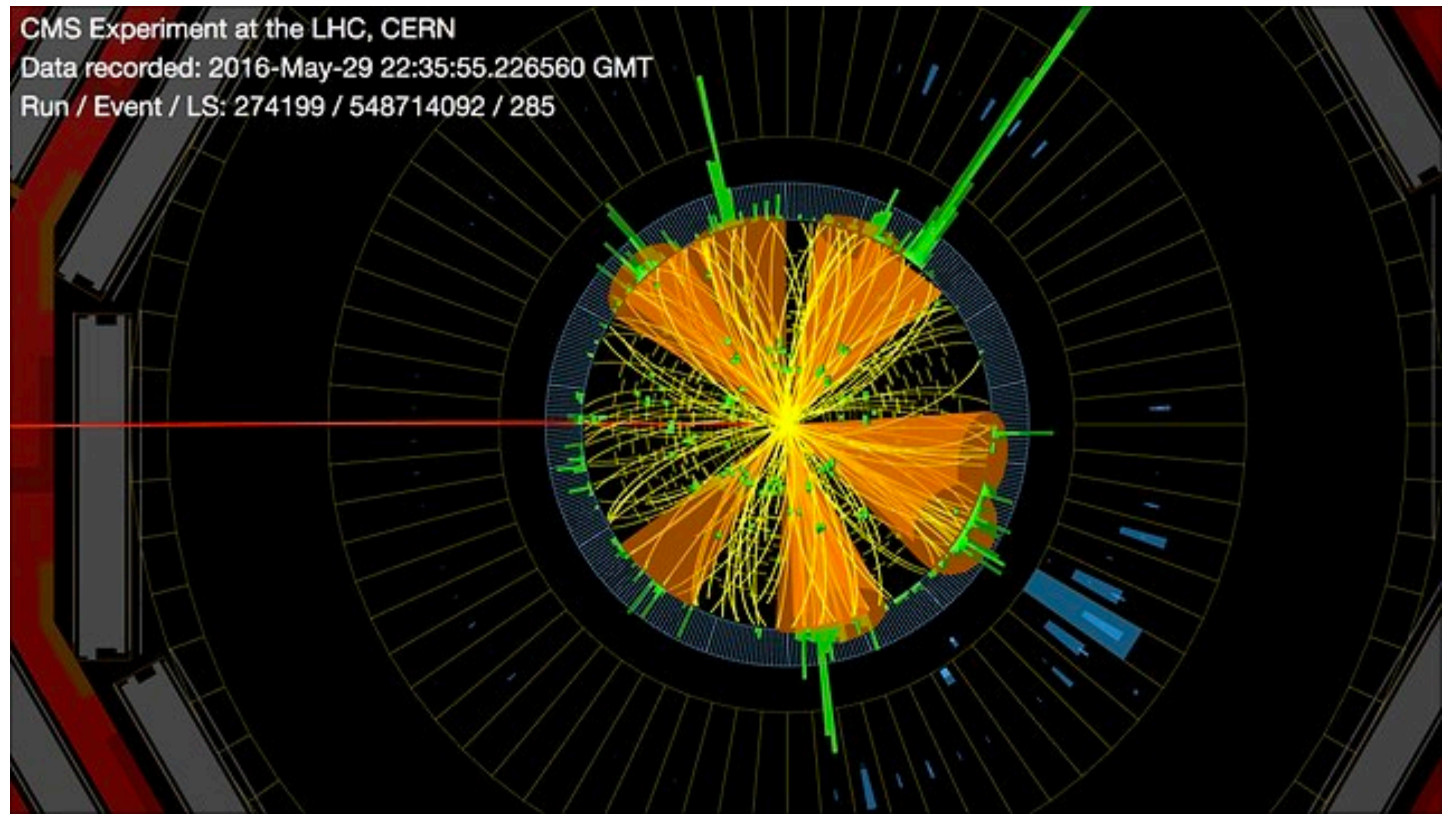
Theory	$a_e = 0.001159652181643(764)$
Experiment	$a_e = 0.00115965218073(28)$

2. **Exciting future of fundamental could be hidden in these many decimals**

Theory	$a_\mu = 0.00116591804(51)$
Experiment	$a_\mu = 0.0011659209(6)$

Test the fundamental principle of QFT

hidden symmetry behind the Lagrangian description



Application to precision measurements

Understanding Quantum Field Theory

Perturbative Quantum Field Theory

New calculation methods

New organization principle

New observables

Rapid development in China

微 扰 量 子 场 论 研 讨 会

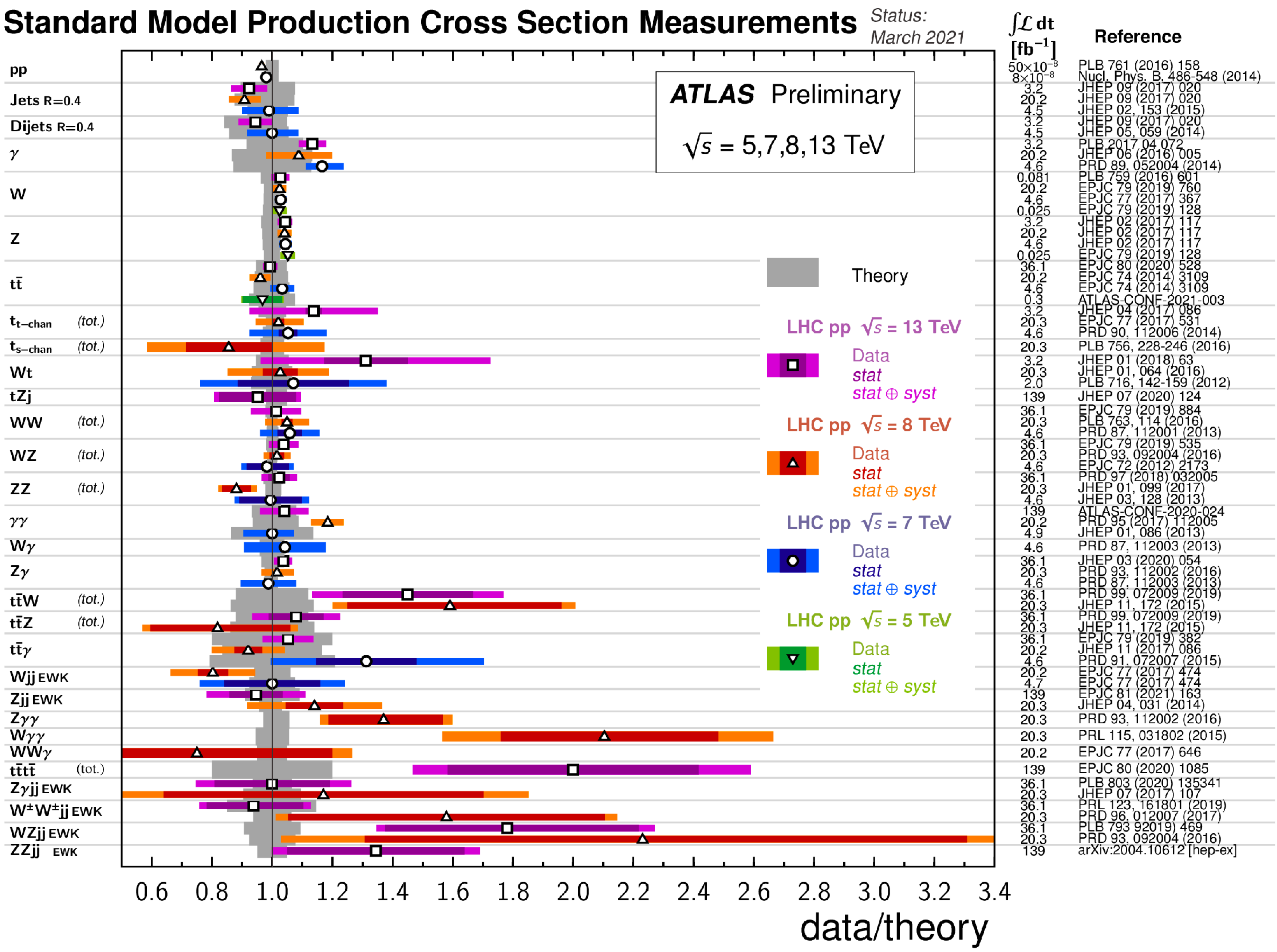
Advertisement: 16th International Symposium on Radiative Corrections (**RADCOR**) will be held at Hangzhou in 2023.

Outline for this talk

- Motivation for precision calculation from the LHC
- Fundamental ideas behind continuous revolution in the past decades
  - Unitarity
  - Analyticity
  - Effective Field Theory
- Recent progress from Higgs to top, from NNLO to N3LO
- Conclusion

Motivation from the LHC

- Remarkable successful run1 and run2 at the LHC, yet new physics signal is still elusive
- New physics signal might be buried in the SM background
- A successful precision program will be vita for a full harvest of the LHC data set, both for precision measurement of the SM parameter of searching for new physics
- Interesting theoretical challenge ahead



Experimentalist’s wishlist

Les Houches 05-09

Process ($V \in \{Z, W, \gamma\}$)	Comments
Calculations completed since Les Houches 2005	
1. $pp \to VV$jet	WWjet completed by Dittmaier/Kallweit/Uwer [4, 5]; Campbell/Ellis/Zanderighi [6]. ZZjet completed by Binoth/Gleisberg/Karg/Kauer/Sanguinetti [7]
2. $pp \to$ Higgs+2jets	NLO QCD to the gg channel completed by Campbell/Ellis/Zanderighi [8]; NLO QCD+EW to the VBF channel completed by Ciccolini/Denner/Dittmaier [9, 10]
3. $pp \to V\,V\,V$	ZZZ completed by Lazopoulos/Melnikov/Petriello [11] and WWZ by Hankele/Zeppenfeld [12] (see also Binoth/Ossola/Papadopoulos/Pittau [13])
4. $pp \to t\bar{t}\,b\bar{b}$	relevant for $t\bar{t}$ ... mputed by Bredenstein/De... ittmaier/Pozzorini [14, 15] and Bevilacq... Cza... apadopoulos/Pittau/Worek [16]
5. $pp \to V$+3jets	calculated by ... herpa [17] and R...] ... ions
Calculations remaining from Les Houches 2005	
6. $pp \to t\bar{t}$+2jets	... $t\bar{t}H$ computed by Bevi... /Czakon/Papadopoulos/Worek [19]
7. $pp \to VV\,b\bar{b}$,	... for VBF $\to H \to VV$, $t\bar{t}H$
8. $pp \to VV$+2jets	...vant for VBF $\to H \to VV$ ...BF contributions calculated by (Bozzi/)Jäger/Oleari/Zeppenfeld [20–22]
NLO calculations added ... in 200...	
9. $pp \to b\bar{b}b\bar{b}$	$q\bar{q}$ channel calculated by Golem collaboration [23]
NLO calculations added to list in 2009	
10. $pp \to V$+4 jets	top pair production, various new physics signatures
11. $pp \to Wb\bar{b}j$	top, new physics signatures
12. $pp \to t\bar{t}t\bar{t}$	various new physics signatures
Calculations beyond NLO added in 2007	
13. $gg \to W^*W^*\ \mathcal{O}(\alpha^2\alpha_s^3)$	backgrounds to Higgs
14. NNLO $pp \to t\bar{t}$	normalization of a benchmark process
15. NNLO to VBF and Z/γ+jet	Higgs couplings and SM benchmark
Calculations including electroweak effects	
16. NNLO QCD+NLO EW for W/Z	precision calculation of a SM benchmark

note we didn’t even think Higgs+3 jets possible

Closed

New Les Houches wishlist 2014

NNLO QCD+NLO EW wishlist

Higgs

Process	known	desired	details
H	$d\sigma$ @ NNLO QCD $d\sigma$ @ NLO EW finite quark mass effects @ NLO	$d\sigma$ @ NNNLO QCD + NLO EW MC@NNLO finite quark mass effects @ NNLO	H branching ratios and couplings
H + j	$d\sigma$ @ NNLO QCD (g only) $d\sigma$ @ NLO EW finite quark mass effects @ LO	$d\sigma$ @ NNLO QCD + NLO EW finite quark mass effects @ NLO	H p_T
H + 2j	σ_{tot}(VBF) @ NNLO(DIS) QCD $d\sigma$(gg) @ NLO QCD $d\sigma$(VBF) @ NLO EW	$d\sigma$ @ NNLO QCD + NLO EW	H couplings
H + V	$d\sigma$ @ NNLO QCD $d\sigma$ @ NLO EW	with $H \to b\bar{b}$ @ same accuracy	H couplings
$t\bar{t}$H	$d\sigma$(stable tops) @ NLO QCD	$d\sigma$(top decays) @ NLO QCD + NLO EW	top Yukawa coupling
HH	$d\sigma$ @ LO QCD (full m_t dependence) $d\sigma$ @ NLO QCD (infinite m_t limit)	$d\sigma$ @ NLO QCD (full m_t dependence) $d\sigma$ @ NNLO QCD (infinite m_t limit)	Higgs self coupling

Almost Closed

At least four community are forms thanks to theoretical-experimental interaction

- QCD fixed order perturbation theory
- Factorization, resummation, effective field theory
- Parton shower and Monte Carlo event generator
- Scattering amplitudes

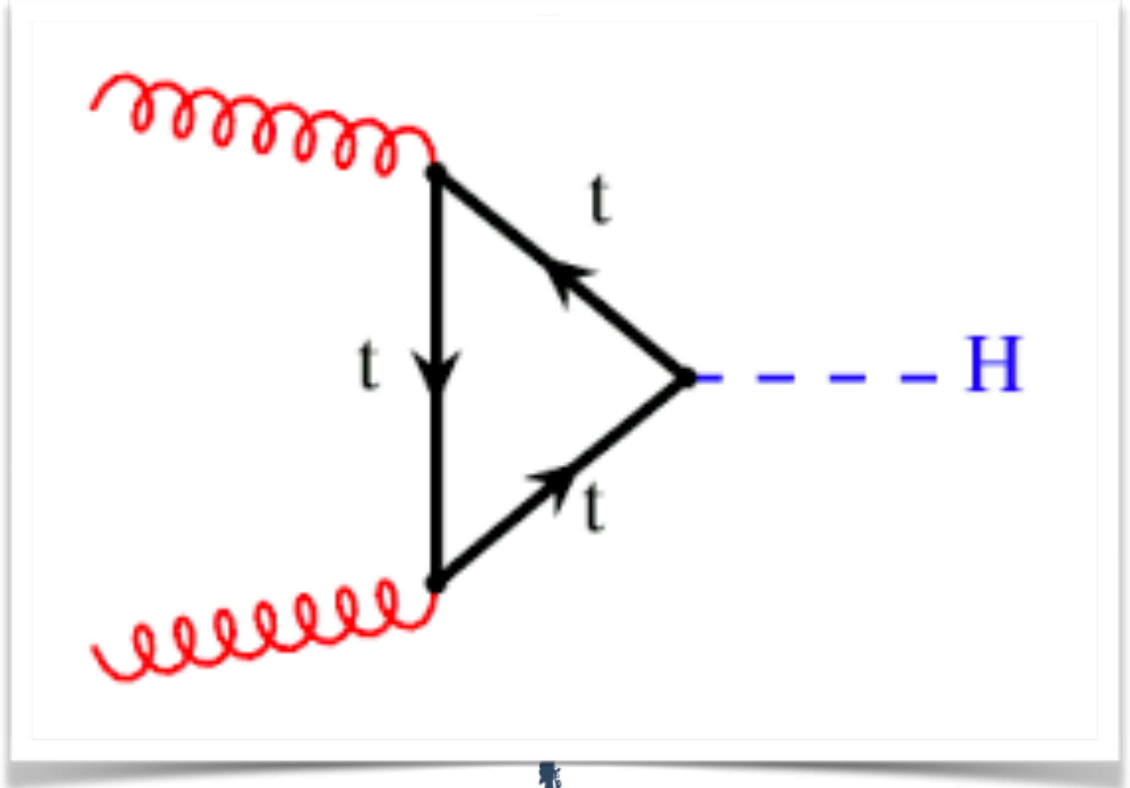
But, are QCD corrections relevant?

Mistlberger, QCD@LHC2016

$$\sigma = 48.58\,\text{pb}^{+2.22\,\text{pb}\,(+4.56\%)}_{-3.27\,\text{pb}\,(-6.72\%)}\,(\text{theory}) \pm 1.56\,\text{pb}\,(3.20\%)\,(\text{PDF}+\alpha_s)\,.$$

$48.58\,\text{pb} =$	$16.00\,\text{pb}$	$(+32.9\%)$	(LO, rEFT)
	$+\,20.84\,\text{pb}$	$(+42.9\%)$	(NLO, rEFT)
	$-\,2.05\,\text{pb}$	(-4.2%)	$((t,b,c)$, exact NLO)
	$+\,9.56\,\text{pb}$	$(+19.7\%)$	(NNLO, rEFT)
	$+\,0.34\,\text{pb}$	$(+0.7\%)$	(NNLO, $1/m_t$)
	$+\,2.40\,\text{pb}$	$(+4.9\%)$	(EW, QCD-EW)
	$+\,1.49\,\text{pb}$	$(+3.1\%)$	(N^3LO, rEFT)

ggH



Theoretical interpretation of experimental measurements would be seriously misleading without the higher order corrections!

The anatomy of perturbation calc.

The master formula:

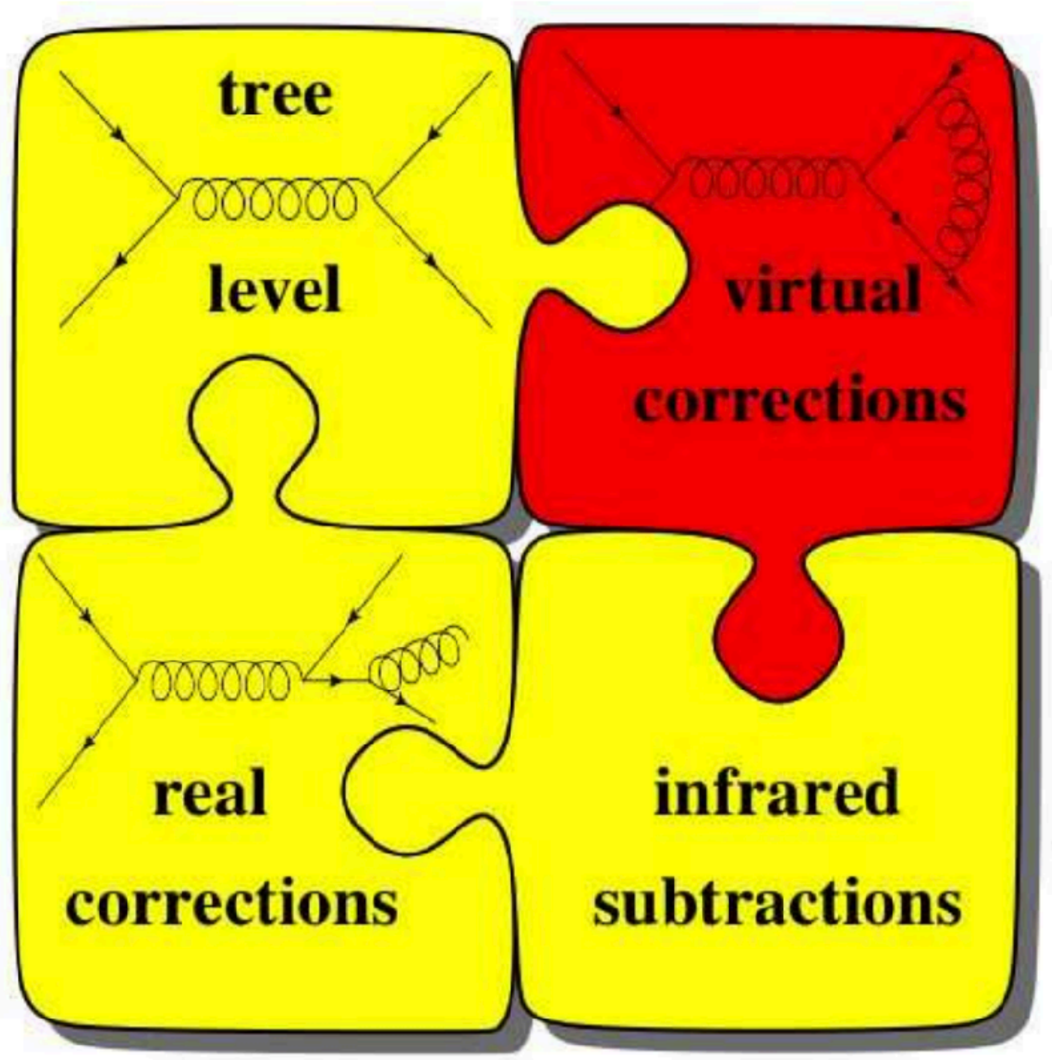
$$\sigma = \sum_{ij} \int dx_1 dx_2 f_{i/H_1}(x_1) f_{j/H_2}(x_2) \sigma_{ij\to abc\ldots} J_a J_b J_c \cdots$$

$$\sigma_{ij\to abc\ldots} = \sigma_0 + \alpha_s \sigma_1 + \alpha_s^2 \sigma_2 + \alpha_s^3 \sigma_3 + \ldots$$

LO NLO NNLO N3LO

Main challenge:

- Multiple-loop integrals
- IR singularities in degenerate phase space configuration

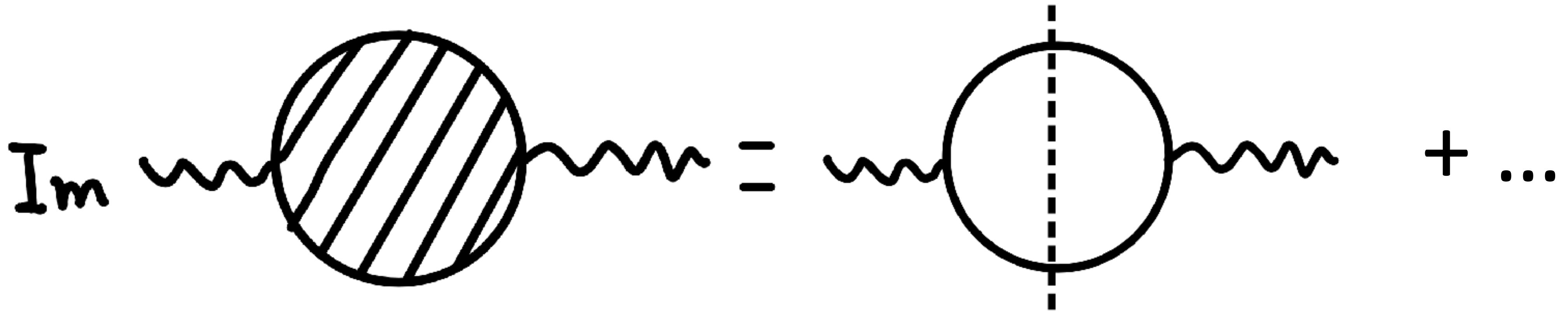


+ more loops and legs at higher order

Generalized unitarity

Unitarity of S-matrix

$$SS^\dagger = 1$$



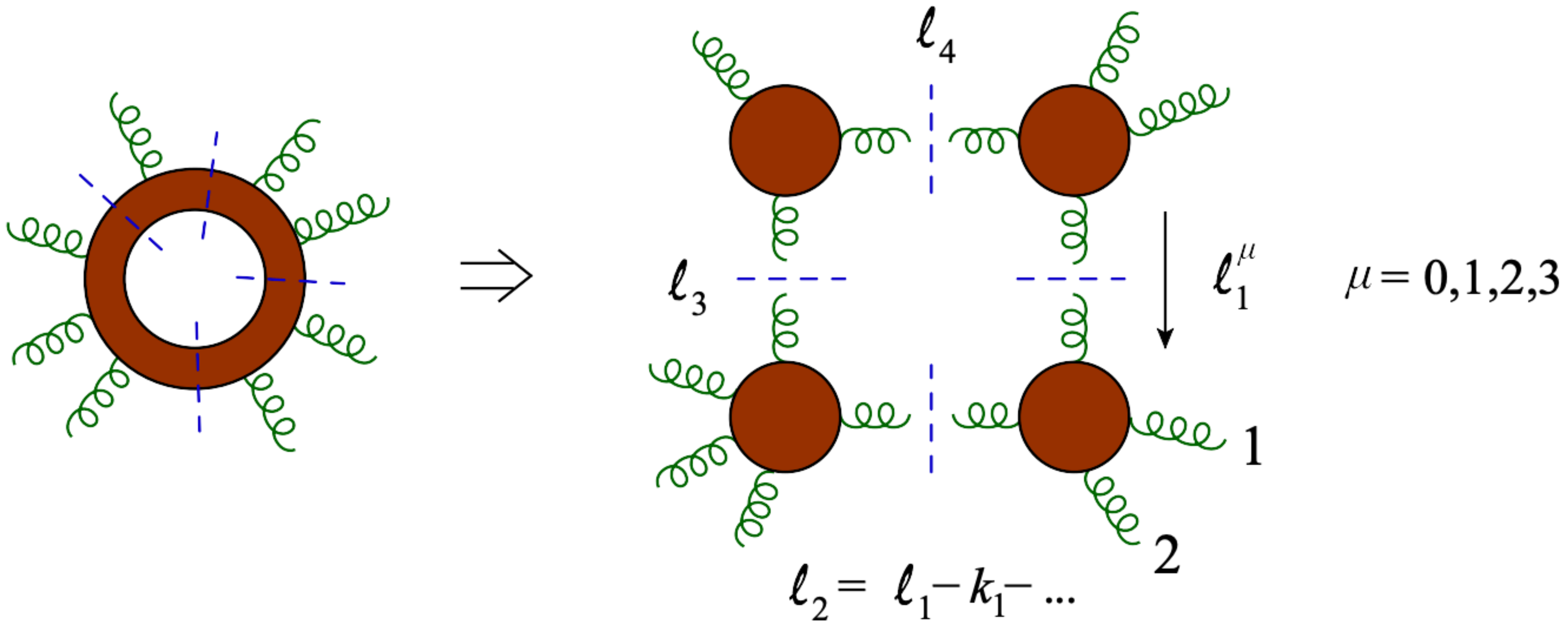
Cutkosky rule

Calculate imaginary part of amplitudes from cross section, and vice versa

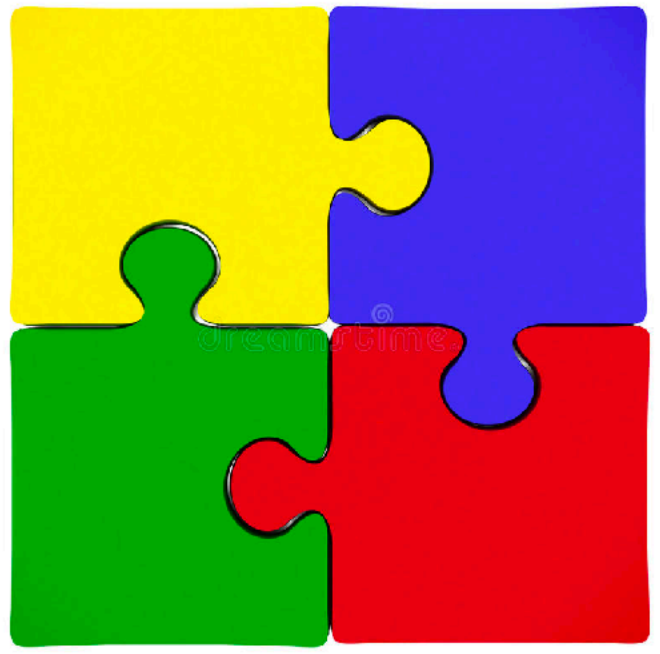
- Only apply to forward scattering
- Sum over inclusive states

Generalized unitarity of Feynman integrals

Bern, Dixon, Dunbar, Kosower, 1995;
Britto, Cachazo, B. Feng, 04',05'



$$\mathrm{FI} = \int d^4 l$$



Analyticity in regulator and momentum

Total derivative vanishes in dimensional regularized integrals

$$0 = \int d^D l \, p^\mu \frac{\partial}{\partial l^\mu} \frac{\mathcal{N}(l; p_1, p_2, \ldots)}{l^2 (l+p_1)^2 (l+p_2)^2 \ldots}$$

- Different Feynman integrals related by **Integration-By-Parts (IBP)** identities.
- Finite number of master integrals (MIs).

$$\mathrm{FI} = \sum_i c_i \mathrm{MI}_i$$

MIs are universal building blocks for Feynman integrals

Causality => Amplitudes as analytic function of external momentum

$$p_i^\mu \frac{\partial}{\partial p_j^\mu} \mathrm{FI}(\{p\}) = \int d^D l \, p_i^\mu \frac{\partial}{\partial p_j^\mu} \frac{\mathcal{N}(l; p_1, p_2, \ldots)}{l^2 (l+p_1)^2 (l+p_2)^2 \ldots}$$

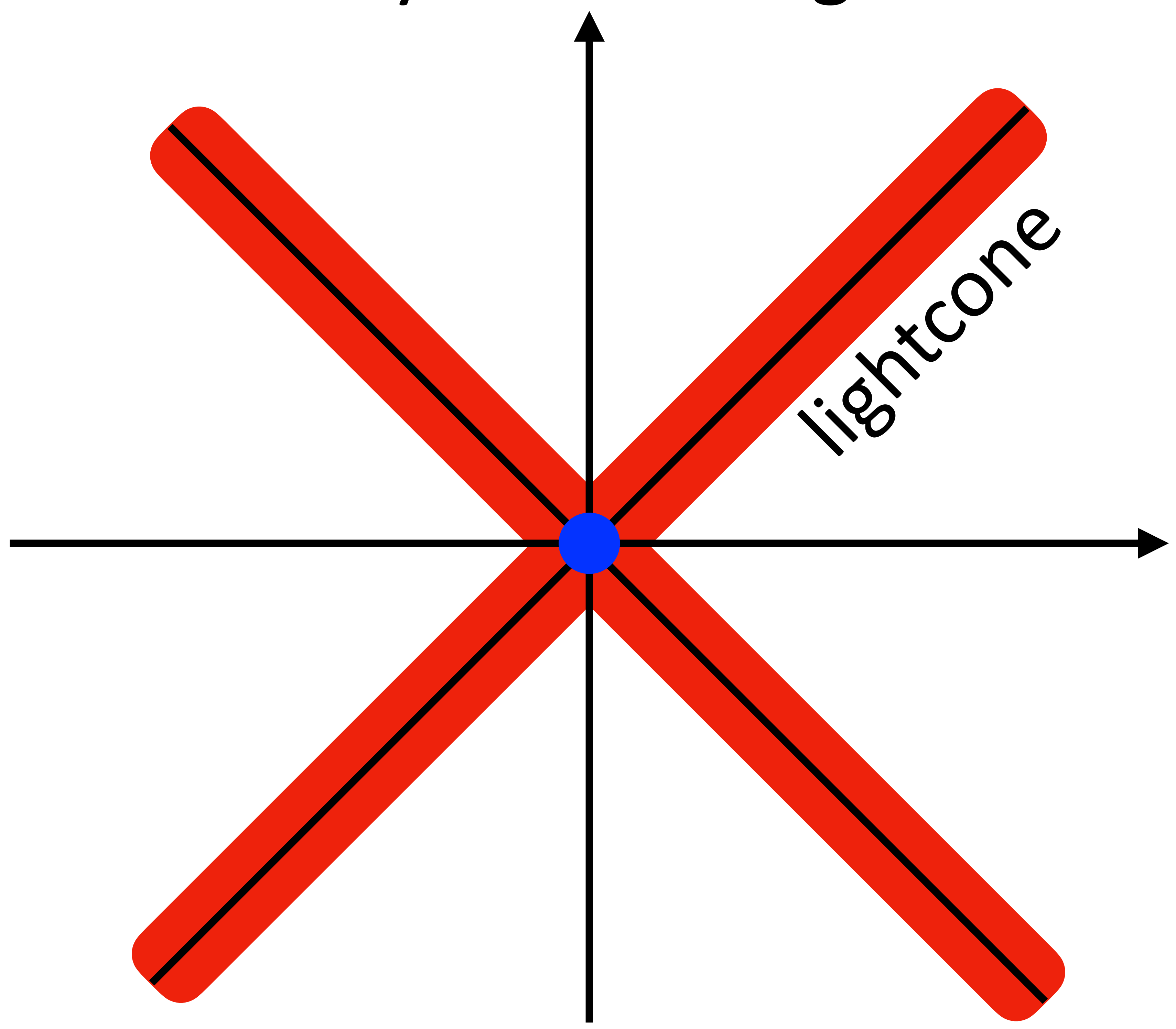
- When applied in conjugation with IBP, leads to closed set of **differential equation** for MIs.
- Often iteration solution in terms of polylogarithms.
- Canonical differenital equation:

$$\frac{\partial}{\partial x} \vec{\mathrm{MI}}(x, \epsilon) = \epsilon dA(x) \vec{\mathrm{MI}}(x, \epsilon)$$

Infrared divergences and SCET

- Scattering amplitudes are infrared divergence beyond LO.
- Origin of infrared divergence: soft and collinear loops
- KLN theorem: soft and collinear divergences cancel between virtual and real radiation corrections for inclusive enough observables
- However, the practical cancellation of IR divergence can be very intricate: open challenge at NNLO and beyond

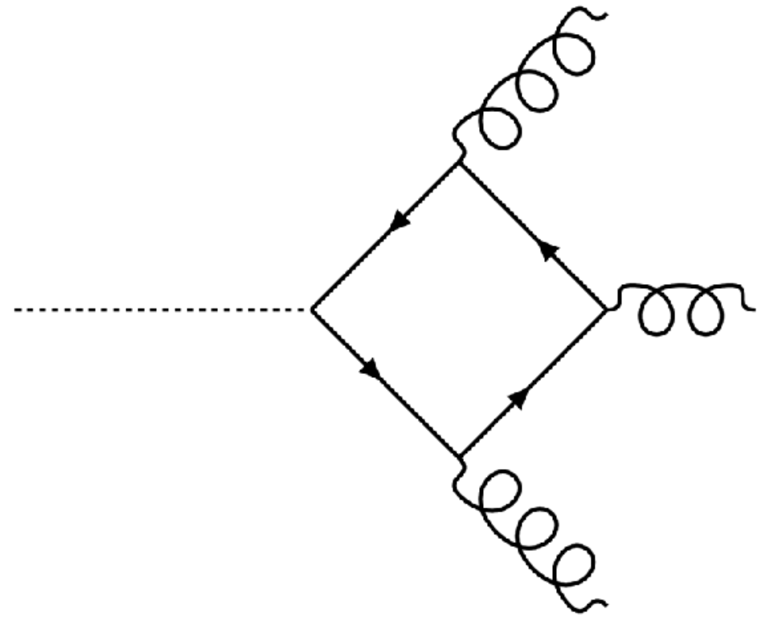
Soft-Collinear Effective Theory: an effective theory near the lightcone



An important insight: infrared divergences are cancelled within a restricted of phase space close to lightcone. Can be carried out within SCET. (J. Gao, C.S. Li, **HXZ**, PRL, 2012)

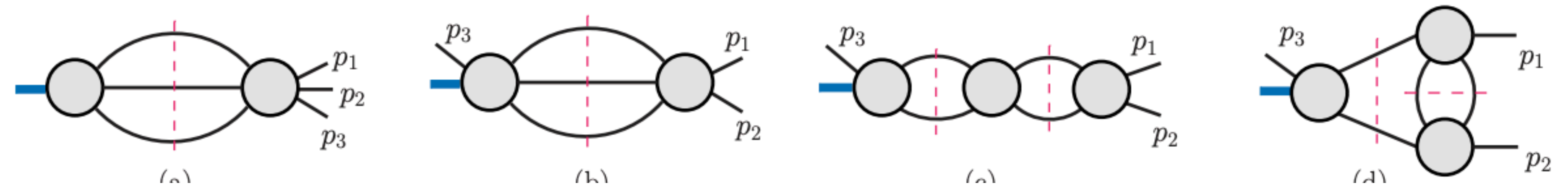
High precision Higgs physics

Q.J. Jin, K. Ren, G. Yang, JHEP, 2021



$$= _{m_t \to \infty} \; c_i \mathcal{O}_i$$

- Effective operators relevant to Higgs + jet production
- Completely operator dimension + Wilson coefficients up to dimension 16!

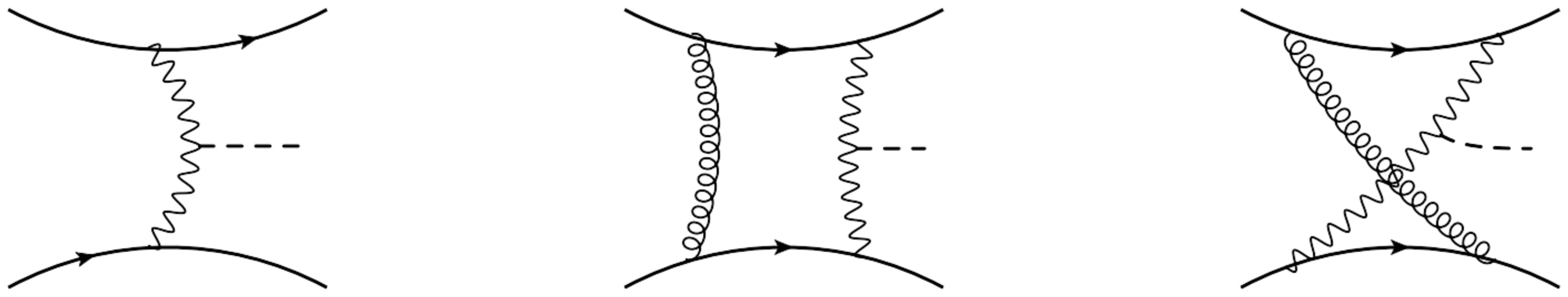


$$Z^{(2)}_{\mathcal{O}_{10,f}}\Big|_{\frac{1}{\epsilon}\text{-part.}} = \frac{N_c^2}{\epsilon}\left(\begin{array}{c|cc|cc} -\frac{34}{3} & 0 & 0 & 0 & 0 \\ \hline -\frac{1}{3} & \frac{269}{72} & 0 & \frac{5}{2} & 0 \\ -\frac{209}{900} & -\frac{5579}{18000} & \frac{712}{125} & \frac{1493}{1200} & \frac{5}{36} \\ \hline -1 & 0 & 0 & \frac{25}{12} & 0 \\ -\frac{19}{36} & \frac{139}{2400} & \frac{499}{800} & -\frac{143}{288} & \frac{2195}{288} \end{array}\right)$$

T. Liu, Melnikov, Penin, PRL, 2019

- Non-factorizable contribution to VBF Higgs production at two loops

$$q_1(p_1) + q_2(p_2) \to q_1(p_3) + q_2(p_4) + H(p_5)$$



Eikonal approximation

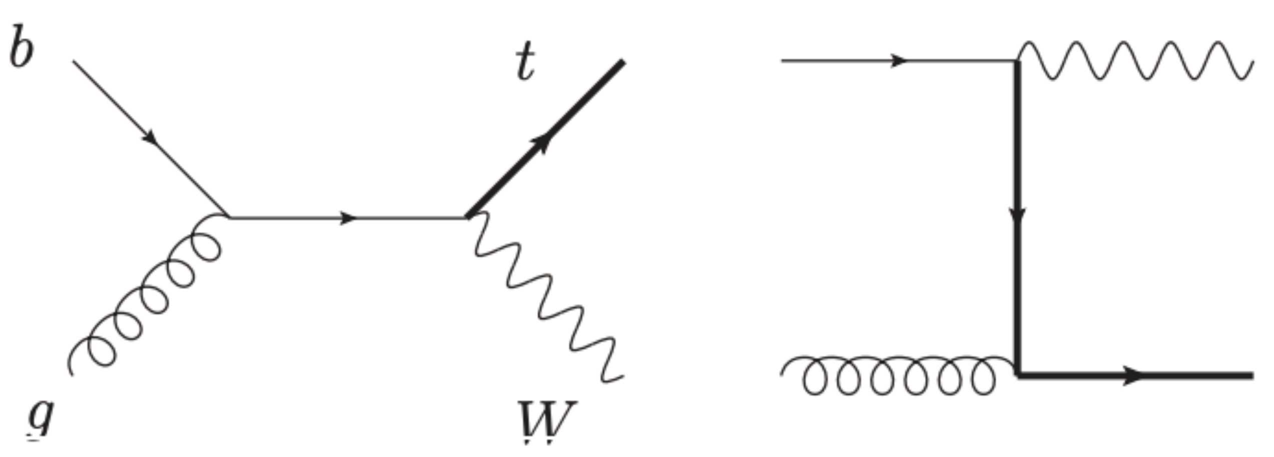
$$\chi^{(1)}(\boldsymbol{q}_3, \boldsymbol{q}_4) = \frac{1}{\pi}\int \frac{\mathrm{d}^2\boldsymbol{k}}{\boldsymbol{k}^2 + \lambda^2} \times \frac{\boldsymbol{q}_3^2 + M_V^2}{(\boldsymbol{k}-\boldsymbol{q}_3)^2 + M_V^2}\,\frac{\boldsymbol{q}_4^2 + M_V^2}{(\boldsymbol{k}+\boldsymbol{q}_4)^2 + M_V^2}$$

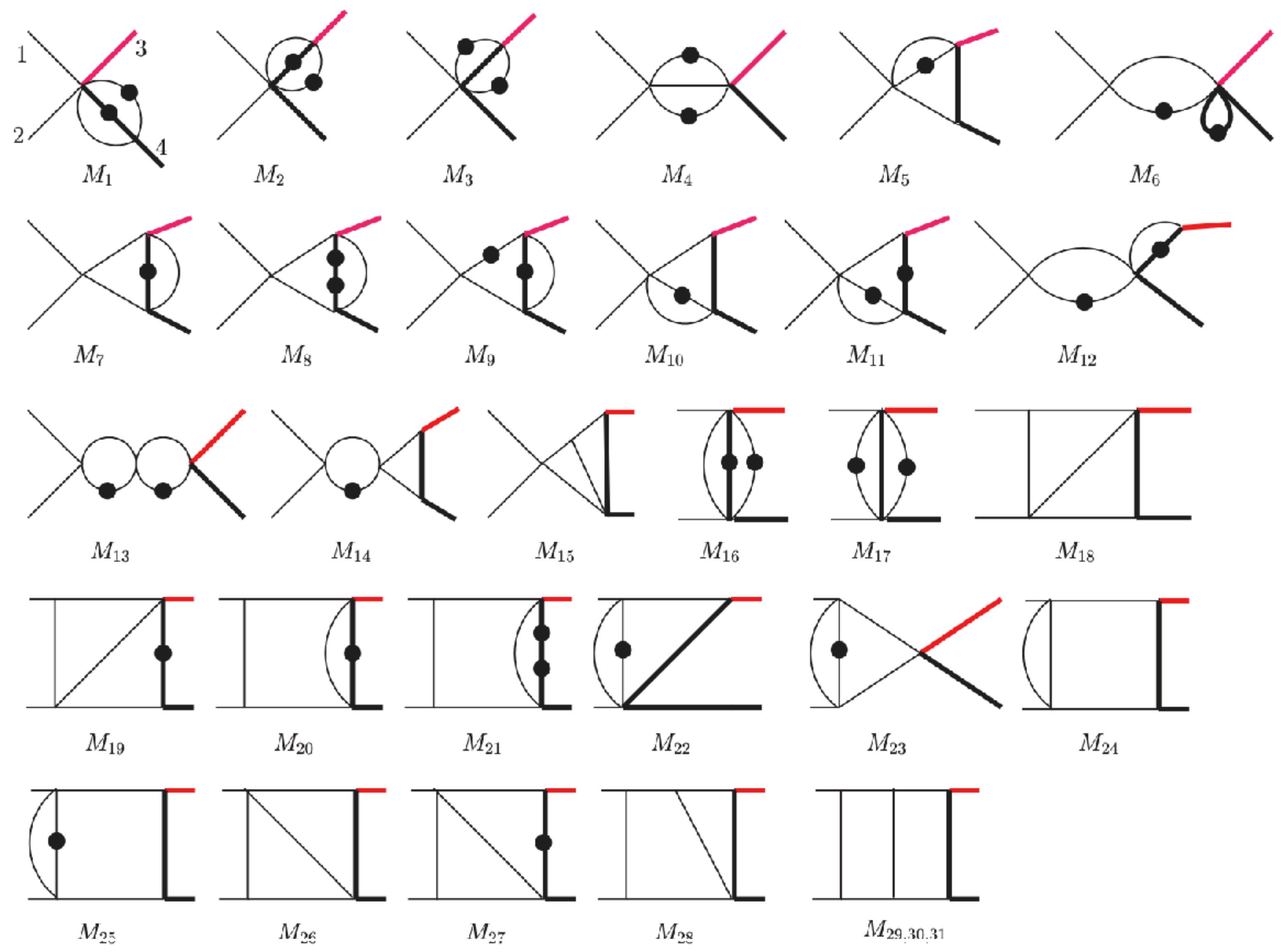
$$\sigma^{\mathrm{LO}}_{\mathrm{VBF}} = 957\ \mathrm{fb}, \quad \sigma^{\mathrm{NNLO,NF}}_{\mathrm{VBF}} = -3.73\ \mathrm{fb}$$

Calculation of master integrals

L.B. Chen, J. Wang, 2106.12093

tW associated production at two loops





31 master integrals calculated analytically in terms of multiple polylogarithms

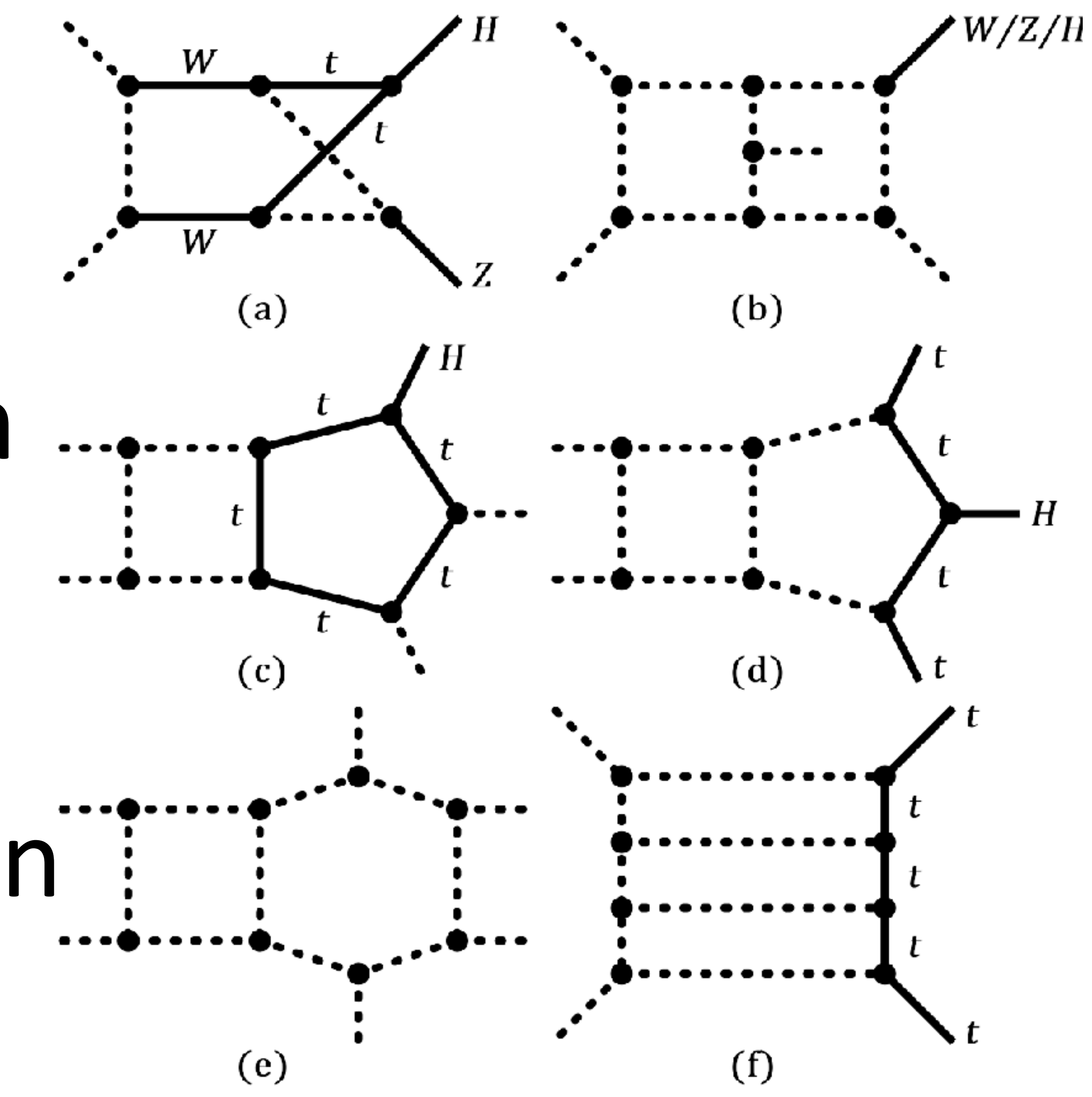
X. Liu, Y.Q. Ma, 2107.01864

Auxiliary mass flow method for numerical calculation of Feynman integrals

$$I_{\rm mod}(\vec{\nu};\eta) = \int \prod_{i=1}^{L} \frac{{\rm d}^D \ell_i}{{\rm i}\pi^{D/2}} \frac{\mathcal{D}_{K+1}^{-\nu_{K+1}} \cdots \mathcal{D}_N^{-\nu_N}}{(\mathcal{D}_1 + {\rm i}\eta)^{\nu_1} \cdots (\mathcal{D}_K + {\rm i}\eta)^{\nu_K}},$$

Analytic continuation to ∞

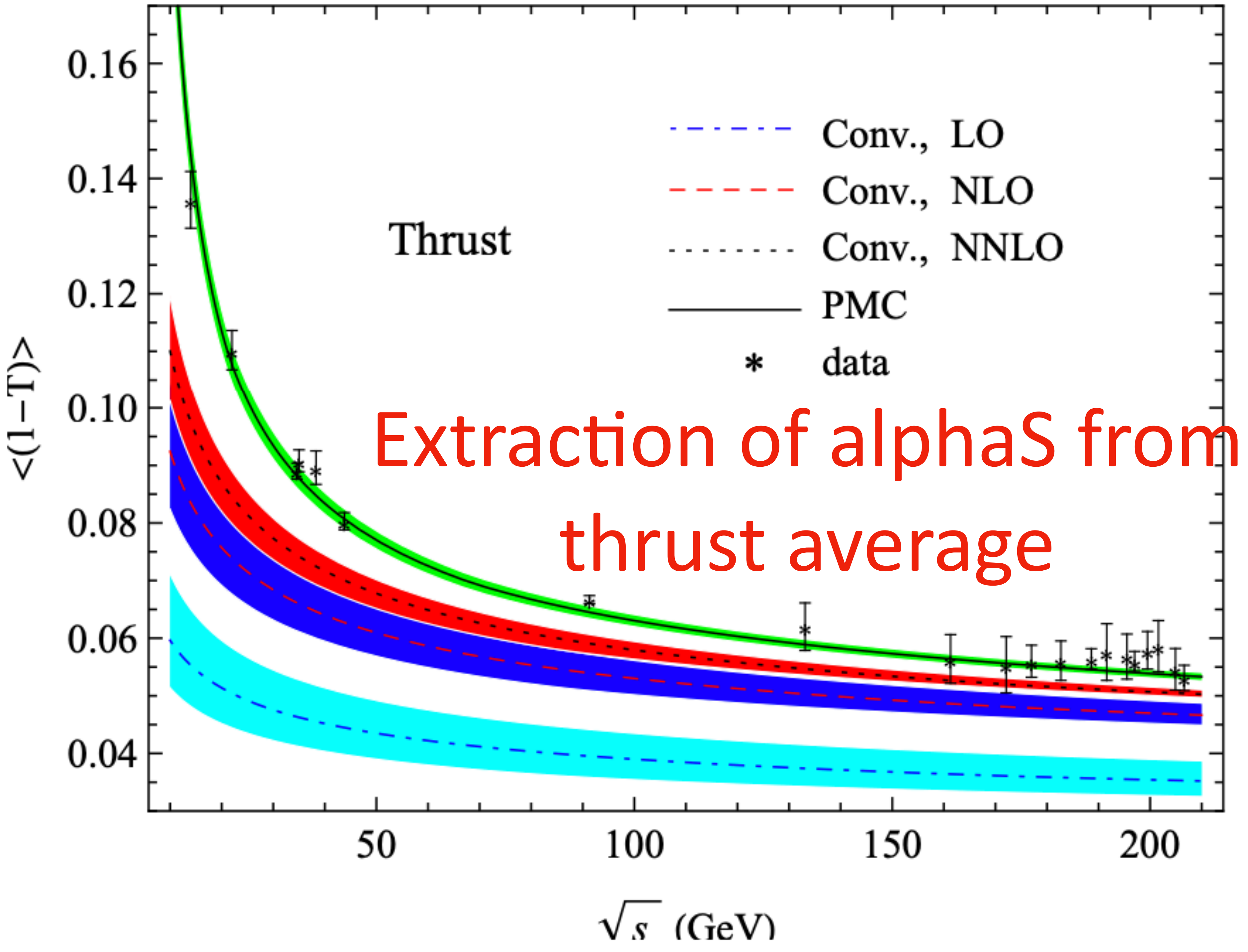
- A large number of two and three loops integrals evaluated to high precision
- All of them are not known before in the literature!
- Phenomenology application promising



alphaS/PDFs

S.Q. Wang, Brodsky, X.G. Wu, J.M. Shen, Giustino, PRD 2019

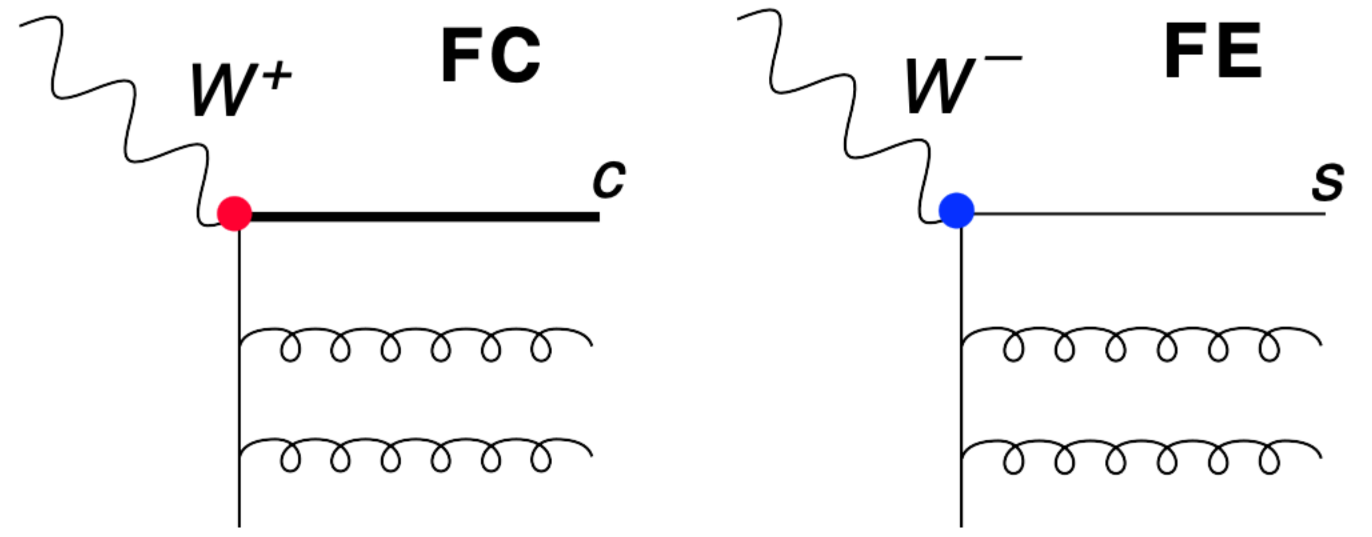
Principle of maximal conformality for scale setting now applied to event shape observable

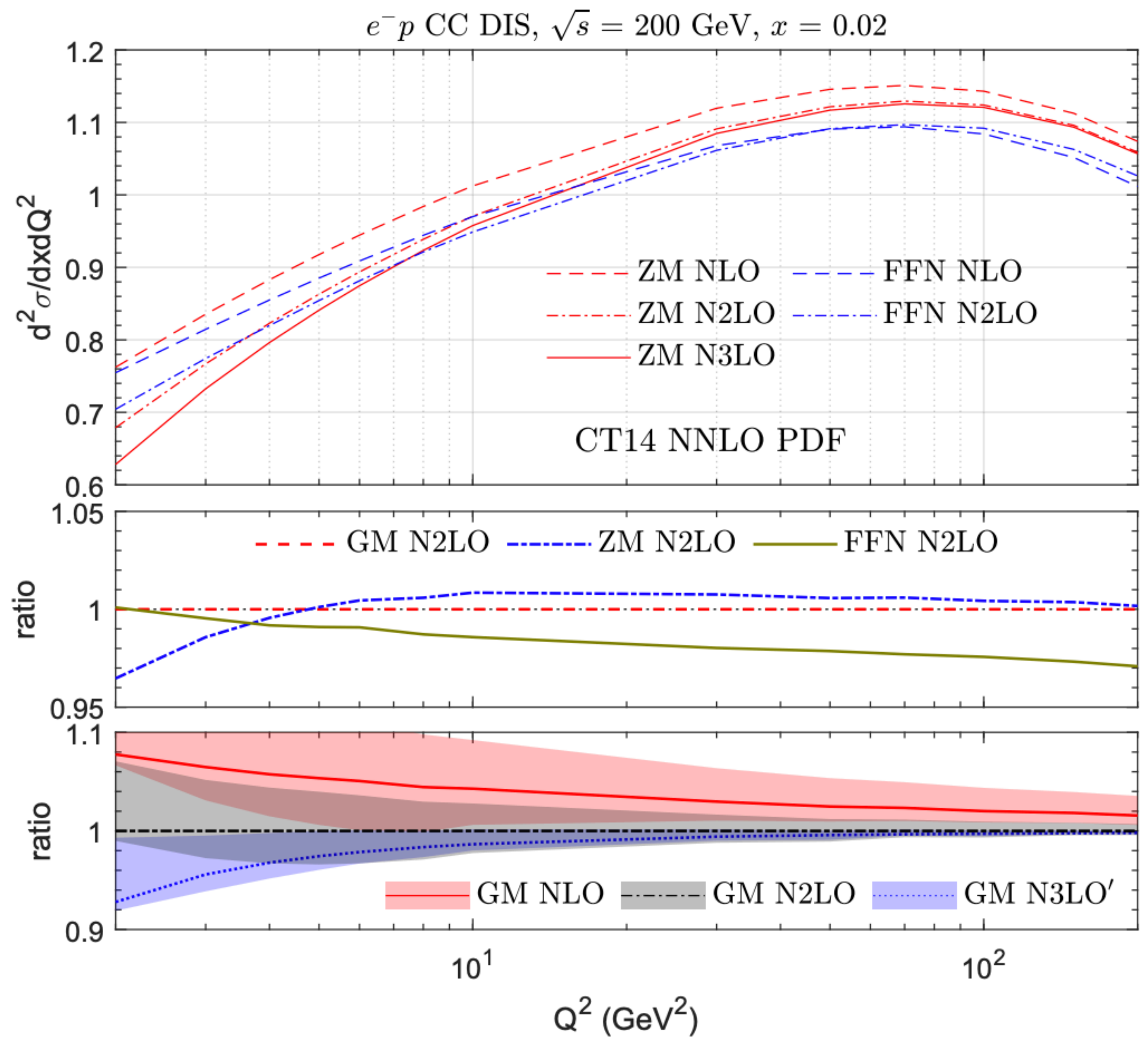


$$\alpha_s(M_Z^2) = 0.1185 \pm 0.0011(\text{Exp.}) \pm 0.0005(\text{Theo.}) = 0.1185 \pm 0.0012,$$

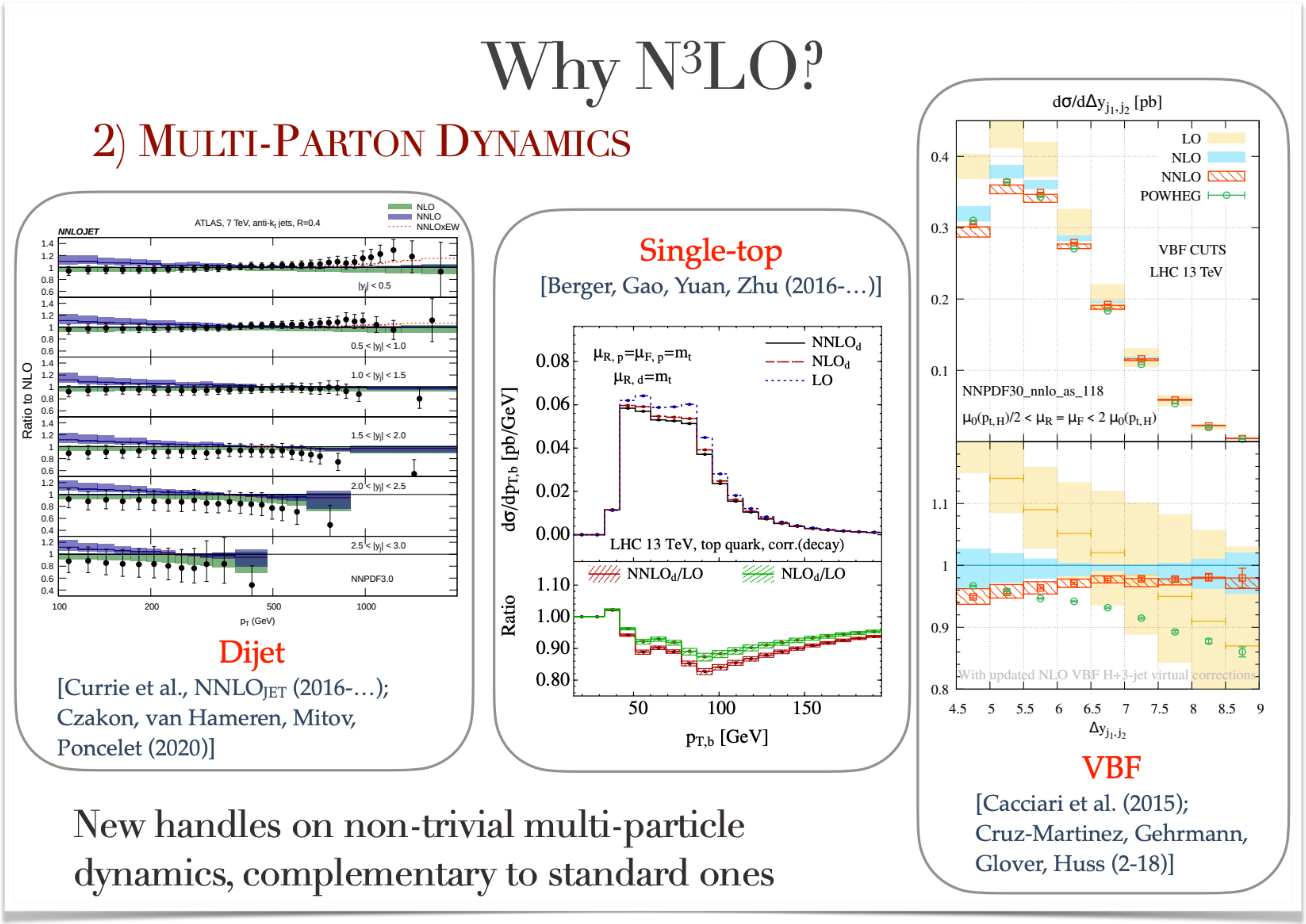
J. Gao, Hobbs, Nadolsky, C.L. Sun, C.-P. Yuan, 2107.00460

Charge-current DIS to probe strange and charm contents of nucleon





Towards N3LO phenomenology



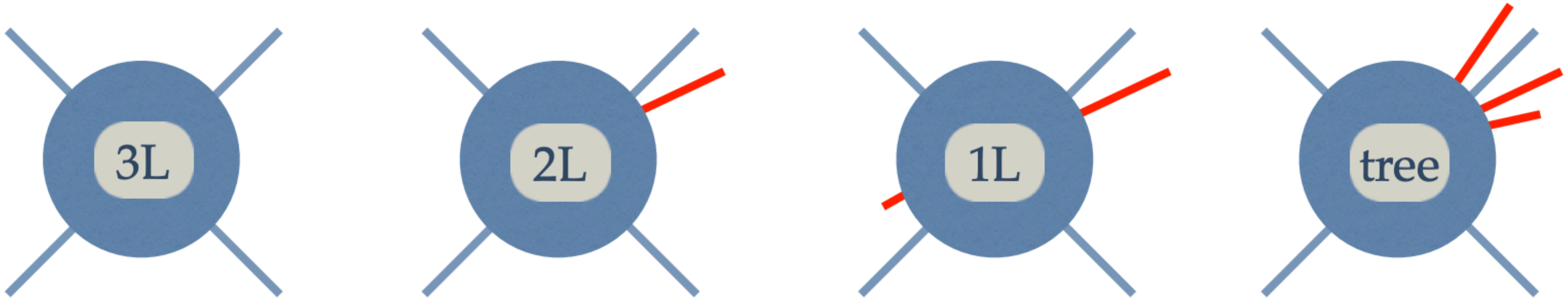
Slides from F. Caola, Snowmass community planning meeting, 2020

Besides phenomenology relevance, also interesting for

- Understanding the IR structure of gauge theory at high orders
- Begin to be sensitive to more intricate effects: power corrections in QCD, EW loops
- Factorization breaking effects are expected to happen at N3LO/N4LO

Catani, de Florian, Rodrigo, 2011
Forshaw, Seymour et al., 2011
Schwartz, Y. Kai, **HXZ**, 2018
Dixon, Hermann, Y. Kai, **HXZ**, 2019

What are needed for N3LO calculation?



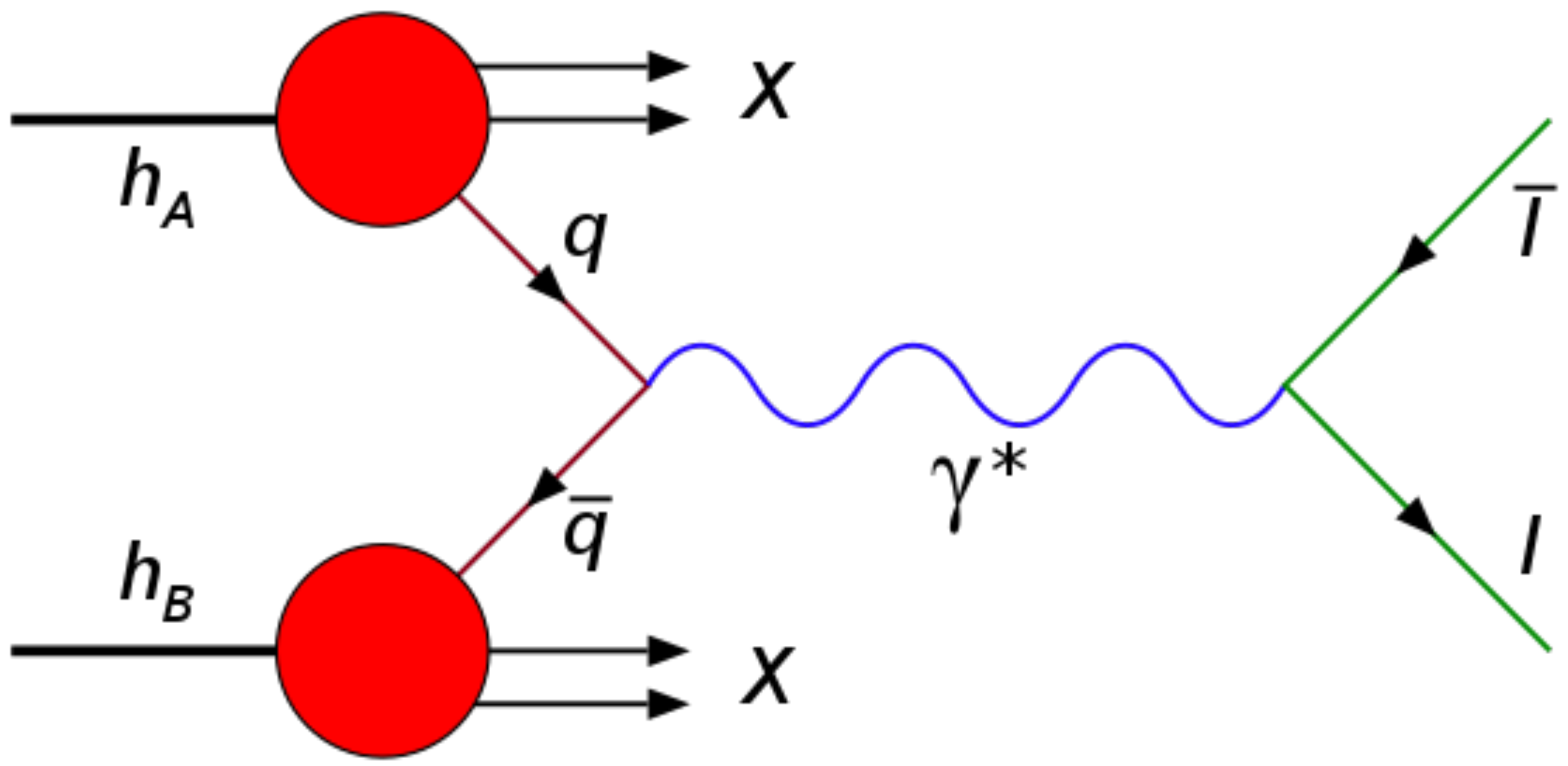
$\sim 1/\epsilon^6 \ldots$ $\quad \sim \int \mathrm{d}\phi_1 1/\epsilon^4 \ldots$ $\quad \sim \int \mathrm{d}\phi_{1,2} 1/\epsilon^2 \ldots$ $\quad \sim \int \mathrm{d}\phi_{1,2,3}\, T$

F. Caola

- Main challenge:
  - Three-loop virtual amplitudes are not known in many cases
  - Cancellation of IR singularities between virtual and real corrections mostly unexplored at N3LO
- Reasonable to focus on the simplest processes first: 2->1 Drell-Yan or Higgs production

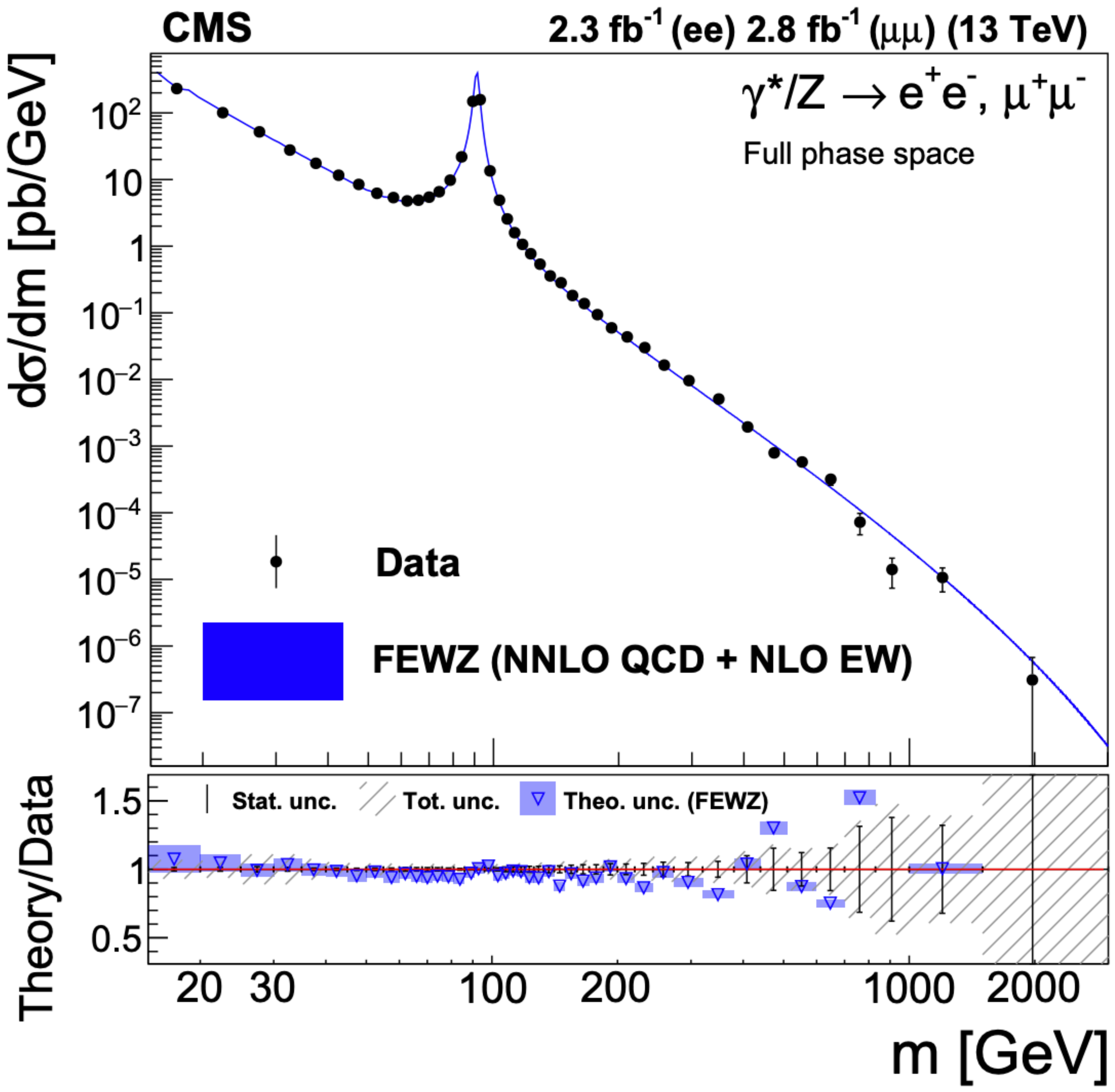
Drell-Yan: the Standard Candle process

Dilepton production in hadron collider



The “Standard Candle” process at hadron collider. Important for:

- PDFs global fit
- Beam calibration
- SM EW parameter determination
- TMD physics
- BSM search
- Dominant uncertainties from luminosity. Normalized cross section < 1% uncertainty!



History of Drell-Yan theory calculation

Duhr, Dulat, Mistlberger, 2020
Inclusive N3LO

Catani, Grazzini, 2007

Melnikov, Petriello, 2006
Full differential NNLO

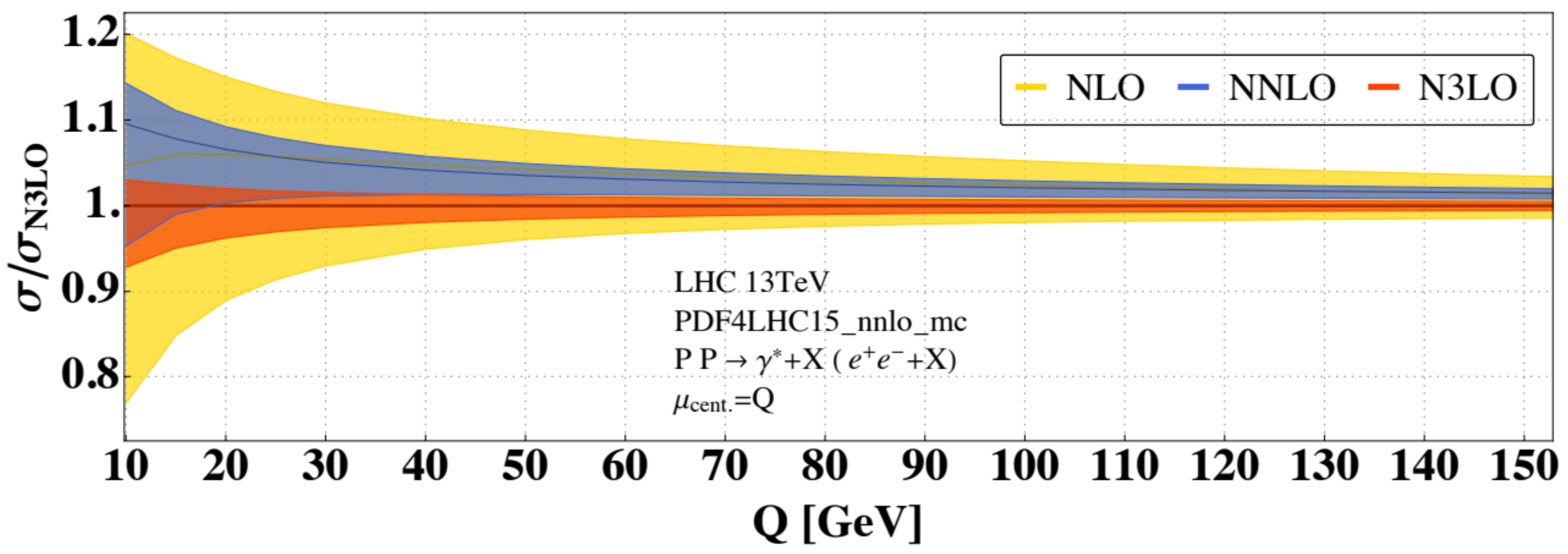
Anastasiou, Dixon, Melnikov, Petriello, 2004
NNLO rapidity

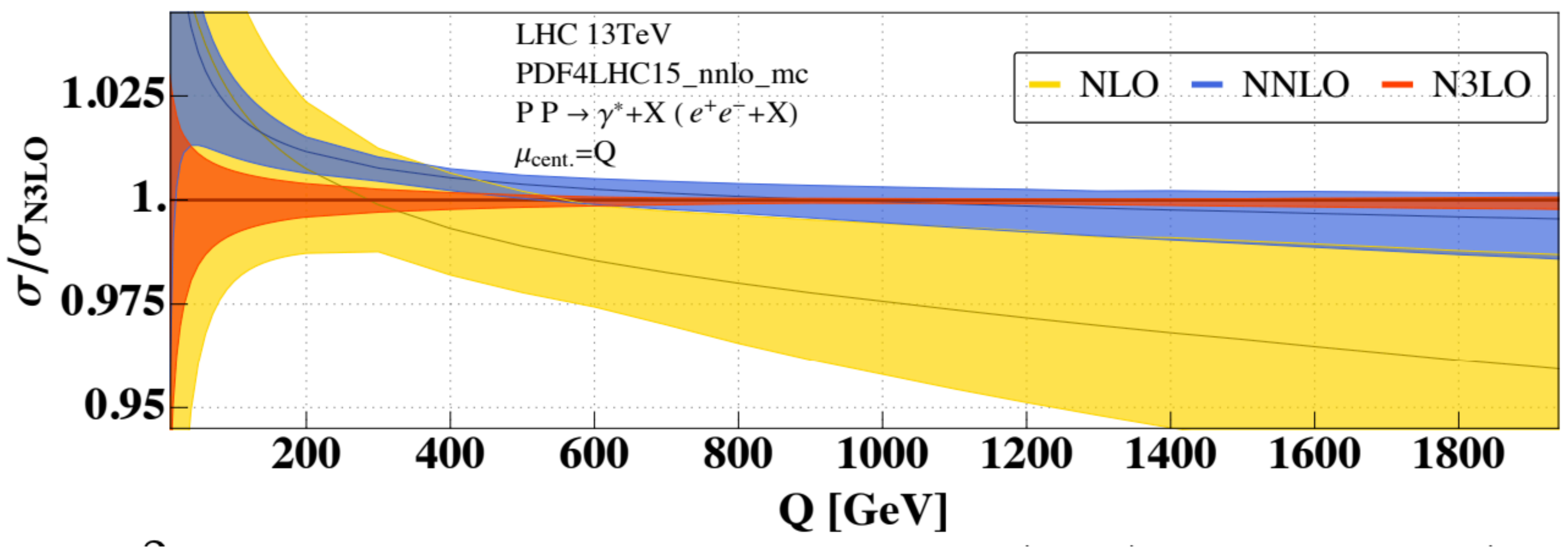
13 years gap

Hamberg, van Neerven, Matsuura, 1991
Inclusive NNLO

Altarelli, Ellis, Martinelli, 1978
NLO

Inclusive Drell-Yan at N3LO





- Scale uncertainties stabilize at N3LO
- Unexpected large corrections at N3LO compared with NNLO (-2%)
- Necessary to have differential distribution for phenomenology application
- If history repeat itself, we will need to wait at least 13 years
- Fortunately, history not always repeat itself

History of Drell-Yan theory calculation

X. Chen, Gehrmann, Glover, Huss, T.Z. Yang, HXZ, 2021, to appear soon
N3LO rapidity

Duhr, Dulat, Mistlberger, 2020
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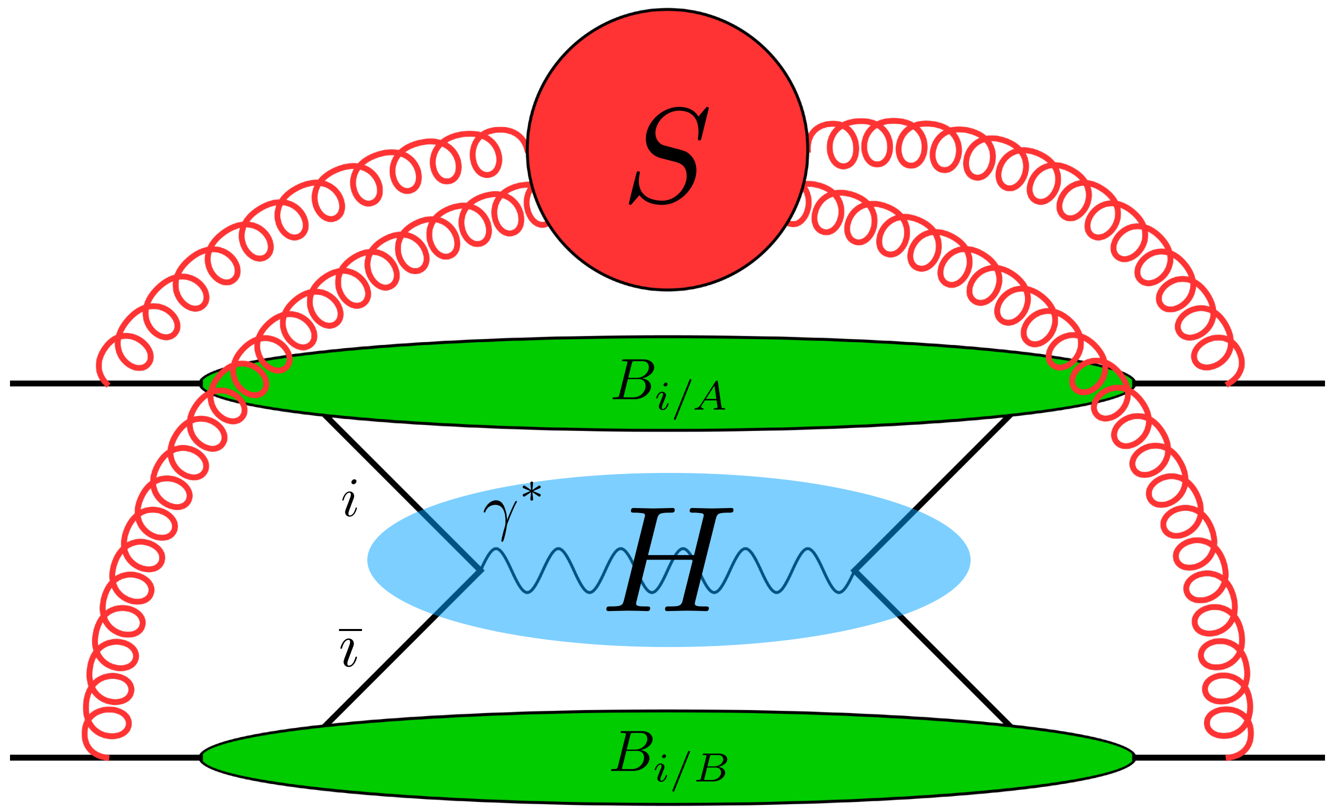
Inclusive NNLO

Altarelli, Ellis, Martinelli, 1978
NLO

IR subtraction at N3LO

- Three-loop virtual corrections have been known for a while. Main difficulty is infrared divergence at N3LO.

- Effective field theory comes to help:
  - Isolate degenerate phase space region where infrared divergence occurs
  - Use effective field theory (SCET) in the degenerate region to simplify calculation



IR subtraction at N3LO

Catani, Grazzini, PRL, 2007

$$\frac{d^2\sigma_{\gamma^*}}{dQ^2dy} = \int_0^{q_T^{\rm cut}} d^2\boldsymbol{q}_T \frac{d^4\sigma_{\gamma^*}}{d^2\boldsymbol{q}_T dQ^2 dy} + \int_{q_T^{\rm cut}} d^2\boldsymbol{q}_T \frac{d^4\sigma_{\gamma^*}}{d^2\boldsymbol{q}_T dQ^2 dy}$$

EFT calculation

Full theory calculation

$$\frac{d^4\sigma_{\gamma^*}}{d^2\boldsymbol{q}_T dQ^2 dy} = \sum_i \frac{\sigma_i^{\rm born}}{E_{\rm CM}^2} \int \frac{d^2\boldsymbol{b}}{(2\pi)^2} e^{i\boldsymbol{q}_T\cdot\boldsymbol{b}} B_{i/A}(x_A,\boldsymbol{b}) \cdot B_{\bar{\imath}/B}(x_B,\boldsymbol{b}) S(\boldsymbol{b}) H(Q^2) + (i\leftrightarrow\bar{\imath}) + \mathcal{O}(q_T^0)$$

NNLOjet collaboration, Gehrmann-De Ridder, Gehrmann, Glover, Huss, Morgan, 2015-2021

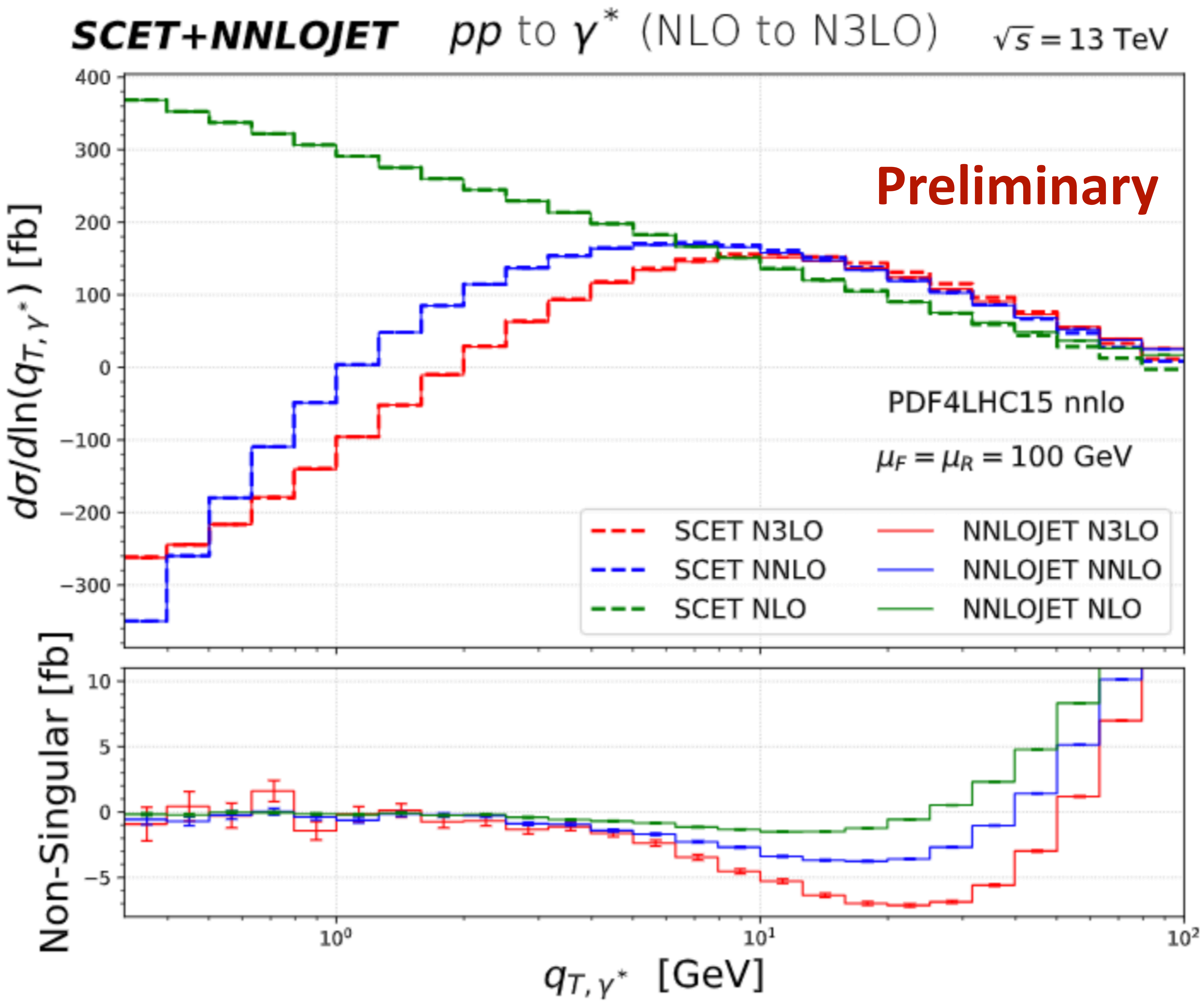
Soft function: Y. Li, **HXZ**, 2016, PRL

M.X. Luo, T.Z. Yang, **HXZ**, Y.J. Zhu, 2019, PRL; 2020, JHEP
Ebert, Mistlberger, Vita, 2020, JHEP

qT distribution of DY lepton pair

X. Chen, Gehrmann, Glover, Huss, T.Z. Yang, **HXZ**, 2021, to appear soon

- Takes a long way to go from amplitudes physical cross sections
- Full theory compared with effective theory in the small qT region
- Excellent agreement, indicating infrared structure at this order is fully and correctly understood
- Highly nontrivial, involve many different ingredients



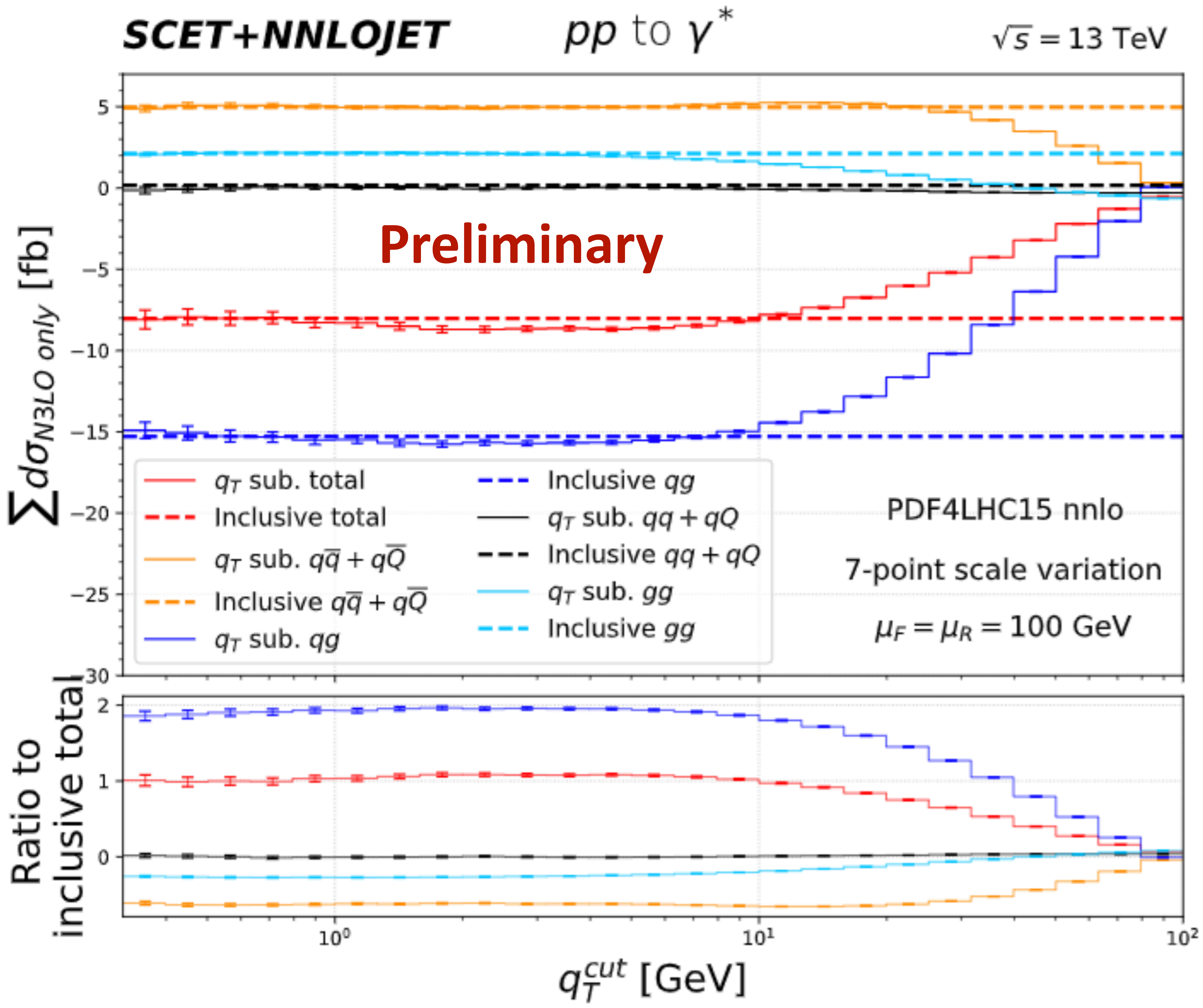
Inclusive cross section

X. Chen, Gehrmann, Glover, Huss, T.Z. Yang, **HXZ**, 2021, to appear soon

- Combination of effective theory and full theory cross section at an infrared cutoff scale qTcut

Fixed Order	$\sigma_{pp\to\gamma^*}$ (fb)	
LO	$339.62^{+34.06}_{-37.48}$	
NLO	$391.25^{+10.84}_{-16.62}$	
NNLO	$390.09^{+3.06}_{-4.11}$	
N3LO	$382.08^{+2.64}_{-3.09}$ from [14]	
N3LO only	q_T-subtraction	Results from [14]
qg	$-15.32(32)$	-15.29
$q\bar{q}+q\bar{Q}$	$+5.08(11)$	$+4.97$
gg	$+2.17(6)$	$+2.12$
$qq+qQ$	$+0.09(13)$	$+0.17$
Total	$-7.98(36)$	-8.03

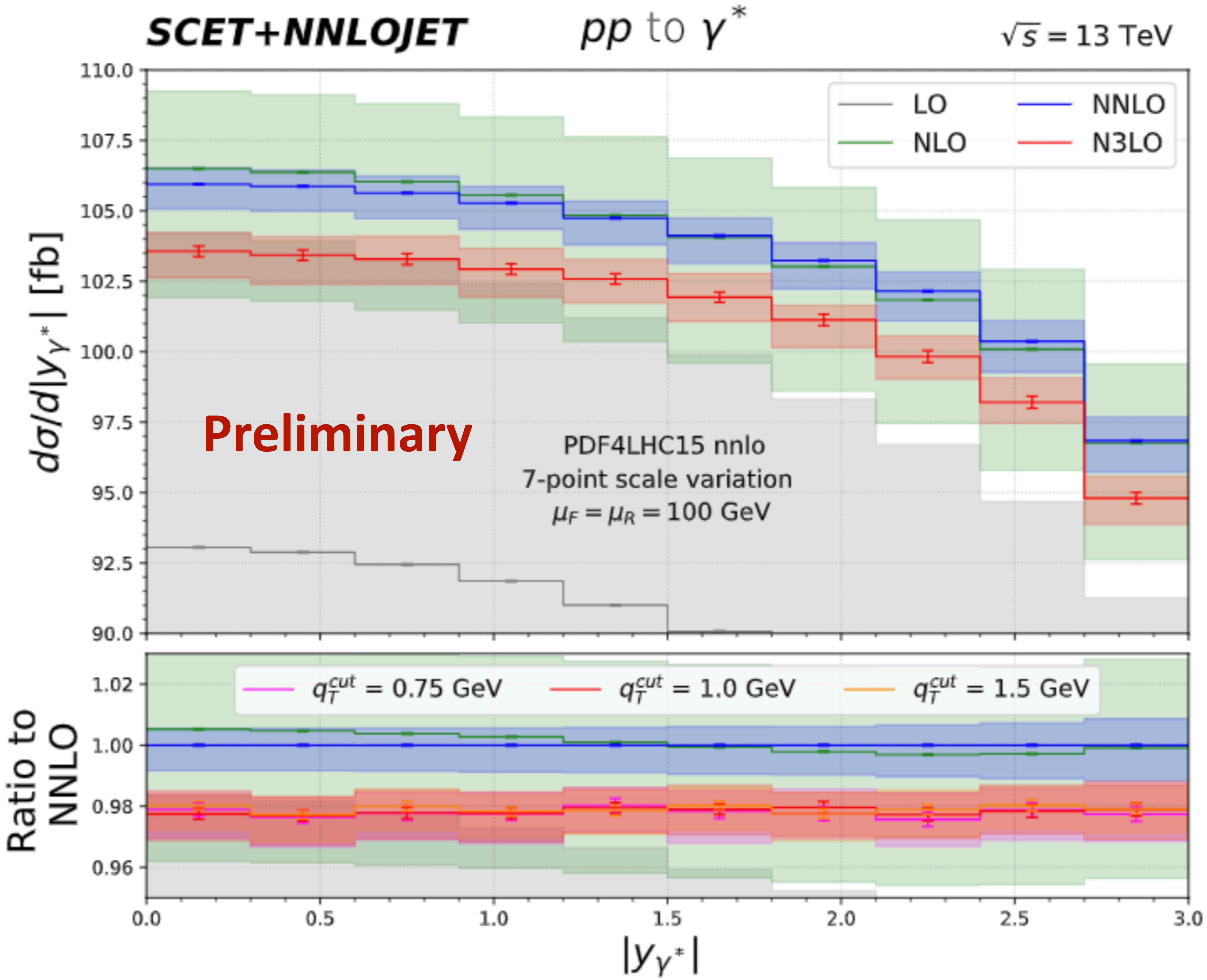
- Large cancellation between qqbar and qg channel at NNLO



Differential rapidity distribution

X. Chen, Gehrmann, Glover, Huss, T.Z. Yang, **HXZ**, 2021, to appear soon

- First differential rapidity distribution for DY at N3LO
- Large corrections (-2%) compared with NNLO over all rapidity region
- N3LO results must take into account in precision phenomenology analysis
- Other effects also become important: N3LO PDF evolution, Mixed QCD-EW



Conclusion and outlook

- Remarkable progress in perturbative Quantum Field Theory powered by simply ideas:
  - Unitarity, analyticity, effective field theory
- Most of the NNLO wishlist closed
- New frontiers
  - 2 -> 3 at NNLO
  - Differential 2->1 at N3LO; towards 2->2 at N3LO
  - The method of effective field theory proved powerful for differential DY at N3LO. Can be applied to other color singlet, such as diboson production (once the virtual amplitudes become available)
  - Many theoretical works to be done, e.g., understand perturbative power corrections to stabilize numerical results

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Exciting times ahead, stay tuned!

Thank you very much for your attention!